Tentamen i integrationsteori 2007–04–12 08.30-13.30
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Lycka till! BB

1. Formulate and prove Fatou's lemma.

2. Prove Lebesgue's theorem on differentiation of the integral of an L^1-function (eg using the Maximal Theorem).

3. a Give the definition of a *monotone class*.

   b prove that if A is an algebra of sets, then the monotone class generated by A is a σ-algebra.

4. Compute
$$\lim_{n\to\infty}\int_0^1 \frac{1+nx^2}{(1+x^2)^n}dx$$

5. Let $f \geq 0$ be a function in $L^1(R)$. Prove that
$$\lim f(x+n) = 0$$
   for almost all x (in $(0,1)$).

6. Let $0 < \alpha < 1$. Give an example of a closed subset with empty interior of the interval $[0,1]$ with measure equal to α.

7. Let f_k be a uniformly bounded sequence of measurable functions on the interval $[0,1]$ and let f be a bounded measurable function on the same interval. Assume

   i $\lim \int f_k = \int f$

   and

   ii $\limsup f_k \leq f$

   everywhere. Prove that

$$\lim \int |f_k - f| = 0.$$

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