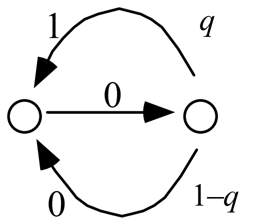
Written exam: Information theory for complex system
Time: Thursday 9 March 2000, 9.00-15.00, room FL63.
Allowed material: anything except other person.
Examiner/teacher: Kristian Lindgren (ext. 3131).

1. Suppose that the initial state ($t = 0$) for the elementary CA rule 18 is described by the finite automaton:

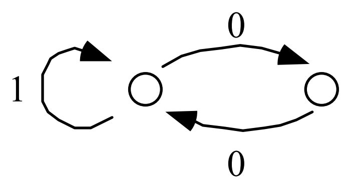


First, suppose that $q = 1/2$. What does the finite automaton look like that describes the state at time $t = 1$? How has the entropy changed between these time steps? What is the entropy at $t = 2$?

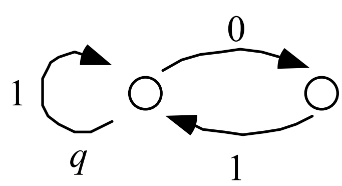
If $q > 1/2$, what is then the entropy s at $t = 2$?

(12 p)

2. Could the automaton below possibly represent the equilibrium state in a one-dimensional spin system, in which the average internal energy u fulfils the constraint $u = \sum_{i_1 \ldots i_m} p(i_1 \ldots i_m)\, h(i_1 \ldots i_m)$?



Suppose instead that the automaton that represents the equilibrium state is as follows:



What spin system (or other one-dimensional system with energy constraints) may this automaton represent? (Must be shown.) How does the probability q connect to temperature β and/or interaction energy constant J ?

(12 p)

3. Consider a binary periodic sequence ...01010101... . If noise is added so that symbols in this sequence are flipped (0 —> 1 and 1 —> 0) with a certain probability $q < ½$, how does the correlation complexity change?

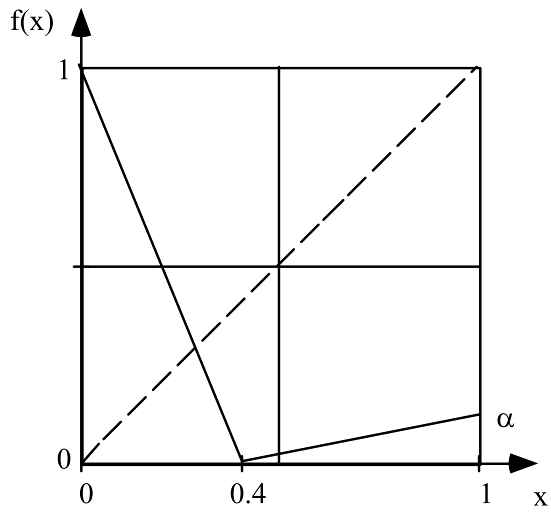
(7 p)

4. Suppose that the concentration of a chemical compound in a one-dimensional system can be described as a concentration peak of a Gaussian form, with an initial width (standard deviation) σ_0. Suppose further that the width σ_0 is much smaller than the length of the system and discuss how information is flowing in (x, r)-space when diffusion is acting on the concentration distribution.

   Start, for example, by showing that diffusion (without chemical reactions) does not destroy the Gaussian form of the distribution, but that the width grows like $\sqrt{(t + b)}$. Then derive an expression for $j_r(r; x, t)$.

(7 p)

5. Let a mapping $f(x)$ be defined by the figure below, where $f(1) = \alpha$ and $f(0.4) = 0$.



   Consider the dynamical system

$$x_{t+1} = f(x_t) .$$

   Describe the behaviour when α is small? At what value on α becomes the system chaotic?

   Suppose that $\alpha = 2/7$. Calculate the Lyapunov exponent λ. What does a generating partition of the interval [0, 1] look like, and what is the measure entropy s_μ ?

   What is the Lyapunov exponent if $\alpha = 2/5$?

(12 p)