Microwave Engineering (MCC121)

Resonators

Outline

- **Series and Parallel Resonant Circuits**
  - Series Resonant Circuits
  - Parallel Resonant Circuits
  - Loaded and Unloaded Q
- **Transmission line resonators**
  - Short-circuited $\lambda/2$ line
  - Short-circuited $\lambda/4$ line
  - Open-circuited $\lambda/2$ line
- **Waveguide cavities**
  - Rectangular waveguie cavities
  - Circular/cylindrical waveguide cavities
- **Dielectric Resonators**
- **Excitation of Resonators**

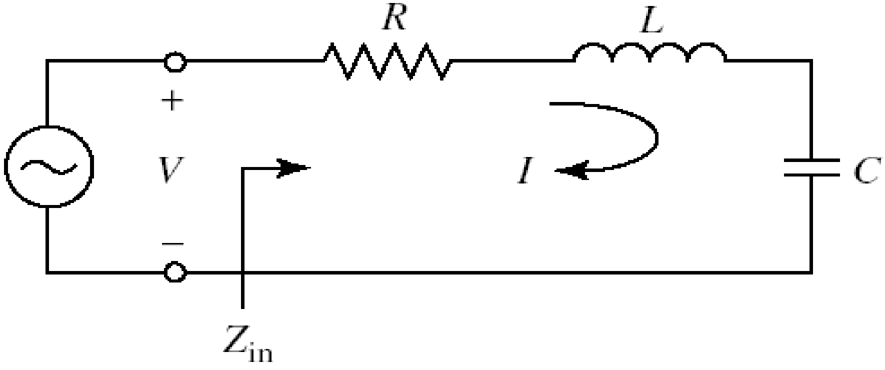
Resonators Applications

- Filters
- Oscillators
- Frequency meters
- Tuned Amplifiers

Series resonant circuits

$$Z_{in} = R + j\omega L - j\frac{1}{C\omega}$$

$$P_{in} = \frac{1}{2}VI^* = \frac{1}{2}Z_{in}|I|^2 = \frac{1}{2}Z_{in}\left|\frac{V}{Z_{in}}\right|^2$$



- **Complex power delivered to the resonator** $$P_{in} = \frac{1}{2}|I|^2\left(R + j\omega L - j\frac{1}{C\omega}\right)$$
- **Power dissipated by the resistor** $$P_{loss} = \frac{1}{2}|I|^2 R$$
- **Average magnetic energy stored in the inductor** $$W_m = \frac{1}{4}|I|^2 L$$
- **Average electric energy stored in the capacitor** $$W_e = \frac{1}{4}|V_c|^2 C = \frac{1}{4}|I|^2\frac{1}{\omega^2 C}$$

$$P_{in} = P_{Loss} + 2j\omega(W_m - W_e)$$

Series resonant circuits

At resonance: $W_e = W_m$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$Q = \omega_0 \frac{2W_m}{P_{Loss}} = \frac{\omega_0 L}{R}$$

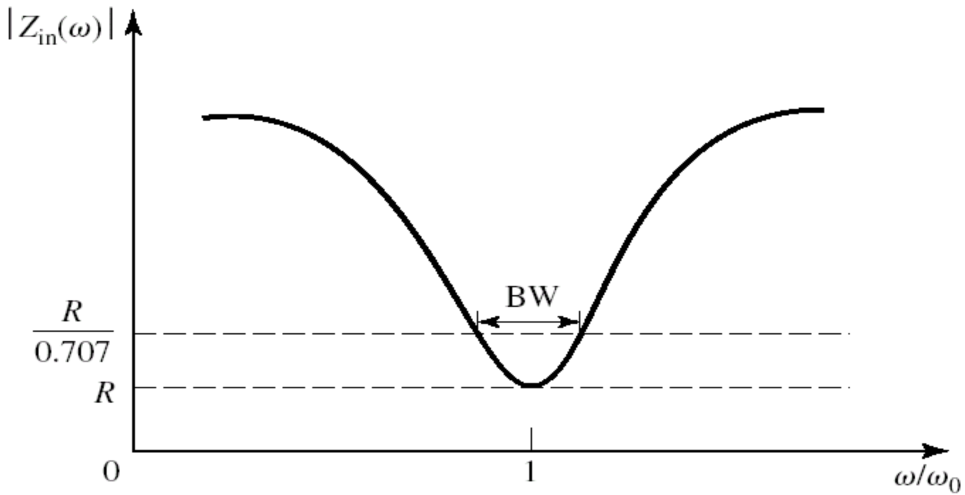
Near resonance: $\omega = \omega_0 + \Delta\omega$

$$Z_{in} = R + j\omega L\left(1 - \frac{1}{\omega^2 LC}\right)$$

$$Z_{in} = R + j\omega L\left(\frac{\omega^2 - \omega_0^2}{\omega^2}\right)$$

$$Z_{in} \approx R + j\omega L\left(\frac{2\omega\Delta\omega}{\omega^2}\right) \approx R + j2\Delta\omega L$$

$$Z_{in} = R\left(1 + j\frac{2Q\Delta\omega}{\omega_0}\right)$$



BW: Frequency interval were $|Zin(\omega)|^2 < 2R^2$

$$\left|1 + j\frac{2Q\Delta\omega}{\omega_0}\right|^2 = 2$$

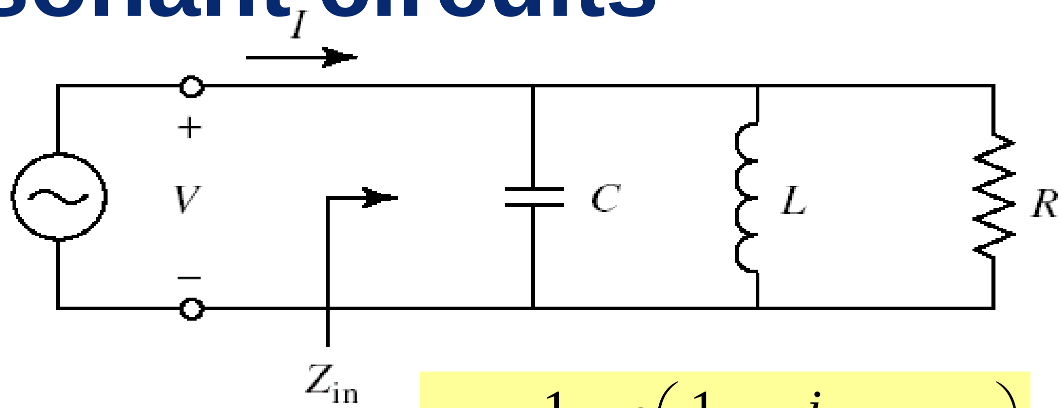
$$1 + 4Q^2\left(\frac{\Delta\omega}{\omega_0}\right)^2 = 2$$

$$\left(\frac{\Delta\omega}{\omega_0}\right)^2 = \frac{1}{4Q^2} \Rightarrow BW = 2\frac{\Delta\omega}{\omega_0} = \frac{1}{Q}$$

Parallel resonant circuits

$$Z_{in} = \left(\frac{1}{R} + \frac{1}{j\omega L} + jC\omega \right)^{-1}$$

$$P_{in} = \frac{1}{2} VI^* = \frac{1}{2} Z_{in} |I|^2 = \frac{1}{2} Z_{in} \left| \frac{V}{Z_{in}} \right|^2$$



- **Complex power delivered to the resonator**

$$P_{in} = \frac{1}{2} |V|^2 \left(\frac{1}{R} + \frac{j}{\omega L} - jC\omega \right)$$

- **Power dissipated by the resistor**

$$P_{loss} = \frac{1}{2} \frac{|V|^2}{R}$$

- **Average magnetic energy stored in the inductor**

$$W_m = \frac{1}{4} |I_L|^2 L = \frac{1}{4} |V|^2 \frac{1}{\omega^2 L}$$

- **Average electric energy stored in the capacitor**

$$W_e = \frac{1}{4} |V|^2 C$$

$$P_{in} = P_{Loss} + 2j\omega (W_m - W_e)$$

Parallel resonant circuits

At resonance: $W_e = W_m$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$Q = \omega_0 \frac{2W_m}{P_{Loss}} = \frac{R}{\omega_0 L} = \omega_0 RC$$

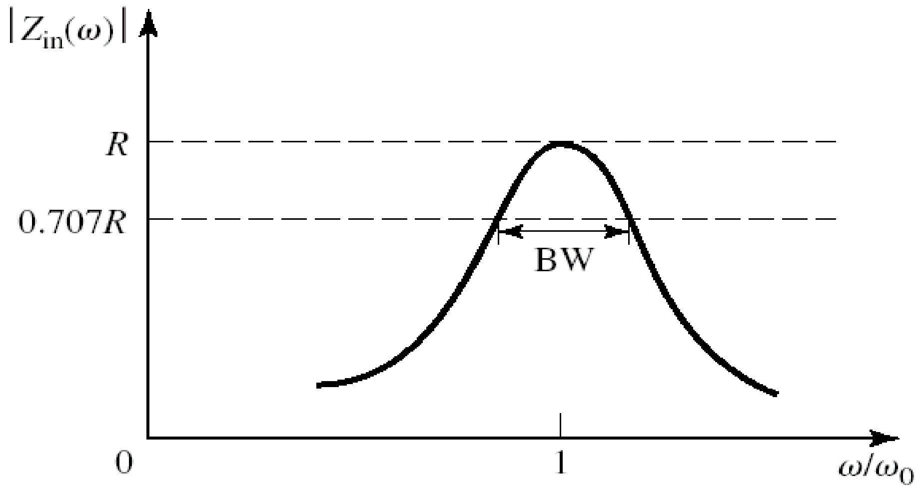
Near resonance: $\omega = \omega_0 + \Delta\omega$

$$\frac{1}{Z_{in}} = \frac{1}{R} + \frac{1}{jL\omega} + jC\omega$$

$$\frac{1}{Z_{in}} \approx \frac{1}{R} + \frac{1 - \frac{\Delta\omega}{\omega_0}}{j\omega_0 L} + j\omega_0 C + j\Delta\omega C$$

$$\frac{1}{Z_{in}} \approx \frac{1}{R} + \frac{j\Delta\omega}{\omega_0 L} + j\Delta\omega C \approx \frac{1}{R} + 2j\Delta\omega C$$

$$Z_{in} = \frac{R}{1 + j\frac{2Q\Delta\omega}{\omega_0}}$$



BW: Frequency interval were $|Zin(\omega)|^2 > 2R^2$

$$\left|1 + j\frac{2Q\Delta\omega}{\omega_0}\right|^2 = 2$$

$$1 + 4Q^2\left(\frac{\Delta\omega}{\omega_0}\right)^2 = 2$$

$$\left(\frac{\Delta\omega}{\omega_0}\right)^2 = \frac{1}{4Q^2} \Rightarrow BW = 2\frac{\Delta\omega}{\omega_0} = \frac{1}{Q}$$

Loaded and unloaded Q factor

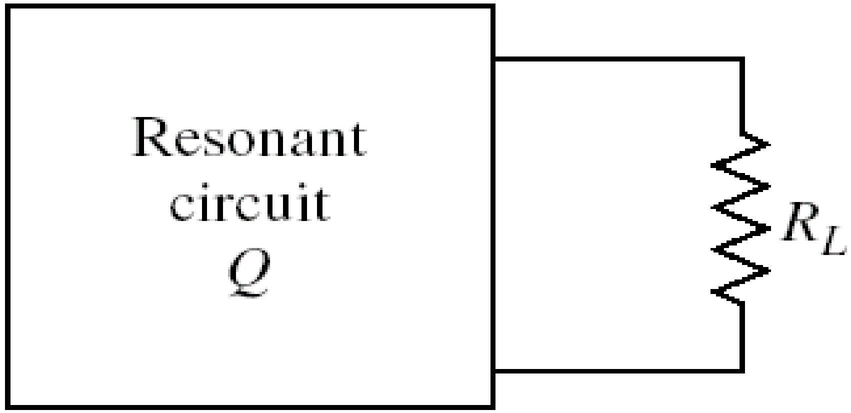
➢ Unloaded Quality Factor: Q

➢ Loaded Quality Factor: Q_L

➢ External Quality Factor: Q_e

(connecting the resonator to an external load)

$$Q_e = \begin{cases} \dfrac{\omega_0 L}{R_L} & \textit{for series circuits} \\ \dfrac{R_L}{\omega_0 L} & \textit{for parallel circuits} \end{cases}$$



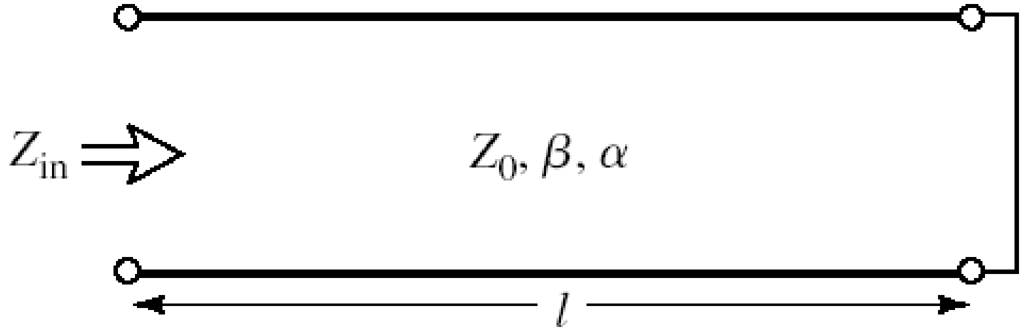
$$\frac{1}{Q_L} = \frac{1}{Q} + \frac{1}{Q_e} \Rightarrow (Q_L \leq Q)$$

Short circuited λ/2 transmission line

$$Z_{in} = Z_0 \frac{Z_L + Z_0 \tanh(\alpha + j\beta)l}{Z_0 + Z_L \tanh(\alpha + j\beta)l}$$

$$Z_L = 0 \Rightarrow Z_{in} = Z_0 \tanh(\alpha + j\beta)l$$

$$Z_{in} = Z_0 \frac{\tan\alpha l + j\tan\beta l}{1 + j\tan\beta l \tan\alpha l}$$



Low losses: $\tan\alpha l \approx \alpha l$

In the vicinity of ω_0 : $\omega = \omega_0 + \Delta\omega$

$$\beta l = \pi + \pi\frac{\Delta\omega}{\omega_0}$$

$$\tan\beta l = \tan\left(\pi + \pi\frac{\Delta\omega}{\omega_0}\right) = \tan\pi\frac{\Delta\omega}{\omega_0} \approx \pi\frac{\Delta\omega}{\omega_0}$$

$$Z_{in} = Z_0\left(\alpha l + j\pi\frac{\Delta\omega}{\omega_0}\right)$$

Identification with Series RLC

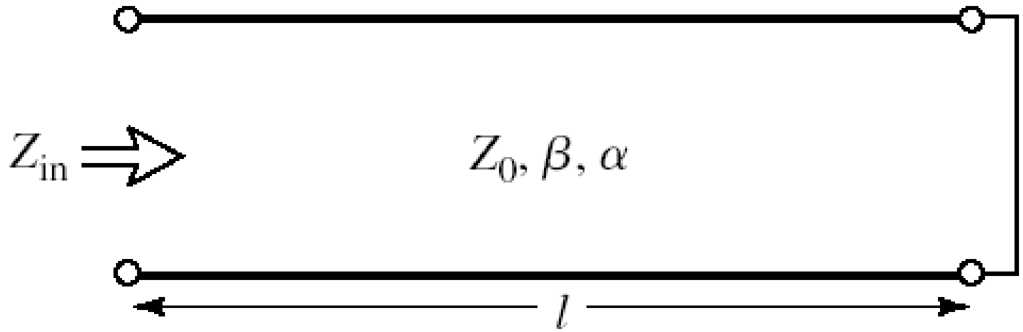
$$R = Z_0 \alpha l \qquad L = \frac{Z_0\pi}{2\omega_0} \qquad C = \frac{1}{\omega_0^2 L} \qquad Q = \frac{\beta}{2\alpha}$$

Short circuited λ/4 transmission line

$$Z_{in} = Z_0 \frac{Z_L + Z_0 \tanh(\alpha + j\beta)l}{Z_0 + Z_L \tanh(\alpha + j\beta)l}$$

$$Z_L = 0 \Rightarrow Z_{in} = Z_0 \tanh(\alpha + j\beta)l$$

$$Z_{in} = Z_0 \frac{\tan \alpha l + j \tan \beta l}{1 + j \tan \beta l \tan \alpha l}$$



Low losses: $\tan\alpha l \approx \alpha l$

In the vicinity of ω_0 : $\omega = \omega_0 + \Delta\omega$

$$\beta l = \frac{\pi}{2} + \frac{\pi}{2}\frac{\Delta\omega}{\omega_0}$$

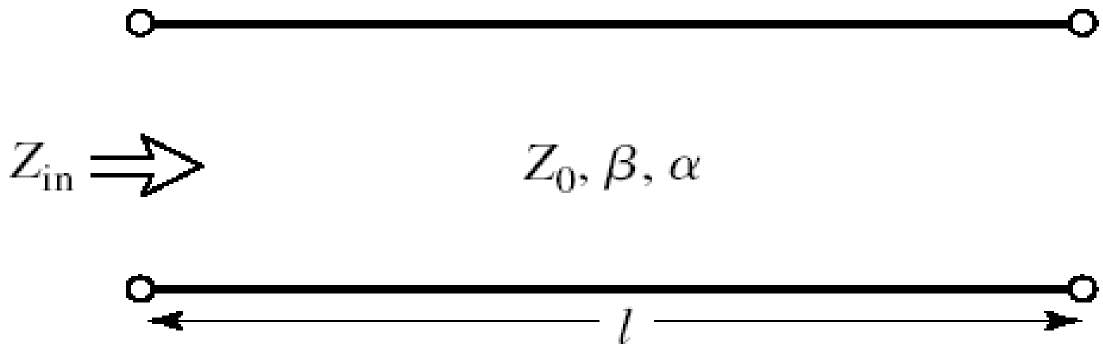
$$\tan \beta l = \tan\left(\frac{\pi}{2} + \frac{\pi}{2}\frac{\Delta\omega}{\omega_0}\right) = \frac{1}{-\tan \pi \frac{\Delta\omega}{2\omega_0}} \approx -\frac{2\omega_0}{\pi\Delta\omega}$$

$$Z_{in} = \frac{Z_0}{\alpha l + j\pi \frac{\Delta\omega}{2\omega_0}}$$

Identification with Parallel RLC

$$R = \frac{Z_0}{\alpha l} \qquad C = \frac{\pi}{4\omega_0 Z_0} \qquad L = \frac{1}{\omega_0^2 C} \qquad Q = \frac{\beta}{2\alpha}$$

Open circuited λ/2 transmission line



$$Z_{in} = Z_0 \frac{Z_L + Z_0 \tanh(\alpha + j\beta)l}{Z_0 + Z_L \tanh(\alpha + j\beta)l}$$

$$Z_L = \infty \Rightarrow Z_{in} = Z_0 \coth(\alpha + j\beta)l$$

$$Z_{in} = Z_0 \frac{1 + j\tan\beta l \tan\alpha l}{\tan\alpha l + j\tan\beta l}$$

Low losses: $\tan\alpha l \approx \alpha l$

In the vicinity of ω_0 : $\omega = \omega_0 + \Delta\omega$

$$\beta l = \pi + \pi\frac{\Delta\omega}{\omega_0}$$

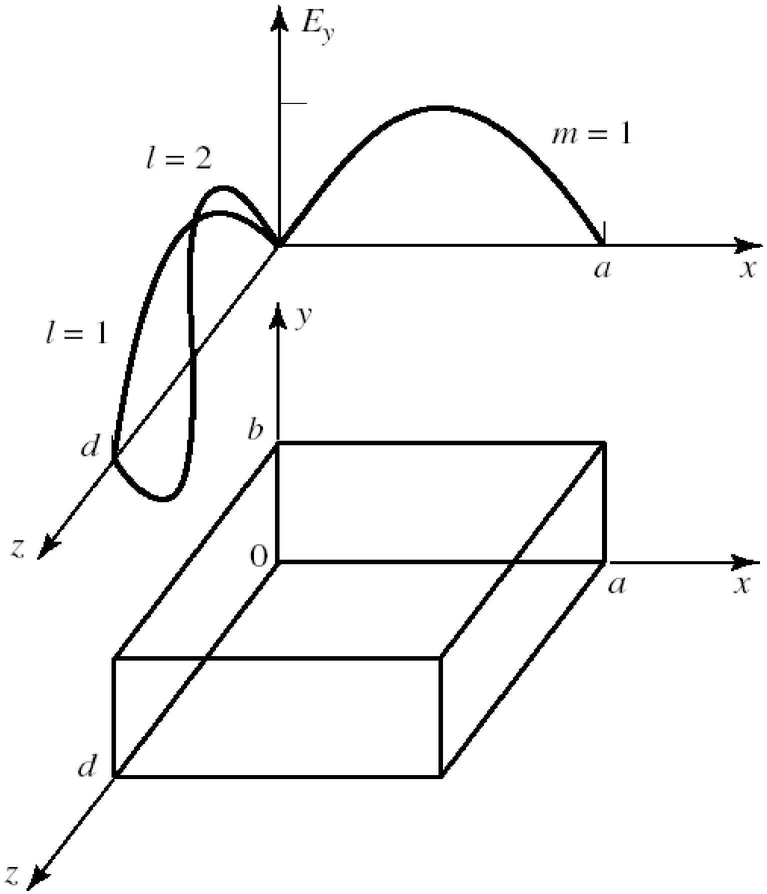
$$\tan\beta l = \tan\left(\pi + \pi\frac{\Delta\omega}{\omega_0}\right) = \tan\pi\frac{\Delta\omega}{\omega_0} \approx \pi\frac{\Delta\omega}{\omega_0}$$

$$Z_{in} = \frac{Z_0}{\alpha l + j\pi\frac{\Delta\omega}{\omega_0}}$$

Identification with Parallel RLC

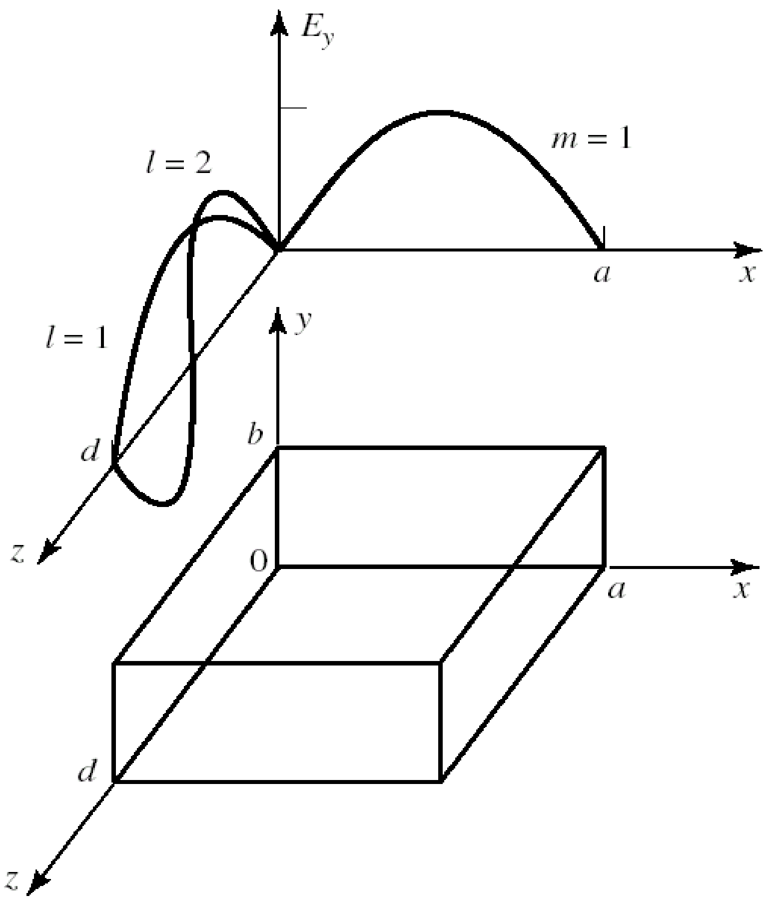
$$R = \frac{Z_0}{\alpha l} \qquad C = \frac{\pi}{2\omega_0 Z_0} \qquad L = \frac{1}{\omega_0^2 C} \qquad Q = \frac{\beta}{2\alpha}$$

Rectangular waveguide cavities



- Waveguide section closed by conductive walls in z direction at z=0 and z=d
- Provide usually better Q values transmission line resonators above 1 GHz.
- Power is dissipated in the metallic walls and dielectric filling the cavity
- Coupling is done by small aperture or small loop

Rectangular waveguide cavities



The E field of a TE_{mn} or TM_{mn} wave can be written as:

$$E_t(x,y,z) = e(x,y)\left[A^+ e^{-j\beta_{mn} z} + A^- e^{+j\beta_{mn} z}\right]$$

$$\beta_{mn} = \sqrt{k^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{m\pi}{a}\right)^2}$$

At z=0 and z=d E=0 (metallic walls short)

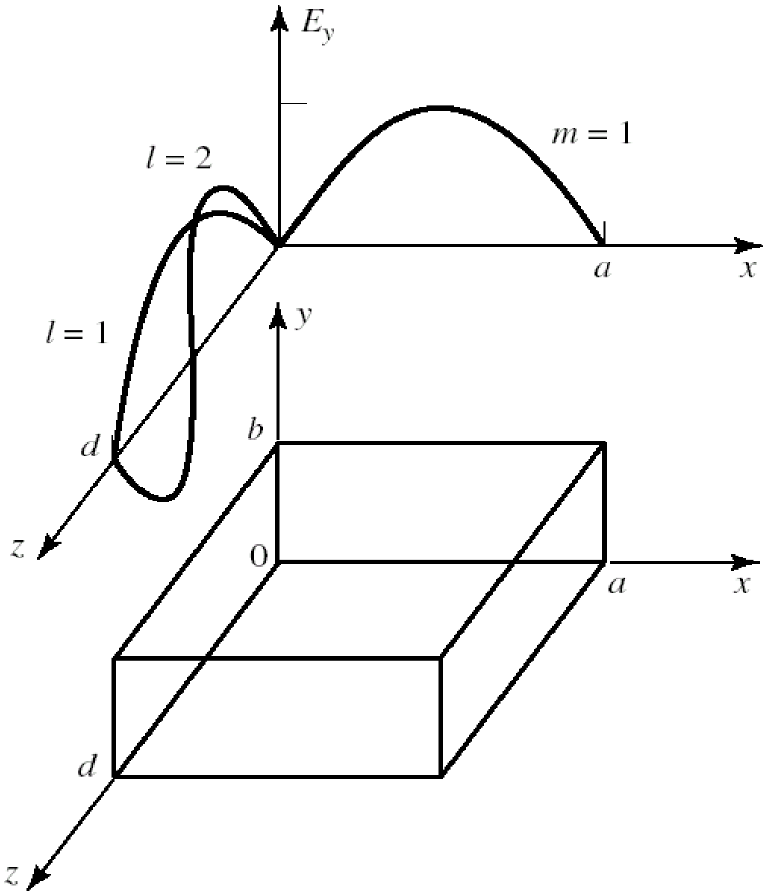
$$A^+ = -A^-$$

$$-e(x,y)A^+ \sin(\beta_{mn} d) = 0$$

$$\Rightarrow \beta_{mn} d = l\pi$$

A rectangluar cavity is similar to a shorted λ/2 transmission line

Rectangular waveguide cavities



Resonant wave numbers and frequency for the TE_{mnl} or TM_{mnl} are given then by

$$k_{mnl} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{l\pi}{d}\right)^2}$$

$$f_{mnl} = \frac{ck_{mnl}}{2\pi\sqrt{\mu_r\varepsilon_r}} = \frac{c}{2\pi\sqrt{\mu_r\varepsilon_r}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{l\pi}{d}\right)^2}$$

If b<a<d the dominant resonant mode is TE_{101} .For a TE_{10l} cavity:

$$E_y = E_0 \sin\frac{\pi x}{a}\sin\frac{l\pi z}{d}$$

$$H_x = \frac{-jE_0}{Z_{TE}}\sin\frac{\pi x}{a}\cos\frac{l\pi z}{d}$$

$$H_z = \frac{j\pi E_0}{k\eta a}\cos\frac{\pi x}{a}\sin\frac{l\pi z}{d}$$

$$E_0 = -2A^+$$

$$Z_{TE} = \frac{k\eta}{\beta}$$

Q value for a Rectangular waveguide cavities of the TE_{10l} mode

➢A resonance:

the time average stored electric energy = time average stored magnetic energy

$$W_e = \frac{\varepsilon}{4}\int_V E_y E_y^* dv = \frac{\varepsilon abd}{16} E_0$$

$$W_m = \frac{\mu}{4}\int_V \left(H_x H_x^* + H_z H_z^*\right) dv = \frac{\varepsilon abd}{16}\left(\frac{1}{Z_{TE}^2} + \frac{1}{k^2\eta^2 a^2}\right) E_0 = W_e$$

➢Losses in the cavity are caused by:

- Finite conductivity, metallic losses
- Non-perfect dielectric, dielectric losses

Q value for a Rectangular waveguide cavities of the TE_{10l} mode

➢Metallic losses:

Using the perturbation theory and bearing in mind that the surface current are given by:

$$\vec{J_S} = \vec{n} \times \vec{H}$$

$$P_{loss,metal} = \frac{R_\delta}{2} \int_{All\,6\,walls} J_s J_s^* dS = \frac{R_\delta}{2} \int_{All\,6\,walls} |H_{\tan}| dS$$

$$R_\delta = \frac{1}{\sigma \delta_s} ; \delta_s = \sqrt{\frac{2}{\omega \mu \varepsilon}}$$

$$P_{loss,metal} = \frac{R_\delta E_0^2 \lambda^2}{8\eta^2} \left[\frac{l^2 ab}{d^2} + \frac{bd}{a^2} + \frac{l^2 a}{2d} + \frac{d}{2a} \right]$$

➢Dielectric losses:

$$P_{loss,diel} = \frac{1}{2} \int_V J E^* dv = \frac{\omega \varepsilon''}{2} \int_V |E|^2 dv \, ; \varepsilon = \varepsilon' + j\varepsilon''$$

$$P_{loss,diel} = \frac{abd\omega\varepsilon'' |E_0|}{8}$$

Q_c with only metallic losses:

$$Q_C = \frac{2\omega_0 W_e}{P_{loss,metal}} = \frac{(kad)^3 b\eta}{2\pi^2 R_\delta} \frac{1}{(2l^2 a^3 b + 2bd^3 + l^2 a^3 d + ad^3)}$$

Q_d with only dielectric losses:

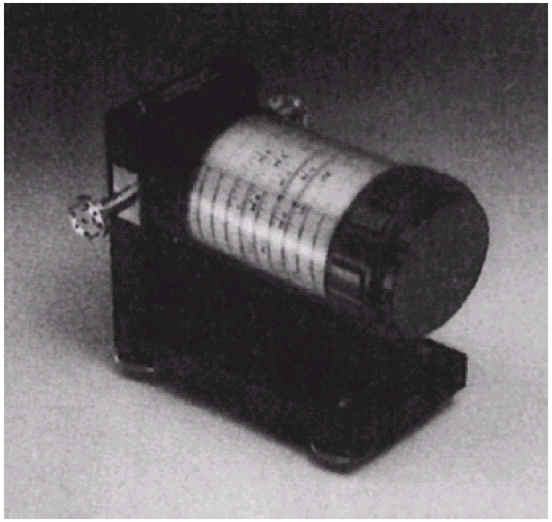
$$Q_d = \frac{2\omega_0 W_e}{P_{loss,diel}} = \frac{\varepsilon'}{\varepsilon''} = \frac{1}{\tan \delta}$$

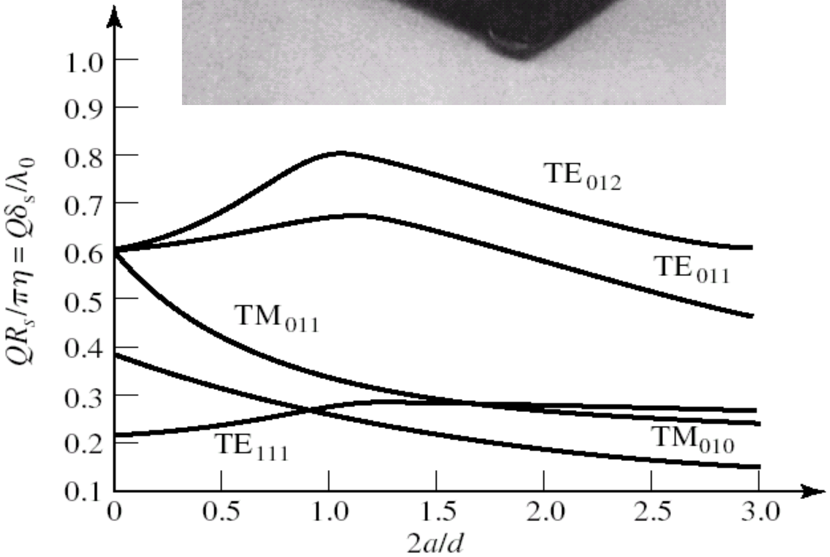
Q taking both into account:

$$Q = \frac{2\omega_0 W_e}{P_{loss,metal} + P_{loss,diel}}$$

$$\frac{1}{Q} = \frac{1}{Q_C} + \frac{1}{Q_d}$$

Circular waveguide cavities





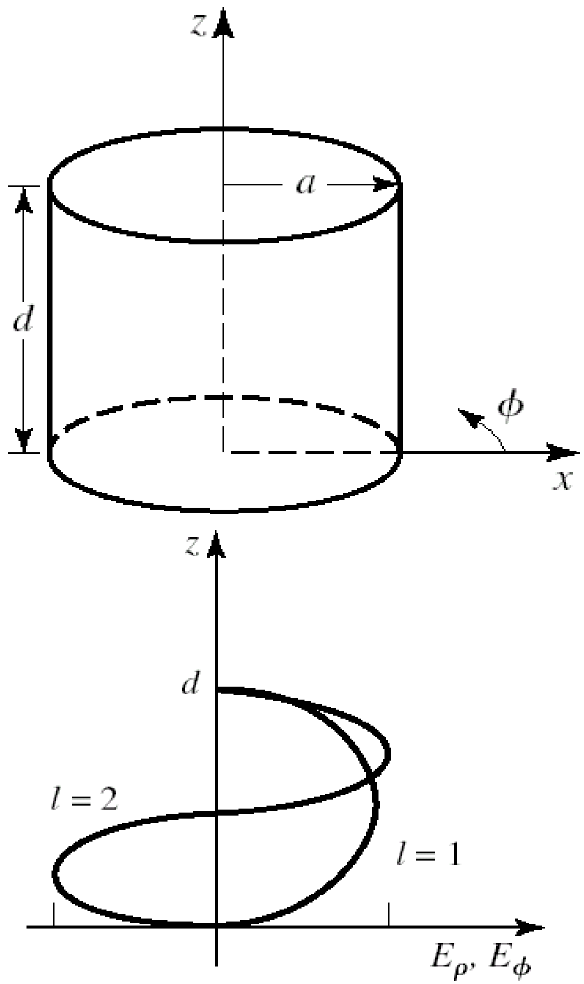
- Circular Waveguide section closed by conductive walls in z direction at z=0 and z=d

- same idea and analysis strategy as for the rectangular waveguide cavity.

- The lowest resonance frequency is obtained for the TE_{111} mode, which correspond to the TE_{11} for a waveguide

- TE_{011} mode is often used for frequency meters because of its much superior Q value over the TE_{111} mode

Cylindrical waveguide cavities



The E field of a TE_{mn} or TM_{mn} wave can be written as:

$$E_t(\rho,\phi,z) = e(\rho,\phi)\left[A^{+}e^{-j\beta_{nm}z} + A^{+}e^{+j\beta_{nm}z}\right]$$

$$\beta_{nm} = \sqrt{k^2 - \left(\frac{p'_{nm}}{a}\right)^2} \quad or \quad \beta_{nm} = \sqrt{k^2 - \left(\frac{p_{nm}}{a}\right)^2}$$

At z=0 and z=d E=0 (metallic walls short)

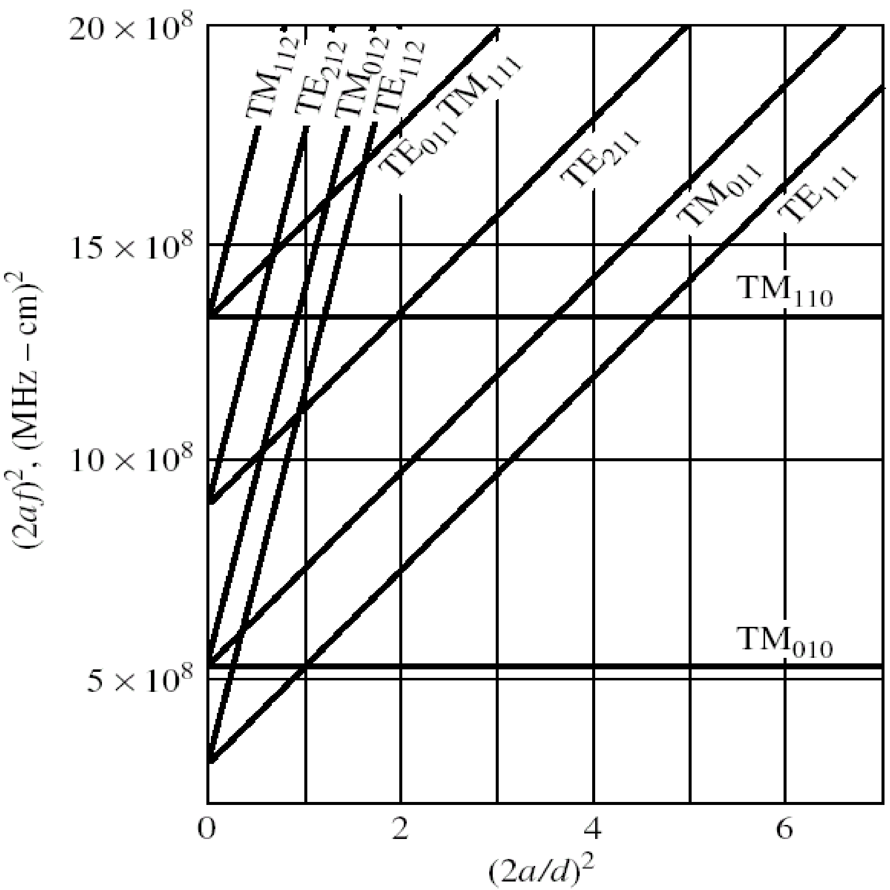
$$A^{+} = -A^{-}$$

$$-e(x,y)A^{+}\sin(\beta_{mn}d) = 0$$

$$\Rightarrow \beta_{nm}d = l\pi$$

A cylindrical cavity length must be an integer number of λ/2 long

Cylindrical waveguide cavities



Resonant frequencies for the TE_{nml} or TM_{nml} are given then by

$$f_{nml,TE} = \frac{c}{2\pi\sqrt{\mu_r \varepsilon_r}} \sqrt{\left(\frac{p'_{nm}}{a}\right)^2 + \left(\frac{l\pi}{d}\right)^2}$$

$$f_{nml,TM} = \frac{c}{2\pi\sqrt{\mu_r \varepsilon_r}} \sqrt{\left(\frac{p_{nm}}{a}\right)^2 + \left(\frac{l\pi}{d}\right)^2}$$

For a given cavity size, use the chart to determine which modes can be excited

Q value for a Rectangular waveguide cavities of the TE_{10l} mode

➢A resonance:

the time average stored electric energy = time average stored magnetic energy

$$W_e = \frac{\varepsilon}{4}\int_V \left(\left|E_\rho\right|^2 + \left|E_\phi\right|^2 \right) \rho d\rho d\phi dz \quad (cylindrical\ coordinates)$$

$$W_e = \frac{\varepsilon k^2 \eta^2 a^4 H_0^2 \pi d}{16\left(p'_{n_m}\right)^2} \left[1 - \left(\frac{n}{p'_{n_m}} \right)^2 \right] J_n^2\left(p'_{nm}\right) = W_m$$

➢Losses in the cavity are caused by:

- •Finite conductivity, metallic losses
- •Non-perfect dielectric, dielectric losses

Q value for a cylindrical waveguide cavities of the TE_{10l} mode

- Metallic losses:

Using the perturbation theory and beaing in mind that the surface current are given by:

$$\vec{J_S} = \vec{n} \times \vec{H}$$

$$P_{loss,metal} = \frac{R_\delta}{2} \int_{walls} J_s J_s^* dS = \frac{R_\delta}{2} \int_{walls} |H_{\tan}| dS$$

$$R_\delta = \frac{1}{\sigma \delta_s} ; \delta_s = \sqrt{\frac{2}{\omega \mu \varepsilon}}$$

$$P_{loss,metal} = \frac{R_\delta}{2} \pi H_0^2 J_n^2 (p'_{nm}) \left\{ \frac{da}{2} \left[1 + \left(\frac{\beta a n}{(p'_{nm})^2} \right)^2 \right] + \left(\frac{\beta a^2}{p'_{nm}} \right)^2 \left(1 - \frac{n^2}{(p'_{nm})^2} \right) \right\}$$

- Dielectric losses:

$$P_{loss,diel} = \frac{1}{2} \int_V J E^* dv = \frac{\omega \varepsilon''}{2} \int_V \left(|E_\rho|^2 + |E_\phi|^2 \right) dv ; \varepsilon = \varepsilon' + j\varepsilon''$$

$$P_{loss,diel} = \frac{\omega \varepsilon'' k^2 \eta^2 a^2 H_0^2 |E_0|}{8 (p'_{nm})^2} \left[1 - \left(\frac{n}{p'_{nm}} \right)^2 \right] J_n^2 (p'_{nm})$$

Q_c with only metallic losses:

$$Q_C = \frac{2\omega_0 W_e}{P_{loss,metal}} = \frac{(ka)^3 \eta a d}{4 (p'_{nm})^2 R_\delta} \frac{1 - \left(\frac{n}{p'_{nm}} \right)^2}{\left\{ \frac{ad}{2} \left[1 + \left(\frac{\beta a n}{(p'_{nm})^2} \right)^2 \right] + \left(\frac{\beta a^2}{p'_{nm}} \right)^2 \left(1 - \frac{n^2}{(p'_{nm})^2} \right) \right\}}$$

$$Q_C \propto 1 / \sqrt{f}$$

Q_d with only dielectric losses:

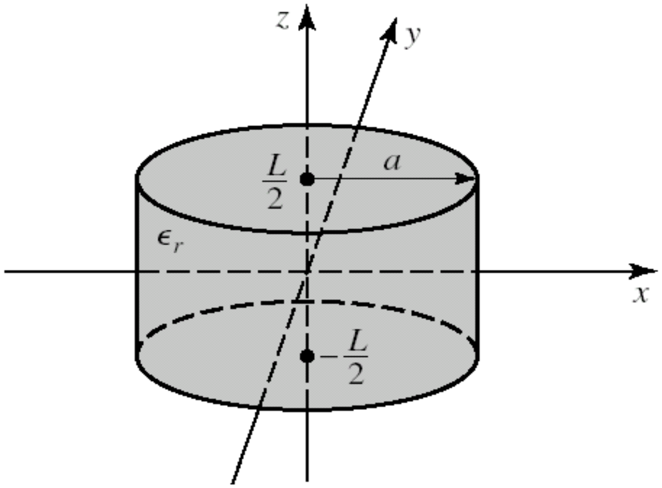
$$Q_d = \frac{2\omega_0 W_e}{P_{loss,diel}} = \frac{\varepsilon'}{\varepsilon''} = \frac{1}{\tan \delta}$$

Q taking both into account:

$$Q = \frac{2\omega_0 W_e}{P_{loss,metal} + P_{loss,diel}}$$

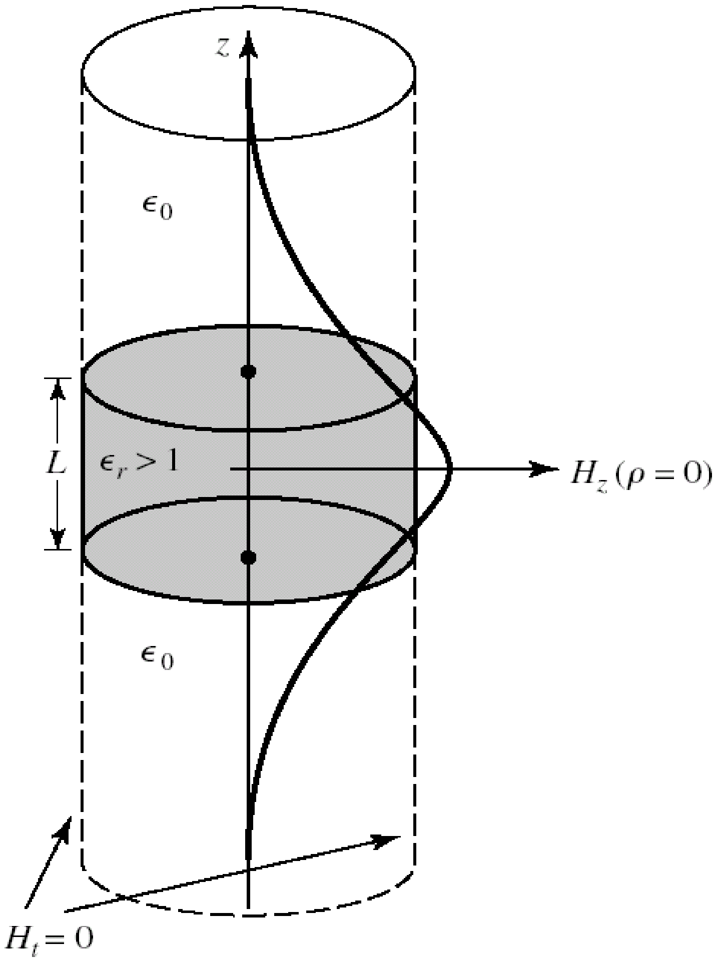
$$\frac{1}{Q} = \frac{1}{Q_C} + \frac{1}{Q_d}$$

Dielectric resonators



- Small cube, disc or hemisphere of low-loss, **high ε material** (in the range of $10\varepsilon_0$ to $100\varepsilon_0$)
- High ε for containing the field in the dielectric with small leakage.
- Same principle of operation as a cavity.
- **Q ~ 1000** but dielectric resonators provide smaller sizes and lower fabrication cost than cavities.
- **Operates in $TE_{01\delta}$ mode**

Dielectric resonators



- **L<λ$_g$/2** where λ_g is the wavelength of the TE_{01} dielectric waveguide mode

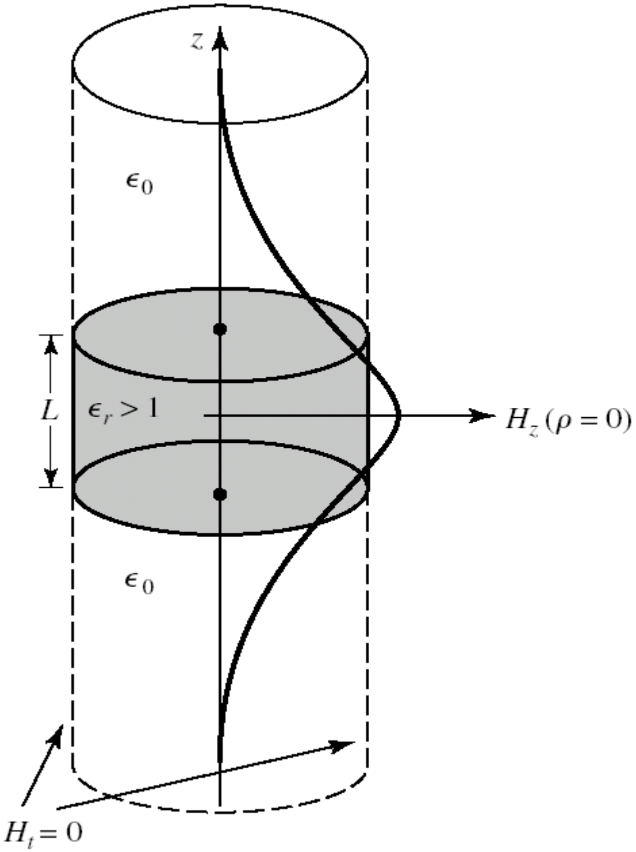
- The equivalent circuit would be a transmission line ended by reactive loads.

- Assume **magnetic walls at *ρ*=a** (i.e. Γ=1).

- Almost true since the incident wave from a high dielectric region to air-filled region is given by:

$$\Gamma = \frac{Z_0 - Z_0/\sqrt{\varepsilon_r}}{Z_0 + Z_0/\sqrt{\varepsilon_r}} = \frac{\sqrt{\varepsilon_r} - 1}{\sqrt{\varepsilon_r} + 1} \rightarrow 1 \text{ for large } \varepsilon_r$$

Dielectric resonators



1. **TE mode : $E_z = 0$**

$$\nabla^2 H_z + k^2 H_z = 0$$

$$k = \begin{cases} \sqrt{\varepsilon_r} k_0 \ \ for\ (|z| < L/2) \\ k_0 \ \ for\ (|z| > L/2) \end{cases}$$

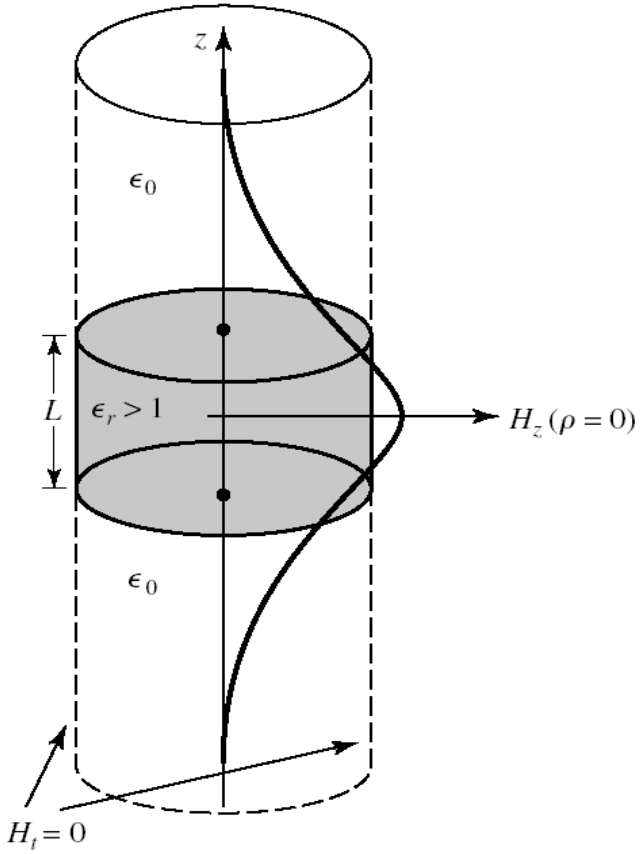
2. There is variation with φ, $\partial/\partial\varphi = 0$

$$E_\phi = \frac{j\omega\mu_0}{k_c^2}\frac{\partial H_Z}{\partial\rho}; \quad H_\rho = -\frac{j\beta}{k_c^2}\frac{\partial H_Z}{\partial\rho}; \quad k_c = \sqrt{k^2 - \beta^2}$$

3. **H_z=0 at ρ=a** and has finite value at **ρ=0**

$$H_z = H_0 J_0(k_c\rho)e^{\pm j\beta z}; \quad k_c = \frac{p_{01}}{a}$$

Dielectric resonators



4. For the transverse fields:

$$E_\phi = \frac{j\omega\mu_0}{k_c} H_0 J_0^{'}(k_c\rho) e^{\pm j\beta z}$$

$$E_\rho = \frac{\pm j\beta}{k_c} H_0 J_0^{'}(k_c\rho) e^{\pm j\beta z}$$

5. Looking at both regions:

$$|z| < \frac{L}{2}$$

$$\beta = \sqrt{\varepsilon_r k_0^2 - k_c^2}$$

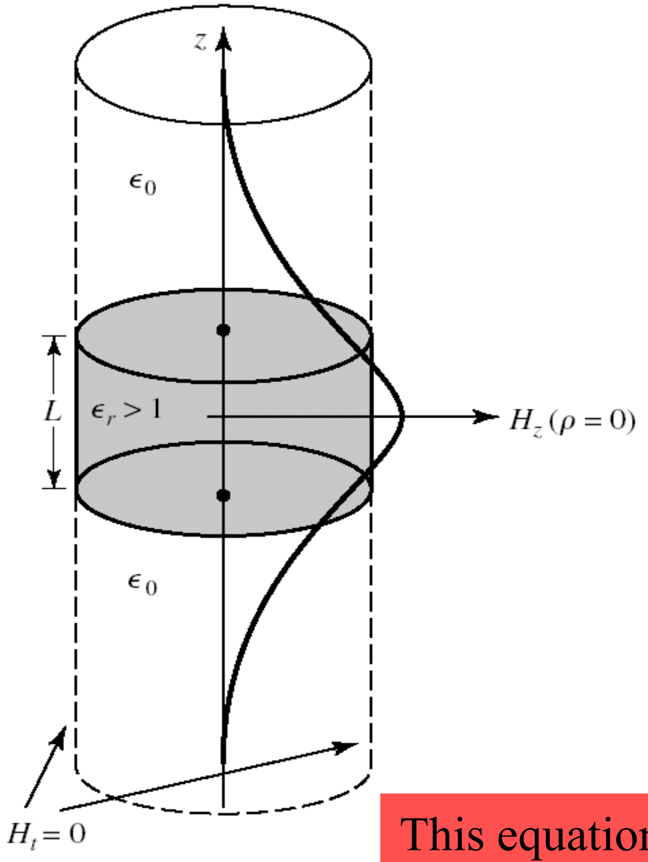
$$Z_d = \frac{E_\phi}{H_\rho} = \frac{\omega\mu_0}{\beta}$$

$$|z| > \frac{L}{2}$$

$$\alpha = \sqrt{k_c^2 - k_0^2}$$

$$Z_a = \frac{E_\phi}{H_\rho} = \frac{j\omega\mu_0}{\alpha}$$

Dielectric resonators



6. Because of **symmetry**: the fields must be even functions about z=0

$$|z| < \frac{L}{2}$$

$$E_\phi = AJ_0^{'}(k_c\rho)\cos\beta z$$

$$H_\rho = \frac{-jA}{Z_d}J_0^{'}(k_c\rho)\sin\beta z$$

$$|z| > \frac{L}{2}$$

$$E_\phi = BJ_0^{'}(k_c\rho)e^{-\alpha|z|}$$

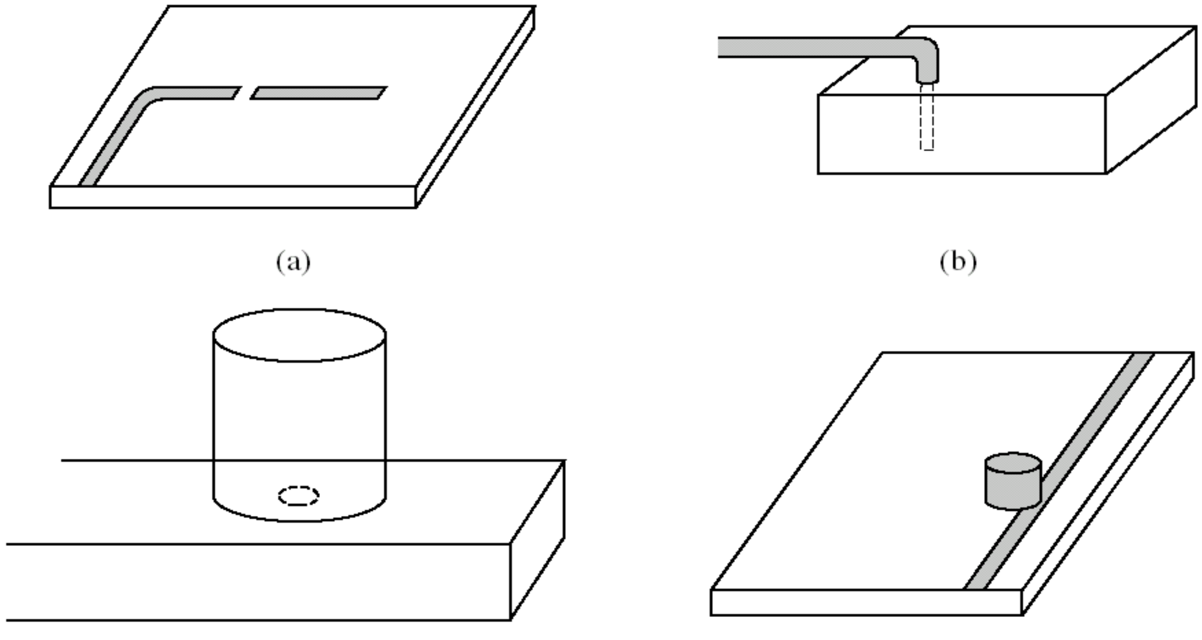
$$H_\rho = \frac{\pm jB}{Z_a}J_0^{'}(k_c\rho)e^{-\alpha|z|}$$

7. Matching the field expressions at z=±L/2:

$$\tan\left(\frac{\beta l}{2}\right) = \frac{\alpha}{\beta}$$

This equation need to be solved numerically to find the resonant frequency
10% accuracy due to the neglection of the fringing fields.

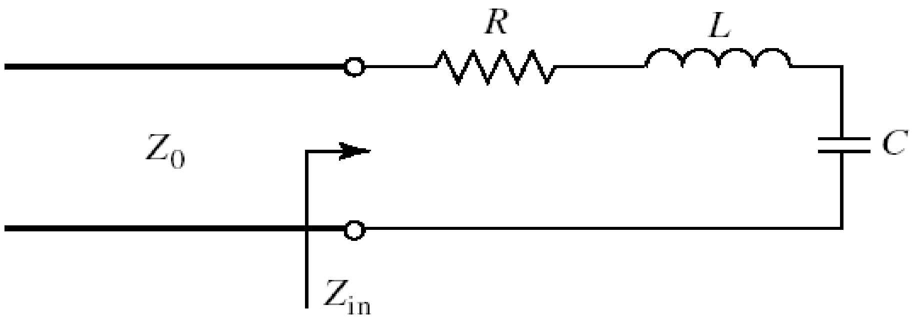
Excitation of Resonators



(a) (b) (c) (d)

Maximum power transferred implies the matching of the resonator to the feed at the resonant frequency : **Critical coupling**

Critical coupling



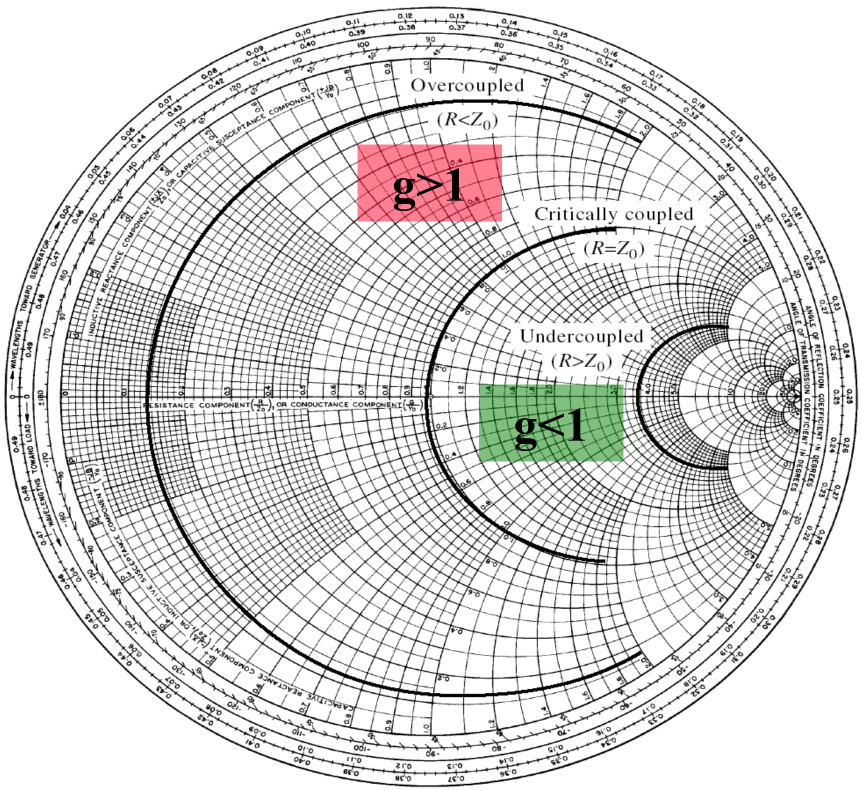
$$Z_{in} = R + j2L\Delta\omega$$

$$Q = \frac{\omega_0 L}{R}$$

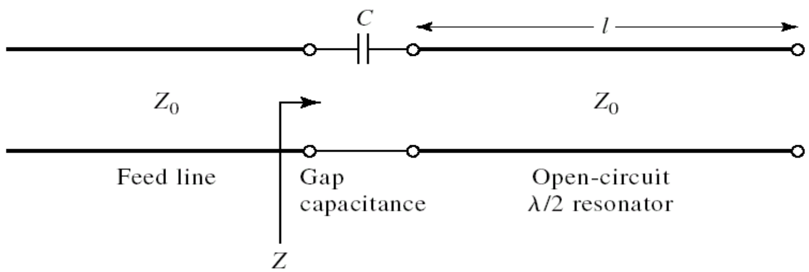
$$Q_e = \frac{\omega_0 L}{Z_0} = \frac{\omega_0 L}{R} = Q \quad (Z_{in} = R \text{ at resonance})$$

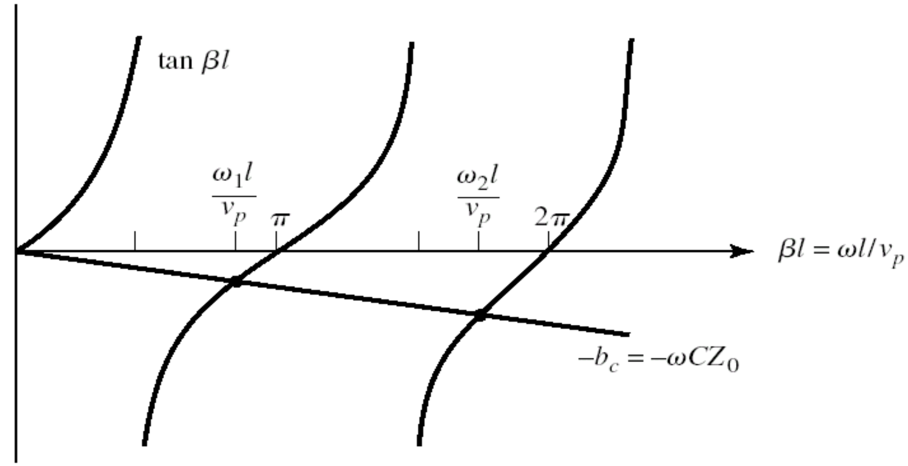
Definition of the coefficient of coupling, g

$$g = \frac{Q}{Q_e}$$



Gap-coupled microstrip resonator





The gap is modeled with single capacitor C of normalized susceptance b_c:

$$\frac{Z}{Z_0} = -j\left[\frac{1}{\omega C} + Z_0 \cot \beta l\right] = -j\left(\frac{\tan \beta l + b_c}{b_c \tan \beta l}\right)$$

At resonance $Z = Z_0$

$$\tan \beta l + b_c = 0$$

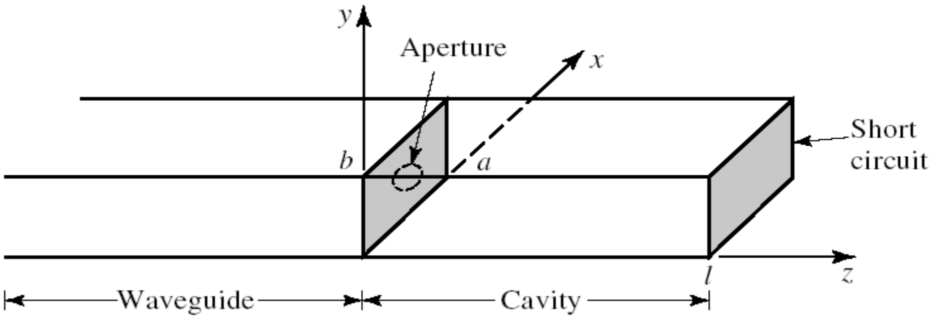
Lowering of the oscillation frequency!!!

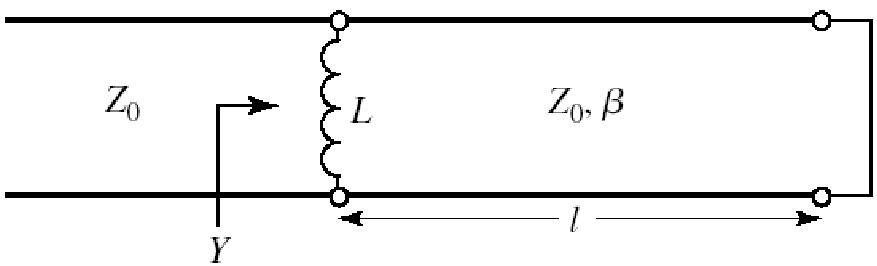
$\omega_0 \rightarrow \omega_1 < \omega_0$

$$b_c = \sqrt{\frac{\pi}{2Q}}$$

$$g = \frac{2Qb_c^2}{\pi}$$

Aperture coupled Cavity





The aperture is modeled as a shunt inductance of reactance $X_{L:}$

$$Z_0 Y = -j\left[\frac{Z_0}{X_L} + \cot \beta l\right] = -j\left(\frac{\tan \beta l + X_L}{X_L \tan \beta l}\right)$$

Antiresonance when Y =0

$$\tan \beta l + X_L = 0$$

As for the coupled microstrip resonator, lowering of the oscillation frequency occur:

$\omega_0 \rightarrow \omega_1 < \omega_0$

$$X_L = Z_0 \sqrt{\frac{\pi k_0 \omega_1}{2Q\beta^2 c}}$$

Particular case :

$l = \lambda_g/2$ for the next resonant mode

No E field in the aperture plane!!