Exam in

RRY125/ASM510 Modern astrophysics

Tid: 30 augusti 2013, kl. 14.00–18.00
Plats: V-salar, Chalmers
Ansvarig lärare: Magnus Thomasson ankn. 8587
(lärare besöker tentamen ca. kl.15.00 och 17.00)

Tillåtna hjälpmedel:
- Typgodkänd räknedosa
- Physics Handbook, Mathematics Handbook
- bifogat formelblad
- ordlista (ej elektronisk)

You may use:
- Chalmers-approved calculator
- Physics Handbook, Mathematics Handbook
- enclosed sheet with formulae
- dictionary (not electronic)

Grades:
The maximum number of points is 30.
Chalmers: Grade 3 requires 12 p, grade 4 requires 18 p, grade 5 requires 24 p.
GU: Grade G requires 12 p, grade VG requires 21 p.

Note: Motivate and explain each answer/solution carefully.

This page is intentionally blank.

1.

Choose the most reasonable of the given values for the following (do not give a motivation):

(a) Surface pressure at Venus (Earth = 1):	A) 0.1	B) 1	C) 10,	D) 100
(b) Size of the nucleus of a comet:	A) 10 km,	B) 1 AU,	C) 1 cm,	D) 1 ly
(c) Age of the Moon	A) 500 Myr,	B) 4 Gyr,	C) 13 Myr,	D) 13 Gyr
(d) Diameter of a red giant:	A) 1 AU,	B) 1 ly,	C) 10^6 km,	D) 1 pc
(e) Central temperature of a main sequence star of spectral class A:	A) 20000 K,	B) $1 \cdot 10^6$ K,	C) $2 \cdot 10^7$ K,	D) 10^9 K
(f) Distance to a quasar:	A) 8.5 kpc,	B) 10^5 ly,	C) 2 Gpc,	D) 13 Gpc
(g) Diameter of a spiral galaxy:	A) 10^{10} km,	B) 20 Gpc,	C) 10^8 ly,	D) 20 kpc
(h) Redshift of the Last Scattering Surface	A) 0	B) 2.735	C) 6	D) 1100

(4 p)

Note: ly=light year

2.

The spectroscopic method has been used to detect a planet around a star: the Calcium H line (396.847 nm) shifts back and forth $\pm 6.6 \cdot 10^{-6}$ nm with a period of 1.5 years. The spectrum shows that the star is a main sequence star with a surface temperature of 7200 K and a radius of $1.1 \cdot 10^9$ m. For such stars, luminosity and mass are related by $\lg(L/L_\odot) = 3.7 \cdot \lg(M/M_\odot)$. (The luminosity of the Sun is $3.84 \cdot 10^{26}$ W.)

Estimate the *mass* and *surface temperature* of the planet (assume that it has an albedo of 0.5)! You may assume that the planet's orbit is circular and seen edge-on, and that the mass of the planet is much smaller than the mass of the star. **(5 p)**

3.

A number of stars are located close together in the sky. They possibly all belong to the same star cluster. The colours and *apparent magnitudes* of six of the stars are given in the table below, together with data for the Sun (for which *absolute magnitudes* are given).

Star	B	V	Colour
A	+0.7	+1.0	blue
B	+9.5	+9.0	yellow
C	+20.0	+20.0	white
D	+15.2	+13.8	red
E	+7.0	+6.9	white
F	+13.5	+12.4	orange
Sun	+5.4	+4.8	yellow (absolute magnitudes)

a.) What type of star cluster is it (open or globular)? Motivate your answer. **(2 p)**
b.) Which one of the six stars does not belong to the star cluster? Motivate your answer. **(2 p)**
c.) Estimate the distance to the cluster! **(1 p)**
d.) Explain what the proton-proton chain and the CNO cycle are (detailed reactions are not needed). Choose one of the stars in the cluster which utilizes the proton-proton chain and one which utilizes the CNO cycle, and explain your choices. **(2 p)**

4.

On 14 August 2013, Japanese amateur astronomer Koichi Itagaki discovered a "new" star, a *nova*, in the constellation Delphinus. The star which became a nova had an apparent magnitude of 17, and increased its brightness 25 000 times.

a.) What was the apparent magnitude of the nova? **(1 p)**

b.) The star which became a nova was a white dwarf in a binary star system. What is a white dwarf (briefly explain the physics without equations)? Could it have become a nova if it were *not* part of a binary system (motivate your answer)? **(2 p)**

5.

Draw a figure showing Hubble's classification of galaxies (sometimes called the "tuning fork"). How do the stellar populations and interstellar medium content differ between spiral and elliptical galaxies? **(3 p)**

6.

The spectrum, in the form of brightness temperature versus frequency, of a molecular cloud has been observed with a radio telescope. The maximum of 12 K is at 110 GHz. Outside this peak, the brightness temperature is 2.7 K. The excitation temperature of the molecule with a spectral line at 110 GHz is 60 K (and we can assume that it is a two-level system). What is the optical depth of the molecular cloud (at 110 GHz)? Why is the brightness temperature 2.7 K and not zero at frequencies where the molecule does not emit radiation? **(2 p)**

7.

Use information given in the attached pages with formulae and constants for the following calculations.

a.) What is the critical density of the Universe? **(1 p)**

b.) Assuming that the Universe is matter-dominated and flat, and with $\Lambda=0$, what is the age of the Universe? (You may make a reasonable assumption of how the density depends on the scale factor. Do *not* just calculate the Hubble time.) **(3 p)**

c.) Based on recent cosmological observations, the age of the Universe is different from what you calculated above. Is it younger or older? Why is there a difference, according to the present cosmological model? **(2 p)**

Astrophysics equations, constants and units

Binary stars, planet+star, etc.

$m_1 r_1 = m_2 r_2$ and $m_1 V_1 = m_2 V_2$	centre of mass
$a = a_1 + a_2$	semi-major axis of relative orbit
$\frac{a^3}{P^2} = \frac{G(m_1 + m_2)}{4\pi^2}$	Keplers 3rd law (for the relative orbit)
$V = V_0 \sin i$	observed velocity
$V_0 = \frac{2\pi a}{P}$	velocity of circular orbit

Radiation, magnitudes, luminosities, etc.

$n_\nu = \frac{8\pi \nu^2}{c^3} \cdot \frac{1}{(e^{h\nu/kT} - 1)}$ [m^{-3} Hz^{-1}] $\qquad n \approx 2{,}03 \cdot 10^7 \cdot T^3$ [m^{-3}]

$U_\nu = \frac{8\pi h \nu^3}{c^3} \cdot \frac{1}{(e^{h\nu/kT} - 1)}$ [J m^{-3} Hz^{-1}] $\qquad U \approx 7{,}56 \cdot 10^{-16} \cdot T^4$ [J m^{-3}]

$I_\nu = \frac{2\pi h \nu^3}{c^2} \cdot \frac{1}{(e^{h\nu/kT} - 1)}$ [W m^{-2} Hz^{-1}] $\qquad I \approx 5{,}67 \cdot 10^{-8} \cdot T^4$ [W m^{-2}]

$I_\nu = \frac{2h \nu^3}{c^2} \cdot \frac{1}{(e^{h\nu/kT} - 1)}$ [W m^{-2} Hz^{-1} sr^{-1}] $\qquad \nu_{max} \approx 5{,}88 \cdot 10^{10} \cdot T$

$\frac{dI_v}{dz} = j_v - \alpha_v I_v \qquad S_v = \frac{j_v}{\alpha_v} \qquad d\tau_v = \alpha_v \, dz$

$I_v = I_{v,\text{bg}} \cdot e^{-\tau_\nu} + S_v \cdot (1 - e^{-\tau_\nu}) \qquad T_\text{b} = T_\text{bg} \cdot e^{-\tau_\nu} + T_\text{ex} \cdot (1 - e^{-\tau_\nu})$

$m = -2{,}5 \lg \frac{F}{F_0}$	m = apparent magnitude, F = observed flux
$m - M = 5 \lg \frac{d}{10\,\text{pc}} + A$	M = absolute magnitude, d = distance, A = extinction
$A = ad$	a = interstellar extinction coefficient
$F = \sigma T^4$	F = flux from surface, T = surface temperature
$L = AF$	L = luminosity, A = emitting area

Stellar structure

$$\frac{dM_r}{dr} = 4\pi r^2 \rho$$

$$\frac{dP}{dr} = -\frac{GM_r}{r^2}\rho$$

$$\frac{dL_r}{dr} = 4\pi r^2 \rho \varepsilon$$

$$\frac{dT}{dr} = -\frac{3}{4a_B c}\frac{\chi\rho}{T^3}\frac{L_r}{4\pi r^2}$$

$$\frac{dT}{dr} = \left(1 - \frac{1}{\gamma}\right)\frac{T}{P}\frac{dP}{dr}$$

Cosmology

$v = H_0 l$ Hubble's law

$1 + z = 1 + \frac{v}{c} = \frac{\lambda_{\text{obs}}}{\lambda_{\text{em}}} = \frac{\nu_{\text{em}}}{\nu_{\text{obs}}} = \frac{a_0}{a}$ redshift

$ds^2 = -c^2 dt^2 + a(t)^2 \left(\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\varphi^2)\right)$ Robertson-Walker metric

$\frac{\dot{a}^2}{a^2} + \frac{kc^2}{a^2} = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3}$ Friedmann equation with cosmological constant

Miscellaneous

$\frac{v}{c} = \frac{\Delta\lambda}{\lambda_0}$ the Doppler effect

$d = \frac{R}{\pi}$ $R = 1$ AU, π = parallax angle ($R = 1$ and $[\pi] = ''$ gives d in pc)

$E_{\text{kin}} = \frac{mv^2}{2}$ kinetic energy

$E_{\text{pot}} = -\frac{GMm}{R}$ potential energy for a point mass m orbiting a point mass M

$E_{\text{kin}} = \frac{M(\Delta v)^2}{2}, \quad E_{\text{pot}} = -\frac{GM^2}{2R}$ (energies for an elliptical galaxy, with some definition of its radius R and velocity dispersion Δv)

$2E_{\text{kin}} + E_{\text{pot}} = 0$ the virial theorem

$V_c = \sqrt{\frac{GM}{R}}$ circular velocity

$\theta \approx 1.22\frac{\lambda}{\text{D}}$ resolution of telescope

$N(t) = N_0 e^{-\lambda t};\ \lambda = \frac{\ln 2}{t_{1/2}}$ radioactive decay

$\frac{dn_{\text{e}}}{dt} = N_{\text{star}}\frac{q}{V} - \alpha n_{\text{e}} n_{\text{p}}$ recombination and ionization equation

$\frac{L_{\text{I}}}{4 \cdot 10^{10} L_{\text{I},\odot}} \approx \left(\frac{V_{\text{max}}}{200\ \text{km/s}}\right)^4$ (the Tully-Fisher relation)

$\frac{L_{\text{V}}}{2 \cdot 10^{10} L_{\text{V},\odot}} \approx \left(\frac{\sigma}{200\ \text{km/s}}\right)^4$ (the Faber-Jackson relation)

$L_{\text{E}} = \frac{4\pi G M m_{\text{p}} c}{\sigma_{\text{T}}} \approx 1.3 \cdot 10^{31} \frac{M}{\text{M}_\odot}(\text{watt}) \approx 30000 \frac{M}{\text{M}_\odot} \text{L}_\odot$ (the Eddington luminosity)

Some mathematics

$x = \ln y \Leftrightarrow y = e^x$ $\quad e^{-x} = \frac{1}{e^x}$ $\quad e^{x+y} = e^x \cdot e^y$

$x = \lg y \Leftrightarrow y = 10^x$ $\quad \lg xy = \lg x + \lg y$ $\quad \lg \frac{x}{y} = \lg x - \lg y$

$f = u + v \qquad f' = u' + v'$

$f = uv \qquad f' = u'v + uv'$

$f = \frac{u}{v} \qquad f' = \frac{u'v - uv'}{v^2}$

$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$ where $y = F(u), u = f(x)$

$\frac{d}{dx}(x^n) = nx^{n-1}$, $\frac{d}{dx}(\ln x) = \frac{1}{x}$ (for $x > 0$), $\frac{d}{dx}(e^x) = e^x$

Constants and units

$G = 6.67 \cdot 10^{-11}$ N m^2 kg^{-2}
$c = 2.9979 \cdot 10^8$ m/s
$\sigma = 5.67 \cdot 10^{-8}$ W m^{-2} K^{-4}
$h = 6{,}62606896 \cdot 10^{-34}$ J s
$k = 1{,}3806504 \cdot 10^{-23}$ J K^{-1}

1 parsec (1 pc) = 3.26 light years = $3.0857 \cdot 10^{16}$ m
1 AU = $1.496 \cdot 10^{11}$ m
1 year = $3.156 \cdot 10^7$ s
1 arcmin (1') = 1°/60. 1 arcsec (1") = 1°/3600.

HI rest frequency ("21 cm line" of atomic hydrogen): 1420.4 MHz

Absolute magnitude of the Sun: +4.8
The solar constant (1 AU from the Sun): 1371 W/m^2
$H_0 = 100h$ km s^{-1} Mpc^{-1}. Use $h = 0.72$

Masses:	Earth: $5.97 \cdot 10^{24}$ kg,	Jupiter: $1.90 \cdot 10^{27}$ kg,	Sun: $1.99 \cdot 10^{30}$ kg
Radii:	Earth: 6378 km,	Jupiter: 71398 km,	Sun: $6.96 \cdot 10^5$ km.