Microwave Engineering MCC121, 7.5hec, 2014

Lecture 3

”a study of microwave circuits provides a deeper physical insight into conventional circuit theory” R E Collin

Outline

- Transmission lines and waveguides (Ch3.1-3.5)
  - Summary of waves on transmission lines (Ch2)
  - Classification of waves (TE, TM, TEM)
  - Field analysis
  - Parallel plate
  - Hollow waveguides
  - Coaxial line

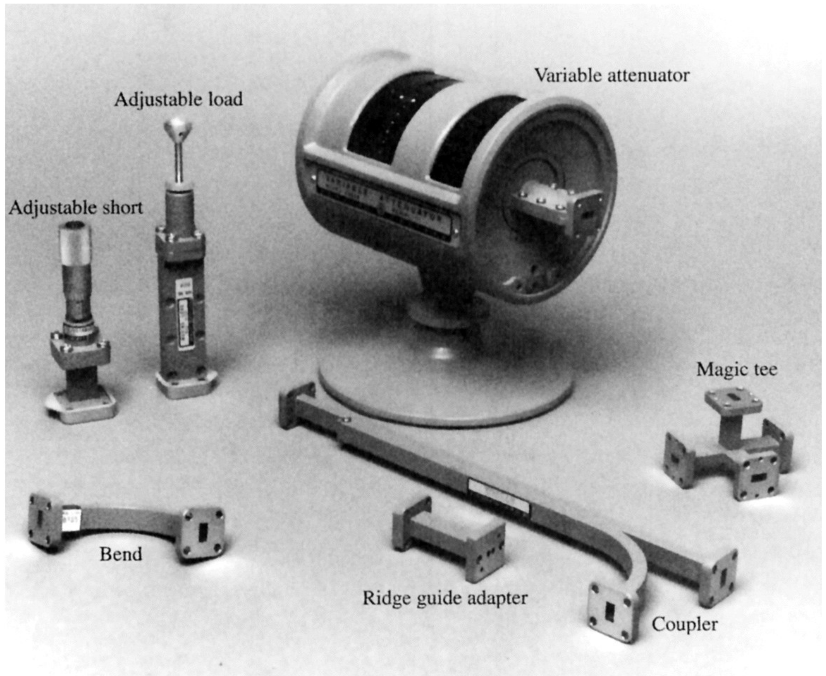


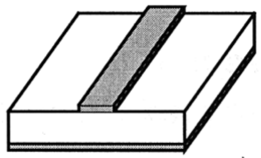
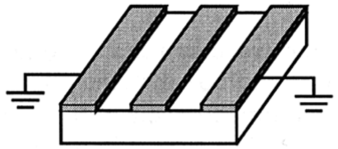
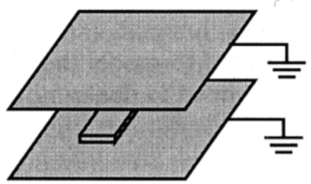
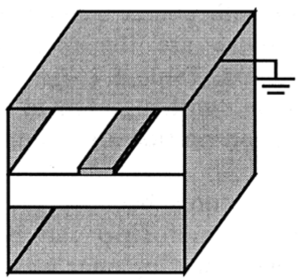
Figure 3.6
David M. Pozar

Objectives

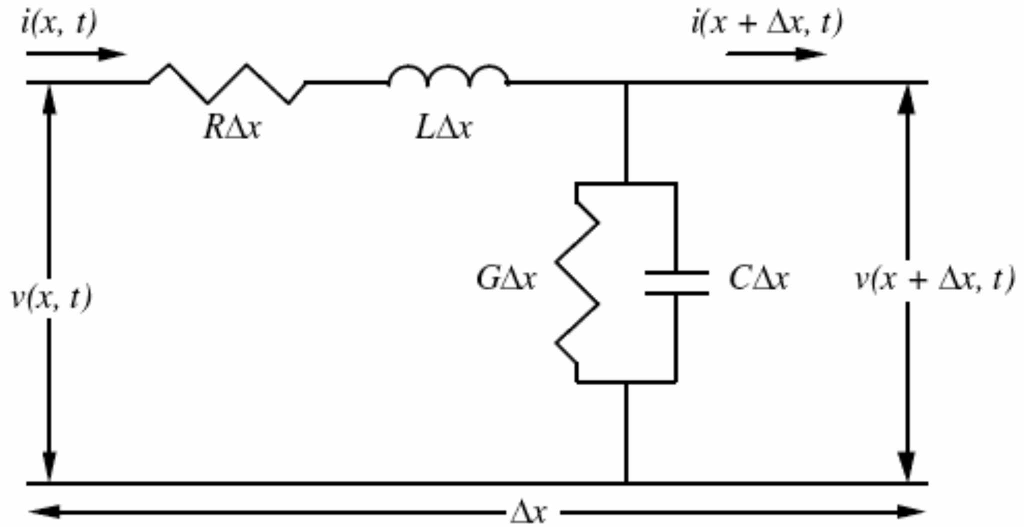
On completion of this course unit you should be able to:

1) Analyse wave propagating properties of guided wave structures (TE, TM, TEM waves, microstrip, stripline, rectangular and circular waveguides, coupled lines)
2) Apply N-port representations for analysing microwave circuits
3) Apply the Smith chart to evaluate microwave networks
4) Design and evaluate impedance matching networks
5) Design, evaluate and characterise directional couplers and power dividers
6) Design and analyse attenuators, phase shifters and resonators
7) Explain basic properties of ferrite devices (circulators, isolators)

Distributed components
Transmission lines

Transmission Line	Structure	Properties
Microstrip		The most common type of transmission line, suitable for both hybrids and monolithic circuits. Moderately dispersive at high frequencies. See Section 1.3.3.
Coplanar waveguide (CPW)		Somewhat lossier and more dispersive than microstrip, but minimizes the parasitic inductance of ground connections. Good transition to coaxial lines. Spurious slotline and microstrip modes are possible. See Section 1.3.4.
Stripline		Does not allow convenient mounting of discrete circuit elements; best for passive components. Difficult to cascade with microstrip or other planar transmission lines. Low loss, TEM, good transition to coax. See Section 1.3.5.
Suspended-substrate stripline (SSSL)		Similar to stripline, but easier to fabricate in many types of circuits. Low loss, low effective dielectric constant, good transition to coax. Waveguide-like modes can be a problem. See Section 1.3.6.

Telegrapher's equations



Distributed Element Model of a Transmission Line

TD

$$\frac{\partial v(z,t)}{\partial z} = -Ri(z,t) - L\frac{\partial i(z,t)}{\partial t}$$

$$\frac{\partial i(z,t)}{\partial z} = -Gv(z,t) - C\frac{\partial v(z,t)}{\partial t}$$

Wave equations

FD

$$\frac{\partial^2 V}{\partial z^2} - \gamma^2 V = 0$$

$$\frac{\partial^2 I}{\partial z^2} - \gamma^2 I = 0$$

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$$

Propagation constant

$$\frac{\partial^2 V}{\partial z^2} - \gamma^2 V = 0$$

$$\frac{\partial^2 I}{\partial z^2} - \gamma^2 I = 0$$

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$V(z) = V^+ e^{-\gamma z} + V^- e^{\gamma z}$$

$$I(z) = I^+ e^{-\gamma z} + I^- e^{\gamma z}$$

Phase velocity: $v_p = \omega / \beta$

Maxwell's equations

$\nabla \cdot \mathbf{D} = \rho_f$ Gauss's law

$\nabla \cdot \mathbf{B} = 0$

$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$ Faraday's law

$\nabla \times \mathbf{H} = \mathbf{J}_f + \frac{\partial \mathbf{D}}{\partial t}$ Ampere's law with Maxwell's correction

Guided waves

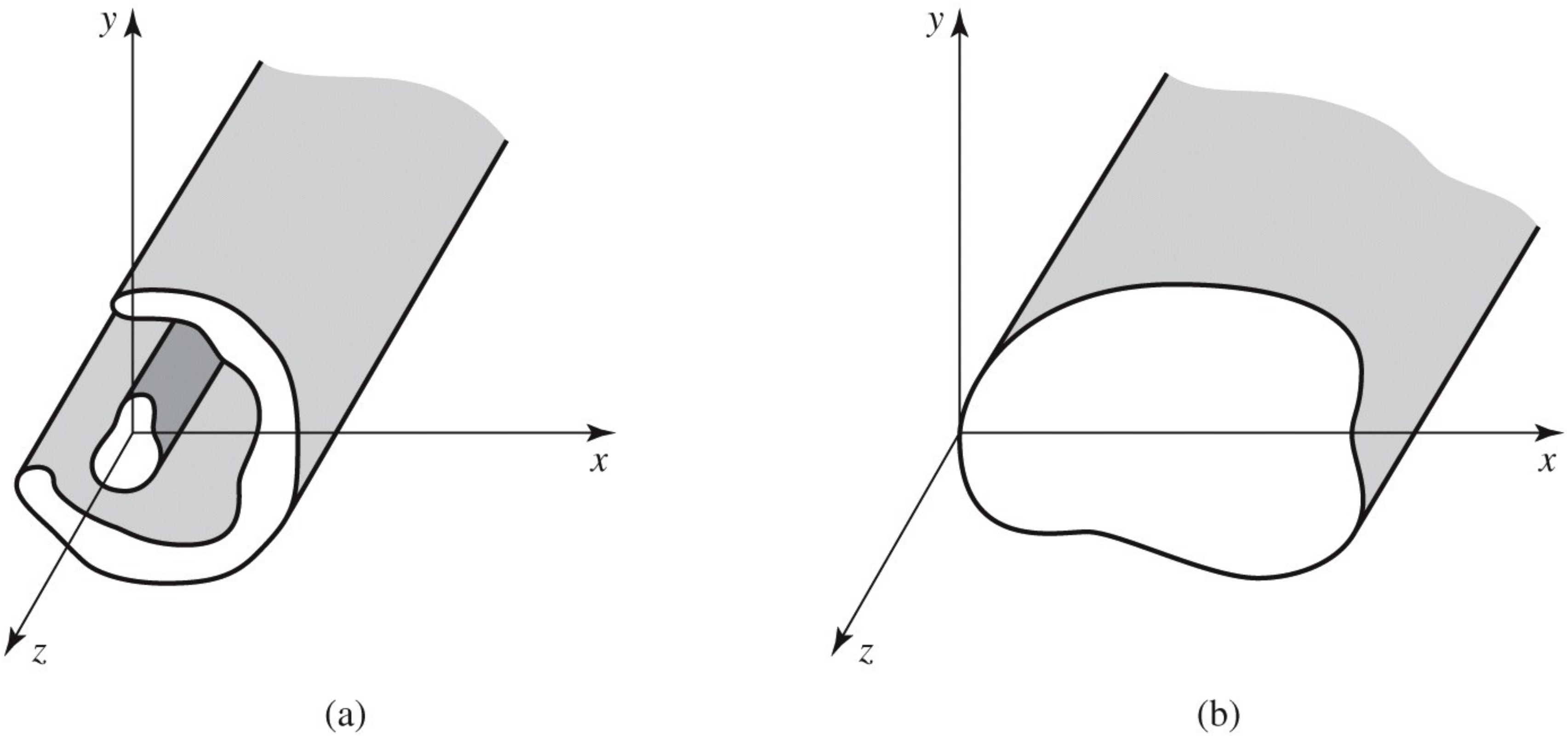


Figure 3.1

Helmholtz equation

- Assume no sources:

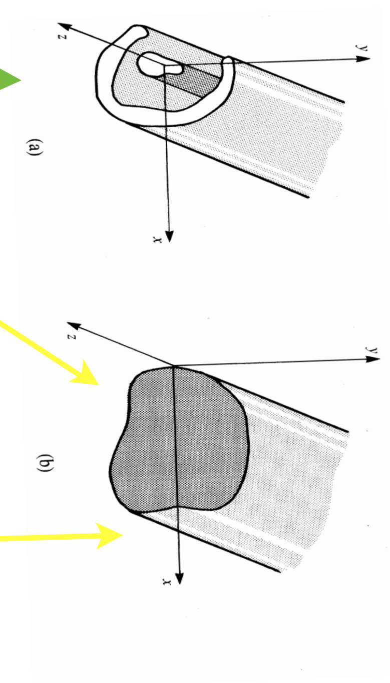
$$\nabla^2 \overline{E} + k_0^2 \overline{E} = 0$$
$$\nabla^2 \overline{H} + k_0^2 \overline{H} = 0$$

$$k = \omega\sqrt{\varepsilon\mu}$$

- Cross section or electrical properties do not vary along z-axis (axial uniformity)
- Separable: assume solution f(z)g(x,y)

Classification of waves

- TEM-Transverse Electromagnetic: no longitudinal field components
- TE-Transverse Electric, or H modes: longitudinal magnetic field component
- TM-Transverse Magnetic, or E modes: longitudinal electric field component



- *On white board: Maxwell equations for TE, TM and TEM waves.*

Summary of modes

- TEM waves

  $E_z = H_z = 0$

  Field is a solution to a transverse gradient of a scalar function $\Phi(x,y)$, which is a solution of a two-dimensional Laplace equation

- TE waves, H modes

  $e_z = 0$

  All field components are derived from h_z

- TM waves, E modes

  $h_z = 0$

  All field components are derived from e_z

- TE and TM

Lossless transmission lines

- Two or more parallel conductors
- Surrounded by a uniform dielectric

➡TEM as principal wave

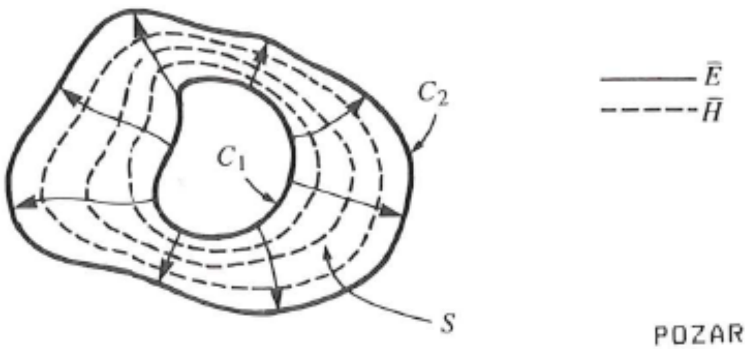
- Microstrip and other planar lines do not have the dielectric medium completely surrounded

➡quasi-TEM waves (low frequency limit)

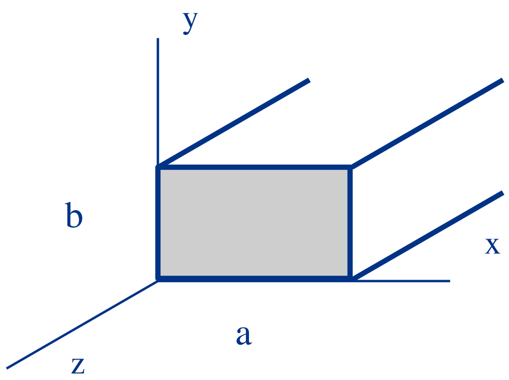
TRANSMISSION LINE PARAMETERS

- L= magnetic flux / total current
- C = total charge per unit length/voltage difference between conductors
- G = total shunt current / voltage difference between conductors

TEM=> electrostatic solution
equivalent circuit parameters



Hollow waveguides



$\bar{e}$ exists if $\bar{h}_z$ exists (TE wave, H mode)

$\bar{h}$ exists if $\bar{e}_z$ exists (TM wave, E mode)

$TE_{n,m}$ and $TM_{n,m}$ modes

The integers n and m pertain to the number of standing-wave interference maxima occuring in the field solutions that describe the variation of the fields along the two transverse coordinates

$f_{c,nm}$ corresponds to cut-off frequency below which the mode does not propagate; it is a geometrical parameter dependent on the waveguide cross-sectional configuration

Propagation factor β

$$\beta = \sqrt{k_0^2 - k_c^2}$$

$$k_0 = 2\pi f \sqrt{\mu_0 \varepsilon},\ k_c = 2\pi f_c \sqrt{\mu_0 \varepsilon}$$

- *In Pozar's book! Read and derive solutions for waves in a rectangular WG.*

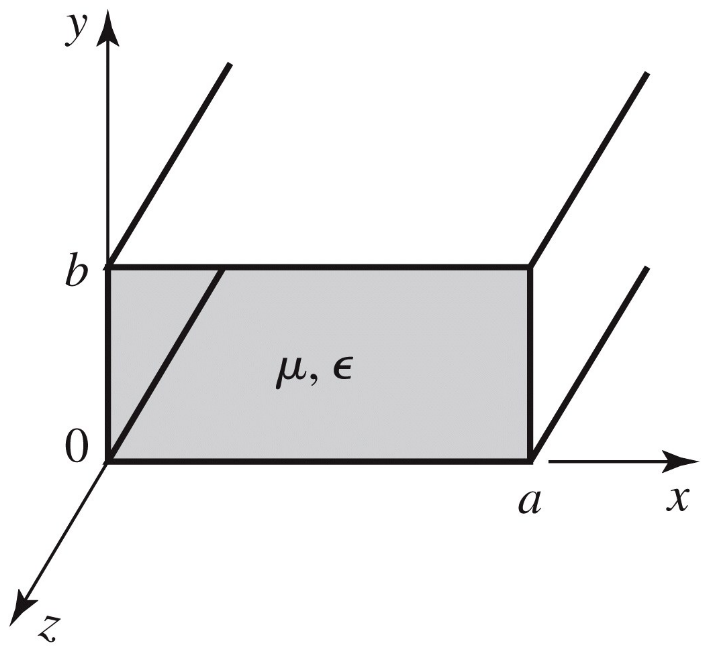


Figure 3.7

Rectangular WG: TE_{nm} modes

$$H_z = A_{nm} \cos\frac{n\pi x}{a} \cos\frac{m\pi y}{b} e^{\mp j\beta_{nm} z}$$

$$H_x = \pm j \frac{\beta_{nm}}{k_{c,nm}^2} A_{nm} \frac{n\pi}{a} \sin\frac{n\pi x}{a} \cos\frac{m\pi y}{b} e^{\mp j\beta_{nm} z}$$

$$H_y = \pm j \frac{\beta_{nm}}{k_{c,nm}^2} A_{nm} \frac{m\pi}{b} \cos\frac{n\pi x}{a} \sin\frac{m\pi y}{b} e^{\mp j\beta_{nm} z}$$

$$E_x = Z_{h,nm} A_{nm} j \frac{\beta_{nm}}{k_{c,nm}^2} \frac{m\pi}{b} \cos\frac{n\pi x}{a} \sin\frac{m\pi y}{b} e^{\mp j\beta_{nm} z}$$

$$E_y = -Z_{h,nm} A_{nm} j \frac{\beta_{nm}}{k_{c,nm}^2} \frac{n\pi}{a} \sin\frac{n\pi x}{a} \cos\frac{m\pi y}{b} e^{\mp j\beta_{nm} z}$$

$$Z_{h,nm} = \frac{k_0}{\beta_{nm}} Z_0$$

Different modes in rectangular WG

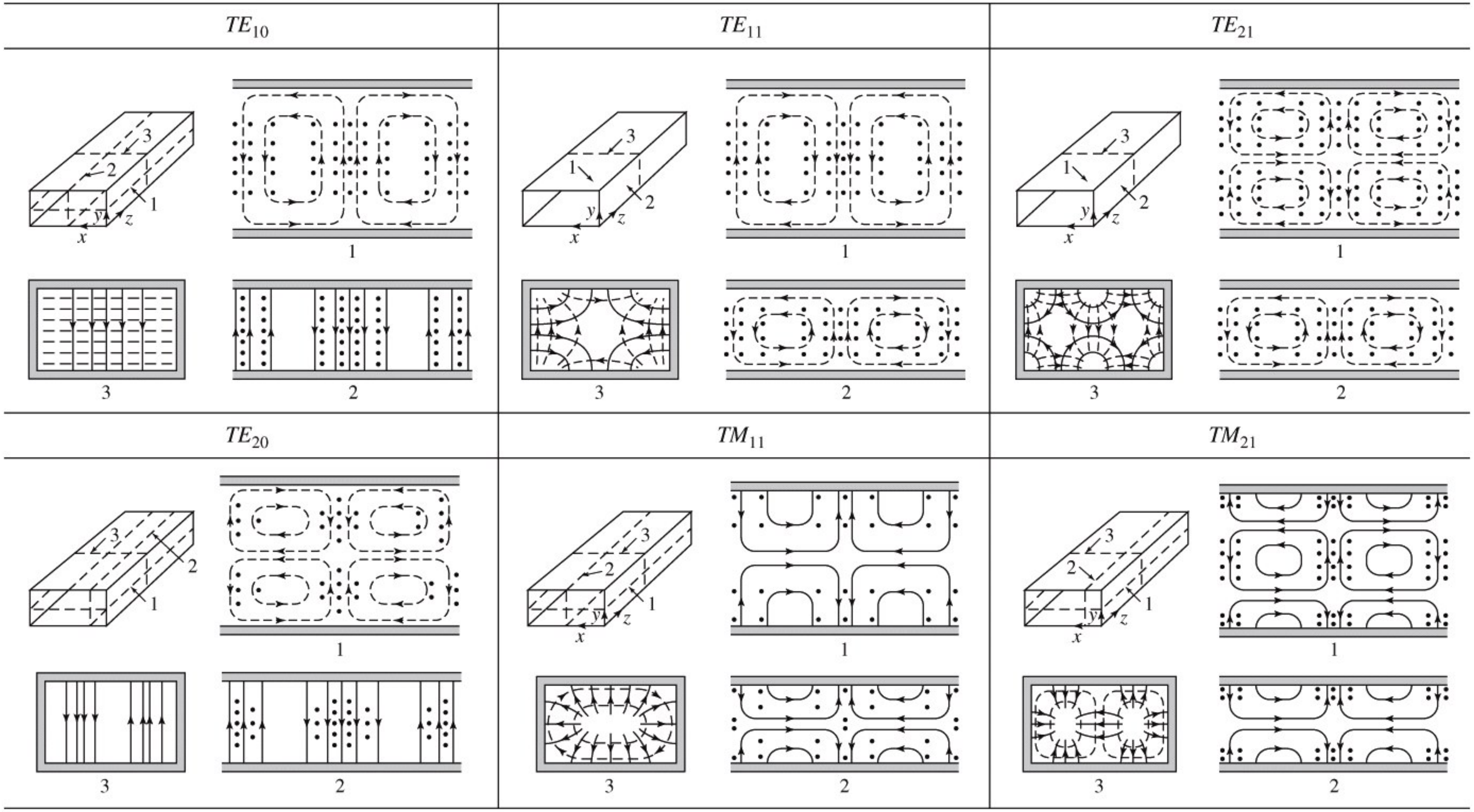


Figure 3.9
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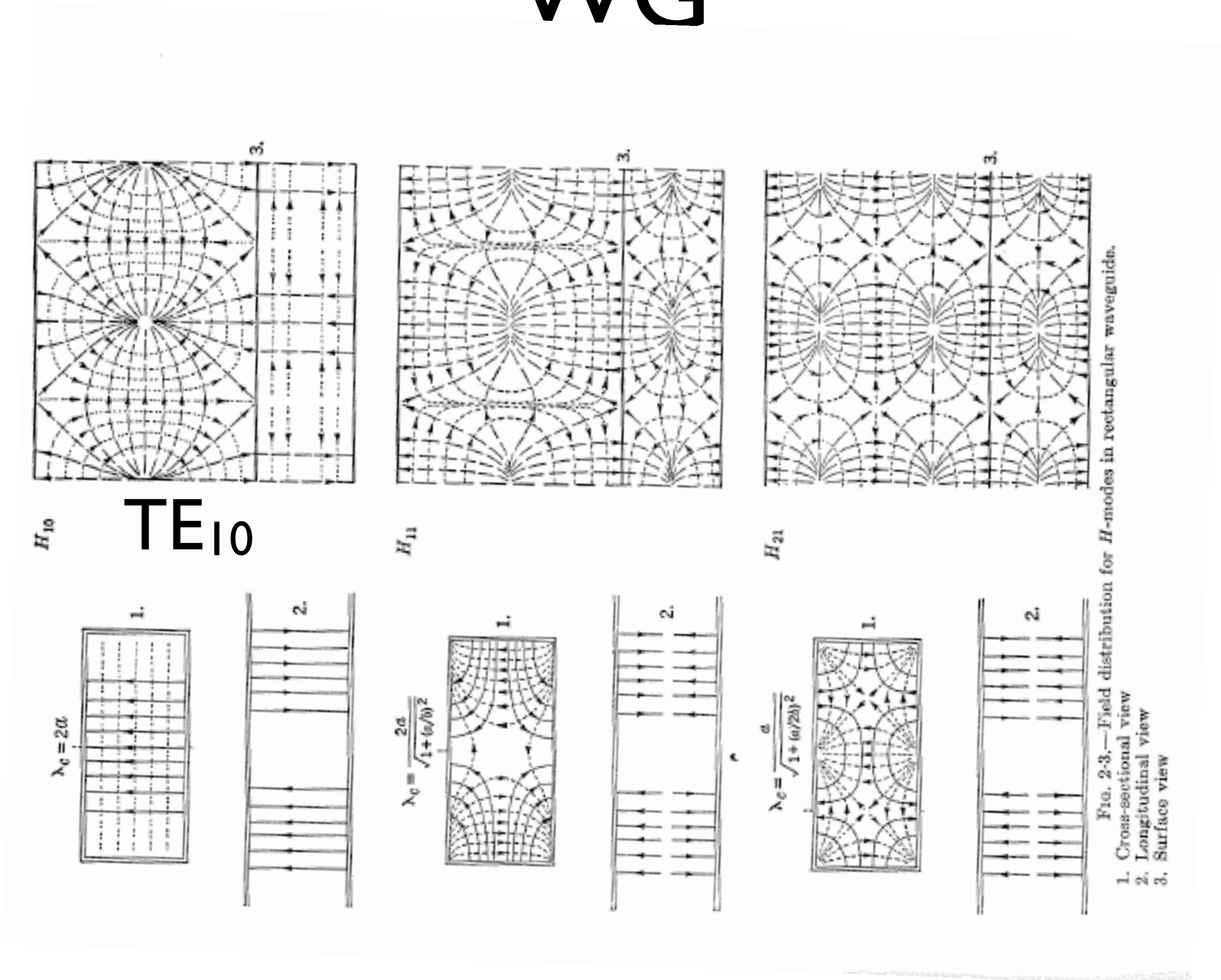
Surface current in rectangular WG



Waves in WGs

- Standard rectangular WG: a=2b
- Single mode bandwidth: one octave bandwidth (c/2a < f < c/a), usually a bit less due to dispersion
- Propagating modes
  - exhibit different propagation constants
- Evanescent modes
  - Important for discontinuities (reactive energy)

Standard WGs

Internal Band Designation	EIA Band Designation	Internal Dimensions (mils)	Internal Dimensions (mm)	Frequency Range (GHz)	TE(10) Cutoff (GHz)	WG Loss Low - High[1] (dB/mm)	Flange Designation	Description	Letter Desig.
WR- 51.0	WR- 51	510 x 255	12.954 x 6.477	15.0 - 22.0	11.6	0.0005 - 0.0004			
WR- 42.0	WR- 42	420 x 170	10.668 x 4.318	17.5 - 26.5	14.0	0.0008 - 0.0006			K
WR- 34.0	WR- 34	340 x 170	8.636 x 4.318	22.0 - 33.0	17.4	0.001 - 0.0007			
WR- 28.0	WR- 28	280 x 140	7.112 x 3.556	26.5 - 40.0	21.1	0.0013 - 0.0009	UG-599/U	Square, Four hole fixing	Ka
WR- 22.4	WR- 22	224 x 112	5.690 x 2.845	33.0 - 50.5	26.3	0.0019 - 0.0013	UG-383/U	Circular, Four hole fixing/doweled	Q
WR- 18.8	WR- 19	188 x 94	4.775 x 2.388	40.0 - 60.0	31.4	0.0023 - 0.0016	UG-383/UM	Circular, Four hole fixing/doweled	U
WR- 14.8	WR- 15	148 x 74	3.759 x 1.880	50.5 - 75.0	39.9	0.0034 - 0.0024	UG-385/U	Circular, Four hole fixing/doweled	V
WR- 12.2	WR- 12	122 x 61	3.099 x 1.549	60.0 - 90.0	48.4	0.0047 - 0.0032	UG-387/U	Circular, Four hole fixing/doweled	E
WR- 10.0	WR- 10	100 x 50	2.540 x 1.270	75.0 - 110.0	59.0	0.0061 - 0.0043	UG-387/UM	Circular, Four hole fixing/doweled	W
WR- 8.0	WR- 8	80 x 40	2.032 x 1.016	90.0 - 140.0	73.8	0.0092 - 0.0059	UG-387/UM	Circular, Four hole fixing/doweled	F
WR- 6.5	WR- 6	65 x 32.5	1.651 x 0.826	110.0 - 170.0	90.8	0.0128 - 0.0081	UG-387/UM	Circular, Four hole fixing/doweled	D
WR- 5.1	WR- 5	51 x 25.5	1.295 x 0.648	140.0 - 220.0	116	0.0185 - 0.0117	UG-387/UM	Circular, Four hole fixing/doweled	G
WR- 4.3	WR- 4	43 x 21.5	1.092 x 0.546	170.0 - 260.0	137	0.0227 - 0.0151	UG-387/UM	Circular, Four hole fixing/doweled	
WR- 3.4	WR- 3	34 x 17	0.864 x 0.432	220.0 - 330.0	174	0.0308 - 0.0214	UG-387/UM	Circular, Four hole fixing/doweled	
WR- 2.8	n/a	28 x 14	0.711 x 0.356	260.0 - 400.0	211	0.0436 - 0.0287	UG-387/UM	Circular, Four hole fixing/doweled	
WR- 2.2	n/a	22 x 11	0.559 x 0.279	330.0 - 500.0	268	0.063 - 0.041	UG-387/UM	Circular, Four hole fixing/doweled	
WR- 1.9	n/a	19 x 9.5	0.483 x 0.241	400.0 - 600.0	311	0.072 - 0.051	UG-387/UM	Circular, Four hole fixing/doweled	
WR- 1.5	n/a	15 x 7.5	0.381 x 0.191	500.0 - 750.0	393	0.105 - 0.073	UG-387/UM	Circular, Four hole fixing/doweled	
WR- 1.2	n/a	12 x 6	0.305 x 0.152	600.0 - 900.0	492	0.159 - 0.104	UG-387/UM	Circular, Four hole fixing/doweled	
WR- 1.0	n/a	10 x 5	0.254 x 0.127	750.0 - 1100.0	590	0.192 - 0.135	n/a		
WR- 0.8	n/a	8 x 4	0.203 x 0.102	900.0 - 1400.0	738	0.292 - 0.188	n/a		
WR- 0.65	n/a	6.5 x 3.25	0.165 x 0.083	1100.0 - 1700.0	908	0.406 - 0.258	n/a		
WR- 0.51	n/a	5.1 x 2.55	0.130 x 0.065	1400.0 - 2200.0	1157	0.586 - 0.369	n/a		

1) The waveguide loss is calculated assuming the conductivity of Gold, and a surface roughness factor of 1.5.
The two values listed represent the loss at the low end and high end of the frequency range.

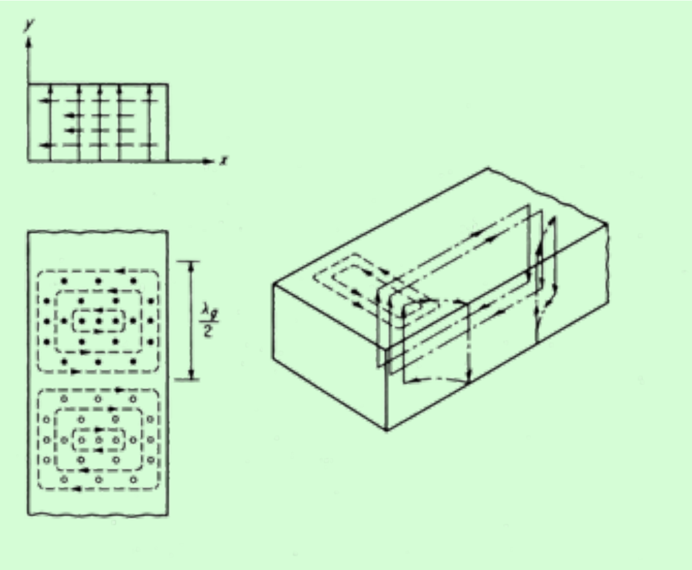
from http://www.vadiodes.com/VDI/pdf/waveguidechart200908.pdf

The concept of impedance

The term impedance was first used by Oliver Heaviside in the19th century to describe the complex ratio V/I in AC circuits. In the 1930's Schelkunoff extended this concept to electromagnetic fields and noted that impedance should be regarded as characteristic of the type of field, as well as medium. The impedance may also be dependent on the direction of the propagating wave. The concept of impedance is an important link between field theory and transmission line theory.

- Intrinsic impedance of the medium, $Z_0 = \eta = \sqrt{\frac{\mu}{\varepsilon}}$
- Wave impedance; this impedance is a characteristic of the particular type of wav[e] $Z_w = E/H$ TEM, TE, TM waves each have different wave impedances; they may depend on the type of the line or guide, the material, and frequency,
- Characteristic impedance is the ratio of voltage to current for a travelling wave; voltage and current are uniquely defined only for a TEM wave; TE and TM waves do not have uniquely defined voltage and current, so the characteristic impedan[ce] for such waves may be defined in various ways. $Z_0 = \sqrt{\frac{L}{C}}$

Dominant TE_{10} mode



$$H_{z,10} = A\cos\frac{\pi x}{a}e^{-j\beta z}$$

$$H_{x,10} = A\frac{j\beta}{k_c}\sin\frac{\pi x}{a}e^{-j\beta z}$$

$$E_{y,10} = -AZ_{h,10}\frac{j\beta}{k_c}\sin\frac{\pi x}{a}e^{-j\beta z}$$

$$k_{c,10} = \frac{\pi}{a},\ \beta_{10} = \sqrt{k_0^2 - \left(\frac{\pi}{a}\right)^2}$$

$$Z_{h,10} = -\frac{E_y}{H_x} = \frac{k_0}{\beta}Z_0$$

$$\lambda_g = \frac{2\pi}{\beta} = \frac{\lambda_0}{\sqrt{1-\left(\lambda_0/2a\right)^2}}$$

$$v_p = \frac{\lambda_g}{\lambda_0}c,\ v_g = \frac{\lambda_0}{\lambda_g}c$$

Attenuation

- Attenuation due to dielectric loss α_d
- Attenuation due to conductor loss, α_c
- Total attenuation: $\alpha = \alpha_c + \alpha_d$

Attenuation due to dielectric loss: homogenous filling

- Propagation constant

$$\gamma = \alpha_d + j\beta = \sqrt{k_c^2 - k^2} = \sqrt{k_c^2 - \omega^2 \mu_o \varepsilon_o \varepsilon_r \left(1 - j\tan\delta\right)} = \sqrt{k_c^2 - k^2 + jk^2 \tan\delta} \approx$$

$$\cong \sqrt{k_c^2 - k^2} + \frac{jk^2 \tan\delta}{2\sqrt{k_c^2 - k^2}} = j\beta + \frac{k^2 \tan\delta}{2\beta}$$

- for TE and TM (Np/m).

$$\alpha_d = \frac{k^2 \tan\delta}{2\beta}$$

- TEM waves (Np/m)=> $\alpha_d = \dfrac{k\tan\delta}{2}$

Perturbation method to calculate loss

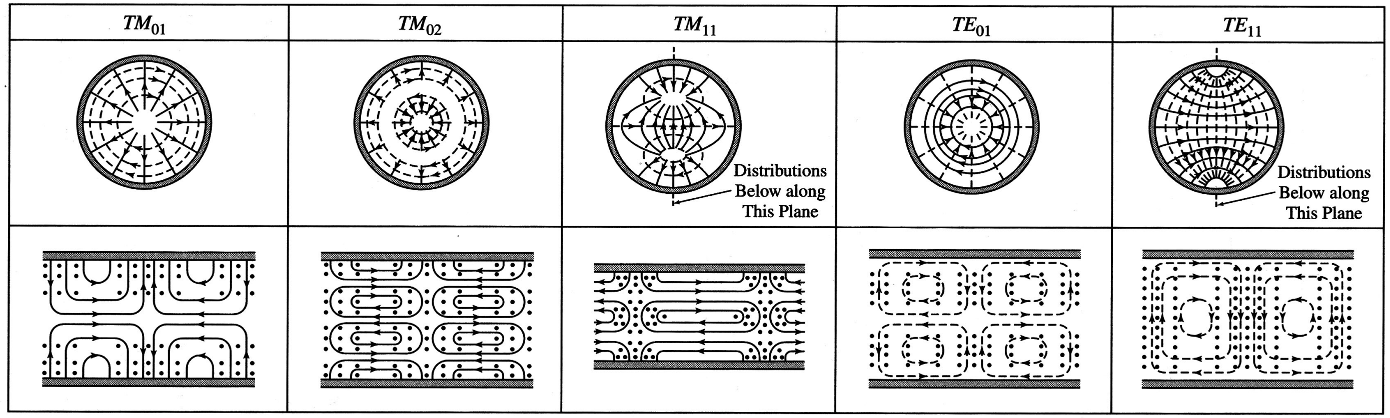
- Assumes that field distribution in lossy line is not different from lossless line.
- Derive method to calculate loss...

$$P(z) = P_o e^{-2\alpha z}$$

$$p_l = \frac{-\partial P}{\partial z} = 2\alpha P(z) \text{ "power loss per unit length"}$$

$$\alpha = \frac{p_l(z)}{2P(z)} = \frac{p_l(z=0)}{2P_o}$$

Circular waveguide



TE modes

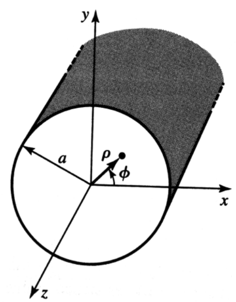
$$e_z = 0, \quad h_z \neq 0$$

$$\nabla_t^2 h_z + k_c^2 h_z = 0$$

$$\left. \frac{\partial h_z}{\partial r} \right|_{r=a} = 0$$

$$h_z(r,\phi) = (B_1 \cos n\phi + B_2 \sin n\phi) J_n(k_c r)$$

$$\left. \frac{dJ_n(k_c r)}{dr} \right|_{r=a} = 0; \; k_{c,nm} = \frac{p'_{nm}}{a}$$



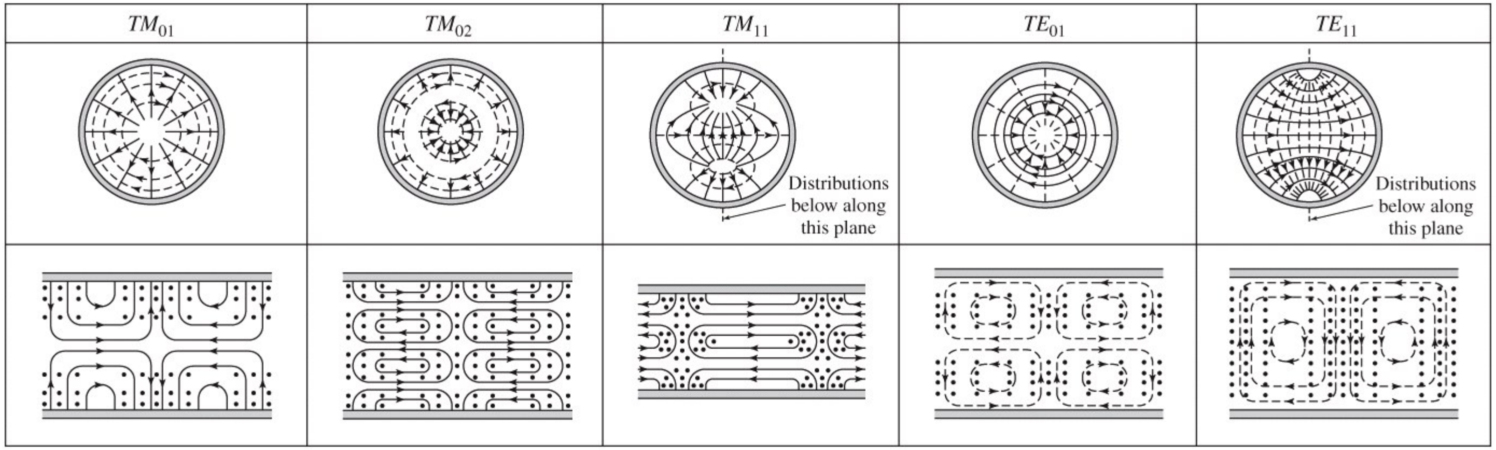


Figure 3.14
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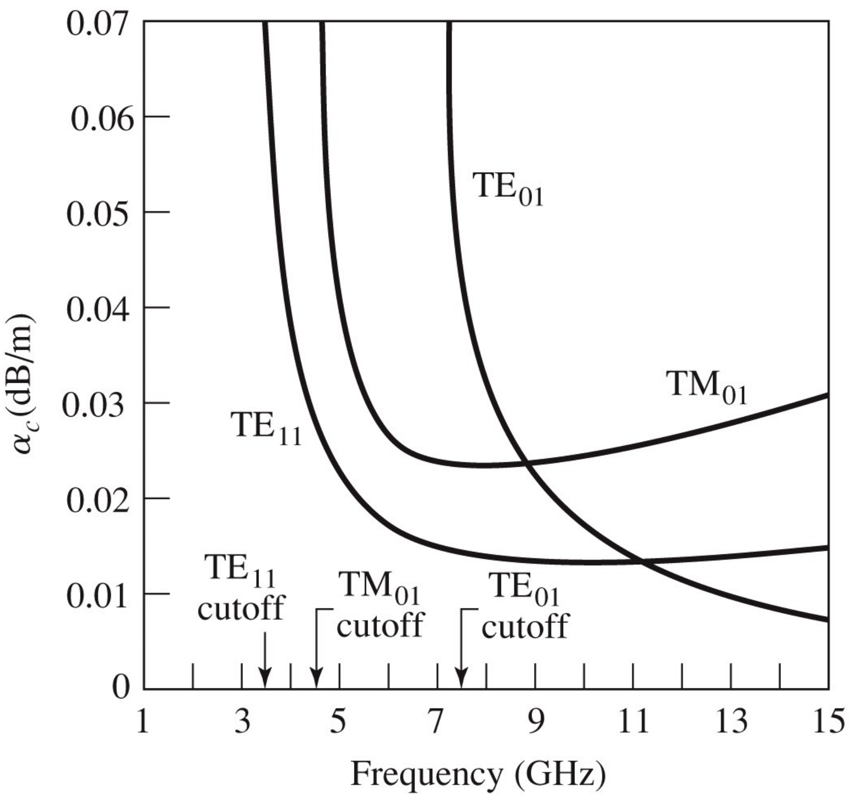


Figure 3.12

Serious attempts to utilise TE01 for long distance communication

Elliptical waveguide

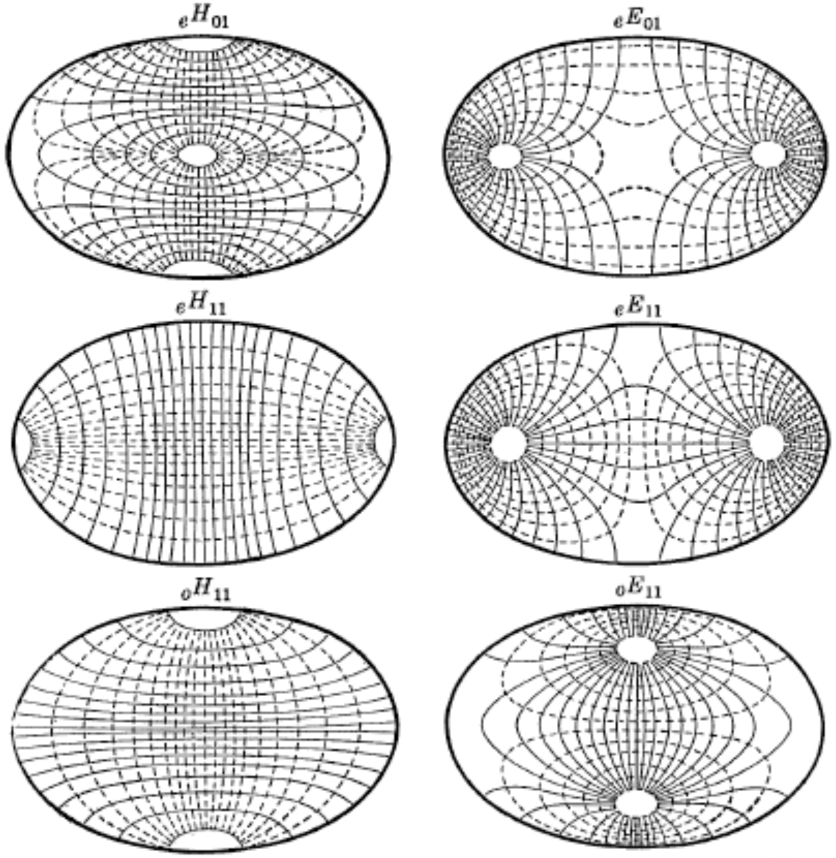
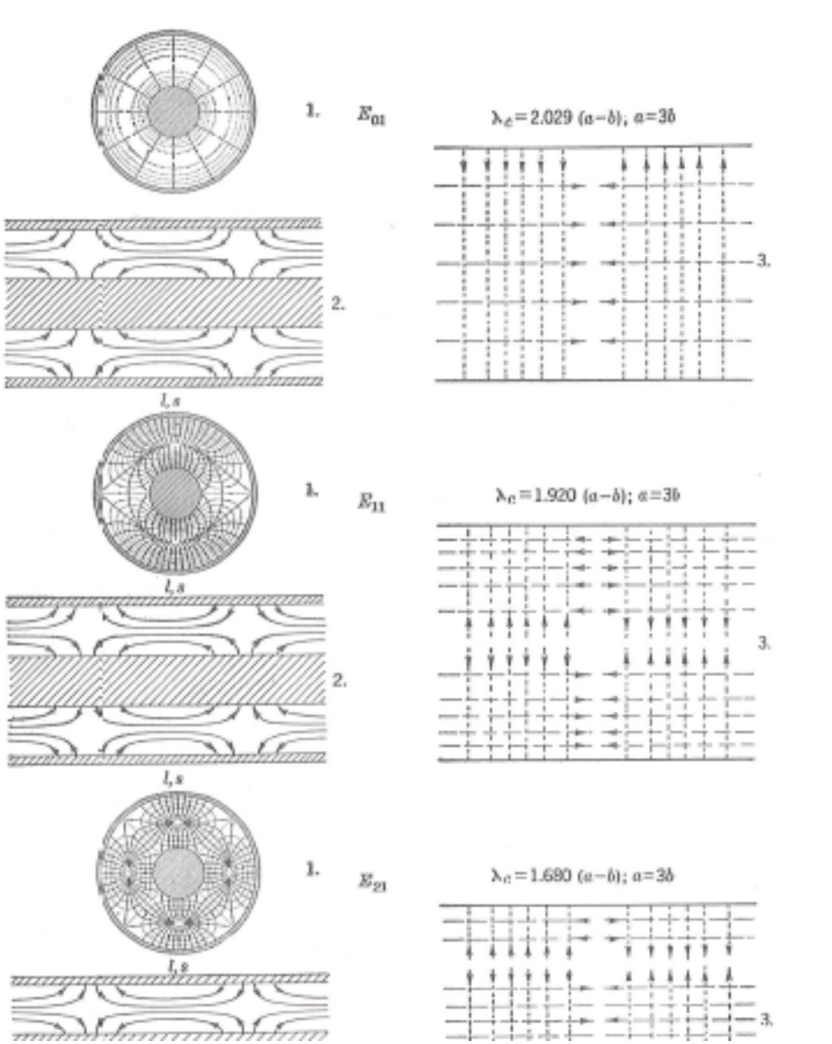


FIG. 2-12.—Field distribution of modes in elliptical waveguide. Cross-sectional view.

Coaxial waveguides



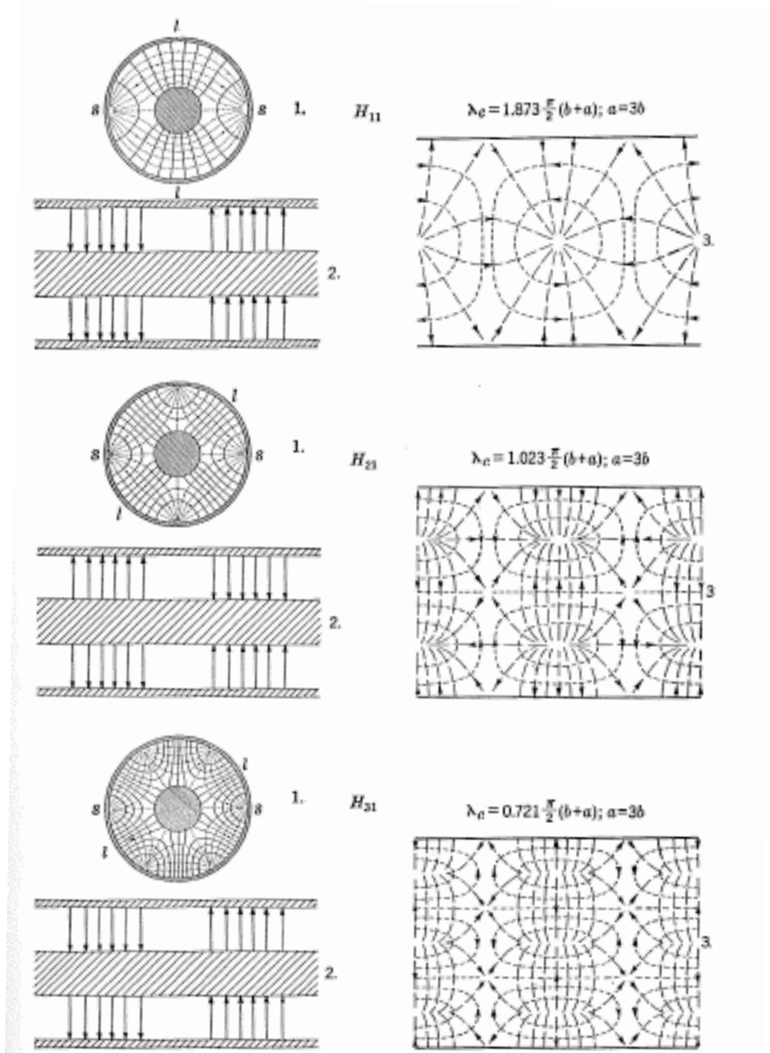
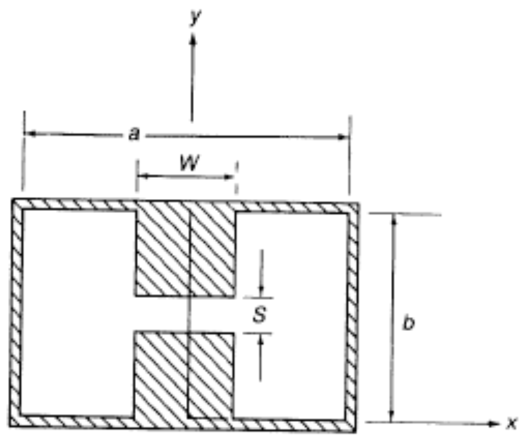


FIG. 2·9.—Field distribution for H-modes in coaxial waveguide.
1. Cross-sectional view
2. Longitudinal view through plane l-l

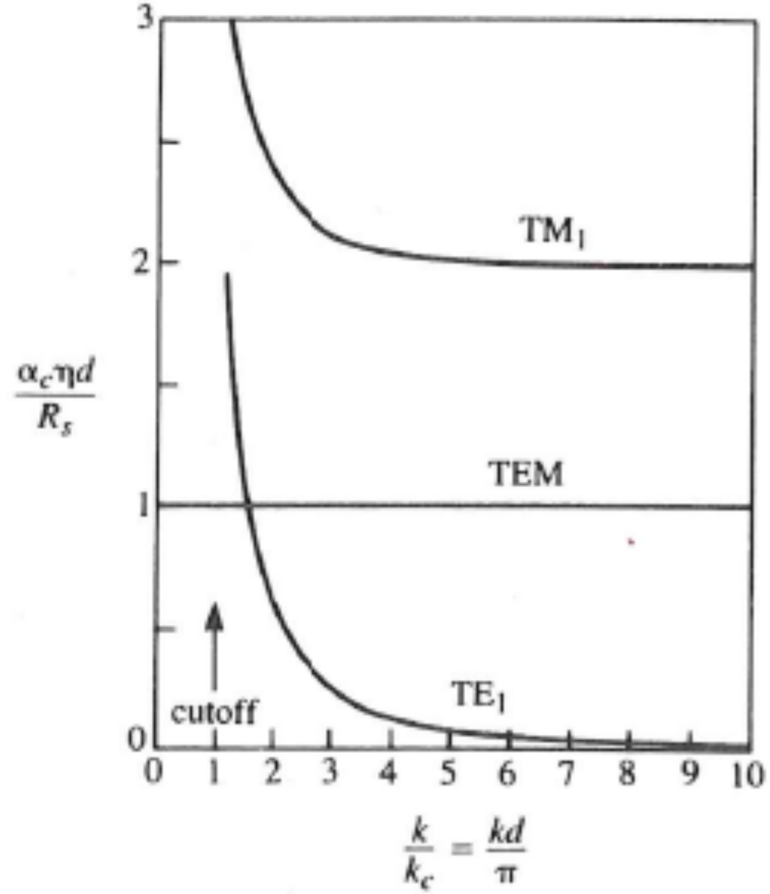
Ridge waveguide

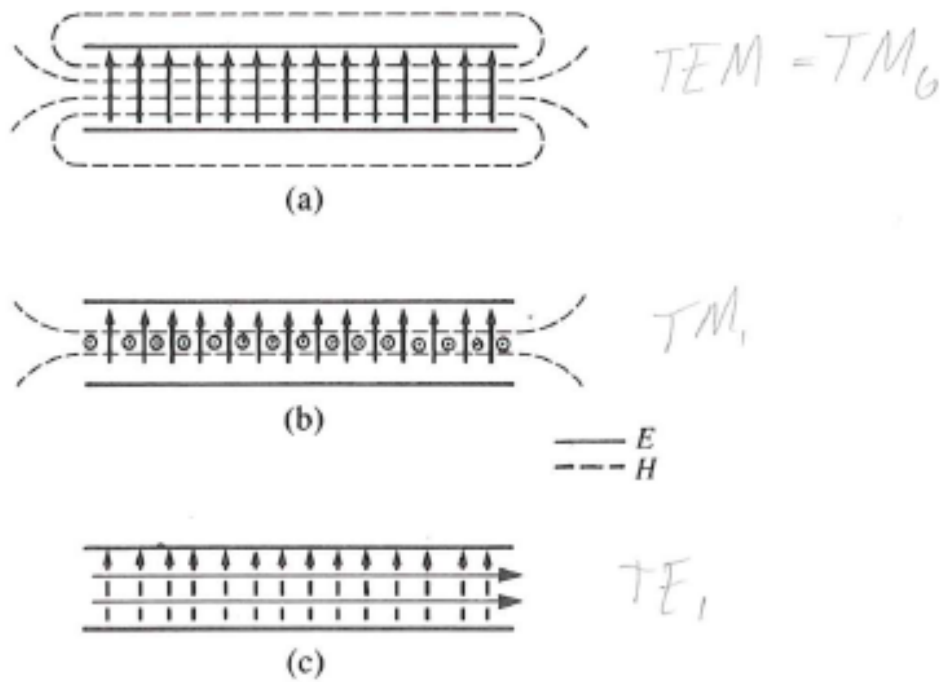
- Better single mode bandwidth



- *On white board: in-homogenous filled parallel plate waveguide.*

Parallel plate waveguide





Summary of lecture 3

- Read chapter 3 (3.1-3.5).
  - TEM, TE, and TM modes
  - Hollow waveguides (TE and TM modes)
  - Field analysis on transmission lines
  - Dispersion, characteristic impedance
- Next: Planar transmission lines such as microstrip, stripline and coplanar lines