Microwave Engineering MCC121, 7.5hec, 2014

Lecture 7

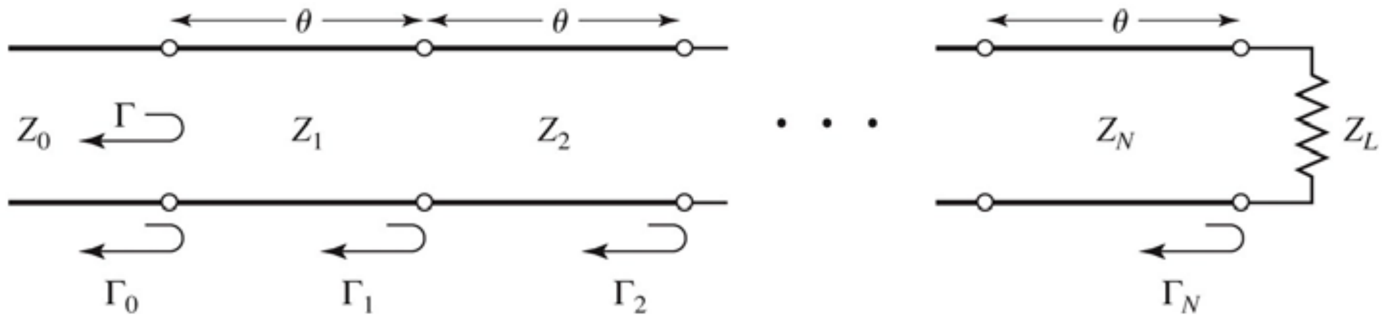


Figure 5.14

Outline

- Summary of stub matching (Ch5)
- Impedance matching cont (Ch5.5-5.9)
  - theory of small reflections
  - transformers based on single and multi section quarter wave lines
  - tapered transmission line transformers

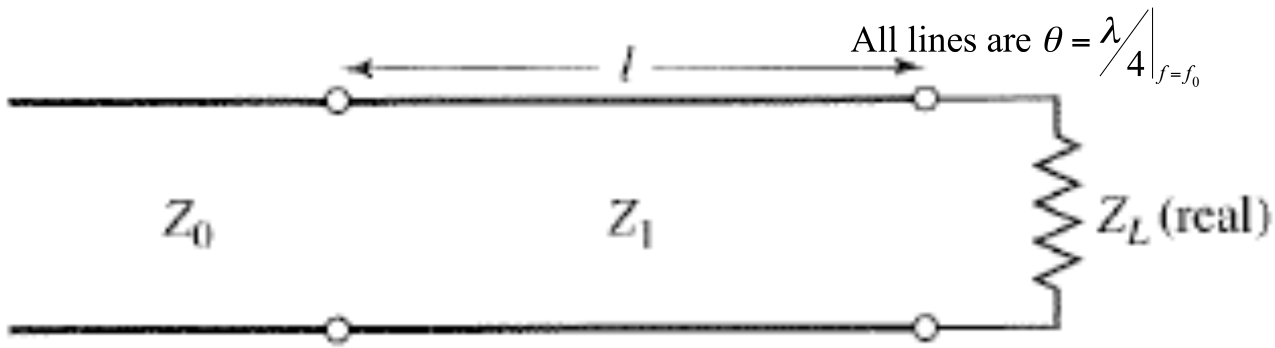
Objectives

On completion of this course unit you should be able to:

- ☑ Analyse wave propagating properties of guided wave structures (TE, TM, TEM waves, microstrip, stripline, rectangular and circular waveguides, coupled lines)
- ☑ Apply N-port representations for analysing microwave circuits
- ☑ Apply the Smith chart to evaluate microwave networks
- ☑ Design and evaluate impedance matching networks
- ☐ Design, evaluate and characterise directional couplers and power dividers
- ☐ Design and analyse attenuators, phase shifters and resonators
- ☐ Explain basic properties of ferrite devices (circulators, isolators)

Transformers

Quarter-wave transformer



$$Z_1 = \sqrt{Z_L Z_0}$$

- *On white board: derive response versus frequency.*

Bandwidth for quarter-wave transformer

$$Z_{in} = Z_2 \frac{Z_L + jZ_2 \tan\theta}{Z_2 + jZ_L \tan\theta}$$

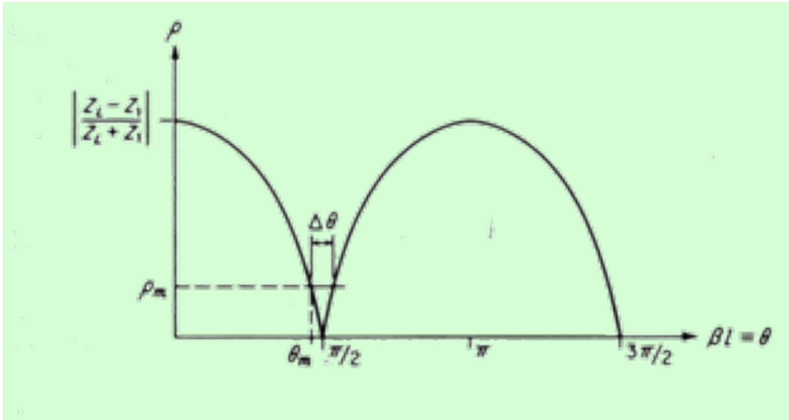
$$\Gamma = \frac{Z_{in} - Z_1}{Z_{in} + Z_1} = \frac{Z_L - Z_1}{Z_L + Z_1 + 2j\sqrt{Z_L Z_1}\tan\theta}$$

$$|\Gamma| = \frac{|Z_L - Z_1|}{\left[(Z_L + Z_1)^2 + 4Z_L Z_1 \tan^2\theta\right]^{1/2}} =$$

$$= \frac{1}{\sqrt{1 + \left(\frac{2\sqrt{Z_L Z_1}}{Z_L - Z_1}\frac{1}{\cos\theta}\right)^2}}$$

For $\frac{\pi}{2} - d < \theta < \frac{\pi}{2} + d \Rightarrow \cos\theta \approx 0, \frac{1}{\cos\theta}$? 1

$$|\Gamma| \cong \frac{|Z_L - Z_1|}{2\sqrt{Z_L Z_1}} |\cos\theta|$$



$$\theta_m = \arccos\left|\frac{2|\Gamma_m|\sqrt{Z_L Z_1}}{(Z_L - Z_1)\sqrt{1 - |\Gamma_m|^2}}\right|$$

Single section quarter wave transformer

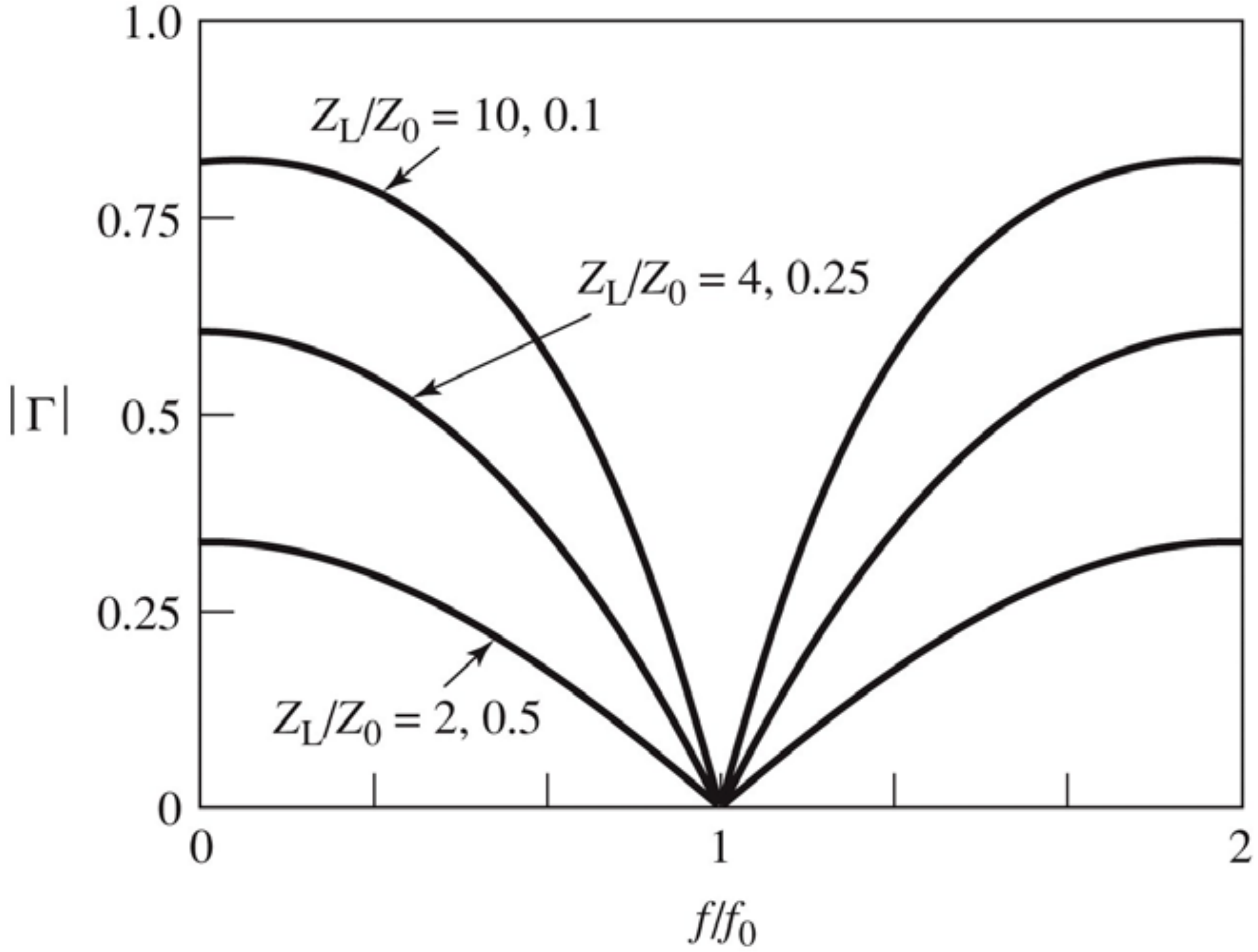


Figure 5.12

Theory of small reflections

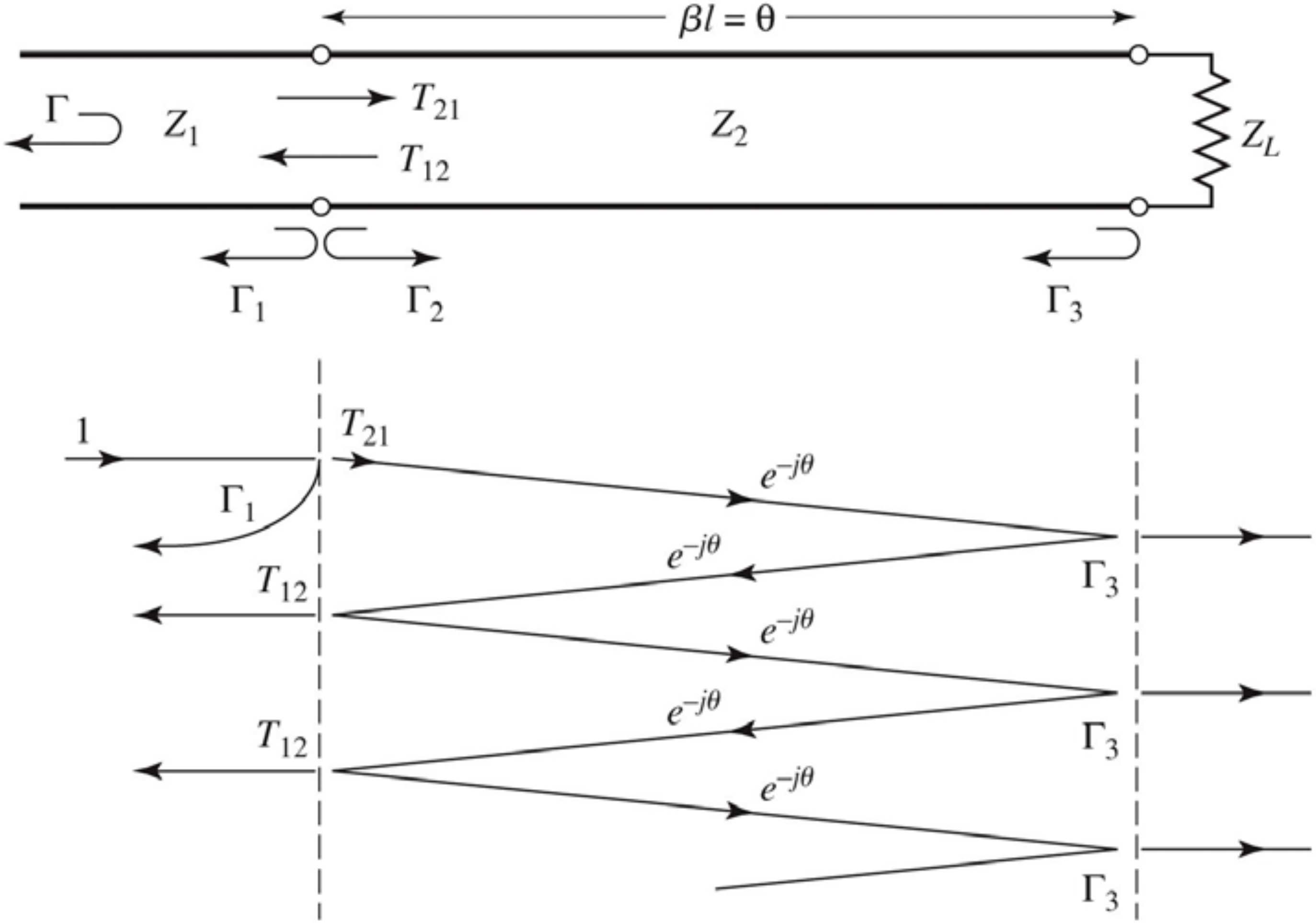
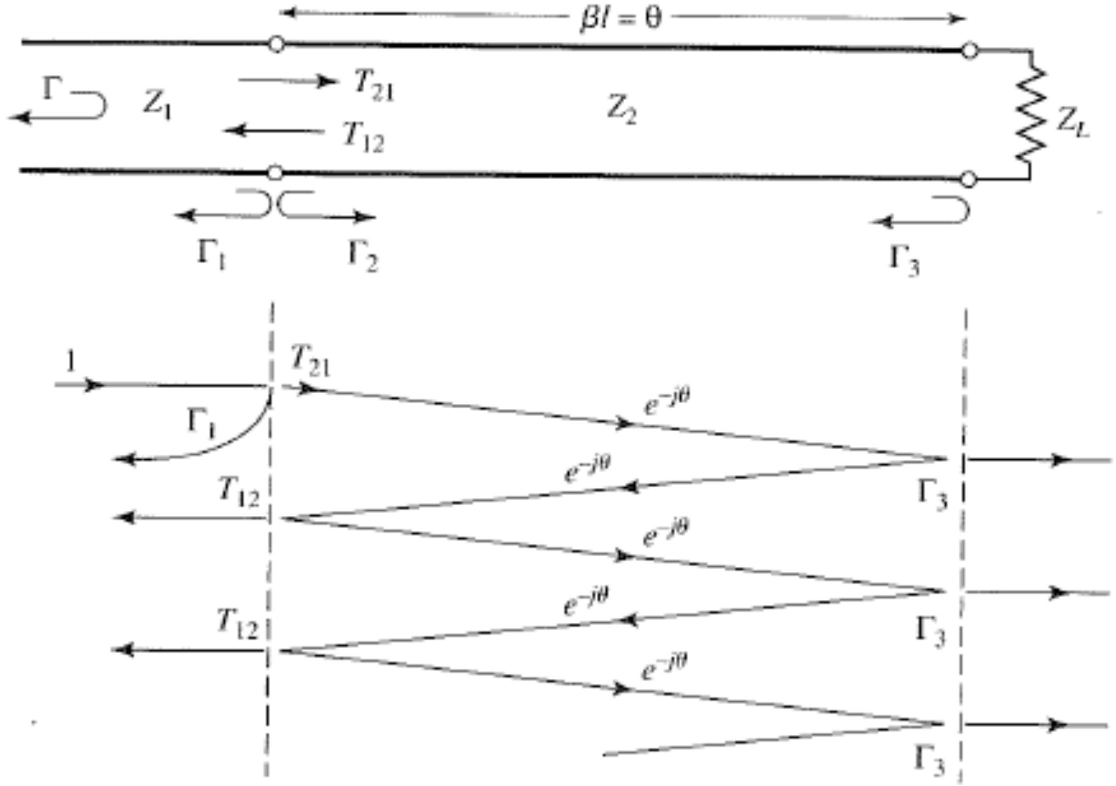


Figure 5.13

- *On white board: derive the overall reflection coefficient for a multi-section transformer, assuming small reflections.*

Theory of small reflections

Assume constant characteristic impedance (frequency independent)
Neglect influence from junctio

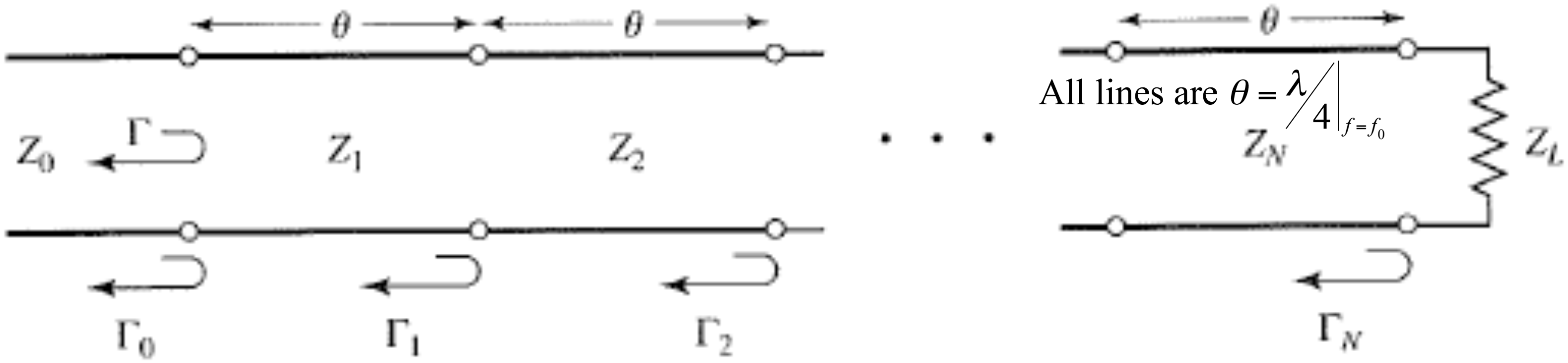


For $\Gamma_1 \ll 1$ and $\Gamma_2 \ll 1$

$$\Gamma \cong \Gamma_1 + \Gamma_3 e^{-2j\theta}$$

For small reflections -> only first order reflections needed

Multisection quarter-wave transformers



$$\Gamma = \rho_0 + \rho_1 e^{-2j\theta} + \rho_2 e^{-4j\theta} + \cdots + \rho_n e^{-2jn\theta} + \cdots + \rho_N e^{-2jN\theta}$$

$$\Gamma_0 = \frac{Z_1 - Z_0}{Z_1 + Z_0} = \rho_0$$

$$\Gamma_1 = \frac{Z_2 - Z_1}{Z_2 + Z_1} = \rho_1$$

....

$$\Gamma_n = \frac{Z_{n+1} - Z_n}{Z_{n+1} + Z_n} = \rho_n$$

....

$$\Gamma_N = \frac{Z_L - Z_N}{Z_L + Z_N} = \rho_N$$

Symmetrical transformer

$$\rho_0 = \rho_N,\ \rho_1 = \rho_{N-1},\ \rho_2 = \rho_{N-2}, \ldots\ldots, \rho_n = \rho_{N-n}, \ldots$$

$$\Gamma = e^{-jN\theta}\left[\rho_0\left(e^{jN\theta} + e^{-jN\theta}\right) + \rho_1\left(e^{j(N-2)\theta} + e^{-j(N-2)\theta}\right) + \cdots + \begin{cases} \rho_{\frac{(N-1)}{2}}\left(e^{j\theta} + e^{-j\theta}\right) & N \text{ odd} \\ \rho_{\frac{N}{2}} & N \text{ even} \end{cases}\right]$$

$$\Gamma = 2e^{-jN\theta}\left[\rho_0 \cos N\theta + \rho_1 \cos(N-2)\theta + \cdots + \begin{cases} \rho_{\frac{(N-1)}{2}} \cos\theta & N \text{ odd} \\ \frac{1}{2}\rho_{\frac{N}{2}} & N \text{ even} \end{cases}\right] \qquad (1)$$

Equation (1) is a cosine series; the function it defines is periodic over the interval π corresponding to the frequency range over which the length of each transformer section changes by a $\lambda/2$.

It is possible to specify Γ in different ways e.g.: Butterworth (maximally flat) or Chebyshev (equal ripple) for the passband characteristics.

Binomial transformer

Butterworth approximation → maximally flat

$$\Gamma = A\left(1+e^{-2j\theta}\right)^N \qquad (1)$$

(N-1)derivatives of $|\Gamma| = \rho$ with respect to frequency vanish (=0) at the matching frequency f_0 where $\theta = \pi/2$

When $\theta = 0$ or $\theta = \pi$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} \overset{(1)}{=} 2^N A \quad \Rightarrow \quad A = 2^{-N}\frac{Z_L - Z_0}{Z_L + Z_0} \qquad (2)$$

Expand (1) by the binomial expansion

$$\Gamma = 2^{-N}\frac{Z_L - Z_0}{Z_L + Z_0}(1+e^{-2j\theta})^N = 2^{-N}\frac{Z_L - Z_0}{Z_L + Z_0}\sum_{n=0}^{N} C_n^N e^{-2jn\theta}, \quad C_n^N = \frac{N!}{(N-n)!n!}$$

$C_n^N = C_{n-N}^N$ symmetry condition is fulfilled

Compare with multisection transformer

$$\Gamma = \rho_0 + \rho_1 e^{-2j\theta} + \rho_2 e^{-4j\theta} + \cdots + \rho_n e^{-2jn\theta} + \cdots + \rho_N e^{-2jN\theta}$$

$$\rho_n = 2^{-N} \frac{Z_L - Z_0}{Z_L + Z_0} C_n^N$$

To calculate Z_n we start with an approximation

$$\ln \frac{1+x}{1-x} = 2\left(x + \frac{x^3}{3} + \frac{x^5}{5} + \cdots \right)$$

$$x = \frac{Z_{n+1} - Z_n}{Z_{n+1} + Z_n} = \rho_n$$

$$\ln \frac{1+x}{1-x} = \ln \frac{1 + \dfrac{Z_{n+1} - Z_n}{Z_{n+1} + Z_n}}{1 - \dfrac{Z_{n+1} - Z_n}{Z_{n+1} + Z_n}} = \ln \frac{Z_{n+1}}{Z_n}$$

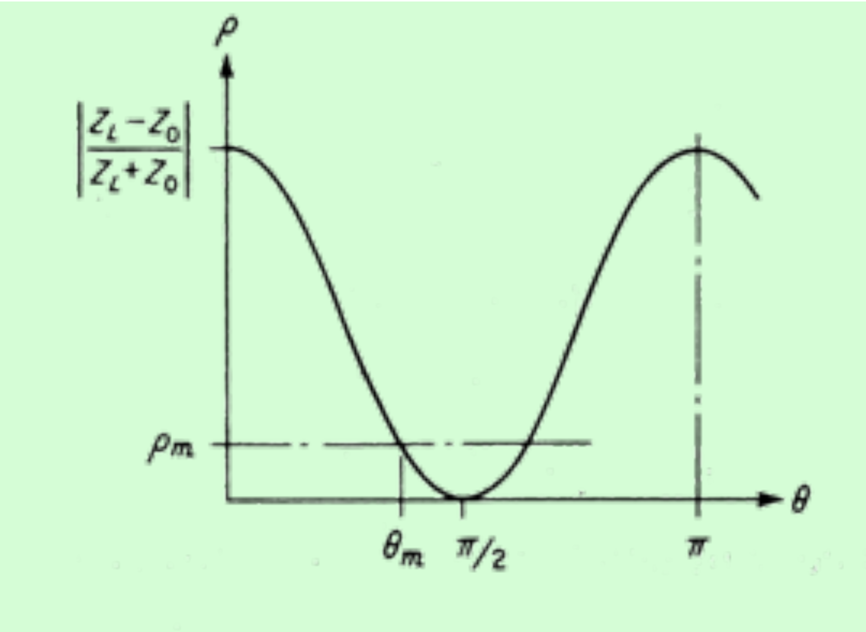
$$\ln \frac{Z_{n+1}}{Z_n} \approx 2 \frac{Z_{n+1} - Z_n}{Z_{n+1} + Z_n} = 2\rho_n = 2^{-N} C_n^N \ln \frac{Z_L}{Z_0}$$

$$\ln \frac{Z_L}{Z_0} = 2 \frac{Z_L - Z_0}{Z_L + Z_0} + \frac{2}{3}\left(\frac{Z_L - Z_0}{Z_L + Z_0} \right)^3 + \cdots \approx 2 \frac{Z_L - Z_0}{Z_L + Z_0}$$

Since the theory is approximate the range of Z_L is limited to

$$0.5\, Z_0 < Z_L < 2\, Z_0$$

Bandwidth (binomial)



$$\left|\Gamma_m\right| = \rho_m = \frac{1}{2}\ln\frac{Z_L}{Z_0}\left(\cos\theta_m\right)^N$$

$$\theta_m = \arccos\left|\frac{2\rho_m}{\ln\frac{Z_L}{Z_0}}\right|^{1/N}$$

$$\frac{\Delta f}{f_0} = \frac{2\left(f_m - f_0\right)}{f_0} = 2 - \frac{4}{\pi}\arccos\left|\frac{2\rho_m}{\ln\frac{Z_L}{Z_0}}\right|^{1/N}$$

- *On white board: design a binomial transformer to match a 50 ohm load to a 100 ohm line, using three sections.*

Ex) Frequency response Binomial transformer

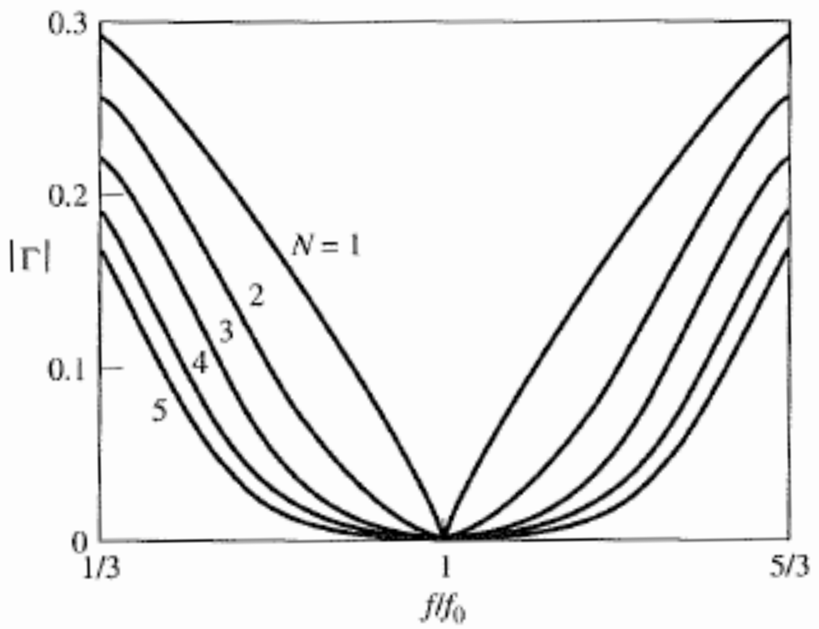
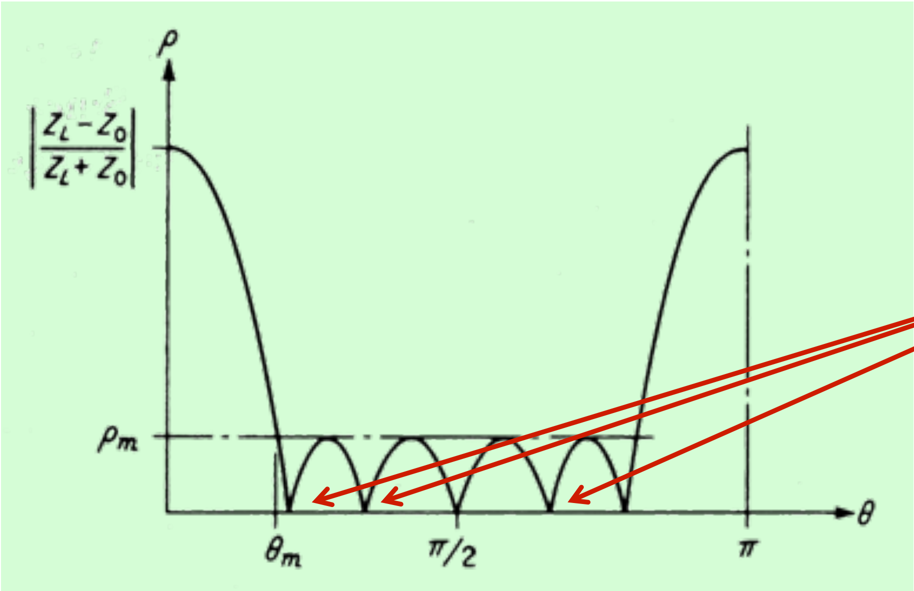


FIGURE 5.15 Reflection coefficient magnitude versus frequency for multisection binomial matching transformers of Example 5.6. $Z_L = 50\ \Omega$ and $Z_0 = 100\ \Omega$.

from Pozar

Chebyshev transformer



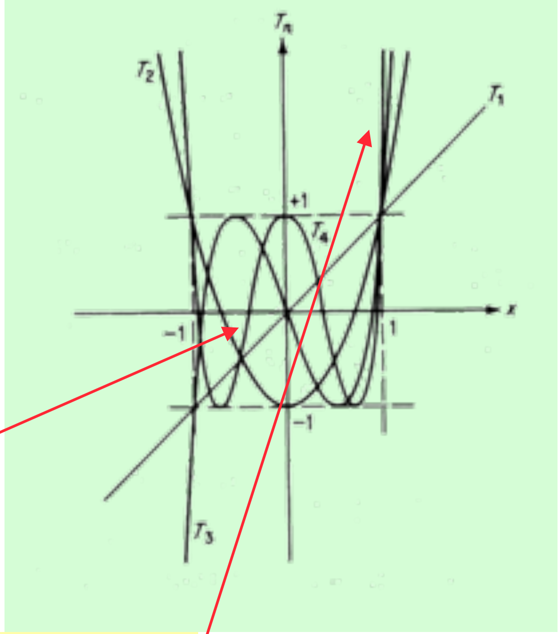
The number of zeros for Γ in the passband corresponds to the number of transformer sections

We permit $|\Gamma|=\rho$ to vary between 0 and ρ_m in an oscillatory manner over the passband, which will be described by a Chebyshev polynomial.

This will provide a considerable increase in bandwidth of the transformer as compared to the Butterworth case.

Chebyshev polynomial of degree *n*, $T_n(x)$

$$T_1(x) = x$$
$$T_2(x) = 2x^2 - 1$$
$$T_3(x) = 4x^3 - 3x$$
.
.
$$T_n(x) = 2xT_{n-1}(x) - T_{n-2}(x)$$



$-1 \le x \le 1, \quad |T_n(x)| \le 1,$ "equal ripple"

$|x| > 1, \quad |T_n(x)| > 1$ increases faster with order higher order n

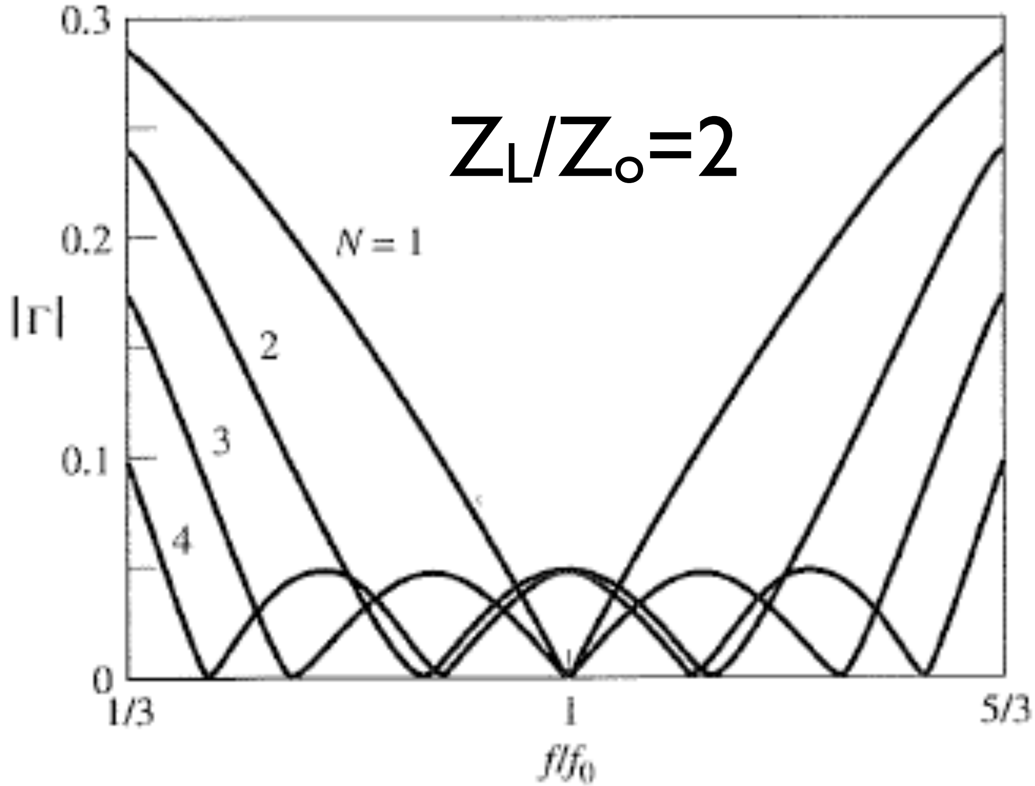
For $|x| < 1$ we can replace x with cosθ

$$T_1(\cos\theta) = \cos\theta$$
$$T_2(\cos\theta) = 2\cos^2\theta - 1 = \cos 2\theta$$
.
$$T_n(\cos\theta) = \cos(n\theta) = \cos(n\arccos x)$$

For $|x| > 1$

$$T_n(\cos\theta) = \cosh(n\text{arccosh}x)$$

Ex) Frequency response Chebyshev transformer



from Pozar, "Microwave Engineering"

Chebysheff transformer, recap

$$\Gamma = 2e^{-jN\theta}\left[\rho_0 \cos N\theta + \rho_1 \cos(N-2)\theta + \cdots + \begin{cases} \rho_{N/2}/2 \ (N \text{ even}) \\ \rho_{N-1/2} \cos\theta (N \text{ odd}) \end{cases}\right] =$$

$$= Ae^{-jN\theta} T_N(\sec\theta_m \cos\theta)$$

A is calculated for $\theta = 0$

$$\Gamma(0) = AT_N(\sec\theta_m) = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$A = \frac{Z_L - Z_0}{Z_L + Z_0}\left[T_N(\sec\theta_m)\right]^{-1}$$

For $|\Gamma_m| = \rho_m$ in the passband $\rho_m = A$ since $T_N(\sec\theta_m \cos\theta)\big|_{\max} = 1$

$$\sec\theta_m = \cosh\left(\frac{1}{N}\operatorname{arccosh}\left(\frac{1}{\rho_m}\left|\frac{Z_L - Z_0}{Z_L + Z_0}\right|\right)\right)$$

Practise as part of Lab 3

Binomial transformer design

TABLE 5.1 Binomial Transformer Design

	N = 2		N = 3			N = 4			
Z_L/Z_0	Z_1/Z_0	Z_2/Z_0	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0	Z_4/Z_0
1.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1.5	1.1067	1.3554	1.0520	1.2247	1.4259	1.0257	1.1351	1.3215	1.4624
2.0	1.1892	1.6818	1.0907	1.4142	1.8337	1.0444	1.2421	1.6102	1.9150
3.0	1.3161	2.2795	1.1479	1.7321	2.6135	1.0718	1.4105	2.1269	2.7990
4.0	1.4142	2.8285	1.1907	2.0000	3.3594	1.0919	1.5442	2.5903	3.6633
6.0	1.5651	3.8336	1.2544	2.4495	4.7832	1.1215	1.7553	3.4182	5.3500
8.0	1.6818	4.7568	1.3022	2.8284	6.1434	1.1436	1.9232	4.1597	6.9955
10.0	1.7783	5.6233	1.3409	3.1623	7.4577	1.1613	2.0651	4.8424	8.6110

	N = 5					N = 6					
Z_L/Z_0	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0	Z_4/Z_0	Z_5/Z_0	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0	Z_4/Z_0	Z_5/Z_0	Z_6/Z_0
1.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1.5	1.0128	1.0790	1.2247	1.3902	1.4810	1.0064	1.0454	1.1496	1.3048	1.4349	1.4905
2.0	1.0220	1.1391	1.4142	1.7558	1.9569	1.0110	1.0790	1.2693	1.5757	1.8536	1.9782
3.0	1.0354	1.2300	1.7321	2.4390	2.8974	1.0176	1.1288	1.4599	2.0549	2.6577	2.9481
4.0	1.0452	1.2995	2.0000	3.0781	3.8270	1.0225	1.1661	1.6129	2.4800	3.4302	3.9120
6.0	1.0596	1.4055	2.4495	4.2689	5.6625	1.0296	1.2219	1.8573	3.2305	4.9104	5.8275
8.0	1.0703	1.4870	2.8284	5.3800	7.4745	1.0349	1.2640	2.0539	3.8950	6.3291	7.7302
10.0	1.0789	1.5541	3.1623	6.4346	9.2687	1.0392	1.2982	2.2215	4.5015	7.7030	9.6228

from Pozar, "Microwave Engineering"

Chebyshev transformer design

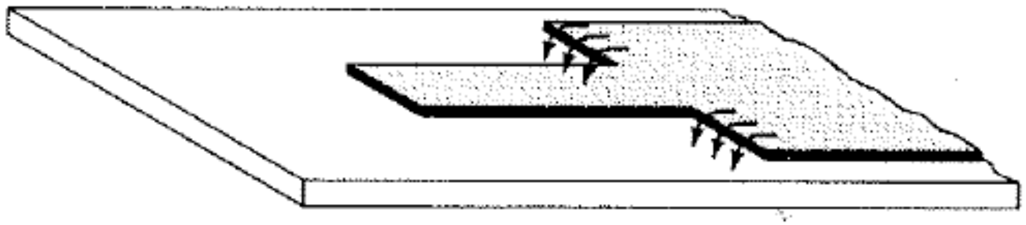
	N = 2				N = 3					
	$\Gamma_m = 0.05$		$\Gamma_m = 0.20$		$\Gamma_m = 0.05$			$\Gamma_m = 0.20$		
Z_L/Z_0	Z_1/Z_0	Z_2/Z_0	Z_1/Z_0	Z_2/Z_0	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0
1.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1.5	1.1347	1.3219	1.2247	1.2247	1.1029	1.2247	1.3601	1.2247	1.2247	1.2247
2.0	1.2193	1.6402	1.3161	1.5197	1.1475	1.4142	1.7429	1.2855	1.4142	1.5558
3.0	1.3494	2.2232	1.4565	2.0598	1.2171	1.7321	2.4649	1.3743	1.7321	2.1829
4.0	1.4500	2.7585	1.5651	2.5558	1.2662	2.0000	3.1591	1.4333	2.0000	2.7908
6.0	1.6047	3.7389	1.7321	3.4641	1.3383	2.4495	4.4833	1.5193	2.4495	3.9492
8.0	1.7244	4.6393	1.8612	4.2983	1.3944	2.8284	5.7372	1.5766	2.8284	5.0742
10.0	1.8233	5.4845	1.9680	5.0813	1.4385	3.1623	6.9517	1.6415	3.1623	6.0920

N = 4

	$\Gamma_m = 0.05$				$\Gamma_m = 0.20$			
Z_L/Z_0	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0	Z_4/Z_0	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0	Z_4/Z_0
1.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1.5	1.0892	1.1742	1.2775	1.3772	1.2247	1.2247	1.2247	1.2247
2.0	1.1201	1.2979	1.5409	1.7855	1.2727	1.3634	1.4669	1.5715
3.0	1.1586	1.4876	2.0167	2.5893	1.4879	1.5819	1.8965	2.0163
4.0	1.1906	1.6414	2.4369	3.3597	1.3692	1.7490	2.2870	2.9214
6.0	1.2290	1.8773	3.1961	4.8820	1.4415	2.0231	2.9657	4.1623
8.0	1.2583	2.0657	3.8728	6.3578	1.4914	2.2428	3.5670	5.3641
10.0	1.2832	2.2268	4.4907	7.7930	1.5163	2.4210	4.1305	6.5950

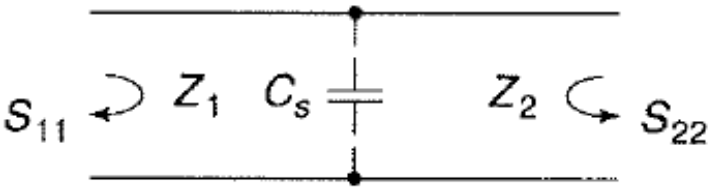
from Pozar, "Microwave Engineering"

Junction capacitance and length compensation

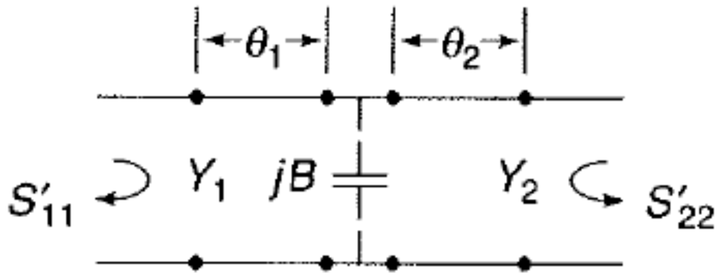


(a)

- *Derive correction.*



(b)



(c)

$$\theta_1 = \frac{BY_1}{Y_1^2 - Y_2^2}; \quad \theta_2 = -\frac{BY_2}{Y_1^2 - Y_2^2}$$

->Compensate length due to fringing field

Tapered transformer

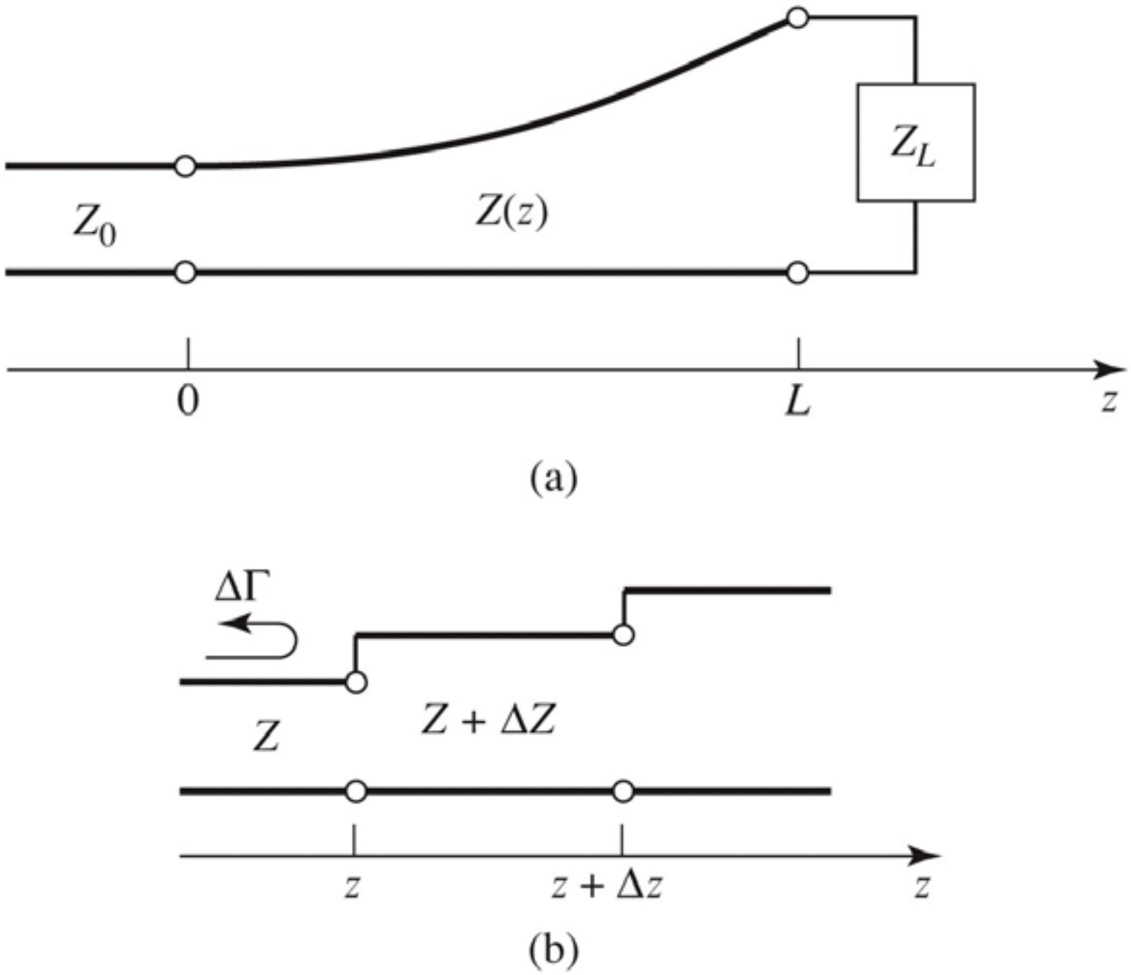
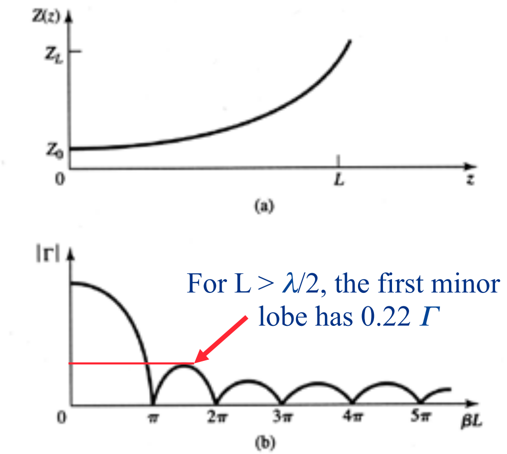


Figure 5.18

$$d\Gamma_{in} = e^{-2j\beta z}\frac{1}{2}\frac{d}{dz}(\ln Z)\,dz$$

$$\Gamma_{in} = \int_0^L d\Gamma_{in} = \frac{1}{2}\int_0^L e^{-2j\beta z}\frac{d}{dz}(\ln Z)\,dz \quad (1)$$

Exponential taper



$$Z(z) = Z_0 e^{\alpha z} \quad 0 < z < L$$

$$Z(z)\big|_{z=0} = Z_0 \quad Z(z)\big|_{z=L} = Z_0 e^{\alpha L} = Z_L$$

$$\alpha = \frac{1}{L}\ln\left(\frac{Z_L}{Z_0}\right)$$

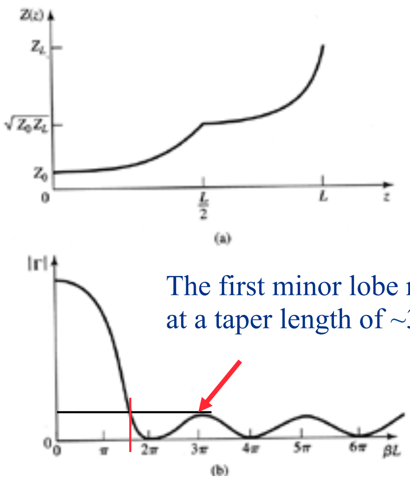
$$\Gamma = \frac{1}{2}\int_0^L e^{-2j\beta z}\frac{d}{dz}\left(\ln e^{\alpha z}\right)dz = \frac{1}{2}\ln\frac{Z_L}{Z_0}e^{-j\beta L}\frac{\sin\beta L}{\beta L}$$

We assume that β is constant and not a function of z

Triangular taper

$$Z(z)=\begin{cases} Z_0 e^{2(z/L)^2 \ln(Z_L/Z_0)} & 0\le z\le L/2 \\ Z_0 e^{\left(4z/L-2z^2/L^2-1\right)\ln(Z_L/Z_0)} & L/2\le z\le L \end{cases}$$

$$\frac{d\ln(Z/Z_0)}{dz}=\begin{cases} \frac{4z}{L^2}\ln\frac{Z_L}{Z_0} & 0\le z\le L/2 \\ \left(\frac{4}{L}-\frac{4z}{L^2}\right)\ln\frac{Z_L}{Z_0} & L/2\le z\le L \end{cases}$$



The first minor lobe maximum occurs at a taper length of ~3λ/2

L=0.815λ

$$\Gamma(\beta l)=\frac{1}{2}\int_{z=0}^{L} e^{-2j\beta z}\frac{d}{dz}\ln\frac{Z}{Z_0}\,dz$$

$$\Gamma_{in}=\frac{1}{2}e^{-j\beta L}\ln\left(\frac{Z_L}{Z_0}\right)\left(\frac{\sin\frac{\beta L}{2}}{\frac{\beta L}{2}}\right)^2$$

Comparison- tapers

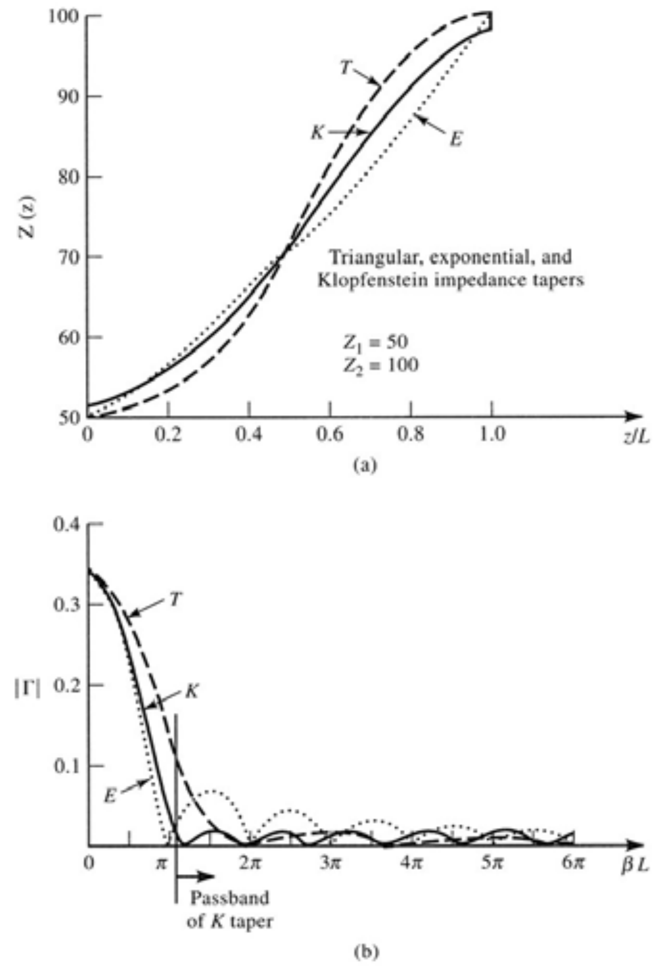


Figure 5.21

Summary of lecture 7

- Read chapter 5 (impedance matching).
  - Quarter wave transformers
  - Theory of small reflections
  - Chebyshev, Binomial transformers
  - Tapered transformers
  - length compensation due to fringing fields at junctions

Further reading

- R. W. Klopfenstein, "A Transmission Line Taper of Improved Design," in Proceedings of the IRE, 1956, vol. 44, no. 1, pp. 31–35.

- A wide range of applets on transmission lines, electromagnetic waves and antennas: http://www.amanogawa.com/index.html