Exam in Course Radar Systems and Applications (RRY080)

21th October 2010, 08.30 a.m.-12.30 p.m., Halls at "Maskin"

Additional materials: Beta and Physics Handbook (or equivalent), Dictionary and Electronic calculator (free-of-choice but no computers allowed).

Leif Eriksson (tel: 4856) will visit around 9:30 and 11:30.

Answers must be given in English.

Clearly show answers (numbers and dimensions!) to each part of each question just after the corresponding solution.

Begin each question on a new sheet of paper!

Each question carries a total of ten points. Where a question consists of different parts (sub-questions), the number of points for each part of the question is indicated in parentheses. **In most cases it should be possible to answer later parts of the question even if you cannot answer one part.**

Grades will be awarded approximately as <20 = Fail, 20-29 = 3, 30-39 = 4, ≥40 = 5 (including a maximum of 6 bonus points; exact boundaries may be adjusted later).

Review of corrected exam papers will take place on the two following occasions:

Monday 8th November, 14:00-15:00
Thursday 11th November, 15:00-16:00

1. Basic terms & principles

a) A radar transmits the following pulse signal (f_0 = 5 GHz, τ = 20 μs, $\gamma = 10^{12}$ s^{-2}):

$$s(t) = \begin{cases} \cos\left(2\pi\left[f_0 t + \gamma t^2\right]\right) & \text{for } |t| \leq \tau/2 \\ 0 & \text{otherwise} \end{cases}$$

What is the time-bandwidth product of the signal, i.e. the product of the pulse length and the signal bandwidth? (2p)

b) What is the smallest range resolution (in meter) of the radar in a)? (2p)

c) What is the required area, at X-band (10 GHz), of a flat metallic plate (in m^2) which gives a maximum radar-cross section of $\sigma = 1000$ m^2? (2p)

d) What is the amplitude ratio corresponding to -20 dB? (1p)

e) What is the far-field beamwidth (in degree) of an antenna, operating at S-band (3 GHz) and with a uniformly illuminated aperture of length 2 m. (2p)

f) You are designing an airborne early warning (AEW) radar system for long-range detection and tracking of aircraft. You are to choose between a design in S band (3 GHz), X band (10 GHz) or Ku band (15 GHz). Which band do you choose? Give a short motivation to your answer. (1p)

2. Synthetic aperture radar resolution

An aircraft has two SAR systems operating in two different frequency bands, P-band with transmit frequencies 250-350 MHz, and C-band with transmit frequencies 4.95-5.05 GHz. Assume that both systems are looking at broadside from the flight direction, i.e. antenna boresight pointing towards zero Doppler frequency.

a) What aperture angle is required for the P-band system to get the same resolution in azimuth as in range? Your solution should include a figure that illustrates the observation geometry and the angles you have used in your calculations (no need to illustrate resolution cells). (2 p)

b) What is the synthetic aperture length at a range of 6 km? (2 p)

c) How long does it take to fly the aperture at a speed of 100 m/s? (1 p)

d) What aperture angle is required for a C-band system to achieve the same resolution as the P-band system and what is the synthetic aperture length at a range of 6 km? (3 p)

e) If the C-band antenna is directed at an angle of 30 deg from the flight direction instead of broadside, what synthetic aperture length is required, at a range of 6 km, to achieve the same resolution as the broadside P-band system? Your solution should include a figure that illustrates the new observation geometry and the angles you have used in your calculations. (2 p)

3. Ambiguity function

Assume that we have a single rectangular constant-frequency pulse with pulse width T. The ambiguity function is $|\chi(\tau, \upsilon)|$, where τ is the delay and υ is the Doppler shift.

a) Determine the complex envelope u(t) for this pulse. Note that the maximum value of the ambiguity function is equal to 1. (2 p)

b) Show that:
$|\chi(\tau,0)| = 1 - (|\tau| / T)$, $|\tau| \leq T$; zero otherwise (2 p)

c) Determine the 3 dB range resolution after matched filtering. (1 p)

d) Show that:
$|\chi(0, \upsilon)| = |\mathrm{sinc}(\pi\upsilon T)|$ (2 p)

e) Determine the 3 dB Doppler resolution after matched filtering. (1 p)

Assume now that T =10 μs and that a burst with 10 pulses at PRF = 1 kHz is used.

f) Determine the 3 dB Doppler resolution after matched filtering. (2 p)

4. Target detection

Marine navigation radars are usually designed with magnetron transmitters and a horizontally rotating waveguide antenna. A new IMO (International Maritime Organisation) standard has recently been adopted which allows solid-state transmitters and thus coherent pulse integration to enhance radar performance. A S-band shipborne radar has the following parameters:

Centre frequency	3.05 GHz
Peak radiated power	30 kW
Antenna length	3.7 m
Antenna height	0.18 m
Antenna rotation rate	45 rpm (revolutions per minute)
Antenna efficiency	50%
Antenna beam width (azimuth/horizontal)	1.8 deg
System noise temperature	900 K
Losses	3 dB

The range scale (max. range on display), pulse length and PRF are selectable according to:

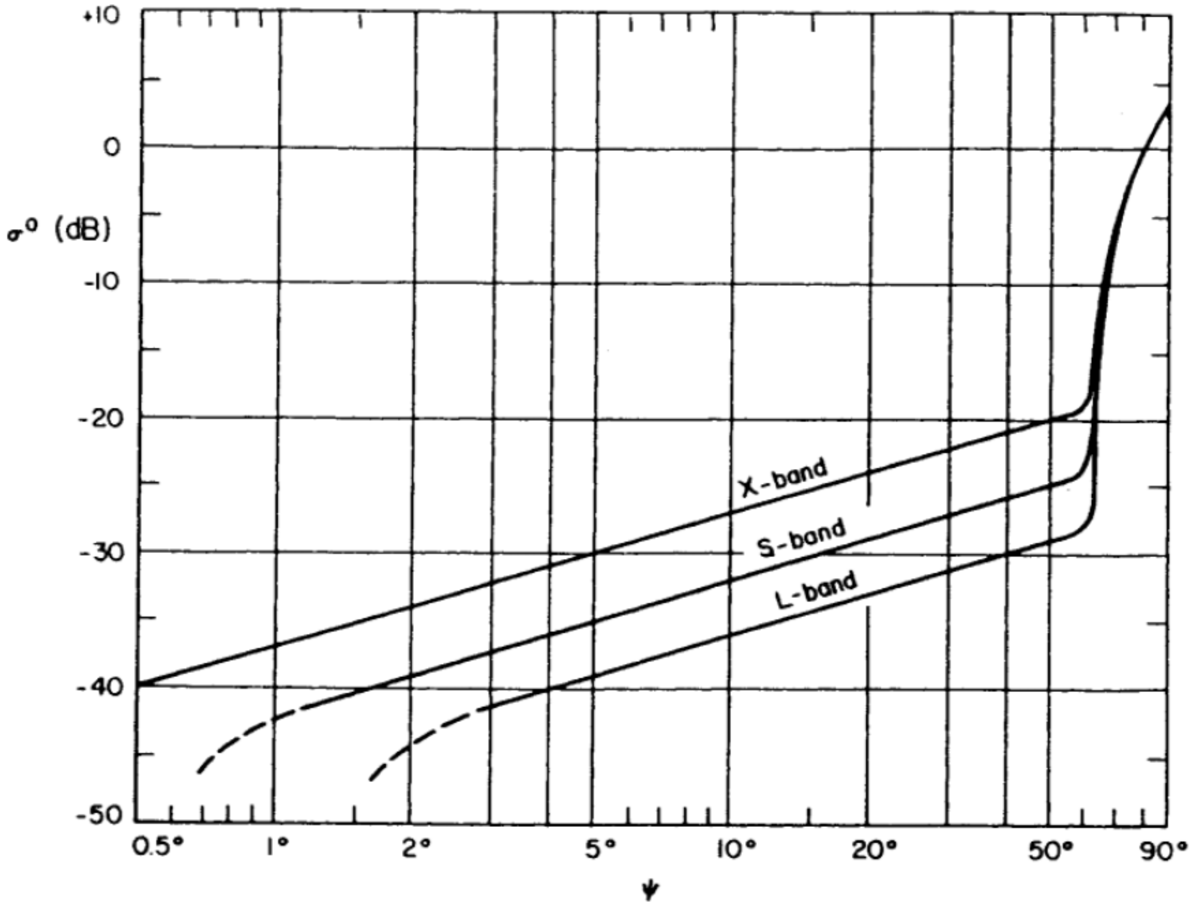
Range scale (n.m., where 1 n.m. = 1852 m)	Pulse length (μs)	PRF (Hz)
0.125, 0.25	0.07	3000
0.5	0.07, 0.15	3000
0.75, 1.5	0.07, 0.15, 0.3	3000, 1500
3	0.15, 0.3, 0.5, 0.7	3000, 1500, 1000
6	0.3, 0.5, 0.7, 1.2	1500, 1000, 600
12, 24	0.5, 0.7, 1.2	1000, 600
48, 96	1.2	600

a) What is the minimum range of the radar system, i.e. the shortest range to a detectable target? (2p)

b) Determine max range of the radar system for a non-fluctuating target with radar-cross section (RCS) 1000 m^2 with P_D = 90% and $P_{FA} = 10^{-8}$. All pulses from a target within the antenna beam in one scan are non-coherently integrated. Neglect multi-path, clutter, filter mismatch. (3p)

c) In ISAR mode, the antenna is fixed and pointing at a non-fluctuating target. Determine the average power requirement for detecting a non-fluctuating target with RCS 0.5 m^2 at range 50 km with P_D = 90% and $P_{FA} = 10^{-6}$. Coherent integration time is 0.5 s. Neglect multi-path, clutter, filter mismatch. (3p)

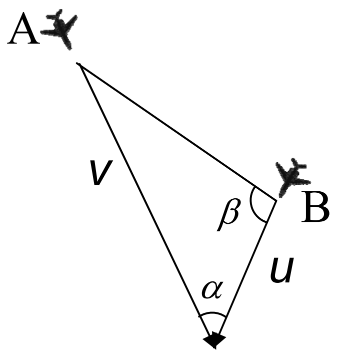
d) The bandwidth of the ISAR mode gives a range resolution of 0.5 m. Is the target detection in c) limited by system noise or sea clutter? Average sea clutter reflectivity is given by the graph on the next page. (2p)



Backscattering coefficient of sea clutter as a function of grazing angle.

5. Pulse-Doppler Radar

An aircraft (A, speed v) is on collision-course with another aircraft (B, speed u) and at constant altitude. Both aircraft have their 10 GHz radars turned on and in high-PRF Pulse-Doppler mode.



a) Assume u = 150 m/s, v = 300 m/s and $\alpha = 70°$. Compute the (non-relativistic) Doppler frequency shift which aircraft B appears to have for the radar in aircraft A. Compute also the Doppler frequency shift which aircraft A appears to have for the radar in aircraft B. (2p)

b) Compute the maximum and minimum Doppler frequency shift that the ground clutter will have in the radar of aircraft A. Compute also the maximum and minimum Doppler frequency shift that the ground clutter will have in the radar of aircraft B. Assume the same parameters as in a). (2p)

c) Will aircraft A be in the clutter-free region of the radar in aircraft B? (1p)

d) Will aircraft B be in the clutter-free region of the radar in aircraft A? (1p)

e) Derive an equation for the maximum bearing (aspect) angle β, for which aircraft A is in the clutter-free region of the radar in aircraft B, expressed as a function of the speed ratio v/u. Plot the maximum bearing angle in the interval v/u = 0 to 2. (4p)

Formulas which may be used at examination of the course Radar Systems and Applications (equation and page numbers refer to the course book by Sullivan)

Radar equation

$$SNR = \frac{P_{peak} G^2 \lambda^2 \sigma \tau}{(4\pi)^3 R^4 k_B T_s C_B L} \quad (1.23)$$

$$SNR = \frac{P_{avg} G^2 \lambda^2 \sigma t_{dwell}}{(4\pi)^3 R^4 k_B T_s C_B L} \quad (1.24)$$

Antennas

$$A_e = \frac{G \lambda^2}{4\pi} \quad (1.18)$$

$$A_e = A\eta \quad \text{(p. 15)}$$

$$R_{far} = \frac{2L^2}{\lambda} \quad \text{(p. 43)}$$

System noise

$$k_B T_{sys} = k_B \left[T_{ant} / L_{radar} + T_{radar} \left(1 - 1/L_{radar}\right) + T_{rcvr} \right] \quad (2.10)$$

$$T_{rcvr} = (F-1) T_0 \quad , F > 1 \quad (2.11)$$

Radar Cross Section

$$\sigma = \lim_{R \to \infty} 4\pi R^2 \frac{|\mathbf{E_s}|^2}{|\mathbf{E_i}|^2} \quad (3.1)$$

$$\sigma = 4\pi \frac{A^2}{\lambda^2}$$

(Broadside RCS of large metallic flat plate, p. 69-70)

Radar clutter

$$\sigma_c = \sigma^0 A \quad \text{(p. 87)}$$

$$\sigma_c = \eta V \quad \text{(p. 88)}$$

Envelope detection of targets embedded in noise (single-pulse decision)

a) Nonfluctuating (steady) target

$$P_D \approx \frac{1}{2} \left[erfc \left(\sqrt{\ln\left(\frac{1}{P_{FA}}\right)} - \sqrt{SNR + \frac{1}{2}} \right) \right]$$

(4.9) corrected

b) Fluctuating target

$$P_D = P_{FA}^{\;1/(1+SNR)} \quad (4.13)$$

Matched filter

$$h(t) = Ku^*(t_m - t) \quad \Leftrightarrow \quad H(\omega) = KU^*(\omega)\exp(-j\omega t_m) \qquad (4.32)$$

Ambiguity function

$$\left|\chi(\tau,\nu)\right| = \left|\int_{-\infty}^{+\infty} u(t)u^*(t-\tau)\exp(j2\pi\nu t)\,dt\right| \qquad \text{(p. 116)}$$

Doppler frequency (this formula takes into account the sign of Doppler frequency)

$$f_d = -\frac{2v}{\lambda} = -\frac{2\cdot(dR/dt)}{\lambda} \qquad (1.32)$$

Maximum unambiguous velocity interval

$$\Delta v_u = \frac{f_R\lambda}{2} \qquad \text{(p. 22)}$$

Inverse SAR

$$\delta_{rpn} = \frac{c}{2B} \qquad (6.4)$$

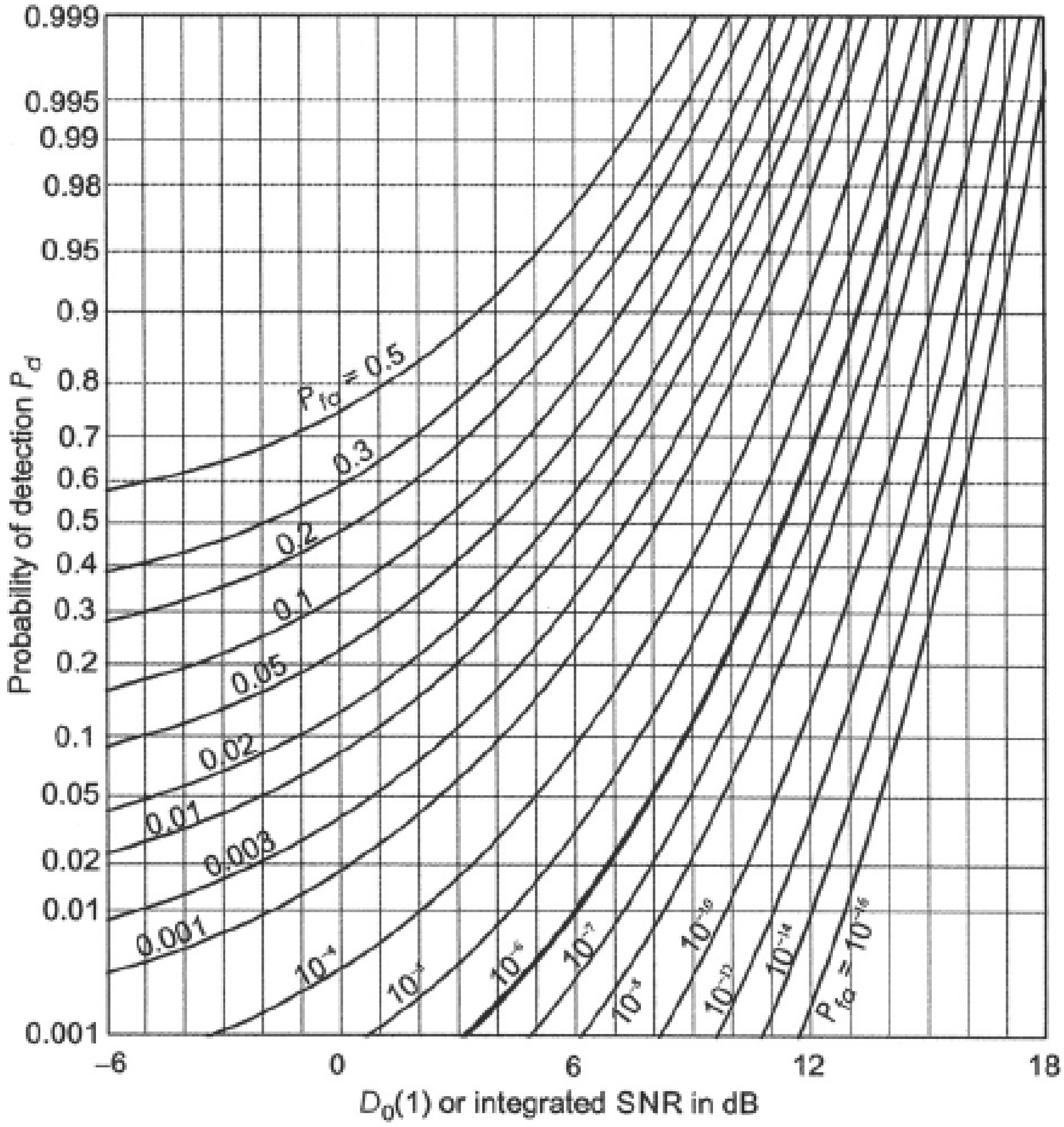
$$\delta_{crpn} = \frac{\lambda}{2\Delta\phi} \qquad (6.5)$$

SAR

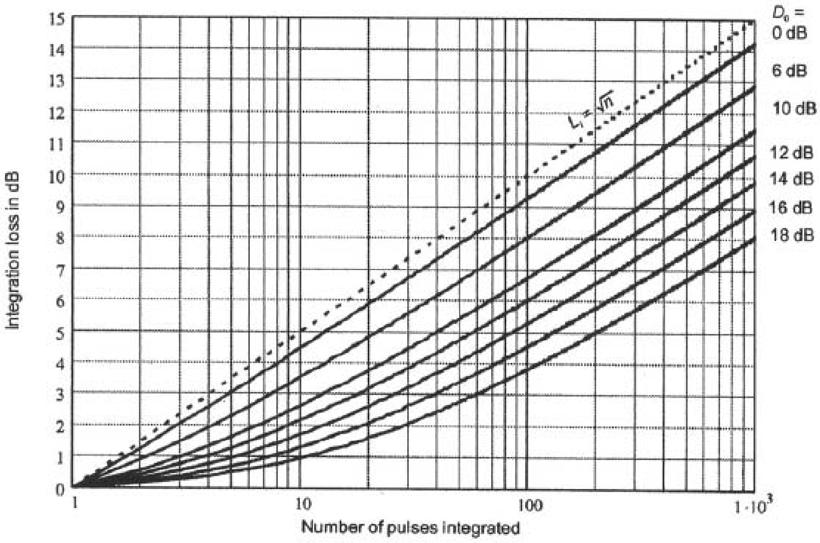
$$\delta_r \approx \frac{c}{2B} \qquad \text{(p. 216)}$$

$$\delta_{cr} \approx \frac{\lambda}{2\Delta\theta} \qquad (7.1)$$

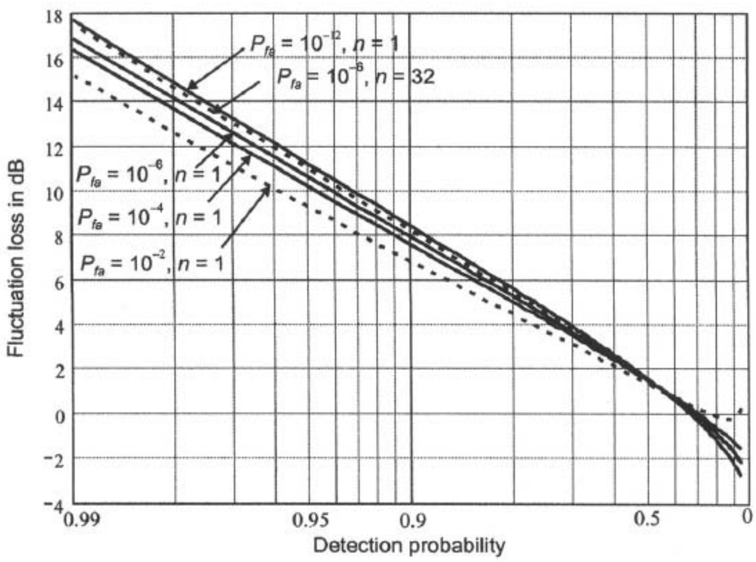
$$CNR = \frac{P_{avg}G^2\lambda^3\sigma^0\delta_r}{2(4\pi)^3R^3k_BT_{sys}LV\cos\psi} \qquad (7.46)$$



Detectability factor for a steady target
[David K. Barton, 2004, Radar System Analysis and Modeling, p. 45]



Integration loss versus number of pulses integrated after envelope detection for different values of output detectability factor $D_0(1)$
[David K. Barton, 2004, Radar System Analysis and Modeling, p. 52]



Fluctuation losses for case 1 target (Swerling-1 model)
[David K. Barton, 2004, Radar System Analysis and Modeling, p. 61]

Solutions to problems of exam 2010-10-21

1. Basic Terms and Principles

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a) $2\gamma\tau^2 = 800$

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b) $\dfrac{c}{2B} = \dfrac{c}{4\gamma\tau} = 3.8\ m$

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c) $\sqrt{\dfrac{\lambda^2\sigma}{4\pi}} = 0.27\ m^2$

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d) $10^{-\frac{20}{20}} = 0.1$

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e) $\dfrac{\lambda}{D} = 2.9^o$

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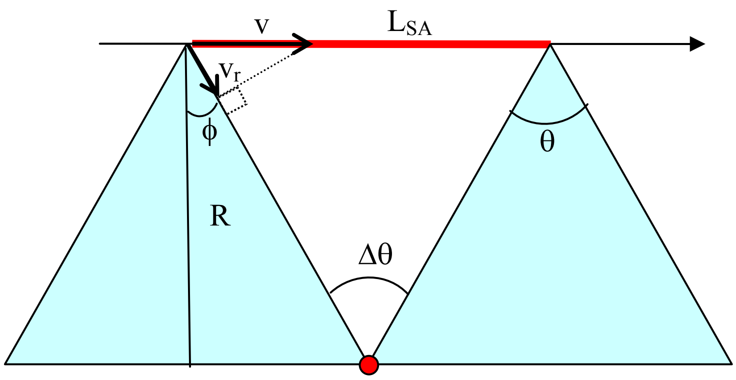
f) Choose S-band to minimize the tropospheric loss.

2. Synthetic aperture radar resolution

a)



B_P = 100 MHz f_P = 300 MHz R = 6 km
v = 100 m/s

Range resolution:

$$\delta_r = \frac{c}{2B}$$

Cross range resolution is determined by Doppler bandwidth:

$$\delta_{cr} = \frac{v}{B_D}$$

$$f_D = -\frac{2v_r}{\lambda} = -\frac{2v\sin\phi}{\lambda} \quad \Rightarrow \quad f_{DMAX} = \frac{2v\sin\frac{\Delta\theta}{2}}{\lambda} \quad f_{DMIN} = -\frac{2v\sin\frac{\Delta\theta}{2}}{\lambda}$$

$$B_D = f_{DMAX} - f_{DMIN} = \frac{4v\sin\frac{\Delta\theta}{2}}{\lambda}$$

We want the same resolution in cross range as in range which gives us the required Doppler bandwidth and the corresponding aperture length:

$$B_{Dreq} = \frac{2Bv}{c} = \frac{4v\sin\frac{\Delta\theta}{2}}{\lambda} \quad \Rightarrow \quad \Delta\theta = 2\arcsin\frac{B}{2f} \quad \Rightarrow \Delta\theta_P = 0.3349 rad$$

b)

$$L_{SA} = 2R\tan\frac{\Delta\theta}{2}$$

$$\Rightarrow \quad L_{SA\,P} = 2.028 km$$

c)
It takes 20 seconds to fly the synthetic aperture.

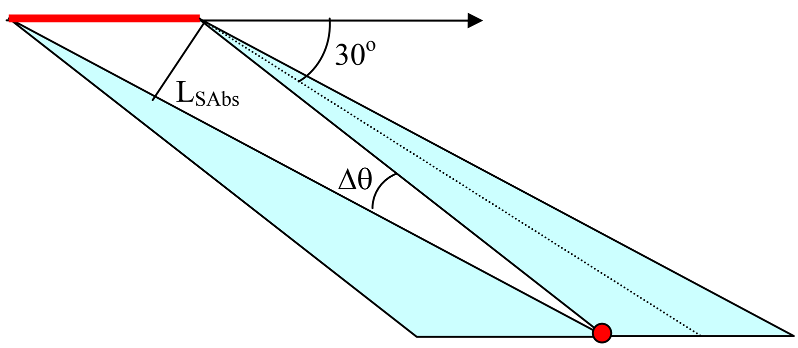
d)
For C-band to achieve the same resolution:

$B_C = 100$ MHz $\qquad$ $f_X = 5$ GHz

$$\Delta\theta_C = 2\arcsin\frac{B}{2f_C} \approx \frac{B}{f_C} = 0.02 rad$$

$$L_{SA_C} = 2R\tan\frac{\Delta\theta_C}{2} \approx R\Delta\theta_C = 120\text{m}$$

e)
At 30 deg angle:



$L_{SA30} \approx L_{SAbs}/\sin 30^o = 240$ m

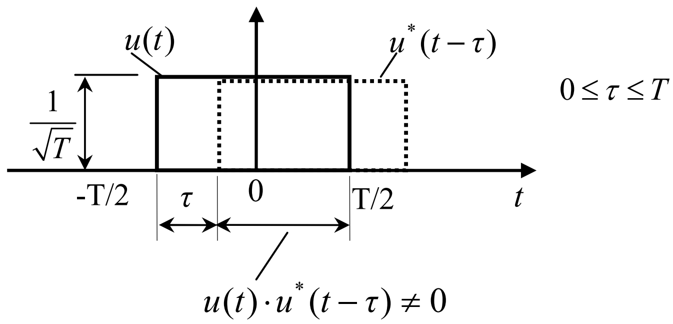
3. Ambiguity function

a) The complex envelope of a single rectangular constant-frequency pulse is given by

$$u(t) = \frac{1}{\sqrt{T}},\ -\frac{T}{2} \le t \le \frac{T}{2};\ \text{zero otherwise}$$

b) Applying the definition of the ambiguity function gives, for $\tau \ge 0$
(figure explains the integration limits)

$$\chi(\tau,\nu) = \int_{-T/2+\tau}^{T/2} \frac{1}{\sqrt{T}}\frac{1}{\sqrt{T}} \exp(j2\pi\nu t)dt$$



$$\chi(\tau,0) = \frac{1}{T}\int_{-\frac{T}{2}+\tau}^{\frac{T}{2}} dt = \frac{1}{T}[t]_{-\frac{T}{2}+\tau}^{\frac{T}{2}} = \frac{1}{T}\left(\frac{T}{2}+\frac{T}{2}-\tau\right) = 1-\frac{\tau}{T}$$

$$|\chi(\tau,0)| = \left|1-\frac{\tau}{T}\right| = \left|1-\frac{|\tau|}{T}\right|,\ 0 \le \tau \le T \qquad (3.1)$$

Repeating the derivation for $\tau \le 0$ gives

$$\chi(\tau,0) = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}+\tau} dt = \frac{1}{T}[t]_{-\frac{T}{2}}^{\frac{T}{2}+\tau} = \frac{1}{T}\left(\frac{T}{2}+\tau+\frac{T}{2}\right) = 1+\frac{\tau}{T}$$

$$|\chi(\tau,0)| = \left|1+\frac{\tau}{T}\right| = \left|1-\frac{|\tau|}{T}\right|,\ -T \le \tau \le 0 \qquad (3.2)$$

Combining (3.1) and (3.2) yields

$|\chi(\tau,0)| = 1-\frac{|\tau|}{T}$, $-T \le \tau \le T$; zero otherwise

c) $\delta_{\tau_{3dB}} = T \cdot 2\left(1-\frac{1}{\sqrt{2}}\right) = 0.586 \cdot T$

d) Applying the definition of the ambiguity function gives,

$$\chi(\tau,\nu)=\int_{-T/2+\tau}^{T/2}\frac{1}{\sqrt{T}}\frac{1}{\sqrt{T}}\exp(j2\pi\nu t)dt$$

$$\chi(0,v)=\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}}e^{j2\pi vt}dt=\frac{1}{T}\left[\frac{1}{j2\pi v}e^{j2\pi vt}\right]_{-\frac{T}{2}}^{\frac{T}{2}}=\frac{1}{j2\pi vT}\left(e^{j\pi vT}-e^{-j\pi vT}\right)=\frac{1}{\pi vT}\sin(\pi vT)$$

$$|\chi(0,v)|=|sinc(\pi vT)|$$

e) $\delta_{f_{D-3dB}}=0.886\left(\frac{1}{T}\right)$

f) Pulse repetition interval: $T_R = 1/PRF = 0.001$ s

$$\delta_{f_{D-3dB}}=0.886\left(\frac{1}{NT_R}\right)=\frac{0.886}{10\cdot 0.001}=88.6\ Hz$$

4. Target detection

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a) Minimum range is defined by minimum pulse length (70 ns in table) since simultaneous transmission and reception is not possible for high-power radars.

$$R \geq \frac{c\tau}{2} = 10.5\ m$$

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b) Assuming max range scale (48 or 96 nm) gives number of pulses per scan

$$n = \frac{1.8^o}{360^0 \frac{45}{60}} \cdot 600 = 4$$

and thus (non-coherent) integration loss of 0.8 dB according to graph in formula sheet. Required SNR (single-pulse) is

$$SNR_{required} = \frac{D_0(1)L_i}{n} \cong 14.2 + 0.8 - 6 = 9\ dB$$

Antenna gain is given by

$$G = \frac{4\pi A}{\lambda^2}\eta = \frac{4\pi \cdot 3.7 \cdot 0.18}{0.098^2} 0.5 = 436 \cong 26.4\ dB$$

The radar equation gives

$$R = \left(\frac{P_{peak} G^2 \lambda^2 \sigma \tau}{(4\pi)^3 k_B T_s C_B L} \cdot \frac{1}{SNR_{required}} \right)^{0.25}$$

or $\frac{(44.8 + 52.8 - 20.2 + 30 - 59.2 - 33.0 + 199.1 - 3 - 9)}{4} dB = 50.6\ dB$ relative 1 m

i.e. 114 km or 62 n.m. (the assumption of max range scale was indeed correct).

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c) $SNR_{required} = 13.2\ dB$ from graph in formula sheet. The radar equation gives

$$P_{avg} = \frac{(4\pi)^3 R^4 k_B T_s C_B L}{G^2 \lambda^2 \sigma t_{dwell}} SNR_{required}$$ or

$$(33 + 188.0 - 199.1 + 3 - 52.8 + 20.2 + 3 + 3 + 13.2)\ dBW = 11.5\ dBW$$ or 14 W

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d) Equivalent backscattering coefficient for noise is given by

$$\sigma_{ne}^{o} = \frac{\sigma}{A_{resolution} SNR_{required}} = \frac{0.5}{0.5 \cdot \frac{1.8}{180}\pi \cdot 50000 \cdot 10^{\frac{13.2}{10}}} = -45.2 dB$$

Sea backscattering coefficient is much less than -45 dB for realistic grazing angles (<0.1 deg) according to the graph. Target detection is hence limited by system noise.

5. Pulse-Doppler radar

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a) Cosine theorem gives $v_{rel} = 286$ m/s and thus $f_{Dop} = \dfrac{2v_{rel}}{\lambda} = 19.1\,kHz$

Both radars experience the same Doppler shift

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b) Min and max Doppler frequency is (A) $\pm\dfrac{2v}{\lambda} = 20.0kHz$ and (B) $\pm\dfrac{2u}{\lambda} = 10.0kHz$

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c) Yes

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d) No

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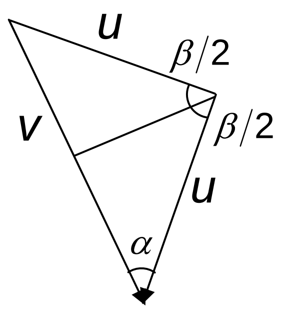
e)



$$\beta = 2\arcsin\left(\frac{v}{2u}\right)$$