Passiva & aktiva
elektriska nät A

fö 97 F

sidor: 59

pris: ~~20:-~~ 30:-

Passiva & aktiva elektriska nät

föreläsnings-anteckningar

Läsper. I -97

Introduction to electric circuits R. C. Dorf

3:e utgåvan

$\boxed{2/9-97}$

Kapitel 1

$V = R \cdot i$ (Ohms lag), R mäts i Ω (ohm) = 1 V/A

$i = G \cdot V$, G = konduktans, $G = \frac{1}{R}$

$R = \frac{1}{G}$

$P = V \cdot i = \frac{V^2}{R}$, $P > 0$ upptagen effekt

$P < 0$ avgiven effekt

Begrepp:

Laddning: Q (Coulomb), 1 elektron = $1{,}602 \cdot 10^{-19}$ C

1 Coulomb har $6{,}24 \cdot 10^{18}$ elektroners laddning

Elektrisk krets

Ström: $i = \frac{dQ}{dt}$, hur mycket laddning som förflyttas m.a.p tiden

Spänning: V, det arbete som åtgår för 1 Coulomb att vandra mellan A & B

Likström: (Direct Current: DC)

Effekt: P, med vilken hastighet energi omvandlas

$$P = \frac{dW}{dt}$$

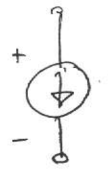
Kapitel 2

Aktivt element: avger energi, ex. transistor

Passivt element: upptar energi, ex. resistans

Oberoende källa: genererar ober. av andra spänningar & strömmar, t. ex batteri.

Spänningskälla $V(t)$

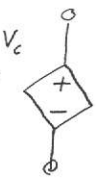


Strömkälla i

Beroende källa: genererar olika spänning- el. strömstyrka beroende på andra strömmar & spänningar
sid. 55

Spänningsber. $V = bV_c$

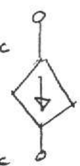
V



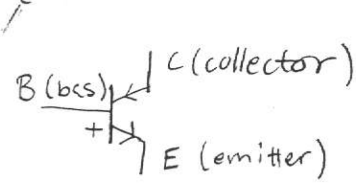
Strömber. $V = ri_c$

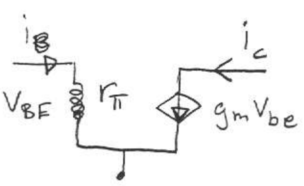
Spänningsber. $i = gv_c$

i



Strömber. $i = di_c$



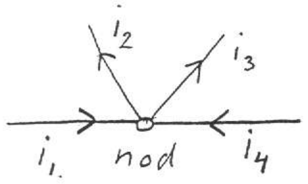


Kapitel 3

Resistiva nät

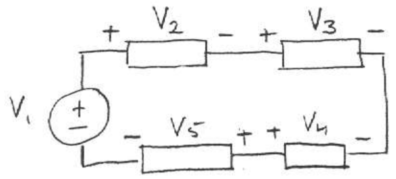
Kirchoffs lagar:

Strömlagen (K.C.L): Samriktade strömmar $\sum i_k = 0$



lika mycket ström in till noden som ut

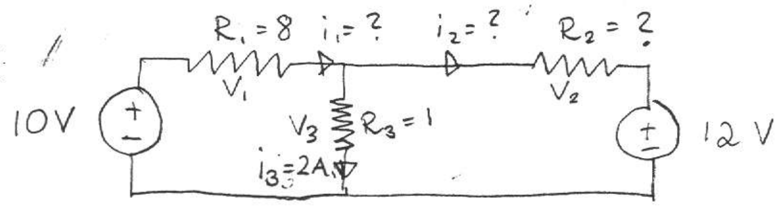
• **Spänningslagen (K.V.L):** Samriktade spänningar $\sum V_k = 0$



$$-V_1 + V_2 + V_3 - V_4 + V_5 = 0$$

Potentialen summa då man gått ett varv = 0

Ex.



Ohms lag: $V_3 = R_3 i_3 = 2V$

K.V.L: $+10V - V_1 - V_3 = 0 \Rightarrow V_1 = 8V$

Ohms lag: $i_1 = \frac{V_1}{R_1} = 1A$

K.C.L: $i_1 - i_2 - i_3 = 0 \Rightarrow i_2 = i_1 - i_3 = -1A$

K.V.L: $V_3 - V_2 - 12 = 0$

$$V_2 = -10V$$

$$R_2 = \frac{V_2}{i_2} = 10\,\Omega$$

4.

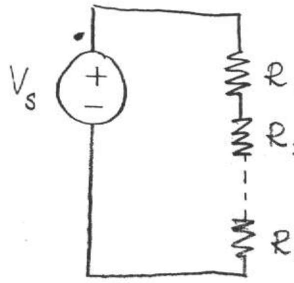
3/9 - 97

Kapitel 5

1) Källtransformering
2) Superposition av linjära element & ober. källa
3) Thévenins teorem
4) Nortons teorem

* hoppa över avsnitt om maximal förstärkning

Spänningsdelning & strömdelning Kap. 3



$$V_s = \sum_{m=1}^{n} i R_m = i \sum_{m=1}^{n} R_m$$

$$V_s = R \cdot i$$

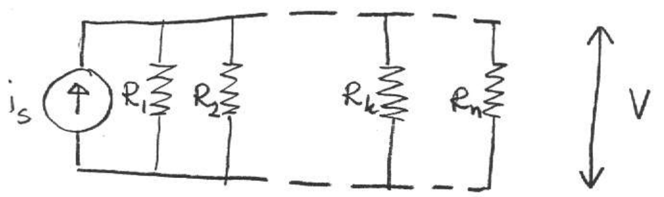
$$R = \sum_{m=1}^{n} R_m$$

$$V_k = i R_k$$

Sp.-delning

$$V_s = i \sum_{m=1}^{n} R_m, \quad \frac{V_k}{V_s} = \frac{R_k}{\sum_{m=1}^{n} R_m} \qquad \left(\frac{V_1}{V_s} = \frac{R_1}{R_1 + R_2} \right)$$

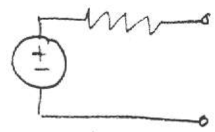
Strömgrening



$$i_s = \frac{V}{R_1} + \frac{V}{R_2} + \ldots \frac{V}{R_n} = \frac{V}{R}, \quad \frac{1}{R} = \sum_{m=1}^{n} \frac{1}{R_m} \qquad \left(\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \right)$$

Konduktans: $G = \frac{1}{R}$, $G = \sum_{m=1}^{n} G_m$

Källtransformering

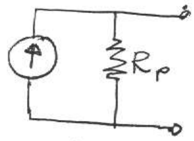


Thévenin

$\longrightarrow$ i_s

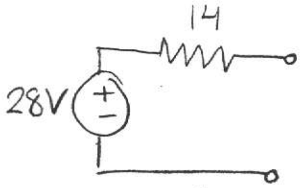
$R_p = R_s$

$i_s = \dfrac{V_s}{R_s}$



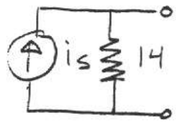
Norton

Ex.



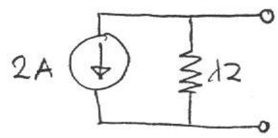
Thévenin

$\longrightarrow$



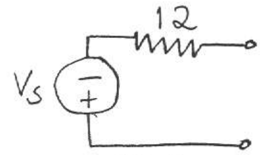
Norton

$i_s = \dfrac{V_s}{R_s} = \dfrac{28}{14} = 2A$



Norton

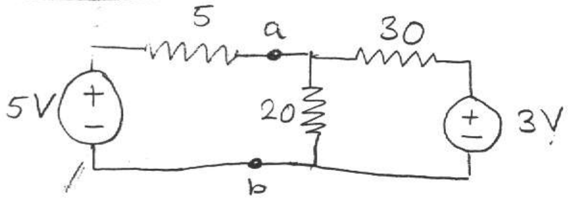
$\longrightarrow$



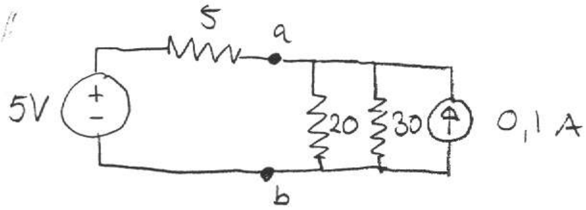
Thévenin

$V_s = i_s R_s = 24V$

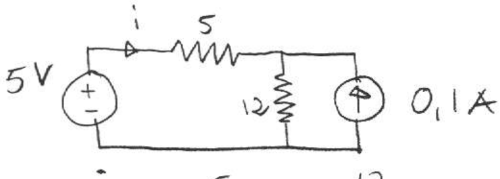
(1)



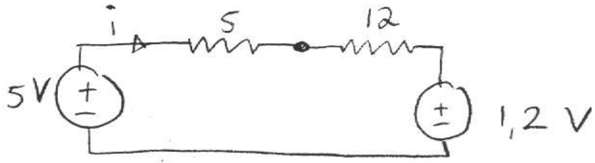
(2)



(3)



(4)



Bestäm i : $V = 5 - 1,2 = 3,8\ V$

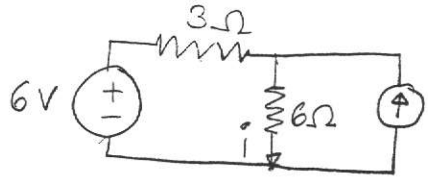
$R = 5 + 12 = 17\ \Omega$

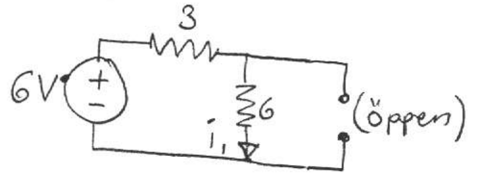
$i = \dfrac{V}{R} = \dfrac{3,8}{17} = 0,224\ A$

6.

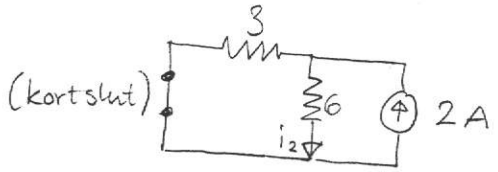
Superpositionsprincipen

Totala effekten från olika oberoende källor i linjära kretsar som verkar samtidigt =
= summan av effekterna från varje ober. källa när spänningskällan kortsluts och strömkällan ersättes med en öppen krets (tomgångspänning)

① 

② 

$i_1 = \frac{6\,V}{(3+6)\,\Omega} = \frac{6}{9}\,A = \frac{2}{3}\,A$

③ 

$i_2 = \frac{R_1 \cdot i_s}{R_1 + R_2} = \frac{3\Omega \cdot 2A}{3\Omega + 6\Omega} = \frac{2}{3}A$

$i_{tot} = i_1 + i_2 = \frac{2}{3} + \frac{2}{3} = \frac{4}{3}\,A$

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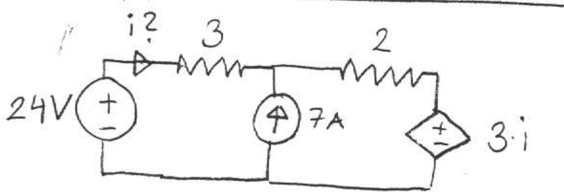
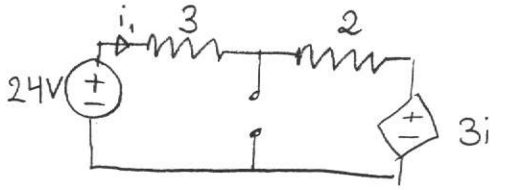
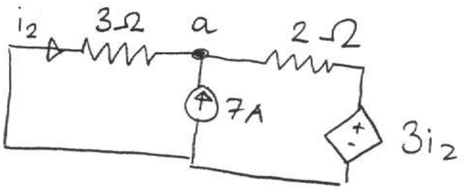
① 

OBS! Koppla aldrig bort beroende källa

② 

K.V.L : $-24V + (3+2)\Omega \cdot i_1 + 3i_1 = 0$

$\Rightarrow i_1 = \frac{24}{8} = 3\,A$

③ 

K.C.L : $-i_2 - 7A + \frac{\overbrace{V_a - 3i_2}^{\text{skillnaden i potential}}}{2\Omega} = 0$

där V_a = potentialen i a

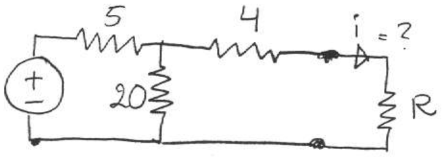
$\Rightarrow V_a = -3i_2$

se resten i boken, (i upplaga 2 sid. 150)
(— " — 3 ?)

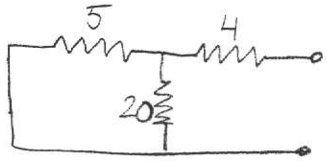
Thévenins teorem

För en resistanskrets med energikällor gäller att kretsen kan förädlas till

1. Bara ober. källor
2. Både ober. & ber. källor
3. Bara ber. källor

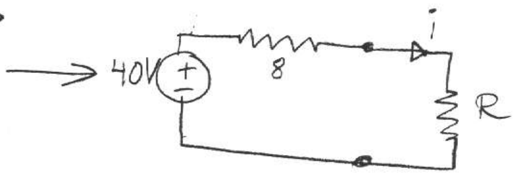


Deaktivera den ober. källan $\Rightarrow R_T$



Gör till en resistans

$$R = \frac{5 \cdot 20}{5+20} + 4 = 8\,\Omega$$



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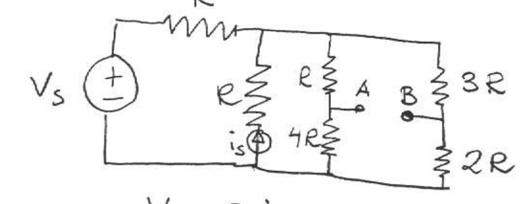
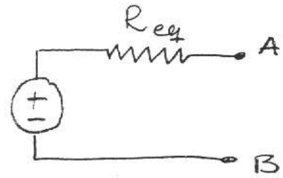
Övning: sök e_{oc} & R_{eq}



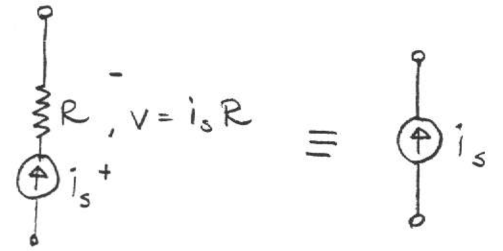
$\Longrightarrow$ e_{oc}



$V_s = R i_s$

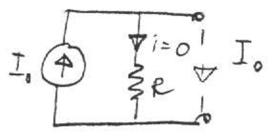
Lösning:

①

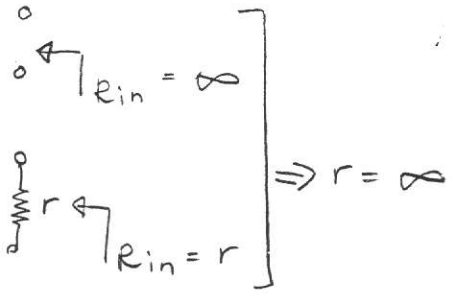


Förklaring

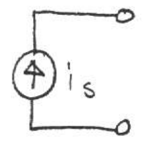
Norton ekv.



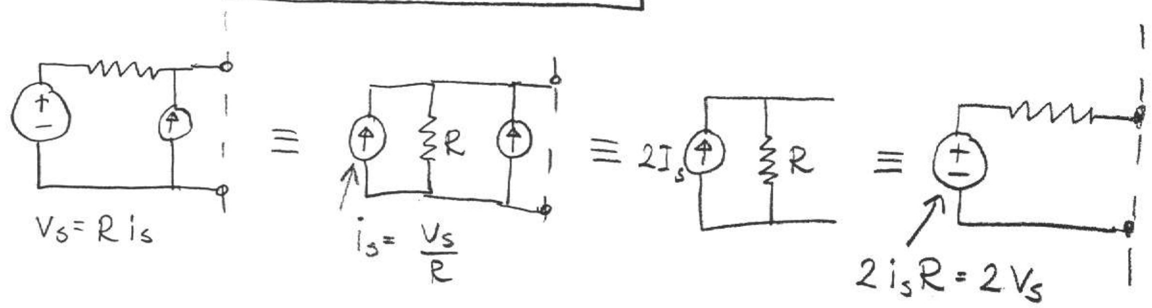
1. Kortslut terminalen $\boxed{I_s = I_0}$
2. Nollställ källor ($I_0 = 0$, $i_0 = 0$.
   $\Rightarrow$ öppen krets



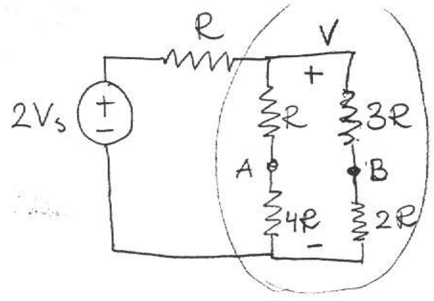
$\Longrightarrow$ Sökt Norton krets



②.



(3)



$$e_{oc} = V_{AB} = V_A - V_B$$

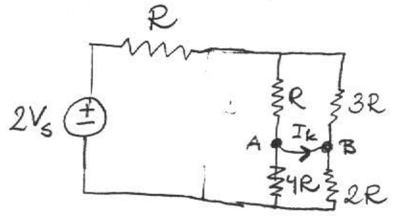
$$\frac{V}{2V_s} = \frac{\frac{5}{2}R}{\frac{5}{2}R + R} = \frac{5}{7} \Rightarrow V = \frac{10}{7} V_s$$

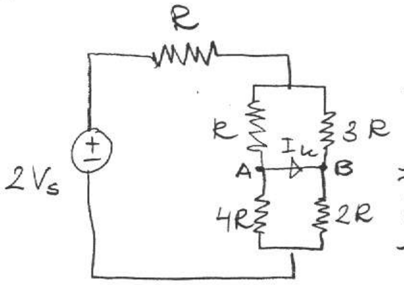
$$V_A = \frac{10}{7} V_s \cdot \frac{4R}{(4+1)R} = \frac{10}{7} \cdot \frac{4}{5} = \frac{8}{7} V_s$$

$$V_B = \frac{10}{7} V_s \cdot \frac{2R}{(2+3)R} = \frac{10}{7} \cdot \frac{2}{5} V_s = \frac{4}{7} V_s$$

$$e_{oc} = V_{AB} = V_A - V_B = \frac{8-4}{7} V_s = \frac{4}{7} V_s \approx 0{,}57\, V_s$$

(4) Finn R_{eq} : Nollställ källor





$$\frac{R \cdot 3R}{R + 3R} = \frac{3}{4} R$$

$$\frac{2 \cdot 4}{2+4} = \frac{4}{3} R$$

$$V_A = \frac{\frac{3}{4}R}{R + \frac{3}{4}R + \frac{4}{3}R} \cdot 2V_s = \frac{9}{12+9+16} \cdot 2V_s = \underline{\underline{\frac{18}{37} V_s}}$$

$$V_B = \frac{\frac{4}{3}R}{R + \frac{3}{4}R + \frac{4}{3}R} \cdot 2V_s = \frac{16}{37} \cdot 2V_s = \underline{\underline{\frac{32}{37} V_s}}$$

$$I_A = \frac{18}{37} \frac{V_s}{R}, \quad I_B = \frac{32}{37} \frac{V_s}{4R}$$

$$I_k = I_A - I_B = \frac{18-8}{37} \frac{V_s}{R} = \underline{\frac{10}{37} \frac{V_s}{R}}$$

$$e_{oc} = I_k R_{eq}, \quad R_{eq} = \frac{e_{oc}}{I_k} = \frac{\frac{4}{7} V_s}{\frac{10}{37} \frac{V_s}{R}} = \underline{\underline{\frac{74}{35}}} R$$

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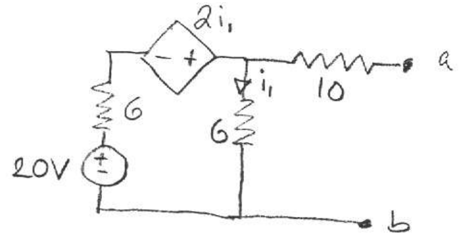
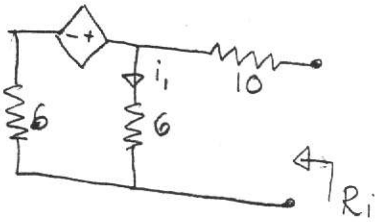
Tomgång med bara beroende källor



① Sök R_i, Nollställ ober. källor



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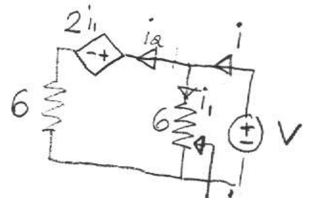


$R_i' = R_i - 10\Omega$ (se ①)

$i_1 = \frac{V}{6}$, K.V.L : $+V - 2i_1 - 6i_2 = 0$

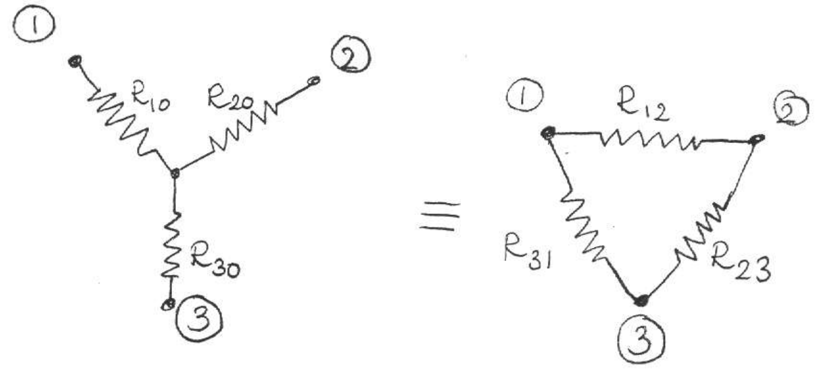
$i_2 = \frac{V - 2i_1}{6} = \frac{V - \frac{2}{6}V}{6} = \frac{V}{9}$

$i = i_1 + i_2 = \left(\frac{1}{6} + \frac{1}{9}\right)V = \underline{\underline{\frac{5}{18}V}}$

$R_i' = \frac{V}{i} = \frac{V}{\frac{5}{18}V} = \frac{18}{5}\Omega$

$R_i = \frac{18}{5} + 10 = \frac{68}{5}\Omega$

Stjärn-triangel transformation



Transformations formler

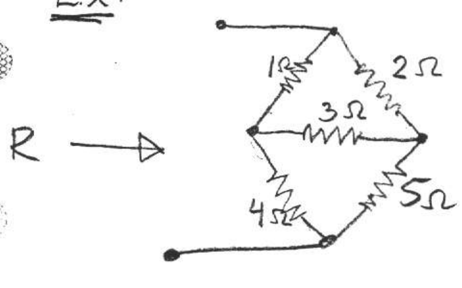
⟸

$$R_{10} = R_{12}R_{31}/R_{00}$$
$$R_{20} = R_{12}R_{23}/R_{00} \quad ; \quad R_{00} = R_{12} + R_{23} + R_{31}$$
$$R_{30} = R_{23}R_{31}/R_{00}$$

⟹

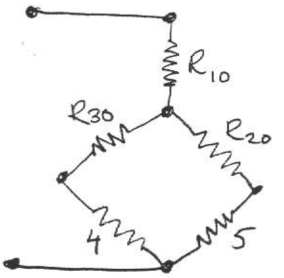
$$R_{12} = R_{10}R_{20}/R_0$$
$$R_{23} = R_{20}R_{30}/R_0 \quad ; \quad \frac{1}{R_0} = \frac{1}{R_{10}} + \frac{1}{R_{20}} + \frac{1}{R_{30}}$$
$$R_{31} = R_{30}R_{10}/R_0$$

<u>Ex.</u>



4 transf. möjligheter
2 trianglar
2 stjärnor

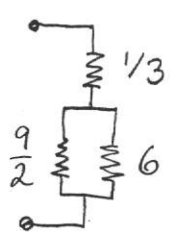
①



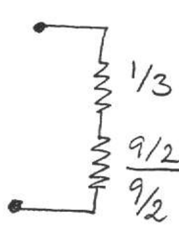
$$R_{10} = \frac{1 \cdot 2}{1+2+3} = \frac{1}{3}$$
$$R_{20} = \frac{2 \cdot 3}{6} = 1$$
$$R_{30} = \frac{1 \cdot 3}{6} = \frac{1}{2}$$

② a) Slå samman seriekopplade resistanser, b) parallellkopplade

a)



b)

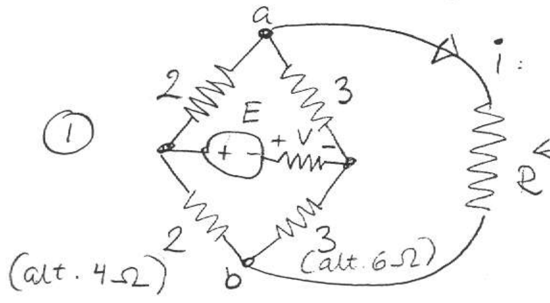


$$\frac{9/2 \cdot 6}{9/2 + 6} = \frac{18}{7}$$

$$R_{tot} = \frac{1}{3} + \frac{18}{7} = \underline{\underline{\frac{61}{21}}}$$

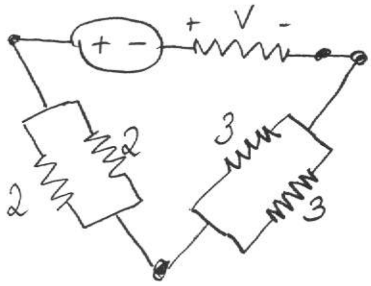
12:

Symmetri

① 

$V_a = V_b \Rightarrow i = 0 \Rightarrow$ motståndet kan plockas bort

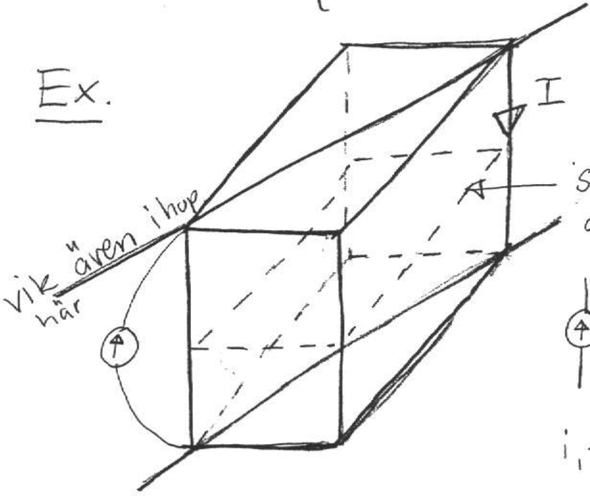
Man kan nu vika den symmetriska delen på mitten

② 

Krav på symmetri:
① symm. graf
② symm. element värden
$\Rightarrow$ symm. spänningar & strömmar

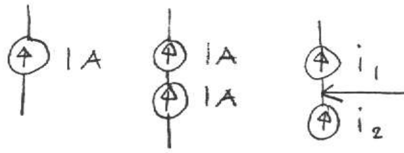
$\Rightarrow \begin{cases} \text{Bryt upp gren m. } i=0 \\ \text{förena noder m. } V_{ab}=0 \end{cases}$

Ex.



alla kanter: 1Ω

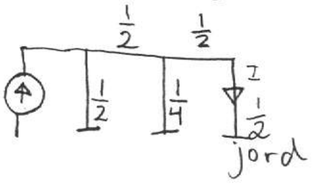
symm. gör att vi kan skära bort undre halvan och koppla ner till jord



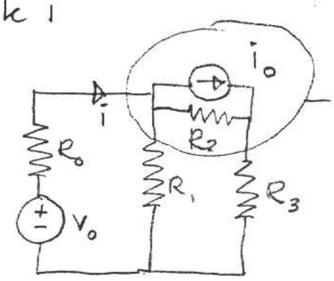
i_3 — anslut var som hels

$i_1 = i_2 = 1\,A$

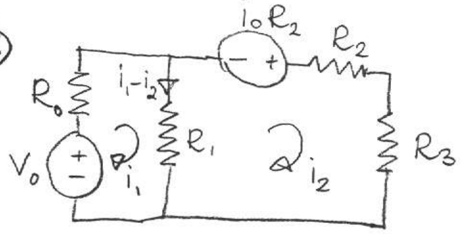
K.C.L: $\underbrace{-i_1 + i_2}_{=0} + i_3 = 0 \Rightarrow i_3 = 0$



Sök i

① 

— Thévenin ekv.

② 

K.V.L

(M = maska)

$$M\,1:\ +V_0 - R_0 i_1 - R_1(i_1 - i_2) = 0$$

$$M\,2:\ i_0 R_2 - (R_2 + R_3) i_2 + R_1 (i_1 - i_2) = 0$$

$$\begin{cases} (R_0 + R_1) i_1 - R_1 i_2 = V_0 \\ -R_1 i_1 + (R_1 + R_2 + R_3) i_2 = i_0 R_2 \end{cases}$$

Lös ekv. m.h.a. matriser

$$\begin{bmatrix} R_0 + R_1 & -R_1 \\ -R_1 & R_1 + R_2 + R_3 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} V_0 \\ R_2 i_0 \end{bmatrix}$$

använd t.ex. Cramers regel för lösning

14.

Allmän maskanalys

obekanta storheter : nätets maskströmmar ! bestäm dessa

Begränsningar:

- Inga strömkällor (omvandla)
- Praktiskt kan vi bara lösa 3x3 -matriser ≙ 3 maskströmmar

Praktisk regel: välj samma rotationsriktning på maskströmmarna

Generaliserad maskanalys

$\mathbb{R} \cdot \mathbb{i} = \mathbb{V}_0$ $\begin{cases} \text{matris ekv.} \\ \text{linj. ekv. syst.} \end{cases}$

$\mathbb{R}$	maskresistans	$(n \times n)$	matris
$\mathbb{i}$	mask ström	$(n \times 1)$	vektor
$\mathbb{V}_0$	källspänning	$(n \times 1)$	vektor

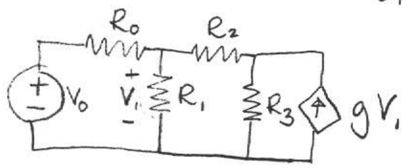
// [→] : Referens riktning för maskströmmar
[− +] : // — " — för spänningskällor

Väljes samma rot. riktning (t.ex. medurs)

Matriselement: Diagonal: $R_{ii} = \sum$ maskresistanser
Ej — " — : $R_{ij} = -\sum$ gemensamma res. (maska i, j)

Vektor element: $V_{0i} = \sum$ spänningskällor i maskan i

Ex.

① 

② 

$$\begin{bmatrix} R_0+R_1 & -R_1 \\ -R_1 & R_1+R_2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} V_0 \\ -gR_3V_1 \end{bmatrix} \leftarrow \text{maskanalys}$$

$V_1 = R_1(i_1 - i_2)$ $\quad -gR_3R_1(i_1-i_2)$

Lös i_1 & i_2

Procedur vid beroende källor

1) Omvandla till spänningskälla
2) Uttryck sp. beroendet i maskströmmen
3) Flytta över termer med maskströmmar

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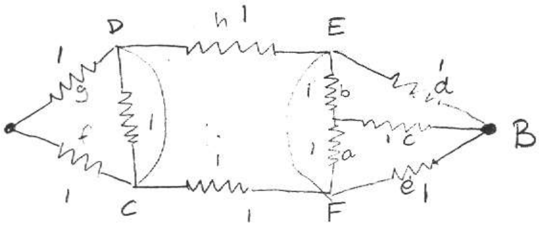
A34-1

3:e uppl.



(Ω)

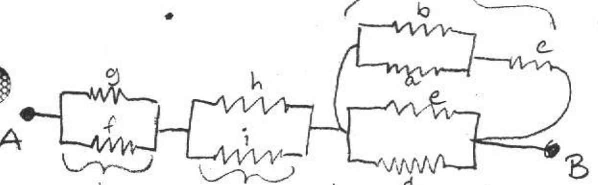
Beräkna den ekvivalenta resistansen mellan A & B.

Symmetri:

$\Rightarrow V_{DC} = 0 \quad \Rightarrow$ Vi kan förbinda dessa

$V_{EF} = 0$

$\frac{1}{2} + 1 = \frac{3}{2}$ (Ω)



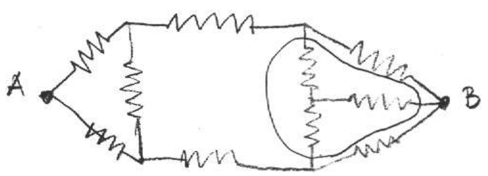
$\frac{1}{2}$ $\quad$ $\frac{1}{2}$

$\frac{1}{2} // \frac{3}{2} = \frac{\frac{3}{2} \cdot \frac{1}{2}}{\frac{1}{2} + \frac{3}{2}} = \frac{3}{8}$

$R_{AB} = 1 + \frac{3}{8} = \underline{\underline{\frac{11}{8}}}\ \Omega$

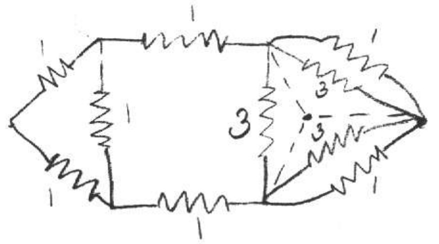
---

alternativ lösning:

① 

Stjärn-triangel transformera

och räkna ut de nya resistanserna

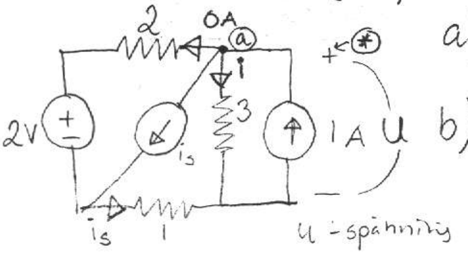
② 

Undvik stj.-tri-transf. om det inte leder till omedelbar förbättring

Fortsätt m.h.a symmetri o.s.v.

16.

A33-1



a) Sök i_s som gör sp.-källan strömlös

b) Hur stor effekt leverar 1A källan?

(*) fel som korr. i efterhand, ej samordnade ref.riktningar för ström & spänning

Lösn.:

a)
$$\begin{cases} \text{K.C.L i @ : } 0 + i_s + i - 1 = 0. \\ \text{K.V.L : } 0 \cdot 2A - 2V - 1 \cdot i_s + 3 \cdot i = 0 \end{cases}$$

Löses $\Rightarrow i_s = \frac{1}{4} A$

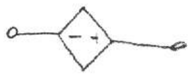
b) Förbrukad $P = \overset{\text{teckenbyte p.g.a. (*)}}{\pm} 1 \cdot U$

levererad $= P = +U$

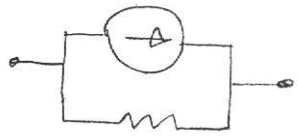
K.C.L. $\left.\begin{array}{l} 0 + i_s + i - 1 = 0 \\ i_s = \frac{1}{4} \end{array}\right\} \Rightarrow i = \frac{3}{4} A$

$U = \frac{3}{4} \cdot 3 = \frac{9}{4} \Rightarrow$ Avgiven effekt $P = U = \frac{9}{4} W$

Nät modifiering för generell maskanalys

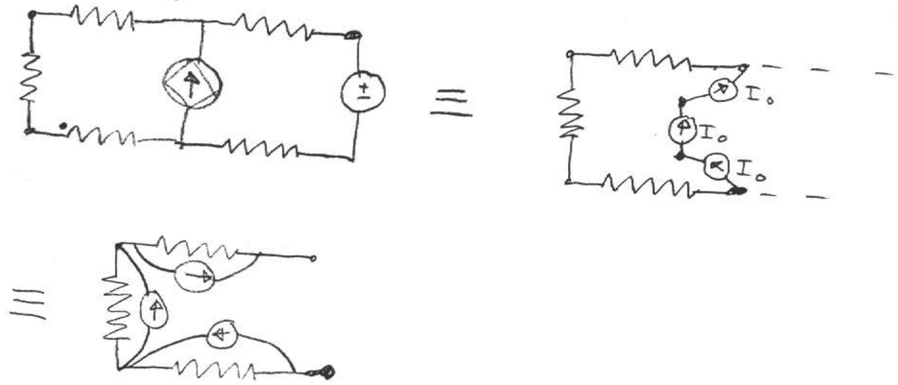


- uttryck i maskströmmar
- behandlas som sp. källa
- flyttas : HL → V.L.

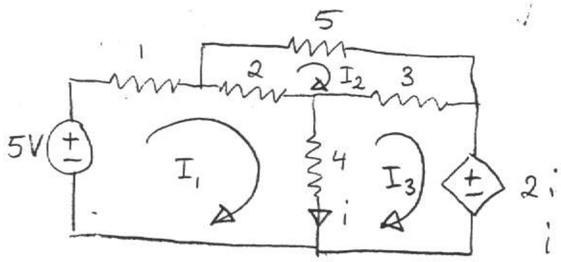


Norton - Thevenin omvandla

Övriga fall



Ex.



(Ω)

Beräkna i

$i = I_1 - I_3$

Lösn.: Maskanalys:

$$\begin{bmatrix} 7 & -2 & -4 \\ -2 & 10 & -3 \\ -4 & -3 & 7 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} +5 \\ 0 \\ -2i \end{bmatrix} \quad i = I_1 - I_3$$

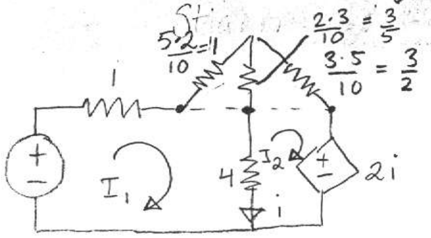
$$\begin{bmatrix} 7 & -2 & -4 \\ -2 & 10 & -3 \\ \underbrace{-2}_{+2-4} & -3 & \underbrace{5}_{7-2} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix}$$

Cramers regel på I_1 & I_3 .....

$$\Rightarrow \quad i = \frac{205}{151} - \frac{130}{151} \simeq 0{,}5\text{ A}$$

alt. lösn. ⟶

18. Stjärn-triangel transformera



o. s. v.

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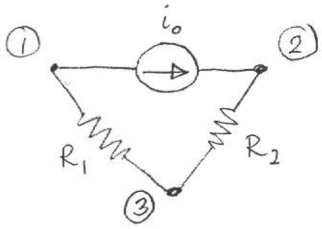
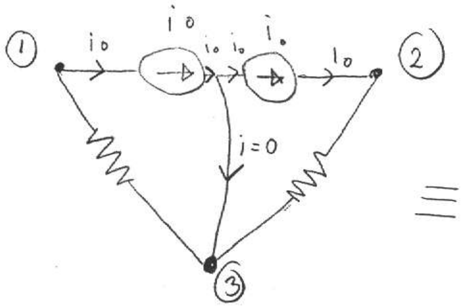
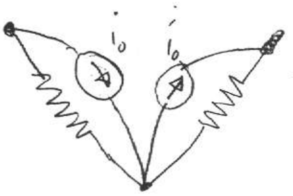
Problem: Gren med strömkälla



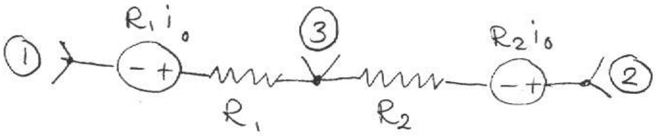
$\equiv$



$\equiv$

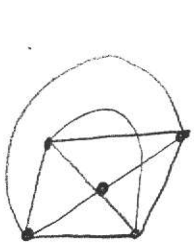


=



Maskanalys, begränsning

- Plana nät, ej korsande ledningar



Lösn.

Sök en dual metod:

$R i = V_0$ Maskanalys

$\boxed{G V = i}$ Nodanalys

Från KCL till nodanalys

- KCL (Noder N-1 + [ref-nod / jord-nod])

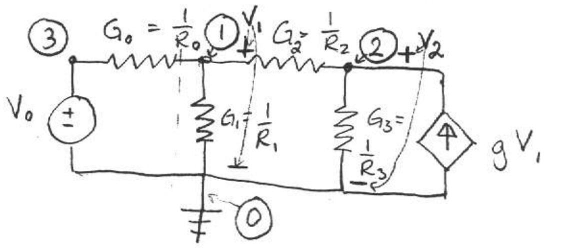
Nodanalys: se stencil

Generaliserad nodanalys (G + I-källor

Specialfall:

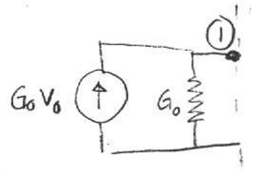
- (O-)Beroende V-källa (Omvandla!)
- Uttryck beroende i nodspänningar
- Flytta från H.L → V.L.

Exempel sid 45 i kursboken, modifierat



OBS! Konduktanser

Th-No-omvandling

① 

② Generell nodanalys

$$\begin{bmatrix} G_0+G_1+G_2 & -G_2 \\ -G_2 & G_2+G_3 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} G_0V_0 \\ gV_1 \end{bmatrix}$$

$$\begin{bmatrix} G_0+G_1+G_2 & -G_2 \\ -G_2 & G_2+G_3 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} G_0V_0 \\ 0 \end{bmatrix}$$

Matriselement

Diagonal: $G_{ii} = \Sigma$ grenkonduktanser, nod i

Ej diagonal: $G_{ij} = -\Sigma$ — " — , mellan nod i & j

Vektorelement: $i_{01} = \Sigma$ strömkällor vid nod i. Plus tecken för ström mot noden.

Procedur vid beroende källa

(1. Omvandla till strömkälla)

(2. Uttrycka strömberoendet i nod spänning

3. Flytta över termer nod sp. till V.L.

spänningsber. sp. källa använd punkt 1+3

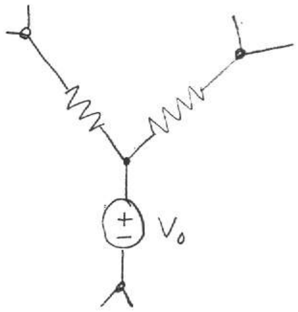
sp. ber strömkälla använd 3

str. ber sp. källa —〃— 1+2+3

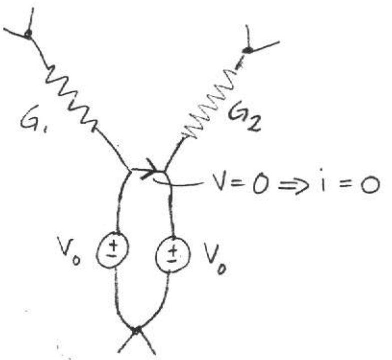
str. ber str. källa —〃— 2+3

Problem: Gren med sp. källa

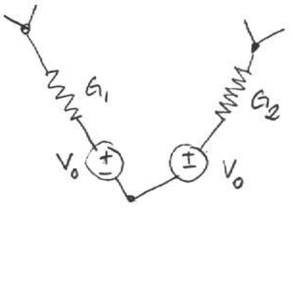
①



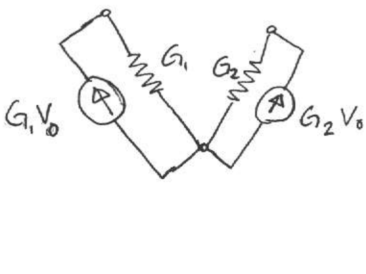
②



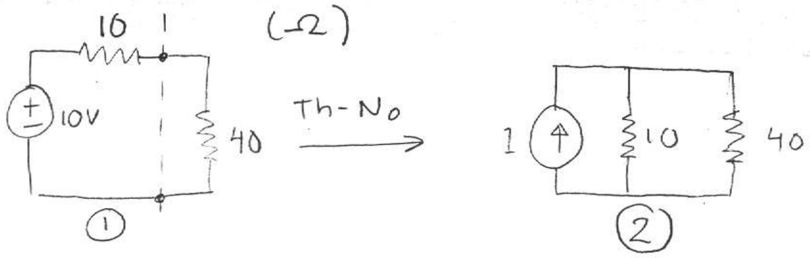
③



④



2a.



① $P_{R10} = \dfrac{V_{R10}}{10} = \dfrac{\left(10\cdot\frac{10}{10+40}\right)^2}{10} = \ldots = \dfrac{2}{5}\,W$

P_{R40} = på samma sätt = $\dfrac{\left(10\cdot\frac{40}{50}\right)^2}{40} = \ldots = \dfrac{8}{5}\,W$

$P_R = \dfrac{2}{5} + \dfrac{8}{5} = 2\,W$

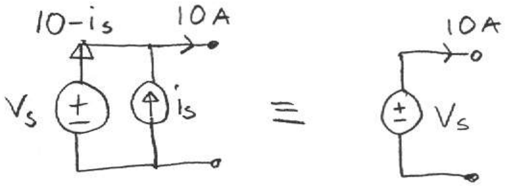
② $P_{R10} = \left(1\cdot\dfrac{40}{40+10}\right)^2\cdot 10 = \ldots = \dfrac{32}{5}\,W$

$P_{R40} = \left(1\cdot\dfrac{10}{10+40}\right)^2\cdot 40 = \dfrac{8}{5}\,W$

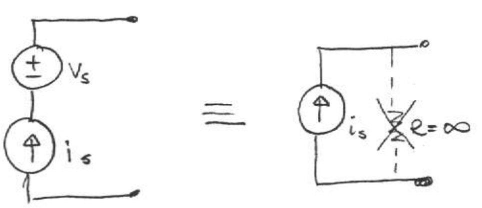
$P_R = \dfrac{32+8}{5} = 8\,W$

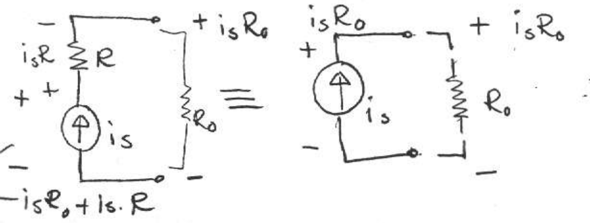
① Levererar 2W , ② levererar 8W VARNING!!

⇒ Th - No utbytbara med avseende på externa effekter men ej m.a.p. de interna effekterna.

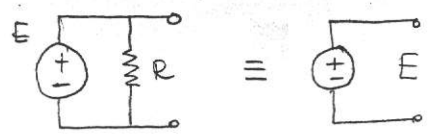


korrekt omvandling, olika effekter



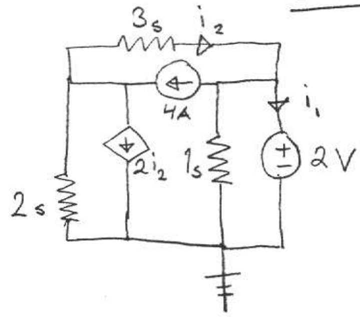


$V - i_sR - i_sR_o = 0$



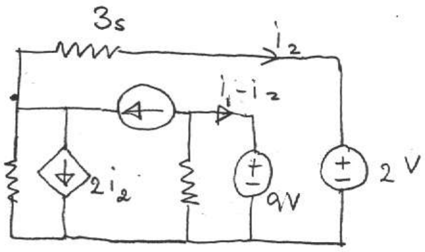
Svårt tal (tentatips) $\boxed{\text{Ex. 4.5-5}}$

①

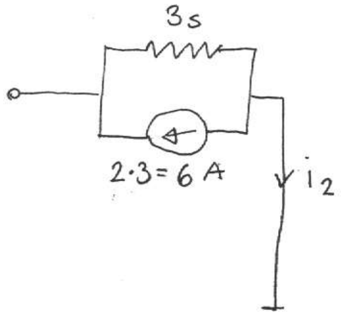


Sök i_1 & i_2

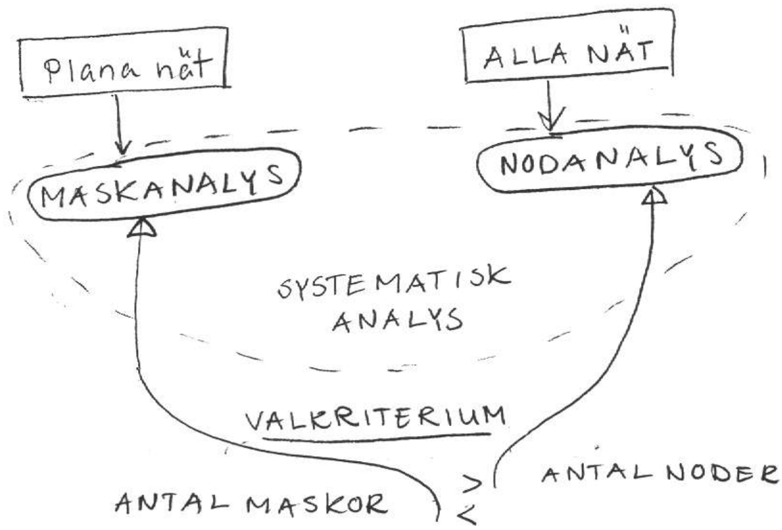
②



③



o.s.v



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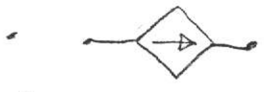
Nodanalys

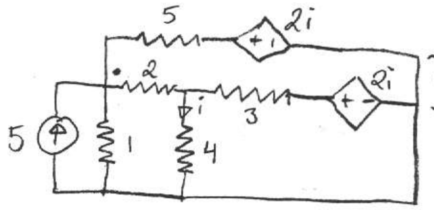
- Använd systematisk nodanalys $\boxed{GV = i_s}$



- uttryck i nodspänning
- behandla som strömkälla
- flytta H.L. → V.L.

- The - No - omvandla

Ex.



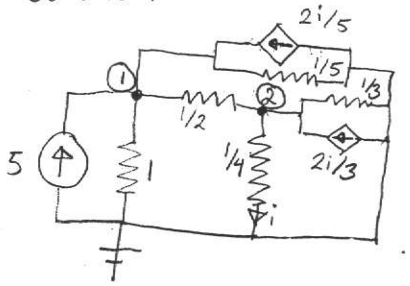
(Ω)

} Thévenin - Norton

Den ber. sp.-källan måste bytas mot en beroende strömkälla

Beräkna m.h.a. nodanalys. Finns 2 noder

[siemens]



Allt uttryckt i konduktanser

Nodanalysen ger:

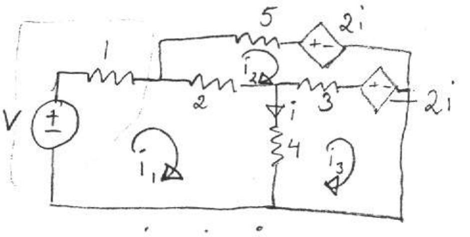
$$\begin{bmatrix} 1+\frac{1}{2}+\frac{1}{5} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{3}+\frac{1}{4}+\frac{1}{2} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 5+\frac{2i}{5} \\ \frac{2i}{3} \end{bmatrix} = \begin{bmatrix} 5+\frac{2}{5}\cdot\frac{V_2}{4} \\ \frac{2}{3}\cdot\frac{V_2}{4} \end{bmatrix}$$

$$\begin{bmatrix} \frac{17}{10} & \overbrace{-\frac{1}{2}-\frac{1}{10}}^{-\frac{6}{10}} \\ -\frac{1}{2} & \underbrace{\frac{13}{12}-\frac{1}{6}}_{\frac{11}{2}} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \end{bmatrix}$$

CRAMER:

$$V_1 = \frac{\begin{vmatrix} \frac{17}{10} & 5 \\ -\frac{1}{2} & 0 \end{vmatrix}}{\begin{vmatrix} \frac{17}{10} & -\frac{6}{10} \\ -\frac{1}{2} & \frac{11}{2} \end{vmatrix}} = \ldots = \frac{300}{151}, \quad i = \frac{V_2}{4} = \frac{75}{151} \approx \underline{\underline{0{,}50}}$$

Lös föregående krets med maskanalys



Th - No - omvandla strömkällan

$i = i_1 - i_3$

Maskanalys ger

$$\begin{bmatrix} 1+2+4 & -2 & -4 \\ -2 & 2+3+5 & -3 \\ -4 & -3 & 3+4 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 5 \\ -2i+2i \\ -2i \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ -2(i_1-i_3) \end{bmatrix}$$

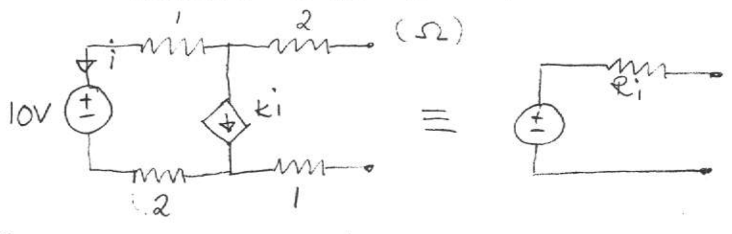
$$\begin{bmatrix} 7 & -2 & -4 \\ -2 & 10 & -3 \\ -4+2 & -3 & 7-2 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix}$$

$$i_1 = \frac{|\quad|}{|\quad|}, \quad i_3 = \frac{|\quad|}{|\quad|}$$

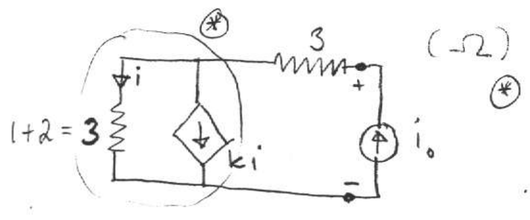
$i = i_1 - i_3$

Dugga uppgift:

Bestäm k så att $R_i = 4\,\Omega$



① Nollställ sp.-källan



(*) No-Th. ej lämpligt då ki ber. av i

K.C.L: $i_0 - ki - i = 0$

K.V.L: $+3i + 3i_0 - V = 0$

$i_0 = (k+1)i$

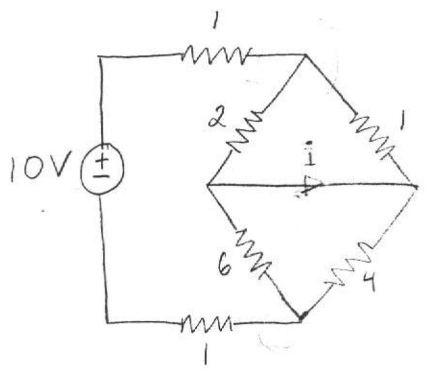
$3i + 3(k+1)i = V$

$$R_i = \frac{V}{i_0} = \frac{(3k+6)\cancel{i}}{(k+1)\cancel{i}} = \frac{3k+6}{k+1} = 4\,\Omega$$

$\Rightarrow \underline{\underline{k = 2}}$

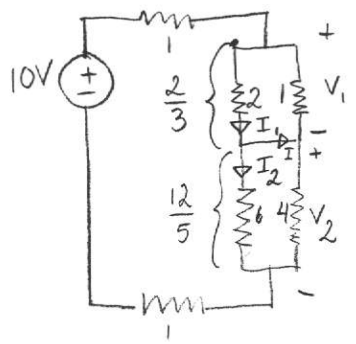
Dugga uppgift 2:

Bestäm strömmen i.



$$\boxed{\begin{array}{l} I \neq 0 \quad \text{ty} \\ R = 0 \\ I \cdot R = V \\ I \cdot 0 = 0 \end{array}}$$

En lösningsmöjlighet:



Sp. delning

$$V_1 = 10 \frac{\frac{2}{3}}{1+\frac{2}{3}+\frac{12}{5}+1} = \frac{\frac{2}{3}}{\frac{15+10+36+15}{15}} = \frac{\cancel{2}^{10}}{\cancel{3}} \cdot \frac{\cancel{15}^{5}}{\cancel{76}_{38}} = \frac{50}{38} = \frac{25}{19}$$

$$V_2 = \text{p.s.s} = V_1 \frac{\frac{12}{5}}{\frac{2}{3}} = \ldots = \frac{90}{19}$$

$$I_1 = \frac{V_1}{2} = \frac{25}{38}$$

$$I_2 = \frac{V_2}{6} = \frac{90}{6 \cdot 19} = \frac{30}{38}$$

$$I = I_1 - I_2 = \frac{25}{38} - \frac{30}{38} = -\frac{5}{38} \approx -0{,}132 \text{ A}$$

$\boxed{19/9-97}$

Kap. 7

Kondensator

$q = CV$

$W_C = \frac{1}{2} CV^2$

$C_p = \sum_{n=1}^{N} C_n$

$\frac{1}{C_s} = \sum_{n=1}^{N} \frac{1}{C_n}$

Induktanser

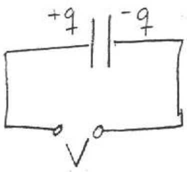
$V = L \frac{di}{dt}$

$W = \frac{1}{2} L i^2$

$\frac{1}{L_p} = \sum_{n=1}^{N} \frac{1}{L_n}$

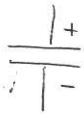
$L_s = \sum_{n=1}^{N} L_n$

Kapacitans



$q = CV$: C = kapacitans

Enheten = F (Farad) = $\frac{\text{Coulomb}}{\text{Volt}}$



$$\boxed{i = C\frac{dv}{dt}}$$

Specialfall:

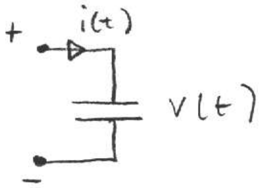
Plattkondensator

$C = \frac{\varepsilon A}{d}$

$\varepsilon = \varepsilon_r \cdot \varepsilon_0$; ε_0 = Permitiviteten i vakuum

ε_r = Relativa permitiviteten

Ex. $V(t)$?



$i = C\frac{dv}{dt} \Rightarrow v(t) = \frac{1}{C}\int_{t_0}^{t} i \, d\tau + v(t_0)$

→

Energin lagrad i kondensatorn

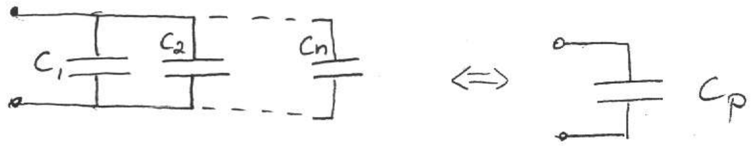
$$W_c = \frac{1}{2} C V^2(t)$$

$$W_c = \frac{1}{2} \frac{1}{C} q^2 \qquad (\text{ty } q = CV)$$

<u>Bevis</u>: $W_c(t) = \int_{-\infty}^{t} v \cdot i \, d\tau$, men $i = C \frac{dv}{d\tau}$

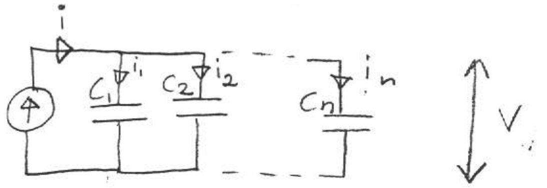
$$W_c = \int_{-\infty}^{t} v \, C \frac{dv}{d\tau} = C \int_{V(-\infty)}^{V(t)} v \, dv = \frac{1}{2} C V(t)^2$$

Parallell koppling med kondensator



$$, \quad C_p = \sum_{n=1}^{N} C_n$$

<u>Bevis</u>:

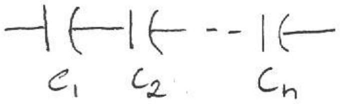


K.C.L : $i_n = C_n \frac{dV}{dt}$

$$i = C_1 \frac{dV}{dt} + C_2 \frac{dV}{dt} + \ldots + C_n \frac{dV}{dt} = (C_1 + C_2 + \ldots C_n) \frac{dV}{dt}$$

$$\Rightarrow i = C_p \frac{dV}{dt} \; ; \quad \boxed{C_p = \sum_{n=1}^{N} C_n}$$

Seriekopplade kondensatorer



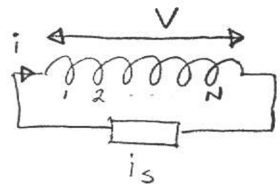
$$= C_s \quad , \quad \frac{1}{C_s} = \sum_{n=1}^{N} \frac{1}{C_n} \quad , \quad V_n = \frac{1}{C_n} \int_{t_0}^{t} i \, d\tau + V_n(t_0)$$

K.V.L : $V = \frac{1}{C_1} \int_{t_0}^{t} i \, d\tau + v(t_0) + \ldots + \frac{1}{C_N} \int_{t_0}^{t} i \, d\tau + V_n(t_0) = \sum_{n=1}^{N} \frac{1}{C_N} \int_{t_0}^{t} i \, d\tau + \sum_{n=1}^{N} V_n(t_0) =$

$$= \frac{1}{C_s} \int_{t_0}^{t} i \, d\tau \; :\Rightarrow \boxed{\frac{1}{C_s} = \sum_{n=1}^{N} \frac{1}{C_n}}$$

Induktans $= L$

$$V = L \frac{di}{dt}$$



$L =$ Henry (H)

$V =$ självinducerande spänning

$\phi =$ magnetiskt flöde

$$\left.\begin{array}{l} V = N \dfrac{d\phi}{dt} \\ N\phi = Li \end{array}\right\} = V = L \frac{di}{dt}$$

Energin lagrad i en spole $= w$

$$W = \frac{1}{2} L i^2$$

Bevis: $p = v \cdot i = L \dfrac{di}{dt}$

$$W = \int_{t_0}^{t} p\,d\tau = L \int_{i(t_0)}^{i(t)} i\,di = \frac{L}{2} i^2(t) - \underbrace{\frac{L}{2} i^2(t_0)}_{=0}$$

Kap. 11

Stationära lösningar för sinussvängningar

$v(t) = V_m \sin(\omega t + \phi)$ $V_m =$ maxvärdet på V

$\underline{V} = V_m e^{i\varphi} = V_m \angle\varphi$ (visare)

$\underline{V} = RI$

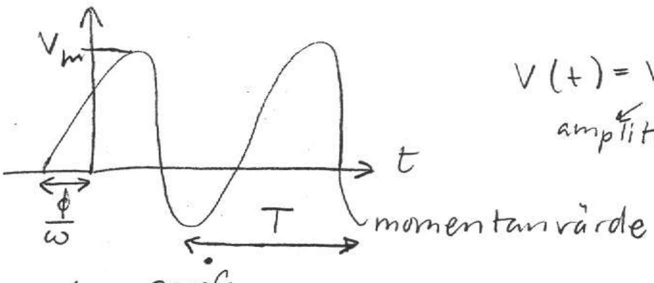
$\underline{V} = j\omega L I$

$\underline{V} = \dfrac{1}{j\omega C} I$

$Z = \dfrac{1}{Y}$ (Resonans)

Ex.

Matematisk beskrivning av sinussignaler



$V(t) = V_m \sin(\omega t + \phi)$

amplitud, vinkelfrekvens, fasvinkel

$\omega = 2\pi f$

(radianer/s)

$T = \frac{2\pi}{\omega} = \frac{1}{f}$ (Periodtiden)

f = frekvensen (Hz = svängningar/s)

Komplexa tal representationer

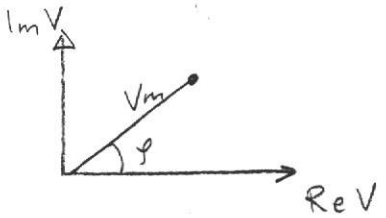
$c = a + ib$

$c = re^{i\theta}$ (exponentialform)

Eulers formel: $\boxed{e^{j\omega t} = \cos\omega t + j\sin\omega t}$

$\boxed{e^{-j\omega t} = \cos\omega t - j\sin\omega t}$

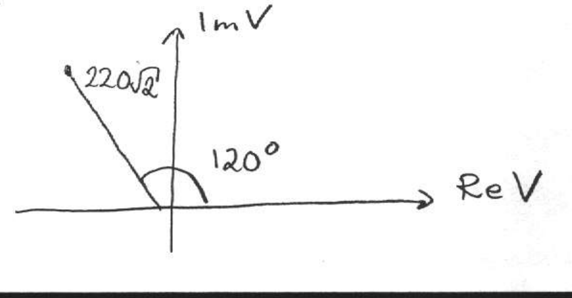
$V = V_m e^{i\varphi} = V_m \angle\varphi = V_m\cos\varphi + jV_m\sin\varphi$



$\tan\varphi = \frac{\mathrm{Im}V}{\mathrm{Re}V}$

Ex. Stickkontakt, $V(t) = \underbrace{220\sqrt{2}}_{max}(\cos 2\pi\cdot 50\cdot t + 120°)$ — effektivvärde; 50 Hz i uttagen; $120° = \frac{2}{3}\pi$

alt. $220\sqrt{2}\angle 120°$ & $f = 50\,Hz$



Ex. $\phi = 1000\,Hz$

$$V = 10\sqrt{3} - j10$$

$$|V|^2 = 10^2(\sqrt{3}^2 + 1) = 4 \cdot 10^2$$

$$V = 20$$

$$\varphi = \arctan\left(\frac{-10}{10\sqrt{3}}\right) = -30^\circ$$

$$V = 20 \angle -30^\circ \quad \text{eller} \quad u(t) = 20\cos(2000\pi t - 30^\circ)$$

Definition av visare

En visare, $\underline{I}$, är ett komplext tal som beskriver amplitud och fas hos en sinusformad signal.

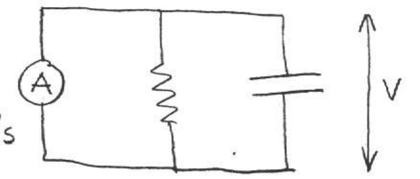
$$\underline{I} = I_m e^{j\theta} = I_m \angle\theta$$

$$i(t) = I_m \cos(\omega t + \theta) = \mathrm{Re}\left\{ I_m e^{j\theta} e^{j\omega t} \right\}$$

Ex

$i = 10\cos\omega t$

$\omega = 100$ rad/s



Bestäm stationära lösningen för V

$$\underline{I} = 10\angle 0^\circ$$

$$\frac{V}{R} + C\frac{dV}{dt} = i$$

Ansätt $v = V_m e^{j(\omega t + \phi)}$

$$\frac{V_m}{R} e^{j(\omega t + \phi)} + j\omega C V_m e^{j(\omega t + \phi)} = 10 e^{j\omega t}$$

$$\left(\frac{1}{R} + j\omega C\right)\underline{V} = \underline{I}$$

$(1+j)\,\underline{V} = \underline{I}$ Sätt in numeriska värden

$$\underline{V} = \frac{\underline{I}}{1+j}$$

$$\underline{V} = \frac{10}{\sqrt{2}} \angle -45^\circ, \quad V(t) = \frac{10}{\sqrt{2}}\cos(\omega t - 45^\circ)$$

Visar relationer mellan R, L & C

1) $\underline{V} = R\underline{I}$ Resistans

$V = V_m \cos(\omega t + \phi) = \text{Re}\, V_m e^{j(\omega t + \phi)}$

$i = \text{Re}\, I_m e^{j(\omega t + \beta)}$

$V_m e^{j(\omega t + \phi)} = R I_m e^{(j\omega t + \beta)}$

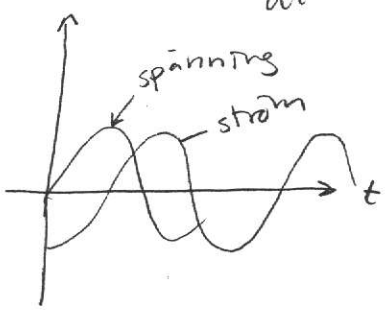
$V_m e^{j\phi} = R I_m e^{j\beta}$

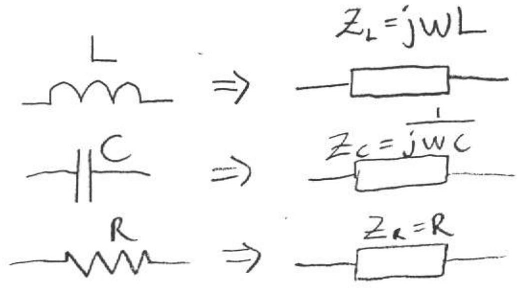
$\underline{V} = R\underline{I}$

2) Spole

$\underline{V} = j\omega L \underline{I} = e^{j90^\circ} \omega L \underline{I}$; $j = e^{j90^\circ}$ spänningen ligger 90° före strömmen

Bevis: $v = L \frac{di}{dt} \Rightarrow \underline{V} = j\omega L \underline{I}$





$u = L\frac{di}{dt}$

$i = C\frac{du}{dt}$

$w = 2\pi f$

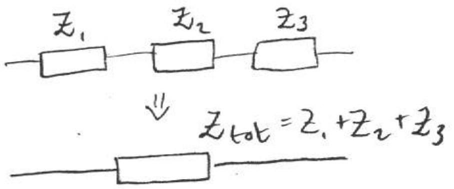
Admittans $Y = \frac{1}{Z} = G + jB$

G = konduktans, B = suspedans $\left(\frac{1}{\Omega}\text{, Siemens, mho}\right)$

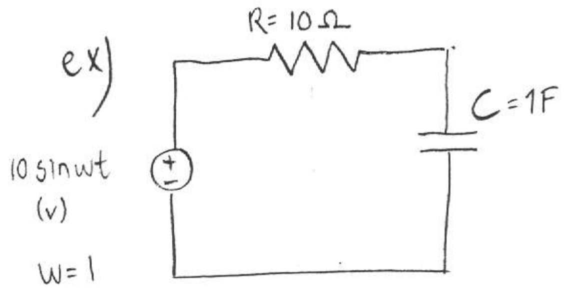
Impedans $Z = R + jX$

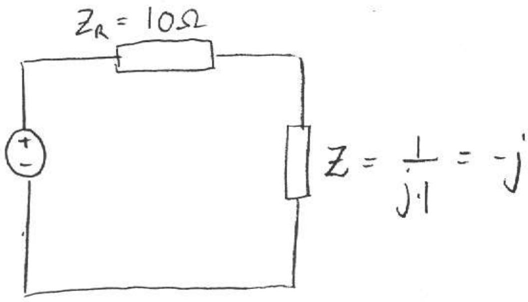
X = Reaktans (Ω)

R = Resistans (Ω)

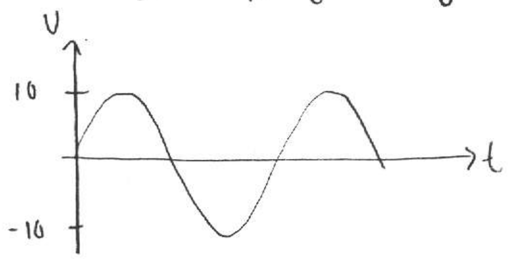


ex)





$$I = \frac{U}{Z} = \frac{U}{Z_R + Z_C} = \frac{10}{10-j} = \frac{10(10+j)}{100+1} = \frac{10}{101}(10+j)$$



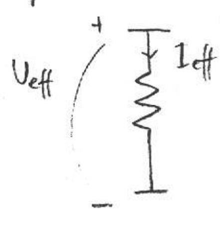
EFFEKT Rent resistiv last: ström o sp. i fas.

$P = U_{topp} \cdot I_{topp}$

MEDELEFFEKT (AKTIVA EFFEKTEN) Medelv. av den överförda effekten P fås genom att integrera över en hel tidsperiod T:

$T = \frac{1}{f} = \frac{2\pi}{w} \Rightarrow w = \frac{2\pi}{T}$

$$P = \frac{1}{T}\int_0^T p(t)\,dt = \frac{1}{T} U_{topp} \cdot I_{topp} \sin wt \sin wt \; dt = \frac{U_{topp} I_{topp}}{2}$$



EFFEKTIVVÄRDE (= RMS - Root Mean Square)

22/9

$I_{eff} = \frac{I_{topp}}{\sqrt{2}}$ $\qquad P = \frac{U_{topp} I_{topp}}{2}$

$U_{eff} = \frac{U_{topp}}{\sqrt{2}}$ för sinusformade

Om ström o sp. inte ligger i fas:

$u(t) = U_{topp} \sin(wt)$

$i(t) = I_{topp} \sin(wt + \theta)$ $\quad \theta$ - skilln. i fas mellan str. o sp.

⇒ momentana effekten

$p(t) = U_{topp} \cdot I_{topp} \sin(wt) \sin(wt+\theta) = \frac{1}{2} U_t \cdot I_t (\cos\theta - \cos(2wt + \theta)) \ldots$

<u>Medeleff</u> av den överförda eff fås genom int över en hel tidsperiod

$P = \frac{1}{T}\int_0^T p(t)\,dt = \frac{1}{2T}\int_0^T U_t \cdot I_t (\cos\theta(1 - \cos(\frac{4\pi}{T}t)) - \sin\theta \sin(\frac{4\pi}{T}t))\,dt =$

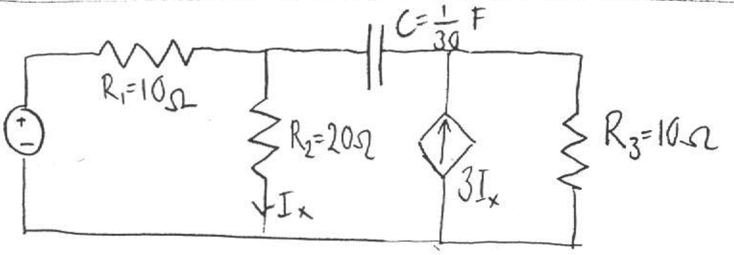
$= \frac{U_t \cdot I_t}{2} \cos\theta + 0 = U_{eff} \cdot I_{eff} \cos\theta$ där $\cos\theta$ kallas effektfaktorn

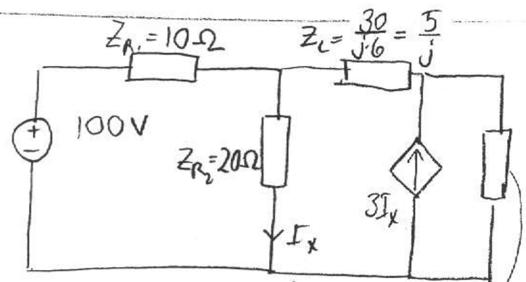
---

12.4-3

$100\cos(6t)$

$w = 6$

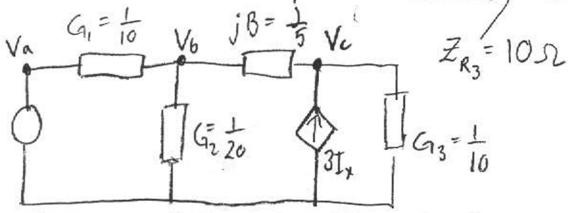




jw-metoden anv.

$\frac{1}{Z} = Y = G + jB$

$V_a = 100V$



Samma värden, fast här admittans

<u>Nodanalys:</u>

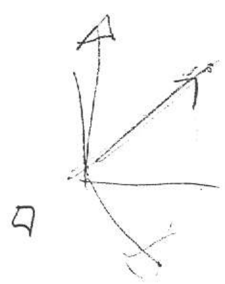
Nod V_c: $\quad 3I_x = V_c \cdot G_3 + (V_c - V_b) \cdot jB$

V_b: $\quad (V_b - V_a) \cdot G_1 + (V_b - V_c) \cdot jB + V_b \cdot G_2 = 0$

$I_x = V_b \cdot G_2$

$\Rightarrow \quad V_b = \frac{V_a G_1}{(G_1 + G_2 + jB) - jB \frac{3G_2 + jB}{G_3 + jB}} = 88 + 16j \ (V) \quad U_{topp} = \sqrt{88^2 + 16^2}$

$P = \frac{I_{topp} \cdot \cancel{U_{topp}}}{2} = \frac{U_{topp}}{I_{topp}} \cdot \frac{U_{topp}}{2} = 200W$



Reaktiva effekten

$$Q = \frac{U_{topp} I_{topp}}{2} \cdot \sin\theta = U_{eff} I_{eff} \sin\theta \quad [VAR]$$

Komplexa effekten

$$S = P + jQ \quad [VA]$$

$$S = U_{eff} \cdot I_{eff}$$

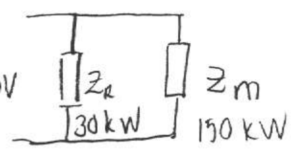
Skenbara effekten

$$|S| = \sqrt{P^2+Q^2} = \frac{U_{topp} \cdot I_{topp}}{2} \quad [VA]$$

P är den i nätets resistanser omsatta värmeenergin per tidsenhet. Viss energi går tillbaks till källan.

127-10

$U_{RMS} = 4000V$



laggning 0,6 (lead 0,6 - neg)

pos. fas, innebär att: $\cos\theta = 0{,}6$

$$\sin\theta = \sqrt{1-\cos^2\theta} = 0{,}8$$

$$U_{topp} = 4000\sqrt{2}\ V$$

$$S = R + jQ$$

S_R: $P_R = 30kW$ $\quad Q_R = U_{RMS} \cdot I_{RMS} \cdot \underbrace{\sin\theta}_{}$

0 ty ingen reaktiv eff på en resistans

S_M: $P_M = 150kVA \cdot \cos\theta = 90kW \quad Q_M = 150\,kVA \sin\theta = 120kVAR$

$$S_{tot} = S_R + S_M = \overset{30+90}{120kW} + j\,120\,kVAR$$

$|I_{RMS}|$

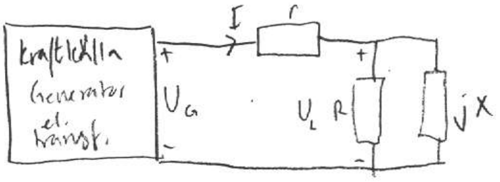
$$S = U_{RMS} \cdot I^*_{RMS} \qquad I^*_{RMS} = \frac{120 + j\,120}{4} = 30 + 30j$$

(4000V- stryk nollorna)

$$|I_{RMS}| = 30\sqrt{2} = \underline{42{,}4\ A}$$

□

Kraftöverföring, faskompensering



kraftledn har resistansen r

$$I_{eff} = \frac{U_G}{r + \frac{RjX}{R+jX}}$$

Om X inte ∃ $(X \to \infty)$

blir den nyttiga effekten

$P = R I_{eff}^2$ o

förlusteffekten $P_F = r I_{eff}^2$

Om X ∃ blir strömmen genom

ledn. större

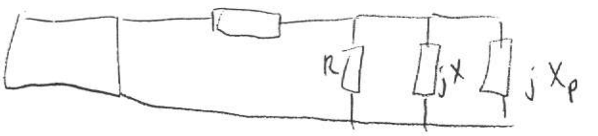
⇒ nyttiga eft minskar (trots att X inte förbr. någon medeleff)

Faskompensering

reaktiva effekten

$Q = U_{eff} I_{eff} \sin\theta$ ger förluster i ledningarna (r)

För att nå låga förluster ska Q elimineras. Detta görs m. parallellkoppl:

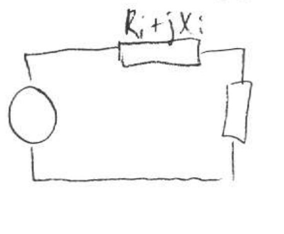


formel: ....

Anpassning

Den till z överförda aktiva eff. blir

$$P = R I_{eff}^2 = R\left|\frac{E}{z_{tot}}\right|^2 = \frac{R E_{eff}^2}{|R+z_i|^2} = \frac{R E_{eff}^2}{(R+R_i)^2 + (X+X_j)^2}$$



Två fall finns (hur anpassn. kan ske)

- R fast $P_{max} = X = -X_j$
- - - - -

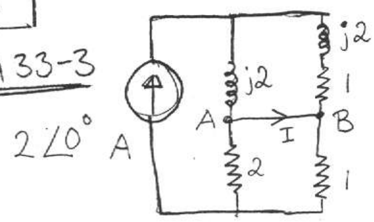
Transformatorn

ideal −"−

$$\frac{V_1}{V_2} = \frac{N_1}{N_2} \quad \frac{i_1}{i_2} = -\frac{N_2}{N_1}$$

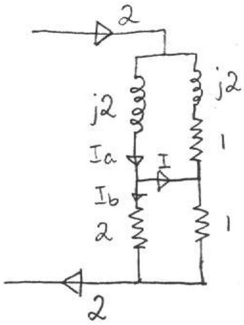
24/9

Ex. A33-3



En kortslutning uppträder mellan punkterna A & B i fig. Beräkna den komplexa kortslutningsströmmen I.
Impedansvärden i Ω.

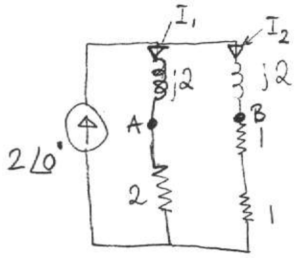
Strömdelning



$$I_a = 2\,\frac{1+2j}{1+2j+2j} = \frac{2(1+2j)}{1+4j}$$

$$I_b = 2\cdot\frac{1}{1+2} = \frac{2}{3}$$

$$I = I_a - I_b = 2\left(\frac{1+2j}{1+4j} - \frac{1}{3}\right) = \ldots = \frac{4}{3}\left(\frac{1+j}{1+4j}\right) \approx 0{,}46\angle{-31^\circ}$$



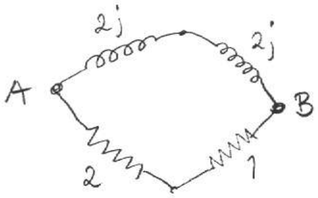
Thevenin ekv. nät

strömdelning:

$$I_1 = 2\cdot\frac{2+j2}{2+2j+2+2j} = 1$$

$$I_2 = \text{p.s.s.} = 1$$

$$V_{AB} = V_A - V_B = 2I_1 - 1\cdot I_2 = 2-1 = \underline{\underline{1}}$$

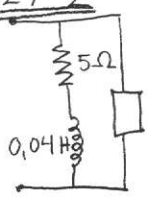


$$Z_{AB} = \frac{(1+2)(1+4j)}{1+2+1+4j} = \frac{3}{4}\cdot\frac{1+4j}{1+j}$$

Kortslutningsström

$$I = \frac{V_{AB}}{Z_{AB}} = \frac{4}{3}\,\frac{1+j}{1+4j} = \ldots = 0{,}46\angle{-31^\circ}$$

Ex A 27-2



$z_{in} = 5\,\Omega$ för alla frekvenser. Bestäm z och ange hur z kan åstadkommas m.h.a. två kretselement.

sätt $z = R + jx$

$$z_{in} = \frac{\left(5 + j\frac{\omega}{25}\right)(R + jx)}{(R+5) + j\left(x + \frac{\omega}{25}\right)} = 5$$

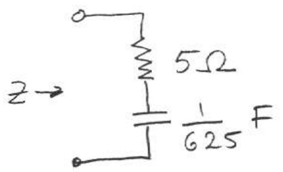
forts. →

forts. $5R - \frac{\omega X}{25} + j\left(5X + \frac{\omega R}{25}\right) = (5R + 25) + j\left(5X + \frac{\omega}{5}\right)$

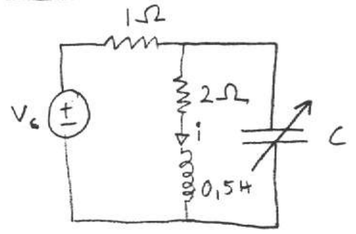
Re: $-\frac{\omega X}{25} = 25$, $X = -\frac{625}{\omega}$ $\left(X = -\frac{1}{\omega C}\right) \Rightarrow C = \frac{1}{625}$ F

Im: $\frac{R}{25} = \frac{1}{5} \Rightarrow \underline{\underline{R = 5\,\Omega}}$

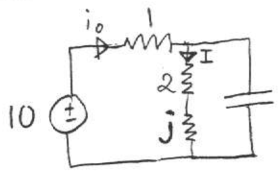
Svar:



Ex. A 31-3



Välj C så att strömmens i:s effektivvärde blir så stort som möjligt och bestäm detta maximalvärde $V_s = 10\cos 2t$ (V)



$\frac{1}{j\omega C} = \frac{-j}{2C}$ $\quad (j\omega L = j \cdot 2 \cdot 0{,}5 = j)$

$$I_0 = \frac{10}{1 + (2+j) \,//\, \frac{-j}{2C}} = 10 \frac{}{1 + \frac{(2+j)\cdot\frac{-j}{2C}}{2 + j\left(1 - \frac{1}{2C}\right)}} = \frac{10\left[2 + j\left(1 - \frac{1}{2C}\right)\right]}{2 + \frac{1}{2C} + j\left(1 - \frac{3}{2C}\right)}$$

Strömdelning

$$I = I_0 \frac{\frac{-j}{2C}}{2 + j\left(1 - \frac{1}{2C}\right)} = \ldots = \frac{-j10}{4C + 1 + j(2C - 3)}$$

$$|I| = \frac{10}{\sqrt{(4C+1)^2 + (2C-1)^2}}$$

Max $|I|$ ⇒ vi söker minsta möjliga nämnare A ⇒ derivera uttrycket innanför rottecknet och finn dess min.värde

$A = (4C+1)^2 + (2C-3)^2$

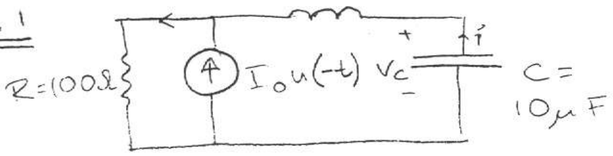
$\frac{dA}{dC} = 2 \cdot 4(4C+1) + 2 \cdot 2(2C-3) = 40C - 4 = 0$

$\Rightarrow C = \frac{1}{10}$ F

$\text{Max}|I| = \frac{10}{\sqrt{(1{,}4)^2 + (2{,}8)^2}}$

$\text{Max } I_{eff} = \frac{\text{Max}|I|}{\sqrt{2}} \approx 2{,}26$ A

10.6.1



⟹ C är fullt uppladdad → ingen ström i denna k

39

$i = I_0$

$V_c = R i$

t>0

$$V_c = L\frac{\partial i}{\partial t} + Ri$$

$$i = -C\frac{dV_c}{dt} \qquad \frac{di}{dt} = -C\frac{d^2V_c}{dt^2}$$

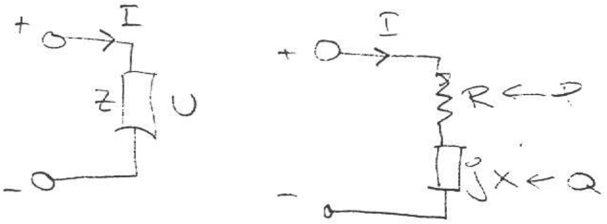
$$V_c = -LC\frac{d^2V_c}{dt^2} - RC\frac{dV_c}{dt}$$

$$V_c(0) = RI_0$$

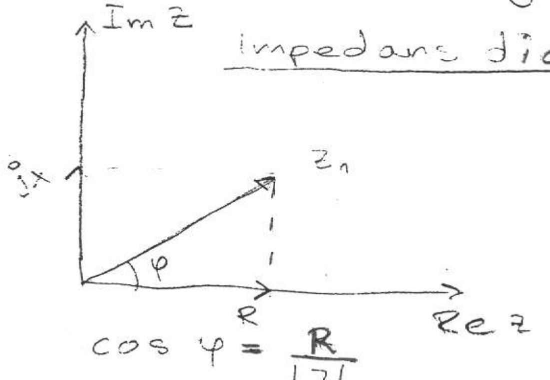
$$i(0) = -C\left.\frac{dV_c}{dt}\right|_{t=0} = 0 \qquad V_c'(0) = 0$$

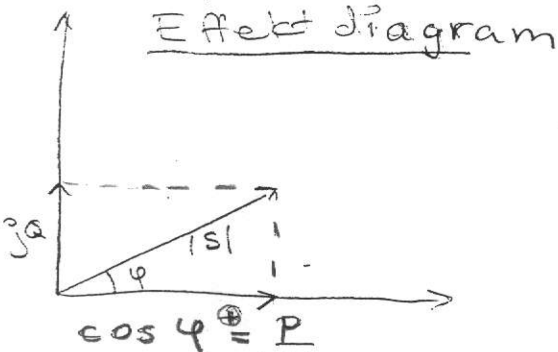
$$V_c = (3 + 6000t)e^{-2000t}$$

29/9-97 Effekt



$$S = I^* U = Z|I|^2 = (R + jx)|I|^2 = P + jQ$$





$$\cos\varphi = \frac{R}{|Z|} \qquad \cos\varphi = \frac{P}{|S|}$$

Tre sätt att teckna effekt:

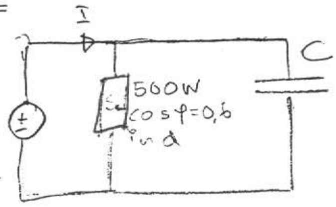
$$U \cdot I^* = Z|I|^2 = \frac{|U|^2}{Z^*}$$

Effektfaktor, η:

$$\eta = \frac{P}{|S|} \quad \begin{cases} \text{ind}: & \varphi > 0 \\ \text{cap}: & \varphi < 0 \end{cases} \qquad 0 \le |\eta| \le 1$$

A 27-3 minimera I

40.



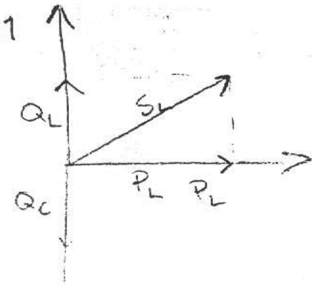
$|U|=220$

50Hz

$\cos\varphi = 0{,}6 \Rightarrow \varphi = +53{,}1$

$S_L = P_L + jQ_L$

500W



$|I|$ minimeras om $Q_L + Q_C = 0$

$P_L = |S_L| \cos\varphi$ $\quad Q_L = P_L \tan\varphi$

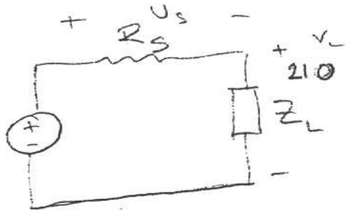
$Q_L = |S_c| \sin\varphi$

$$S = \frac{|U|^2}{Z^*} \Rightarrow jQ_L = \frac{|U|^2}{(jX_L)^*} \Rightarrow Q_L = \frac{|U|^2}{X_L}$$

$$Q_c = \frac{|U|^2}{X_c} = \frac{|U|^2}{-\frac{1}{\omega C}} \Rightarrow C = \frac{-Q_c}{\omega|U|^2} = \frac{Q_L}{\omega|U|^2}$$

$$\Rightarrow \frac{P_L \tan\varphi}{\omega|U|^2} = \frac{500 \cdot 1{,}33}{2\pi 50 \cdot 220^2} \approx 43{,}8\,\mu F$$

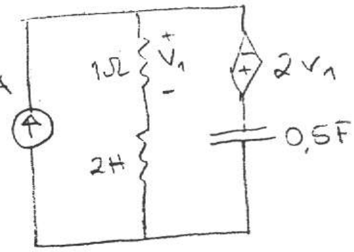
Ex



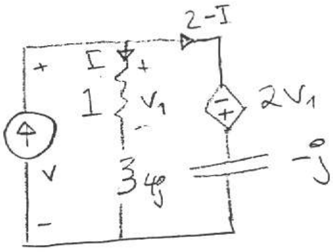
$220 = U_s + 210$

$220 + U_s + U_L$ komplext

A 32-2



Bestäm den komplexa effekten som strömkällan avger.



$V_1 = I \cdot 1$

KVL: $2V_1 - (-j(2-I)) + I(1+4j) = 0$

$2I + 2j - jI + I(1+4j) = 0$

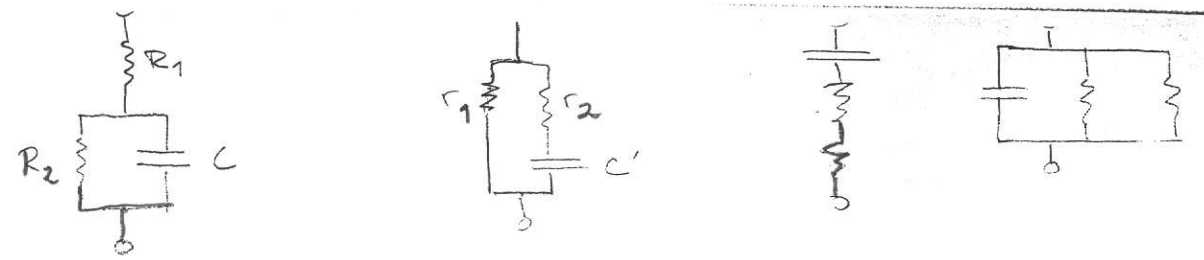
$$I = \frac{-j2}{3+3j}$$

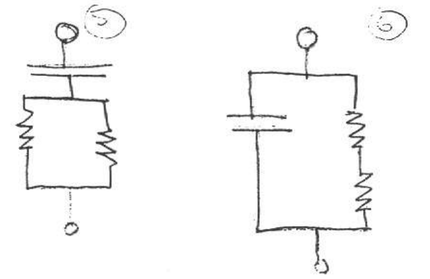
$$V = I(1+4j) = -\frac{2j(1+4j)}{3+3j}$$

Upptagen effekt: $S_I = \frac{1}{2} V(-2)^* = \ldots = -1 + j1{,}67$ VA

Strömkällan avger: $1 - j1{,}67$ VA

Ex





② & ⑥ imp → 0 i ∞

③ & ⑤ imp → ∞ i ∞

Välj ①

$f=0$: $R_1+R_2=90 \Rightarrow R_2=64$ $\quad$ R_2 försvinner i ∞

$f=\infty$: $R_1=26$

$$Z = R_1 + \frac{R_2}{1+j\omega R_2 C}$$

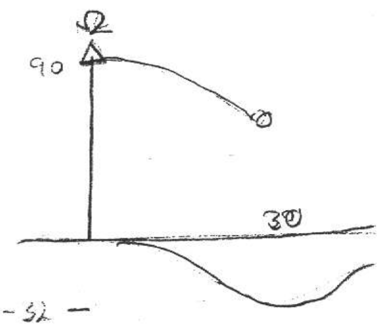
$$X = \operatorname{Im} Z = \operatorname{Im} \frac{R_2(1-j\omega R_2 C)}{1+(\omega R_2 C)^2} = \frac{-\omega R_2^2 C}{1+(\omega R_2 C)^2}$$

ωRC är dimensionslöst

Sätt $U=\omega RC$

$$X = -k\frac{U}{1+U^2}$$

$$\frac{d\left(\frac{1}{x}\right)}{du} = -\frac{1}{k}\frac{d}{du}\left(u+\frac{1}{u}\right) = -\frac{1}{k}\left(1-\frac{1}{u^2}\right) = 0$$

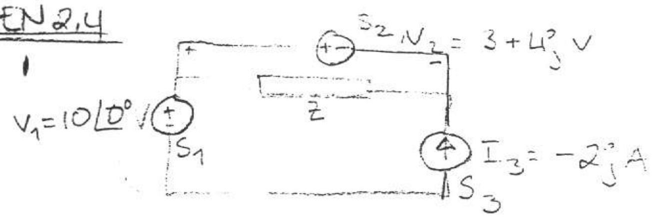


$u=1$ Min $\omega_0 R_2 C = 1$

$$C = \frac{1}{\omega_0 R_2} = \frac{1}{2\pi\, 30\cdot 10^6\cdot 64} \approx 82{,}9\ \text{pF}$$

PAEN 2.4

Effektivvärde



Sök Z om

$S_1 = 20j$ VA (förbrukad e

$S_2 = 3+4j$ VA

$S_3 = -8-14j$ VA

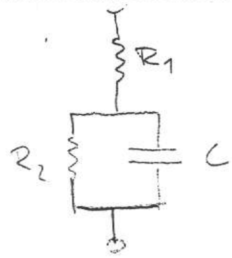
$$\sum S_i = 0$$

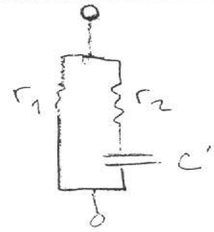
$$S_Z = -(S_1+S_2+S_3) = -20j-3-4j+8+14j = 5-10j \text{ VA}$$

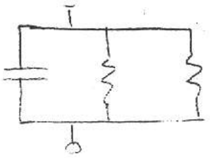
$$S_Z = \frac{|U_Z|^2}{Z^*} \Rightarrow Z = \frac{|U_Z|^2}{S_Z^*} = \frac{|3+4j|^2}{5+10j} = \frac{25}{5+10j} = \frac{5}{1+2j}$$

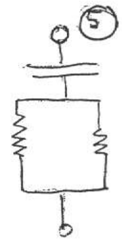
$$= \frac{5(1-2j)}{1+2^2} = 1+2j$$

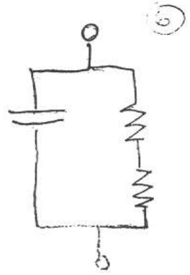
42. **Ex**







⑤


⑥


④ & ⑥ imp → 0 i ∞

③ & ⑤ imp → ∞ i ∞

Välj ①

$f=0: \; R_1+R_2=90 \Rightarrow R_2=64$ $\qquad$ R_2 försvinner i ∞

$f=\infty: \; R_1=26$

$$Z = R_1 + \frac{R_2}{1+j\omega R_2 C}$$

$$X = \mathrm{Im}\, Z = \mathrm{Im}\, \frac{R_2(1-j\omega R_2 C)}{1+(\omega R_2 C)^2} = \frac{-\omega R_2^2 C}{1+(\omega R_2 C)^2}$$

ωRC är dimensionslöst

sätt $U = \omega RC$

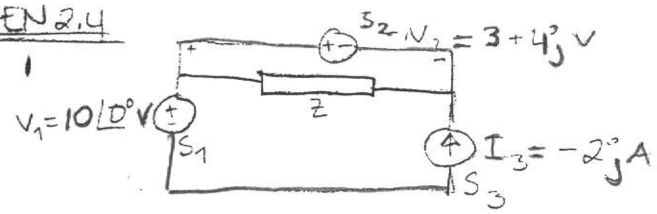
$$x = -k \frac{U}{1+U^2}$$

$$\frac{d\left(\frac{1}{x}\right)}{du} = -\frac{1}{k}\frac{d}{du}\left(u + \frac{1}{u}\right) = -\frac{1}{k}\left(1-\frac{1}{u^2}\right) = 0$$

$u=1$ $\quad$ Min $\quad$ $\omega_0 R_2 C = 1$

$$C = \frac{1}{\omega_0 R_2} = \frac{1}{2\pi 30 \cdot 10^6 \cdot 64} \approx 82{,}9\,pF$$

PAEN 2.4 $\qquad$ Effektivvärde



Sök Z om

$S_1 = 20j$ VA (förbrukad)

$S_2 = 3+4j$ VA

$S_3 = -8-14j$ VA

$$\sum S_i = 0$$

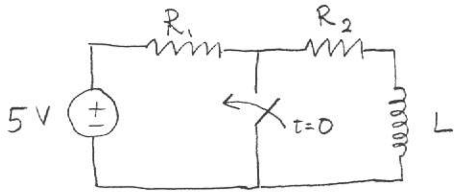
$$S_z = -(S_1+S_2+S_3) = -20j-3-4j+8+14j = 5-10j \text{ VA}$$

$$S_z = \frac{|U_z|^2}{Z^*} \Rightarrow Z = \frac{|U_z|^2}{S_z^*} = \frac{|3+4j|^2}{5+10j} = \frac{25}{5+10j} = \frac{5}{1+2j}$$

$$= \frac{5(1-2j)}{1+2^2} = 1+2j$$

26/9-97

Ex. 8.5-1

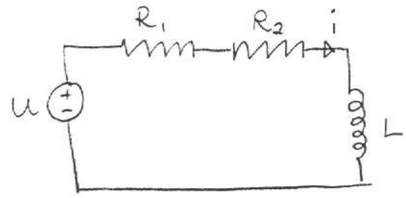


$R_1 = 2k\Omega$

$R_2 = 3k\Omega$

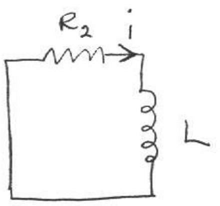
$L = 100mH$

$t<0$



$$i = \frac{u}{R_{tot}} = \frac{u}{R_1 + R_2}$$

$i(0) = 1mA$



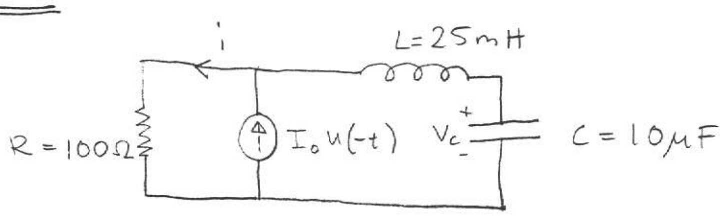
$$\text{K.V.L: } 0 = R_2 i + L\frac{di}{dt}$$

$$i = I_0 e^{-\frac{Rt}{L}}$$

$$i(0) = I_0 = 1mA$$

$$i = \begin{cases} 10^{-3} e^{-3\cdot 10^4 t} & (A) \quad t \geq 0 \\ 10^{-3} & (A) \quad t<0 \end{cases}$$

Ex. 10.6.1



$t<0: \quad i = I_0 \quad , V_c = Ri$

$t>0: \quad V_c = L\frac{di}{dt} + Ri$

$$i = -C\frac{dV_c}{dt}$$

$$\frac{di}{dt} = -C\frac{d^2V_c}{dt^2}$$

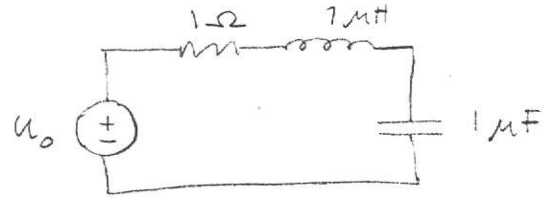
$$V_c = -LC\frac{d^2V_c}{dt^2} - RC\frac{dV_c}{dt}$$

$$V_c(0) = RI_0 \quad , i(0) = -C\frac{dV_c}{dt}\bigg|_{t=0} = 0 \quad , V_c'(0) = 0$$

$$V_c = (3+6000t)e^{-2000t}$$

3/10-97

A 35-3

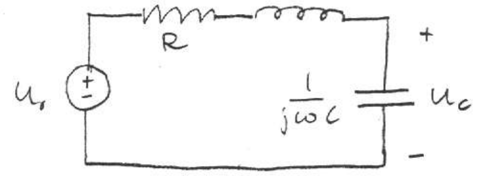


Generatorn har variabel frekvens. Vid vilken vinkelfrekvens är spänningen över kondensatorn maximal, och hur stor är denna spänning.

$u_{oe} = 100\text{ V}$
↑
effektivvärde

lösning:

(effektivvärden)



spänningsdelning

$$u_c = \frac{\frac{1}{j\omega C}}{R + j\omega L + \frac{1}{j\omega L}} u_o = \frac{u_o}{1 - \omega^2 LC + j\omega RC}$$

$$|u_c| = \frac{|u_o|}{\sqrt{(1-\omega^2 LC)^2 + (\omega RC)^2}}$$

- Derivera A
- Kvadratisk form

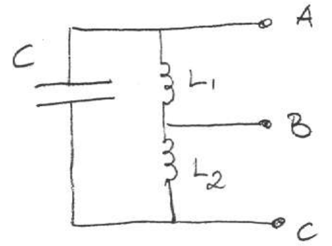
$$\frac{dA}{d(\omega^2)} = 2(1-\omega^2 LC)(-LC) + (RC)^2 \Rightarrow \omega^2 = \frac{1}{LC}\left(1 - \frac{R^2}{2\left(\frac{L}{C}\right)}\right) = \frac{1}{10^{-12}}\left(1 - \frac{1}{2}\right) \Rightarrow$$

$$\Rightarrow \omega_0 = 0{,}707 \cdot 10^6 \text{ rad/s.}$$

$$\omega_0 \Rightarrow A = \frac{R^2 C}{L}\left(1 - \frac{R^2}{4LC}\right) \Rightarrow \underline{\underline{|u_c|_{max} = 115\text{ V}}}$$

45.

Ex.



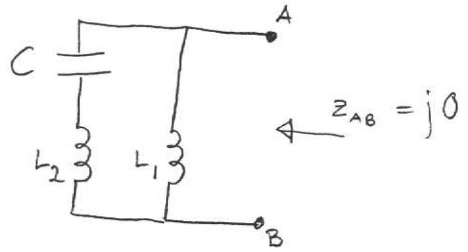
Bestäm $f_3(f_1, f_2)$

— " — $\frac{L_2}{L_1}$

$Z_{AB}(f_2) = j0$

$Z_{BC}(f_1) = j0$

$Z_{AC}(f_3) = j\infty$



(1) $\omega^2 = \frac{1}{L_2 C}$ ty $u_{L_1} = 0$ vid ω_2

p.s.s.

(2) $\omega_1^2 = \frac{1}{L_1 C}$

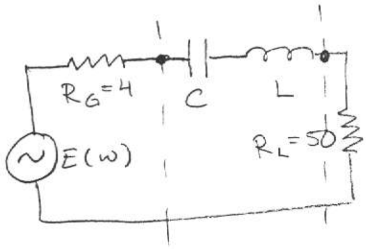
parallellresonans

(3) $\omega_3^2 = \frac{1}{(L_1 + L_2)C}$

$\frac{1}{\omega_1^2} + \frac{1}{\omega_2^2} = (L_1 + L_2)C = \frac{1}{\omega_3^2}$

$\underline{\underline{\omega_3^2 = \frac{\omega_1^2 \omega_2^2}{\omega_1^2 + \omega_2^2}}}$

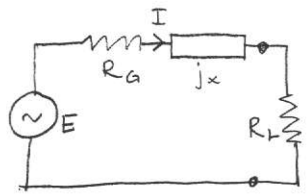
(2) % (1) : $\frac{\omega_1^2}{\omega_2^2} = \frac{L_2 \cancel{C}}{L_1 \cancel{C}}$, $\underline{\underline{\frac{L_2}{L_1} = \frac{\omega_1^2}{\omega_2^2}}}$



Bandpass filter

Bestäm max effekt i last

samt bandbredd



$|I|_{max}$ då $x=0$

$$\Rightarrow \omega_0 = \frac{1}{\sqrt{LC}}$$

resonans

$$\text{Max } P_{R_L} = |I|^2_{max} R_L =$$

$$= \left(\frac{|E|}{R_G+P_L}\right)^2 R_L = \frac{|E|^2}{(200)^2}\cdot 150 = |E|^2 \underline{\underline{\frac{3}{800} \text{ W}}}$$

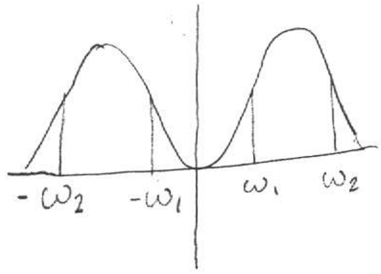
Halverad effekt för $|I| = \frac{|I|_{max}}{\sqrt{2}}$

$$\Rightarrow R_G + R_L = |x_L|$$

$$\pm(R_G+R_L) = \omega_0 L - \frac{1}{\omega_0 C} \quad \ldots\ldots$$

$$\omega_{00} = \pm \frac{R_G+R_L}{2L} \pm \sqrt{\left(\frac{R_G+R_L}{2L}\right)^2 + \frac{1}{LC}}$$

$$B = \omega_{00}^H - \omega_{00}^V = 2\,\frac{R_G+R_L}{2L} = \frac{R_G+R_L}{L} = \underline{\underline{\frac{200}{L}}} \text{ rad/s}$$



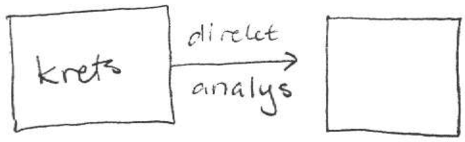
8/10-97

Laplacetransformer

kap. 14.1-14.9

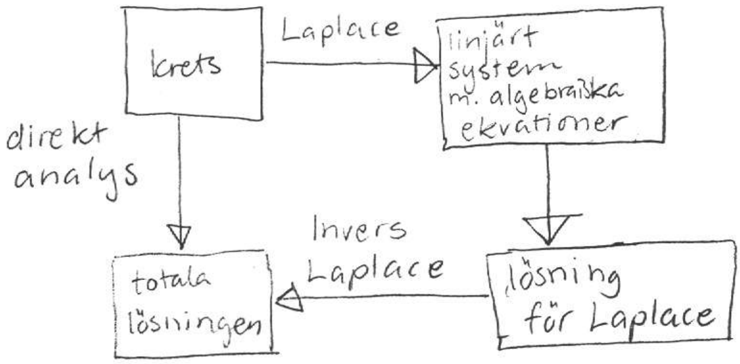
Varför behövs Laplace ?

linjära ODE system med konstanta koefficienter

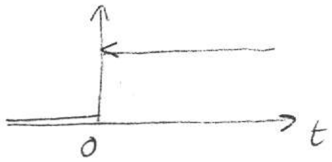


$$\frac{dx}{dt} = Ax + Bf(t)$$

Alternativ lösning:

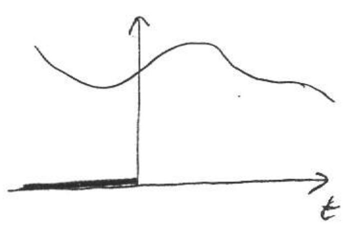


Ex. $f(t) \quad t > 0$



$$L(f,s) \overset{def.}{=} \int_0^\infty f(t)e^{-st}\,dt \quad , s = \delta + j\omega \quad , \delta > 0$$

$$|f(t)| \le Me^{\alpha t} \quad , M > 0 \quad , \alpha > 0 \qquad \alpha < \delta$$



48.

$$f(t) = e^{-at}$$

$$F(s) = \int_0^\infty e^{-at} e^{-st} \, dt = \left. \frac{-e^{-(s+a)t}}{s+a} \right|_0^\infty = \frac{1}{s+a}$$

$f(t)$	$F(s)$
1) stegfunktion	$\frac{1}{s}$
2) e^{-at}	$\frac{1}{s+a}$
3) $\sin(\omega t)$	$\frac{\omega}{s^2+\omega^2}$

$$G(s) = \int_0^\infty f'(t) e^{-st} \, dt = f(t) e^{-st} \Big|_0^\infty - \int_0^\infty f(t) e^{-st} \, dt = \left[f(t) e^{-st} \xrightarrow[t \to \infty]{} 0 \right] =$$

$$= -f(0) + sF(s)$$

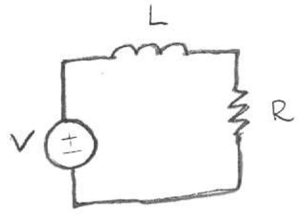
$$\boxed{G(s) = sF(s) - f(0)}$$

$f(t)$	$F(s)$
4) $f \to F(s)$ $f' \to G(s)$	$G(s) = sF(s) - f(0)$

$$\begin{array}{ccc} f_1'(t) & f_2'(t) & \alpha_1 f_1(t) + \alpha_2 f_2(t) \longrightarrow F(s) \\ \downarrow & \downarrow & \\ F_1(s) & F_2(s) & F(s) = \alpha_1 F_1(s) + \alpha_2 F_2(s) \end{array}$$

$$L(t^n f(t)) = (-1)^n \frac{d^n F(s)}{ds^n}$$

$$L\left(\int_0^t f(u) \, du \right) = \frac{F(s)}{s} \quad , t > 0$$



$$v = L\frac{di}{dt} + Ri$$

$$V(s) = L\underbrace{\mathcal{L}\left(\frac{di}{dt}\right)}_{sI(s)-i(0)} + RI(s)$$

$$V(s) = L[sI(s) - i(0)] + RI(s)$$

$$\boxed{I(s) = \frac{V(s) + Li(0)}{R + Ls}}$$

$$I(j\omega) = Z(j\omega)V(j\omega)$$

50. $F(s) = \int_0^\infty f(t) e^{-st}\, dt = \lim_{s\to\infty} sF(s)$

$$sF(s) = \int_0^\infty f(t)\, s e^{-st}\, dt = \left[s = \frac{1}{r} \right] = \int_0^\infty f(t) \frac{1}{r} e^{-\frac{t}{r}}\, dt = \left[\begin{matrix} u = \frac{t}{r} \\ t = ru \end{matrix} \right] =$$

$$= \int_0^\infty f(ru) e^{-st}\, du = f(0) \int_0^\infty e^{-u}\, du = f(0)$$

$\lim\limits_{\substack{r\to 0 \\ u>0}} f(ru) = f(0)$ $\quad \varepsilon > 0 \quad \delta > 0 : \ 0 < r < \delta \Rightarrow$

$\Rightarrow |f(ru) - f(0)| < \varepsilon$

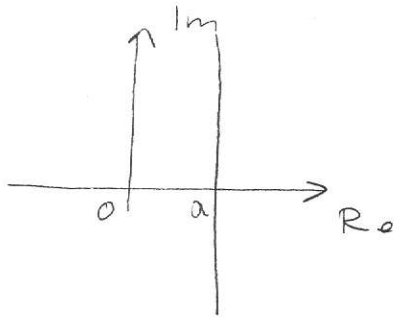
$\lim\limits_{t\to\infty} f(t) = \lim\limits_{s\to 0} sF(s)$

$f(t) = \sin t$

$f(t) \to F(s)$

$F(s) \to f(t)$

$f(t) \overset{def}{=} \frac{1}{2\pi j} \int_{a-j\infty}^{a+j\infty} F(s) e^{st}\, ds$



$$F(s) = \frac{4}{(s+1)^2(s+2)} = \frac{A}{(s+1)^2} + \frac{B}{s+1} + \frac{C}{s+2}$$

$$4 = A(s+2) + B(s+1)(s+2) + C(s+1)^2 = As + 2A + B(s^2+3s+2) +$$
$$+ C(s^2+2s+1) = (B+C)s^2 + (A+3B+2C)s + 2A + 2B + C$$

$B + C = 0 \Rightarrow C$

$A + 3B + 2C = 0$

$A + 3B + 2(-B) = A + B = 0$

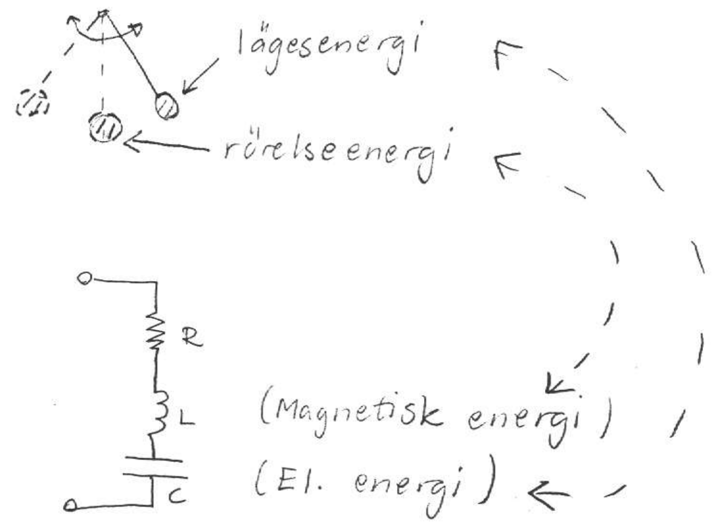
$4 = 2A + 2B + C = 2A + 2(-A) + A = A$

$$\boxed{\begin{matrix} B = -A \\ A = 4 \\ B = -4 \\ C = 4 \end{matrix}}$$

10/10 - 97

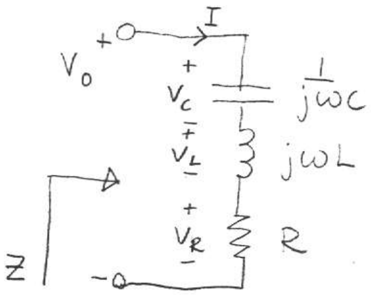
Resonans:

En pendling av energi mellan två energitillstånd.



Resonans 2:

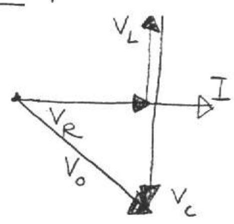
En krets med $\begin{Bmatrix}\text{admittansen } Y \\ \text{impedansen } Z\end{Bmatrix}$ är i resonans om $\begin{Bmatrix}\text{Im } Y \\ \text{Im } Z\end{Bmatrix} = 0$



$$Z = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

Visardiagram

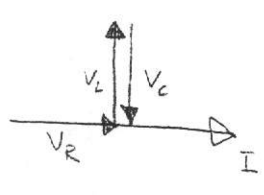
Ex. 1



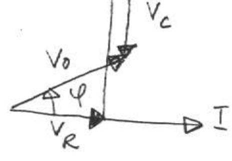
$\omega < \omega_0 \Rightarrow$ under resonansfrekvens

Ex. på resonans

$\omega = \omega_0 \Rightarrow$ resonans



Ex 3



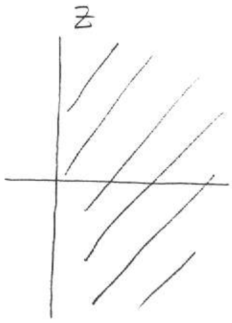
$\omega > \omega_0 \Rightarrow$ över resonansfrekvens

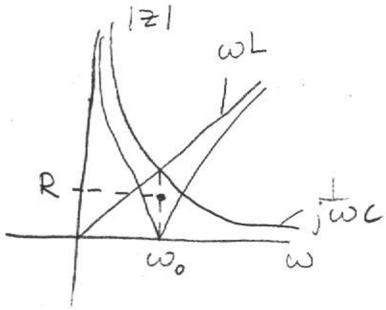
OBS! Mycket stora spänningar/strömmar uppträder vid resonans

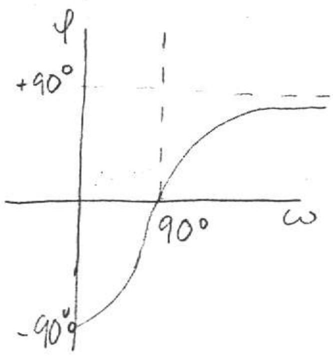
$$|Z|^2 = R^2 + X^2$$

$$X = \left(\omega L - \frac{1}{\omega C}\right)$$

$$\varphi = \arctan \frac{X}{R}$$



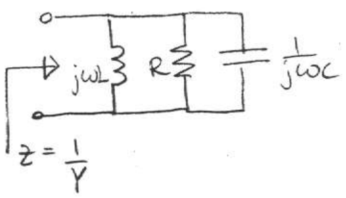




$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$f_0 = \frac{1}{2\pi} \cdot \frac{1}{\sqrt{LC}}$$

$$Q = \frac{\omega_0 L}{R}$$ serie resonans



$$Q = \frac{R}{\omega_0 L}$$ parallellresonans

Tre sätt att räkna resistans

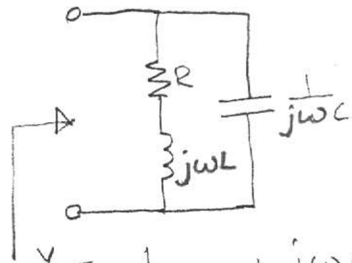
1. $Z = \frac{a+jb}{c+jd} = \frac{(a+jb)(c-jd)}{c^2+d^2} = \frac{ac+bd+j(bc-ad)}{N}$; $\mathrm{Im}\, Z = 0$ $\Rightarrow \boxed{ad = bc}$

2. $\angle Z = \arctan \frac{b}{a} - \arctan \frac{d}{c}$, $\mathrm{Im}\, Z = 0 \Rightarrow \angle Z = 0 \Rightarrow \boxed{\frac{b}{a} = \frac{d}{c}}$

3. $Z = \frac{a+jb}{c+jd} = K\angle 0°$; <u>Ekv.</u> : $a+jb = Kc + jKd$

$$\begin{matrix} a = Kc \\ b = Kd \end{matrix} \Rightarrow \boxed{K = \frac{a}{c} = \frac{b}{d}}$$

Ex. Bestäm resonansfrekvens

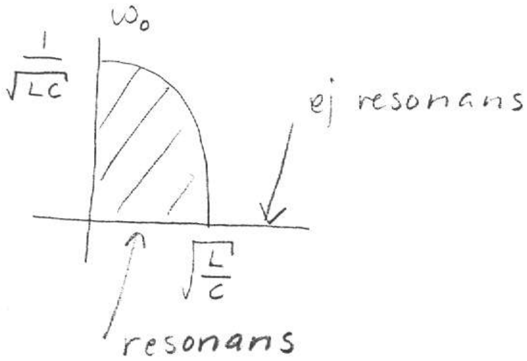


$$Y = \frac{1}{R + j\omega L} + j\omega C = \frac{R - j\omega L}{R^2 + \omega^2 L^2} + j\omega C$$

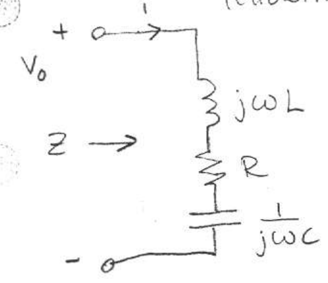
$$\operatorname{Im} Y = 0 \quad , \quad \frac{\cancel{\omega_0} L}{R^2 + \omega_0^2 L^2} = \cancel{\omega_0} C$$

$$\omega_0^2 = \frac{\frac{L}{C} - R^2}{L^2} = \frac{1}{LC} - \left(\frac{R}{L}\right)^2$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \sqrt{1 - \frac{R^2}{\left(\frac{L}{C}\right)}}$$



(effektivvärden)



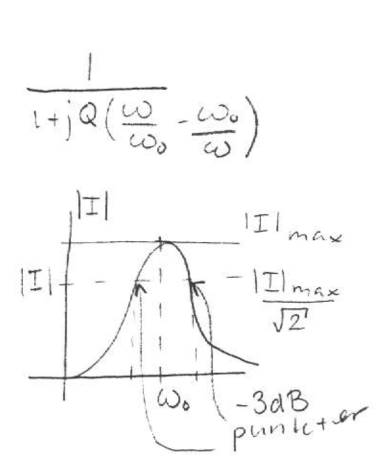
$$Z = R + j\left(\omega L - \frac{1}{\omega C}\right) = R\left[1 + j\left(\frac{\omega L}{R} - \frac{1}{\omega R C}\right)\right] = \left\| \omega_0^2 = \frac{1}{LC} \right\| =$$

$$= R\left[1 + j\underbrace{\frac{\omega_0 L}{R}}_{Q = \frac{\omega_0 L}{R}}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)\right]$$

Q = quality factor

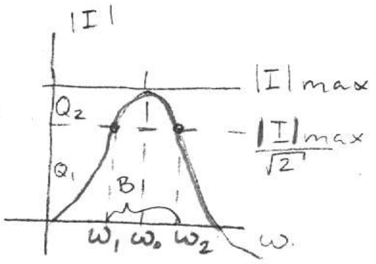
$$I = \frac{V_0}{Z} = \frac{V_0}{R} \frac{1}{1 + jQ\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)}$$

Studera $|I|$



$P_R = R|I|^2$

$Q_2 > Q_1$



$$-3dB: \quad Q\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) = \pm 1$$

$$\omega^2 \pm \frac{\omega_0}{Q}\omega - \omega_0^2 = 0$$

$$\omega_{1,2} = \left(\overset{+}{(-)}\sqrt{1 + \frac{1}{4Q^2}} \pm \frac{1}{2Q}\right)\omega_0$$

$$B = \omega_2 - \omega_1 = \frac{\omega_0}{Q} = \frac{\omega_0}{\left(\frac{\omega_0 L}{R}\right)} = \frac{R}{L}$$

↑Bandbredd

Relativ bandbredd, $b = \dfrac{B}{\omega_0} = \dfrac{1}{Q}$, $bQ = 1$

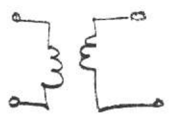
ex
$$\boxed{Q = 100 \Rightarrow b = 1\,\%}$$

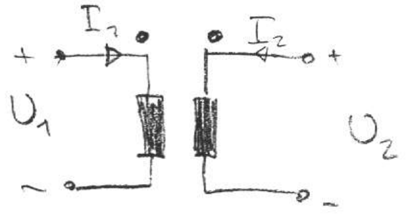
15/10-97 **Transformatorn**

Lenz lag $V = n \frac{d\phi}{dt}$

Allt flöde genom båda spolarna

$L_1 \to \infty$ $L_2 \to \infty$

$\Rightarrow$ ideal trafo. 



$N = \frac{n_1}{n_2}$

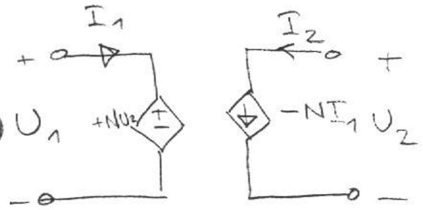
$$\boxed{\begin{aligned} U_1 &= \pm N U_2 \\ I_1 &= \mp \frac{I_2}{N} \end{aligned}}$$

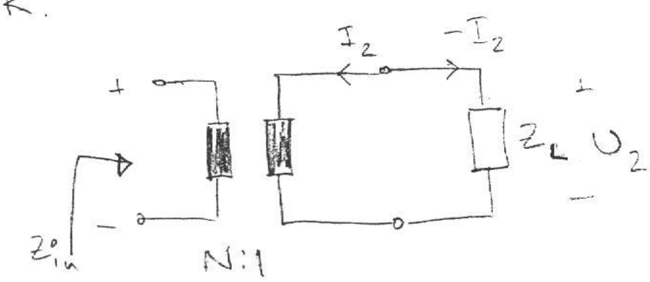
- Trafon förlustfri

• • pos. sp. ger pos sp. vid pricken

Hantering

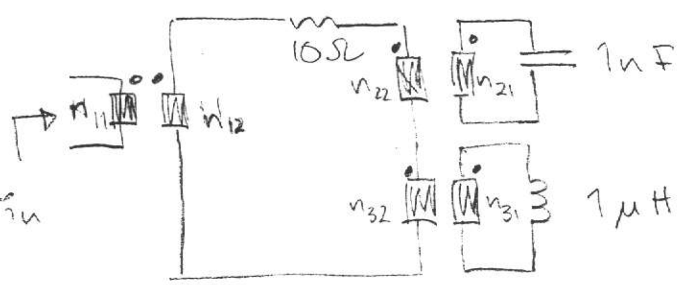
1. Räkna som vanligt, lägg till trafo ekv.
2. —||— + anv. b.k.





- $Z_{in} = \frac{U_1}{I_1} = \frac{N U_2}{-I_2/N} = N^2 \frac{U_2}{-I_2} = N^2 Z_L$

Ex



$N_1 = \frac{n_{11}}{n_{12}}$ $N_3 = \frac{n_{31}}{n_{32}}$

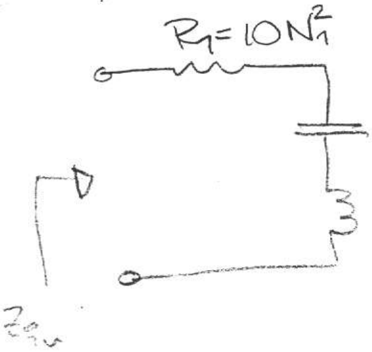
$N_2 = \frac{n_{21}}{n_{22}}$

Best. N_1 N_2 N_3 s.a resonansfrekvens = 10 MHz

bandbred 5%

$Z_0 = 100\Omega$ vid resonans

VÄND



$Z_C = \left(\frac{N_1}{N_2}\right)^2 \frac{1}{j\omega C}$

$Z_L = \left(\frac{N_1}{N_3}\right)^2 j\omega L$

$Z_{in}(\omega_0) = R_1 = 10N_1^2 = 100$ $\qquad \underline{N_1 = \pm\sqrt{10}}$

$$f_0 = \frac{1}{2\pi}\frac{1}{\sqrt{L'C'}} = \frac{1}{2\pi}\frac{1}{\sqrt{\left(\frac{N_1}{N_3}\right)^2 \cdot L \cdot \left(\frac{N_2}{N_1}\right)^2 C}} = \frac{1}{2\pi}\frac{1}{\sqrt{LC}}\frac{N_3}{N_2} = 10^7$$

$$\Rightarrow \frac{N_3}{N_2} = \ldots \frac{2\pi}{\sqrt{2}}$$

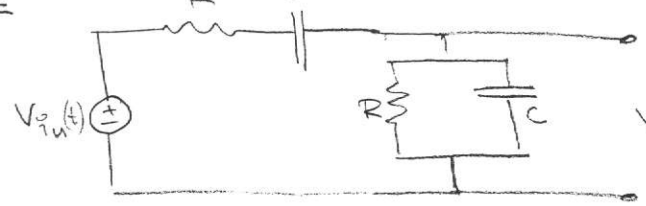
$\underline{N_2 = \frac{1}{2\sqrt{\pi}}}$

Q-värde $\quad Q = \frac{1}{5}$

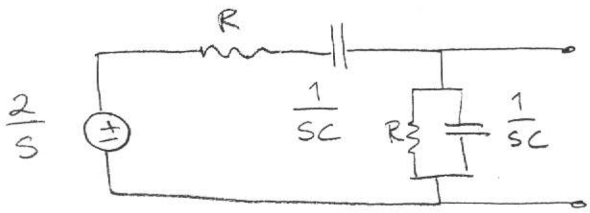
$$\frac{\omega_0 L'}{R_1} = \frac{2\pi 10^7 \left(\frac{N_1}{N_3}\right)^2 L}{10N_1^2} = \frac{2\pi}{N_3^2}$$

$\underline{N_3 = \sqrt{\frac{\pi}{10}}}$

Ex



Best. $V_{ut}(t)$ då insignalen är en stegf. med ampl. $2V_0$. Beg. energi saknas.



Sp. delning

$$\frac{V_{ut}}{V_{in}} = \frac{\frac{R \cdot \frac{1}{sC}}{R + \frac{1}{sC}}}{\left(R + \frac{1}{sC}\right) + \left(\frac{R \cdot \frac{1}{sC}}{R + \frac{1}{sC}}\right)} \Rightarrow \ldots = \frac{2}{RC}\frac{1}{s^2 + \frac{3}{RC}s + \frac{1}{(RC)^2}} \subset V_{ut}(t)$$

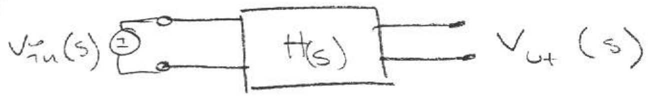
partial.uppdeln.

$$\Rightarrow s = \left(-\frac{3}{2} \pm \sqrt{\frac{9}{4} - \frac{4}{4}}\right)\frac{1}{RC}$$

$$V_{ut}(s) = \frac{2}{\sqrt{5}}\left(\frac{1}{s + \frac{3+\sqrt{5}}{2RC}} - \frac{1}{s + \frac{3-\sqrt{5}}{2RC}}\right) \subset$$

$$= \frac{2}{\sqrt{5}}\left(e^{-\frac{3-\sqrt{5}}{2RC}t} - e^{-\frac{3+\sqrt{5}}{2RC}t}\right.$$

Ex



Impulsfunktionsvaret från en överf. funk H(s) är

$V_{ut}(t) = e^{-t}\cos 2t + ae^{-10t}$

Best motsv. stegfunktion svar samt ett värde på a så att stegfunktionssvaret i $V_{ut}(t)$ saknar en stegfunktions komponent.

In: $\delta(t) \Rightarrow h(t)$ ut

$$h(t) \supset H(s) = \frac{s+1}{(s+1)^2+(2)^2} + \frac{a}{s+10}$$

In: $\theta(t)$ stegfunktion

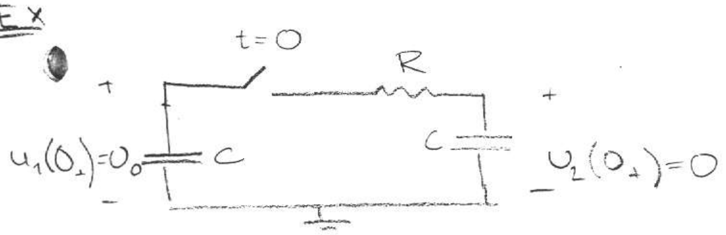
$$V_{ut}(s) = \frac{1}{s}H(s) \Rightarrow \frac{1}{s}\frac{s+1}{(s+1)^2+4} + \frac{a}{s}\frac{1}{s+10} \Rightarrow$$

Partialbråks uppdelning $= \ldots = \frac{1}{5}\left[2\frac{2}{(s+1)^2+4} - \frac{s+1}{(s+1)^2+4} + \frac{1}{s}\right] + \frac{a}{10}\left[\frac{1}{s} - \frac{1}{s+}\right.$

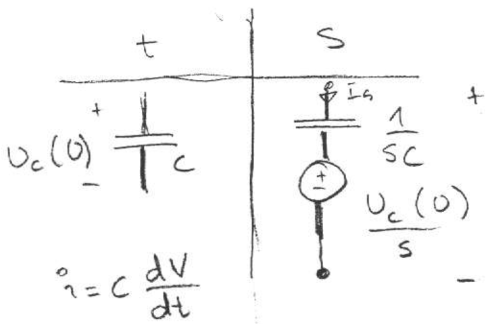
Stegfunktions komp. $\Rightarrow \left(\frac{1}{5} + \frac{a}{10}\right)\frac{1}{s} = 0$ om $a = -2$

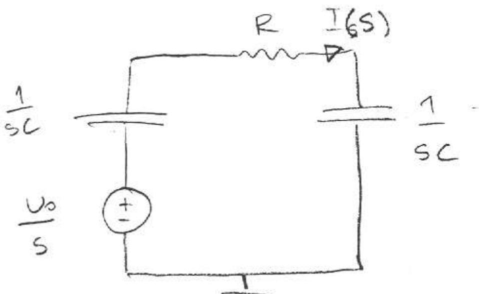
$\circledast \subset \frac{1}{5}\left[2\sin 2t - \cos 2t\right]e^{-t} + \frac{1}{5}e^{-10t} = V_{ut}(t)$

Ex



Best. laddningens tidsfunk. $q(t)$ för den t=0 oladdade kondensator samt tiden för halvladdni





$$I(s) = \frac{\frac{U_0}{s}}{R + \frac{2}{sC}} = \frac{U_0}{R} \frac{1}{s + \frac{2}{RC}}$$

$$q(t) = \int_0^t i(t)\,dt \supset Q(s) = \frac{I(s)}{s} = \frac{U_0}{R}\frac{1}{s}\frac{1}{s+\frac{2}{RC}} = \dots =$$

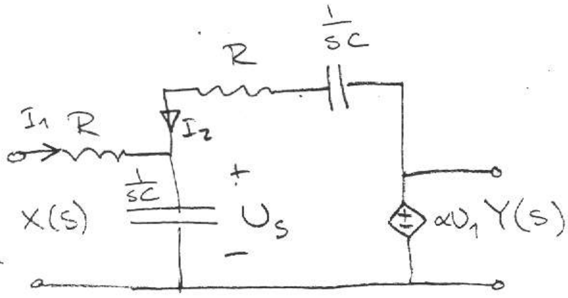
$$\frac{U_0}{R}\left(\frac{RC/2}{s} - \frac{RC/2}{s+2/RC}\right) \subset \frac{U_0 C}{2}\sigma(t)\left(1 - e^{-\frac{2t}{RC}}\right) = q(t)$$

Halveringstid.: $e^{-\frac{2t_0}{RC}} = \frac{1}{2} \Rightarrow t_0 = \frac{1}{2} RC \ln 2 \approx 0{,}347 RC$

Ex

Bestäm $H(s) = \dfrac{Y(s)}{X(s)}$

För vilket värde på α är nätet stabilt (polen måste vara i v.h.p).



$$I_1 = \frac{X - U_1}{R}; \quad I_2 = \frac{(\alpha-1)U_1}{R + \frac{1}{sC}} \qquad U_1 = (I_1 + I_2)\cdot\frac{1}{sC}$$

$$U_1 = \frac{1 + sRC}{(sRC)^2 + (3-\alpha)sRC + 1}\cdot X \qquad \frac{Y}{X} = \alpha\,\frac{s + \frac{1}{RC}}{s^2 + (3+\alpha)\frac{s}{RC} + \frac{1}{(RC)^2}}$$

$$p_{1,2} = \left(-\frac{3-\alpha}{2} \pm \sqrt{\left(\frac{3-\alpha}{2}\right)^2 - 1}\right)\frac{1}{RC}$$

$p_{1,2}$ i v.h.p om $\alpha < 3$