Exam in

RRY125/ASM510 Modern astrophysics

Tid: 16 december 2013, kl. 08.30–12.30
Plats: V-salar, Chalmers
Ansvarig lärare: Magnus Thomasson ankn. 8587
(lärare besöker tentamen ca. kl.09.00 och 11.00)

Tillåtna hjälpmedel:
- Typgodkänd räknedosa, eller annan räknedosa med nollställt minne
- Physics Handbook, Mathematics Handbook
- bifogat formelblad
- ordlista (ej elektronisk)

You may use:
- Chalmers-approved calculator, or other calculator with cleared memory
- Physics Handbook, Mathematics Handbook
- enclosed sheet with formulae
- dictionary (not electronic)

Grades:
The maximum number of points is 30.
Chalmers: Grade 3 requires 12 p, grade 4 requires 18 p, grade 5 requires 24 p.
GU: Grade G requires 12 p, grade VG requires 21 p.

Note: Motivate and explain each answer/solution carefully.

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1.
Choose the most reasonable of the given values for the following (do not give a motivation):

(a) Surface pressure at Mars (Earth = 1):	A) 100,	B) 2,	C) 0.5,	D) 0.01
(b) Size of the nucleus of a comet:	A) 10 km,	B) 1 AU,	C) 10 m,	D) 1 ly
(c) Age of planet Venus	A) 500 Myr,	B) 13 Myr,	C) 4 Gyr,	D) 13 Gyr
(d) Diameter of a red giant (ly=light year):	A) 1 lyU,	B) 1 AU,	C) 10^5 km,	D) 1 pc
(e) Central temperature of a main sequence star of spectral class M:				
	A) 10^9 K,	B) $6 \cdot 10^7$ K,	C) $9 \cdot 10^6$ K,	D) 3000 K
(f) Distance to a quasar (ly=light year):	A) 10^{10} ly,	B) 10^7 AU,	C) 20 kpc,	D) 13 Gpc
(g) Density parameter $\Omega_{M,0}$ (matter) for the present cosmological model:				
	A) 0.04,	B) 0.3,	C) 0.7,	D) 1.0
(h) Redshift of the Last Scattering Surface	A) 0	B) 2.735	C) 6	D) 1100

(4 p)

2.
A result from radiative transfer theory for stellar atmospheres is the following:
$I_\nu(0,1) \approx B_\nu(\tau_\nu = 1)$, where I_ν is a function of τ_ν and of $\mu =$ ~~arc~~$\cos(\theta)$, and *B* is Planck's law. Use this result to explain the formation of absorption lines in stellar spectra. **(2 p)**

3.
a.) Draw a Hertzsprung-Russel (HR) diagram. Label each axis in two different ways and mark the positions of different types of stars. **(2 p)**
b.) Describe how you can use an HR diagram to determine age and distance to a globular cluster in the Milky Way. **(2 p)**

4.
A spiral galaxy is observed to have a redshift corresponding to a velocity of 720 km/s and an apparent magnitude of 9. What is its absolute magnitude? Assume that the galaxy consists of only Sun-like stars. How many stars does it contain? **(2 p)**

5.
The relativistic equation of state for a degenerate electron gas is $P = K_2 \rho^{4/3}$, where
$K_2 = \frac{1.24 \cdot 10^{10}}{{\mu_e}^{4/3}}$ (and $\mu_e = \frac{2}{1+X}$, where *X* is the hydrogen mass fraction).
a.) Make a very *crude ("order of magnitude") estimate* of the mass of a star composed of such matter. **(2 p)**
b.) What type of star is it? What happens if such a star attracts more mass from a companion, and why is that process important for cosmology? **(2 p)**

6.
Assume that you have discovered a planetary system where the most massive planet is a Saturn-sized planet in circular orbit (radius 8 AU) around a Sun-like star.
a.) What is the maximum Doppler shift of the Calcium H line (396.847 nm)? **(1 p)**
b.) How far from the centre of the star is the centre of mass of the system? **(1 p)**

7.
Explain the meaning of the "Horizon problem" in modern cosmology and how it is resolved.
(1 p)

8.
Give an example of supportive evidence for the "unification theory" for active galactic nuclei (AGNs). Give also one example of why the unification theory can not explain all of the observed variety in AGN-types. **(2 p)**

9.
Derive *Friedmann's equation* using Newtonian physics plus the result from general relativity that the energy per unit mass is $E = -kc^2/2$. Explain very briefly what *k* is. (Neglect Λ.)

Then solve Friedmann's equation (i.e., find $a(t)$) for the time *before* matter-radiation equality. (Make a reasonable simplification of Friedmann's equation first, and explain why you do it, and make a reasonable assumption of how the density depends on the scale factor.)
(5 p)

10.
A star (5 solar masses) has started to evolve away from the main sequence (MS). When the star arrives at its nect destination in the HR diagram it has a surface temperature of 5000 K and is shining at 5% of its Eddington luminosity. Estimate the Kelvin-Helmholtz timescale for star. It would typically take the star 350 000 years to make the transition away from the MS - can you give a brief explanation to why your result differs from this? **(4 p)**

Astrophysics equations, constants and units

Binary stars, planet+star, etc.

$m_1 r_1 = m_2 r_2$ and $m_1 V_1 = m_2 V_2$	centre of mass
$a = a_1 + a_2$	semi-major axis of relative orbit
$\dfrac{a^3}{P^2} = \dfrac{G(m_1 + m_2)}{4\pi^2}$	Keplers 3rd law (for the relative orbit)
$V = V_0 \sin i$	observed velocity
$V_0 = \dfrac{2\pi a}{P}$	velocity of circular orbit

Radiation, magnitudes, luminosities, etc.

$_{\nu} = \frac{\nu}{3} \cdot \frac{1}{(e^{\nu/} \quad 1)}$ m^{-3} Hz^{-1} $\approx 2{,}0 \cdot 10 \quad 3$ m^{-3}

$_{\nu} = \frac{\nu^3}{3} \cdot \frac{1}{(e^{\nu/} \quad 1)}$ m^{-3} Hz^{-1} $\approx \quad , \quad \cdot 10 \quad 4$ m^{-3}

$I_{\nu} = \frac{2 \quad \nu^3}{} \cdot \frac{1}{(e^{\nu/} \quad 1)}$ W m^{-2} Hz^{-1} $I \approx \quad , \quad \cdot 10 \quad 4$ W m^{-2}

$I_{\nu} = \frac{2 \quad \nu^3}{} \cdot \frac{1}{(e^{\nu/} \quad 1)}$ W m^{-2} Hz^{-1} sr^{-1} $\nu_a \approx \quad , \quad \cdot 10$

$— = \quad - \quad I \qquad = — \qquad \tau =$

$I \;=\; I_{,} \cdot \quad ^{\nu} \quad \cdot (1 - \quad ^{\nu}) \qquad = \quad \cdot \quad ^{\nu} \quad \cdot (1 - \quad ^{\nu})$

$m = -2{,}5 \lg \dfrac{F}{F_0}$	m = apparent magnitude, F = observed flux
$m - M = 5 \lg \dfrac{d}{10\ \text{pc}} + A$	M = absolute magnitude, d = distance, A = extinction
$A = ad$	a = interstellar extinction coefficient
$F = \sigma T^4$	F = flux from surface, T = surface temperature
$L = AF$	L = luminosity, A = emitting area

Stellar structure

$$\frac{}{} = \quad {}^{2}\rho$$

$$\frac{P}{} = -\frac{}{\;\;{}_{2}}\rho$$

$$\frac{}{} = \quad {}^{2}\rho$$

$$\frac{}{} = -\frac{}{ac}\frac{\rho}{{}_{3}}\frac{}{{}_{2}}$$

$$\frac{}{} = \quad 1 - \frac{1}{} \quad \frac{}{P}\frac{P}{}$$

Cosmology

$= \;_{0}$ Hubble's law

$1 \qquad = 1 - = \frac{{}_{0}\quad{}_{s}}{} = \frac{\nu}{\nu_{0}\quad{}_{s}} = \frac{{}_{0}}{}$ redshift

${}^{2} = -c^{2} \quad t^{2} \quad a(t)^{2} \quad \frac{}{1} \qquad {}^{2}(\quad \theta^{2} \quad s \;{}^{2}\theta \quad {}^{2})$ Robertson-Walker metric

$\frac{}{} \quad \frac{}{} = \frac{}{3}\rho \quad \frac{}{3}$ Friedmann equation with cosmological constant

Miscellaneous

$\frac{v}{c} = \frac{\Delta\lambda}{\lambda_0}$ — the Doppler effect

$d = \frac{R}{\pi}$ — $R = 1$ AU, π = parallax angle ($R = 1$ and π = " gives d in pc)

$E_{\text{kin}} = \frac{mv^2}{2}$ — kinetic energy

$E_{\text{pot}} = -\frac{GMm}{R}$ — potential energy for a point mass m orbiting a point mass M

$E_{\text{kin}} = \frac{M(\Delta v)^2}{2}, \quad E_{\text{pot}} = -\frac{GM^2}{2R}$ — (energies for an elliptical galaxy, with some definition of its radius R and velocity dispersion Δv)

$2E_{\text{kin}} + E_{\text{pot}} = 0$ — the virial theorem

$V_c = \sqrt{\frac{GM}{R}}$ — circular velocity

$\theta \approx 1.22\frac{\lambda}{\text{D}}$ — resolution of telescope

$(t) = \ _0 \ \ \frac{=}{1/} \ ^2$ — radioactive decay

— = s ar — — recombination and ionization equation

$\frac{}{\cdot 10^{10}\ \odot} \approx \frac{}{200\ /s}^4$ — ( ) s r r a o

$\frac{}{2 \cdot 10^{10}\ \odot} \approx \frac{}{200\ /s}^4$ — (a r ac s) o r a o

$= \frac{c}{} \approx 1.\ \cdot 10^{31} \frac{}{\odot} (\ a\ ) \approx\ 0000 \frac{}{\odot}\ \odot$ (o os

Some mathematics

$x = \ln y \Leftrightarrow y = e^x$ $\qquad e^{-x} = \frac{1}{e^x}$ $\qquad e^{x+y} = e^x \cdot e^y$

$x = \lg y \Leftrightarrow y = 10^x$ $\qquad \lg xy = \lg x + \lg y$ $\qquad \lg \frac{x}{y} = \lg x - \lg y$

= =

= =

=– = $\frac{-}{2}$

— = —— r (=), = ( )

—( ) = $^1_,$ —($^1_)$ = (or ,0) —( ) =

Constants and units

$G = 6.67 \cdot 10^{-11}$ N m^2 kg^{-2}

$c = 2.9979 \cdot 10^8$ m/s

= $5.67 \cdot 10^{-8}$ W m^{-2} K^{-4}

= , 2 0 ^{34} ·s10

k = 1, 0 0 ^{23}10□K^{-1}

1 parsec (1 pc) = 3.26 light years = $3.0857 \cdot 10^{16}$ m

1 AU = $1.496 \cdot 10^{11}$ m

1 year = $3.156 \cdot 10^7$ s

1 arcmin (1') = 1□/60. 1 arcsec (1") = 1□/3600.

HI rest frequency ("21 cm line" of atomic hydrogen): 1420.4 MHz

Absolute magnitude of the Sun: □4.8

The solar constant (1 AU from the Sun): 1371 W/m^2

H_0 = 100h km s^{-1} Mpc^{-1}. Use h = 0.72

Masses: Earth: $5.97 \cdot 10^{24}$ kg, □upiter: $1.90 \cdot 10^{27}$ kg, Sun: $1.99 \cdot 10^{30}$ kg

Radii: Earth: 6378 km, □upiter: 71398 km, Sun: $6.96 \cdot 10^5$ km.

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(1) a) D b) A c) C d) B e) C f) A g) B h) D

(2) See textbook, section 2.4.3

(4) $v = H \cdot d \Rightarrow d = v/H = 720/72 = 10$ Mpc

$m - M = 5 \lg \frac{d}{10\text{pc}} \Rightarrow M = m - 5\lg\frac{10 \cdot 10^6}{10} = 9 - 30 = \underline{\underline{-21}}$

The Sun at 10 Mpc: $m_s = M_s + 5\lg\frac{d}{10\text{pc}} = 4.8 + 30 = 34.8$

The galaxy, with N Sun-like stars: $m = -2.5\lg\frac{F}{F_0} =$

$= -2.5\lg\frac{N F_s}{F_0} = -2.5\lg N - 2.5\lg\frac{F_s}{F_0} = -2.5\lg N + m_s$

So: $9 = -2.5\lg N + 34.8 \Rightarrow \underline{N = 2 \cdot 10^{10}}$

(5) a) Simplify $\frac{dP}{dr} = -\frac{GM_r}{r^2}\rho \longrightarrow$ Roughly $\frac{P}{R} = \frac{GM}{R^2}\rho$

Use $P = K_2 \rho^{4/3}$ and $\rho = M/(\frac{4\pi}{3}R^3)$. {or $\rho \approx M/R^3$}

$\Rightarrow M = \left(\frac{K_2}{G}\right)^{3/2}\left(\frac{3}{4\pi}\right)^{1/2}$. $K_2 = \frac{1.24 \cdot 10^{10}}{\mu_e^{4/3}}$, $\mu_e = \frac{2}{1+X}$

Use $X = 0$ {no hydrogen; other values 0-1 also ok}, $\mu_e = 2$

$\Rightarrow K_2 = 4.92 \cdot 10^9$ [SI] $\Rightarrow M = 3 \cdot 10^{29}$ kg $= 0.15\, M_\odot$

b) White dwarf. Supernova (Ia). Distance measurements.

(9) See textbook, section 10.4. k describes curvature

Radiation dominated era: $\rho = \rho_{R,0}\left(\frac{a_0}{a}\right)^4$.

$k \approx 0$ (density term larger than curvature term when $a \to 0$), so Friedmann's equation:

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho_{R,0}\left(\frac{a_0}{a}\right)^4 \Rightarrow \frac{da}{dt} = \sqrt{\frac{8\pi G \rho_{R,0}}{3}}\, a_0^2 \frac{1}{a} \Rightarrow$$

$$\Rightarrow \int_0^a a\, da = \sqrt{\frac{8\pi G \rho_{R,0}}{3}}\, a_0^2 \int_0^t dt \Rightarrow \underline{a(t) = \left(\frac{32\pi G \rho_{R,0}}{3}\right)^{1/4} a_0\, t^{1/2}}$$

3a) Please see the book for sample HR diagrams. Locate the positions of the Main Sequence, Giants, super Giants, white dwarfs.

b) Age: identify the "knee" in the MS, distance: Estimate apparent magnitudes of identifiable stellar

6) $V_* = \frac{m_p}{m_*} V_p$; $V_p = \frac{2\pi r}{P}$; $V_p^2 = \frac{GM}{R}$ $\left(P = \frac{2\pi R}{V_p}\right)$

$\Delta\lambda = \frac{V_*}{c}\lambda_0$

$m_p r_p = m_* r_*$ $r_p = 8AU - r_*$

$r_* = \frac{m_p 8AU}{m_* + m_p}$

7) Horizon problem: opposite regions of the Universe have not been in contact with each other since they are outside the "horizon". How can the Universe be homogeneous? Solution: Inflation. Explain why the inflation theory is the solution.

8) For example: radio galaxies and QSOs – jet orientation
Sey 1 & Sey 2 – dust obscuration & hidden emission lines
superluminal motion

Evidence against: Evolution; different population of radio sources at different radio powers.
Torus properties change with luminosity

10.) From MS to Giants. Use final T to estimate R together with 5% of L_{Edd}. This will give the final radius and the total contraction energy. The KH time scale is the time potential energy is released during contraction in the absence of nuclear reactions. This energy release is $E_g/L \approx$
$\approx \frac{GM^2}{RL}$ $\Rightarrow$ $t \approx 900$ yrs.

Why the discrepancy? Only the He core collapses →
→ alteration of time scale