CHALMERS TEKNISKA HÖGSKOLA 2008-03-13
Mikroteknologi och nanovetenskap Kl: 08.30-12.30
V-building

Exam in FMI036, Superconductivity and low temperature physics

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Allowed aids: Tefyma, Physics Handbook, Stand Math Tables and similar handbooks, calculator, and one A4 sheet with handwritten notes.
Max points: 30p
Grading: Grade 3: 5p on problem 1 and 15p in total, (Grade 4: 20p, Grade 5: 25p)
Motivate your answer in a logical way. You are welcome to illustrate with readable diagrams.
Answer in Swedish or English.

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1. Short questions to test the understanding of concepts.
Give short descriptions or definitions (use diagrams if appropriate) of the following:

a) How large is the free energy difference between the superconducting and the normal state for a low T_C superconductor?
b) What are the main differences between a type I and type II superconductor, what determines the type?
c) What is the isotope effect for superconductors, how can it be explained?
d) What is a vortex in a superconductor, and which properties does it have?
e) Sketch the current voltage characteristic of an (unshunted) SIS Josephson junction. Give current and voltage scales and describe contributions by Cooper-pairs and quasi-particles.
f) Describe the main differences between the High T_C superconductors and ordinary superconductors.
g) What is Bose Einstein condensation and how can you make a connection to the two-fluid model?
h) Sketch the low temperature phase diagram (P vs. T) for ^{3}He. Which are the different phases? Give the most important temperatures and pressures.
i) Describe the specific heat of a metal at low temperature, draw a figure and indicate temperature dependences and the origin of different parts of the specific heat.
j) Describe the difference between a primary and a secondary thermometer. Give one example of each.

(1p per question)

2. From Ginzburg-Landau to London.
The current density in a superconductor is given by the second Ginzburg-Landau equation

$$\vec{J}_S = \frac{ie\hbar}{2m}\left(\Psi^*\vec{\nabla}\Psi - \Psi\vec{\nabla}\Psi^*\right) - \frac{2e^2}{m}\vec{A}\Psi^*\Psi$$

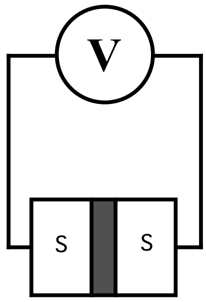
a) Derive the second London equations from the second GL equation. (2p)
b) Derive the London penetration depth from the second London equations. (2p)

3. BCS Theory
The BCS ground state is given by $|BCS\rangle = \prod_k \left(u_k + e^{i\theta} v_k c^\dagger_{k+s,\uparrow} c^\dagger_{-k+s,\downarrow}\right)|0\rangle$.

a) Explain the different parts of the BCS ground state. What is |0> and θ, what do the two operators c and $c^\dagger$ do? What is the meaning of u_k and v_k, how do they vary as a function of energy. (2p)
b) From the BCS theory we also get an expression for the excitation energy, $E(k) = \sqrt{\varepsilon(k)^2 + \Delta^2}$ where $\varepsilon(k)$ is the kinetic energy, and Δ is the energy gap. Derive an expression for the Density of states for excitations, *N(E)*. (2p)

4. Josephson effect

Two superconductors of the same material are separated by a thin tunnel barrier and a voltage is applied between them as shown in the figure below. The system can be described by two coupled Schrödinger equations, where μ is the chemical potential of the material, Ψ_1 and Ψ_2 are the orderparameters for the left and the right side respectively.



$$i\hbar\frac{\partial}{\partial t}\Psi_1 = (\mu + eV)\Psi_1 + K\Psi_2$$

$$i\hbar\frac{\partial}{\partial t}\Psi_2 = (\mu - eV)\Psi_2 + K\Psi_1$$

a) Derive the two Josephson relations (3p)
b) Derive an expression for the Josephson inductance from the two Josephson relations. (1p)

5. Superfluid Helium

a) Derive the value of the quantum of circulation in a superfluid vortex. (1p)
b) Draw the phase diagram, for a mixture of ^{3}He and ^{4}He, give important temperatures and concentrations. (1p)
c) Describe what different symmetry breaking mechanisms that can be found in superfluid Helium 3 and explain how the A and the B phase relates to these mechanisms. (2p)

6. Joule Thompson cooling

The Joule-Thompson coefficient is given by $\mu = \left(\frac{dT}{dP}\right)_H = \frac{1}{C_p}\left[V - T\left(\frac{dV}{dT}\right)_p\right]$

a) Explain the concept of inversion temperature for a gas and how it relates to the Joule-Thompson coefficient. (2p)
b) Derive an expression for the inversion temperature of a nonideal gas. (2p)

Good Luck !