Name:

Exam in the course Antenna Engineering 2012-05-25

ANTENNA ENGINEERING (SSY100)
(E4) 2011/12 (Period IV)

Friday 25 May 1400-1800 hours.

Teachers: Adjunct Prof Jan Carlsson, Prof Per-Simon Kildal, Associate Prof Jian Yang, Post Doc Esperanza Alfonso

Questions: Jan Carlsson, tel. 0703 665169

The exam consists of 2 parts. Part A is printed on colored paper and must be solved without using the textbook. When you have delivered the colored text and the solutions of Part A (latest 17:00), the textbook can be used for Part B of the exam.

You are allowed to use the following:

For Part A:	**Pocket calculator of your own choice**
For Part B only:	**Mathematical tables including Beta**
	Pocket calculator of your own choice
	Kildal's compendium "Foundations of Antennas: A Unified Approach for LOS and Multipath"

(The textbook can contain own notes and marks on its original printed pages. No other notes are allowed.)

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För del B:	Matematiska Tabeller inkluderad Beta
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Name:

PART A (must be delivered before textbook can be used)

1.0 Incremental sources and BOR1 antennas (25p)

In the problems below we do not require correct constants in the far field formulas.

1.1. When we present far fields of antennas we choose a special convenient coordinate system. Give the name of this coordinate system. Make a sketch of it inside a rectangular xyz coordinate system, showing the three special coordinates and their associated unit vectors. Write the names of each of the coordinates and what kinds of surfaces that are described by each one of them being constant, and the remaining two varying. (3p)

1.2. We have two kinds of fundamental incremental current sources, from which all radiation originates. Write the expressions for the far field functions of each of these two sources, when they are polarized in y-direction for radiation along the z-axis. Explain the shapes of the far field functions in the E- and H-planes. (5p)

1.3. Write the expression for the far field functions of the same two incremental sources when they are polarized in x-direction for radiation along the z-axis. Explain the E- and H-planes. (3p)

1.4. Write the <u>general vector expressions</u> for the far field functions for the two types of sources. (5p)

1.5. There is a third classical incremental source, which appears as a linear combination of the two previously mentioned sources. Write the name of this source, and give the far field in the spherical coordinate system when it is y-polarized. Explain the shapes in E- and H-planes. (3p)

1.6. Explain the mechanical and electromagnetic characteristics of BOR1 antennas, and write the general form of their far field functions. How do we need to excite them in order to ensure BOR1 far fields? (3p)

1.7. Explain the conditions to get zero cross polarization in the 45 deg plane for BOR1 antennas. Can we use this explanation to obtain the cross-polar performance of the three incremental sources considered above? (3p)

Name:

2.0 Aperture theory and fundamental directivity limitations (25p)

2.1. Consider a planar aperture of area A. Write the formula for the maximum directivity of large apertures. We always evaluate this in dB. Write the formula in dBi. (3p)

2.2. The above formula cannot be used for small antennas. What is the maximum available directivity of small antennas? Explain approximately at which diameter we have to switch between the two formulas. (3p)

2.3. Explain the three conditions to achieve maximum directivity of an aperture. Which quantity do we use to measure how close the directivity is to the maximum available one? (3p)

2.4. Explain the differences between radiation patterns of circular and rectangular apertures. Make a sketch. Do you remember the level of the 1st sidelobe for both cases when the apertures are excited for maximum directivity? (3p)

2.5. The far field function of a planar aperture can be described as a product of two factors. Explain. (3p)

2.6. Explain the two incremental sources we normally use for aperture antennas, and when and how we use them. (3p)

2.7. Aperture theory can be applied to many different antenna types. Describe briefly why we can use it to determine far field functions of planar arrays. Hint: There are two alternative sum expressions for far field functions of linear and planar arrays. (4p)

2.8. Consider a very small rectangular slot oriented in x-direction in an infinite ground plane. Write its far field function (correct constants are not required), and explain its directivity compared to an incremental source in free space. (3p)

Name:

PART B (You can use the textbook to solve this problem, but only after PART A has been delivered)

3.0 Diversity antenna (25p)

We have access to a two element antenna that we would like to use as a diversity antenna in order to improve the performance of a radio unit that is used in a rich isotropic multipath environment (RIMP). The CDF curves for the two individual branches and that of the selection combined signal are shown in figure 3.1. The two elements are of different types but we know that when element 2 is characterized as an isolated element (i.e. without the other element present) it is perfectly matched at the considered frequency and lossless.

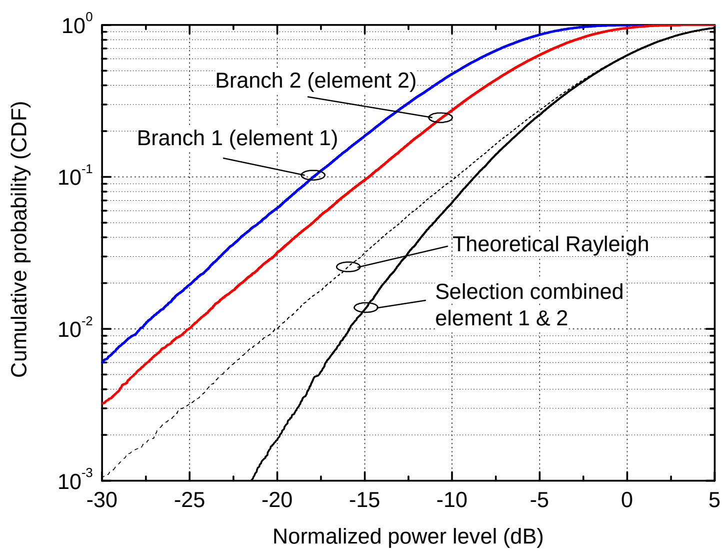

Figure 3.1. CDF:s for the two individual branches and selection combined signal, respectively.

3.1 What are the effective and apparent diversity gains, respectively? Assume selection combining and a CDF level of 1%. (3p)

3.2 What is the absolute value of the complex correlation coefficient between the two branches? (3p)

3.3 Assume that we can measure the two elements separately, i.e. without the other element present. Then, what are the measured normalized power levels for each element at a CDF level of 1%? Assume RIMP environment. (4p)

3.4 What is the radiation efficiency for element 1 when it is measured without the other element present? (4p)

3.5 Instead of using the antenna whose data is shown in Fig. 3.1 we have the option to use another two-port antenna. The only information we have about this new antenna is that it is lossless and that it has the equivalent circuit shown in the figure below. The values of the impedances in the figure are $Z_{11} = Z_{22} = 82.4 + j32.1$; $Z_{12} = Z_{21} = 76.1 - j0.7$ ohm .

What are the effective and apparent diversity gains of this antenna if selection combining and a CDF-level of 1% are assumed? (6p)

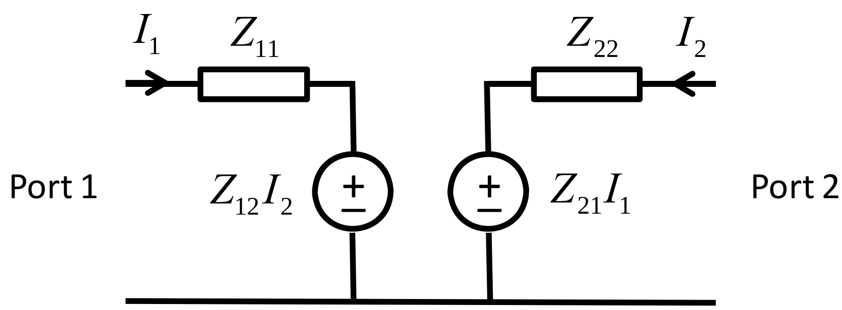

Figure 3.2. Equivalent circuit for diversity antenna.

3.6 When transmitting we excite Port 1 with a generator with 50 ohm internal impedance and Port 2 is either left open or short circuited. Determine how much power is radiated for the two cases. Give the answer in terms of the available power from the generator. (5p)

Name:

4.0 Design of planar dipole array (25p)

You shall now design a planar array operating at 1GHz with required broadside directivity of at least 30 dBi. The aperture of the array is quadratic with equal element spacing and equal number of elements in both *x* and *y* directions.

It should be possible to scan array to ±30º from broadside in both E- & H-planes. The array elements are horizontal dipoles which are half wavelength long and located at a height of quarter wavelength above infinite PEC ground plane, oriented in y-direction.

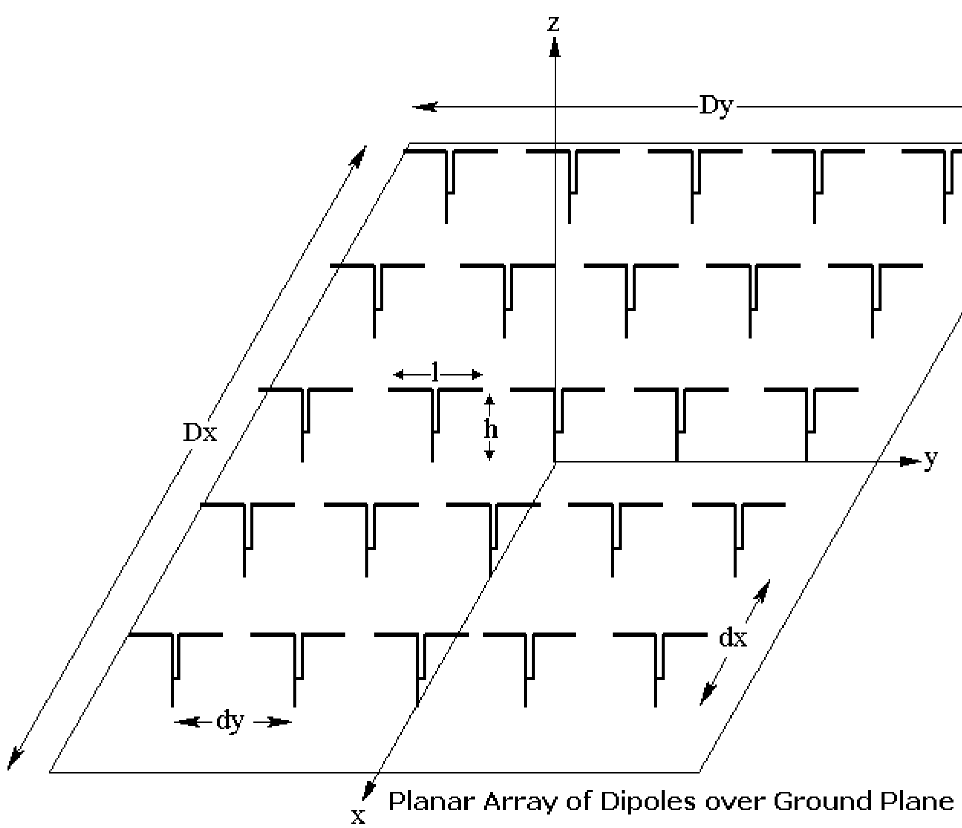

Figure 4.1 Planar array of dipoles over ground plane

4.1 Identify the co-polar unit vector in the center of the main beam of the array, and E- and H-planes. (3p)

4.2 Write down the isolated far field function of a single array element by assuming sinusoidal current distribution on the dipoles. (3p)

4.3 Find dimensions of quadratic aperture, the element spacing and number of elements so as to avoid grating lobes. (3p)

4.4 Assume the uniform amplitude excitation, and linear phase progression $\Delta\Phi_x$ and $\Delta\Phi_y$ in x and y directions, respectively. Write down the expression of array factor using element-by-element sum.(4p)

4.5 Find direction of steered main lobe when $\Delta\Phi_x = -60°$ and $\Delta\Phi_y = -90°$. Determine the directivity in the new main beam direction, and determine if you have grating lobes or not. (4p)

4.6 Locate new dipoles in the empty space between existing dipoles, so that there are almost double as many dipoles as before, and assume that these new dipoles are excited in the same way as the initial dipoles and for the same directions. Make a sketch. What is now a) the directivity of the broadside beam, b) the directivity of the beam that is phase-steered to the same direction as in §4.5. (4p)

4.7 Each dipoles of the array is connected to a transmit/receive module that is digitally controlled to give the correct amplitude and phase. The dipoles were matched for the original element spacing. What happened then with the realized gain when we doubled the number of dipoles. Please explain. (4p)

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Exam in Antenna Engineering May 2012
Solution to problem 1.0 by Per-Simon Kildal

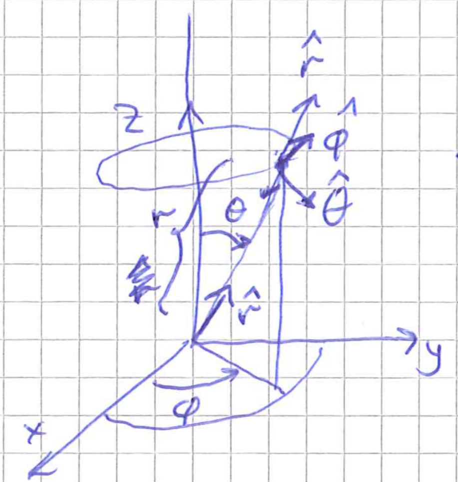
1.0 Incremental sources and BOR_1 antennas (25p)

1.1. (3p)

Name: Spherical* coordinate system.

Coordinates: Polar* angle θ. Constant θ defines a cone*.
Azimuth* angle φ. Constant φ — " — a plane*.
Radius* r. Constant r — " — a sphere*

(Number of points = Integer (N_{stars} / 3)
Errors give "negative" star)



$\hat{r} \perp \hat{\theta} \perp \hat{\varphi}$

* *

1.2. (5p)

Incremental electric current: $\vec{J}_y = I_y \hat{y}$

Far field: $\vec{G}_e(\theta,\varphi) = C\, I_y \big(\underbrace{\cos\theta \sin\varphi\, \hat{\theta}}_{\text{E-plane}} + \cos\varphi\, \hat{\varphi}\big)$ **

Incremental magnetic current: $\vec{M} = E_y \hat{x}$

Far field: $\vec{G}_m(\theta,\varphi) = C\, E_y \big(\sin\varphi\, \hat{\theta} + \underbrace{\cos\theta \cos\varphi\, \hat{\varphi}}_{\text{H-plane}}\big)$ **

* { Far field of electric current is $\cos\theta$-shaped in E-plane and uniform in H-plane.

Far field of magnetic current is $\cos\theta$-shaped in H-plane and uniform in E-plane. }

(Number of points = number of stars)

1.3 (3p)

Electric, polarized in x-dir : $\vec{G}_e(\theta,\varphi) = C I_x (\underbrace{\cos\theta\cos\varphi\hat{\theta}}_{E\text{-plane}} - \sin\varphi\hat{\varphi})$ *

Magnetic, ——— " ——— : $\vec{G}_m(\theta,\varphi) = C E_x (\cos\varphi\hat{\theta} - \cancel{\sin\varphi}\ \underbrace{\cos\theta\sin\varphi\hat{\varphi}}_{H\text{-plane}})$ *

(Number of points = number of stars)
(Correctly stated E- and H-plane = 1 star)

1.4 (5p)

Electric current $\vec{J}$. Far field $\vec{G}_J(\hat{r}) \propto \vec{J} - (\vec{J}\cdot\hat{r})\hat{r}$

Magnetic current $\vec{M}$. Far field $\vec{G}_M(\hat{r}) \propto \hat{r} \times \vec{M}$ or $\vec{M} \times \hat{r}$

(2p per correct answer. 1 p extra if correct constants $C_k = -jk/4\pi$)

1.5 (3p)

Huygen's source

$$\vec{G}_H(\theta,\varphi) = \underbrace{(1+\cos\theta)}_{=2\cos^2(\theta/2)}(\sin\varphi\hat{\theta} + \cos\varphi\hat{\varphi})$$

Equal E- and H-planes

1.6 (3p)

* Mechanical : Rotationaly symmetric structures around z-axis.
  Electromagnetic : Contains only basic $\cos\varphi$ and $\sin\varphi$ variations of fields around z-axis.

* Excitation : By BOR_1 waveguide modes, i.e. such as TE_{11} mode in circular waveguide,
  or by incremental sources of BOR_1 type on axis. (transverse incremental electric and magnetic dipoles, & Huygens's sources.

General form : $\vec{G}(\theta,\varphi) = G_E(\theta)\sin\varphi\hat{\theta} + G_H(\theta)\cos\varphi\hat{\varphi}$

$$* \begin{cases} G_E(\theta) = \text{E-plane function} \\ G_H(\theta) = \text{H-plane function} \end{cases}$$

(Number of points = number of stars)

1.7 (3p)

~~Zero~~ Co-, and Crosspolar function in 45° plane : ~~$G_{xp45}(\theta) = G_E$~~

$$G_{xp}(\theta) = \frac{1}{2}\left(G_E(\theta) - G_H(\theta)\right) \quad *$$

$$G_{co}(\theta) = \frac{1}{2}\left(G_E(\theta) + G_H(\theta)\right)$$

Zero crosspol $\Rightarrow G_{xp}(\theta) = 0 \Rightarrow G_E(\theta) = G_H(\theta)$ *

Thus, E- and H-plane functions must be equal in both amplitude and phase. *

2.0 Aperture theory and fundamental directivity limitations (25p)

2.1. Consider a planar aperture of area A. Write the formula for the maximum directivity of large apertures. We always evaluate this in dB. Write the formula in dBi. (3p)

2.2. The above formula cannot be used for small antennas. What is the maximum available directivity of small antennas? Explain approximately at which diameter we have to switch between the two formulas. (3p)

2.3. Explain the three conditions to achieve maximum directivity of an aperture. Which quantity do we use to measure how close the directivity is to the maximum available one? (3p)

2.4. Explain the differences between radiation patterns of circular and rectangular apertures. Make a sketch. Do you remember the level of the 1st sidelobe for both cases when the apertures are excited for maximum directivity? (3p)

2.5. The far field function of a planar aperture can be described as a product of two factors. Explain. (3p)

2.6. Explain the two incremental sources we normally use for aperture antennas, and when and how we use them. (3p)

2.7. Aperture theory can be applied to many different antenna types. Describe briefly why we can use it to determine far field functions of planar arrays. Hint: There are two alternative sum expressions for far field functions of linear and planar arrays. (4p)

2.8. Consider a very small rectangular slot oriented in x-direction in an infinite ground plane. Write its far field function (correct constants are not required), and explain its directivity compared to an incremental source in free space. (3p)

2.0 Aperture theory and fundamental directivity limitations

2.1 (3p)

$$D_{max} = \frac{4\pi}{\lambda^2} A \quad *$$

$$(D_{max})_{dBi} = \underset{*}{10} \log \underbrace{\left(\frac{4\pi}{\lambda^2} A \right)}_{*}$$

2.2 (3p)

The max directivity is achieved by Huygen's source, * for which

$$D_{max} = 3^{*}, \quad \text{i.e.} \quad 4.8\,dBi^{*}$$

2.3 (3p)

Conditions for max directivity of planar aperture

i) Uniform amplitude over aperture *
ii) Constant phase —"— *
ii) Constant polarization —"— *

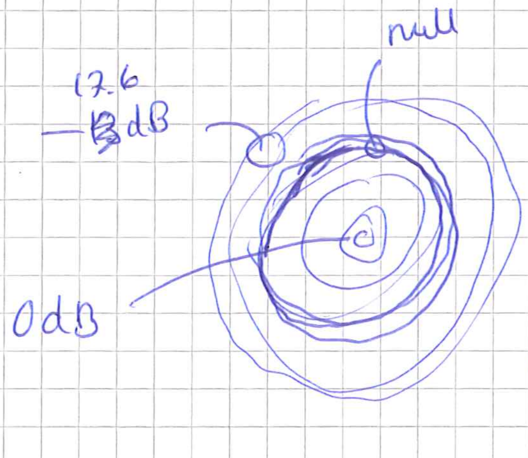
$\}$ of aperture distribution or "excitation" of an array.

2.4 (3p)

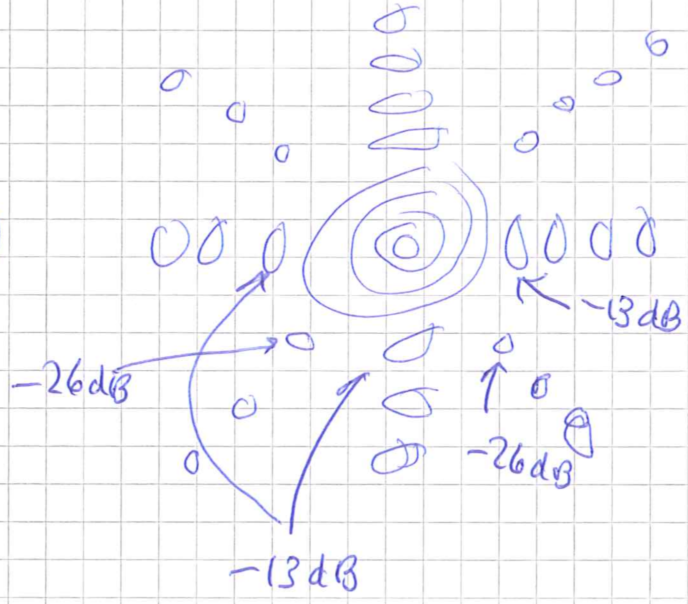
Radiation patterns of circular apertures have circular sidelobes around the main beam, but they can have different levels in E- and H-planes. First sidelobe level for max directivity −17.6 dB

Radiation patterns of rectangular apertures have sidelobes in the two principle planes only. The sidelobes in the diagonal planes are a sum of the relative dB-levels of the sidelobes in the two principle planes. First sidelobe level for max directivity −13 dB.

2.4 continues. Illustration of radiation pattern for uniform aperture distribution:



circular aperture



rectangular aperture

2.5. (3p)

a) Incremental source factor. (~~Magnetic current~~ *

b) Spatial Fourier transform of aperture distribution. *
$k_x = k\sin\theta\cos\phi$ $k_y = k\sin\theta\sin\phi$. *

c) This is magnetic current for PEC apertures, and Huygen's source for free space apertures (reflectors, horns, lenses) *

2.6. (3p)

*** * *

* The Huygen's source gives useable results both in front of and behind the aperture, but it is more correct near main beam.

The magnetic current can be imaged in PEC if the latter is planar and infinite. Then, the solution is valid for $z > 0$ only.

27. (4p)

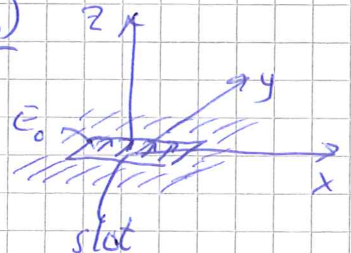
Planar arrays generate a discrete but planar aperture field. The aperture field distribution corresponds to the excitation function of the elements.

The far field function can be described ~~as a product of the aperture exci~~ in two ways:

a) as an element-by-element summation of the far field functions of all elements. This results in array factor times element factor.

b) as a grating-lobe sum expression. Here, each of the grating-lobe far field terms is appearing from a smooth aperture distribution equal to the continuous excitation function, in the direction of the grating lobe, and with the element far field function replacing the incremental source factor of the aperture.

28 (3p)



$$\vec{M} = E_0 \hat{y} \times \hat{z} = E_0 \hat{x}$$

Far field: $\vec{G}(\theta,\varphi) \propto 2E_0 \left(\sin\varphi\, \hat{\theta} + \cos\theta \cos\varphi\, \hat{\varphi} \right)$

due to imaging

H-plane

valid for $\theta < 90°$.

$\vec{G}(\theta,\varphi) = 0$ for $\theta > 90°$ (shadow region)

The directivity is 3dB (factor 2) larger than incremental magnetic dipole in free space, because the slot only radiation in half space; thus, total 4.8 dBi.

Name:

PART B (You can use the textbook to solve this problem, but only after PART A has been delivered)

3.0 Diversity antenna (25p)

We have access to a two element antenna that we would like to use as a diversity antenna in order to improve the performance of a radio unit that is used in a rich isotropic multipath environment (RIMP). The CDF curves for the two individual branches and that of the selection combined signal are shown in figure 3.1. The two elements are of different types but we know that when element 2 is characterized as an isolated element (i.e. without the other element present) it is perfectly matched at the considered frequency and lossless.

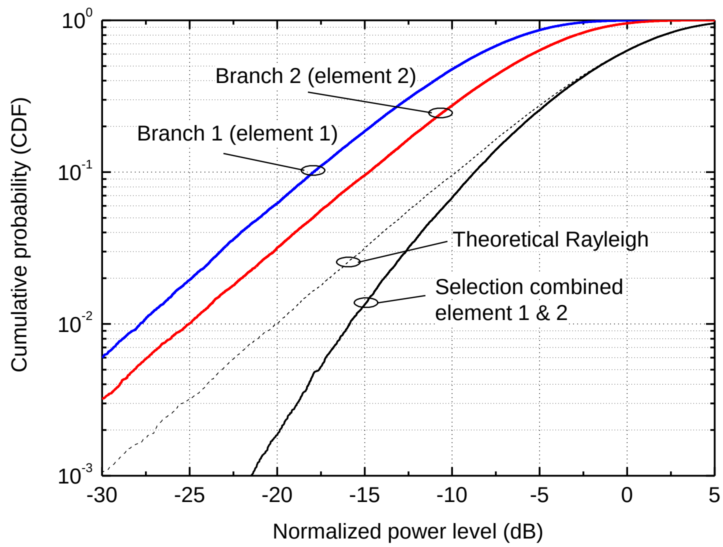


Figure 3.1. CDF:s for the two individual branches and selection combined signal, respectively.

3.1 What are the effective and apparent diversity gains, respectively? Assume selection combining and a CDF level of 1%. (3p)

3.2 What is the absolute value of the complex correlation coefficient between the two branches? (3p)

3.3 Assume that we can measure the two elements separately, i.e. without the other element present. Then, what are the measured normalized power levels for each element at a CDF level of 1%? Assume RIMP environment. (4p)

3.4 What is the radiation efficiency for element 1 when it is measured without the other element present? (4p)

3.5 Instead of using the antenna whose data is shown in Fig. 3.1 we have the option to use another two-port antenna. The only information we have about this new antenna is that it is lossless and that it has the equivalent circuit shown in the figure below. The values of the impedances in the figure are $Z_{11} = Z_{22} = 82.4 + j32.1$; $Z_{12} = Z_{21} = 76.1 - j0.7$ ohm .

What are the effective and apparent diversity gains of this antenna if selection combining and a CDF-level of 1% are assumed? (6p)

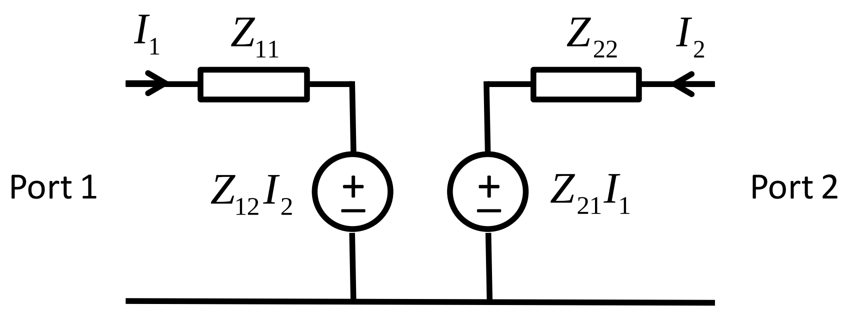


Figure 3.2. Equivalent circuit for diversity antenna.

3.6 When transmitting we excite Port 1 with a generator with 50 ohm internal impedance and Port 2 is either left open or short circuited. Determine how much power is radiated for the two cases. Give the answer in terms of the available power from the generator. (5p)

Diversity antenna (20p)

We have access to two dipole antennas that we would like to use as a diversity antenna in order to improve the performance of a radio unit that is used in an isotropic multipath environment. The CDF curves for the two antennas measured separately (i.e. without the other antenna present) as well as the CDF when the two antennas are placed close to each other, as they would be when used on the radio, and using selection combining are shown in the figure below. We also know that antenna 2 has an efficiency of 0 dB.

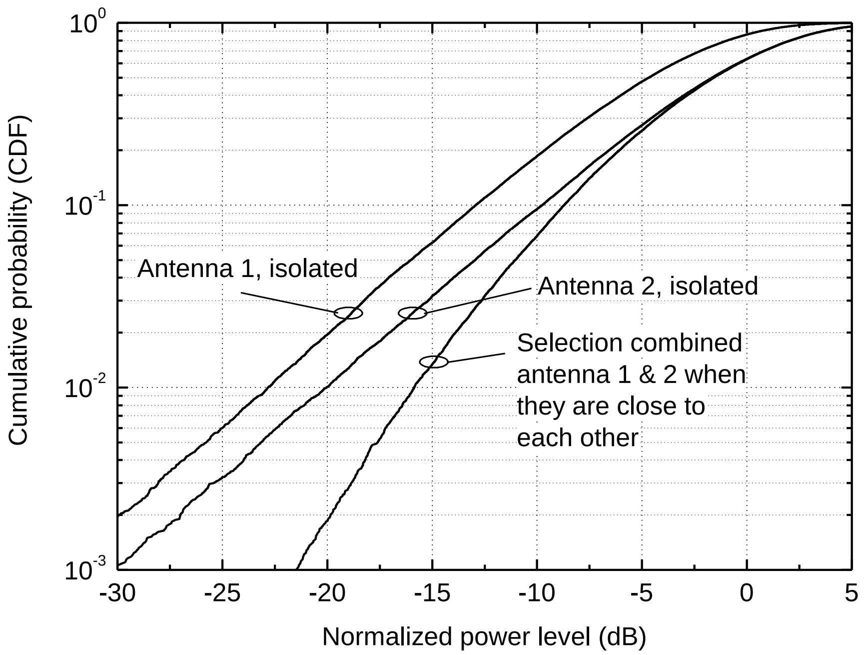


Figure 1. CDF:s for the two dipole antennas measured separately without the other antenna present and when placed close to each other when selection combination is used, respectively.

a. Why are the CDF curves for antenna 1 and 2 different and what does this difference represents?

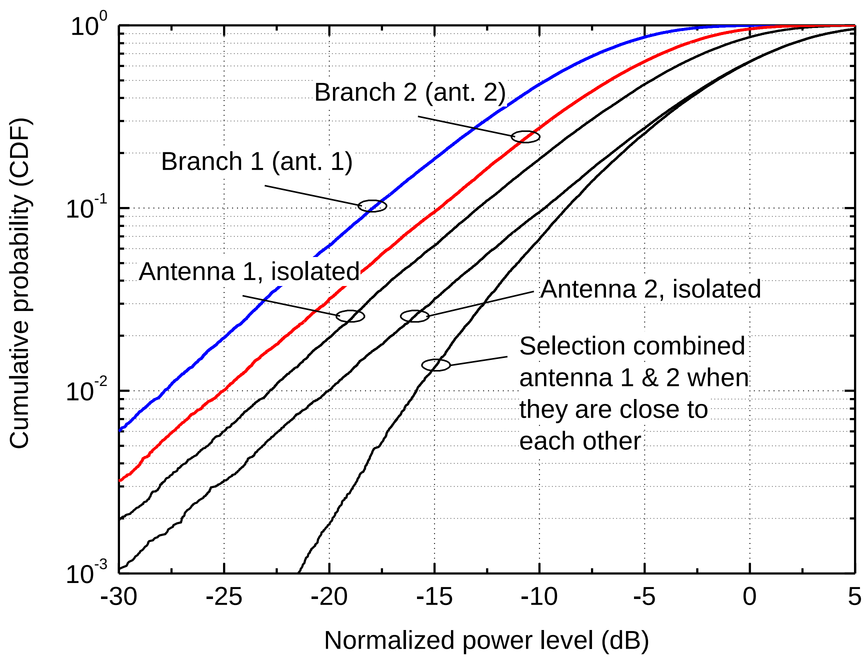
Solution: (2p)
The reason for the different CDF curves is that the two antennas have different efficiencies. The horizontal distance between the two curves represents the difference in efficiency, from the figure it can be seen to be 3 dB.

b. In the figure, sketch the CDF curves for the two branches when the antennas are placed close to each other.

Solution: (4p)

See figure.

- The curves for the two branches should be shifted to the left (2p)
- The two curves should be shifted equally much (2p)



c. What is the gain of using the two antennas and selection combining if we compare with using only antenna 1? Assume a CDF level of 1%.

Solution: (2p)

From the figure the diversity gain can be seen to be approx. 7 dB

d. What is the gain of using the two antennas and selection combining if we compare with using an <u>ideal reference antenna</u>? Assume a CDF level of 1%.

Solution: (3p)

- From the figure the diversity gain can be seen to be approx. 4 dB (2p)
- It should be realized that antenna 2 represents an ideal reference since it has 0 dB efficiency (1p)

e. If the efficiency for each branch is assumed to decrease 5 dB when the antennas are closely spaced compared to when they are far away from each other, what is the absolute value of the complex correlation coefficient between the two branches?

Solution: (3p)

The apparent diversity gain is given by equation 3.12 in the book, i.e. $G_{app} = 10\sqrt{1-|\rho|^2}$ (note that it is Ok to use other, better, formulas for the apparent diversity gain). Since the efficiency has decreased 5 dB due to that the antennas are closely spaced as compared to when they are far apart the apparent diversity gain is 5+4=9 dB (from d). Thus, we have

$$G_{app} = 10^{9/10} = 10\sqrt{1-|\rho|^2} \Rightarrow |\rho| = 0.61$$

f. Now, we have got an offer from an antenna vendor to buy a diversity antenna instead of using the two dipoles. The only information we have is that the antenna is lossless and that it has the equivalent circuit shown in the figure below. The values of the impedances in the figure are; $Z_{11} = Z_{22} = 82.4 + j32.1$; $Z_{12} = Z_{21} = 76.1 - j0.7$ ohm.

What are the effective and apparent diversity gains of this antenna if selection combining and a CDF-level of 1% are assumed?

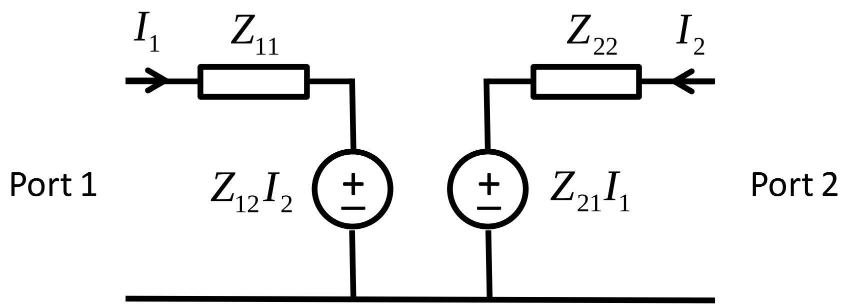
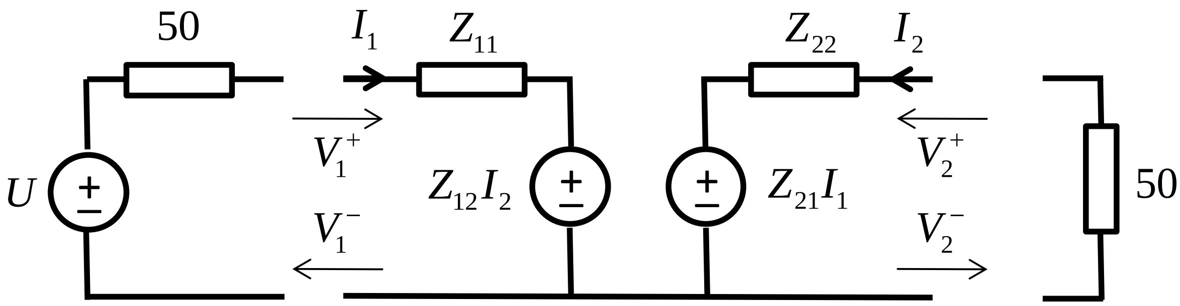


Figure 2. Equivalent circuit for diversity antenna.

Solution: (6p)

The apparent diversity gain can be obtained from the knowledge of the correlation coefficient which in turn, since the antenna is lossless, can be calculated from the S-parameters as given by equation 3.10 in the book (note that there is an error in the book, it should be a square root in the denominator, it is however Ok if the expression in the book is used). Thus, the first thing we have to do is to determine the S-parameters. For the given problem $Z_{11} = Z_{22}$ and $Z_{12} = Z_{21}$ so that $S_{11} = S_{22}$ and $S_{12} = S_{21}$.

S_{11} and S_{21} can be determined from the following circuit;



Since port 2 is terminated in 50 ohm $V_2^+ = 0$ and we have the following relations;

$$S_{11} = \frac{V_1^-}{V_1^+} \;,\; S_{21} = \frac{V_2^-}{V_1^+} \;,\; V_2^- = V_2$$

We also know that $V_1^+ = \dfrac{U}{2} \;\Rightarrow\; V_1^- = V_1 - \dfrac{U}{2}$

We can now express the S-parameters in the total voltages as;

$$S_{11} = S_{22} = \frac{2V_1}{U} - 1 \;,\; S_{12} = S_{21} = \frac{2V_2}{U}$$

The currents in the left and right loops can be determined as;

$$I_1 = \frac{U - Z_{12}I_2}{Z_{11} + 50} \;,\; I_2 = \frac{-Z_{21}I_1}{Z_{22} + 50}$$

Solving for the currents gives;

$$I_1 = U\frac{Z_{22} + 50}{(Z_{11} + 50)(Z_{22} + 50) - Z_{12}Z_{21}} \;,\; I_2 = -U\frac{Z_{21}}{(Z_{11} + 50)(Z_{22} + 50) - Z_{12}Z_{21}}$$

We can now calculate the port voltages as;

$$V_1 = Z_{11}I_1 + Z_{12}I_2 = U\frac{Z_{11}(Z_{22} + 50) - Z_{12}Z_{21}}{(Z_{11} + 50)(Z_{22} + 50) - Z_{12}Z_{21}}$$

$$V_2 = -50I_2 = U\frac{50Z_{21}}{(Z_{11} + 50)(Z_{22} + 50) - Z_{12}Z_{21}}$$

The S-parameters are now given by;

$$S_{11} = S_{22} = 2\frac{Z_{11}(Z_{22} + 50) - Z_{12}Z_{21}}{(Z_{11} + 50)(Z_{22} + 50) - Z_{12}Z_{21}} - 1 = 0.10 + j0.42$$

$$S_{12} = S_{21} = \frac{100Z_{21}}{(Z_{11} + 50)(Z_{22} + 50) - Z_{12}Z_{21}} = 0.43 - j0.35$$

Now, when we have the S-parameters we can compute the absolute value of the complex correlation coefficient by using equation 3.10 in the book (or rather the correct version of the equation).

$$|\rho| = \frac{\left|S_{11}^* S_{12} + S_{21}^* S_{22}\right|}{\sqrt{\left(1-\left(|S_{11}|^2 + |S_{21}|^2\right)\right)\left(1-\left(|S_{22}|^2 + |S_{12}|^2\right)\right)}} = 0.41$$

The apparent diversity gain is given by equation 3.12 in the book;

$$G_{app} = 10\sqrt{1-|\rho|^2} = 9.12 = 9.60 \text{ dB}$$

The effective diversity gain is defined as the apparent diversity gain multiplied with the radiation efficiency, equation 3.11. We have by using equation 3.7;

$$G_{eff} = e_{rad} G_{app} = \left(1-|S_{11}|^2 - |S_{21}|^2\right) 10\sqrt{1-|\rho|^2} = 4.61 = 6.64 \text{ dB}$$

Name:

4.0 Design of planar dipole array (25p)

You shall now design a planar array operating at 1GHz with required broadside directivity of at least 30 dBi. The aperture of the array is quadratic with equal element spacing and equal number of elements in both *x* and *y* directions.

It should be possible to scan array to ±30º from broadside in both E- & H-planes. The array elements are horizontal dipoles which are half wavelength long and located at a height of quarter wavelength above infinite PEC ground plane, oriented in y-direction.

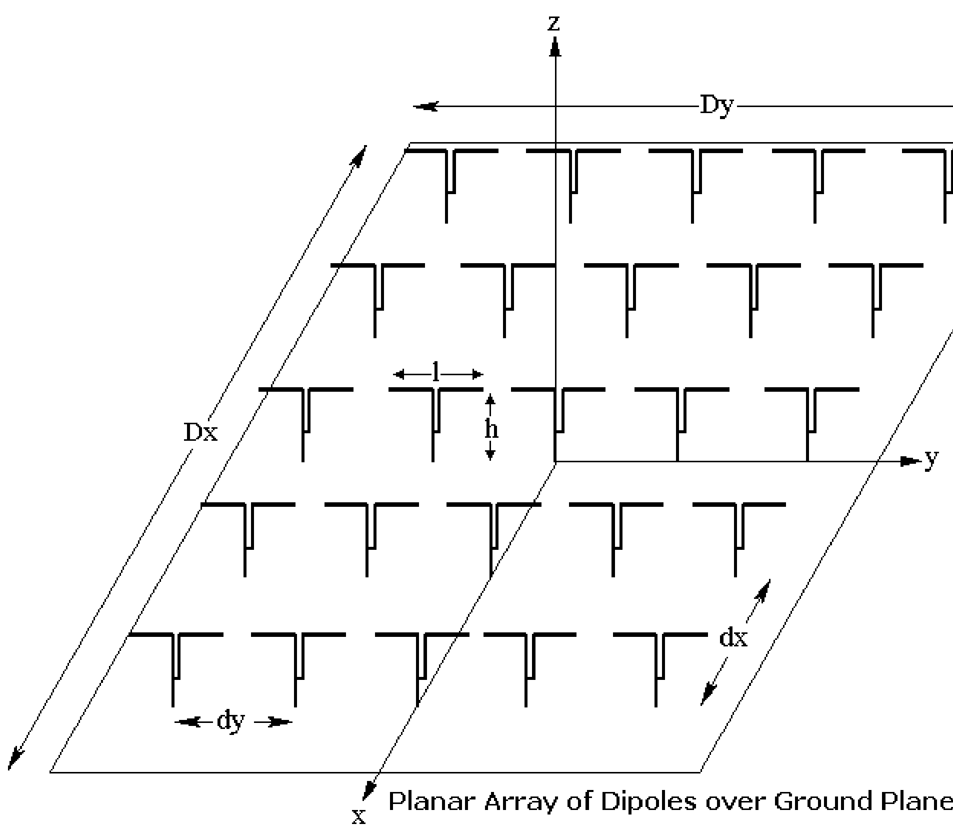


Figure 4.1 Planar array of dipoles over ground plane

4.1 Identify the co-polar unit vector in the center of the main beam of the array, and E- and H-planes. (3p)

A: Copolar Unit Vector:- $\hat{y}$

E-plane: φ=90°

H-plane: φ=0°

4.2 Write down the isolated far field function of a single array element by assuming sinusoidal current distribution on the dipoles. (3p)

A: The dipole is y-polarized and λ/2 long, located at height of λ/4 above ground plane. The far field function of such dipole is a product of incremental source factor, Fourier transform of current distribution on dipole and ground plane factor for dipole located horizontally on PEC ground plane.

Using equation 4.70, the y polarized incremental source is written as,

$$G_{id}(\theta,\varphi) = C_k \eta I_o l(\cos\theta \sin\varphi\hat{\theta} + \cos\varphi\hat{\varphi})$$

Using equation (5.11), Fourier transform of sinusoidal current distribution is given as,

$$\tilde{j}(k\hat{l} \bullet \hat{r}) = \frac{2}{k}\frac{\left[\cos(kl\hat{l} \bullet \hat{r}/2) - \cos(kl/2)\right]}{\left[1-(\hat{l} \bullet \hat{r})^2\right]\sin(kl/2)}$$

$$\hat{l} \bullet \hat{r} = \sin\theta\sin\varphi$$

$$kl/2 = \frac{2\pi}{\lambda}\frac{\lambda}{2}\frac{1}{2} = \frac{\pi}{2}$$

$$\therefore \tilde{j}(k\hat{l} \bullet \hat{r}) = \frac{2}{k}\frac{\cos(\frac{\pi}{2}\sin\theta\sin\varphi)}{\left[1-(\sin\theta\sin\varphi)^2\right]}$$

Using equation (5.64), the ground plane contribution due to imaging is given as,

$$Ground(\theta) = 2j\sin(kh\cos\theta) = 2j\sin(\frac{\pi}{4}\cos\theta)$$

Thus, the complete far field function of the array element is written as,

$$G_{hd}(\theta,\varphi) = C_k \eta I_o l \frac{4}{k} j \cdot \frac{\left[\cos(\frac{\pi}{2}\sin\theta\sin\varphi)\right]}{\left[1-(\sin\theta\sin\varphi)^2\right]} \cdot \sin(\frac{\pi}{4}\cos\theta) \cdot (\cos\theta\sin\varphi\hat{\theta} + \cos\varphi\hat{\varphi})$$

4.3 Find dimensions of quadratic aperture, the element spacing and number of elements so as to avoid grating lobes. (3p)

A: The array is quadratic having equal dimensions in x and y directions. So $D_x=D_y=L$. The required directivity is at least 30dBi along broadside.

$$\therefore D_o = \frac{4\pi}{\lambda^2}A = \frac{4\pi}{\lambda^2}L^2$$

$$\therefore L = \sqrt{\frac{D_o\lambda^2}{4\pi}} = \sqrt{\frac{10^{30/10}\lambda^2}{4\pi}} = 8.92\lambda$$

Element spacing to avoid grating lobes is given as,

$$d_x = d_y \le \frac{\lambda}{1+\left|\sin\theta_0\right|+\frac{\lambda}{L}}, \text{ (sufficient condition)}$$

$$\therefore d_x \le \frac{\lambda}{1+\left|\sin(30)\right|+\frac{\lambda}{8.92\lambda}}$$

$$\therefore d_x = 0.62\lambda$$

And for this array the element spacing is equal in x and y directions.

$$d_x = d_y = d_a = 0.62\lambda$$

$$N_x = N_y = \frac{L}{d_a} = 14.38$$

$$\therefore N = 15\cdot 15 = 225$$

4.4 Assume the uniform amplitude excitation, and linear phase progression $\Delta\Phi_x$ and $\Delta\Phi_y$ in x and y directions, respectively. Write down the expression of array factor using element-by-element sum.(4p)

A: Using equation (10.66) & (10.67), and assuming the phase of the center element is zero, the phase progression in x-y direction can be written as,

$$\Phi(x,y) = \frac{\Delta\Phi_x}{d_x}\cdot x + \frac{\Delta\Phi_y}{d_y}\cdot y$$

$$\therefore \Phi(x,y) = \frac{1}{d_a}(\Delta\Phi_x\cdot x + \Delta\Phi_y\cdot y)$$

Now, using equation (10.62), element location can be written as with r_c=0,

$$r_{mn} = (m-\frac{15+1}{2})\cdot d_a\hat{x} + (n-\frac{15+1}{2})\cdot d_a\hat{y}$$

$$\therefore r_{mn} = (m-8)\cdot d_a\hat{x} + (n-8)\cdot d_a\hat{y}$$

$$\therefore r_{mn}\cdot\hat{r} = da\sin\theta\left[(m-8)\cos\varphi + (n-8)\sin\varphi\right]$$

$$\therefore \Phi_{mn} = \frac{1}{d_a}\cdot\left(\Delta\Phi_x\cdot(m-8)\cdot d_a + \Delta\Phi_y\cdot(n-8)\cdot d_a\right)$$

$$\therefore \Phi_{mn} = \left(\Delta\Phi_x\cdot(m-8) + \Delta\Phi_y\cdot(n-8)\right)$$

The complete array factor using element by element sum is written as,

$$AF(\hat{r}) = \sum_{n=1}^{15}\sum_{m=1}^{15} A_{mn}e^{j\Phi_{mn}}e^{jkr_{mn}\cdot\hat{r}}$$

With A_{mn}=1 for uniform amplitude excitation,

$$AF(\theta,\varphi) = \sum_{n=1}^{15}\sum_{m=1}^{15} 1\cdot e^{j\left(\Delta\Phi_x\cdot(m-8)+\Delta\Phi_y\cdot(n-8)\right)}e^{jkd\sin\theta\left[(m-8)\cos\varphi+(n-8)\sin\varphi\right]}$$

where $d_a = 0.62\lambda$

Name:

4.5 Find direction of steered main lobe when $\Delta\Phi_x = -60°$ and $\Delta\Phi_y = -90°$. Determine the directivity in the new main beam direction, and determine if you have grating lobes or not. (4p)

A: Using equation (10.75),

$$\tan\varphi_o = \frac{k_{\Phi y}}{k_{\Phi x}} = \frac{\dfrac{90}{d_y}}{\dfrac{60}{d_x}} = \frac{3}{2}$$

$$\therefore \varphi_o = 56.31^o$$

$$\sin\theta_o = \frac{\sqrt{\left(\dfrac{\frac{\pi}{2}}{0.62\lambda}\right)^2 + \left(\dfrac{\frac{\pi}{3}}{0.62\lambda}\right)^2}}{k}$$

$$\therefore \theta_o = \sin^{-1}\sqrt{\frac{\left(\frac{\pi}{3}\right)^2 + \left(\frac{\pi}{2}\right)^2}{0.62^2 * 4\pi^2}} = 29^o$$

No grating lobes.

4.6 Locate new dipoles in the empty space between existing dipoles, so that there are almost double as many dipoles as before, and assume that these new dipoles are excited in the same way as the initial dipoles and for the same directions. Make a sketch. What is now a) the directivity of the broadside beam, b) the directivity of the beam that is phase-steered to the same direction as in §4.5. (4p)

A: a) $D_0 = \dfrac{4\pi}{\lambda^2} A = \dfrac{4\pi}{\lambda^2}(15 \cdot 0.62\lambda)^2 = 30.4\text{dBi}$

b) $D_0 = \dfrac{4\pi}{\lambda^2} A\cos\theta_0 = \dfrac{4\pi}{\lambda^2}(15 \cdot 0.62\lambda)^2 \cos\theta_0 = 29.8\text{dBi}$

4.7 Each dipoles of the array is connected to a transmit/receive module that is digitally controlled to give the correct amplitude and phase. The dipoles were matched for the original element spacing. What happened then with the realized gain when we doubled the number of dipoles. Please explain. (4p)

A: The realized gain for the denser array will be decreased, since the spacing is just half of the original, the mutual couplings among the elements are stronger, which leads to stronger mismatch for the scanned port impedance. More transmitted power reflected by the antenna. So the realized gain is decreased. We had a deep discussion in the lecture for this question.