

Home assignment 2, Gravitation & Cosmology, 2010

To be handed in Friday, December 10

- 1.a. The German astronaut Thomas Reiter has spent almost 6 months aboard the ISS. Calculate the accumulated difference in elapsed time on the ISS and on earth after this period of time.
- b. Are general relativistic effects significant for time-keeping in the GPS system?
2. Consider a D -dimensional space-time with the metric

$$ds^2 = \frac{1}{y^2}(-dt^2 + dy^2 + dx^i dx^i),$$

defined in the region $y > 0$. Is this space-time maximally symmetric? Show, or argue convincingly, that the constant time slices are maximally symmetric spaces of negative curvature. For the special case $D = 3$, find a coordinate transformation that relates the metric $ds^2 = y^{-2}(dx^2 + dy^2)$ to the metric of the Poincaré disk given in the first home assignment (the “radius” a and overall constants in front of the metric are inessential).

Hint: the conformal transformation

$$\zeta = i \frac{1+z}{1-z}$$

maps the unit disk to the upper half plane.

3. Prove that for a given space-time with N Killing vectors $\xi_{(I)}^\mu(x)$, $I = 1, \dots, N$ the scalars $P_{(I)} \equiv \xi_{(I)}^\mu P_\mu$ are constants of motion for a freely falling particle. Draw some conclusion from this concerning geodesic motion in the Schwarzschild metric.