Introduction to Microwave Ferrite components

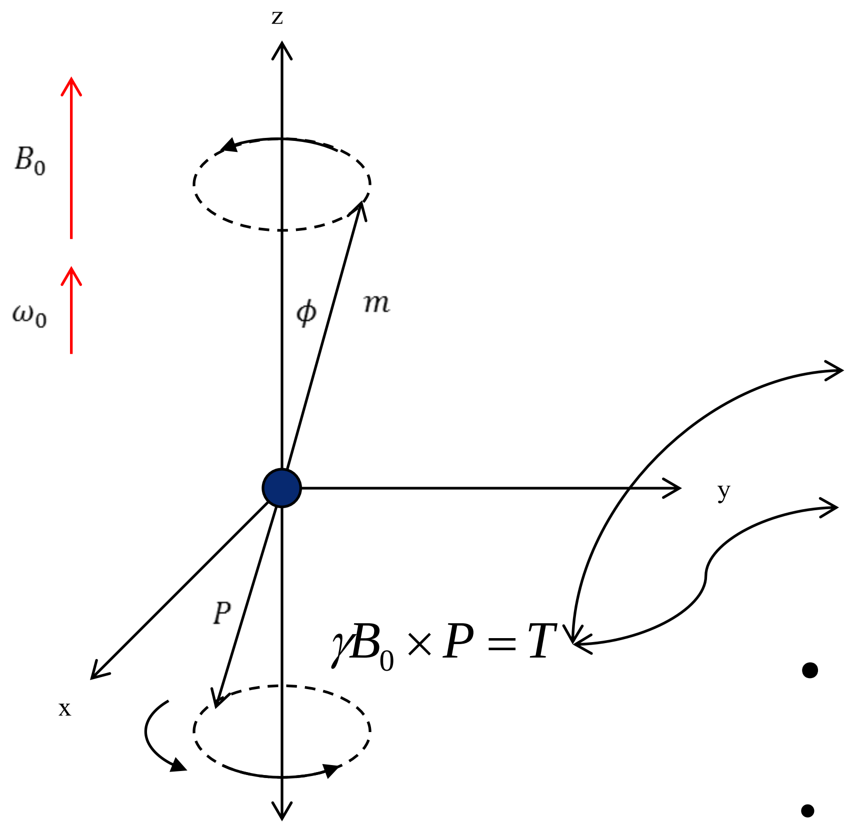
Vincent Desmaris

Outline

- **Microwave Propagation in a Ferrite Medium**
- **Faraday Rotation**
- **Gyrator**
- **Isolators**
- **Circulators**

Simplest model – Larmor Precession

One single spinning electron in a static "magnetic field" B_0



P = angular momemtum

m = magnetic dipole moment

γ=m/P gyromagnetic ratio

$$T = m \times B_0$$

$$\frac{dP}{dt} = T \Rightarrow \quad \omega_0 = \gamma B_0$$

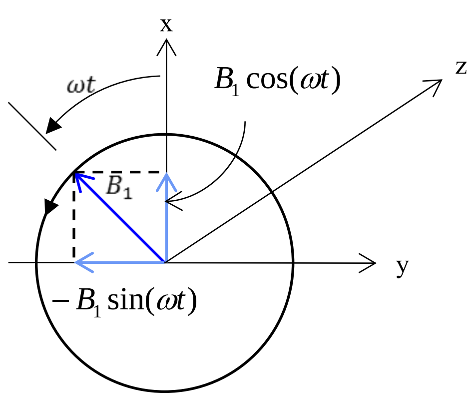
- $\frac{dP}{dt}$ is $\perp$ to $P \rightarrow$ leads to precession of moment:
- Larmor precession **independent of ϕ**

More complete model

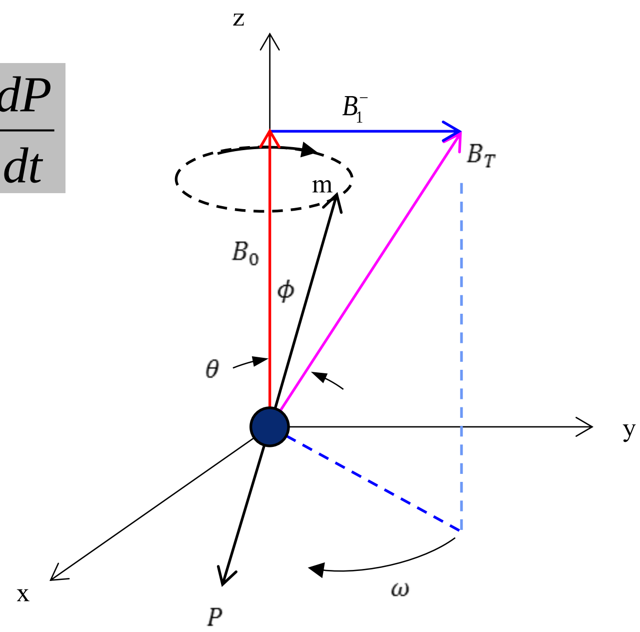
Superposition of circularly polarised ac B_1 in plane perpendicular to B_0

- Free precession (static B_0) → **Forced** precession (static B_0 + ac B_1)
- Resultant torque T → equation of motion
- **Left Circular Polarisation**→ "negative" direction of rotation
  - Dipole moment m forced to **precess** about z with angular frequency ωt
  - Precession angle ϕ must be **less** than θ to ensure that torque will precess in CCW direction

$$B_1^- = B_1(\hat{x} + j\hat{y})$$



$$T = m \times B_T = \frac{dP}{dt}$$



$$m^- = m_0 \tan\phi = \frac{-\gamma m_0 B_1}{\omega_0 + \omega}$$

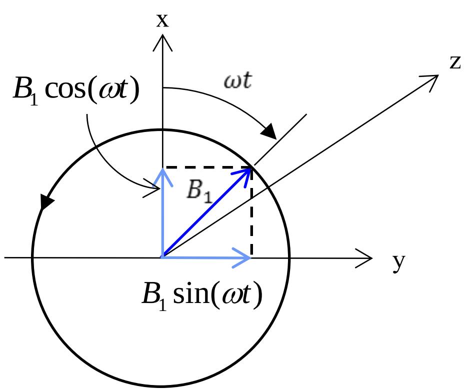
Component of m rotating in sync with B_1^-

More complete model

Superposition of circularly polarised ac B_1 in plane perpendicular to B_0

- Free precession (static B_0) → **Forced** precession (static B_0 + ac B_1)
- Resultant torque T → equation of motion
- **Right Circular Polarisation**→ "positive" direction of rotation
  - Dipole moment m forced to **precess** about z with angular frequency ωt
  - Precession angle ϕ must be **greater** than θ to ensure that torque will precess in CW direction

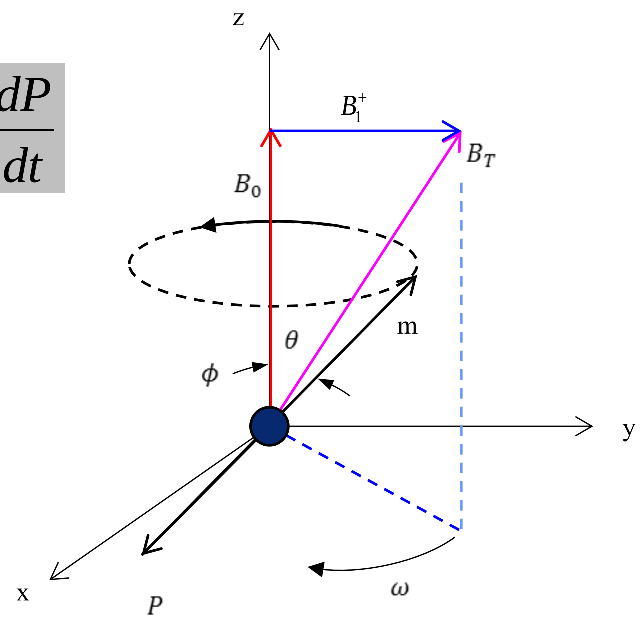
$$B_1^- = B_1(\hat{x} + j\hat{y})$$



$$T = m \times B_T = \frac{dP}{dt}$$

$$m^+ = m_0 \tan\phi = \frac{+\gamma m_0 B_1}{\omega_0 + \omega}$$

Component of m rotating in sync with B_1^+



Generalization to the whole ferrite

Ferrite= N effective electrons spinning per unit volume
→ Macroscopic viewpoint: consider density of dipoles as "*smeared out*"
→ Total magnetic dipole *M:*

$$M = N \cdot m$$

If $B_0 >> 1$, and $B_0 >> B_1$

$$\mu^+ = \mu_0 \left(1 + N \cdot m^+\right) = \mu_0 \left(1 + \frac{\gamma \mu_0 M_0}{\omega_0 - \omega}\right)$$

$$\mu^- = \mu_0 \left(1 + N \cdot m^-\right) = \mu_0 \left(1 + \frac{\gamma \mu_0 M_0}{\omega_0 + \omega}\right)$$

Fundamental result, depending on polarization, circularly polarized TEM waves experience **different permeability** (i.e. propagation constants) depending on type of polarization

Mathematical Modelling of Ferrites

- Magnetisation M: "magnetic polarisation"
  - Vector field describing density of magnetic dipole moments
- Magnetic Susceptibility χ_m:
  - Degree of M in response to applied field H

$$M = \chi_m . H$$

- χ_m is a tensor, and so is μ:

$$B = \mu_0 (H + M)$$
$$\Longrightarrow B = \mu_0 (1 + \chi_m) H$$

$$\therefore \begin{bmatrix} \boldsymbol{B_x} \\ \boldsymbol{B_y} \\ \boldsymbol{B_z} \end{bmatrix} = \boldsymbol{\mu_0} \begin{bmatrix} \mathbf{1} + \boldsymbol{\chi_{xx}} & \boldsymbol{\chi_{xy}} & \mathbf{0} \\ \boldsymbol{\chi_{yx}} & \mathbf{1} + \boldsymbol{\chi_{yy}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \end{bmatrix} \begin{bmatrix} \boldsymbol{H_x} \\ \boldsymbol{H_y} \\ \boldsymbol{H_z} \end{bmatrix}$$

- For small signals:
  - $\chi_{xx} = \chi' - j\chi'' = \chi$
  - $\chi_{xy} = \chi_{yx} = j(K' - jK'') = jK$

Plane Wave in Ferrite Medium (1/3)

- Prior to ferrite loaded waveguide structures → simple analysis of PW in infinite ferrite medium

1. Assume Plane Wave
   - $\frac{\partial}{\partial x} = \frac{\partial}{\partial Y} = 0$

   → $E = E_0 e^{-j\beta z}$

   $H = H_0 e^{-j\beta z}$

2. Source Free Medium
   - $\nabla \cdot E = \nabla \cdot H = 0$

   → $\nabla \times E = -j\omega\mu H$

   $\nabla \times H = j\omega\varepsilon E$

   Maxwell's Equations

3. Assume lossless media
   - $\chi_{xx} = \chi' - j\chi'' = \chi$
   - $\chi_{xy} = \chi_{yx} = j(K' - jK'') = jK$

   } Small signal analysis

Plane wave in ferrite medium (2/3)

$$\begin{bmatrix} \beta^2 - \omega^2 \varepsilon\mu_0 (1+\chi) & -j\omega^2 \varepsilon\mu_0 K \\ j\omega^2 \varepsilon\mu_0 K & \beta^2 - \omega^2 \varepsilon\mu_0 (1+\chi) \end{bmatrix} \begin{bmatrix} H_{0x} \\ H_{0y} \end{bmatrix} = 0$$

Note that this is an eigenvalue problem, eigenvalues are obtained by setting the determinant of the matrix to zero

$$\beta_+ = \omega\sqrt{\varepsilon\mu_0}\sqrt{(1+\chi)+K}$$

$$\beta_- = \omega\sqrt{\varepsilon\mu_0}\sqrt{(1+\chi)-K}$$

Obtain two solutions for propagation constant β $\rightarrow$ *natural modes*

Plane wave in ferrite medium (3/3)

1. The two modes of propagations corresponds to circularly polarized waves

$$\beta_+ \Rightarrow \text{positive circularly polarized wave}$$

$$\beta_- \Rightarrow \text{negative circularly polarized wave}$$

2. Both waves propagates with a **different** phase constant

3. If the propagation is **not** along the static magnetic field, still 2 modes of propagations exists with different phase constants but not circularly polarized
   - Linearly polarised wave along B_0 - plane of polarisation will **rotate** since $|\beta_+| \neq |\beta_-|$

   → **Faraday Rotation – non-reciprocal**

Faraday Rotation (1/3)

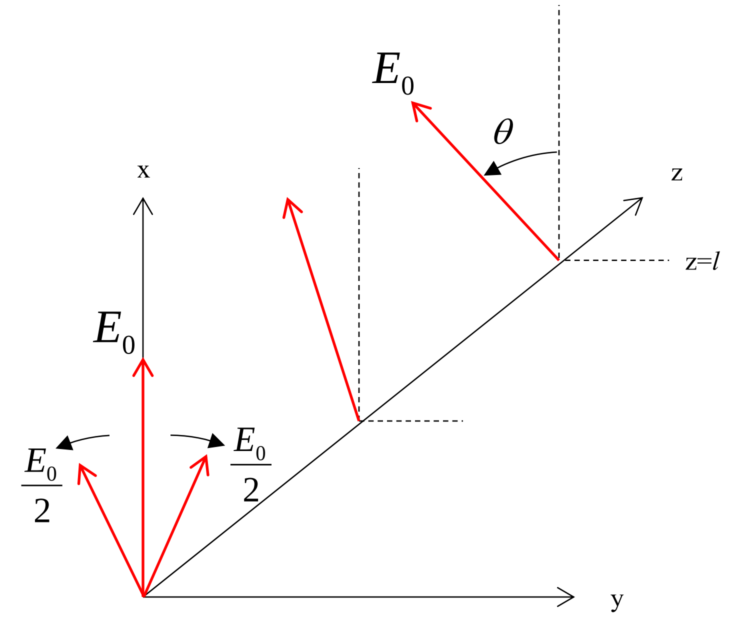
- Consider a TEM wave linearly polarised in *x* at *z=0*: $E = \hat{x}E_0$
- The wave propagates in an infinite, lossless medium with a static magnetic field B_0 aligned in the *z* direction.

- Decompose the linearly polarised wave into two circularly polarised waves:

$$E = \hat{x}E_0 = \frac{E_0}{2}(\hat{x} + j\hat{y}) + \frac{E_0}{2}(\hat{x} - j\hat{y})$$

LHCP RHCP

- LHCP propagates with $e^{-j\beta_{-}z}$
- RHCP propagates with $e^{-j\beta_{+}z}$
  - i.e. different phase constants



Faraday Rotation (2/3)

- The resultant wave at distance *z*:

$$E = E_0\left[\hat{x}\cos\left(\frac{\beta_+ - \beta_-}{2}z\right) - \hat{y}\cos\left(\frac{\beta_+ - \beta_-}{2}z\right)\right]e^{-j\frac{\beta_+ - \beta_-}{2}z}$$

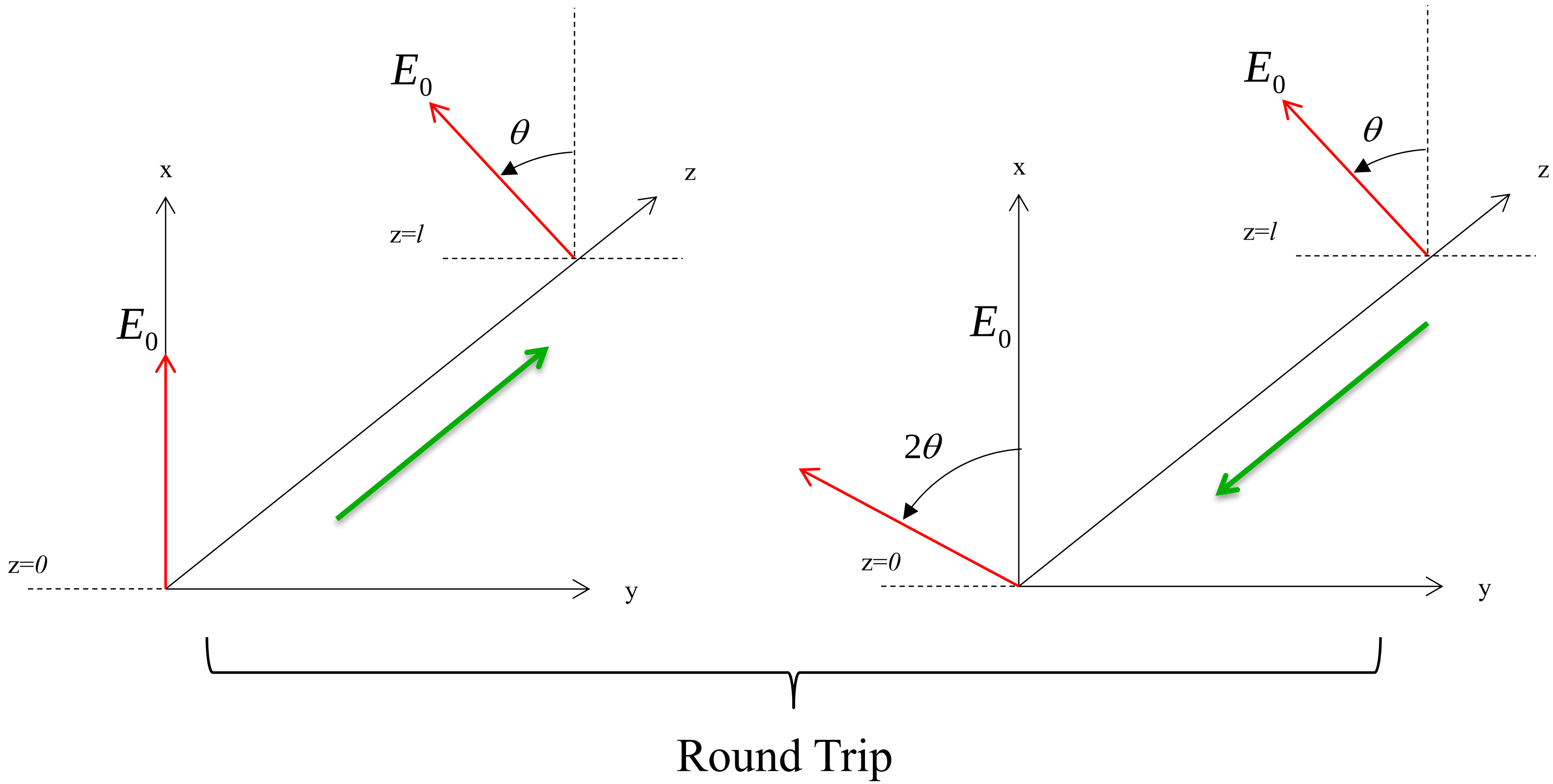
- The two components (LHCP and RHCP) have equal amplitudes, with the phase difference between them given by:

$$\theta = \tan^{-1}\frac{E_y}{E_x} = \tan^{-1}\left(-\tan^{-1}\left(\frac{\beta_+ - \beta_-}{2}z\right)\right) = -\left(\frac{\beta_+ - \beta_-}{2}\right)z$$

- **Resultant Wave:** still linearly polarised but plane of polarisation is **rotated** by angle $\theta \rightarrow$ **Faraday Rotation**

Faraday Rotation (3/3)

- Faraday rotation is **non-reciprocal** – if the direction of propagation is reversed, the plane of polarisation continues to rotate in the same direction
- i.e. in a round-trip (i.e. forward and reversed), the total rotation is 2θ



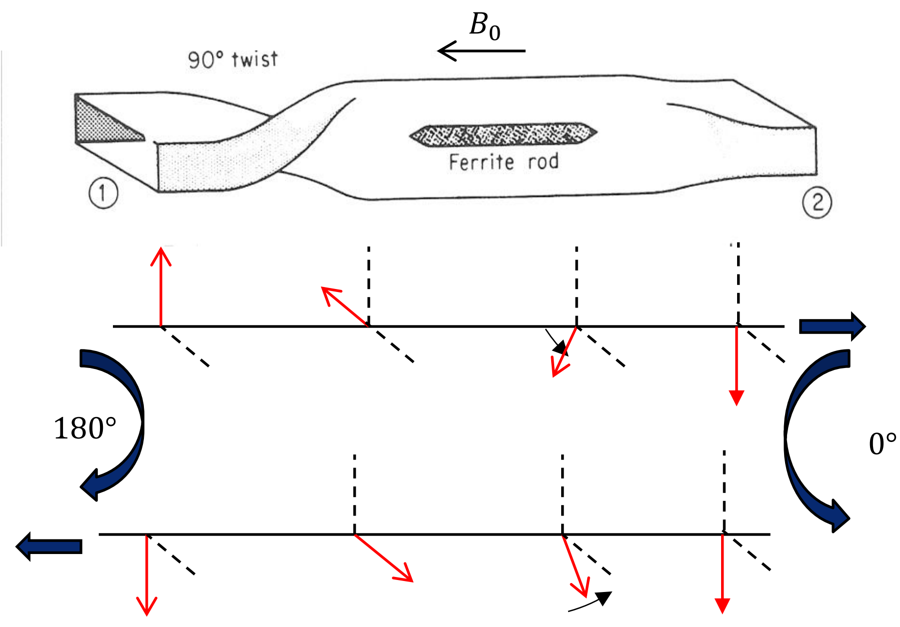
Gyrators

- 2 port devices
- 180 degrees phase shift for transmission from port 1 to port 2 as compared to port 2 to port 1
- Take advantage of Faraday rotation
- Not very useful as single device, but handy as building block of isolators and circulators

$$\left[S_{ideal\ gyrator}\right]=\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}$$

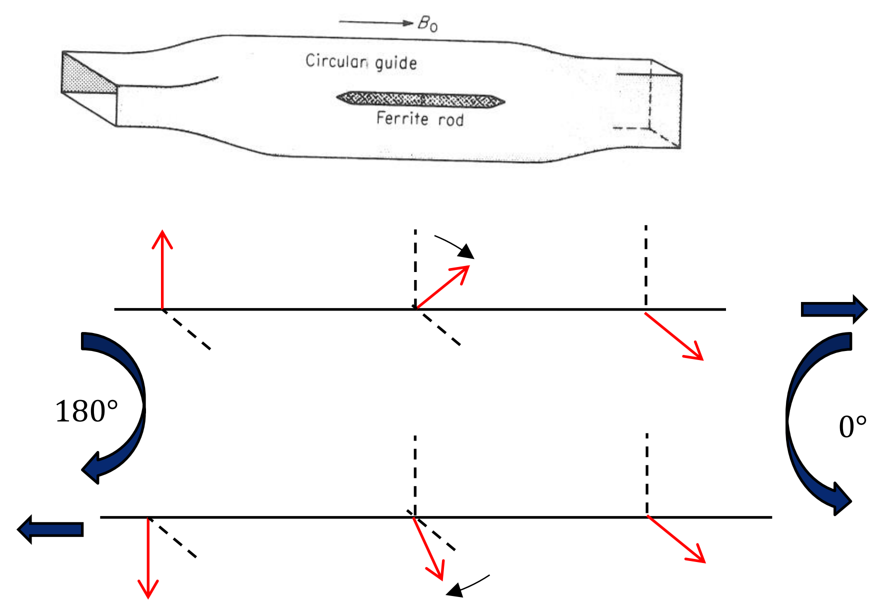
Simple Gyrators

- Rectangular guide with twist



- Forward: $\Delta\phi = 180°$
- Reverse: $\Delta\phi = 0°$

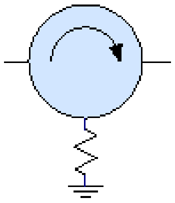
- Straight rectangular guide



- Forward: $\Delta\phi = 90°$
- Reverse: $\Delta\phi = 90°$

Isolators

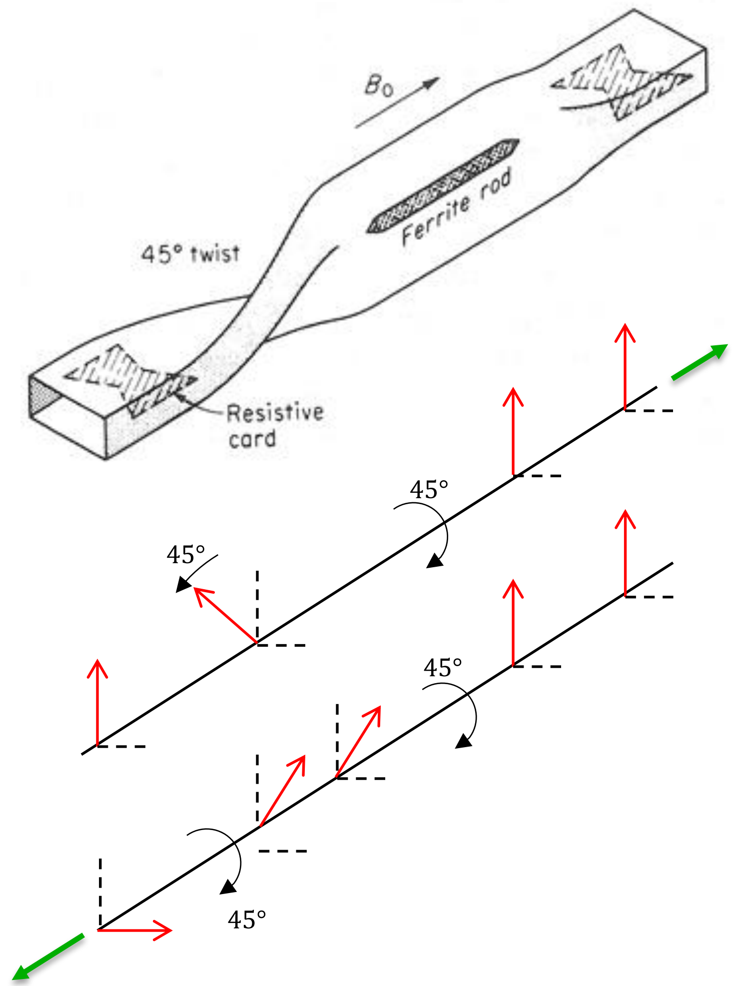
- 2 port devices
- Permit unattenuated transmission from Port 1 to Port 2 but high attenuation from Port 2 to Port 1
  - Typical forward transmission loss $\sim 1\ dB$
  - Typical reverse attenuation $\sim 20 - 30\ dB$ for $10\%\ BW$
- Takes advantage of Faraday rotation
- Applications:
  - Common for coupling microwave signal generator to load network
  - All available power delivered to the load
  - Reflections from load not transmitted back to the output terminals



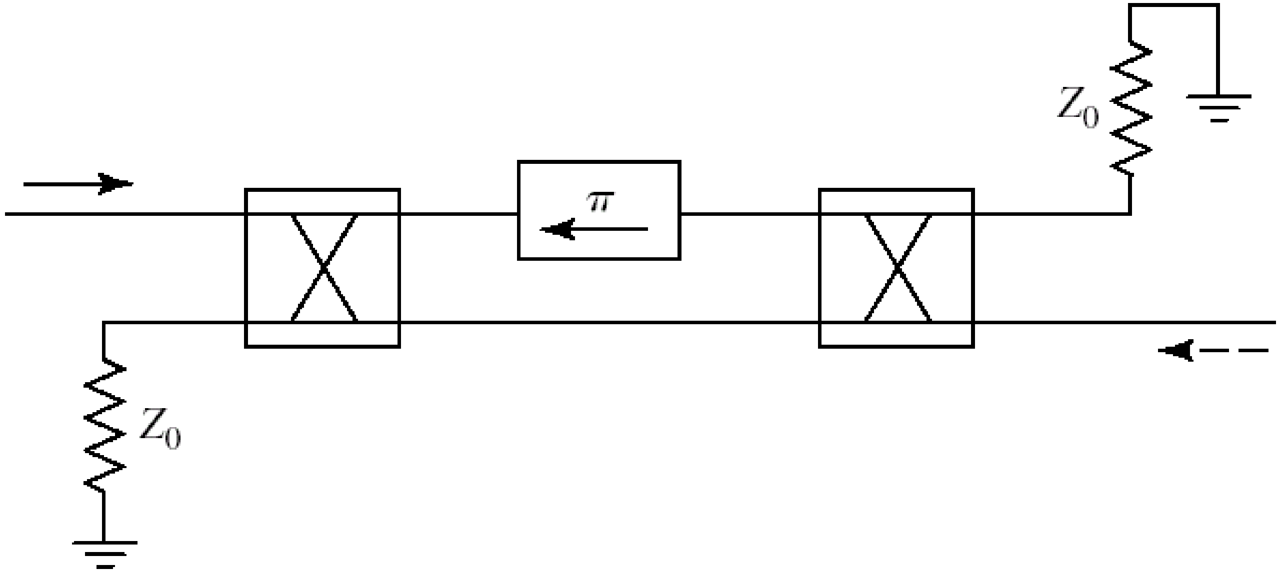
$$\left[S_{ideal\ isolator}\right] = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$

Simple Isolator

- Design:
- Gyrator (or similar) with *45°* twist and *45°* Faraday Rotation
- Forward:
  - Polarisation rotated by *+45°* (CCW)
  - Polarisation rotated again by ferrite rod by *-45°* (CW)
  - Emerges at output with desired polarisation
- Reverse:
  - Polarisation rotated by ferrite rod and twist by *90°*
  - At input the polarisation is parallel to resistive card → absorption
- Two cards needed to account for multiple reflecions



Another simple Isolator ?

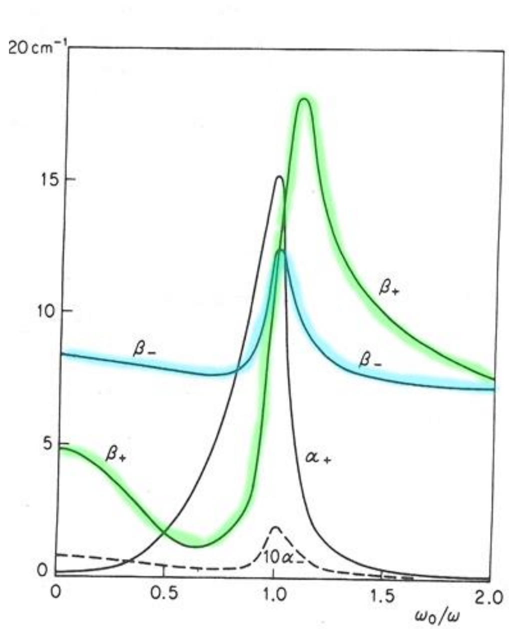


Resonance Isolator

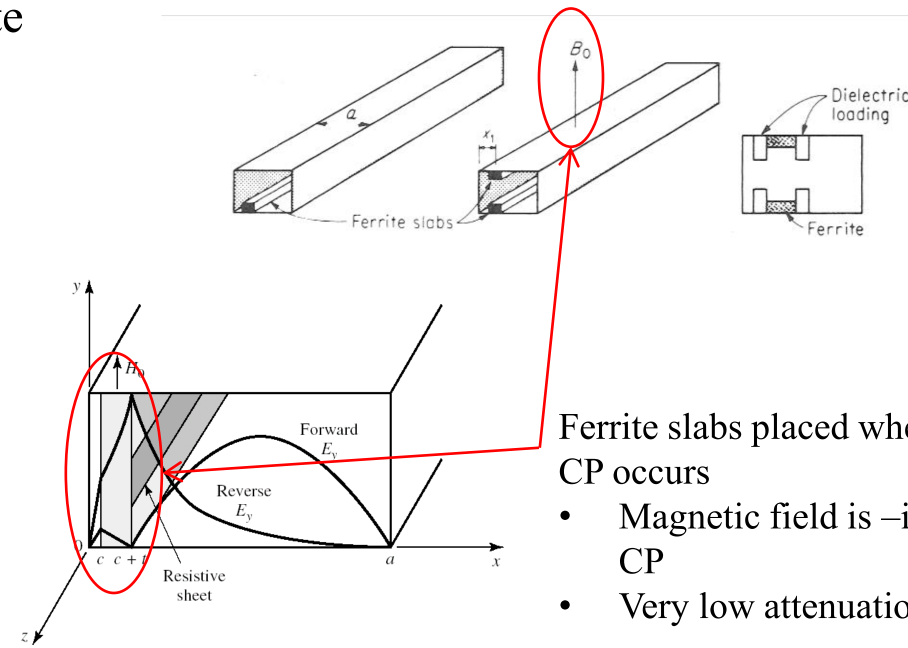
- Attenuation constants:

$$(\alpha_\pm) = \frac{\omega^2 \epsilon \mu''_\pm}{2\beta_\pm}$$

- Varying frequency ω about ferrite resonance frequency ω_0 :



- CP condition is inherent property of dominant TE_{10} mode at two positions in rectangular waveguide

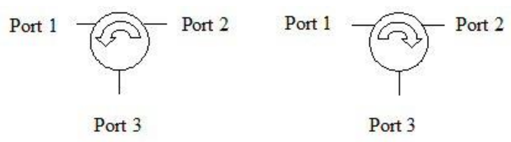


Ferrite slabs placed where CP occurs

- Magnetic field is –ive CP
- Very low attenuation

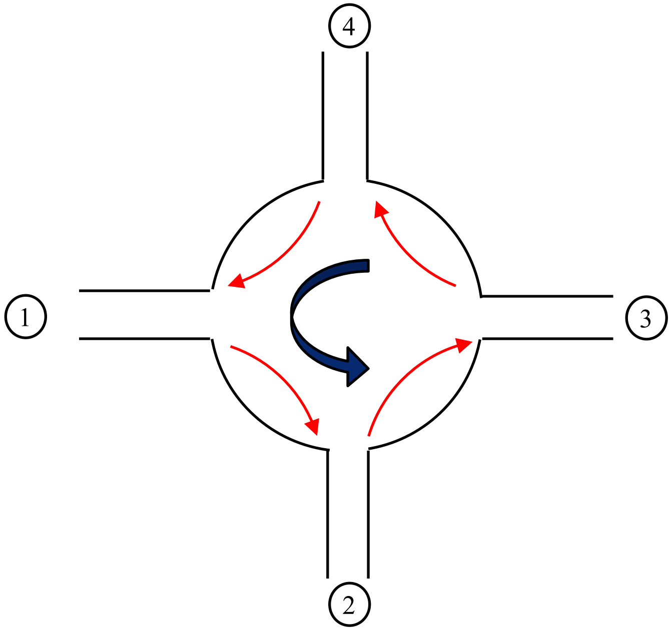
Circulators

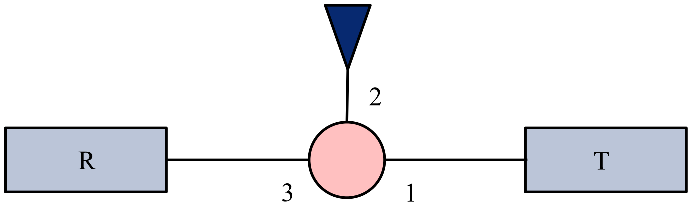
- Multiport devices ($n > 2$)
- Wave incident on port *1* couples **only** to port *n+1*



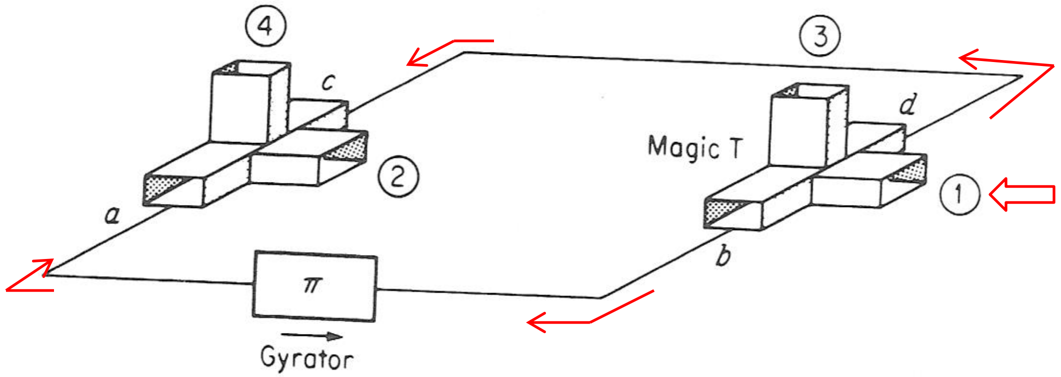
- Useful for coupling receiver and transmitter to the same antenna in radar system or separate input and output ports of negative resistance amplifiers

$$[S_{ideal\,circulator}] = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$





4 Port Circulators

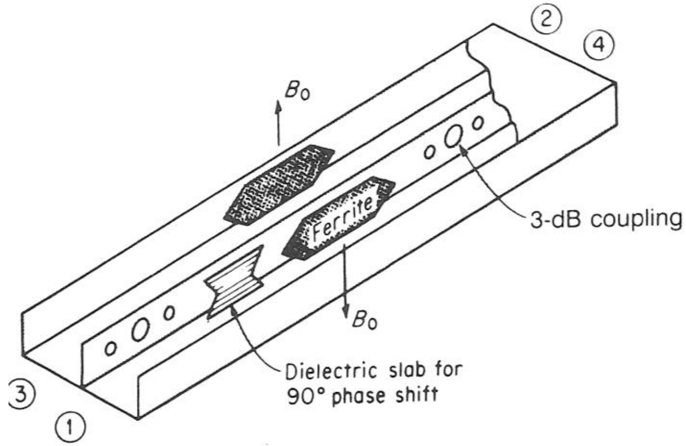


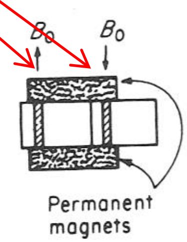
- Two magic tees *or* hybrid junctions with gyrator

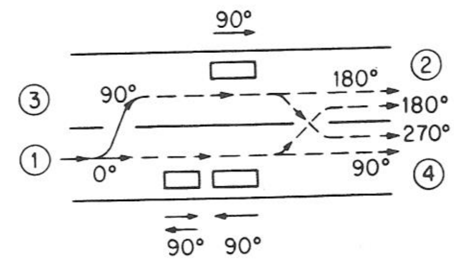
- Incident at Port 1
  - Split between *b* and *d* in amp and phase
  - Arrive in-phase at *a* and *c*
  - → Emerge from Port 2
- Incident at Port 2
  - Split between *a* and *c*
  - Gyrator → $\phi_b = \Delta\phi_{a-b} + \pi$
  - $\Delta\phi_{b-d} = \pi$
  - Combine and emerge at Port 3
- Incident at Port 3
  - Split between *b* and *d* with $\Delta\phi_{b-d} = \pi$
  - Combine at *a* and *c* and exit Port 4
- Incident at Port 4
  - Split between *a* and *c* with $\Delta\phi_{a-c} = \pi$
  - Gyrator → $\phi_b = \Delta\phi_{a-b} + \pi$
  - $\Delta\phi_{b-d} = 0\ (2\pi)$
  - Combine and emerge at Port 1

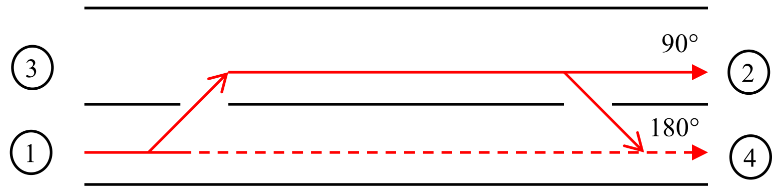
4 Port Circulators

- *Phase Shifter:*
  - Thin ferrite slab located in WG where magnetic field of TE_{10} is circularly polarised
  - Biasing field in *y* direction
  - ∴ RCP for one direction, LCP for other
  - $\beta_+ \neq \beta_- \rightarrow \Delta\phi = \frac{\pi}{2}$
- *3dB directional couplers*
  - Location of holes ($\frac{\lambda}{4}$ apart) splits power evenly between two outputs







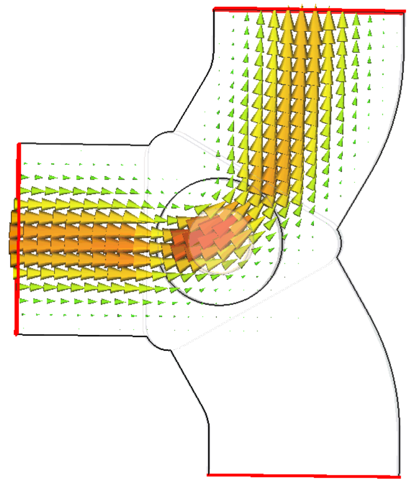


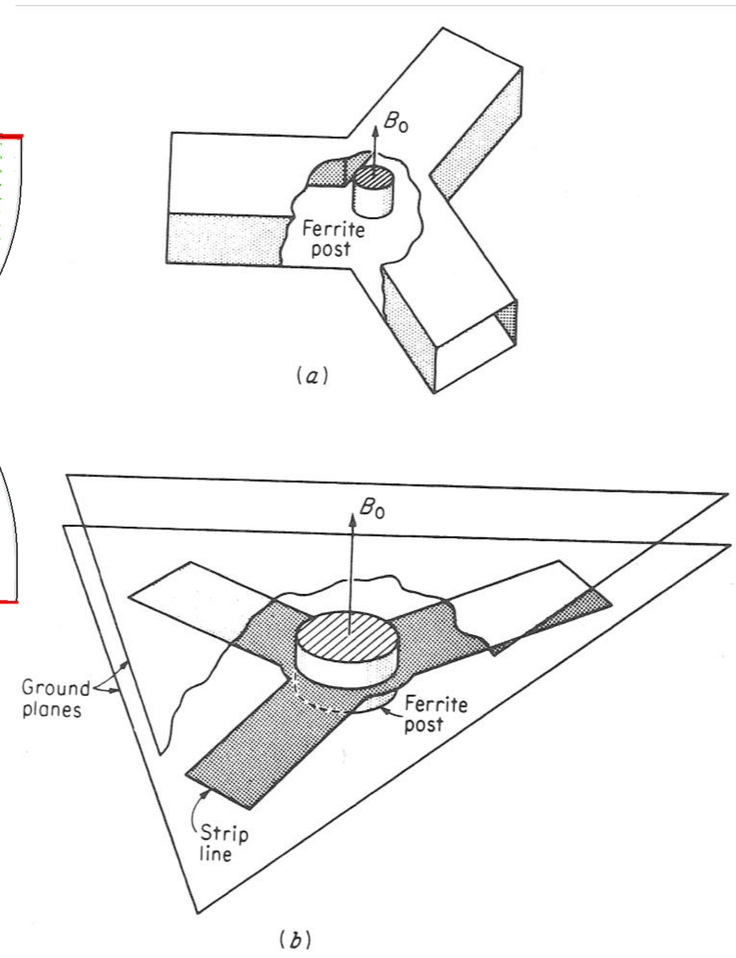
3 Port Circulators

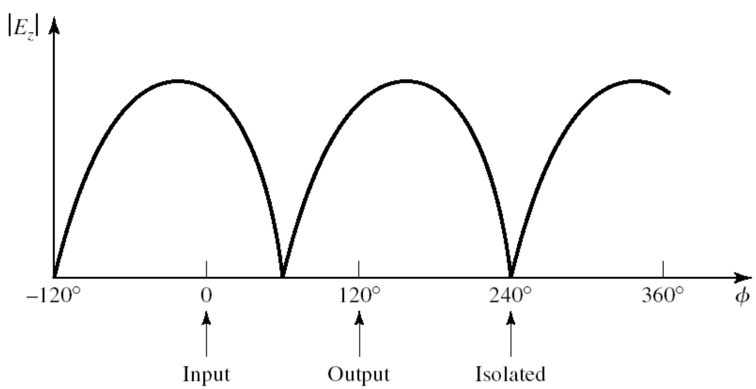
- Y-Junction Circulator

$$\left[S_{3-port}\right]=\begin{bmatrix}0 & 0 & 1\\ 1 & 0 & 0\\ 0 & 1 & 0\end{bmatrix}$$

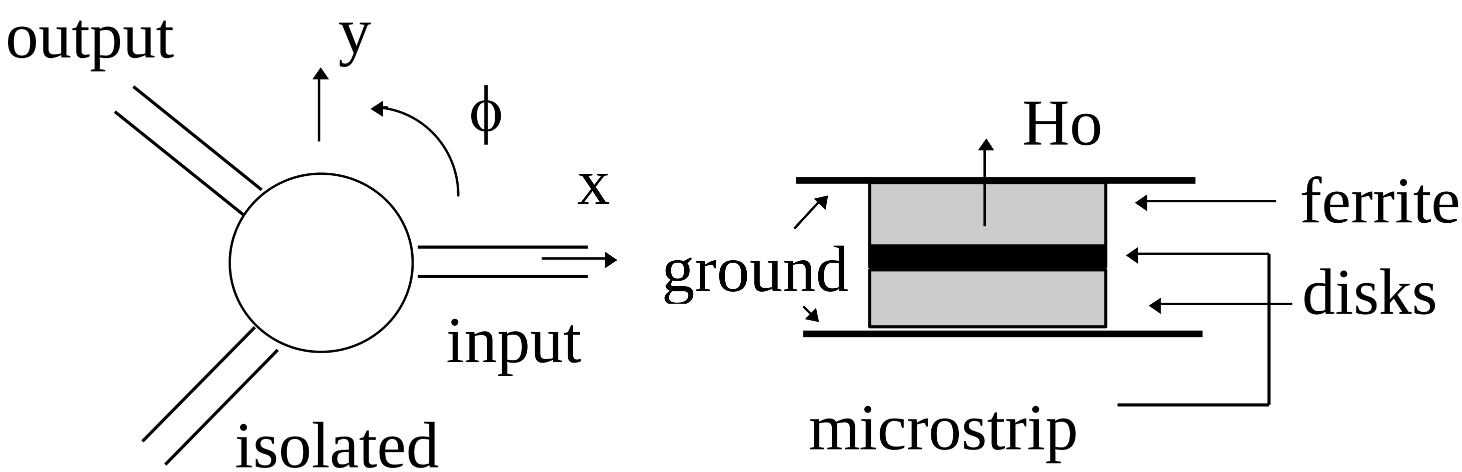
- Ferrite post – static B_0 field provides non-reciprocal property







Junction Circulator



- Field Analysis
- Assume *E* field only in *z* direction - E_z
- E_z is antisymmetric
- Applied DC magnetic field **also** only in *z* direction

Junction Circulator

- Susceptibility Tensor
  - Degree of magnetisation M in response to applied magnetic field: $M = \chi H$

$$\begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix} = \underbrace{\begin{bmatrix} \chi_{xx} & \chi_{xy} & 0 \\ \chi_{yx} & \chi_{yy} & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\text{Magnetic susceptibility tensor of ferrite}} \begin{bmatrix} H_x \\ H_y \\ H_z \end{bmatrix}$$

- Permeability Tensor
  - Ability of material to support B field: $B = \mu_0(H + M) = \mu_0(1 + \chi)H = \mu H$
  - In cylindrical coordinates:

$$\rightarrow \begin{bmatrix} B_\rho \\ B_\phi \\ B_z \end{bmatrix} = \underbrace{\begin{bmatrix} \mu & j\kappa & 0 \\ -j\kappa & \mu & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\text{Magnetic permeability tensor of ferrite}} \begin{bmatrix} H_\rho \\ H_\phi \\ H_z \end{bmatrix}$$

Junction Circulator

- Applying Maxwell's Equations for solution:

$$\frac{\partial^2 E_z}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial E_z}{\partial \rho} + \frac{1}{\partial \rho^2}\frac{\partial^2 E_z}{\partial \phi^2} + k^2 E_z = 0$$

$$where\ k = \omega\sqrt{\varepsilon\mu_0\frac{(\mu^2 - \kappa^2)}{\mu}}$$

- This is the same situation as for a TM mode solution in a cylindrical waveguide, but with different propagation constant *k*

Junction Circulator

- A more general solution for E_z is given (by analogy with TM mode solution):

$$E_z = \sum_{n=0}^{\infty} \left(a_n e^{-jn\phi} + b_n e^{+jn\phi}\right) J_n(k\rho)$$

Circulate in $+\phi$ Circulate in $-\phi$ Bessel

- Corresponding solution for ϕ component of H field:

$$H_\phi = -j\frac{\omega\varepsilon}{k}\sum_{n=0}^{\infty}\left(a_n\left[J_n'(k\rho) - \frac{nK}{k\mu\rho}J_n(k\rho)\right]e^{-jn\phi} \ldots\right.$$
$$\left. + b_n\left[J_n'(k\rho) + \frac{nK}{k\mu\rho}J_n(k\rho)\right]e^{+jn\phi}\right)$$

- Interpretation: waves circulating around ferrite disk in $\pm\phi$ directions
  - The H_ϕ field is different for the two sets of waves → **non-reciprocal** behaviour
  - Essential property for a circulator

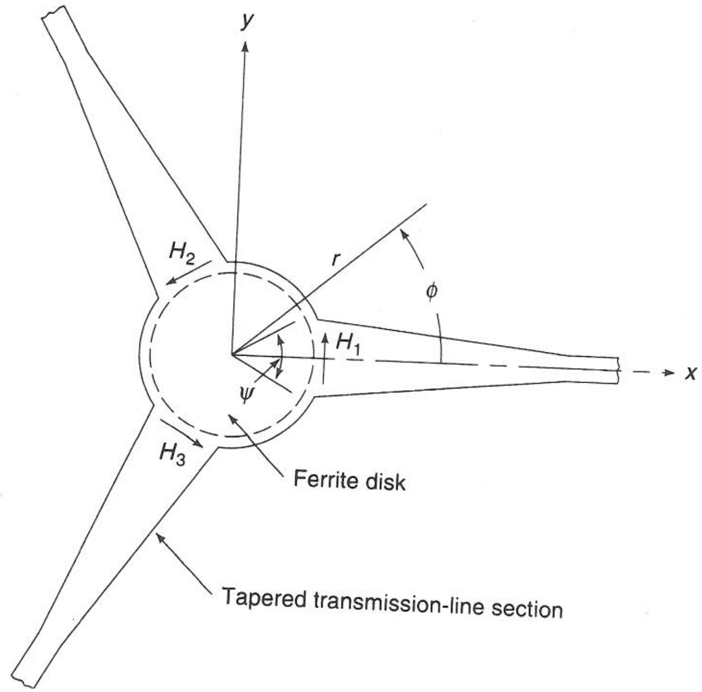
Junction Circulator

- Boundary Conditions

1. Continuous fields with TEM waves at ports
2. No E_z field for $\phi = \frac{4\pi}{3}$ (isolated port)
   - This ensures no coupling at port 3 – only coupling from 1→2
3. No H field outside coupling regions $(r = a)$

$$H_\phi = \begin{cases} H_1 & \frac{\psi}{2} < \phi < \frac{\psi}{2} \\ H_2 & \frac{2\pi}{3} - \frac{\psi}{2} < \phi < \frac{2\pi}{3} + \frac{\psi}{2} \\ 3 & \frac{4\pi}{3} - \frac{\psi}{2} < \phi < \frac{4\pi}{3} + \frac{\psi}{2} \end{cases}$$

- In practice – enough normally to take care of the first **3-6** propagation modes



References

1. Collin, foundations for microwave Engineering, 2000
2. Pozar, “Microwave Engineering”, 2004
3. Lax, Button, “microwave ferrites and ferrimagnetics”, 1962