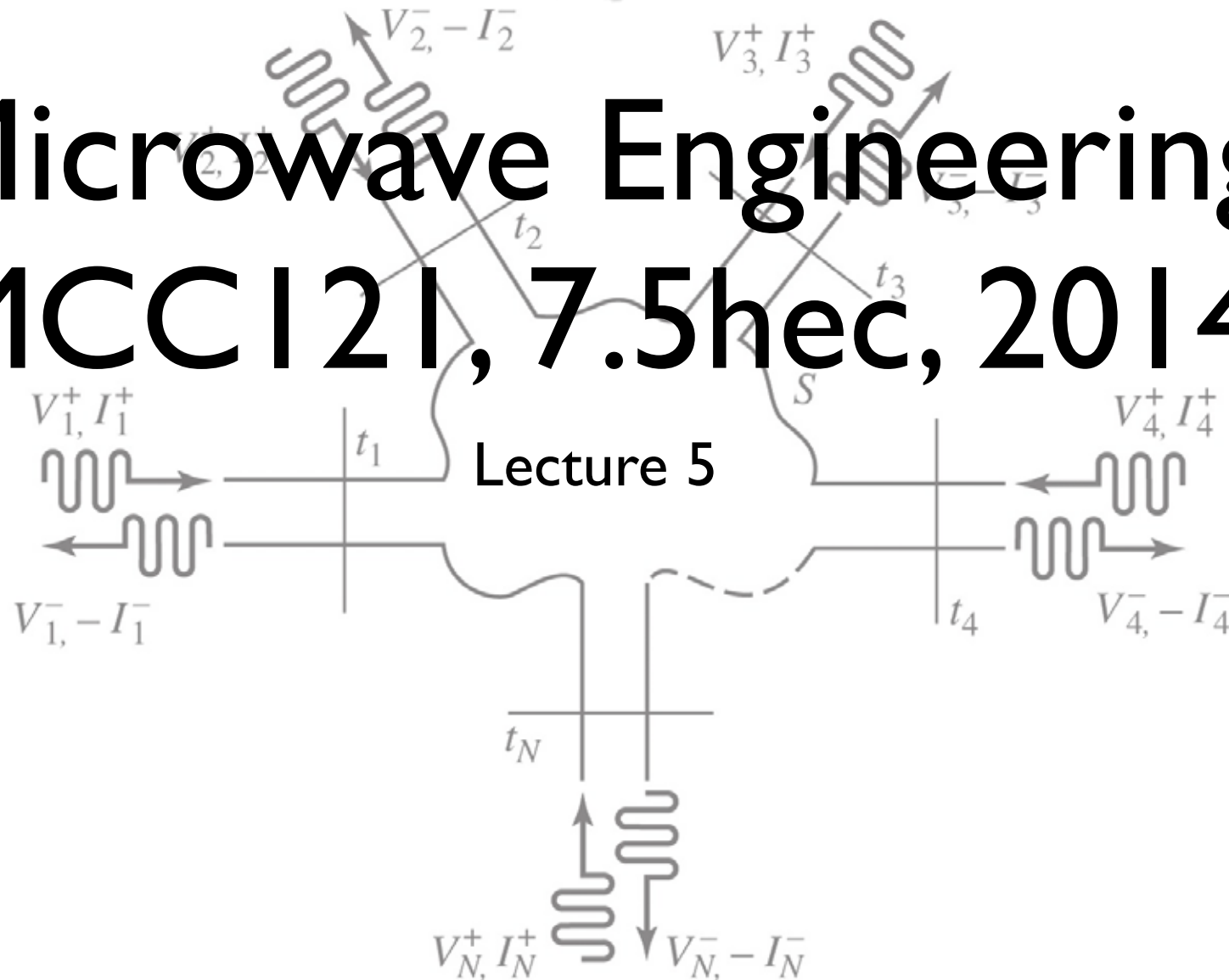


*"a study of microwave circuits provides a deeper physical insight into conventional circuit theory" R E Collin*

# Microwave Engineering

## MCCI21, 7.5hec, 2014



Challenging  
Stimulating  
Rewarding

# Notice

- Don't forget to register for the labs!

# Outline

- Summary of transmission lines (Ch3)
- Circuit theory for waveguiding systems (Ch4)
  - Impedance matrix
  - Reciprocal, lossless networks
  - Scattering matrix
  - ABCD matrix

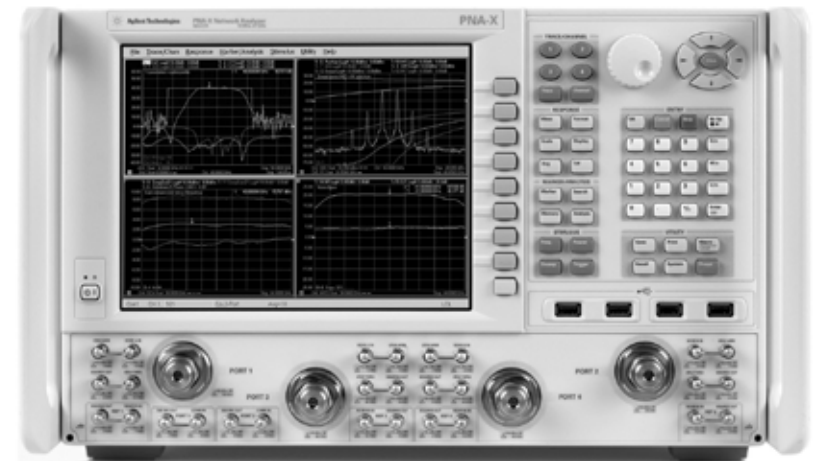


Figure 4.7

# Objectives

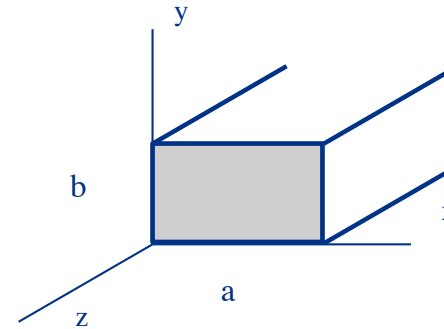
On completion of this course unit you should be able to:

- Analyse wave propagating properties of guided wave structures (TE, TM, TEM waves, microstrip, stripline, rectangular and circular waveguides, coupled lines)
- Apply N-port representations for analysing microwave circuits
- Apply the Smith chart to evaluate microwave networks
- Design and evaluate impedance matching networks
- Design, evaluate and characterise directional couplers and power dividers
- Design and analyse attenuators, phase shifters and resonators
- Explain basic properties of ferrite devices (circulators, isolators)

# Hollow waveguides

$\bar{e}$  exists if  $\bar{h}_z$  exists (TE wave, H mode)  
 $\bar{h}$  exists if  $\bar{e}_z$  exists (TM wave, E mode)

$TE_{n,m}$  and  $TM_{n,m}$  modes



The integers  $n$  and  $m$  pertain to the number of standing-wave interference maxima occurring in the field solutions that describe the variation of the fields along the two transverse coordinates

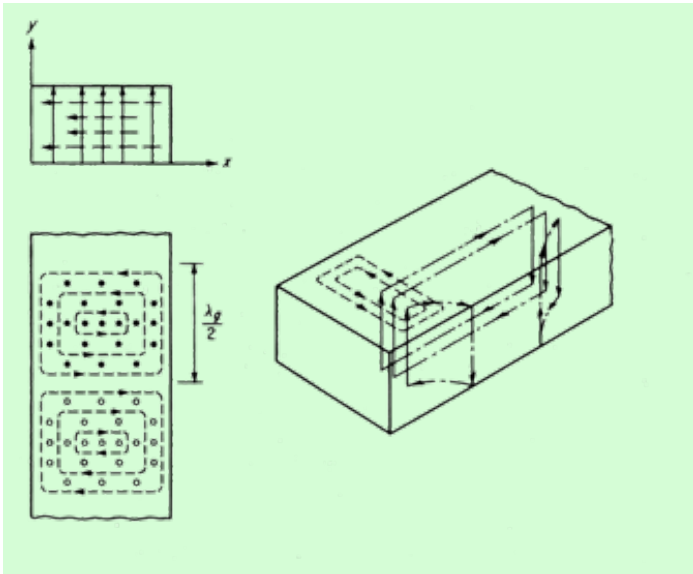
$f_{c,nm}$  corresponds to cut-off frequency below which the mode does not propagate; it is a geometrical parameter dependent on the waveguide cross-sectional configuration

Propagation factor  $\beta$

$$\beta = \sqrt{k_0^2 - k_c^2}$$

$$k_0 = 2\pi f \sqrt{\mu_0 \epsilon}, k_c = 2\pi f_c \sqrt{\mu_0 \epsilon}$$

# Dominant TE<sub>10</sub> mode



$$H_{z,10} = A \cos \frac{\pi x}{a} e^{-j\beta z}$$

$$H_{x,10} = A \frac{j\beta}{k_c} \sin \frac{\pi x}{a} e^{-j\beta z}$$

$$E_{y,10} = -AZ_{h,10} \frac{j\beta}{k_c} \sin \frac{\pi x}{a} e^{-j\beta z}$$

$$k_{c,10} = \frac{\pi}{a}, \beta_{10} = \sqrt{k_0^2 - \left(\frac{\pi}{a}\right)^2}$$

$$Z_{h,10} = -\frac{E_y}{H_x} = \frac{k_0}{\beta} Z_0$$

$$\lambda_g = \frac{2\pi}{\beta} = \frac{\lambda_0}{\sqrt{1 - (\lambda_0/2a)^2}}$$

$$v_p = \frac{\lambda_g}{\lambda_0} c, v_g = \frac{\lambda_0}{\lambda_g} c$$

# The concept of impedance

The term impedance was first used by Oliver Heaviside in the 19th century to describe the complex ratio  $V/I$  in AC circuits. In the 1930's Schelkunoff extended this concept to electromagnetic fields and noted that impedance should be regarded as characteristic of the type of field, as well as medium. The impedance may also be dependent on the direction of the propagating wave. The concept of impedance is an important link between field theory and transmission line theory.

- Intrinsic impedance of the medium,  $Z_0 = \eta = \sqrt{\frac{\mu}{\epsilon}}$
- Wave impedance; this impedance is a characteristic of the particular type of wave. TEM, TE, TM waves each have different wave impedances; they may depend on the type of the line or guide, the material, and frequency,  $Z_w = E/H$
- Characteristic impedance is the ratio of voltage to current for a travelling wave; voltage and current are uniquely defined only for a TEM wave; TE and TM waves do not have uniquely defined voltage and current, so the characteristic impedance for such waves may be defined in various ways.  $Z_0 = \sqrt{\frac{L}{C}}$

# Circuit theory

- At low frequencies: Kirchoff's laws apply
- At high frequencies: propagation effects important
- Still! it is possible to utilise equivalent voltages and currents. But with the main difference that such voltage/current waves are not always uniquely defined.

*"a study of microwave circuits provides a deeper physical insight into conventional circuit theory" R E Collin*

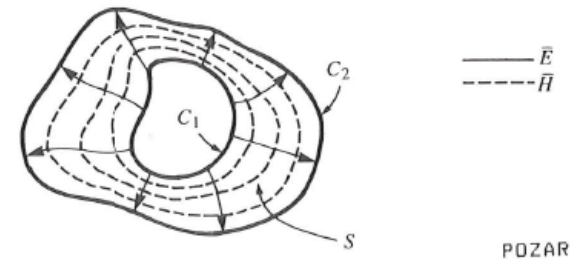


# Circuit theory cont.

- Hard and often impossible to measure  $v(t)$  and  $i(t)$  at high frequencies

$$V = \int_{+}^{-} \vec{E} \cdot d\vec{l}, I = \oint_C \vec{H} \cdot d\vec{l}$$

- TEM Wave



- A unique impedance can be defined. Circuit theory directly applicable!

# Waveguides: TE and TM modes

- *On white board: Show that voltage,  $V$ , is not unique but depends on  $x, y$ . For a TE<sub>10</sub> mode in a rectangular waveguide.*

# Equivalent voltage and current waves

- Power transmitted is given by an integral involving the transverse electric and magnetic fields only
- In a loss free guide supporting several modes, the power transmitted is the sum of that contributed by each mode individually
- The transverse field vary with distance along the guide according to  $e^{\pm j\beta z}$
- The transverse magnetic field is related to the transverse electric field by a simple constant

$$Z_{\omega} \mathbf{h} = \mathbf{a}_z \times e$$

# Fictitious transmission lines

$$V = \sum_{n=1}^N \left( V_n^+ e^{-j\beta_n z} + V_n^- e^{j\beta_n z} \right)$$

$$I = \sum_{n=1}^N \left( I_n^+ e^{-j\beta_n z} - I_n^- e^{j\beta_n z} \right) = \sum_{n=1}^N \left( Y_n V_n^+ e^{-j\beta_n z} - Y_n V_n^- e^{j\beta_n z} \right)$$

- A waveguide supporting N propagating modes, can be represented as N fictitious transmission lines supporting equivalent voltages and current waves
- Hence, when several modes are supported, the number of electrical ports will exceed the number of physical ports

# Equivalent voltage, current and impedance

- Only for a particular mode, so  $Z_{\omega} \mathbf{h} = \mathbf{a}_z \times e$
- Eq.  $V$  and  $I$  should be defined so their product gives the power flow of the particular mode
- The ratio should be equal to the characteristic impedance of the line (this selection is arbitrary, often equal to wave impedance)

# Comment: Impedance concept

- *On white board: Discuss common definitions such as  $Z_{PI}$ ,  $Z_{VI}$ , and  $Z_{VP}$ .*

# Characteristic impedance ch4.1

1. Power-current definition:  $Z_{pi} = 2P/I^*$
2. Power-voltage definition:  $Z_{pv} = V^*V/2P$
3. Voltage-current definition:  $Z_{vi} = V/I = \text{sqrt}(Z_{pv} \times Z_{pi})$

E. Wollack, "TCHEB x: Homogeneous Stepped Waveguide Transformers," NRAO, EDTN Memo Series, vol. 176, 1996.

S.A. Schelkunoff, "Impedance concept in waveguides," Q. Appl. Math, vol. 2, no. 1, 1944.

# Rectangular and circular waveguide impedances normalised to the wave impedance

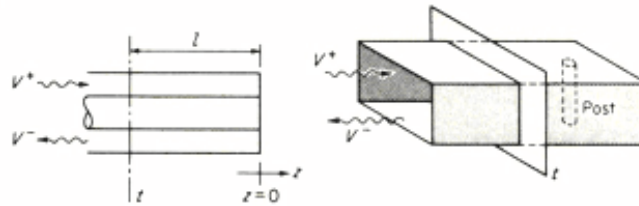
TABLE 2  
WAVEGUIDE IMPEDANCE BASIS

| Impedance  | $\text{TE}_{10}^{\square}$  | $\text{TE}_{11}^{\circ}$                      |
|------------|-----------------------------|---|
| Definition | $[\eta\lambda_g/\lambda_o]$ | $[\eta\lambda_g/\lambda_o]$                   |
| $Z_{EH}$   | 1                           | 1   |
| $Z_{PI}$   | $\pi^2 b/8a$                | $\pi(1 - s_{11}^2)/8$                         |
| $Z_{VI}$   | $\pi b/2a$                  | $\int_0^{s_{11}} J_0(x) dx / J_1(s_{11}) - 1$ |
| $Z_{VP}$   | $2b/a$                      | 2   |

$\eta = (\mu_o/\epsilon_o)^{1/2} \simeq 377\Omega$  is the intrinsic impedance of free-space.



# One-port circuit



- energy can enter or leave through a single propagation line
- Introduce input impedance,  $Z_{in}$

# Impedance description

Assume now perfectly conductive walls,  
 $\sigma = \infty$ ,  $E_{tan} = 0$  on all walls but  $t$ .

$$\frac{1}{2} \oint_t \bar{E} \times \bar{H} \cdot \bar{a}_z dS = P_{loss} + 2j\omega(W_m - W_e)$$

At the terminal plane  $t$  the transverse fields are

$$\begin{aligned} \bar{E}_t &= K_1^{-1}(V^+ + V^-)\bar{e} = K_1^{-1}V\bar{e} \\ \bar{H}_t &= K_2^{-1}(I^+ - I^-)\bar{h} = K_2^{-1}I\bar{h} \end{aligned}$$

Thus

$$\frac{1}{2} (K_1 K_2^*)^{-1} V I^* \int_t \bar{e} \times \bar{h}^* \cdot \bar{a}_z dS = \frac{1}{2} V I^* = P_{loss} + 2j\omega(W_m - W_e)$$

We have now  $V = Z_{in} I$

$$Z_{in} = \frac{P_{loss} + 2j\omega(W_m - W_e)}{\frac{1}{2} I I^*} = R + jX$$


$$Z_{in} = f(P_{loss}, W_m - W_e)$$

If  $W_m > W_e$   $\longrightarrow$   $X > 0$ , inductive one-port

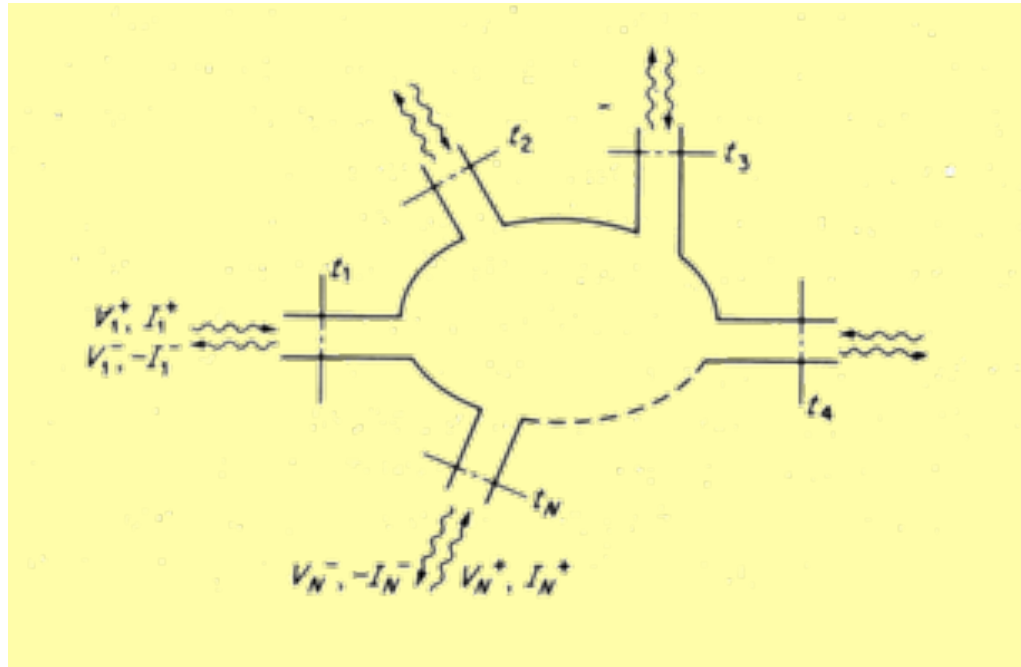
If  $W_m < W_e$   $\longrightarrow$   $X < 0$ , capacitive one-port

# Foster's reactance theorem

$$\frac{\partial X}{\partial \omega} = \frac{4(W_m + W_e)}{II^*} > 0$$

-  The slope of the reactance function must always be positive for a lossless circuit (reactive termination)

# N-port circuits



N-port microwave circuit: if each guide only supports one mode

# Impedance matrix

- let the terminal planes be chosen sufficiently far from the junction  $\Rightarrow$  only dominant incident and reflected waves.  
 $\Rightarrow$  equivalent voltages and currents
- Use total current as independent variables and total voltages as dependent variables, hence linear combination can be written as:

$$\begin{bmatrix} V_1 \\ V_2 \\ \cdot \\ \cdot \\ V_N \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} & \cdot & \cdot & z_{1N} \\ z_{21} & z_{22} & \cdot & \cdot & z_{2N} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ z_{N1} & z_{N2} & \cdot & \cdot & z_{NN} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \cdot \\ \cdot \\ I_N \end{bmatrix}$$

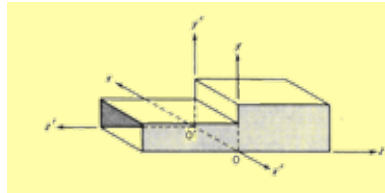
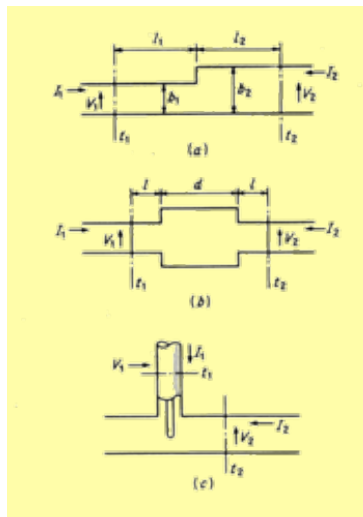
# Properties

- Non reciprocal circuit:  $Z_{ij} \neq Z_{ji}$  unsymmetrical impedance matrix (  $2N^2$  parameters)
- Reciprocal circuit:  $Z_{ij} = Z_{ji} \Rightarrow$  symmetrical impedance matrix (  $N(N+1)$  parameters)
- Lossless circuit: symmetrical and imaginary  $[Z]$  (  $N(N+1)/2$  parameters)
- Same applies to  $[Y] = [Z]^{-1}$

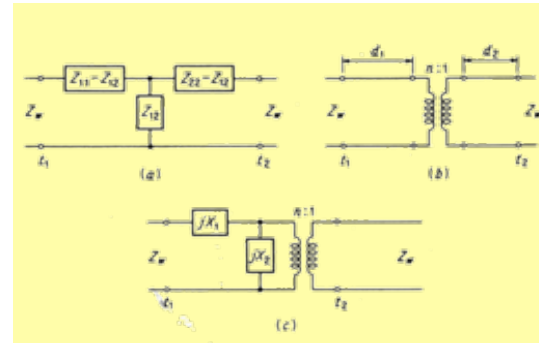
$$\begin{bmatrix} V_1 \\ V_2 \\ \cdot \\ \cdot \\ V_N \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} & \cdot & \cdot & z_{1N} \\ z_{21} & z_{22} & \cdot & \cdot & z_{2N} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ z_{N1} & z_{N2} & \cdot & \cdot & z_{NN} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \cdot \\ \cdot \\ I_N \end{bmatrix}$$

- *On white board: Derivation of matrix properties for N-port circuits*

# Two-port junctions



$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$



Common! especially transistor amplifiers involves 2-port theory



- *On white board: Example with Z-matrix (T and Pi networks)*

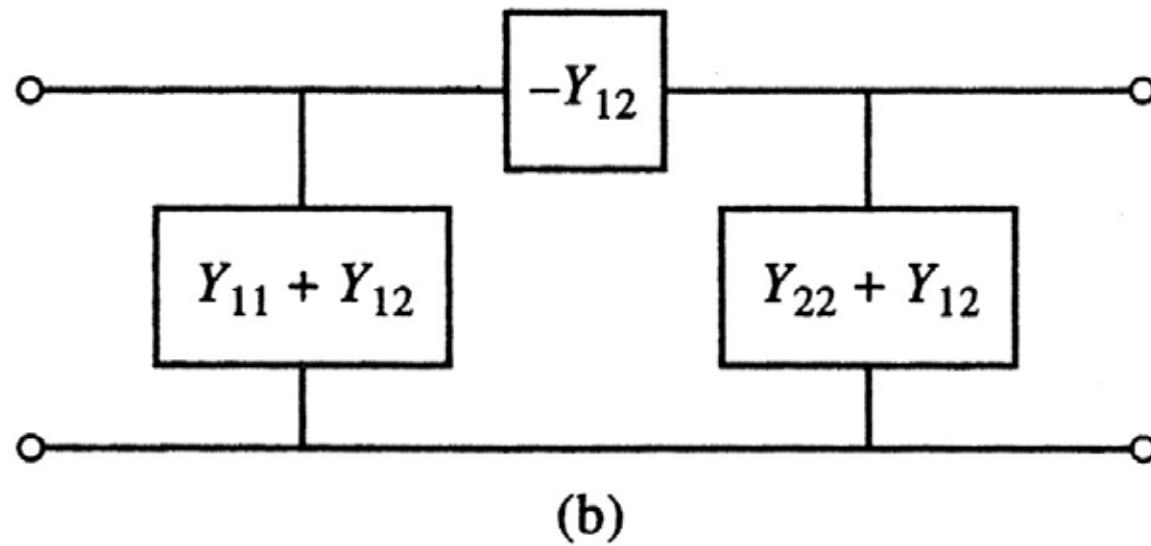
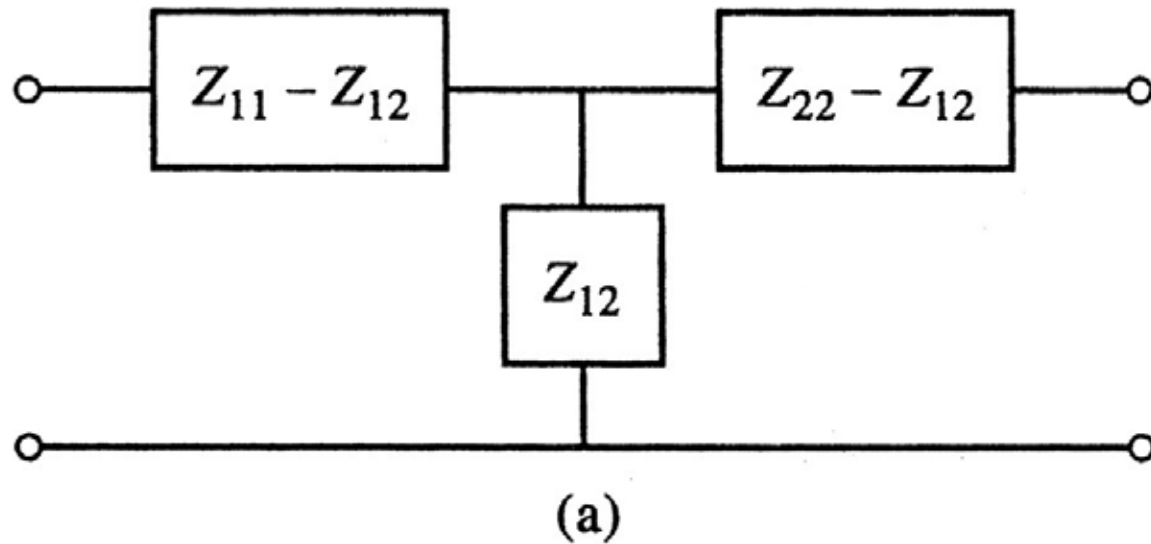
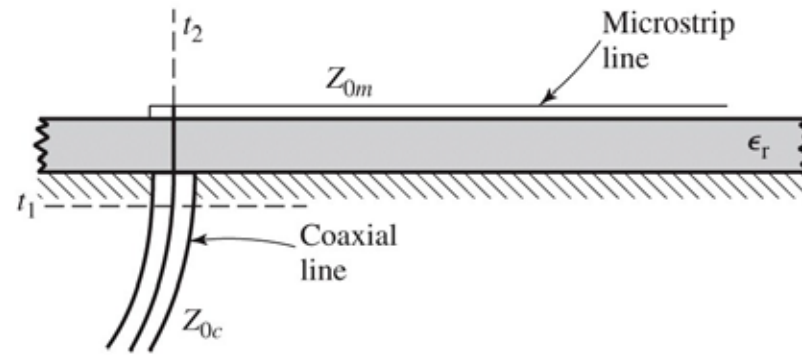
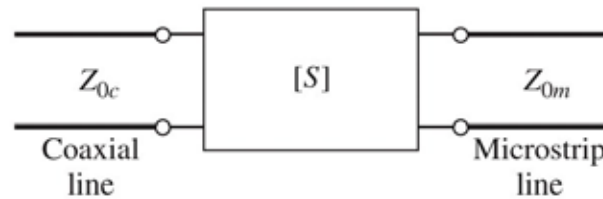


Figure 4.13  
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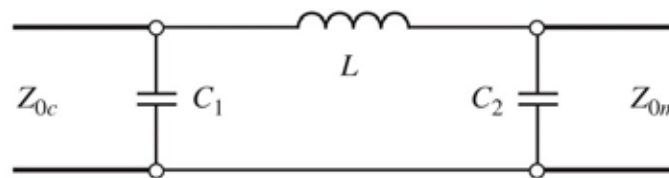
# ex) lossless & reciprocal 2-port



(a)

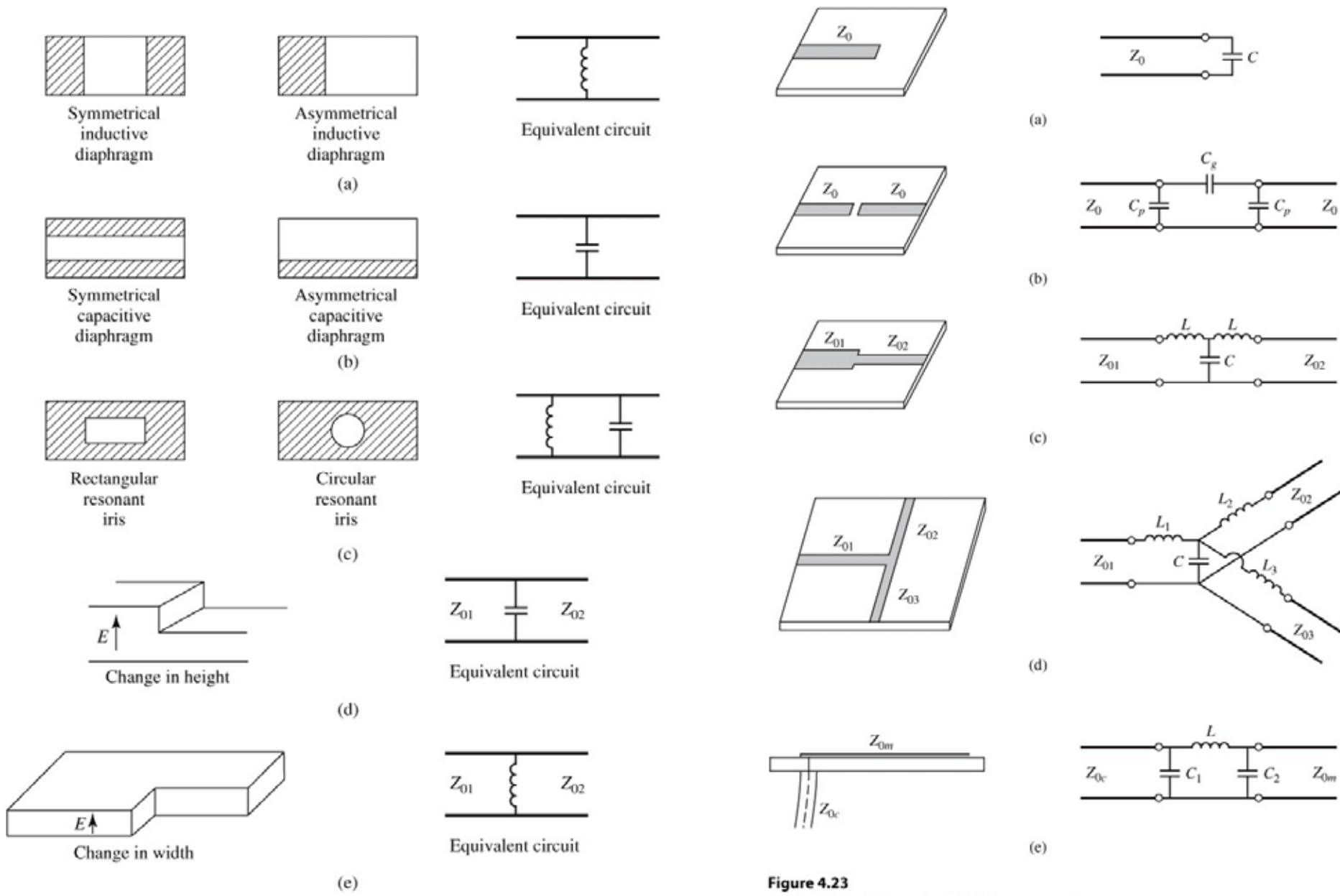


(b)



(c)

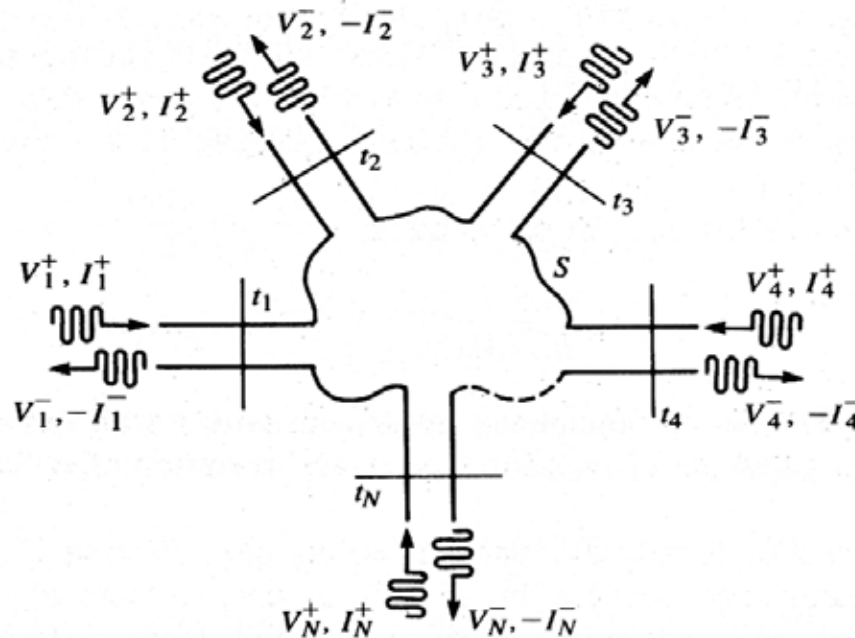
Figure 4.12  
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**Figure 4.22**  
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**Figure 4.23**  
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# Scattering matrix [S]



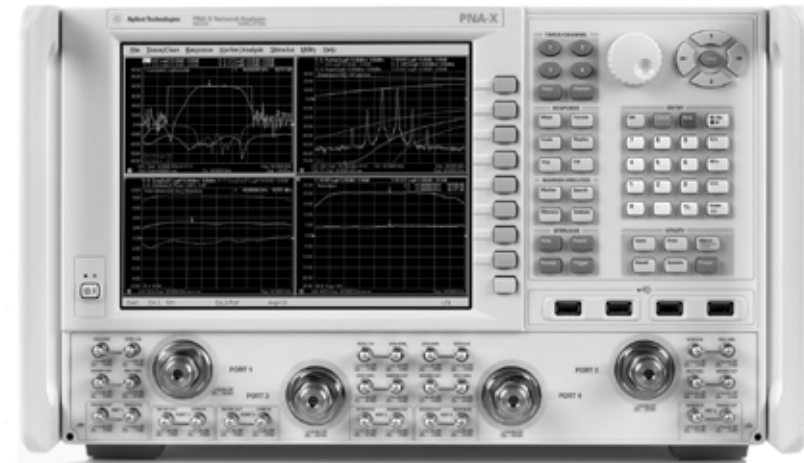
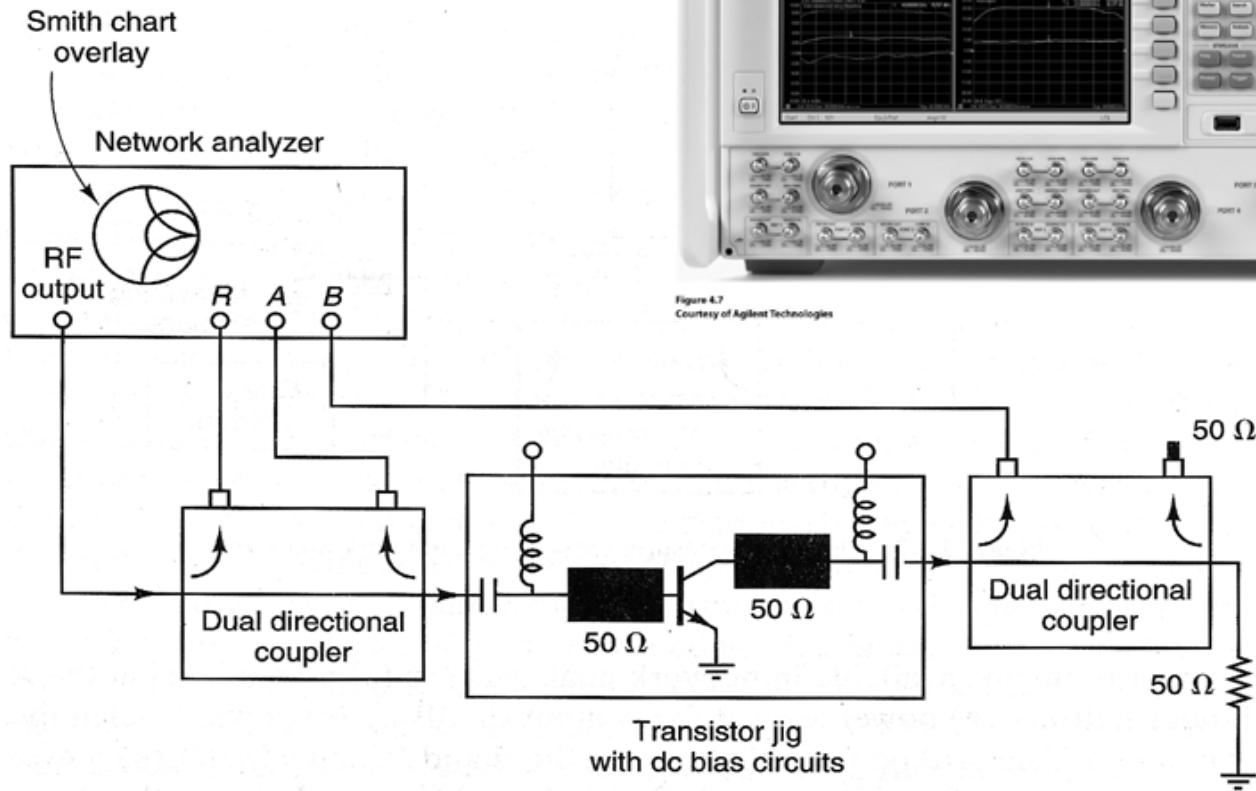
$$s_{11} = \frac{V_1^-}{V_1^+}; \quad s_{n1} = \frac{V_n^-}{V_1^+}, \quad n = 2, 3, \dots, N$$

$$\begin{bmatrix} V_1^- \\ V_2^- \\ \cdot \\ \cdot \\ V_N^- \end{bmatrix} = \begin{bmatrix} s_{11} & s_{12} & \cdot & \cdot & s_{1N} \\ s_{21} & s_{22} & \cdot & \cdot & s_{2N} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ s_{N1} & s_{N2} & \cdot & \cdot & s_{NN} \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \\ \cdot \\ \cdot \\ V_N^+ \end{bmatrix}$$

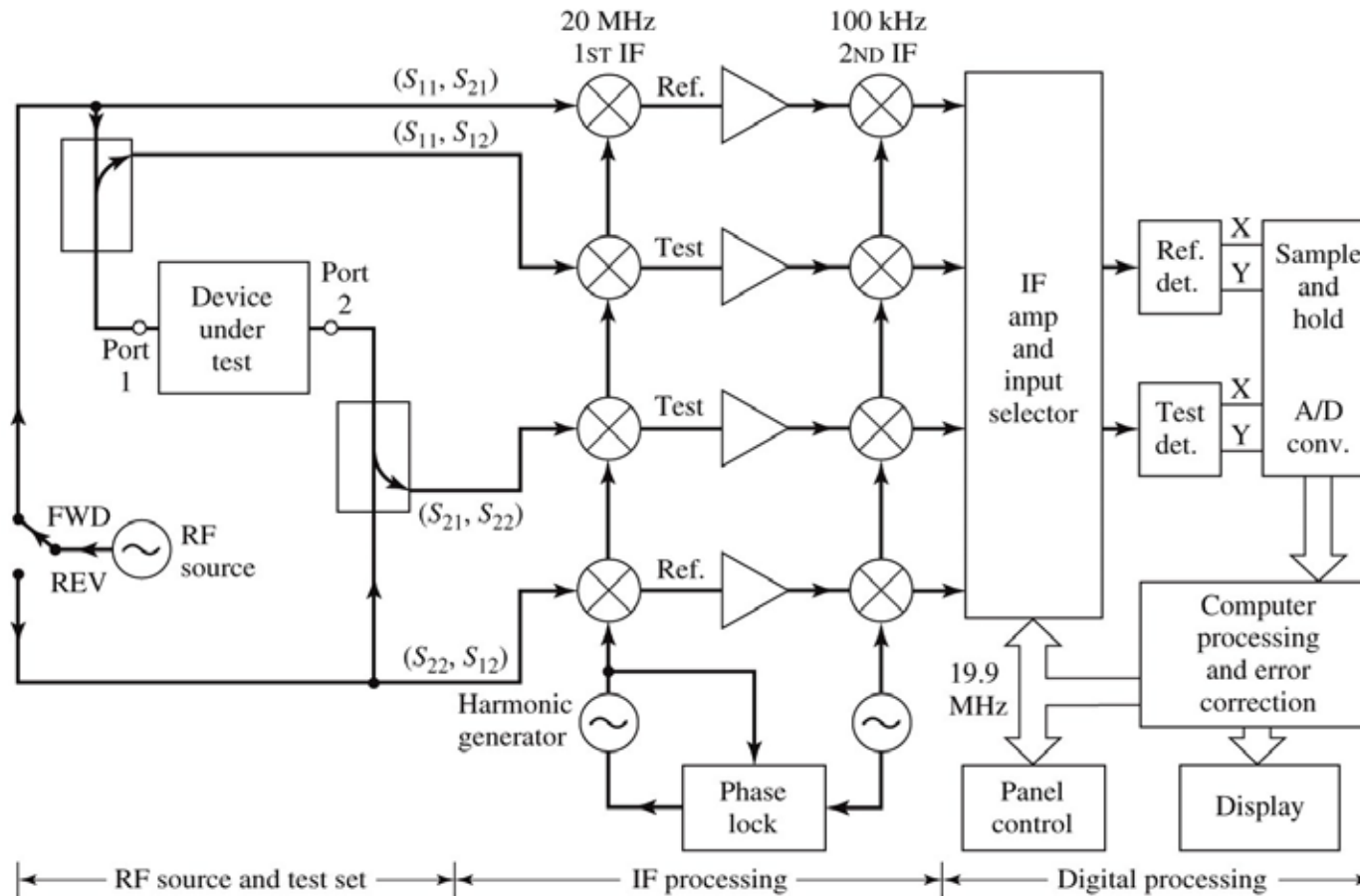
$$[V^-] = [S][V^+] \quad \text{and} \quad s_{ij} = \left. \frac{V_i^-}{V_j^+} \right|_{V_n^+ = 0, n \neq j}$$

[S] can be measured using a Vector Network Analyser (VNA), even at very high frequencies.

# S-parameter test set-up



# Vector Network Analyzer



Unnumbered 4 p188  
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# Properties of the S-matrix

- Reciprocal if  
( $[S]$  symmetric)

$$[S] = [S]^t$$

- Lossless if:  
( $[S]$  is unitary,  $[U]$  is  
the unit diagonal  
matrix)

$$[S]^t [S]^* = [U]$$



- *On white board: Scattering matrix for a lossless circuit*

# Scattering matrix [S]

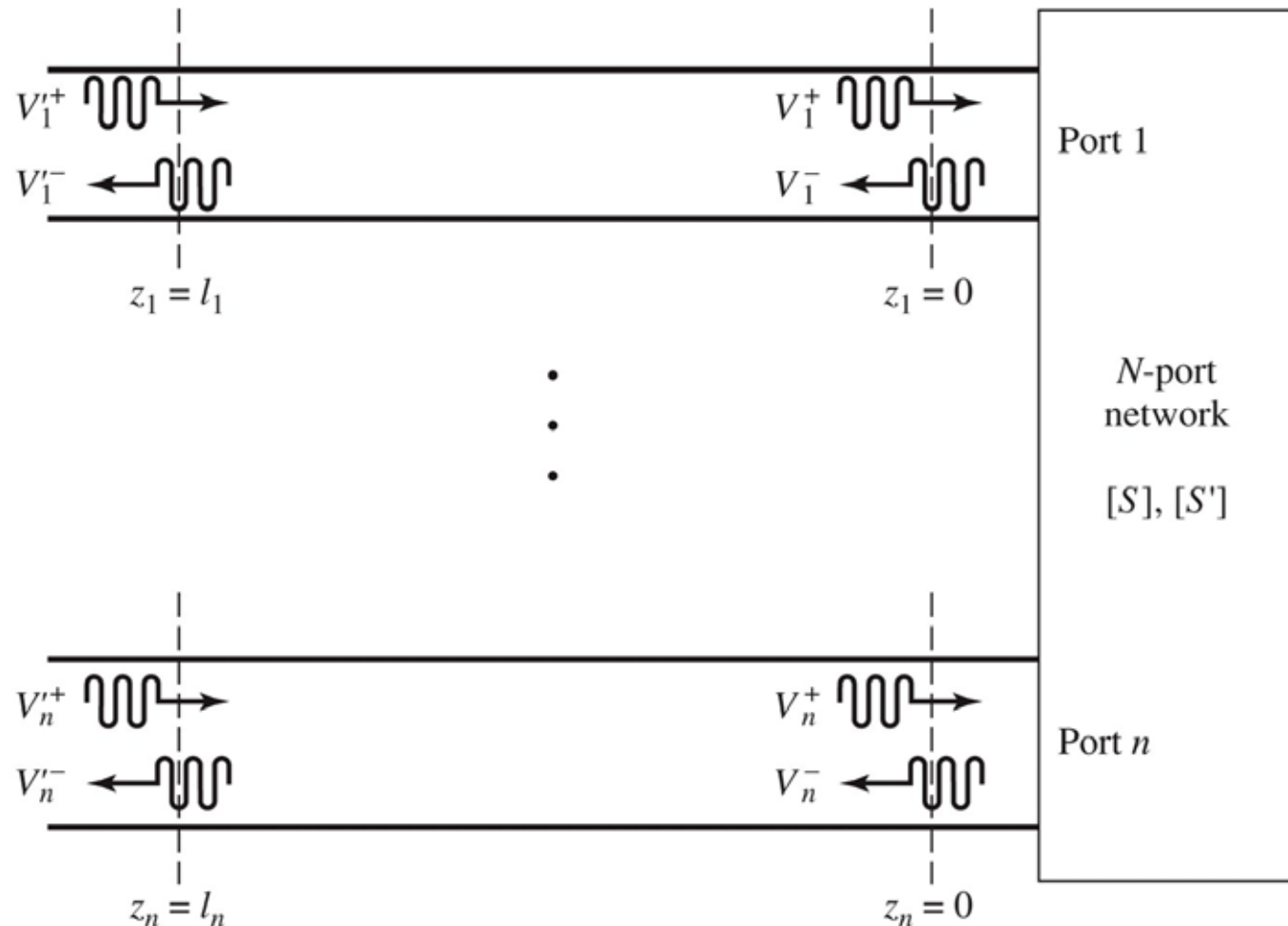
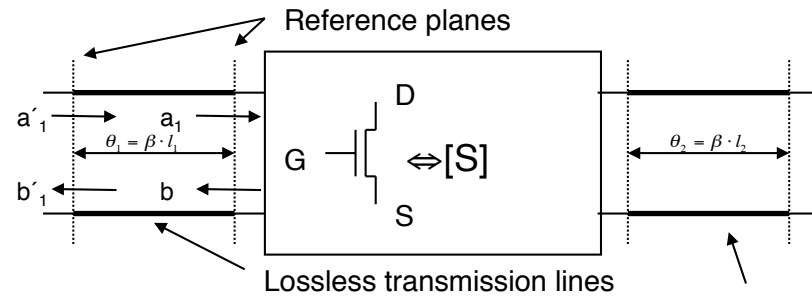


Figure 4.9  
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# Shift of the reference plane

- Two port case:

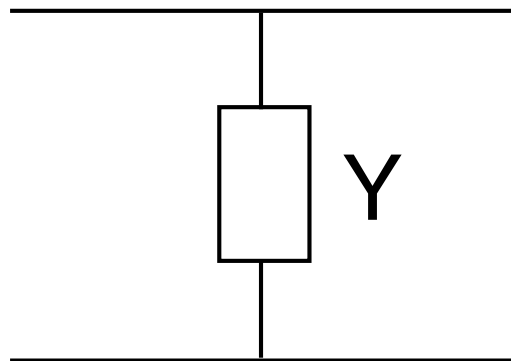


$$[S'] = [\Phi] \cdot [S] \cdot [\Phi], \quad [\Phi] = \begin{bmatrix} e^{-j \cdot \beta \cdot l_1} & 0 \\ 0 & e^{-j \cdot \beta \cdot l_2} \end{bmatrix}$$

$$\text{or } [S] = [\Phi]^{-1} \cdot [S'] \cdot [\Phi]^{-1}$$

$$[S] = \begin{bmatrix} e^{j \cdot \beta \cdot l_1} & 0 \\ 0 & e^{j \cdot \beta \cdot l_2} \end{bmatrix} \cdot [S'] \cdot \begin{bmatrix} e^{j \cdot \beta \cdot l_1} & 0 \\ 0 & e^{j \cdot \beta \cdot l_2} \end{bmatrix}$$

- *On white board: Example, define  $[S]$  for a shunt admittance*



# Cascaded components

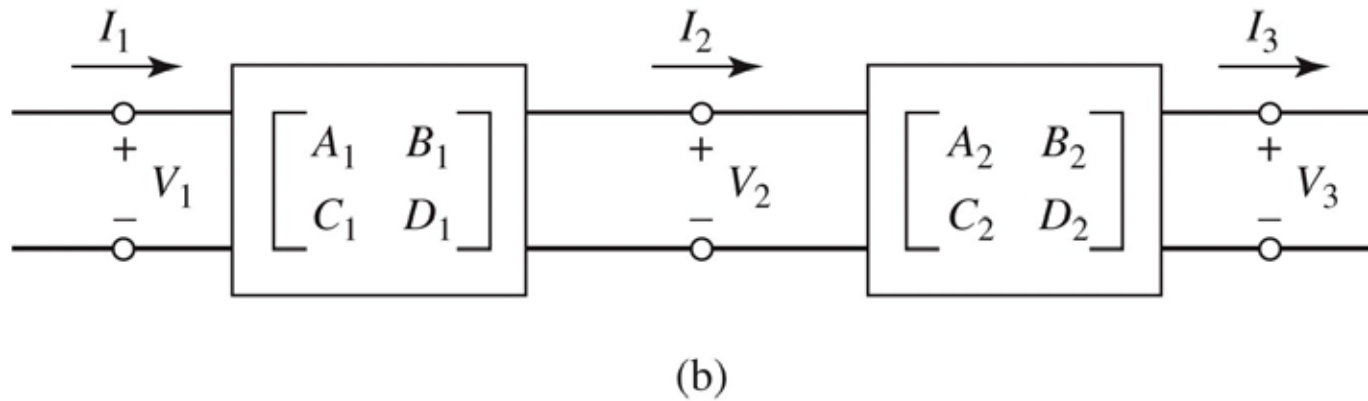
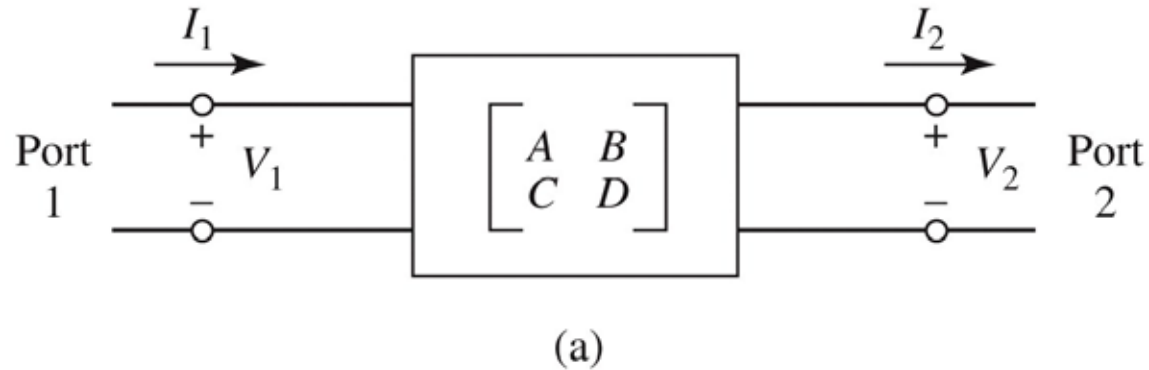
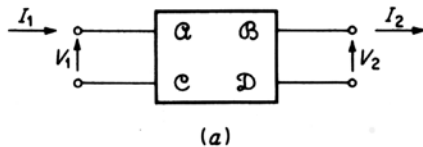


Figure 4.11  
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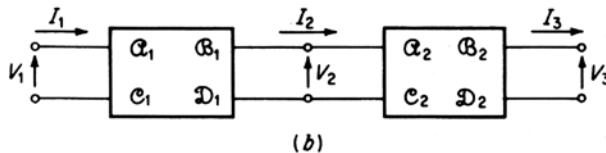
*How to define  $[ ]$ ? so new  $[ ]$  is a matrix multiplication.*

# Cascaded components

- For cascaded components a convenient way to describe the connection is to use transmission matrices (sometimes called ABCD matrices)



$$\begin{aligned} V_1 &= AV_2 + BI_2 \\ I_1 &= CV_2 + DI_2 \end{aligned} \quad \begin{pmatrix} V_1 \\ I_1 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} V_2 \\ I_2 \end{pmatrix}$$

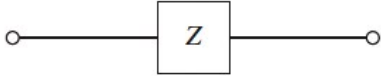
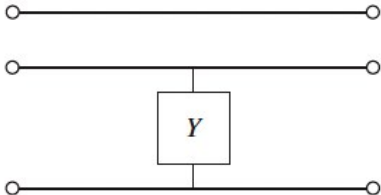
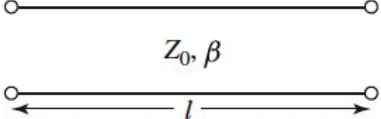
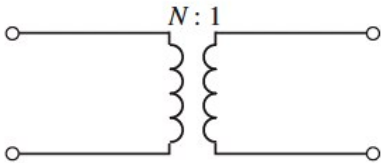
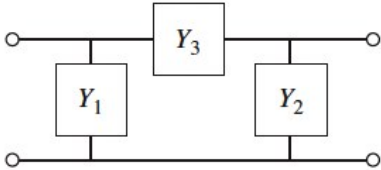
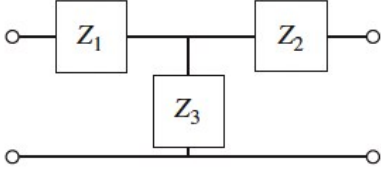


$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} A_1 & B_1 \\ C_1 & D_1 \end{pmatrix} \begin{pmatrix} A_2 & B_2 \\ C_2 & D_2 \end{pmatrix}$$

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} z_{11} & \frac{(z_{11}z_{22} - z_{12}^2)}{z_{12}} \\ z_{12} & z_{12} \\ 1 & \frac{z_{22}}{z_{12}} \\ z_{12} & z_{12} \end{pmatrix}$$

For reciprocal junctions  $AD-BC=1$

**TABLE 4.1** *ABCD* Parameters of Some Useful Two-Port Circuits

| Circuit   | <i>ABCD</i> Parameters   |  |
|---|--|--|
|    | $A = 1$<br>$C = 0$   | $B = Z$<br>$D = 1$   |
|    | $A = 1$<br>$C = Y$   | $B = 0$<br>$D = 1$   |
|    | $A = \cos \beta l$<br>$C = jY_0 \sin \beta l$                      | $B = jZ_0 \sin \beta l$<br>$D = \cos \beta l$                      |
|    | $A = N$<br>$C = 0$   | $B = 0$<br>$D = \frac{1}{N}$                                       |
|   | $A = 1 + \frac{Y_2}{Y_3}$<br>$C = Y_1 + Y_2 + \frac{Y_1 Y_2}{Y_3}$ | $B = \frac{1}{Y_3}$<br>$D = 1 + \frac{Y_1}{Y_3}$                   |
|  | $A = 1 + \frac{Z_1}{Z_3}$<br>$C = \frac{1}{Z_3}$                   | $B = Z_1 + Z_2 + \frac{Z_1 Z_2}{Z_3}$<br>$D = 1 + \frac{Z_2}{Z_3}$ |

**Table 4.1**

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# Conversion table

|      | S  | z  | y  | h   | ABCD   |
|------|--|--|--|---|--|
| S    | $S_{11}$ $S_{12}$<br>$S_{21}$ $S_{22}$   | $S_{11} = \frac{(z'_{11}-1)(z'_{22}+1) - z'_{12}z'_{21}}{\Delta_1}$<br>$S_{12} = \frac{2z'_{12}}{\Delta_1}$<br>$S_{21} = \frac{2z'_{21}}{\Delta_1}$<br>$S_{22} = \frac{(z'_{11}+1)(z'_{22}-1) - z'_{12}z'_{21}}{\Delta_1}$ | $S_{11} = \frac{(1-y'_{11})(1+y'_{22}) + y'_{12}y'_{21}}{\Delta_2}$<br>$S_{12} = \frac{-2y'_{12}}{\Delta_2}$<br>$S_{21} = \frac{-2y'_{21}}{\Delta_2}$<br>$S_{22} = \frac{(1+y'_{11})(1-y'_{22}) + y'_{12}y'_{21}}{\Delta_2}$ | $S_{11} = \frac{(h'_{11}-1)(h'_{22}+1) - h'_{12}h'_{21}}{\Delta_3}$<br>$S_{12} = \frac{2h'_{12}}{\Delta_3}$<br>$S_{21} = \frac{-2h'_{21}}{\Delta_3}$<br>$S_{22} = \frac{(1+h'_{11})(1-h'_{22}) + h'_{12}h'_{21}}{\Delta_3}$ | $\frac{A'+B'-C'-D'}{\Delta_4}$ $\frac{2(A'D'-B'C')}{\Delta_4}$<br><br>$\frac{2}{\Delta_4}$ $\frac{-A'+B'-C'+D'}{\Delta_4}$ |
| z    | $z'_{11} = \frac{(1+S_{11})(1-S_{22}) + S_{12}S_{21}}{\Delta_5}$<br>$z'_{12} = \frac{2S_{12}}{\Delta_5}$<br>$z'_{21} = \frac{2S_{21}}{\Delta_5}$<br>$z'_{22} = \frac{(1-S_{11})(1+S_{22}) + S_{12}S_{21}}{\Delta_5}$                                 | $z_{11}$ $z_{12}$<br><br>$z_{21}$ $z_{22}$   | $\frac{y_{22}}{ y }$ $\frac{-y_{12}}{ y }$<br><br>$\frac{-y_{21}}{ y }$ $\frac{y_{11}}{ y }$   | $\frac{ h }{h_{22}}$ $\frac{h_{12}}{h_{22}}$<br><br>$\frac{-h_{21}}{h_{22}}$ $\frac{1}{h_{22}}$   | $\frac{A}{C}$ $\frac{\Delta g}{C}$<br><br>$\frac{1}{C}$ $\frac{D}{C}$  |
| y    | $y'_{11} = \frac{(1-S_{11})(1+S_{22}) + S_{12}S_{21}}{\Delta_6}$<br>$y'_{12} = \frac{-2S_{12}}{\Delta_6}$<br>$y'_{21} = \frac{-2S_{21}}{\Delta_6}$<br>$y'_{22} = \frac{(1+S_{11})(1-S_{22}) + S_{12}S_{21}}{\Delta_6}$                               | $\frac{z_{22}}{ z }$ $\frac{-z_{12}}{ z }$<br><br>$\frac{-z_{21}}{ z }$ $\frac{z_{11}}{ z }$   | $y_{11}$ $y_{12}$<br><br>$y_{21}$ $y_{22}$   | $\frac{1}{h_{11}}$ $\frac{-h_{12}}{h_{11}}$<br><br>$\frac{h_{21}}{h_{11}}$ $\frac{ h }{h_{11}}$   | $\frac{D}{B}$ $\frac{-\Delta g}{B}$<br><br>$\frac{-1}{B}$ $\frac{A}{B}$  |
| h    | $h'_{11} = \frac{(1+S_{11})(1+S_{22}) - S_{12}S_{21}}{\Delta_7}$<br>$h'_{12} = \frac{2S_{12}}{\Delta_7}$<br>$h'_{21} = \frac{-2S_{21}}{\Delta_7}$<br>$h'_{22} = \frac{(1-S_{11})(1-S_{22}) - S_{12}S_{21}}{\Delta_7}$                                | $\frac{ z }{z_{22}}$ $\frac{z_{12}}{z_{22}}$<br><br>$\frac{-z_{21}}{z_{22}}$ $\frac{1}{z_{22}}$  | $\frac{1}{y_{11}}$ $\frac{-y_{12}}{y_{11}}$<br><br>$\frac{y_{21}}{y_{11}}$ $\frac{ y }{y_{11}}$  | $h_{11}$ $h_{12}$<br><br>$h_{21}$ $h_{22}$  | $\frac{B}{D}$ $\frac{-\Delta g}{D}$<br><br>$\frac{-1}{D}$ $\frac{C}{D}$  |
| ABCD | $A' = \frac{(1+S_{11})(1-S_{22}) + S_{12}S_{21}}{2S_{21}}$<br>$B' = \frac{(1+S_{11})(1+S_{22}) - S_{12}S_{21}}{2S_{21}}$<br>$C' = \frac{(1-S_{11})(1-S_{22}) - S_{12}S_{21}}{2S_{21}}$<br>$D' = \frac{(1-S_{11})(1+S_{22}) + S_{12}S_{21}}{2S_{21}}$ | $\frac{z_{11}}{z_{21}}$ $\frac{ z }{z_{21}}$<br><br>$\frac{1}{z_{21}}$ $\frac{z_{22}}{z_{21}}$   | $\frac{-y_{22}}{y_{21}}$ $\frac{-1}{y_{21}}$<br><br>$\frac{- y }{y_{21}}$ $\frac{-y_{11}}{y_{21}}$   | $\frac{- h }{h_{21}}$ $\frac{-h_{11}}{h_{21}}$<br><br>$\frac{-h_{22}}{h_{21}}$ $\frac{-1}{h_{21}}$  | $A$ $B$<br><br>$C$ $D$   |

$\Delta_1 = (z'_{11}+1)(z'_{22}+1) - z'_{12}z'_{21}$   
 $\Delta_2 = (1+y'_{11})(1+y'_{22}) - y'_{12}y'_{21}$   
 $\Delta_3 = (h'_{11}+1)(h'_{22}+1) - h'_{12}h'_{21}$   
 $\Delta_4 = A' + B' + C' + D'$   
 $\Delta_5 = (1-S_{11})(1-S_{22}) - S_{12}S_{21}$   
 $\Delta_6 = (1+S_{11})(1+S_{22}) - S_{12}S_{21}$   
 $\Delta_7 = (1-S_{11})(1+S_{22}) + S_{12}S_{21}$   
 $\Delta_8 = AD - BC$

$z'_{11} = z_{11}/Z_0, z'_{12} = z_{12}/Z_0, z'_{21} = z_{21}/Z_0, z'_{22} = z_{22}/Z_0$   
 $y'_{11} = y_{11}/Z_0, y'_{12} = y_{12}/Z_0, y'_{21} = y_{21}/Z_0, y'_{22} = y_{22}/Z_0$   
 $h'_{11} = h_{11}/Z_0, h'_{12} = h_{12}, h'_{21} = h_{21}, h'_{22} = h_{22}/Z_0$   
 $A' = A, B' = B/Z_0, C' = CZ_0, D' = D$   
 $|z| = z_{11}z_{22} - z_{12}z_{21}$   
 $|y| = y_{11}y_{22} - y_{12}y_{21}$   
 $|h| = h_{11}h_{22} - h_{12}h_{21}$

See Gonzalez, page 62.



# Summary of lecture 5

- Read chapter 4.
  - Impedance (equivalent voltage / current)
  - N-ports, matrix representations
  - Properties for lossless and reciprocal circuits
- Next: Impedance transformation and matching (ch5)

# Further reading

- R. Bauer and P. Penfield, “De-Embedding and Unterminating,” IEEE Transactions on Microwave Theory and Techniques, vol. 22, no. 3, pp. 282–288, 1974.
- D.A. Frickey, “Conversions between S, Z, Y, H, ABCD, and T parameters which are valid for complex source and load impedances,” IEEE Transactions on Microwave Theory and Techniques, vol. 42, no. 2, pp. 205–211, 1994.