Department of Applied Physics, Chalmers, Göteborg

FKA121/FIM540 Computational Physics

Time: 11 December 2010, 14:00 - 18:00
Place: V-building
Teacher: Göran Wahnström, 031 - 827264, 076 - 10 10 523
Göran will be available to answer questions at 15:00
No allowed materials or tools (besides pencil etc)

For more information on the grading and on the inspection of the outcome of the exam, please see the homepage.

1. Consider Newton's equation of motion for a harmonic oscillator

$$m\frac{dx^2(t)}{dt^2} = -kx(t)$$

   Rewrite this as a set of first-order ODEs. (3p)

2. This could be a part of a Matlab program for E1 Solar system. Can you add what is missing at the question mark.

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%---- rhs.m: function file for use with ode23 ---------------------

%% rhs.m: returns right hand side of 1st order ODE "d(rv)/dt = f(t,rv)".
function out = rhs(t,rv); % input: time vector t and state vector rv

global G m1 m2; % make constants from main file available in this file
% (need similar declaration in main file)

r12 = rv(3:4) - rv(1:2); % vector from body 1 to body 2
d12 = norm(r12);  % distance between body 1 and body 2

v1  = rv(5:6);   % "dr1/dt = v1"
v2  = rv(7:8);   % "dr2/dt = v2"
a1  = G*m2*r12/d12^3;   % "dv1/dt = a1"
a2  = ?;  % "dv2/dt = a2"

out = [v1;v2;a1;a2];    % return all components of right hand side (column vector)
```

   (3p)

3. The equation

$$\frac{d^2y(t)}{dt^2} = f(y,t)$$

can be solved using the Verlet algorithm

$$y_{n+1} = 2y_n - y_{n-1} + h^2 f_n + O(h^4)$$

where

$$f_n = f(y_n, t_n)$$

The Verlet algorithm can also be written on the "velocity Verlet" form

$$\begin{aligned} y_{n+1} &= y_n + hv_n + (h^2/2)f_n \\ v_{n+1} &= v_n + (h/2) \ ? \end{aligned}$$

This algorithm was used when you solved the Fermi-Pasta-Ulam problem. Can you add what is missing at the question mark. (3p)

4. Consider a one-dimensional integral

$$I = \int_0^1 dx f(x)$$

that you would like to solve using the Monte Carlo method. Explain, in words, how the error can be reduced using a weight function

$$p(x) \ \text{with} \int_0^1 p(x)dx = 1$$

(5p)

5. Explain the idea behind error estimate for correlated values using block averaging. (5p)

6. Consider the wave equation

$$\frac{\partial^2 y}{\partial t^2} = c^2 \left(\frac{\partial^2 y}{\partial x^2} - \epsilon L^2 \frac{\partial^4 y}{\partial x^4} \right)$$

with the boundary conditions, for $x = 0$ and $x = L$,

$$y = \frac{\partial^2 y}{\partial x^2} = 0$$

and discretize $y(x, t)$ according to

$$y_j^n = y(x_j, t_n)$$

Describe how the boundary conditions can be implemented. (5p)

7. Describe the difference between the Jacobi and the Gauss-Seidel relaxation method for the solution of an elliptic PDE. (4p)