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FKA121/FIM540 Computational Physics

1. This could be a part of a Matlab program for E1 Solar system. Can you add what is missing at the question mark.

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%---- rhs.m: function file for use with ode23 ---------------------

%% rhs.m: returns right hand side of 1st order ODE "d(rv)/dt = f(t,rv)".
function out = rhs(t,rv); % input: time vector t and state vector rv

global G m1 m2; % make constants from main file available in this file
% (need similar declaration in main file)

r12 = rv(3:4) - rv(1:2); % vector from body 1 to body 2
d12 = norm(r12);  % distance between body 1 and body 2

v1  = rv(5:6);  % "dr1/dt = v1"
v2  = rv(7:8);  % "dr2/dt = v2"
a1  = G*m2*r12/d12^3;  % "dv1/dt = a1"
a2  = ?;  % "dv2/dt = a2"

out = [v1;v2;a1;a2];   % return all components of right hand side (column vector)
```

   (3p)

2. The equation
$$\frac{d^2y(t)}{dt^2} = f(y,t)$$
can be solved using the Verlet algorithm
$$y_{n+1} = 2y_n - y_{n-1} + h^2 f_n + O(h^4)$$
where
$$f_n = f(y_n, t_n)$$
The Verlet algorithm can also be written on the ”velocity Verlet” form
$$\begin{aligned} y_{n+1} &= y_n + hv_n + (h^2/2)f_n \\ v_{n+1} &= v_n + (h/2) \ \ ? \end{aligned}$$
This algorithm was used when you solved the Fermi-Pasta-Ulam problem. Can you add what is missing at the question mark. (3p)

3. Consider a one-dimensional integral
$$I = \int_0^1 dx f(x)$$
Explain how you evaluate this by simple Monte Carlo integration. This should include an explanation of how you estimate the error. (4p)

4. Consider a one-dimensional integral

$$I = \int_0^1 dx f(x)$$

that you would like to solve using the Monte Carlo method. Explain, in words, how the error can be reduced using a weight function

$$p(x) \quad \text{with} \int_0^1 p(x) dx = 1$$

(5p)

5. Explain the idea behind error estimate for correlated values using block averaging. (4p)

6. Consider the wave equation

$$\frac{\partial^2 y}{\partial t^2} = c^2 \left(\frac{\partial^2 y}{\partial x^2} - \epsilon L^2 \frac{\partial^4 y}{\partial x^4} \right)$$

with the boundary conditions, for $x = 0$ and $x = L$,

$$y = \frac{\partial^2 y}{\partial x^2} = 0$$

and with $y(x, t)$ discretized according to

$$y_j^n = y(x_j, t_n)$$

Describe how the boundary conditions can be implemented. (5p)

7. Describe the difference between the Jacobi and the Gauss-Seidel relaxation method for the solution of an elliptic PDE. (4p)