Exam in Remote Sensing (RRY 055)

Place & time:	Hörsalsvägen, 2009-03-14.
Teacher contact:	Samuel Brohede (0704-835870) will visit around 9.30 and 11.30.
Language:	Answers can be given in either Swedish or English.
Extent:	Exam has 5 questions and 6 pages.
Points:	The exam carries 50 points, where 20 points are needed to be approved.
Answers:	Begin each question on a new sheet of paper.
Allowed aid:	Calculators, basic math and physics formulae collections (such as Physics Handbook) and enclosed sheet of formulae.

Important: Write short and concise answers. A long answer with a lot of irrelevant information will not help you, rather the opposite as it shows that you are uncertain about the answer. Answers shall be ordered following the question numbering. Begin each question on a new sheet of paper.

1. Miscellaneous (12 p)

a Describe the most commonly used orbit type for earth observation satellites. Why is this orbit type used? (3 p)

b Give two examples, inside environmental science, when remote sensing is a much better choice than in-situ observations. A short motivation shall be included. (2 p)

c Why is the sky blue and the Sun red at twilight and why is the skylight partly polarized at an angle 90^o from the Sun? (2 p)

d Give two reasons to why it is most favorable for the human eye to observe the world at wavelengths around 550 nm. (1 p)

e Draw a figure that shows scattering from a Lambertian surface. (1 p)

f What does DIFFERENTIAL spectroscopy mean (as in DOAS) and give two advantages compared to using absolute intensities. (1 p)

g Measurements of an atmospheric gas layer is performed using a hot solid black-body as the background radiation source. Which spectrum in figure **??** corresponds best to be observed? If you instead remove the background source and observe the gas molecules in emission, which spectrum would then best correspond to the observed? (Give motivations to both answers) (2 p)

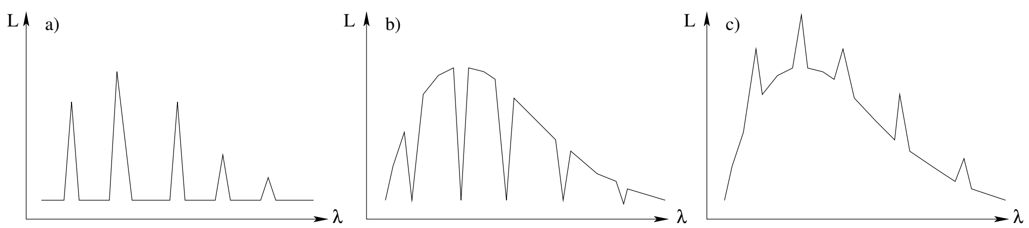

Figure 1: Observed spectra

2. The radiative transfer equation (10 p)

The most general form of the radiative transfer equation can be written as

$$\frac{\mathrm{d}I}{\mathrm{d}l} = -\gamma I + S.$$

a Describe the meaning of the variables of this equation, including stating possible units, and (when applicable) give what physical processes that must be considered? (8 p)

b Write down an expression for the special case of observing the brightness temperature of sea surface emission transmitted through a homogeneous cloud layer using a downwardlooking instrument (situated above the cloud top). In your expression, use the temperature of the sea surface, T_{sea}, and the temperature and optical depth of the cloud; T_{cloud} and τ_{cloud}. The influence from the air and the cosmic background radiation can be ignored. (2 p)

3. Aerial photography (8 p)

An old-fashion camera is used for remote sensing of trees from a balloon at 1 km altitude. The photographic film has the characteristic curve in Fig **??**. The optics has the focal length of 100 mm with f/number 1.2.

a Study figure Fig **??**. How large fraction of light (in percent) will be able to penetrate through the film in fully exposed regions? Hint: the optical density, D, corresponds to the optical depth, τ, when using Beer-Lambert law in log (base 10) rather than ln (base e). (2 p)

b Describe and draw how the characteristic curve would change if you instead use a film with higher speed and higher contrast. (1 p)

c Given that the median grain size of the film is 1 μm, calculate the rezel size (the size of the grain projected on the ground) and determine if the system is under- or over sampled. Hint: use a typical wavelength for visible light. (2 p)

d The relief displacement of a single tree on the film is measured to be 1 mm and you know that the tree is situated 300 m from the principal point. Estimate the height of this tree and write something about the uncertainty of the estimate. (2 p)

e Estimating the height of trees in a forest from relief displacements is non-trivial and cumbersome since it is required that both the top and bottom of the trees are visible in the photo. Suggest changes in the observation system to overcome this problem so that the height of the canopies can be more efficiently monitored. (1 p)

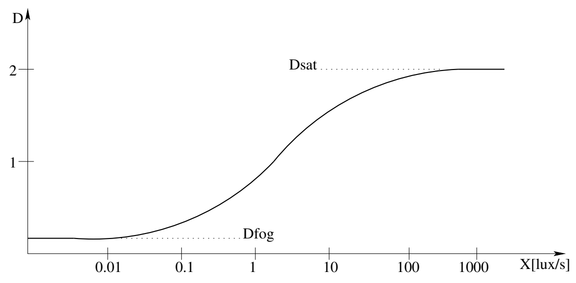

Figure 2: Characteristic curve

4. IR-spectroscopy (10 p)

In Mexico city there are 5 million cars releasing CO. A solar spectrum is shown in Fig **??**, measured with an FTIR in Mexico city on April 2003 at a spectral resolution of 0.5 cm^{-1}, when the sun was 80^o above the horizon. The spectrum corresponds to the relative light intensity on the y-axis and wavenumber (cm^{-1}) on the x-axis. Part of the structures correspond to CO.

a What is the principle of an FTIR (describe hardware and formulas)? (3 p)

b Which type of detector is suitable to measure light at 2100 cm^{-1}. How does it work approximately? (2 p)

c Convert 2100 cm^{-1} to wavelength in μm. (1 p)

d Calculate the average CO concentration in both number density [molecules cm^{-3}] and mixing ratio [ppm] inside Mexico city, assuming a height of the mixing layer of 1500 m (meaning that all CO molecules are trapped below this height). Use the calibration spectrum of CO in Fig **??**. In Mexico city the temperature was 25^oC and pressure 75 kPa. Describe the calculation clearly and motivate simplifications in the calculations. (4 p)

5. Airborne SAR (10 p)

An airborne SAR system has a flight altitude $z = 4$ km. The illuminated swath corresponds to incidence angles θ between 20 and 50 degrees ($\theta = 0$ corresponds to a nadir looking system). The radar operates at a frequency $f = 5.3$ GHz and has a broadside looking antenna with a length $D = 2$ m. The speed of the aircraft is $v = 150$ m/s. Assume that the ground is flat.

a A trihedral corner reflector with a side length $a = 1$ m is located within the swath at an incidence angle $\theta = 30$ degrees. The measured ratio between the received power P_r and transmitted power P_t for the trihedral is $P_r/P_t = -115$ dB. A field, 50 m by 50 m large, is located at an incidence angle of $\theta = 45$ degrees. For this field, P_r/P_t is measured to be -113.5 dB. From these measurement, what is the estimated normalized radar cross section (σ^0) of the field at the incidence angle 45 degrees? Help: The radar cross section of a trihedral is given by

$$\sigma = \frac{4\pi a^4}{3\lambda^2}$$

(4 p)

b The field in **a** is an agricultural field after harvest. At the time of the measurements described above it was covered by a thick layer of dirt, which was relatively smooth and dry. After the measurement a flock of cows were let loose in the field. The cows trampled the ground and made it more rough, and then left the field. At the same time there was an extended period of rain. Discuss how these two events changed the radar cross section of the field. (3 p)

c Calculate the minimum and maximum length of the synthetic aperture. (2 p)

d The flight altitude of the system is changed to 7 km. How does this effect the azimuth resolution? (1 p)

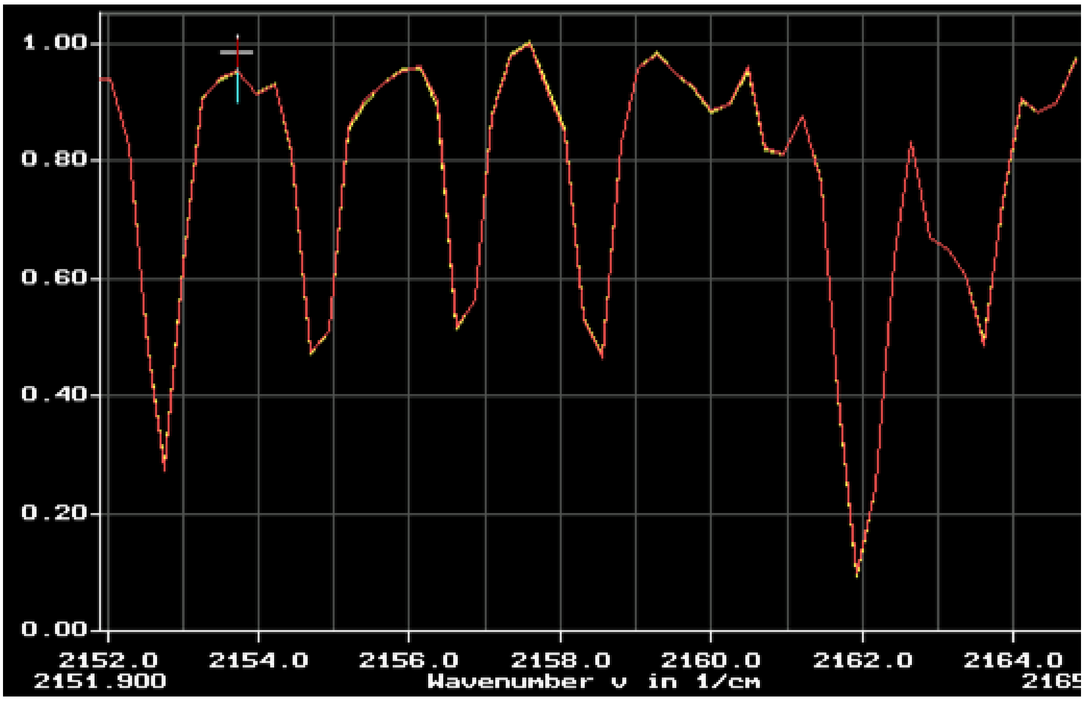

Figure 3: A solar spectrum measured by an FTIR in Mexico city.

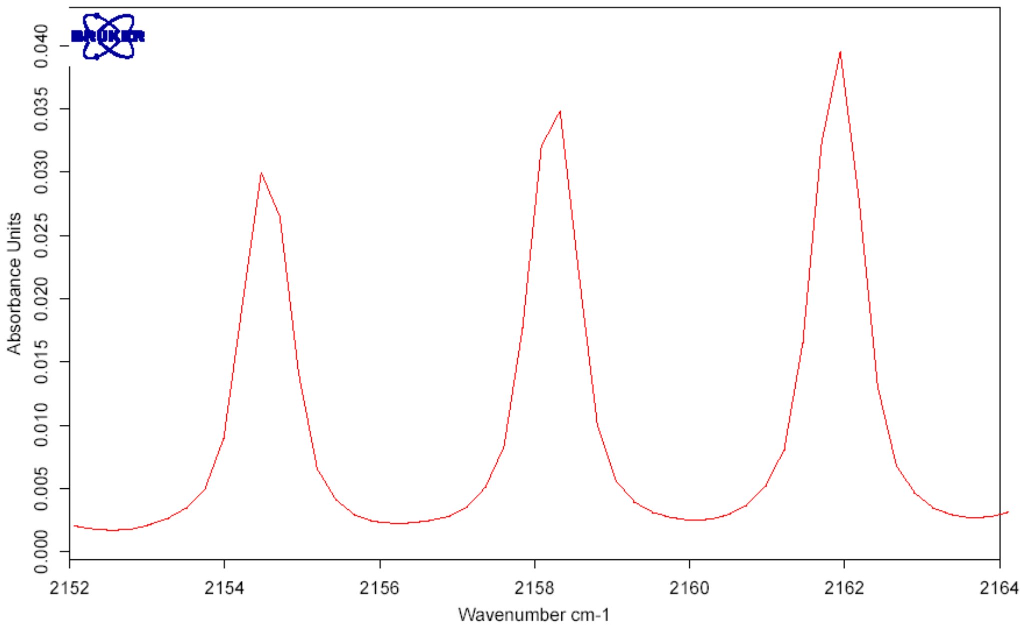

Figure 4: Absorbance spectrum (base 10) of CO corresponding to $2.43 \cdot 10^{15}$ [molecules/cm^3] in a 1 m cell

Formulae

Solid angle of an unit sphere

$$\int d\Omega = 2\int_{\theta=0}^{\pi/2}\int_{\phi=0}^{2\pi}\sin(\theta)\,d\theta\,d\phi = 4\pi$$

Ideal gas law

$$N = \frac{PV}{k_BT}, \qquad \frac{\rho T}{P} = \frac{M}{R}$$

Energy of a photon

$$E = hf$$

Wavenumber as spectroscopic unit

$$\tilde{\nu} = 1/\lambda$$

Stokes Vector

$$\begin{aligned}
\mathbf{s} &= [S_0\ S_1\ S_2\ S_3]^T \\
S_0 &= \langle E_{0x}^2\rangle + \langle E_{0y}^2\rangle \\
S_1 &= \langle E_{0x}^2\rangle - \langle E_{0y}^2\rangle \\
S_2 &= \langle 2E_{0x}^2E_{0y}^2\cos(\phi_y-\phi_x)\rangle \\
S_3 &= \langle 2E_{0x}^2E_{0y}^2\sin(\phi_y-\phi_x))\rangle
\end{aligned}$$

Degree of polarization

$$\frac{\sqrt{S_1^2+S_2^2+S_3^2}}{S_0}$$

Refractive index

$$n = n' - in'' = \sqrt{\epsilon_r}$$

Angular frequency

$$\omega = 2\pi f$$

Complex (angular) wavenumber

$$k = \frac{\omega n}{c}$$

Absorption length

$$l_a = \frac{c}{2\omega n''} = \frac{1}{\gamma_a}$$

Snells law

$$n_1'\sin(\theta_1) = n_2'\sin(\theta_2)$$

Fresnel coefficients ($\varepsilon_1 = 1$)

$$\begin{aligned}
\Gamma_\perp &= \frac{\cos(\theta_1) - \sqrt{\epsilon_{r2} - \sin^2(\theta_1)}}{\cos(\theta_1) + \sqrt{\epsilon_{r2} - \sin^2(\theta_1)}} \\
\Gamma_\parallel &= \frac{\sqrt{\epsilon_{r2} - \sin^2(\theta_1)} - \epsilon_{r2}\cos(\theta_1)}{\sqrt{\epsilon_{r2} - \sin^2(\theta_1)} + \epsilon_{r2}\cos(\theta_1)} \\
r &= |\Gamma|^2
\end{aligned}$$

Irradiance / Exitance

$$E \text{ or } M = \int_{\theta=0}^{\pi/2}\int_{\phi=0}^{2\pi} L\cos(\theta)\sin(\theta)\,d\theta\,d\phi$$

BRDF / surface reflectivity

$$\begin{aligned}
L(\theta_1,\phi_1) &= R(\theta,\phi,\theta_1,\phi_1)F(\theta,\phi)\cos(\theta) \\
M &= r(\theta,\phi)F(\theta,\phi)\cos(\theta) \\
r(\theta,\phi) &= \int_{\theta=0}^{\pi/2}\int_{\phi=0}^{2\pi} R\cos(\theta)\sin(\theta)\,d\theta\,d\phi
\end{aligned}$$

Blackbody radiation

$$B(f,T) = \frac{2hf^3}{c^2}\frac{1}{e^{hf/k_BT}-1}$$

Rayleigh-Jeans approximation

$$\begin{aligned}
&(hf/k_BT \ll 1) \Rightarrow \\
&B(f,T) \approx 2k_BTf^2/c^2
\end{aligned}$$

Absorption coefficient

$$\gamma_a = N\sigma_a = N\left\{\sum[sF] + \sigma_{\text{cont}}\right\}$$

Doppler broadening

$$\begin{aligned}
F_d &= \frac{1}{\sqrt{\pi}w_d}\exp(-((f-f_0)/w_d)^2) \\
w_d &= \frac{f_0}{c}\sqrt{\frac{2RT}{M}}
\end{aligned}$$

Pressure broadening

$$\begin{aligned}
F_p &= \frac{1}{\pi}\frac{w_p}{(f-f_0)^2 + w_p^2} \\
w_p &= w_0P\left(\frac{T}{T_0}\right)^{-n}
\end{aligned}$$

Scattering coefficient

$$\gamma_s = N\sigma_s$$

Rayleigh scattering

$$\sigma_s = \frac{128\pi^5 d^6}{3\lambda^4}$$

Optical thickness

$$\tau(l_1,l_2) = \int_{l_1}^{l_2}\gamma(l)dl$$

Beer-Lambert's law

$$I = I_0\exp(-\tau)$$

DOAS

$$N = \frac{ln(I_1/I_2)}{h[\sigma(\lambda_2) - \sigma(\lambda_1)]}$$

Radiative transfer without scattering

$$\begin{aligned} I(h) &= I_0 e^{-\tau(0,h)} + \int_0^h \gamma_a B e^{-\tau(l,h)} \mathrm{d}l \\ T_b &= T_b^0 e^{-\tau(0,h)} + \int_0^h \gamma_a T e^{-\tau(l,h)} \mathrm{d}l \\ I^{out} &= I^{in} e^{-\tau} + B(1 - e^{-\tau}) \end{aligned}$$

Photographic scaling factor

$$s = \frac{f}{h}$$

f/number

$$f/number = \frac{f}{D}$$

Photogrammetry

$$x' = -\frac{fx}{h-z}, \qquad y' = -\frac{fy}{h-z}$$

Michelson interferometry

$$\begin{aligned} I &= \frac{I_0}{2}\left[1 + \cos(2kl)\right] \\ V &= \frac{I_{max} - I_{min}}{I_{max} + I_{min}} \\ I(l) &= \int_0^\infty I(f)\cos(2\pi l f/c)\mathrm{d}f \end{aligned}$$

Diffraction limited angular resolution

$$\sin(\theta) \geq \frac{\lambda}{D}$$

Antenna relationships

$$\begin{aligned} G &= \eta D_0 = \eta \frac{4\pi}{\Omega_A} \\ \Omega_A &= \frac{\lambda^2}{A_e} \end{aligned}$$

Radiometer sensitivity

$$\Delta T_b = C \frac{T_{sys}}{\sqrt{\Delta f \Delta t}}$$

Noise power

$$P_N = k_B B T_a$$

Footprint of radar altimeter

$$r = \sqrt{ch\Delta t}$$

Radar equation

$$\frac{P_r}{P_t} = \frac{\lambda^2 G^2}{(4\pi)^3 \eta h^4} \sigma^o A$$

Radar Doppler shift

$$f_D = -2v_r/\lambda$$

Weighting functions

$$\begin{aligned} y &= \int_0^h K(z)x(z)\mathrm{d}z + \varepsilon \\ \mathbf{y} &= \mathbf{Kx} + \varepsilon \end{aligned}$$

Optimal weighting

$$\begin{aligned} \hat{x} &= \frac{\sigma_2^2 x_1 + \sigma_1^2 x_2}{\sigma_1^2 + \sigma_2^2} \\ \hat{\sigma} &= \frac{\sigma_1 \sigma_2}{\sqrt{\sigma_1^2 + \sigma_2^2}} \end{aligned}$$

Answers to exam in Remote Sensing 2009-03-14

Question 1:

a The polar low earth orbit (LEO), usually between 500 and 1000 km. The polar orbit can give (nearly) global coverage (depending on orbit inclination and sensor swath width). For earth observation, the relatively low altitude give better spatial resolution than orbits at higher altitudes. The orbit is still high enough to avoid heavy atmospheric drag.

b Examples of remote sensing advantages

- Global/near global coverage (E.g. Satellite ozone monitoring).
- Reach remote (hostile) areas (Arctic sea ice measurement).
- More time efficient (Forest biomass observations with SAR)
- Along path integration/averaging (Air pollutions with DOAS).
- Non-invasive (Aerosol measurements).

c The answer two all three subquestions is Rayleigh scattering.

- Rayleigh (molecular) scattering favors blue light due to the λ^{-4} dependence. Hence diffuse skylight will appear blue.
- At sunset the light travels much longer through the atmosphere and the blue light will be significantly extinct when looking directly into the sun, since it is scattered out of the line of sight. This will make the sun appear redish.
- In the phase function (angular distribution) of Rayligh scattering it is seen that the scattered light is totally polarized at 90 degrees from the direction of the incoming beam. The reason why the skylight is only partially polarized at 90 degrees from the sun is due to multiple scattering.

d The two main reasons are:

- (1) The atmosphere is transparent in this region (atmospheric window).
- (2) The sun has its maximum intensity (Wien's displacement law) here.
- Since the resolution (due to diffraction) is wavelength dependent one could also argue that if the eye was sensitive to only longer wavelengths (IR or microwaves) the spatial resoluion would be lower (or the dimensions of the eye have to be much larger).

e Lambertian scattering distributes the scattered radiation equally in all directions (half sphere) independent of the incoming direction.

f Differential means that the difference between two (or more) wavelengths are studied in order to extract the high frequency part of the signal. Two major advantages:

- The low frequency signal, containing Rayleigh/Mie scattering or instrumental functions, can be ignored.
- No need for absolute calibration.

g The answer to the first question is (b), absorption lines in a black-body curve. The second anser is (a), emission lines against a cold sky. Note that a gas at low pressure does NOT radiate like a full black-body but the emission peaks follow the 'outline' of a black-body at the same temperature.

Question 2:

a:

- I: Spectral radiance. Specifies the intensity of the radiation. A possible unit is [W/(m^2 Hz sr)]. At long wavelengths can be given as the brightness temperature, with unit [K]. (3p)
- l: Distance along the propagation path. (1p)
- γ: Extinction coefficient. Unit is [1/m] (or [1/cm] etc.). Describes the combined strength of scattering and absorption. (2p)
- S: Source term. Possible units are [W/(m^3 Hz sr)] and [K/m]. Represents the power added, along the propagation path, by scattering and emission. (2p)

b: Cosmic background radiation and emission from the cloud-free atmosphere can be ignored. Recieved radiation has three components: (1) Upwelling radiation from the cloud layer

$$T_b = T_{cloud}(1 - e^{-\tau_{cloud}}), \tag{1}$$

(2) emission from the sea surface attnuated by the cloud layer,

$$T_b = T_{sea}(1 - r)e^{-\tau_{cloud}}, \tag{2}$$

where r is the reflection coefficient of the surface, and (3) downwelling radiation reflected by the surface followed by attenuation in the cloud layer,

$$T_b = T_{cloud}(1 - e^{-\tau_{cloud}})re^{-\tau_{cloud}}. \tag{3}$$

This is a sufficient answer. An alternative answer, giving the sum directly is:

$$T_b = T_{sea}(1 - r)e^{-\tau_{cloud}} + T_{cloud}(1 - e^{-\tau_{cloud}})(1 + re^{-\tau_{cloud}}). \tag{4}$$

Question 3:

a Calculate transmittance, T, using Beer-Lambert law (base ten) for D=2:

$$T = \frac{I}{I_0} = 10^{-D} = 0.01 = 1\% \tag{5}$$

b Moved to the left and steeper

c First, calculate rezel size:

$$\frac{\text{Pixel}}{\text{Rezel}} = \frac{f}{H} \Rightarrow \text{Rezel} = \frac{\text{Pixel} \cdot H}{f} = 1\text{cm} \tag{6}$$

Size of diffraction pattern on the ground, l_g, at 500 nm is given by:

$$l_g = \frac{\lambda \cdot H}{D} \Rightarrow \frac{\lambda \cdot H \cdot f_{number}}{f} \approx 6\text{mm} \tag{7}$$

This means undersampling since Rezel $< l_g$

d The relief discplacement, z' is the spatial difference on the film between the top and root of the tree. x is the distance from the principal point and z the tree top altitude, which gives:

$$z' = \frac{f \cdot x}{H - z} - \frac{f \cdot x}{H} \Rightarrow z = \frac{1}{\frac{z'}{f \cdot x} + \frac{1}{H}} \approx 32\text{m} \quad (8)$$

The uncertainty decreases with decreasing altitude or increasing distance from principal point.

e Stereography

Question 4:

- Interference is measured using a moving mirror. The spectrum is calculated by FFT. See course book and handouts for details.
- A photovoltaic semiconductor (InSb) is the best but a photoconductive MCT detector also works. The course book for details.
- Wavenumber[1/cm] =1/(λ[cm]) $\Rightarrow$ 2100cm^{-1} = 1/2100cm = 4.74μm
- First, the integrated concentration (slant column), C is calculated at a suitable wavelength. Here we use the line at 2158cm^{-1}. We use Beer-Lambert law and read in FTIR spectrum that the transmittance, T is 50% (it is assumed that all absorption is due to CO).

$$T = \frac{I}{I_0} = 10^{\sigma \cdot C} \Rightarrow C = \frac{-log(T)}{\sigma} \quad (9)$$

The cross section, σ, of CO at this wavenumber is calculated using the absorbance, A, and the known concentration, c', and path length, l, in the calibration spectrum (and again Beer-Lambert law , base 10).

$$A = -log(\frac{I}{I_0}) = \sigma \cdot c' \cdot l \Rightarrow \sigma = \frac{A}{c \cdot l} \quad (10)$$

This gives the final solution for slant column C:

$$C = \frac{-log(T) \cdot c' \cdot l}{A} = \frac{-log(0.5) \cdot 2.43 \cdot 10^{15} \cdot 100}{0.035} = 2.1 \cdot 10^{8}[\text{molecules/cm}^2] \quad (11)$$

The vertical column is simply $cos(10) \cdot C$ and to get the average concentration (per cm^3) you divide by the height of the mixing layer (in cm) , this gives the average concentration of CO, $c = (cos(10) \cdot C)/150000 = 1.4 \cdot 10^{13}[\text{molecules/cm}^3]$. It is assumed that no CO is situated above the mixing layer. The average mixing ratio is calculated through division with the total atmospheric concentration of molecules, c_{atm}, using the common gas law and the local temperature and pressure in Mexico city:

$$c_{atm} = \frac{n}{V} = \frac{P}{k_B T} = \frac{75}{298 \cdot 1.38 \cdot 10^{-23}} = 1.8 \cdot 10^{25}[\text{molecules/m}^3] = 1.8 \cdot 10^{19}[\text{molecules/cm}^3] \quad (12)$$

Hence the mixing ratio in ppm is: $(c/c_{atm}) \cdot 10^6 = 0.75\text{ppm}$

Question 5:

a The cross section of the trihedral is given by ($a = 1$ m, $\lambda = c/f = 3 \cdot 10^8 / 5.3 \cdot 10^9$ m $= 5.67$ cm)

$$\sigma_{tri} = \sigma^0_{tri} A_{tri} = \frac{4\pi a^4}{3\lambda^2} = 1307.4\text{m}^2$$

The range distance to the trihedral is by: $h_{tri} = z/\cos\theta_{tri} = 4/\cos 30°\text{m} = 4618.8$ m. Similarly, $h_f = z/\cos\theta_f = 5656.9$ m.
The radar equation for the trihedral and fields give

$$(P_r/P_t)_{tri} = \frac{\lambda^2 G^2}{(4\pi)^3 \eta h^4_{tri}} \sigma_{tri} \quad (13)$$

$$(P_r/P_t)_f = \frac{\lambda^2 G^2}{(4\pi)^3 \eta h^4_f} \sigma^0_f A_f \quad (14)$$

Combining Eqs. **??** and **??** we get ($A_f = 50^2$ m^2 $= 2500$ m^2)

$$\frac{(P_r/P_t)_f}{(P_r/P_t)_{tri}} = \frac{\sigma^0_f A_f h^4_{tri}}{h^4_f \sigma_{tri}} \Rightarrow$$

$$\sigma^0 = \frac{(P_r/P_t)_f}{(P_r/P_t)_{tri}} \frac{h^4_f \sigma_{tri}}{A_f h^4_{tri}} = 1.6621\text{m}^2/\text{m}^2$$

b In the first case, a smooth field, the power scattered back to the reflector will be low, since the scattering will be near specular. When the roughness increases more scattering will be in the direction of the radar. The increased moisture will increase the dielectric constant, and thus the strength of the scattering. Therefore both the increased roughness and the increased moisture will **increase** the strength of the backscatter.

c The length of the synthetic aperture (L) at the range distance $h$is given by $L = \theta_A h$, where θ_A is the antenna beamwidth (see the solutions to problem 30 on the course web page). Thus the maximum and minimum L is given for the maximum and minimum range distance, respectively. The antenna beamwidth is given by $\theta = \lambda/D = 1.6216$ degrees. We get

$$L_{max} = \theta_A h_{max} = \theta_A z/\cos\theta_{far} = 176.1\text{m}$$
$$L_{min} = \theta_A h_{min} = \theta_A z/\cos\theta_{near} = 120.5\text{m}$$

The change in flight altitude does not affect the azimuth resolution.