Exam in

RRY125/ASM510 Modern astrophysics

Tid: 21 december 2012, kl. 14.00–18.00
Plats: V-salar, Chalmers
Ansvarig lärare: Magnus Thomasson ankn. 8587
(lärare besöker tentamen ca. kl.15.00 och 17.00)

Tillåtna hjälpmedel:
- Typgodkänd räknedosa
- Physics Handbook, Mathematics Handbook
- bifogat formelblad
- ordlista (ej elektronisk)

You may use:
- Chalmers-approved calculator
- Physics Handbook, Mathematics Handbook
- enclosed sheet with formulae
- dictionary (not electronic)

Grades:
The maximum number of points is 30.
Chalmers: Grade 3 requires 12 p, grade 4 requires 18 p, grade 5 requires 24 p.
GU: Grade G requires 12 p, grade VG requires 21 p.

Note: Motivate and explain each answer/solution carefully.

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1.

Choose the most reasonable of the given values for the following (do not give a motivation):

(a) Surface pressure at Venus (Earth = 1):	A) 100	B) 10	C) 1,	D) 0.1
(b) Age of the Moon:	A) 500 Myr,	B) 1 Gyr,	C) 4 Gyr,	D) 10 Gyr
(c) Diameter of a red giant (ly=light year):	A) 1 AU,	B) 1 ly,	C) 10^5 km,	D) 1 pc
(d) Central temperature of a main sequence star of spectral class A:	A) 20000 K,	B) $2 \cdot 10^6$ K,	C) $2 \cdot 10^7$ K,	D) 10^9 K
(e) Redshift of a nearby cluster of galaxies:	A) 0.0001,	B) 0.01,	C) –0.5,	D) 3.0
(f) Distance to a quasar (ly=light year):	A) 10^5 ly,	B) 10^6 AU,	C) 10 Gpc,	D) 3 Gpc
(g) Density parameter $\Omega_{\Lambda,0}$ ("dark energy") for the present cosmological model:	A) 0.04,	B) 0.3,	C) 0.7,	D) 1.0
(h) Redshift of the Last Scattering Surface	A) 0	B) 2.735	C) 6	D) 1100

(4 p)

2.

Clouds of HI typically have temperatures of 80 K. Such a cloud is observed with a radio telescope. What is the observed brightness temperature if the cloud is optically very thin or very thick, resp.? Take the cosmic microwave background radiation into account. **(2 p)**

3.

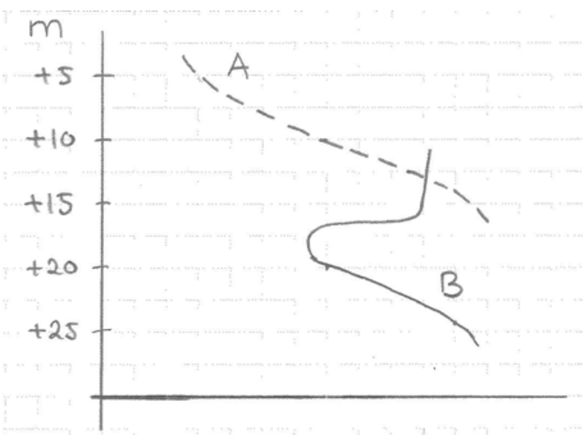
The spectroscopic method has been used to detect a planet around a star: the Calcium H line (396.847 nm) shifts back and forth ±0.000077 nm with a period of 11.6 days. The star's observed parallax is 0.4" and its apparent magnitude is 3.1. The spectrum shows that the star is a main sequence star, for which luminosity and mass are related by $\lg(L/L_\odot) = 3.7 \cdot \lg(M/M_\odot)$. The luminosity of the Sun is $3.84 \cdot 10^{26}$ W.

Estimate the surface temperature of the planet (assume it has an albedo of 0.7)! You may assume that the planet's orbit is seen edge-on. **(5 p)**

4.

The figure shows a Hertzsprung-Russell diagram for two star clusters. The y-axis shows the apparent magnitude.

a.) What is on the x-axis? Give *two* possibilities, and explain briefly what they are. **(1 p)**
b.) Which of the two clusters is nearest? **(1 p)**
c.) Which of the clusters is oldest? Motivate your answer briefly. **(1 p)**
d.) How are open and globular star clusters distributed in the Milky Way? **(1 p)**
e.) Which main sequence are stars burning hydrogen through the CNO-cycle? **(1 p)**



5.

The figure shows the observed radial velocity as a function of radius (in arcsec) for a spiral galaxy seen at an inclination of 45°. A supernova of Type Ia explodes in the galaxy. At its maximum brightness, its apparent magnitude is 14.2. Such supernovae have absolute magnitude –19.3.

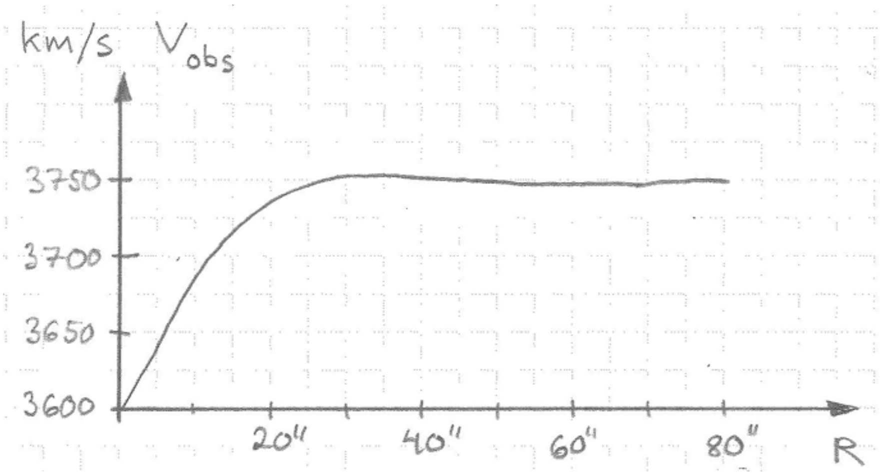
a.) Estimate the mass of the galaxy. **(2 p)**

b.) Estimate the luminosity of the galaxy (expressed in some suitable unit). **(1 p)**

c.) Assume that the Universe is matter-dominated and flat, and with $\Lambda=0$. Calculate the age of the Universe with the information given above and in the figure. (You may make a reasonable assumption of how the density depends on the scale factor.)

Hint: Do *not* just calculate the Hubble time. **(4 p)**

d.) Briefly describe the mechanism behind supernovae type Ia and type II, resp. Why are Type Ia supernovae better for distance measurements than Type II? **(3 p)**



6.

Explain why studies of the cosmic deuterium abundance are important in cosmology. What is the conclusion from such studies (exact numbers not needed)? **(2 p)**

7.

Put the following events during the evolution of the Universe in time order, starting with the event that happened first:

A. Reionization
B. Matter domination begins
C. Formation of quasars
D. Cosmic microwave background radiation emitted from the last scattering surface.
E. Primordial nucleosynthesis **(1 p)**

8.

What was the temperature of the cosmic microwave background radiation when typical distances between galaxies were half as large as now? **(1 p)**

Astrophysics equations, constants and units

Binary stars, planet+star, etc.

$m_1 r_1 = m_2 r_2$ and $m_1 V_1 = m_2 V_2$	centre of mass
$a = a_1 + a_2$	semi-major axis of relative orbit
$\frac{a^3}{P^2} = \frac{G(m_1 + m_2)}{4\pi^2}$	Keplers 3rd law (for the relative orbit)
$V = V_0 \sin i$	observed velocity
$V_0 = \frac{2\pi a}{P}$	velocity of circular orbit

Radiation, magnitudes, luminosities, etc.

$n_\nu = \frac{8\pi\nu^2}{c^3} \cdot \frac{1}{(e^{h\nu/kT} - 1)}$ [m^{-3} Hz^{-1}] $\qquad n \approx 2{,}03 \cdot 10^7 \cdot T^3$ [m^{-3}]

$U_\nu = \frac{8\pi h\nu^3}{c^3} \cdot \frac{1}{(e^{h\nu/kT} - 1)}$ [J m^{-3} Hz^{-1}] $\qquad U \approx 7{,}56 \cdot 10^{-16} \cdot T^4$ [J m^{-3}]

$I_\nu = \frac{2\pi h\nu^3}{c^2} \cdot \frac{1}{(e^{h\nu/kT} - 1)}$ [W m^{-2} Hz^{-1}] $\qquad I \approx 5{,}67 \cdot 10^{-8} \cdot T^4$ [W m^{-2}]

$I_\nu = \frac{2h\nu^3}{c^2} \cdot \frac{1}{(e^{h\nu/kT} - 1)}$ [W m^{-2} Hz^{-1} sr^{-1}] $\qquad \nu_{max} \approx 5{,}88 \cdot 10^{10} \cdot T$

$\frac{dI_\nu}{dz} = j_\nu - \alpha_\nu I_\nu \qquad S_\nu = \frac{j_\nu}{\alpha_\nu} \qquad d\tau_\nu = \alpha_\nu \, dz$

$I_\nu = I_{\nu,\mathrm{bg}} \cdot e^{-\tau_\nu} + S_\nu \cdot (1 - e^{-\tau_\nu}) \qquad T_\mathrm{b} = T_\mathrm{bg} \cdot e^{-\tau_\nu} + T_\mathrm{ex} \cdot (1 - e^{-\tau_\nu})$

$m = -2{,}5 \lg \frac{F}{F_0}$	m = apparent magnitude, F = observed flux
$m - M = 5 \lg \frac{d}{10\,\mathrm{pc}} + A$	M = absolute magnitude, d = distance, A = extinction
$A = ad$	a = interstellar extinction coefficient
$F = \sigma T^4$	F = flux from surface, T = surface temperature
$L = AF$	L = luminosity, A = emitting area

Stellar structure

$$\frac{dM_r}{dr} = 4\pi r^2 \rho$$

$$\frac{dP}{dr} = -\frac{GM_r}{r^2}\rho$$

$$\frac{dL_r}{dr} = 4\pi r^2 \rho \varepsilon$$

$$\frac{dT}{dr} = -\frac{3}{4a_{\mathrm{B}}c}\frac{\chi\rho}{T^3}\frac{L_r}{4\pi r^2}$$

$$\frac{dT}{dr} = \left(1 - \frac{1}{\gamma}\right)\frac{T}{P}\frac{dP}{dr}$$

Cosmology

$v = H_0 l$ Hubble's law

$1 + z = 1 + \frac{v}{c} = \frac{\lambda_{\mathrm{obs}}}{\lambda_{\mathrm{em}}} = \frac{\nu_{\mathrm{em}}}{\nu_{\mathrm{obs}}} = \frac{a_0}{a}$ redshift

$ds^2 = -c^2 dt^2 + a(t)^2 \left(\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\varphi^2)\right)$ Robertson-Walker metric

$\frac{\dot{a}^2}{a^2} + \frac{kc^2}{a^2} = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3}$ Friedmann equation with cosmological constant

Miscellaneous

$\frac{v}{c} = \frac{\Delta\lambda}{\lambda_0}$ the Doppler effect

$d = \frac{R}{\pi}$ $R = 1$ AU, π = parallax angle ($R = 1$ and $[\pi] = ''$ gives d in pc)

$E_{\text{kin}} = \frac{mv^2}{2}$ kinetic energy

$E_{\text{pot}} = -\frac{GMm}{R}$ potential energy for a point mass m orbiting a point mass M

$E_{\text{kin}} = \frac{M(\Delta v)^2}{2}, \quad E_{\text{pot}} = -\frac{GM^2}{2R}$ (energies for an elliptical galaxy, with some definition of its radius R and velocity dispersion Δv)

$2E_{\text{kin}} + E_{\text{pot}} = 0$ the virial theorem

$V_c = \sqrt{\frac{GM}{R}}$ circular velocity

$\theta \approx 1.22\frac{\lambda}{\text{D}}$ resolution of telescope

$N(t) = N_0 e^{-\lambda t}; \; \lambda = \frac{\ln 2}{t_{1/2}}$ radioactive decay

$\frac{dn_{\text{e}}}{dt} = N_{\text{star}}\frac{q}{V} - \alpha n_{\text{e}} n_{\text{p}}$ recombination and ionization equation

$\frac{L_{\text{I}}}{4\cdot 10^{10} L_{\text{I},\odot}} \approx \left(\frac{V_{\text{max}}}{200\text{ km/s}}\right)^4$ (the Tully-Fisher relation)

$\frac{L_{\text{V}}}{2\cdot 10^{10} L_{\text{V},\odot}} \approx \left(\frac{\sigma}{200\text{ km/s}}\right)^4$ (the Faber-Jackson relation)

$L_{\text{E}} = \frac{4\pi G M m_{\text{p}} c}{\sigma_{\text{T}}} \approx 1.3\cdot 10^{31}\frac{M}{\text{M}_\odot}(\text{watt}) \approx 30000\frac{M}{\text{M}_\odot}\text{L}_\odot$ (the Eddington luminosity)

Some mathematics

$x = \ln y \Leftrightarrow y = e^x$ $\qquad e^{-x} = \frac{1}{e^x}$ $\qquad e^{x+y} = e^x \cdot e^y$

$x = \lg y \Leftrightarrow y = 10^x$ $\qquad \lg xy = \lg x + \lg y$ $\qquad \lg \frac{x}{y} = \lg x - \lg y$

$f = u + v \qquad f' = u' + v'$

$f = uv \qquad f' = u'v + uv'$

$f = \frac{u}{v} \qquad f' = \frac{u'v - uv'}{v^2}$

$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$ where $y = F(u), u = f(x)$

$\frac{d}{dx}(x^n) = nx^{n-1}$, $\frac{d}{dx}(\ln x) = \frac{1}{x}$ (for $x > 0$), $\frac{d}{dx}(e^x) = e^x$

Constants and units

$G = 6.67 \cdot 10^{-11}$ N m^2 kg^{-2}
$c = 2.9979 \cdot 10^8$ m/s
$\sigma = 5.67 \cdot 10^{-8}$ W m^{-2} K^{-4}
$h = 6{,}62606896 \cdot 10^{-34}$ J s
$k = 1{,}3806504 \cdot 10^{-23}$ J K^{-1}

1 parsec (1 pc) = 3.26 light years = $3.0857 \cdot 10^{16}$ m
1 AU = $1.496 \cdot 10^{11}$ m
1 year = $3.156 \cdot 10^7$ s
1 arcmin (1') = 1°/60. 1 arcsec (1") = 1°/3600.

HI rest frequency ("21 cm line" of atomic hydrogen): 1420.4 MHz

Absolute magnitude of the Sun: +4.8
The solar constant (1 AU from the Sun): 1371 W/m^2
$H_0 = 100h$ km s^{-1} Mpc^{-1}. Use $h = 0.72$

Masses:	Earth: $5.97 \cdot 10^{24}$ kg,	Jupiter: $1.90 \cdot 10^{27}$ kg,	Sun: $1.99 \cdot 10^{30}$ kg
Radii:	Earth: 6378 km,	Jupiter: 71398 km,	Sun: $6.96 \cdot 10^5$ km.

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(1)

a)	A	e)	B
b)	C	f)	D
c)	A	g)	C
d)	C	h)	D

(2) $T_b = T_{bg} e^{-\tau} + T_{ex}(1-e^{-\tau})$

$\tau \ll 1 \Rightarrow T_b = T_{bg} = 2.7\ K$. $\tau \gg 1 \Rightarrow T_b = T_{ex} = 80\ K$

(3) We need the star's luminosity L, and the planets distance from the star, a.

$\pi = 0.4'' \Rightarrow d = 1/0.4 = 2.5\ pc$

$m - M = 5 \lg \frac{d}{10pc} \Rightarrow M = m - 5\lg \frac{d}{10pc} = 3.1 - 5 \lg \frac{2.5}{10} = 6.11$

Let the star have a luminosity of N times that of the Sun. The flux is then N times the flux from the Sun. Study the star from 10 pc distance. Then

$$M = -2.5 \lg \frac{NF_\odot}{F_0} = -2.5\lg N - 2.5 \lg \frac{F_\odot}{F_0} = -2.5 \lg N + M_\odot =$$

$$= -2.5 \lg N + 4.8 = 6.11 \Rightarrow N = 0.299$$

Thus $L = 0.299\, L_\odot$. Write $\mathcal{M}$ for mass

$\lg(L/L_\odot) = 3.7 \lg(\mathcal{M}/\mathcal{M}_\odot) \Rightarrow \mathcal{M}/\mathcal{M}_\odot = 0.721$

Kepler's 3rd law: $a = \left(\frac{G\mathcal{M}}{4\pi^2} P^2\right)^{1/3} = 1.345 \cdot 10^{10} = 0.0899\ AU$

Thermal balance, assuming the planet is isothermal:

$$\frac{L}{4\pi a^2}(1-A)\pi R^2 = 4\pi R^2 \sigma T^4 \Rightarrow T = \left(\frac{0.299 L_\odot (1-A)}{16\pi a^2 \sigma}\right)^{1/4} =$$

$$= \underline{508\ K}$$

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(4) a) T, B-V, spectral class, colour (see book)

b) A (lowest magnitude)

c) B (evolved stars)

d) See book

e) Upper part of M.S.

(5.) a) $V_{obs} = V \sin i \Rightarrow V = 150/\sin 45^0 = 212$ km/s

$V = \sqrt{GM/R} \Rightarrow M = RV^2/G$. $R = ?$

From Supernova: $m - M = 5 \lg d/10\,pc \Rightarrow$

$\Rightarrow 14.2 + 19.3 = 5 \lg d/10\,pc \Rightarrow d = 50.1$ Mpc

$R = 50.1 \cdot 10^6 \cdot 80'' \cdot \frac{1}{3600} \cdot \frac{\pi}{180}$ pc $= 19431$ pc $= 6.00 \cdot 10^{20}$ m

$M = RV^2/G = 4.04 \cdot 10^{41}$ kg $= 2.0 \cdot 10^{11} M_\odot$

b) Tully-Fisher:

$$\frac{L_I}{4 \cdot 10^{10} L_{I,\odot}} \approx \left(\frac{V_{max}}{200\,km/s}\right)^4 \Rightarrow L_I = 5.0 \cdot 10^{10} L_{I,\odot}$$

c) $k = 0$, $\Lambda = 0$, $H_0 = \frac{V}{d} = \frac{3600 \cdot 10^3}{50.1 \cdot 10^6 \cdot 3.0857 \cdot 10^{16}} = 2.33 \cdot 10^{-18}\,s^{-1}$

Friedmann: $\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3} \rho$. Matter: $\rho = \rho_{0,c} \left(\frac{a_0}{a}\right)^3$

$$\Rightarrow \frac{da}{dt} = \sqrt{\frac{8\pi G}{3} \rho_{0,c} a_0^3}\; a^{-1/2} \Rightarrow \int_0^a a^{1/2} da = \sqrt{\frac{8\pi G}{3} \rho_{0,c} a_0^3} \int_0^t dt$$

$$\Rightarrow a(t) = \left(\frac{3}{2}\right)^{2/3} \left(\frac{8\pi G}{3} \rho_{c,0} a_0^3\right)^{1/3} t^{2/3} = \left(\frac{3}{2}\right)^{2/3} \left(H_0^2 a_0^3\right)^{1/3} t^{2/3}$$

$$\Rightarrow \frac{a(t)}{a_0} = \left(\frac{3}{2} H_0 t\right)^{2/3}. \quad a(t) = a_0 \Rightarrow t = \frac{2}{3} H_0^{-1} = 2.86 \cdot 10^{17}\,s$$

$= \underline{9.1\ Gyr}$

d.) See the book

(6) See the book. Non-baryonic dark matter needed

(7) E, B, D, C, A

(8) $(1+z =) \frac{a_0}{a} = \frac{T}{T_0} \Rightarrow T = 2 T_0 = 5.47$ K.