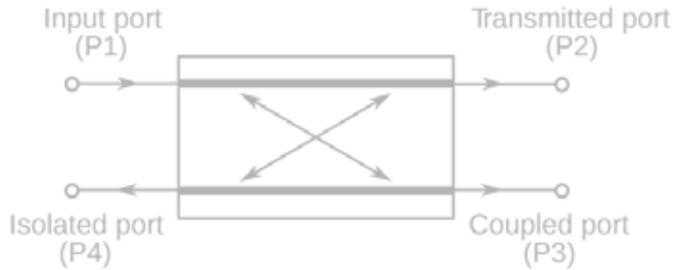
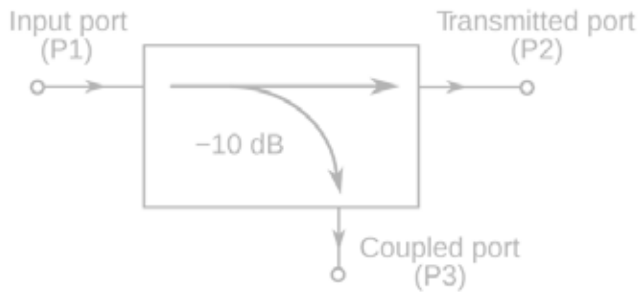
Microwave Engineering MCC121, 7.5hec, 2014

Lecture 10
Couplers





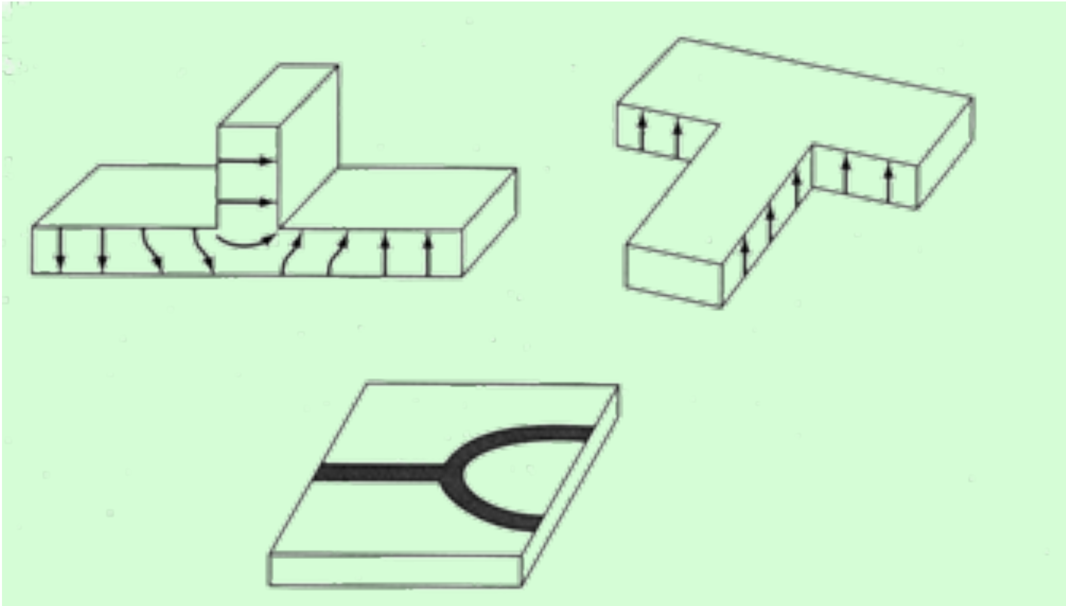
State-of-the-art
Challenging
Stimulating
Rewarding

Outline

- Passive microwave devices (7)
  - directional couplers (7.5-7.9)
  - hybrids
  - power dividers

Info

- 2/12 L10 Directional couplers (Ch7)
- 9/12 L11 Periodic structures (Ch8.1)
- 11/12 L12 (KY) Microwave measurement techniques
- 16/12 L13 (VD) Isolators, circulators, ferrites (Ch 9)
- 18/12 L14 Reserve (excursion?)



Power dividers

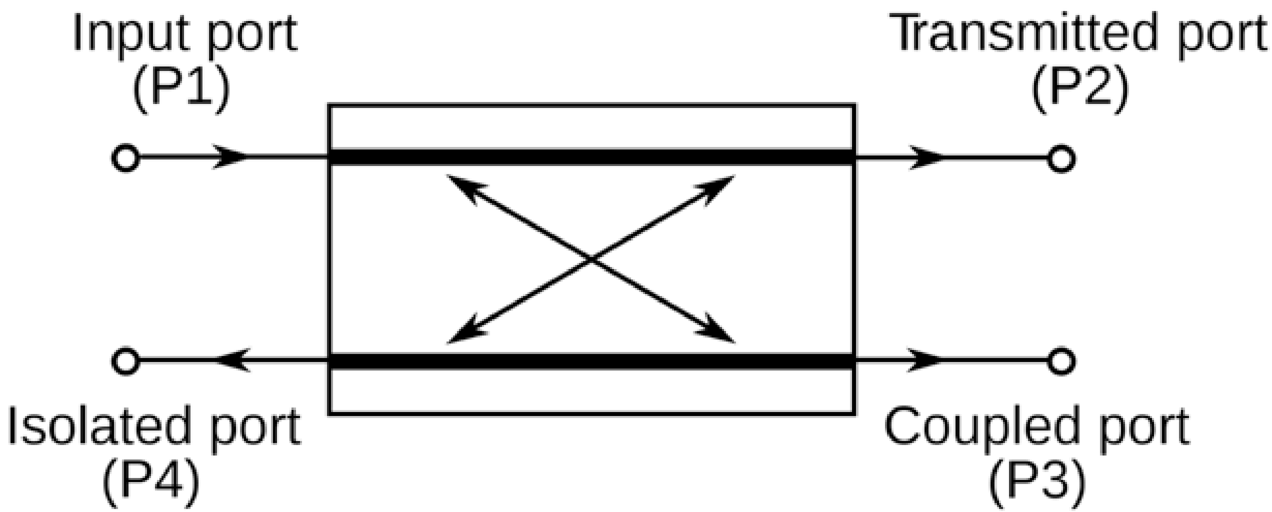
Reciprocal 3-port : Power dividers or combiners

- Power divider is used to divide input power among several outputs
- We want:
  - reciprocal
  - lossless
  - matched

$$[s] = \begin{bmatrix} 0 & s_{12} & s_{13} \\ s_{12} & 0 & s_{23} \\ s_{13} & s_{23} & 0 \end{bmatrix}$$

Impossible!
must relax one of the conditions

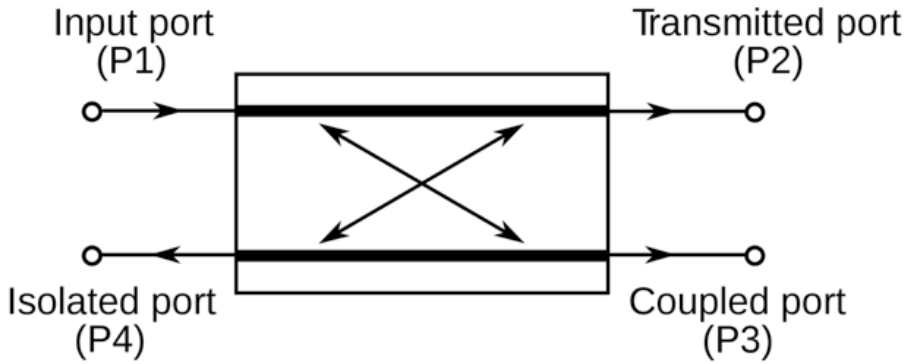
- *On white board: Derive properties for a passive reciprocal 3-port*

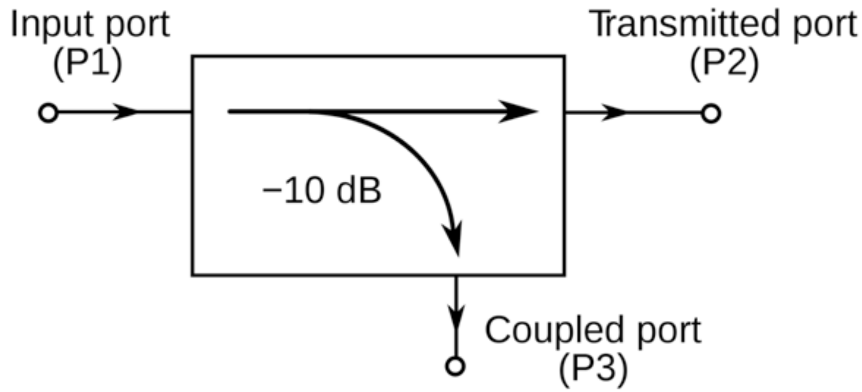


Directional couplers

Properties

- All ports matched
- Ex) Incident power at port 1 couples to port 2 and 3, but not into port 4. Hence, ports 1 & 4 are uncoupled





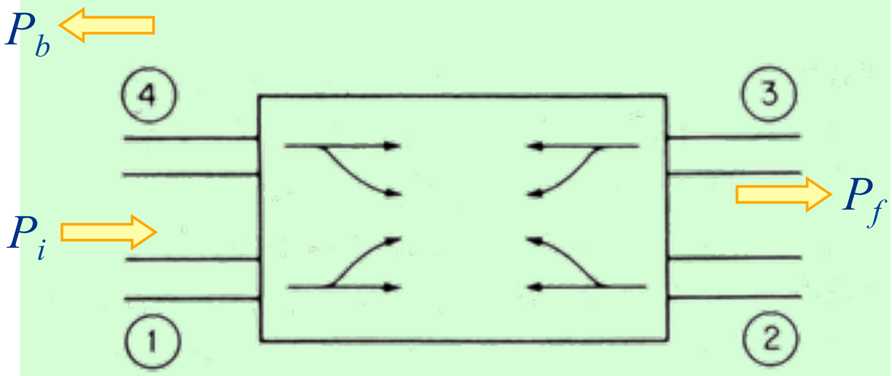
Applications

- Power monitoring
- Impedance measurement (reflectivity)
- Power dividers (distributing networks)

directional coupler parameters

- Coupling
- Directivity (difference in signal levels in dB between the coupled port and isolated port)
- Isolation (difference in signal levels in dB between the input port and isolated port)

Directional couplers



$P_{in} \Rightarrow 1, P_{out} \Rightarrow 2,3$ port 4 is isolated

$P_{in} \Rightarrow 2, P_{out} \Rightarrow 1,4$ port 3 is isolated

$P_{in} \Rightarrow 3, P_{out} \Rightarrow 1,4$ port 2 is isolated

$P_{in} \Rightarrow 4, P_{out} \Rightarrow 2,3$ port 1 is isolated

f: forward

b: backward

i: incident

We define:

Coupling: $C = 10\log\frac{P_i}{P_f}$

Directivity: $D = 10\log\frac{P_f}{P_b}$

Isolation: $I = 10\log\frac{P_i}{P_b}$

$D = I - C$

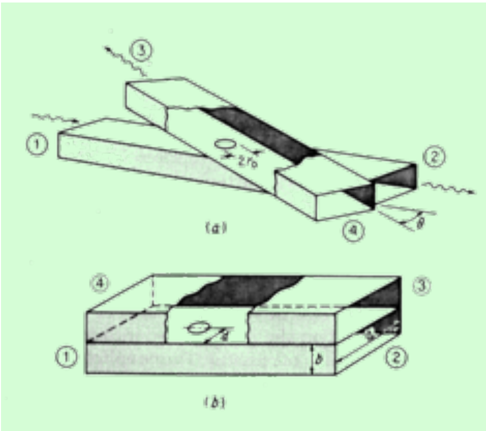
(in dB)

C=3 dB, I=30 dB, D=27 dB??

C=30 dB, I=30 dB, D=0 dB??

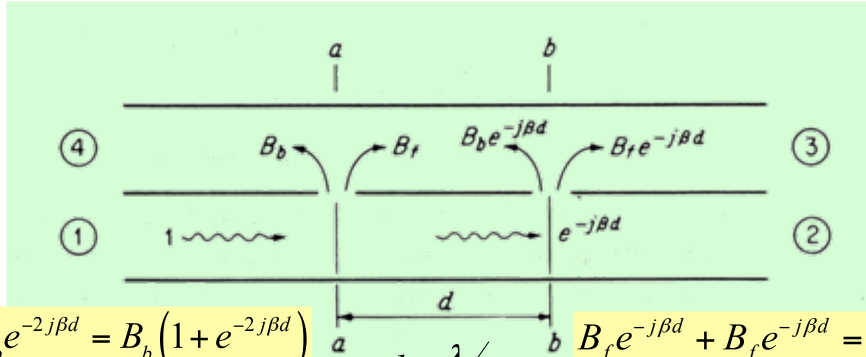
- *On white board: S-matrix for an ideal directional coupler (infinite isolation and perfectly matched).*

Example of couplers



Bethe hole coupler

Two hole coupler



$$B_b + B_b e^{-2j\beta d} = B_b \underbrace{\left(1 + e^{-2j\beta d}\right)}_{0}$$

$$2\beta d = n\pi, \quad n = 1,3,5,\ldots$$

$$d = \lambda/4$$

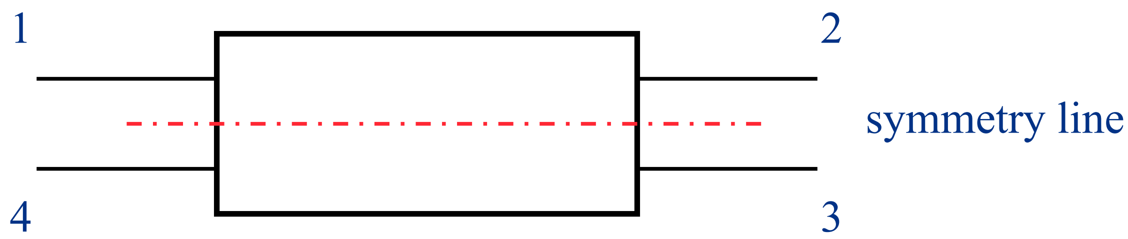
$$B_f e^{-j\beta d} + B_f e^{-j\beta d} = 2B_f e^{-j\beta d}$$

$$C = -20\log 2\left|B_f\right|$$

$$D = 20\log\frac{2\left|B_f\right|}{\left|B_b\right|\left|1 + e^{-2j\beta d}\right|} = 20\log\frac{\left|B_f\right|}{\left|B_b\right|} + 20\log\left|\frac{1}{\cos\beta d}\right|$$

Even and odd mode method

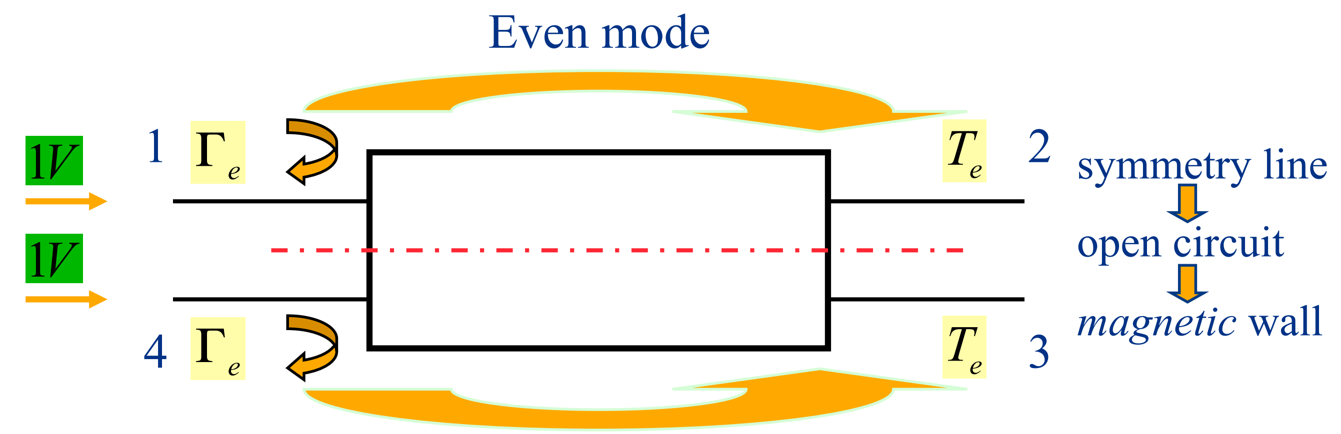
Consider a **linear**, **reciprocal** 4-port with a **symmetry** line as marked

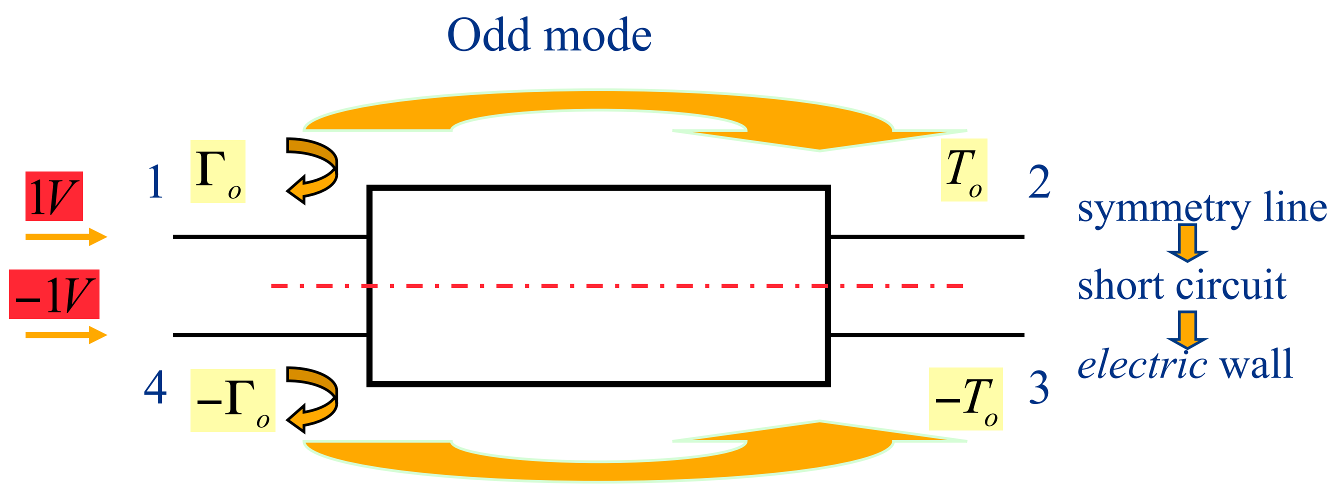


We will analyze this circuit by using the *even* and *odd mode* method. The method is based on two excitations: even and odd, applied to the ports on opposite sides of the symmetry line (in our case port 1 and 4). The even excitation corresponds to two voltages equal in amplitude and phase, e.g. +1V. The odd excitation corresponds to two voltages equal in amplitude but with 180° phase difference (+1V, and –1 V).

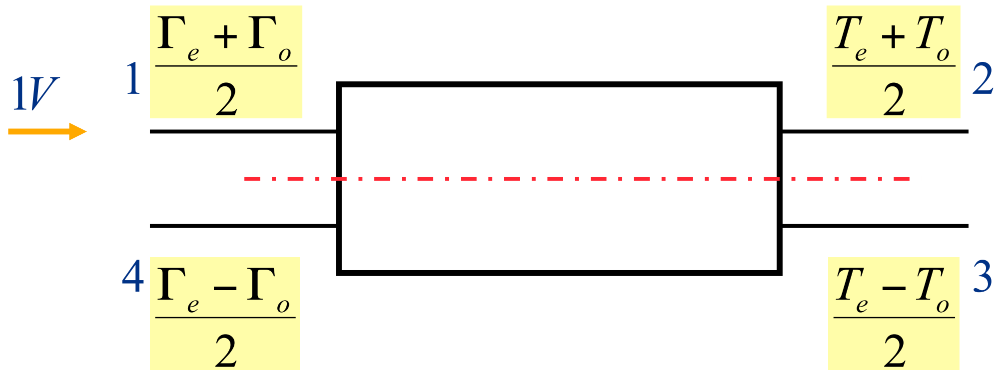
By applying the **even excitation** to the ports 1(+1 V), and 4(+1 V) the symmetry line will act as an **open circuit** or as we say **magnetic wall**.

By applying the **odd excitation** to the ports 1(+1V), and 4(-1V) the symmetry line will act as a **short circuit** or as we say **electric wall**.


Even mode
1V
1V
1
4
Γe
Te
2
3
symmetry line
open circuit
magnetic wall


Odd mode
1V
–1V
1
4
Γo
–Γo
To
–To
2
3
symmetry line
short circuit
electric wall

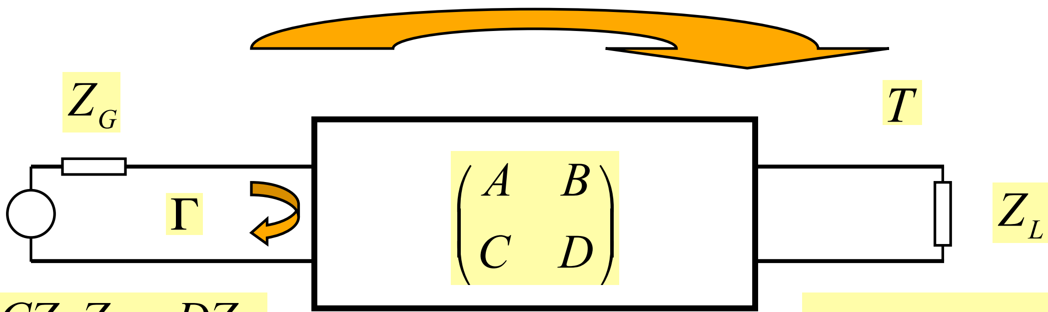
We superimpose now both excitations:



We have only excitation in port 1 and can calculate the reflected and transmitted waves in all ports.

This means that the analysis of a reciprocal, linear 4-port with a symmetry property can be performed by analyzing two 2-ports in two excitation modes and superposition of the results.

Γ and T for the 2-ports can be easily calculated from the cascade matrix analysis

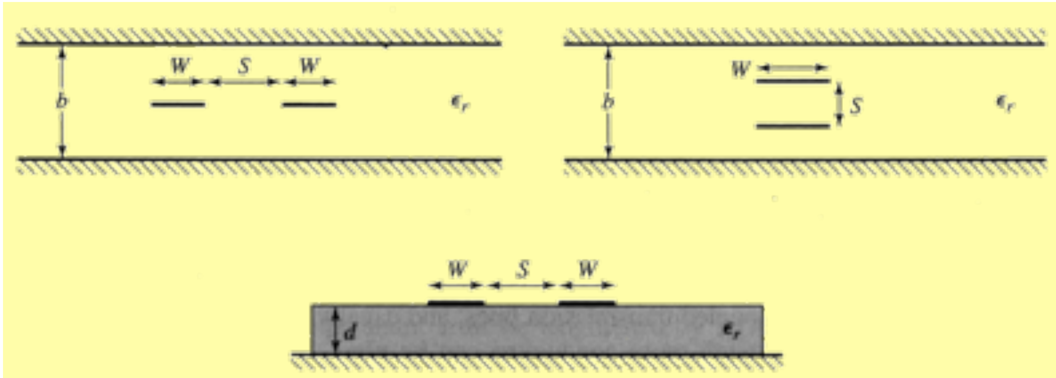


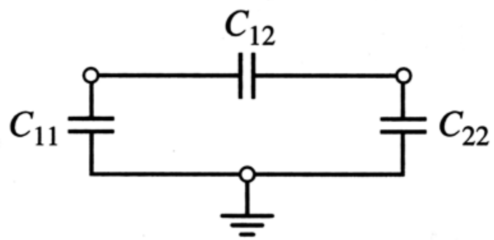
$$\Gamma = \frac{AZ_L + B - CZ_GZ_L - DZ_G}{AZ_L + B + CZ_GZ_L + DZ_G}$$

$$T = \frac{Z_G + Z_L}{AZ_L + B + CZ_GZ_L + DZ_G}$$

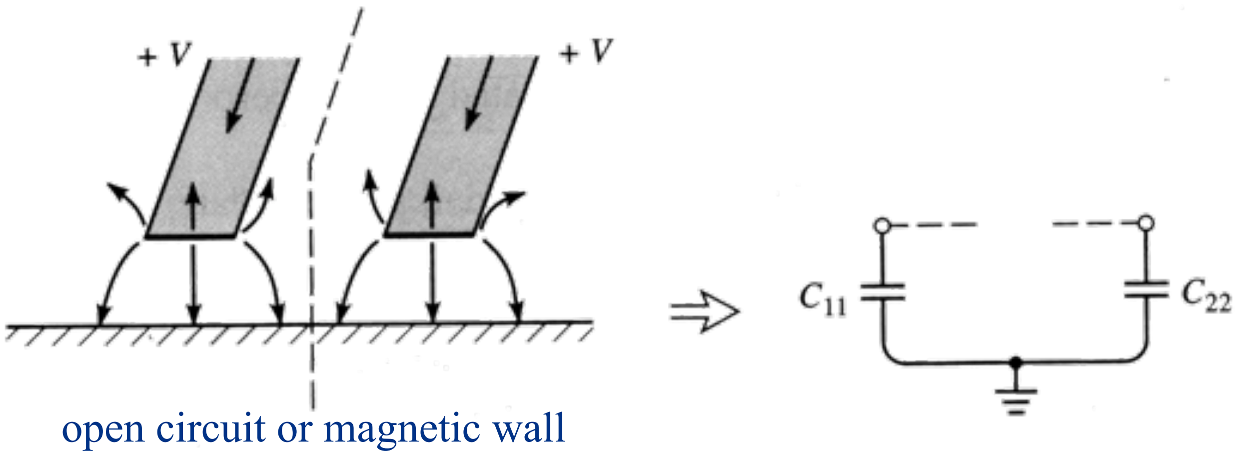
Coupled line theory

TEM
propagation





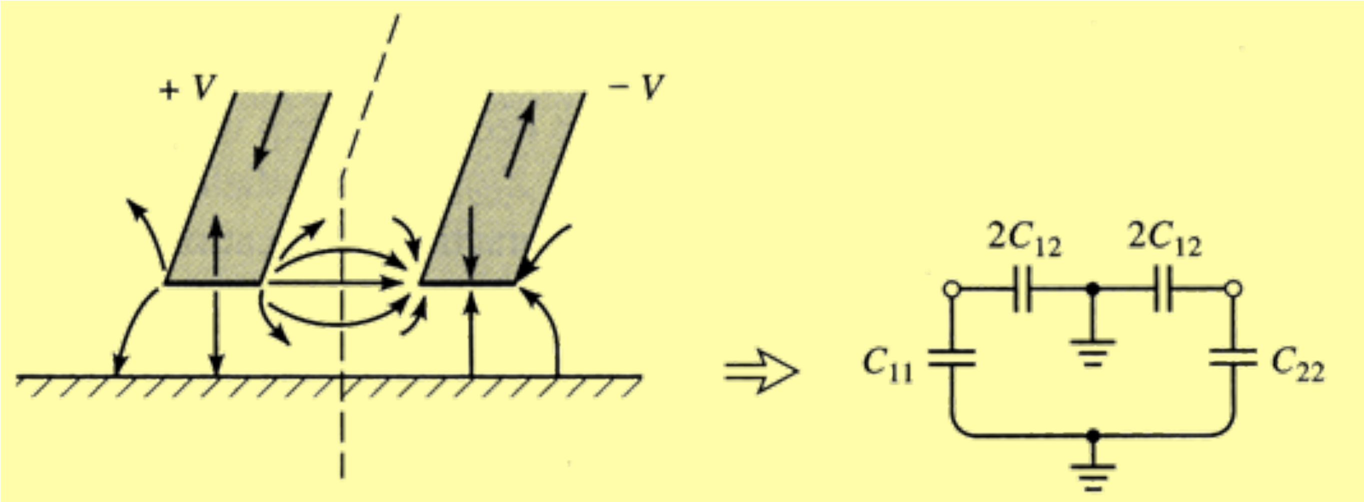
Even mode



$$C_e = C_{11} = C_{22}$$

$$Z_e = \sqrt{\frac{L}{C_e}} = \frac{1}{vC_e}$$

odd mode

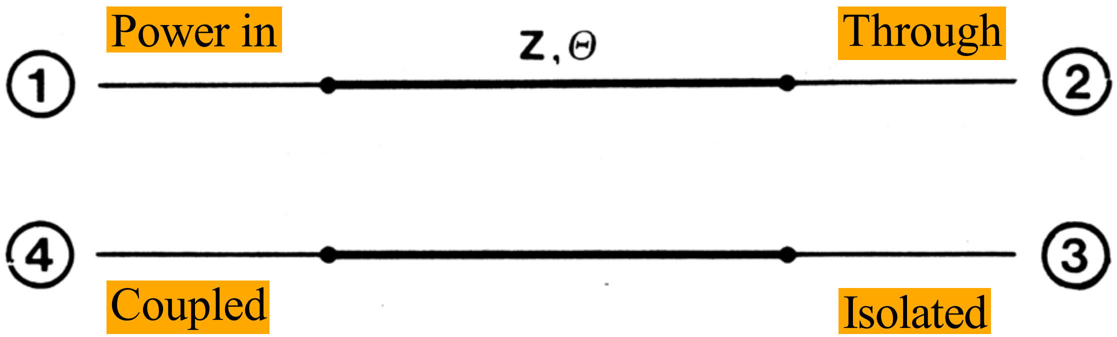


short circuit or electric wall

$$C_o = C_{11} + 2C_{12} = C_{22} + 2C_{12}$$

$$Z_o = \sqrt{\frac{L}{C_o}} = \frac{1}{vC_o}$$

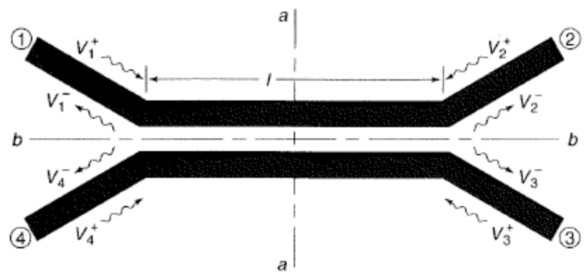
Coupled Transmission Line directional coupler (CTL coupler)



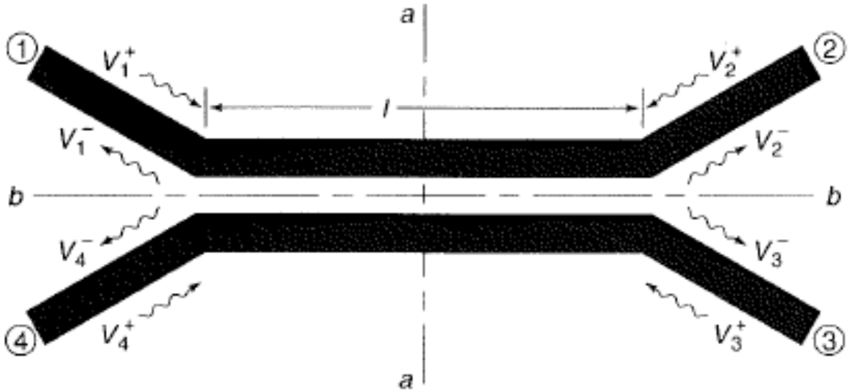
We see that the power is coupled to port 4; thus the power to the coupled port is propagating in the opposite direction as compared to the direct (through) port.

Another name of the coupler is Backward Wave Directional Coupler.

Analysis by the even and odd mode method.

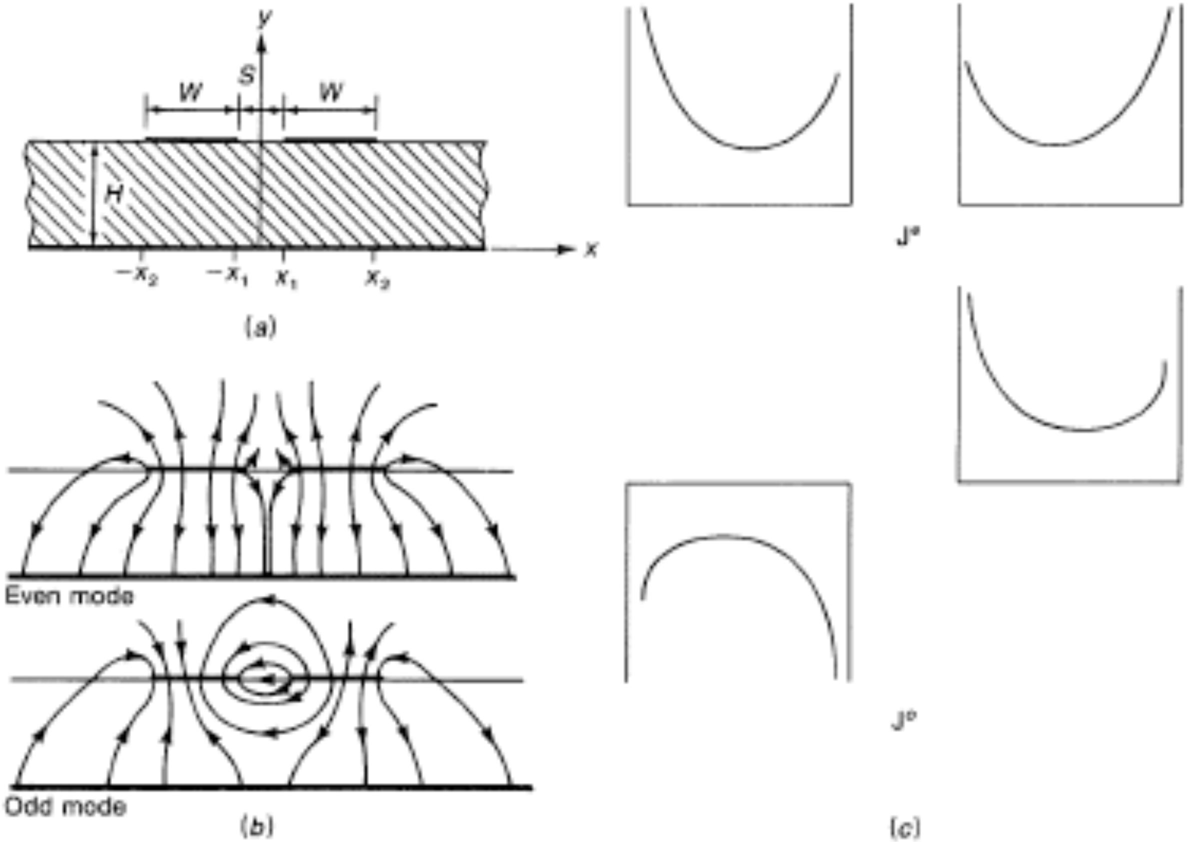


- *On white board: coupled line directional coupler.*



Coupled MS lines

Current distribution



Even mode : Strips being at the same potential, V.

Odd mode : Strips being at the opposite potential, +V and -V.

Coupled MS lines - dispersion

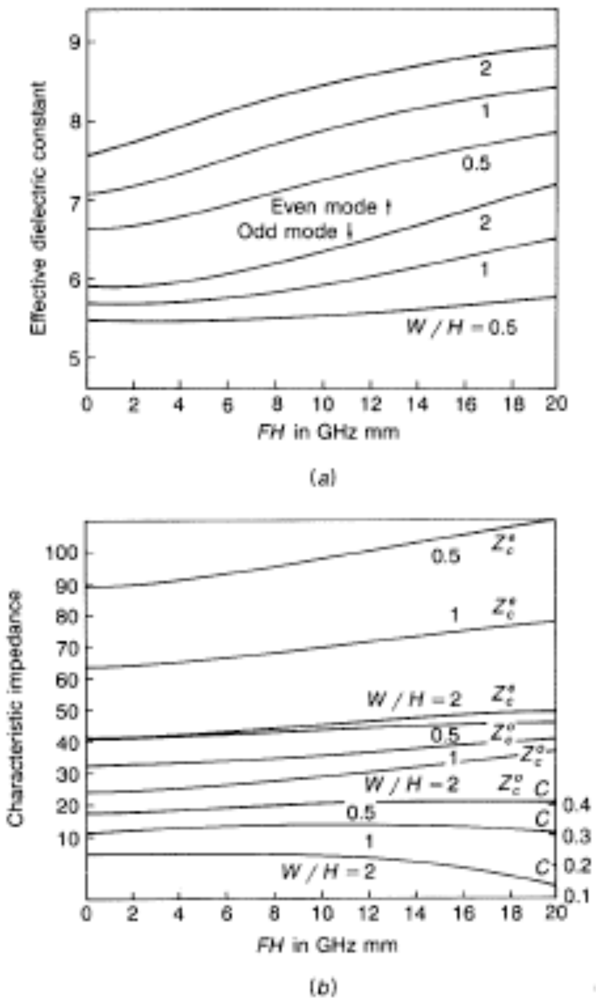


FIGURE 3.31
Dispersion characteristics of a coupled microstrip line on an alumina substrate. $S/H = 0.25$, $\epsilon_r = 9.7$. (*a*) Even- and odd-mode effective dielectric constant; (*b*) even- and odd-mode characteristic impedance and coupling coefficient C.

Used in directional couplers

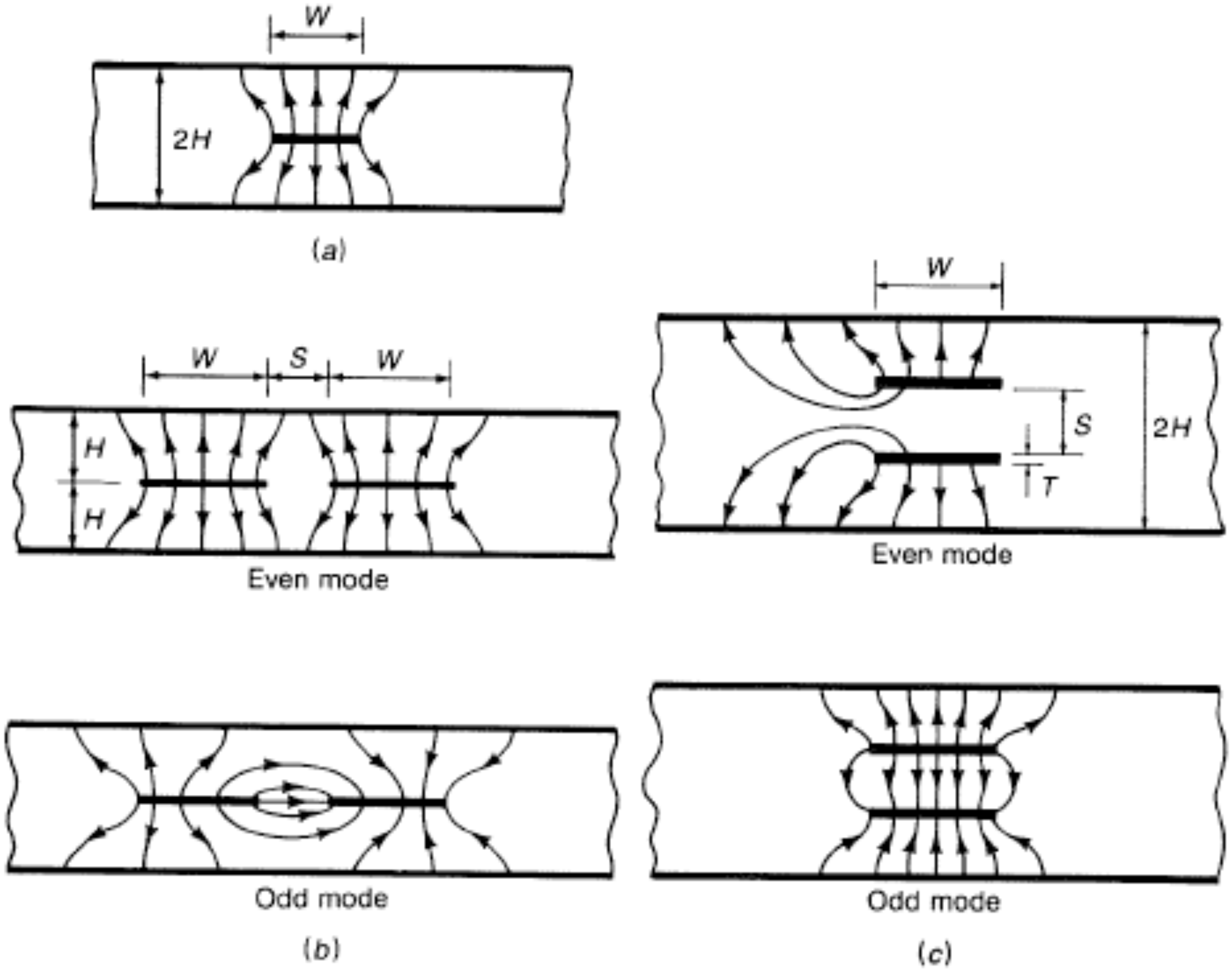
Important parameters:

-Even- and odd-mode effective dielectric constants

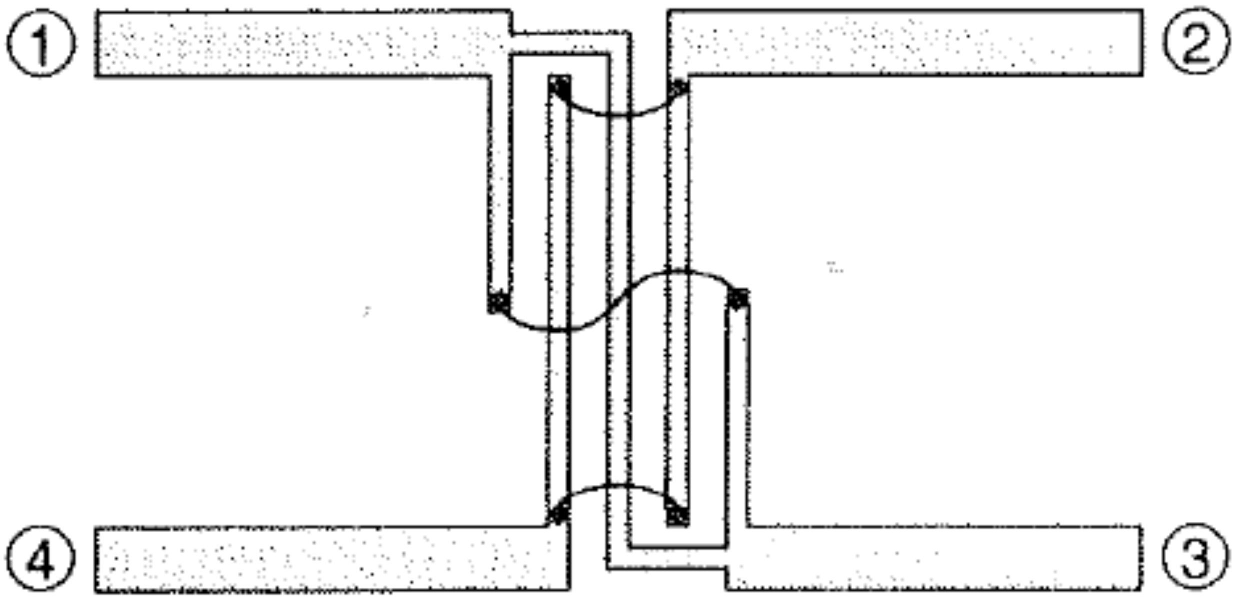
-Even- and odd-mode characteristic impedances

$$C = \frac{Z_c^e - Z_c^o}{Z_c^e + Z_c^o}$$

Coupled strip lines



Lange coupler



Stronger coupling
Compact and broadband (MMICs)

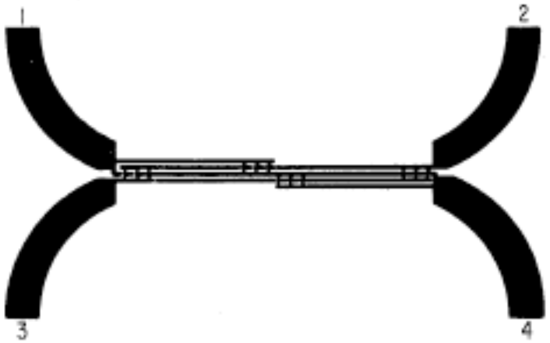


Fig. 1. Interdigitated 3-dB coupler.

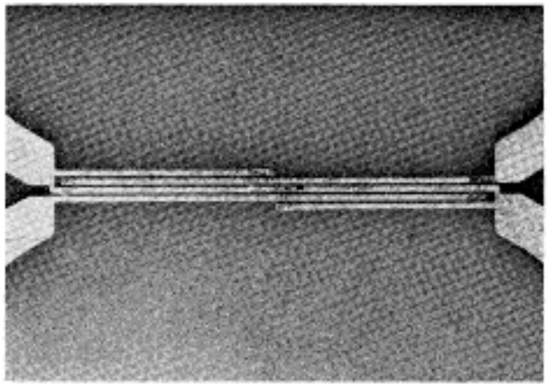


Fig. 2. Interdigitated quadrature hybrid.

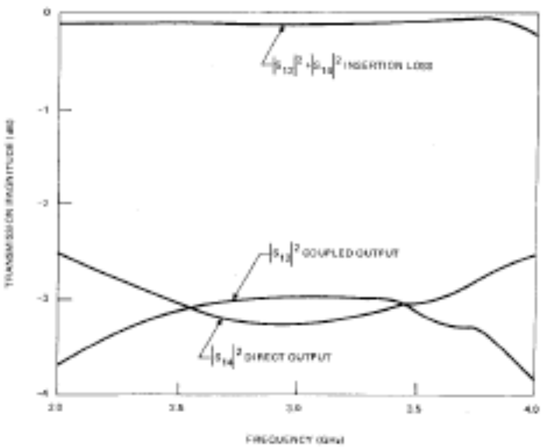
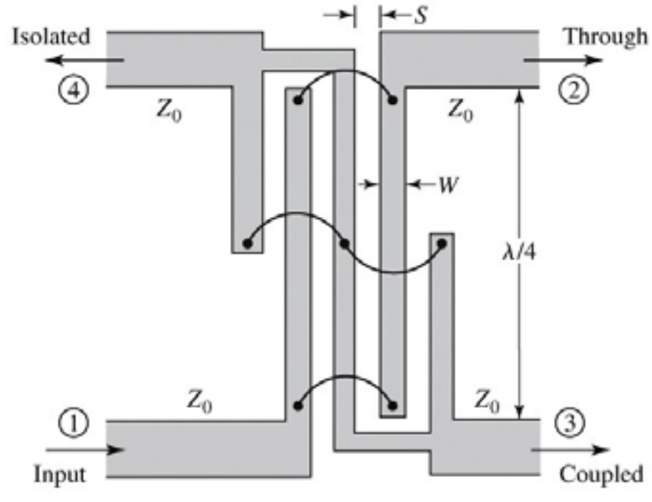
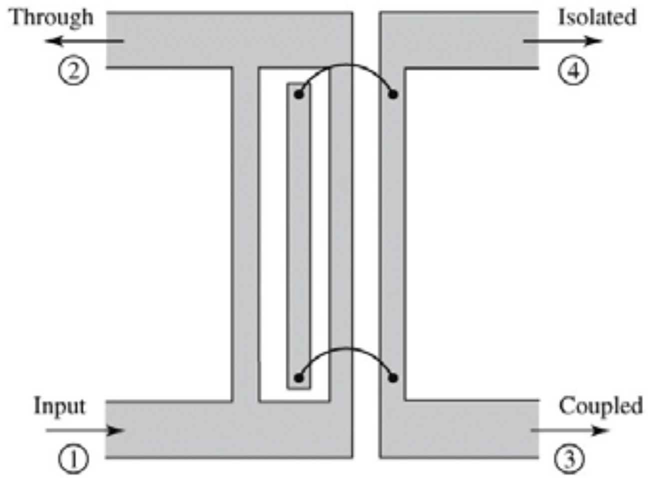


Fig. 3. Coupler response and insertion loss.



(a)

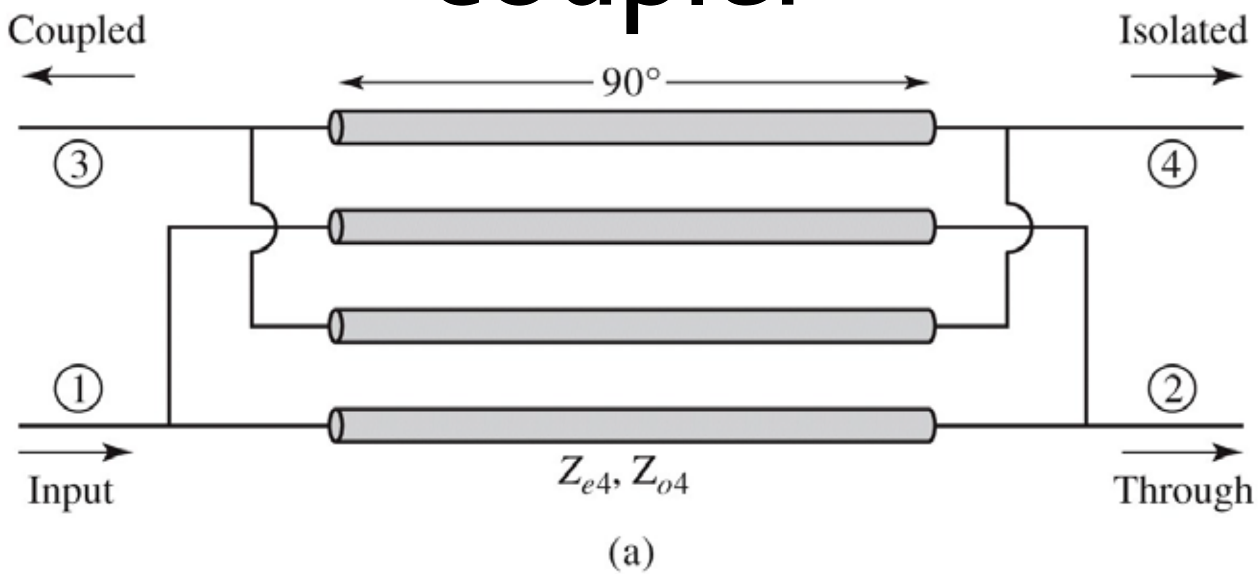


(b)

Figure 7.38

J. Lange, "Interdigitated Stripline Quadrature Hybrid (Correspondence)," IEEE Transactions on Microwave Theory and Techniques, vol. 17, no. 12, pp. 1150–1151, 1969.

Eq. circuit for unfolded Lange coupler



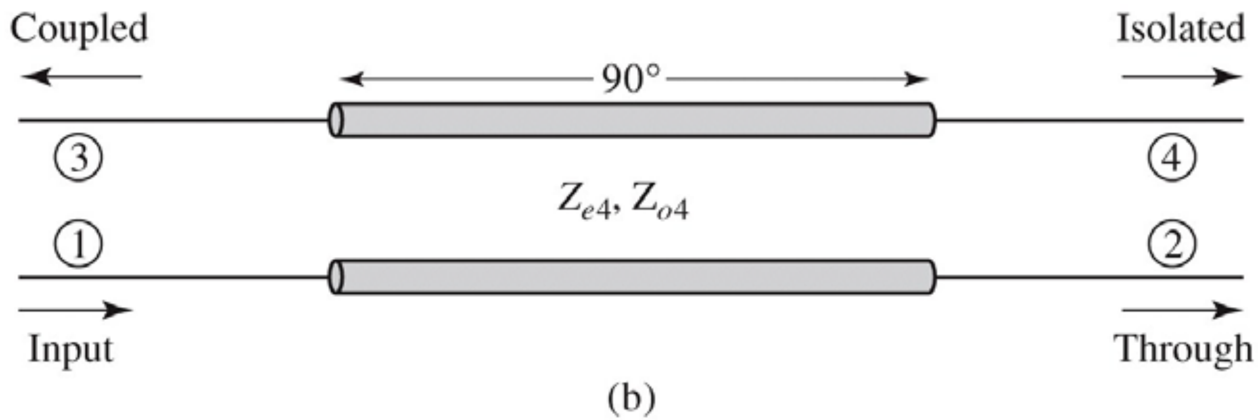
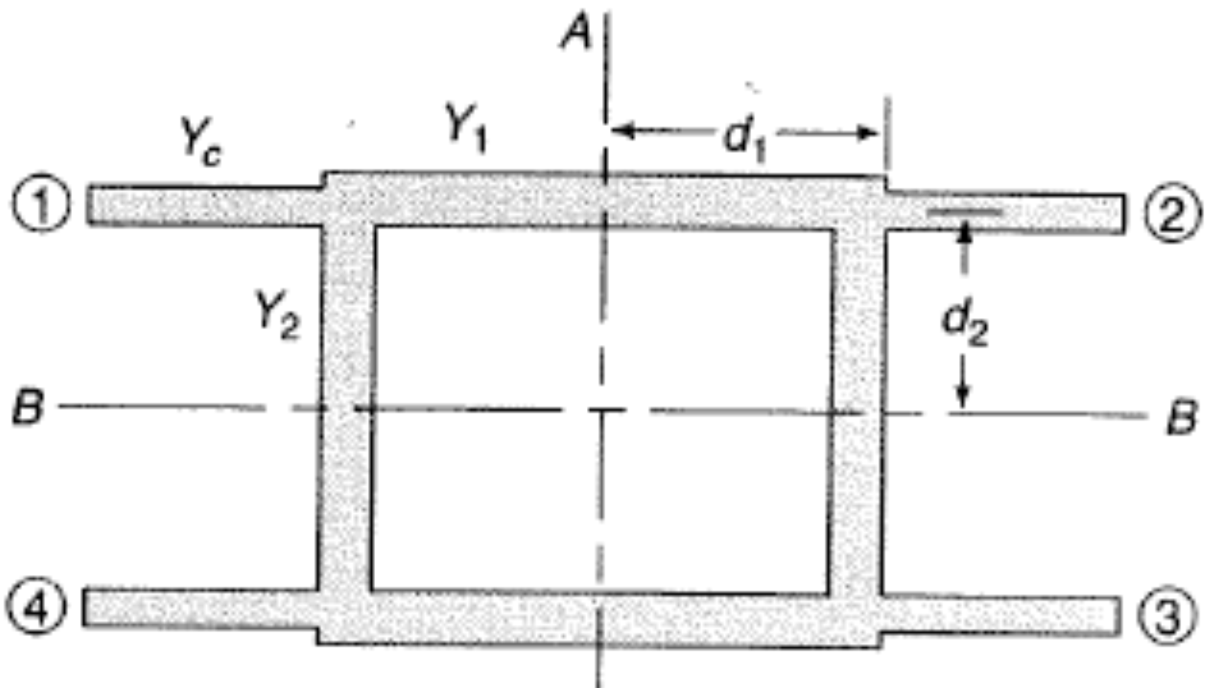
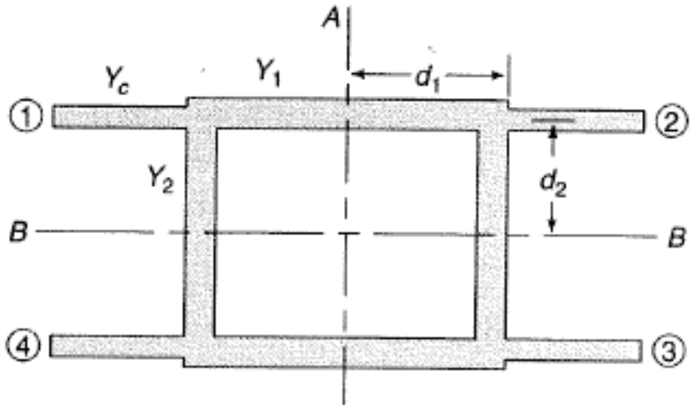


Figure 7.39

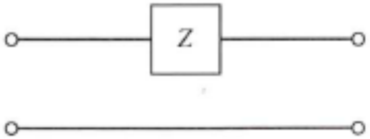
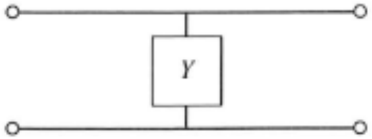
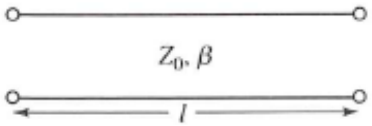
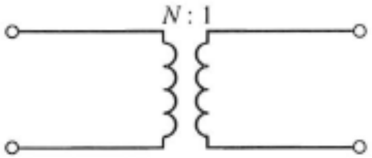
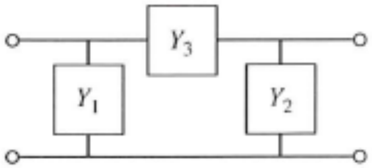
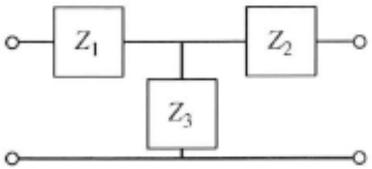


Branch line coupler

- *On white board: Branch-line directional coupler.*



ABCD parameters of some useful two-ports

Circuit	ABCD Parameters	
	$A = 1$ $C = 0$	$B = Z$ $D = 1$
	$A = 1$ $C = Y$	$B = 0$ $D = 1$
	$A = \cos \beta l$ $C = jY_0 \sin \beta l$	$B = jZ_0 \sin \beta l$ $D = \cos \beta l$
	$A = N$ $C = 0$	$B = 0$ $D = \frac{1}{N}$
	$A = 1 + \frac{Y_2}{Y_3}$ $C = Y_1 + Y_2 + \frac{Y_1 Y_2}{Y_3}$	$B = \frac{1}{Y_3}$ $D = 1 + \frac{Y_1}{Y_3}$
	$A = 1 + \frac{Z_1}{Z_3}$ $C = \frac{1}{Z_3}$	$B = Z_1 + Z_2 + \frac{Z_1 Z_2}{Z_3}$ $D = 1 + \frac{Z_2}{Z_3}$

Input impedance of terminated lossless transmission lines:

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan \beta\ell}{Z_0 + jZ_L \tan \beta\ell} \quad \text{(arbitrary load)}$$

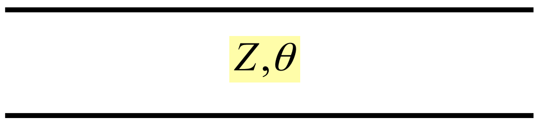
$$Z_{in} = jZ_0 \tan \beta\ell \quad \text{(short-circuited line)}$$

$$Z_{in} = -jZ_0 \cot \beta\ell \quad \text{(open-circuited line)}$$

from Pozar, "microwave engineering"

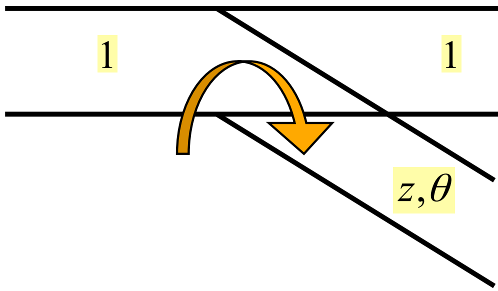
Cascade matrix for some common circuits

Transmission line



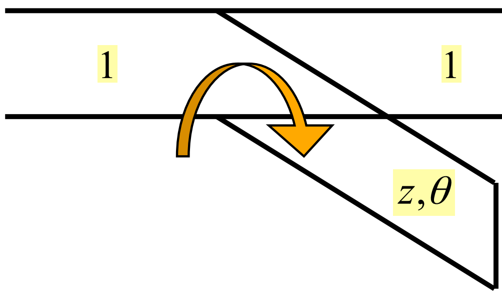
$$\begin{bmatrix} \cos\theta & jZ\sin\theta \\ jY\sin\theta & \cos\theta \end{bmatrix}$$

Parallel coupled open stub



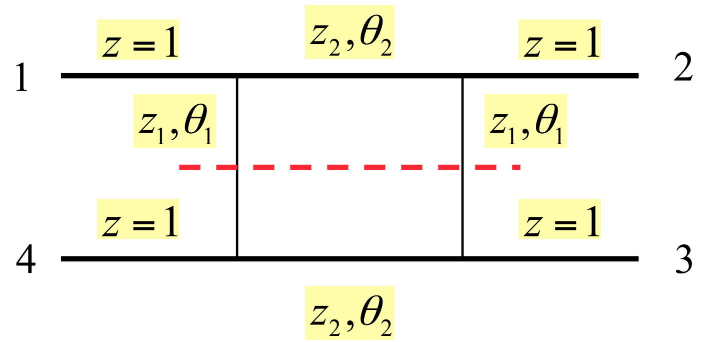
$$\begin{bmatrix} 1 & 0 \\ jy\tan\theta & 1 \end{bmatrix}$$

Parallel coupled shorted stub

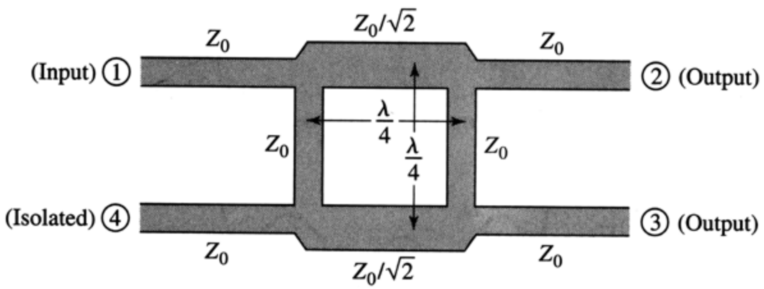


$$\begin{bmatrix} 1 & 0 \\ -jy\text{cotan}\theta & 1 \end{bmatrix}$$

Branch line coupler



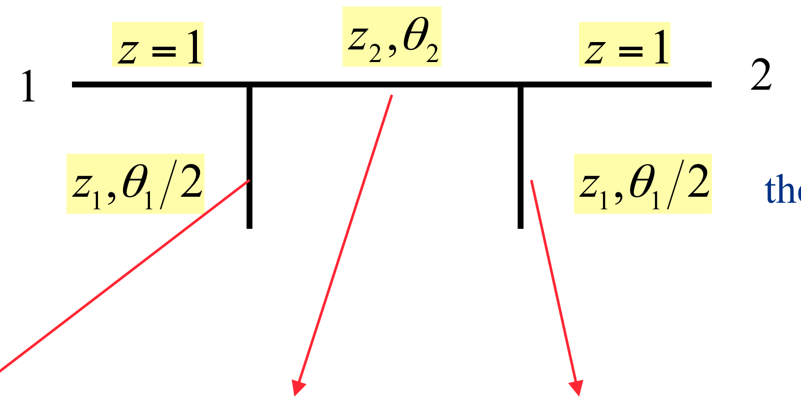
symmetry line



The analysis will be performed by using the even and odd mode method

Even mode

Even mode



the lines are open

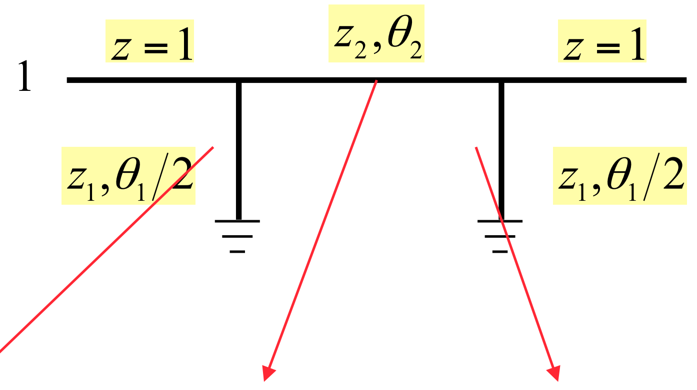
$$\begin{bmatrix} A_e & B_e \\ C_e & D_e \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ jy_1 \tan\frac{\theta_1}{2} & 1 \end{bmatrix} \begin{bmatrix} \cos\theta_2 & jz_2 \sin\theta_2 \\ jy_2 \sin\theta_2 & \cos\theta_2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ jy_1 \tan\frac{\theta_1}{2} & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} -z_2 y_1 \tan\frac{\theta_1}{2}\sin\theta_2 + \cos\theta_2 & jz_2 \sin\theta_2 \\ j\left(2y_1 \cos\theta_2 \tan\frac{\theta_1}{2} + y_2 \sin\theta_2 - z_2 y_1^2 \tan^2\frac{\theta_1}{2}\sin\theta_2\right) & -z_2 y_1 \tan\frac{\theta_1}{2}\sin\theta_2 + \cos\theta_2 \end{bmatrix}$$

$$\Gamma_e = \frac{A_e + B_e - C_e - D_e}{A_e + B_e + C_e + D_e} \qquad T_e = \frac{2}{A_e + B_e + C_e + D_e}$$

Odd mode

Odd mode



$$\begin{bmatrix} A_o & B_o \\ C_o & D_o \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -jy_1\text{cotan}\frac{\theta_1}{2} & 1 \end{bmatrix} \begin{bmatrix} \cos\theta_2 & jz_2\sin\theta_2 \\ jy_2\sin\theta_2 & \cos\theta_2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -jy_1\text{cotan}\frac{\theta_1}{2} & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} z_2y_1\text{cotan}\frac{\theta_1}{2}\sin\theta_2 + \cos\theta_2 & jz_2\sin\theta_2 \\ j\left(-2y_1\cos\theta_2\text{cotan}\frac{\theta_1}{2} + y_2\sin\theta_2 - z_2y_1^2\text{cotan}^2\frac{\theta_1}{2}\sin\theta_2\right) & z_2y_1\text{cotan}\frac{\theta_1}{2}\sin\theta_2 + \cos\theta_2 \end{bmatrix}$$

$$\Gamma_o = \frac{A_o + B_o - C_o - D_o}{A_o + B_o + C_o + D_o} \qquad T_o = \frac{2}{A_o + B_o + C_o + D_o}$$

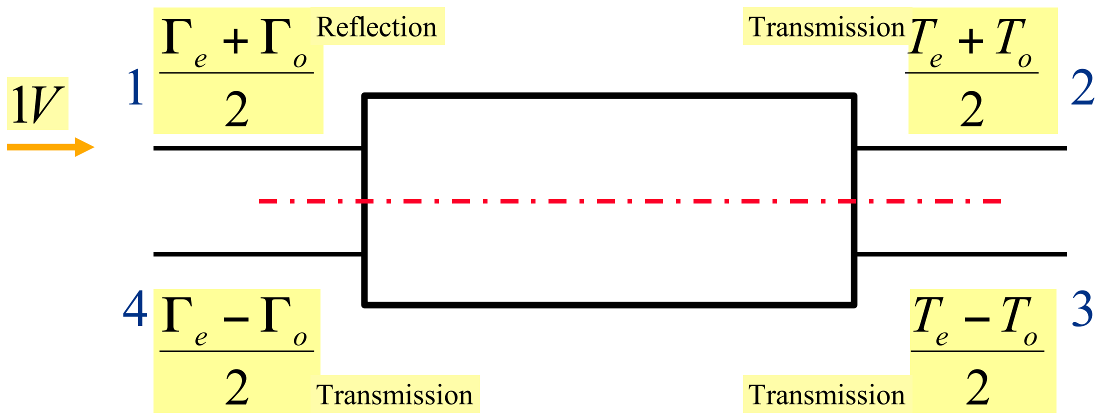
We now select

$$\theta_1\big|_{f=f_0} = \frac{\pi}{2} \quad \theta_2\big|_{f=f_0} = \frac{\pi}{2}$$

Thus, we obtain at center frequency

$$\Gamma_e = \frac{j\left(z_2 - y_2 + z_2 y_1^2\right)}{-2z_2 y_1 + j\left(z_2 + y_2 - z_2 y_1^2\right)}; \quad T_e = \frac{2}{-2z_2 y_1 + j\left(z_2 + y_2 - z_2 y_1^2\right)}$$

$$\Gamma_o = \frac{j\left(z_2 - y_2 + z_2 y_1^2\right)}{2z_2 y_1 + j\left(z_2 + y_2 - z_2 y_1^2\right)}; \quad T_o = \frac{2}{2z_2 y_1 + j\left(z_2 + y_2 - z_2 y_1^2\right)}$$



The coupler will be perfectly matched if $\left.\frac{\Gamma_e + \Gamma_o}{2}\right|_{f=f_0} = 0$

This also means, that $B_e = C_e$ and $B_o = C_o$ and $\left.\frac{\Gamma_e - \Gamma_o}{2}\right|_{f=f_0} = 0$

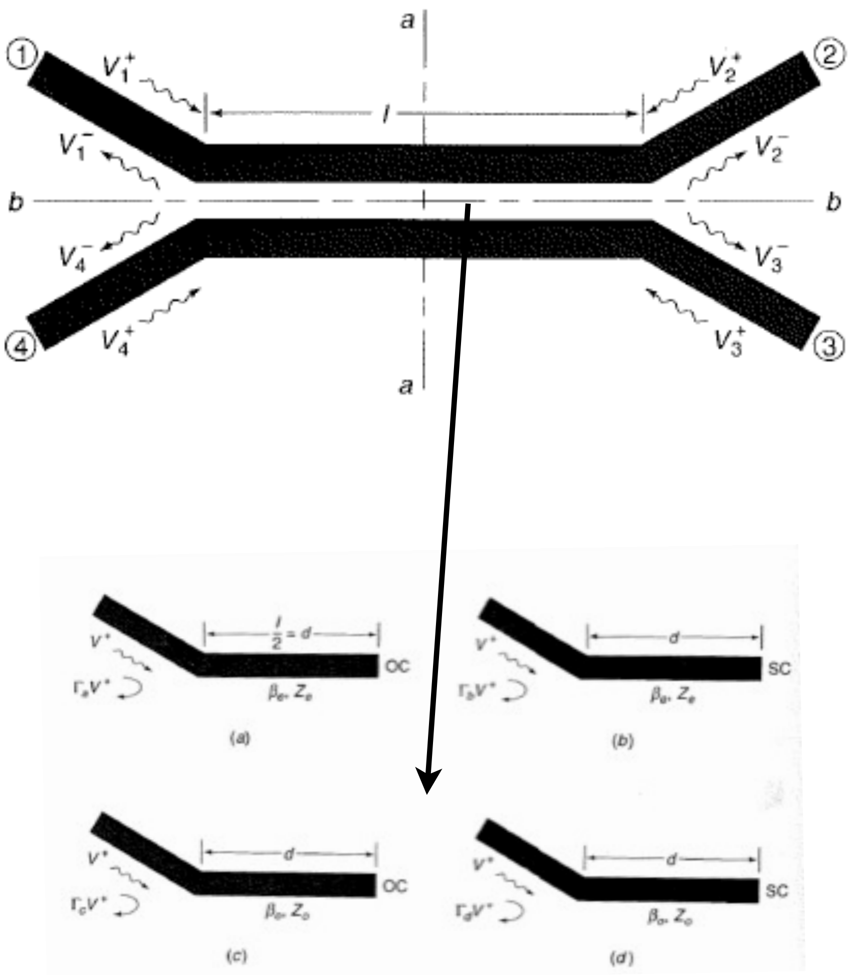
Perfect isolation!!!!!

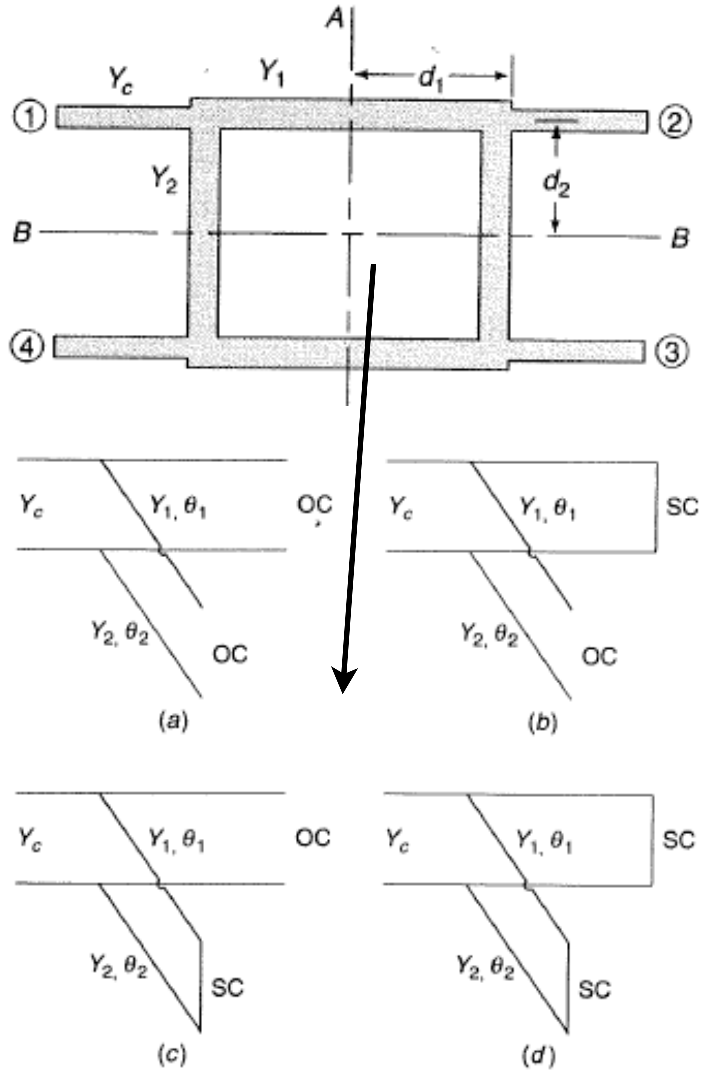
Coupling to port 3 is equal to $C = 20\log\frac{1}{\left|\frac{T_e - T_o}{2}\right|} = 20\log\frac{1}{\sqrt{1-z_2^2}}\,[dB]$ $\quad z_2 = \sqrt{\frac{10^{C/10}-1}{10^{C/10}}}$

From the condition for Γ_e $\quad z_1 = \sqrt{\frac{z_2^2}{1-z_2^2}}$

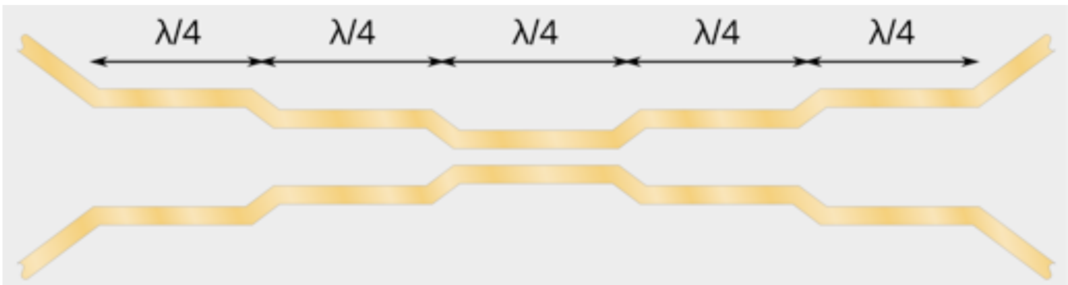
C[dB]	Z_1	Z_2
3	50.0Ω	35.3Ω
6	86.6Ω	43.3Ω
10	150.0Ω	47.4Ω

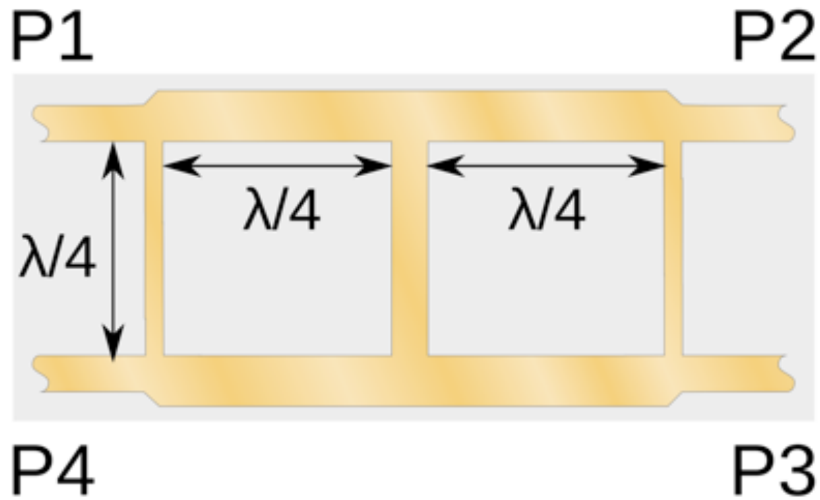
Symmetry one step further





Directional couplers





Several sections to improve bandwidth

Hybrids

- Hybrids are special cases of directional couplers, where the coupling factor is 3-dB
  - 90° phase difference or quadrature hybrid (symmetrical coupler)
  - 180° phase difference (antisymmetrical coupler)

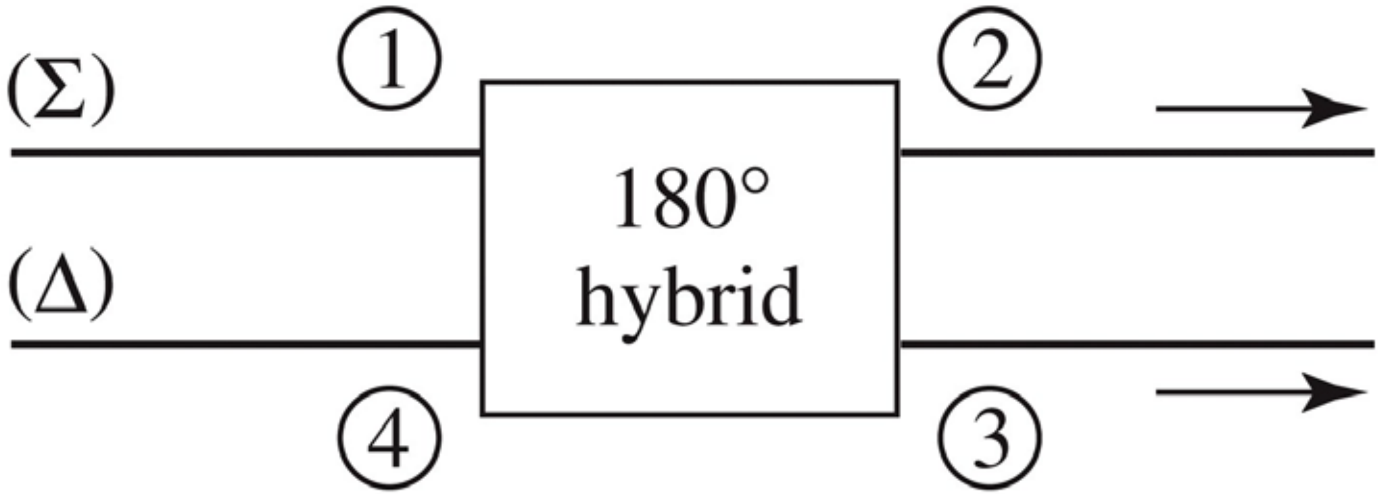
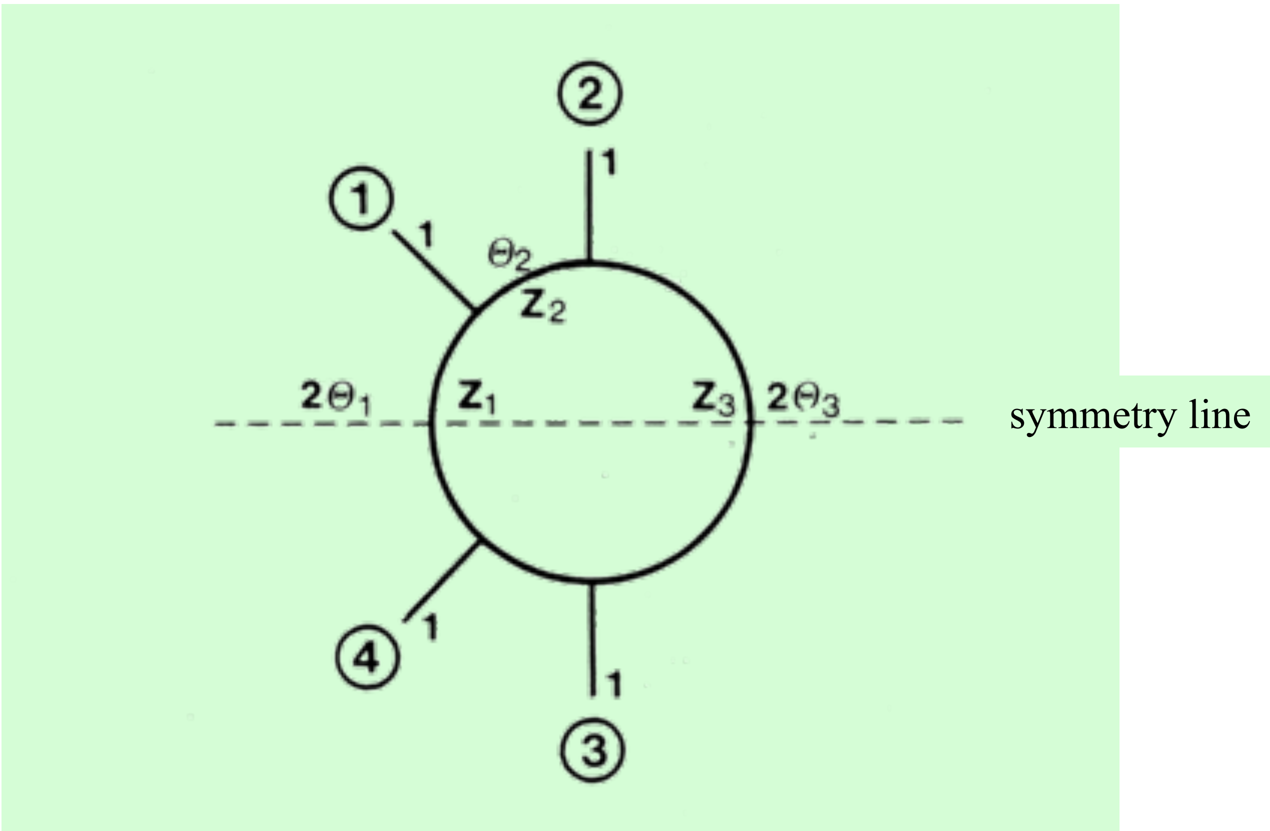
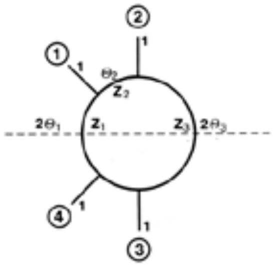


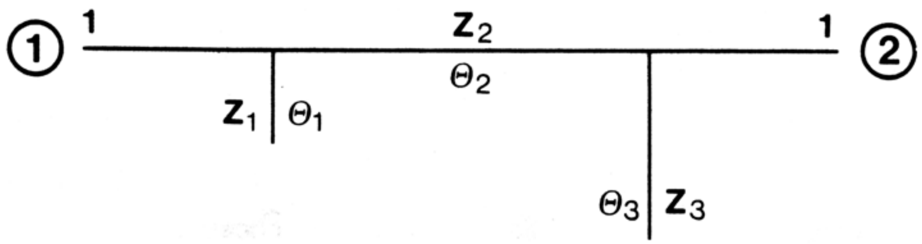
Figure 7.41

Rat race coupler





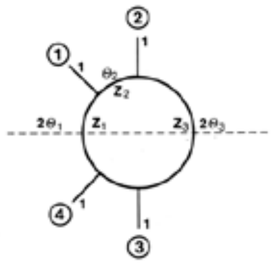
Rat Race, even mode



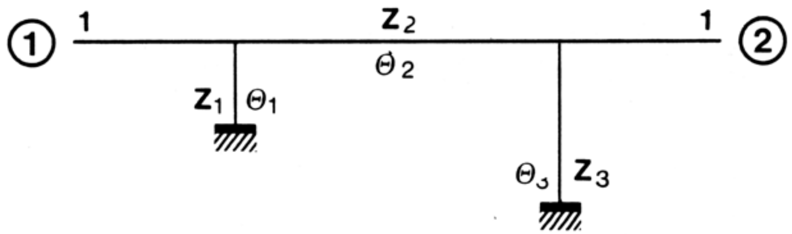
$$\begin{bmatrix} A_e & B_e \\ C_e & D_e \end{bmatrix} =$$

$$= \begin{bmatrix} \cos\theta_2 - z_2 y_3 \sin\theta_2 \tan\theta_3 & jz_2 \sin\theta_2 \\ j\left(y_1 \tan\theta_1 \cos\theta_2 + y_2 \sin\theta_2 - y_1 z_2 y_3 \tan\theta_1 \sin\theta_2 \tan\theta_3 + y_3 \cos\theta_2 \tan\theta_3\right) & \cos\theta_2 - z_2 y_1 \sin\theta_2 \tan\theta_1 \end{bmatrix}$$

$$\Gamma_e = \frac{A_e + B_e - C_e - D_e}{A_e + B_e + C_e + D_e}; \quad T_e = \frac{2}{A_e + B_e + C_e + D_e}$$



Rat Race, odd mode



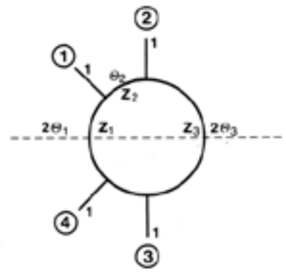
$$\begin{bmatrix} A_o & B_o \\ C_o & D_o \end{bmatrix} =$$

$$= \begin{bmatrix} \cos\theta_2 + z_2 y_3 \sin\theta_2 \operatorname{cotan}\theta_3 & j z_2 \sin\theta_2 \\ j\left(-y_1 \operatorname{cotan}\theta_1 \cos\theta_2 + y_2 \sin\theta_2 - y_1 z_2 y_3 \operatorname{cotan}\theta_1 \sin\theta_2 \operatorname{cotan}\theta_3 - y_3 \cos\theta_2 \operatorname{cotan}\theta_3\right) & \cos\theta_2 + z_2 y_1 \sin\theta_2 \operatorname{cotan}\theta_1 \end{bmatrix}$$

$$\Gamma_o = \frac{A_o + B_o - C_o - D_o}{A_o + B_o + C_o + D_o}; \quad T_o = \frac{2}{A_o + B_o + C_o + D_o}$$

Let us select $\begin{array}{l}\theta_1 = 45°, \theta_2 = 90°, \theta_3 = 135° \text{ all at } f = f_0 \\ Z_1 = Z_2 = Z_3 = \sqrt{2}\end{array}$, this will give us a 3 dB case

We obtain $\begin{array}{l}\Gamma_e = -j\frac{1}{\sqrt{2}}, T_e = -j\frac{1}{\sqrt{2}} \\ \Gamma_o = j\frac{1}{\sqrt{2}}, T_o = -j\frac{1}{\sqrt{2}}\end{array}$



The signal in port

(incident signal in port 1)

$$1: \quad \frac{\Gamma_e + \Gamma_o}{2} = 0 \Rightarrow \text{ perfect match}$$

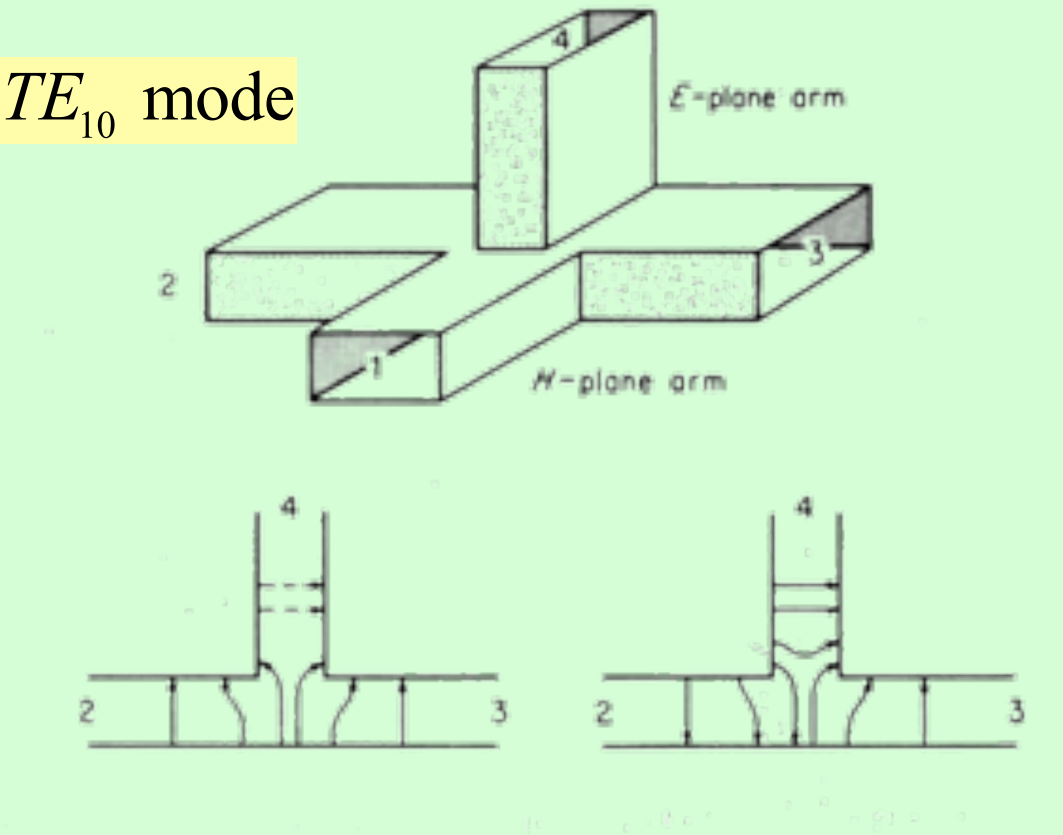
$$2: \quad \frac{T_e + T_o}{2} = -j\frac{1}{\sqrt{2}} \text{, half of the incoming signal with } 90° \text{ delay}$$

$$3: \quad \frac{T_e - T_o}{2} = 0, \text{ perfect isolation}$$

$$4: \quad \frac{\Gamma_e - \Gamma_o}{2} = -j\frac{1}{\sqrt{2}} \text{ half of the incoming signal with } 90° \text{ delay}$$

Magic T

TE_{10} mode



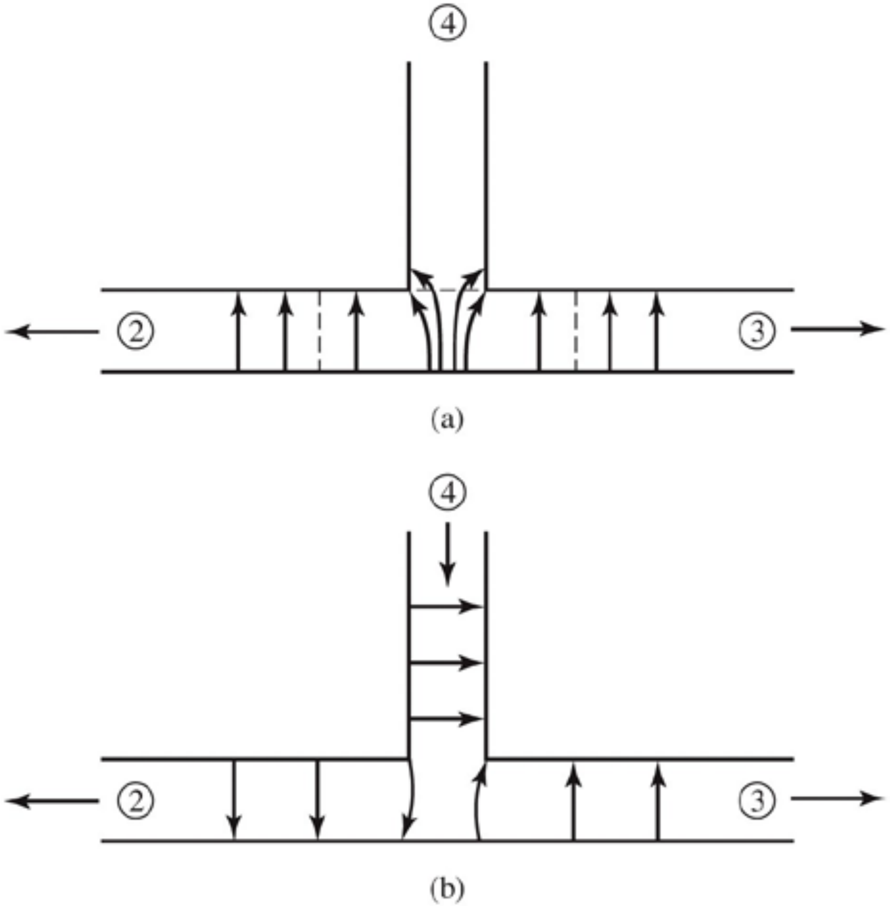


Figure 7.50

$$s_{41} = 0\,; \quad s_{21} = s_{31}$$

$$s_{14} = 0\,; \quad s_{42} = -s_{43}$$

Hybrid-T junction

We add matching elements outside the junction to obtain

$$s_{11} = s_{44} = 0$$

$$[s] = \begin{bmatrix} 0 & s_{12} & s_{12} & 0 \\ s_{12} & s_{22} & s_{23} & s_{24} \\ s_{12} & s_{23} & s_{33} & -s_{24} \\ 0 & s_{24} & -s_{24} & 0 \end{bmatrix}$$

$$[s] = \begin{bmatrix} 0 & s_{12} & s_{12} & 0 \\ s_{12} & s_{22} & s_{23} & s_{24} \\ s_{12} & s_{23} & s_{33} & -s_{24} \\ 0 & s_{24} & -s_{24} & 0 \end{bmatrix}$$

The circuit is lossless

$$\left.\begin{array}{l} \sum(\text{row } 2)*(\text{row } 2)^* = 1 \Rightarrow \quad |s_{12}|^2 + |s_{22}|^2 + |s_{23}|^2 + |s_{24}|^2 = 1 \\ \sum(\text{row } 3)*(\text{row } 3)^* = 1 \Rightarrow \quad |s_{12}|^2 + |s_{23}|^2 + |s_{33}|^2 + |s_{24}|^2 = 1 \end{array}\right\}$$

$$|s_{22}|^2 - |s_{33}|^2 = 0 \Rightarrow |s_{22}| = |s_{33}|$$

$$\sum(\text{row } 1)*(\text{row } 1)^* = 1 \Rightarrow \quad 2|s_{12}|^2 = 1 \Rightarrow |s_{12}| = \frac{1}{\sqrt{2}}$$

$$\sum(\text{row } 4)*(\text{row } 4)^* = 1 \Rightarrow \quad 2|s_{24}|^2 = 1 \Rightarrow |s_{24}| = \frac{1}{\sqrt{2}}$$

$$1 + |s_{22}|^2 + |s_{23}|^2 = 1 \Rightarrow |s_{22}|^2 + |s_{23}|^2 = 0 \Rightarrow s_{22} = s_{23} = 0$$

$$[s] = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & -1 & 0 \end{bmatrix}$$

Summary of lecture 10

- Read chapter 7 (couplers).
  - Directional couplers
  - Power dividers
  - Magic T
- Next: lecture on periodic structures

Further reading

- Schiffman, B.M.: 'A new class of broad-band microwave 90-degree phase shifters', IRE Trans. Microw. Theory Tech., 1958, 6, (2), pp. 232 – 237
- S. Cohn and R. Levy, "History of Microwave Passive Components with Particular Attention to Directional Couplers," IEEE Transactions on Microwave Theory and Techniques, vol. 32, no. 9, pp. 1046–1054, 1984.