Tentamen i integrationsteori 2007–01–19 08.30-13.30
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Lycka till! BB

1. Formulate and prove Fatou's lemma.

2. Prove Lebesgue's theorem on differentiation of the integral of an L^1-function (eg using the Maximal Theorem).

3. a Give the definition of a *monotone class*.

   b prove that if A is an algebra of sets, then the monotone class generated by A is a σ-algebra.

4. a Let, for f and g in $L^1(R)$,

   $$h(x) = \int f(x-t)g(t)dt.$$

   Assume that f is also bounded. Prove that h is continuous and lies in L^1.

   b Prove that if A and B are measurable sets of positive measure on the line, then $A + B = \{a + b; a \in A and\, b \in B\}$ contains some interval.

5. Let f be a nonnegative measurable function on X and let μ be a positive measure on X. Prove that

   $$\sum_1^\infty \mu(\{x; f(x) \geq n\}) = \int [f(x)] d\mu(x),$$

   where $[a]$ is the *integer part* of a, i e the largest integer smaller than or equal to a.

6. Let f be a real valued measurable function on X and let μ be a finite positive measure on X. Put $F(t) = \mu(\{x; f(x) \leq t\})$

   a) Prove that F is increasing and right continuous.

   Let μ_F be the Lebesgue-Stieltjes measure defined by F.

   b) Prove that

   $$\int_R \phi d\mu_F = \int_X \phi \circ f d\mu$$

   if ϕ is nonnegative and measurable.