CHALMERS UNIVERSITY OF TECHNOLOGY — 2010-03-11
Microtechnology och nanoscience — 14.00-18.00
V-building

Exam in FMI036, Superconductivity and low temperature physics

Teacher: Per Delsing, phone: 772 3317, 070-3088317
Allowed aids: Tefyma, Physics Handbook, Stand Math Tables and similar handbooks, calculator, and one A4 sheet with handwritten notes.
Max points: 30p
Grading: Grade 3: 5p on problem 1 and 15p in total, (Grade 4: 20p, Grade 5: 25p)
Motivate your answer in a logical way. Illustrate with readable diagrams and/or figures.

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1. Short questions to test the understanding of concepts.

Give short descriptions or definitions of the following, use diagrams or figures if appropriate:

Superconductors

a) Explain why a superconductor does not have zero resistance at high frequencies.

b) The penetration depth λ and the coherence length ξ are two important length scales in superconductors that follow from the Ginzburg-Landau equations. Define and describe the two length scales?

c) Explain how a vortex comes about in a superconductor and describe its properties. Make a drawing of a vortex in a superconductor and plot the spatial dependence in the vicinity of the vortex core of the magnetic flux density and the pair density.

d) Explain in simple terms how the electron-phonon interaction can give rise to an attractive interaction between electrons that leads to (Cooper-)pairing.

e) Sketch the BCS-density of states for excitations in an ordinary superconductor such as tin. Draw a figure and give important values.

f) Describe how electronic properties of High-Tc superconductor, $YBa_2Cu_3O_{7-x}$, depend on the oxygen concentration, x. Draw general phase diagram and indicate important phase transitions.

Helium

g) Sketch the low temperature phase diagram for ^4He. Which are the different phases? Give the most important temperatures and pressures.

h) What is the meaning of the second sound in a superfluid? What is different from ordinary sound?

Cryogenics

i) Explain the concept of Residual Resistance Ratio (RRR). Give two examples of how big RRR can be in different materials, and what information you get from RRR.

j) How can a dilution refrigerator be used for cooling, what temperatures and cooling powers can be reached?

2. Thermodynamics of superconductors.

For a bulk superconductor the magnetic field is expelled from almost the whole superconductor, and the thermodynamical critical filed can be calculated from the difference in free energy between the normal metal and the superconductor. $\Delta F = -\mu_0 \int_0^{Hc} M_{av} dH = \frac{\mu_0}{2} H_C^2$, where M_{av} is the average magnetization of the superconductor.

For a thin film where the thickness, t, is of the order of the penetration depth, λ, the magnetic flux is only partly expelled and therefore the average magnetization is different. Calculate the critical filed for the thin film assuming that the flux density within the film ($-t/2<x<t/2$) is given by

$B = \mu_0 H \frac{\cosh(x/\lambda)}{\cosh(t/2\lambda)}$, where x is the distance from the center of the thin film. (4p)

3. BCS theory

The energy expectation value for the BCS ground state is given by:

$$\langle E \rangle = \langle \Psi_{BCS} | H_{BCS} | \Psi_{BCS} \rangle = \sum_k 2v_k^2 \varepsilon_k + \sum_{k,k'} v_k u_{k'} u_k v_{k'} V_{k,k'}$$

where u_k and v_k are the probability amplitudes for having the state k empty or filled respectively. $V_{k,k'}$ is the interaction energy between one particle in state k and one particle in state k', and ε_k is the kinetic energy of a particle in state k.

a) Define the k dependent energy gap $\Delta_k = \sum_{k'} u_{k'} v_{k'} V_{k,k'}$ and derive an expression for the Δ_k as a function of probability amplitudes by minimizing $\langle E \rangle$ with respect to the probability amplitudes. (3p)

b) Introduce the excitation energy $E_k = \dfrac{\varepsilon_k}{u_k^2 - v_k^2}$ and express E_k as a function of Δ_k (1p)

4. Josephson tunneling.

A Josephson junction can be described by the following set of coupled time-dependent Schrödinger equations:

$$-i\hbar \frac{\partial}{\partial t} \Psi_1 = \frac{qV}{2} \Psi_1 + K\Psi_2$$

$$-i\hbar \frac{\partial}{\partial t} \Psi_2 = -\frac{qV}{2} \Psi_2 + K\Psi_1$$

where K describes the coupling between left and right electrode, q is the charge of a Cooper-pair and V is the voltage applied to the junction. Derive the two Josephson equations (4p)

5. Superfluid Helium

a) Describe how a Bose Einstein condensation comes about? What is the condensate? (1p)

b) Derive the value of the quantum of circulation in a superfluid vortex. (1p)

c) Describe what different symmetry breaking mechanisms that can be found in superfluid Helium 3 and explain how the superfluid A and the B phases relate to these mechanisms. (2p)

6. Low temperature properties of materials.

When an electron gas is heated the Fermi distribution changes and some of the electrons close to the Fermi energy get a higher kinetic energy. From a thermodynamical point of view, this means that energy is stored in the electron gas. Calculate the electronic specific heat of the electrons in a metal, assuming that the density of states is constant close to the Fermi energy. (4p)

Good luck !