

Microwave Engineering

MCC121, 7.5hec, 2014

Lecture 6

State-of-the-art
Challenging
Stimulating
Rewarding

Future?

Thursday, 15 November 2012 | Logh

microwave road

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Events

- 2012.11.08 - 2012.11.08 | 15.00
Mini-mässa torsdagen den 8 november, 2012

News

- Successful update on assembly methods above 50GHz
- Microwaves in medical instruments
- MWR at Elektronik 2011 in Gothenburg

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- May, 2010
- April, 2010
- January, 2010
- December, 2009

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Outline

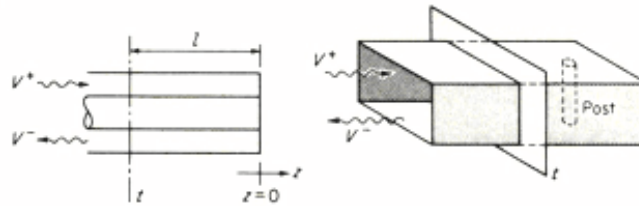
- Summary of n-port representations (Ch4)
- Impedance matching (Ch5.1-5.4)
 - Smith chart
 - Stub matching
 - Lumped element matching

Objectives

On completion of this course unit you should be able to:

- Analyse wave propagating properties of guided wave structures (TE, TM, TEM waves, microstrip, stripline, rectangular and circular waveguides, coupled lines)
- Apply N-port representations for analysing microwave circuits
- Apply the Smith chart to evaluate microwave networks
- Design and evaluate impedance matching networks
- Design, evaluate and characterise directional couplers and power dividers
- Design and analyse attenuators, phase shifters and resonators
- Explain basic properties of ferrite devices (circulators, isolators)

One-port circuit



- energy can enter or leave through a single propagation line
- Introduce input impedance, Z_{in}

Impedance description

Assume now perfectly conductive walls,
 $\sigma = \infty$, $E_{tan} = 0$ on all walls but t .

$$\frac{1}{2} \oint_t \bar{E} \times \bar{H} \cdot \bar{a}_z dS = P_{loss} + 2j\omega(W_m - W_e)$$

At the terminal plane t the transverse fields are

$$\begin{aligned} \bar{E}_t &= K_1^{-1}(V^+ + V^-)\bar{e} = K_1^{-1}V\bar{e} \\ \bar{H}_t &= K_2^{-1}(I^+ - I^-)\bar{h} = K_2^{-1}I\bar{h} \end{aligned}$$

Thus

$$\frac{1}{2} (K_1 K_2^*)^{-1} V I^* \int_t \bar{e} \times \bar{h}^* \cdot \bar{a}_z dS = \frac{1}{2} V I^* = P_{loss} + 2j\omega(W_m - W_e)$$

We have now $V = Z_{in} I$

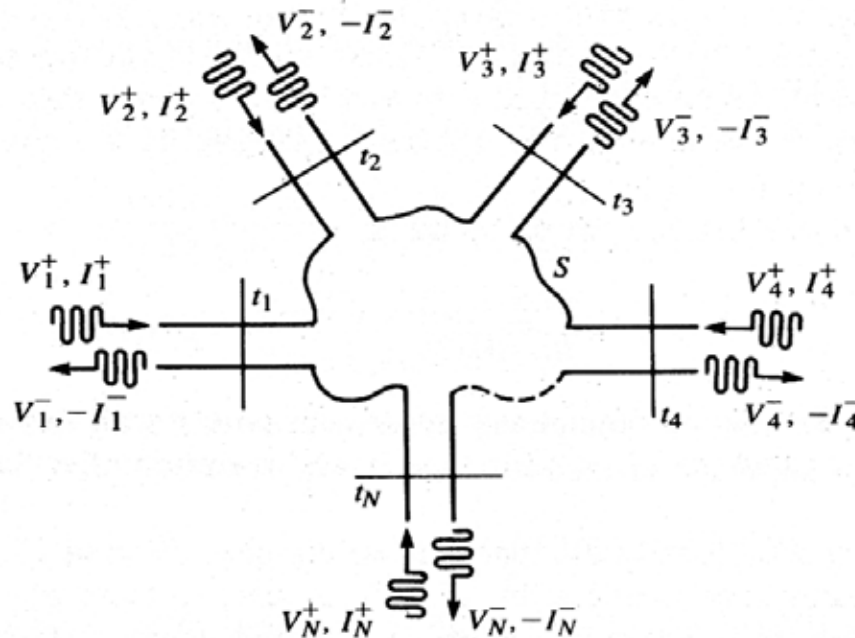
$$Z_{in} = \frac{P_{loss} + 2j\omega(W_m - W_e)}{\frac{1}{2} I I^*} = R + jX$$

$$Z_{in} = f(P_{loss}, W_m - W_e)$$

If $W_m > W_e$ \longrightarrow $X > 0$, inductive one-port

If $W_m < W_e$ \longrightarrow $X < 0$, capacitive one-port

Scattering matrix [S]



$$s_{11} = \frac{V_1^-}{V_1^+}; \quad s_{n1} = \frac{V_n^-}{V_1^+}, \quad n = 2, 3, \dots, N$$

$$\begin{bmatrix} V_1^- \\ V_2^- \\ \cdot \\ \cdot \\ V_N^- \end{bmatrix} = \begin{bmatrix} s_{11} & s_{12} & \cdot & \cdot & s_{1N} \\ s_{21} & s_{22} & \cdot & \cdot & s_{2N} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ s_{N1} & s_{N2} & \cdot & \cdot & s_{NN} \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \\ \cdot \\ \cdot \\ V_N^+ \end{bmatrix}$$

$$[V^-] = [S][V^+] \quad \text{and} \quad s_{ij} = \left. \frac{V_i^-}{V_j^+} \right|_{V_n^+ = 0, n \neq j}$$

[S] can be measured using a Vector Network Analyser (VNA), even at very high frequencies.

Impedance matrix

- let the terminal planes be chosen sufficiently far from the junction \Rightarrow only dominant incident and reflected waves.
 \Rightarrow equivalent voltages and currents
- Use total current as independent variables and total voltages as dependent variables, hence linear combination can be written as:

$$\begin{bmatrix} V_1 \\ V_2 \\ \cdot \\ \cdot \\ V_N \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} & \cdot & \cdot & z_{1N} \\ z_{21} & z_{22} & \cdot & \cdot & z_{2N} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ z_{N1} & z_{N2} & \cdot & \cdot & z_{NN} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \cdot \\ \cdot \\ I_N \end{bmatrix}$$

Properties

- Non reciprocal circuit: $Z_{ij} \neq Z_{ji}$ unsymmetrical impedance matrix ($2N^2$ parameters)
- Reciprocal circuit: $Z_{ij} = Z_{ji} \Rightarrow$ symmetrical impedance matrix ($N(N+1)$ parameters)
- Lossless circuit: symmetrical and imaginary $[Z]$ ($N(N+1)/2$ parameters)
- Same applies to $[Y] = [Z]^{-1}$

$$\begin{bmatrix} V_1 \\ V_2 \\ \cdot \\ \cdot \\ V_N \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} & \cdot & \cdot & z_{1N} \\ z_{21} & z_{22} & \cdot & \cdot & z_{2N} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ z_{N1} & z_{N2} & \cdot & \cdot & z_{NN} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \cdot \\ \cdot \\ I_N \end{bmatrix}$$

Properties of the S-matrix

- Reciprocal if
([S] symmetric)

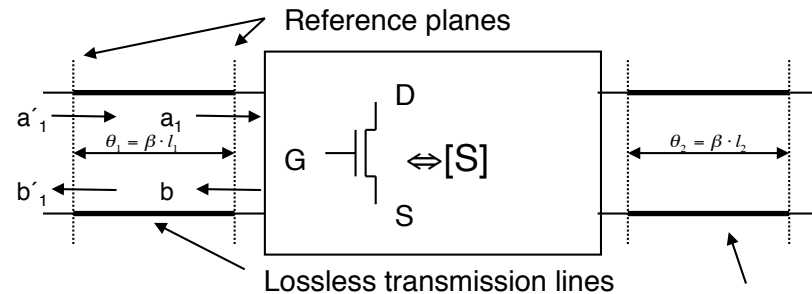
$$[S] = [S]^t$$

- Lossless if:
([S] is unitary, [U] is
the unit diagonal
matrix)

$$[S]^t [S]^* = [U]$$

Shift of the reference plane

- Two port case:



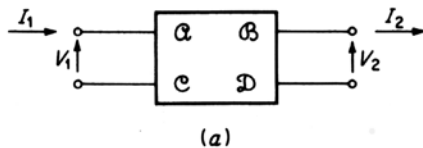
$$[S'] = [\Phi] \cdot [S] \cdot [\Phi], \quad [\Phi] = \begin{bmatrix} e^{-j \cdot \beta \cdot l_1} & 0 \\ 0 & e^{-j \cdot \beta \cdot l_2} \end{bmatrix}$$

$$\text{or } [S] = [\Phi]^{-1} \cdot [S'] \cdot [\Phi]^{-1}$$

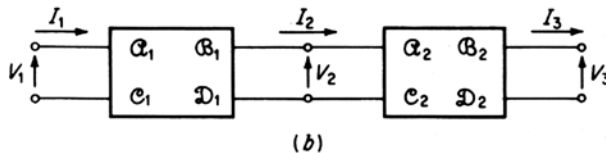
$$[S] = \begin{bmatrix} e^{j \cdot \beta \cdot l_1} & 0 \\ 0 & e^{j \cdot \beta \cdot l_2} \end{bmatrix} \cdot [S'] \cdot \begin{bmatrix} e^{j \cdot \beta \cdot l_1} & 0 \\ 0 & e^{j \cdot \beta \cdot l_2} \end{bmatrix}$$

Cascaded components

- For cascaded components a convenient way to describe the connection is to use transmission matrices (sometimes called ABCD matrices)



$$\begin{aligned} V_1 &= AV_2 + BI_2 \\ I_1 &= CV_2 + DI_2 \end{aligned} \quad \begin{pmatrix} V_1 \\ I_1 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} V_2 \\ I_2 \end{pmatrix}$$



$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} A_1 & B_1 \\ C_1 & D_1 \end{pmatrix} \begin{pmatrix} A_2 & B_2 \\ C_2 & D_2 \end{pmatrix}$$

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} z_{11} & \frac{(z_{11}z_{22} - z_{12}^2)}{z_{12}} \\ z_{12} & z_{12} \\ 1 & \frac{z_{22}}{z_{12}} \\ z_{12} & z_{12} \end{pmatrix}$$

For reciprocal junctions $AD-BC=1$

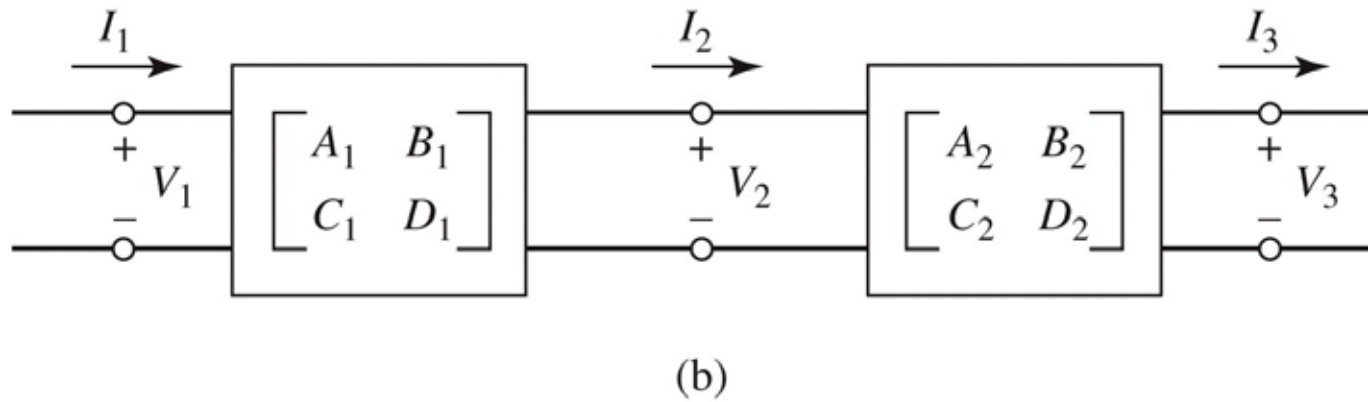
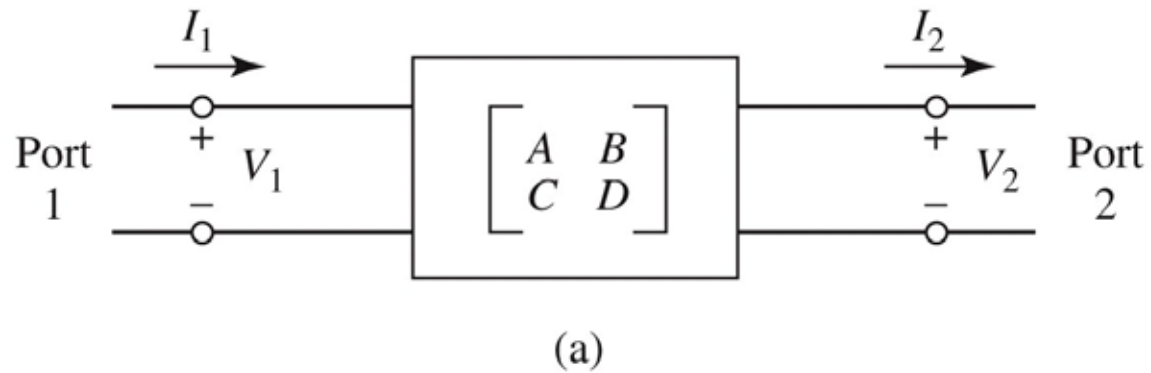


Figure 4.11
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TABLE 4.1 *ABCD* Parameters of Some Useful Two-Port Circuits

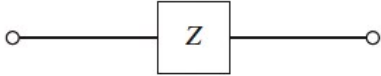
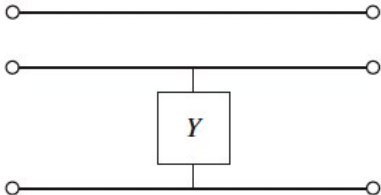
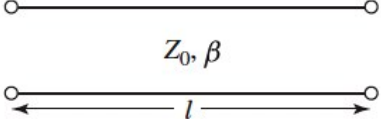
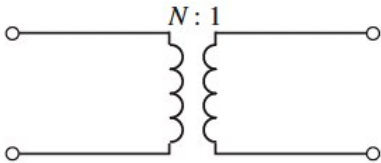
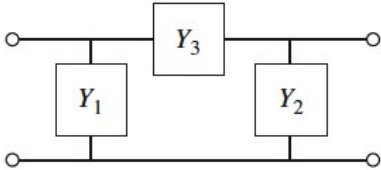
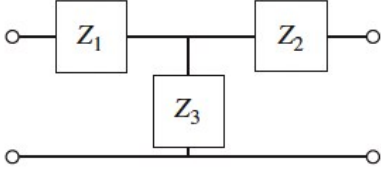
Circuit	<i>ABCD</i> Parameters	
	$A = 1$ $C = 0$	$B = Z$ $D = 1$
	$A = 1$ $C = Y$	$B = 0$ $D = 1$
	$A = \cos \beta l$ $C = jY_0 \sin \beta l$	$B = jZ_0 \sin \beta l$ $D = \cos \beta l$
	$A = N$ $C = 0$	$B = 0$ $D = \frac{1}{N}$
	$A = 1 + \frac{Y_2}{Y_3}$ $C = Y_1 + Y_2 + \frac{Y_1 Y_2}{Y_3}$	$B = \frac{1}{Y_3}$ $D = 1 + \frac{Y_1}{Y_3}$
	$A = 1 + \frac{Z_1}{Z_3}$ $C = \frac{1}{Z_3}$	$B = Z_1 + Z_2 + \frac{Z_1 Z_2}{Z_3}$ $D = 1 + \frac{Z_2}{Z_3}$

Table 4.1

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Conversion table

	S	z	y	h	ABCD
S	S_{11} S_{12} S_{21} S_{22}	$S_{11} = \frac{(z'_{11}-1)(z'_{22}+1) - z'_{12}z'_{21}}{\Delta_1}$ $S_{12} = \frac{2z'_{12}}{\Delta_1}$ $S_{21} = \frac{2z'_{21}}{\Delta_1}$ $S_{22} = \frac{(z'_{11}+1)(z'_{22}-1) - z'_{12}z'_{21}}{\Delta_1}$	$S_{11} = \frac{(1-y_{11})(1+y_{22}) + y_{12}y_{21}}{\Delta_2}$ $S_{12} = \frac{-2y_{12}}{\Delta_2}$ $S_{21} = \frac{-2y_{21}}{\Delta_2}$ $S_{22} = \frac{(1+y_{11})(1-y_{22}) + y_{12}y_{21}}{\Delta_2}$	$S_{11} = \frac{(h_{11}-1)(h_{22}+1) - h_{12}h_{21}}{\Delta_3}$ $S_{12} = \frac{2h_{12}}{\Delta_3}$ $S_{21} = \frac{-2h_{21}}{\Delta_3}$ $S_{22} = \frac{(1+h_{11})(1-h_{22}) + h_{12}h_{21}}{\Delta_3}$	$\frac{A'+B'-C'-D'}{\Delta_4}$ $\frac{2(A'D'-B'C')}{\Delta_4}$ $\frac{2}{\Delta_4}$ $\frac{-A'+B'-C'+D'}{\Delta_4}$
z	$z'_{11} = \frac{(1+S_{11})(1-S_{22}) + S_{12}S_{21}}{\Delta_5}$ $z'_{12} = \frac{2S_{12}}{\Delta_5}$ $z'_{21} = \frac{2S_{21}}{\Delta_5}$ $z'_{22} = \frac{(1-S_{11})(1+S_{22}) + S_{12}S_{21}}{\Delta_5}$	z_{11} z_{12} z_{21} z_{22}	$\frac{y_{22}}{ y }$ $\frac{-y_{12}}{ y }$ $\frac{-y_{21}}{ y }$ $\frac{y_{11}}{ y }$	$\frac{ h }{h_{22}}$ $\frac{h_{12}}{h_{22}}$ $\frac{-h_{21}}{h_{22}}$ $\frac{1}{h_{22}}$	$\frac{A}{C}$ $\frac{\Delta g}{C}$ $\frac{1}{C}$ $\frac{D}{C}$
y	$y'_{11} = \frac{(1-S_{11})(1+S_{22}) + S_{12}S_{21}}{\Delta_6}$ $y'_{12} = \frac{-2S_{12}}{\Delta_6}$ $y'_{21} = \frac{-2S_{21}}{\Delta_6}$ $y'_{22} = \frac{(1+S_{11})(1-S_{22}) + S_{12}S_{21}}{\Delta_6}$	$\frac{z_{22}}{ z }$ $\frac{-z_{12}}{ z }$ $\frac{-z_{21}}{ z }$ $\frac{z_{11}}{ z }$	y_{11} y_{12} y_{21} y_{22}	$\frac{1}{h_{11}}$ $\frac{-h_{12}}{h_{11}}$ $\frac{h_{21}}{h_{11}}$ $\frac{ h }{h_{11}}$	$\frac{D}{B}$ $\frac{-\Delta g}{B}$ $\frac{-1}{B}$ $\frac{A}{B}$
h	$h'_{11} = \frac{(1+S_{11})(1+S_{22}) - S_{12}S_{21}}{\Delta_7}$ $h'_{12} = \frac{2S_{12}}{\Delta_7}$ $h'_{21} = \frac{-2S_{21}}{\Delta_7}$ $h'_{22} = \frac{(1-S_{11})(1-S_{22}) - S_{12}S_{21}}{\Delta_7}$	$\frac{ z }{z_{22}}$ $\frac{z_{12}}{z_{22}}$ $\frac{-z_{21}}{z_{22}}$ $\frac{1}{z_{22}}$	$\frac{1}{y_{11}}$ $\frac{-y_{12}}{y_{11}}$ $\frac{y_{21}}{y_{11}}$ $\frac{ y }{y_{11}}$	h_{11} h_{12} h_{21} h_{22}	$\frac{B}{D}$ $\frac{-\Delta g}{D}$ $\frac{-1}{D}$ $\frac{C}{D}$
ABCD	$A' = \frac{(1+S_{11})(1-S_{22}) + S_{12}S_{21}}{2S_{21}}$ $B' = \frac{(1+S_{11})(1+S_{22}) - S_{12}S_{21}}{2S_{21}}$ $C' = \frac{(1-S_{11})(1-S_{22}) - S_{12}S_{21}}{2S_{21}}$ $D' = \frac{(1-S_{11})(1+S_{22}) + S_{12}S_{21}}{2S_{21}}$	$\frac{z_{11}}{z_{21}}$ $\frac{ z }{z_{21}}$ $\frac{1}{z_{21}}$ $\frac{z_{22}}{z_{21}}$	$\frac{-y_{22}}{y_{21}}$ $\frac{-1}{y_{21}}$ $\frac{- y }{y_{21}}$ $\frac{-y_{11}}{y_{21}}$	$\frac{- h }{h_{21}}$ $\frac{-h_{11}}{h_{21}}$ $\frac{-h_{22}}{h_{21}}$ $\frac{-1}{h_{21}}$	A B C D

$\Delta_1 = (z'_{11}+1)(z'_{22}+1) - z'_{12}z'_{21}$
 $\Delta_2 = (1+y_{11})(1+y_{22}) - y_{12}y_{21}$
 $\Delta_3 = (h_{11}+1)(h_{22}+1) - h_{12}h_{21}$
 $\Delta_4 = A'+B'+C'+D'$
 $\Delta_5 = (1-S_{11})(1-S_{22}) - S_{12}S_{21}$
 $\Delta_6 = (1+S_{11})(1+S_{22}) - S_{12}S_{21}$
 $\Delta_7 = (1-S_{11})(1+S_{22}) + S_{12}S_{21}$
 $\Delta_8 = AD - BC$

$z'_{11} = z_{11}/Z_0, z'_{12} = z_{12}/Z_0, z'_{21} = z_{21}/Z_0, z'_{22} = z_{22}/Z_0$
 $y'_{11} = y_{11}/Z_0, y'_{12} = y_{12}/Z_0, y'_{21} = y_{21}/Z_0, y'_{22} = y_{22}/Z_0$
 $h'_{11} = h_{11}/Z_0, h'_{12} = h_{12}, h'_{21} = h_{21}, h'_{22} = h_{22}/Z_0$
 $A' = A, B' = B/Z_0, C' = CZ_0, D' = D$
 $|z| = z_{11}z_{22} - z_{12}z_{21}$
 $|y| = y_{11}y_{22} - y_{12}y_{21}$
 $|h| = h_{11}h_{22} - h_{12}h_{21}$

See Gonzalez, page 62.

Smith chart

Impedance transformation and matching

- to match an arbitrary load to a given transmission line
- to present a certain impedance to a device (embedding impedances)

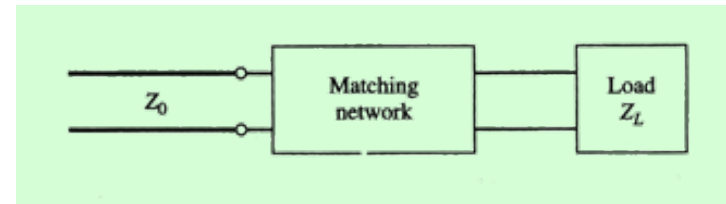
 For low VSWR, energy transfer or design goals

Low VSWR results in better power handling capability

Distributed components

- Single, double or triple stubs
- Transformers

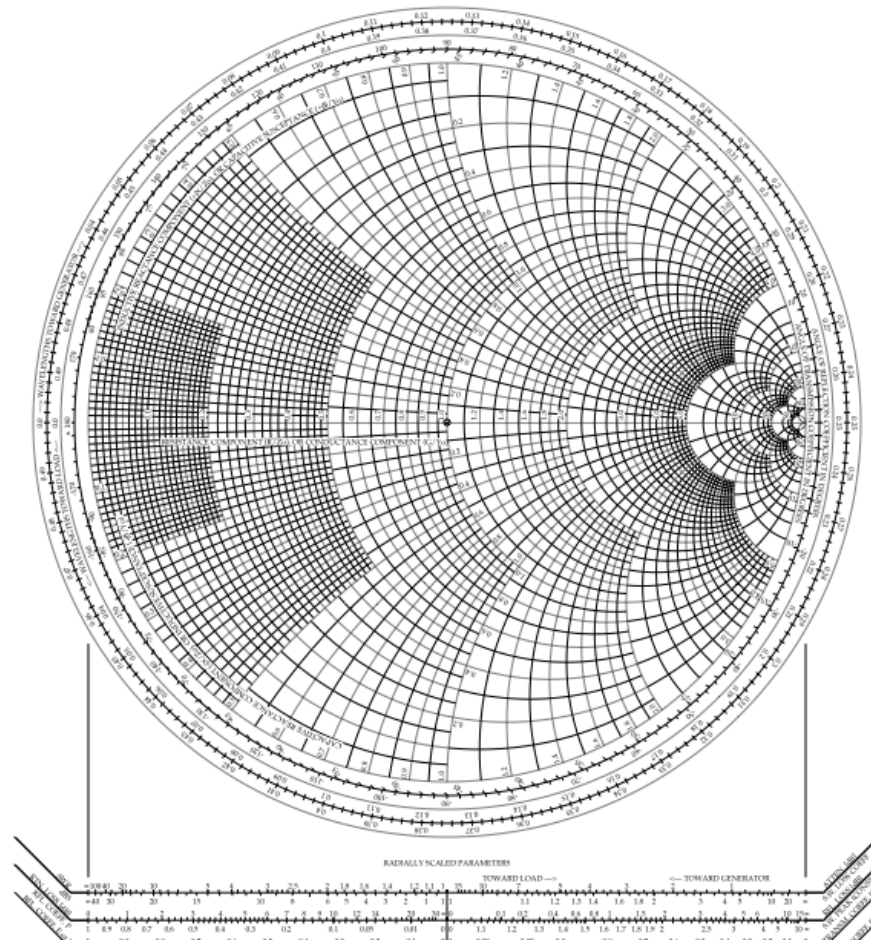
Discrete components



The Smith Chart (SC)

- Proposed 1939 by Philip H. Smith as a graphical aid to analyse and design matching networks
- Mr. Smith worked at Bell Telephone labs with impedance matching of antennas (for AM broadcasting)
- Today, still a powerful tool as part of the design process in order to find suitable circuit topologies etc

Z or impedance SC

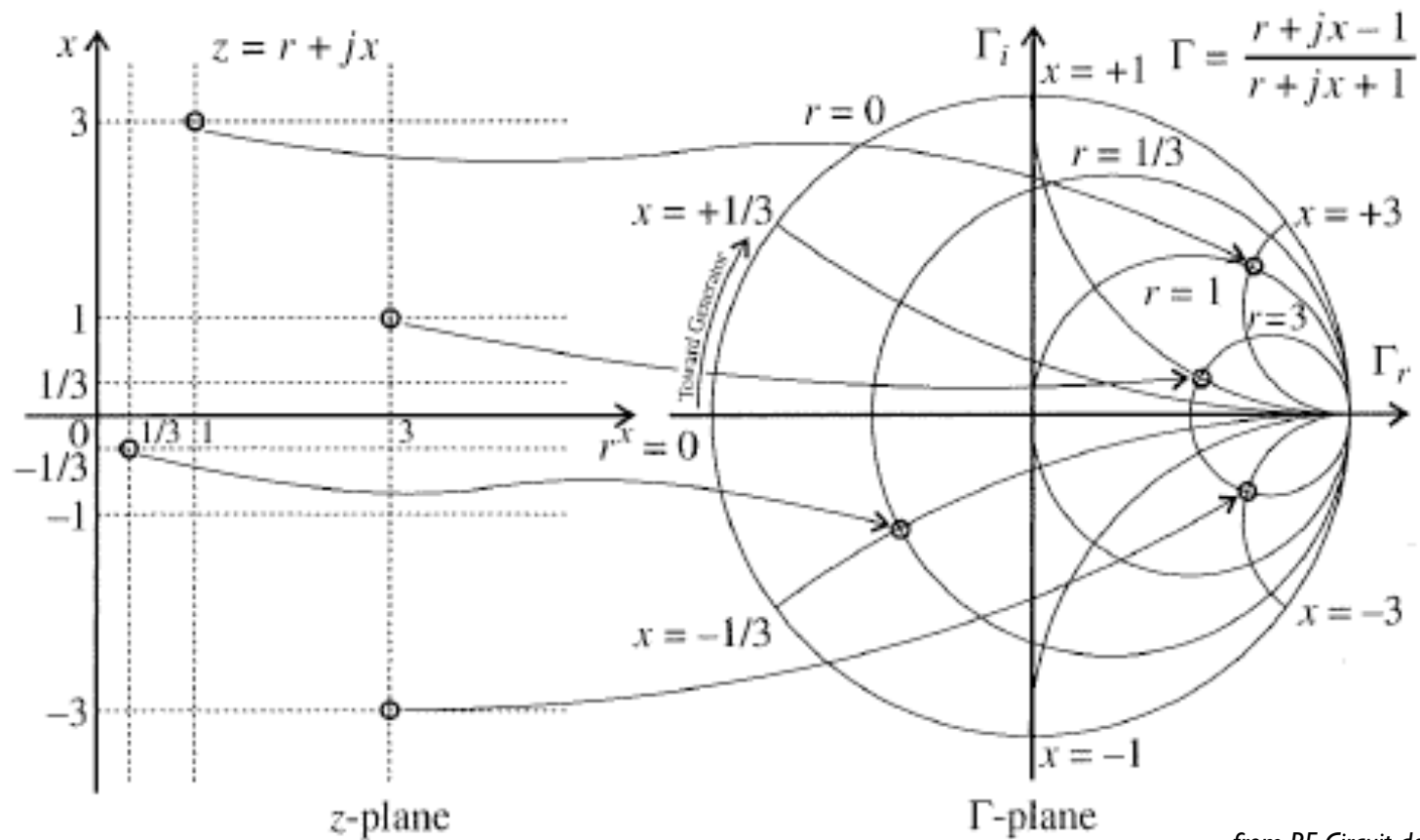


Conformal mapping (Möbius)

- Z Smith chart: $\Gamma = \frac{z-1}{z+1}$

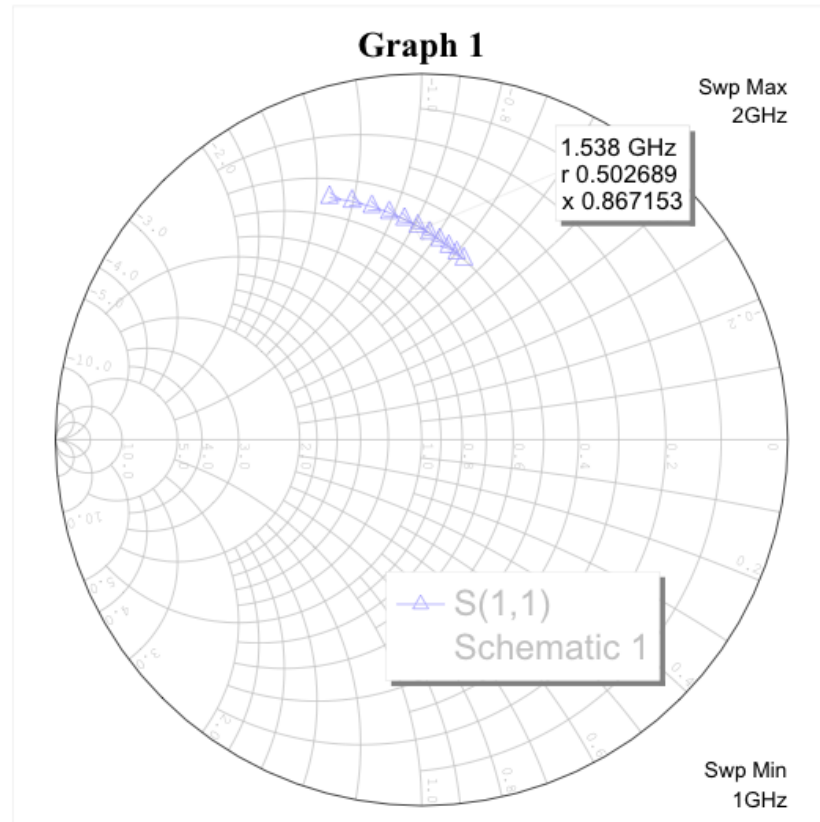
- Y Smith chart: $\Gamma = \frac{1-y}{1+y}$

Complex impedance transformed to complex reflection plane



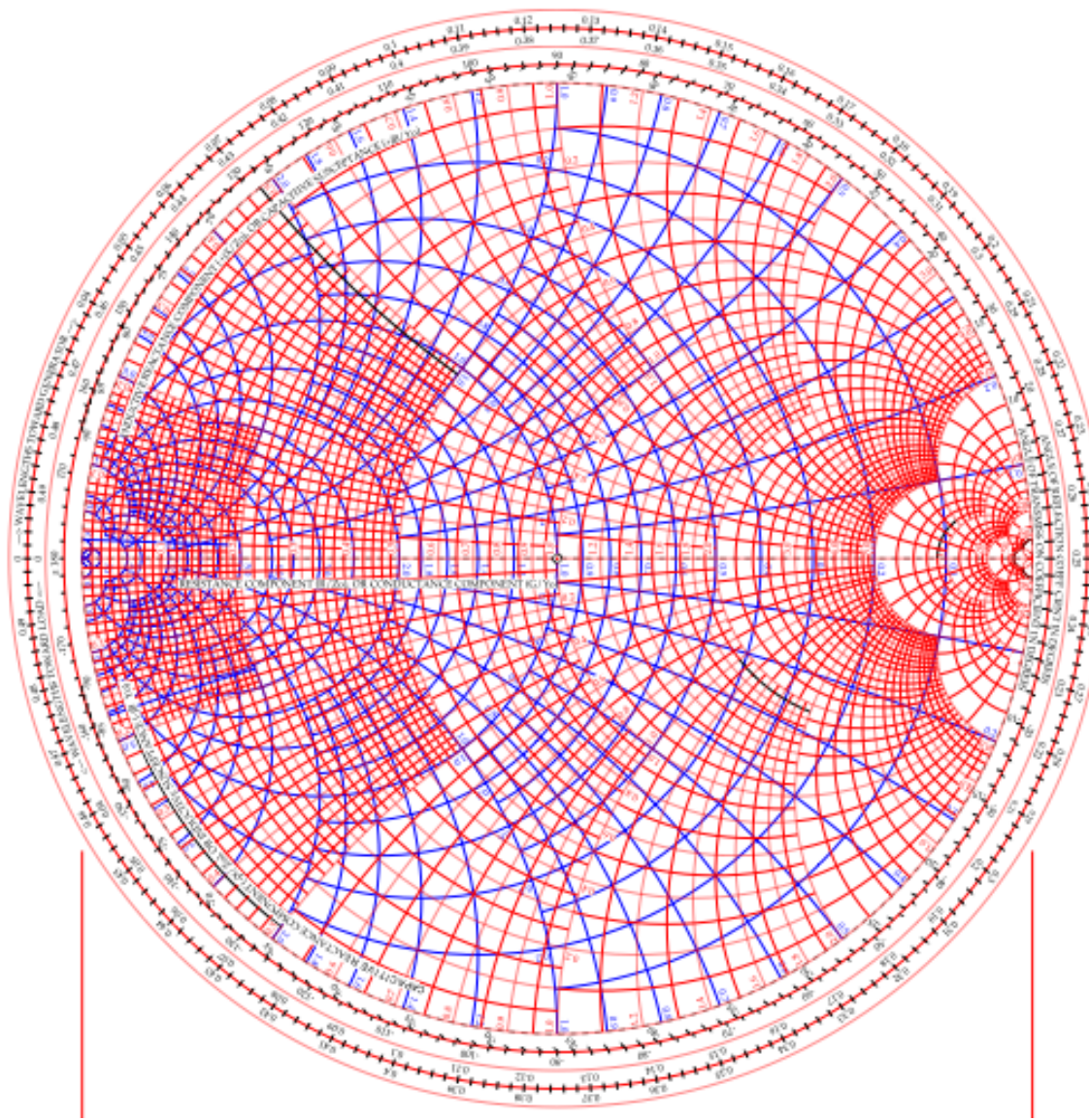
from RF Circuit design, Ludvig and Bogdanov

Y or admittance SC



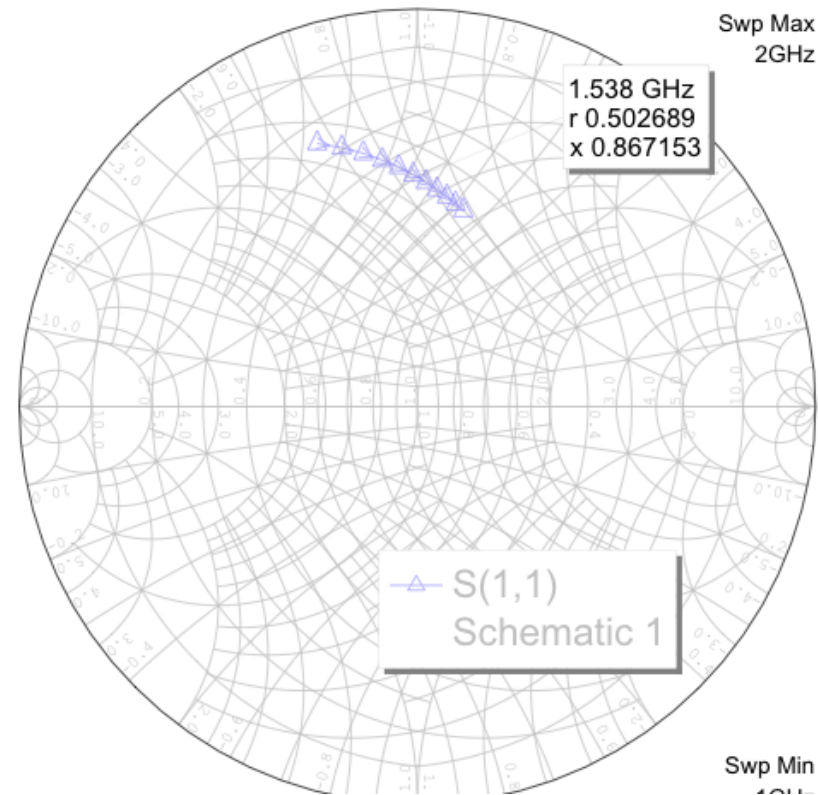
The Smith Chart

- Complex plane for the reflection coefficient.
 - Normalised contours for resistances/ conductances and reactances/susceptances
 - Upper half->inductive, lower half->capacitive
- Common practice to plot S-parameters in Smith charts. E.g. Vector network analysers or design tools



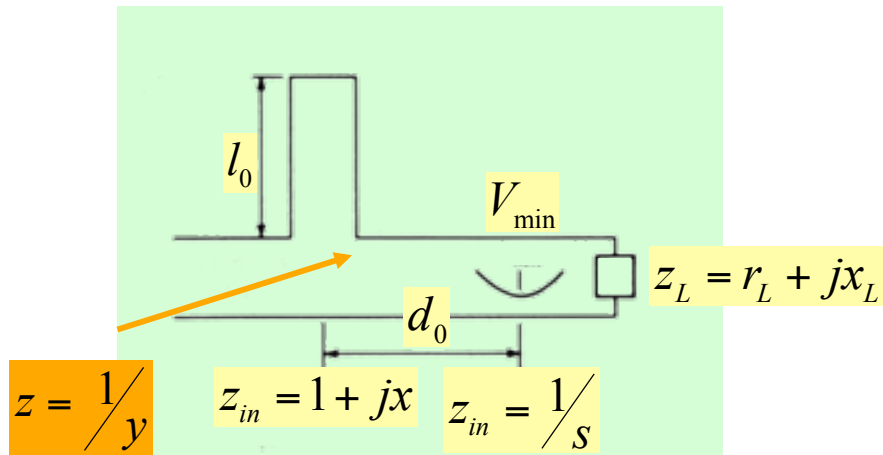
ZY Smith Chart

- Z for series connections



- Y for parallel connections

Single stub matching (series)



$$y = \frac{(g_L + jb_L) + jt}{1 + jt(g_L + jb_L)}; \quad t = \tan \beta d$$

$$z = \frac{1}{y} = r + jx$$

$$r = \frac{g_L(1+t^2)}{g_L^2 + (b_L + t)^2}; \quad x = \frac{g_L^2 t - (1 - tb_L)(b_L + t)}{g_L^2 + (b_L + t)^2}$$

To get the match we need the real part of the equation, $r = 1$

$$t = \frac{b_L \pm \sqrt{g_L [(1 - g_L)^2 + b_L^2]}}{g_L - 1}; \quad g_L \neq 1$$

If $g_L = 1 \rightarrow t = -b_L/2$

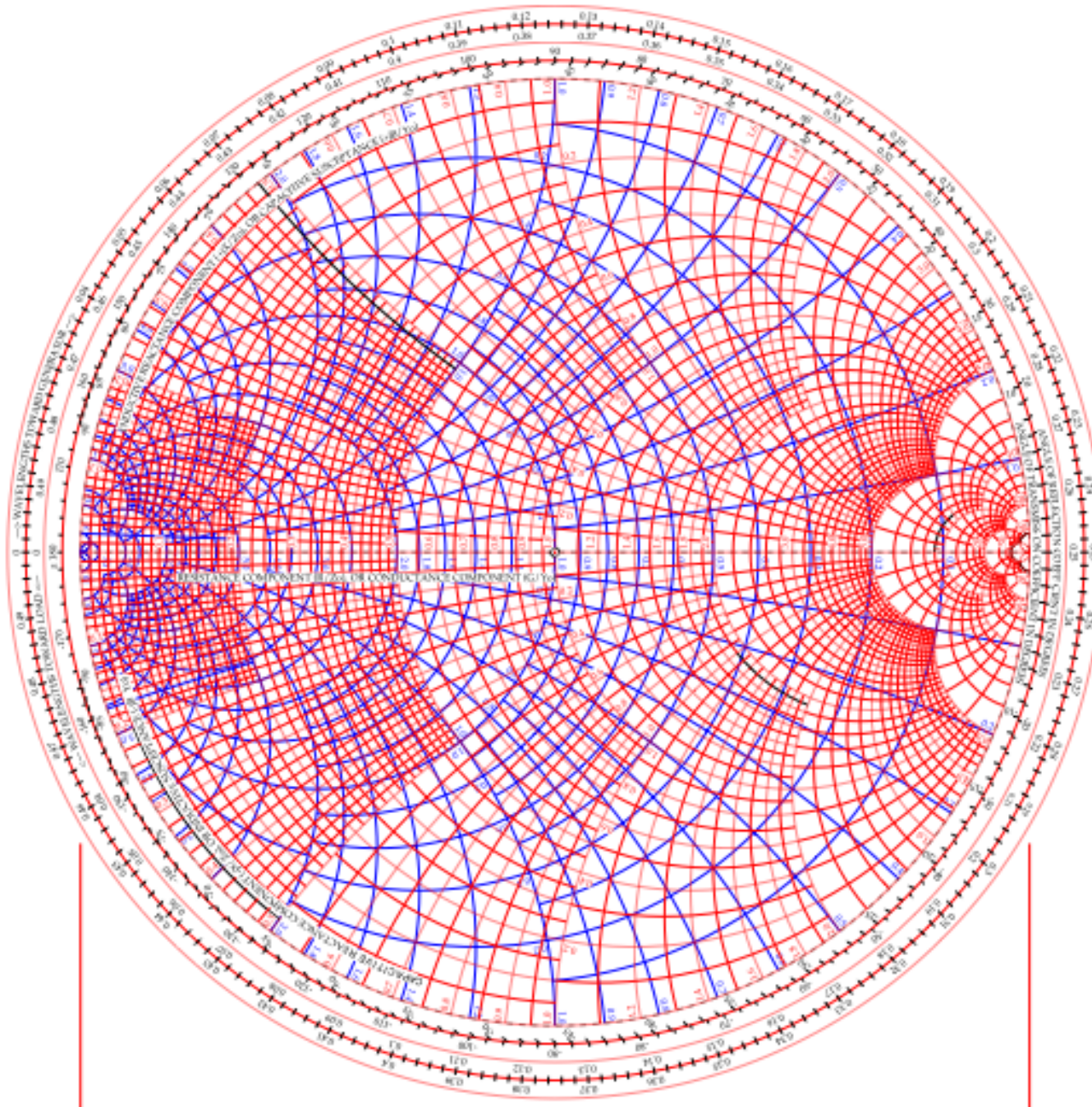
$$d = \frac{\lambda}{2\pi} \arctan t, \quad t \geq 0$$

$$d = \frac{\lambda}{2\pi} (\pi + \arctan t), \quad t < 0$$

$$\text{For shorted stub } l_0 = -\frac{\lambda}{2\pi} \arctan \left(\frac{X}{Z_0} \right)$$

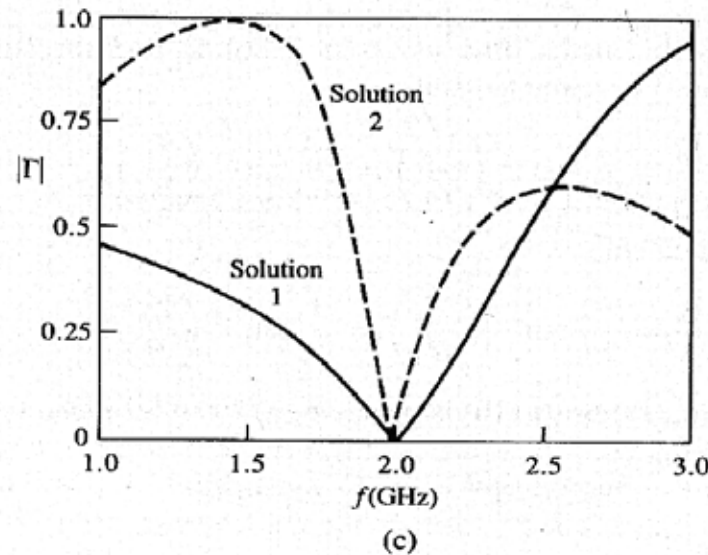
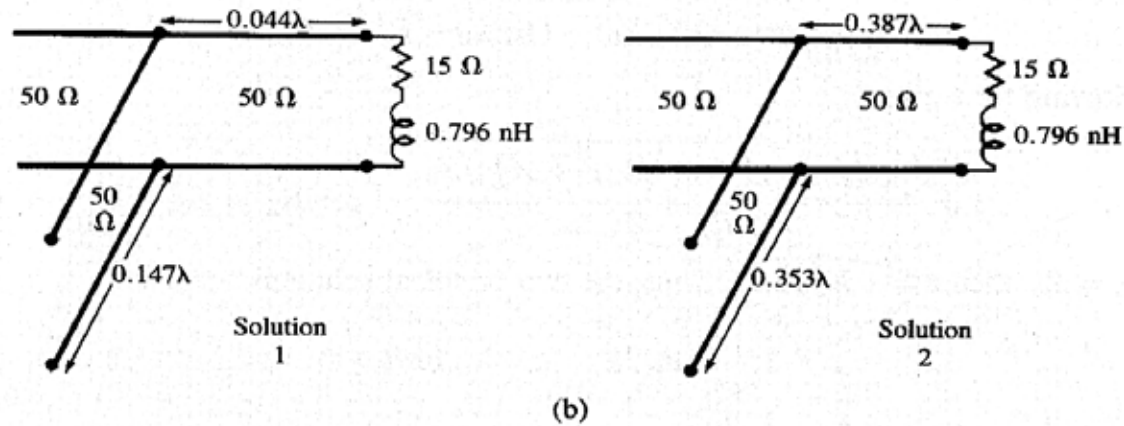
$$\text{For open stub } l_0 = \frac{\lambda}{2\pi} \arctan \left(\frac{Z_0}{X} \right)$$

- *On white board: Use SC to match $z=2+j1.6$ using an open series stub*



- *On white board: Use SC to match $z=0.3+j0.2$ using an open parallel stub*

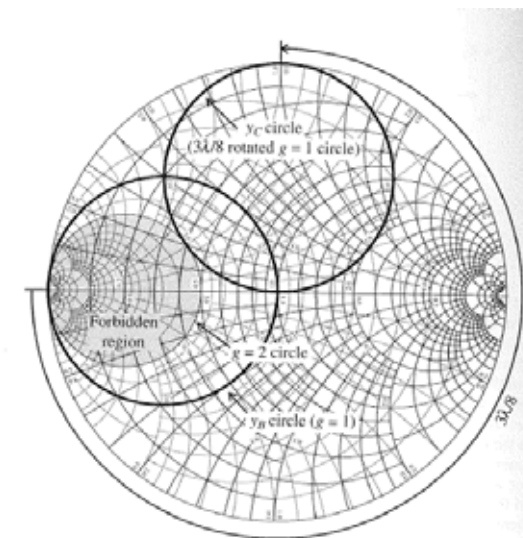
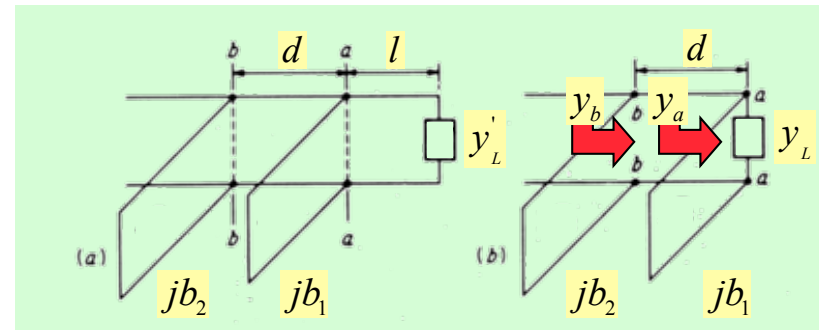
Transmission line matching



Electrically small circuit \rightarrow wider bandwidth!

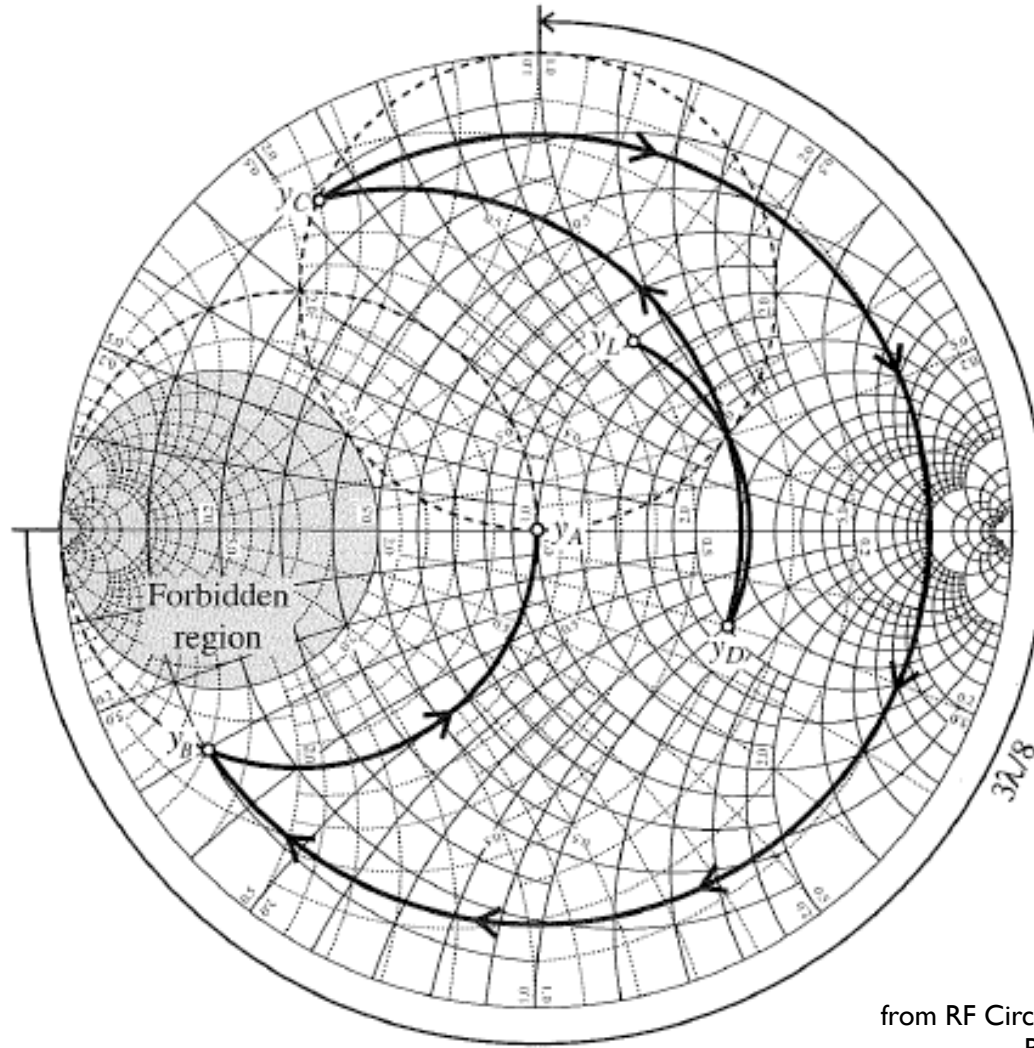
Double-stub tuner

- Rotate $g=1$ circle counter clock wise, so the first stub (jb_1) can transform your load to the rotated circle
- require no variable length between load and stubs
- But!, forbidden region



from RF Circuit design, Ludwig and Bogdanov

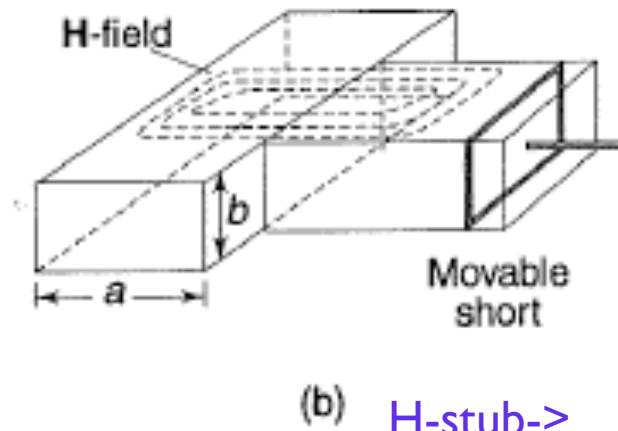
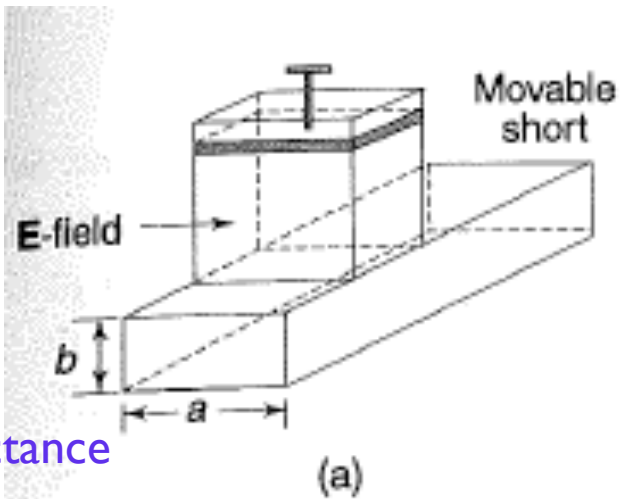
- *On white board: Use SC to match $l + j$ using double, shorted stubs. Distance between stubs $3/8$ wavelengths, and $1/8$ wavelength between load and first stub.*



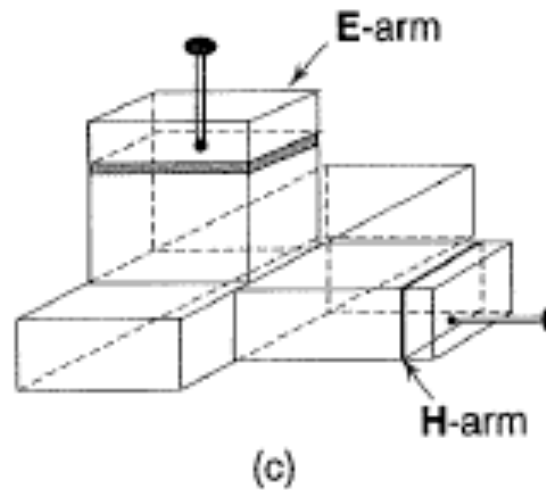
from RF Circuit design, Ludvig and Bogdanov

E-H Tuners

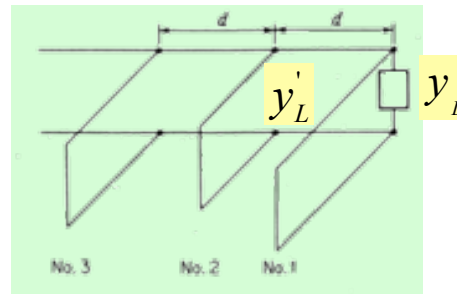
E-stub->
series reactance



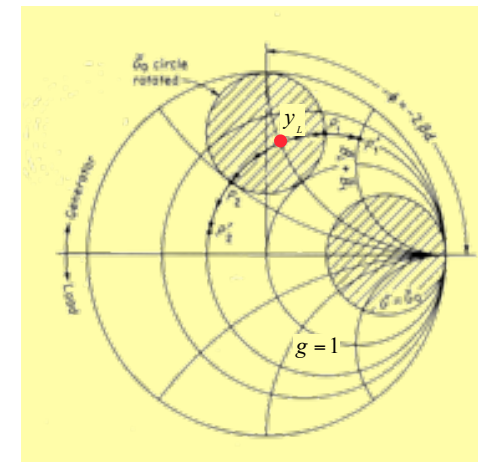
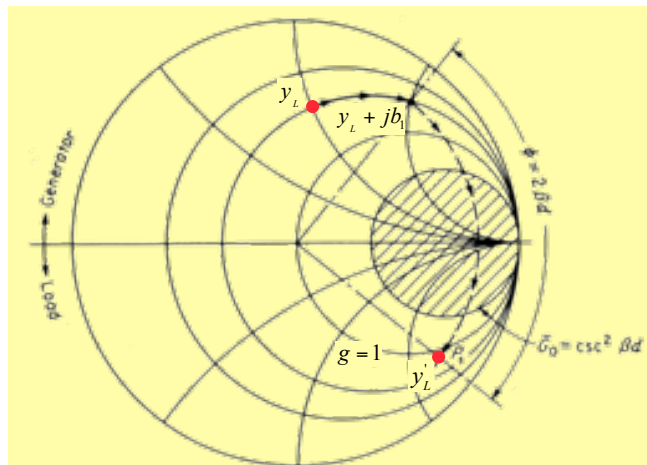
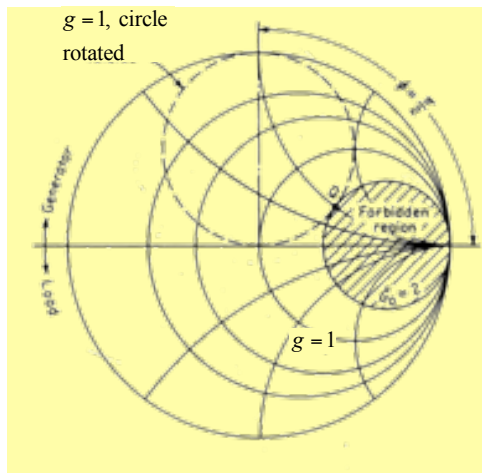
H-stub->
parallel susceptance



Triple-stub tuner

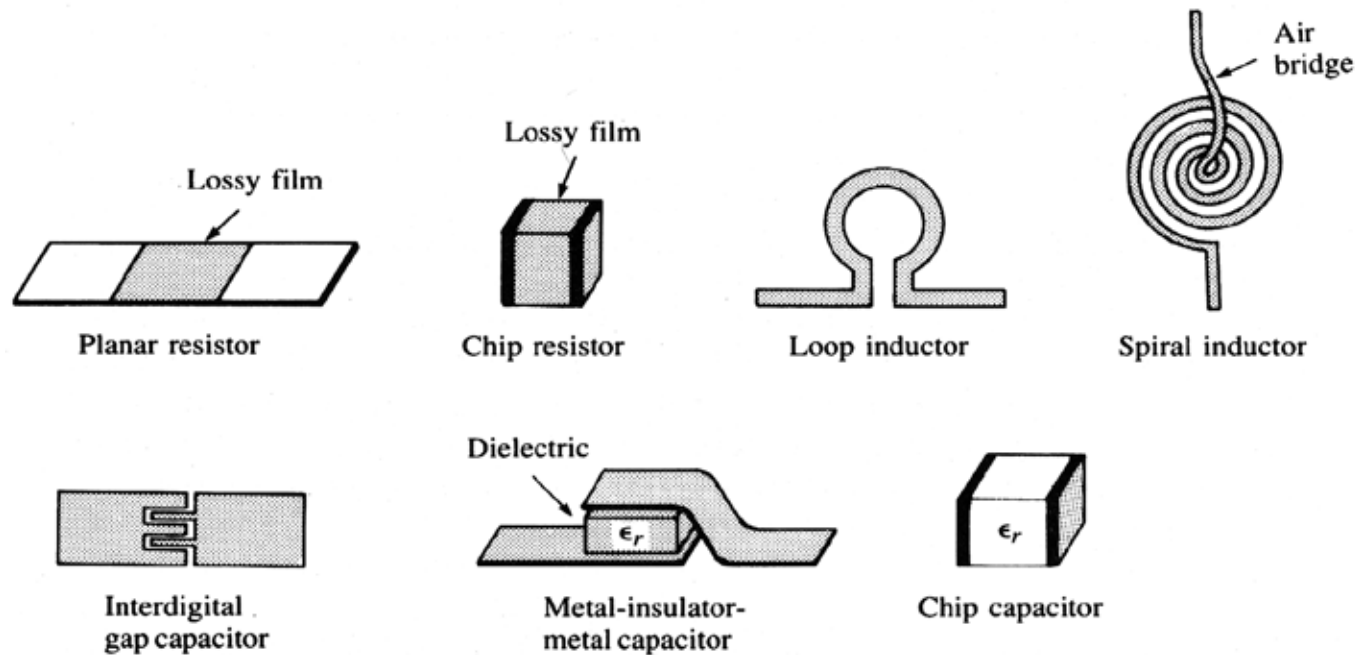


Why?



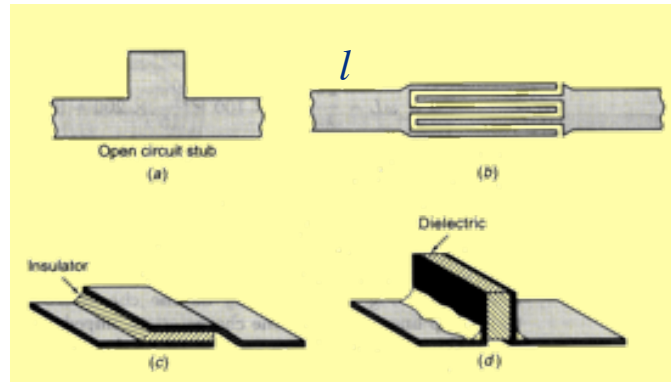
Function of stub 1 is to ensure, that the $y'_L = g'_L + jb'_L$ has $g'_L < \csc^2 bd$

Matching with lumped elements



Capacitors

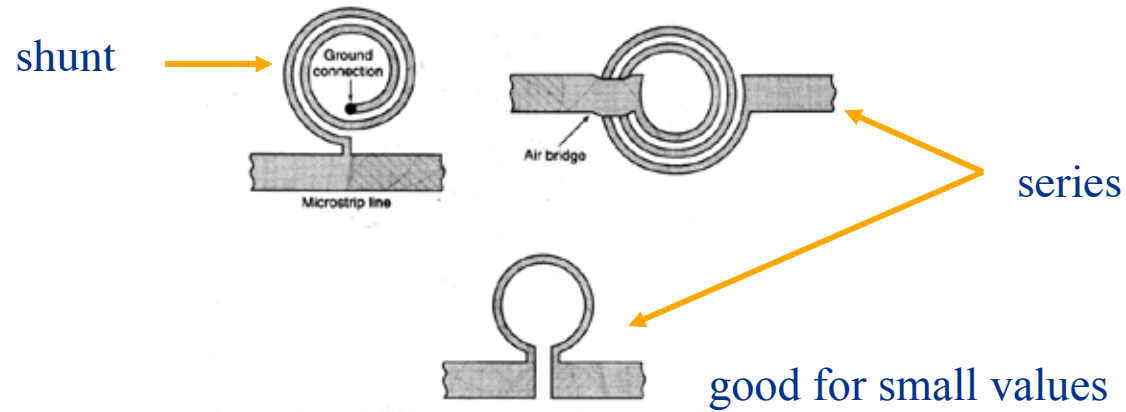
$$l = \lambda/4, C < 1 \text{ pF}$$



Interdigital capacitor, MMIC compatible, $C < 5-6 \text{ pF}$

MIM(metal-insulator-metal),
MMIC compatible, $C < 20-30 \text{ pF}$

Inductors



We assume $l = \lambda$ and TEM wave

$$Z_c = \sqrt{\frac{L}{C}}, \beta = \omega\sqrt{LC}$$

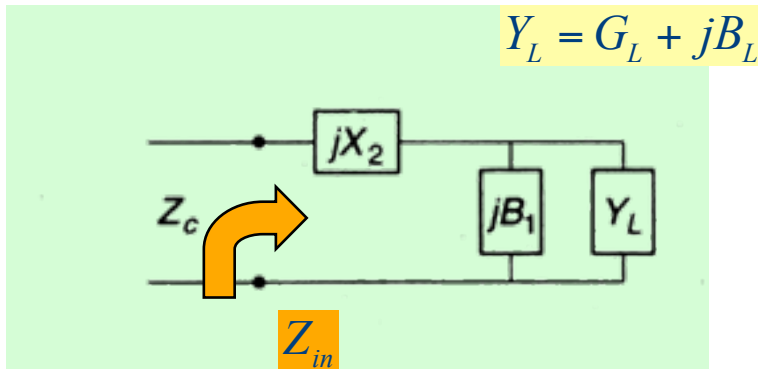
$$X_L = \omega L = \beta Z_c = k_0 Z_{c0} \Omega/m \quad (1)$$

Eq (1) shows that narrow lines (high impedance) should be used for inductors since

$$L \nearrow \text{ if } Z_{c0} \nearrow$$

Matching circuits

$$R_L > Z_c$$

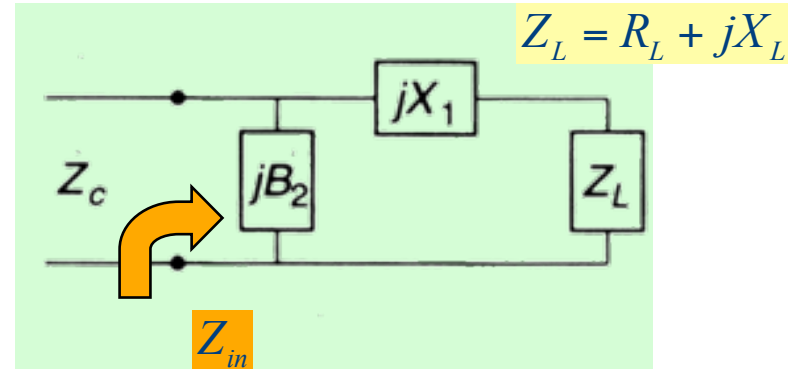


$$Z_{in} = Z_c$$

$$X_2 = \pm \left(\frac{Z_c (1 - Z_c G_L)}{G_L} \right)^{\frac{1}{2}}$$

$$B_1 = -B_L \pm \left(\frac{G_L (1 - Z_c G_L)}{Z_c} \right)^{\frac{1}{2}}$$

$$R_L < Z_c$$



$$Z_{in} = Z_c$$

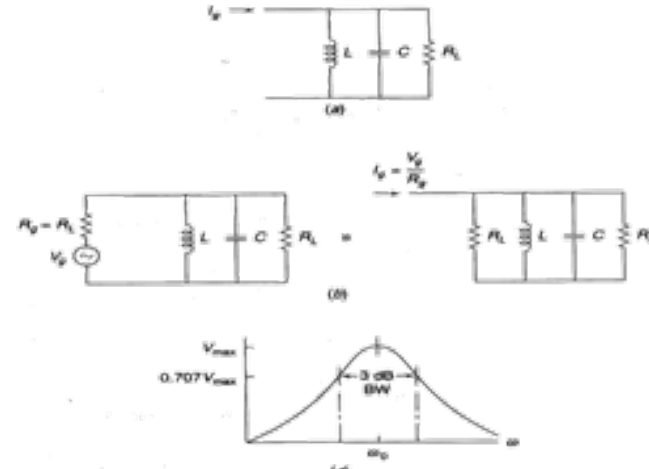
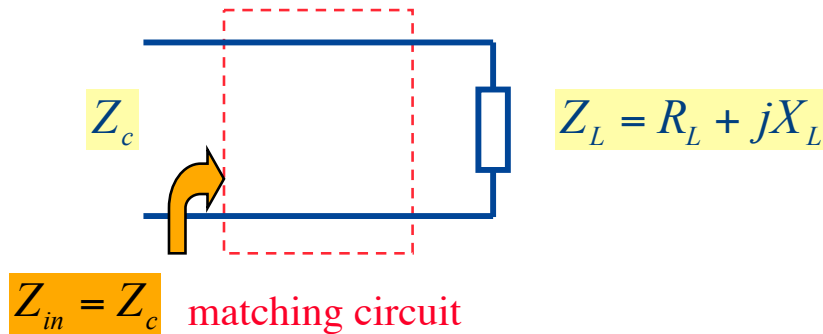
$$X_1 = \pm \left(R_L (Z_c - R_L) \right)^{\frac{1}{2}} - X_L$$

$$B_2 = \pm \frac{1}{Z_c} \left(\frac{Z_c - R_L}{R_L} \right)^{\frac{1}{2}}$$

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- *On white board: Use SC to match $100 + j100$ ohm using L and C, to a 50 ohm transmission line.*

Circuit Q and bandwidth



Reactive components in Z_L and matching circuit form a resonance circuit loaded with R_L and Z_c with a quality factor Q:

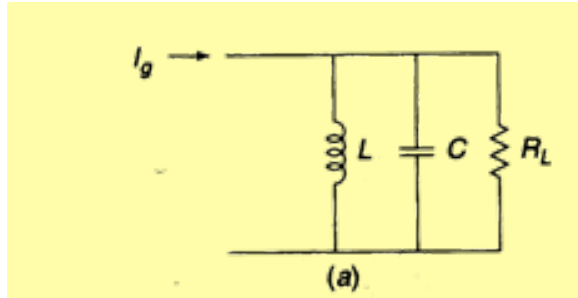
$$Q = \frac{\omega(\text{average stored electric and magnetic energy})}{\text{power loss}}$$

At resonance:

$$W_m = W_e$$

$$Q = \omega \frac{2W_e}{P_{loss}} = \omega \frac{2W_m}{P_{loss}}$$

The bandwidth of the circuit is the frequency band, Δf over which $\frac{1}{2}$ or more (3 dB) of the maximum power is delivered to the load (it is inversely proportional to the loaded Q)



$$V_{R_L} = \frac{I_g}{Y_{in}} = \frac{I_g}{G_L + j\omega C - j/\omega L}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$Y_{in} = G_L + j\omega C \left(\frac{\omega^2 - \omega_0^2}{\omega^2} \right)$$

At resonance $\omega = \omega_0$

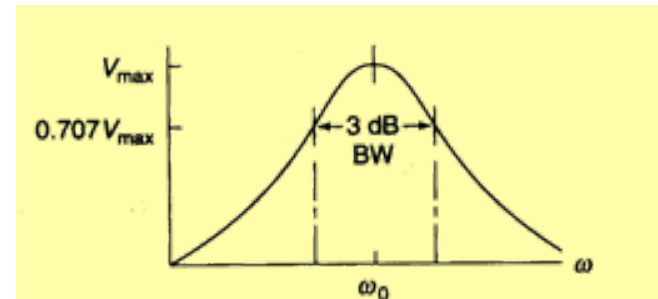
$$V_{R_L} = \frac{I_g}{G_L} \rightarrow V_{\max} |_{load} \rightarrow P_{\max}$$

At the band edges

$$j\omega C \frac{\omega^2 - \omega_0^2}{\omega^2} = jG_L$$

$$|V_{R_L}| = \frac{I_g}{\sqrt{2}G_L} \rightarrow P = 1/2 P_{\max}$$

$$Q = \frac{R_L}{\omega_0 L} = R_L \omega_0 C = \frac{\omega_0 C}{G_L}$$



$Q > 10$

$$\begin{aligned}
 Y_{in} &= G_L + j\omega C \left(\frac{\omega^2 - \omega_0^2}{\omega^2} \right) \approx G_L + j\omega C \frac{2\omega(\omega - \omega_0)}{\omega^2} = \\
 &= G_L + j\omega_0 C \frac{2(\omega - \omega_0)}{\omega_0} = G_L \left(1 + 2jQ \frac{\Delta\omega}{\omega_0} \right)
 \end{aligned}$$

The 3-dB fractional BW:

$$2Q \frac{\Delta\omega}{\omega_0} = 2Q \frac{BW}{2} = 1 \rightarrow BW = \frac{1}{Q}$$

In the matching problems there are generally two solutions possible:

- narrowband design \Rightarrow high Q-value
- broadband design \Rightarrow low Q-value

Summary of lecture 6

- Read chapter 5 (impedance matching).
 - Smith chart
 - Single, double and triple stub matching
 - Discrete elements for matching
- Next: Impedance transformation (ch5)

Further reading

- A. Inan, “Remembering Phillip H. Smith on his 100th birthday,” Antennas and Propagation Society International Symposium, 2005 IEEE, vol. 3, pp. 129–132 vol. 3B, Jun. 2005.
- R. M. Fano, Theoretical limitations on the broadband matching of arbitrary impedances, no. 41. 1948.