Exam in Course Radar Systems and Applications (RRY080)

25th August 2010, 08.30 a.m.-12.30 p.m., Halls at "Maskin"

Additional materials: Beta and Physics Handbook (or equivalent), Dictionary and Electronic calculator (free-of-choice but no computers allowed).

For questions, ask the examination staff to contact Lars Ulander on mobile 0709-277152 or Annelie Wyholt on mobile 0704-613991 between 09.30-10.00 and 11.00-11.30.

Answers must be given in English.

Clearly show answers (numbers and dimensions!) to each part of each question just after the corresponding solution.

Begin each question on a new sheet of paper!

Each question carries a total of ten points. Where a question consists of different parts (sub-questions), the number of points for each part of the question is indicated in parentheses. **In most cases it should be possible to answer later parts of the question even if you cannot answer one part.**

Grades will be awarded approximately as <20 = Fail, 20-29 = 3, 30-39 = 4, ≥40 = 5 (exact boundaries may be adjusted later).

Review of corrected exam papers will take place on the two following occasions:

Friday 10th September, 15:15-16:00
Thursday 16th September, 13:15-15:00

1. Basic terms & principles

A radar transmits the following pulse (f_0 = 15 GHz, τ = 10 μs, $\gamma = 2\pi \cdot 10^{13}$ Hz/s):

$$s(t) = \begin{cases} \cos\left(2\pi f_0 t + \gamma t^2\right) & \text{for } |t| \le \tau/2 \\ 0 & \text{otherwise} \end{cases}$$

a) What is the bandwidth of the signal? (2p)

b) What is the pulse compression ratio, i.e. the ratio of pulse width before and after compression (matched) filtering? (2p)

c) What is the maximum free-space RCS of a dihedral corner reflector composed of two orthogonal square metallic plates, both with a side length $a \gg \lambda$ = radar wavelength? (2p)

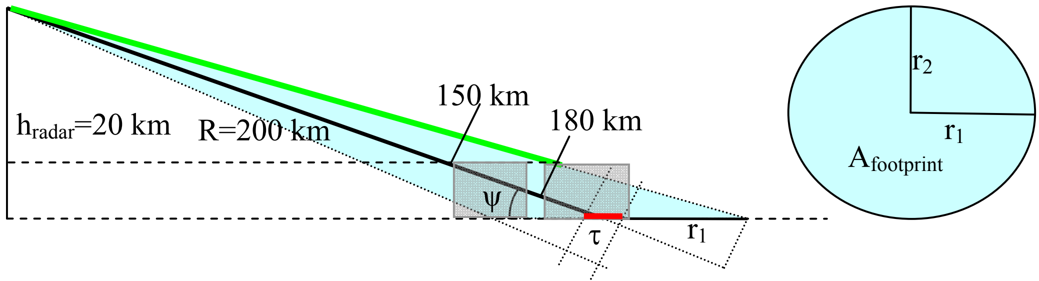
The peak output power from a radar transmitter is 100 kW which is reduced by 3 dB due to losses in the cabling to the antenna.

d) What is the average transmit power input to the antenna if the duty cycle is 2%? (2p)

e) What is the peak power density at 100 km distance if the antenna gain is 30 dB? (2p)

2. Attenuation and Backscatter Interference due to Rainstorm

An airborne radar has h_{radar} = 20 km, vertical lobe width $\theta = 2^o$, horizontal lobe width $\varphi = 4^o$ and pulse width τ = 100 ns. While observing terrain with σ^0 = -25 dB at a slant range R = 200 km, it encounters a large rainstorm with rainfall rate of 30 mm/h located between R_{storm} and R_{storm} + 25 km, and height 5 km.



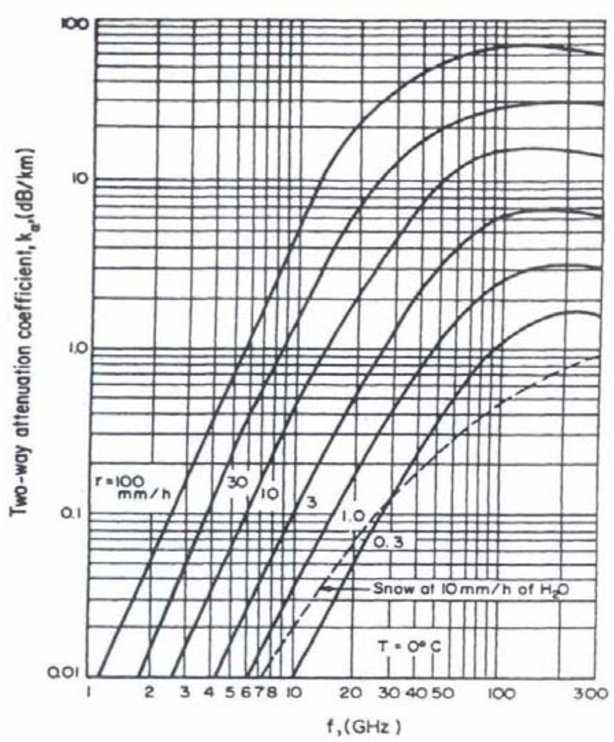
a) Estimate the attenuation (dB) due to the rain for f = 10 GHz (X-band) and 16 GHz (Ku-band); and for R_{storm} = 150 km and 180 km. (2p)

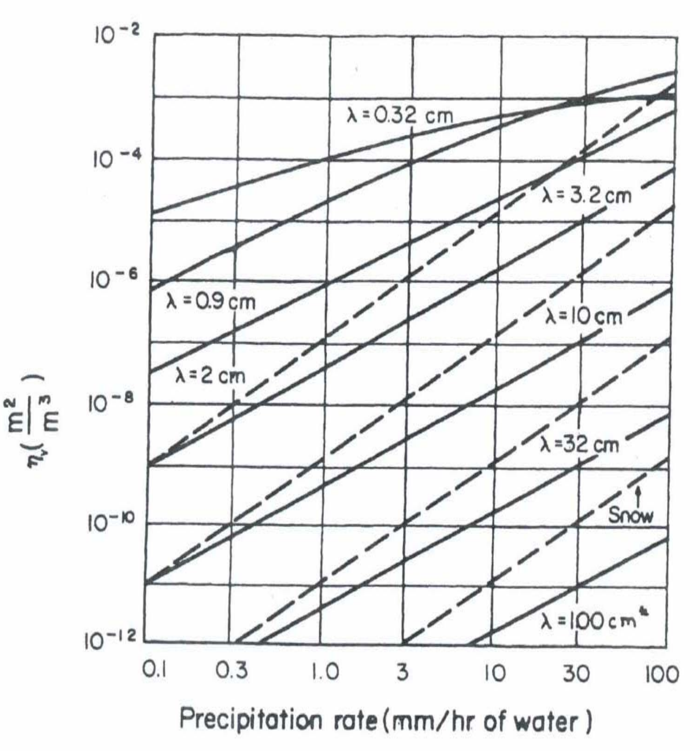
b) What is the radar cross section of the terrain assuming that it is horizontal and flat? Use the low grazing angle approximation (ψ<<1 radians) for the portion of the terrain area struck by the radar pulse. (3p)

c) Estimate the ratio of received power from the terrain and from the rain at R_{storm} = 150 km and 180 km for X-band and Ku-band. (4p)

d) Would you, the designer, have chosen X-band or Ku-band for the radar? (1p)

Hint: The radar cross section of rain is given by $\sigma_{rain}=\eta_V V$, where V is the illuminated volume of rain in the echo (similar to the relation between σ and σ^0)





3. Noise figure and system noise temperature

a) Show that, if we for a radar system define the noise figure $F_2 = (S/N)_{in}/(S/N)_{out}$ and if $T_{antenna} = T_{radar}$, then $T_{receiver} = (F_2 - 1)T_{antenna}$. T stands for temperature, S is the signal power and N is the noise power. Subscript "in" stands for input, and "out" stands for output. (3p)

b) Show that the noise figure $F = (S/N)_{in}/(S/N)_{out} = (N_{out}/N_{in})(S_{in}/S_{out}) = (N_{out}/N_{in})(1/G_{LNA})$ if $T_{antenna} = T_{radar} = T_0$. G_{LNA} is the gain of the low noise amplifier. (3p)

Consider a sensitive radar observing targets against deep space ($T_{antenna}$ = 3K), with an LNA cooled with liquid helium to $T_{receiver}$ = 4.3K. Assume that the losses L_{radar} = 0 dB. Use a standard temperature T_0 = 290 K.

c) What is the noise figure in decibel? (2p)

d) Calculate the system temperature T_{system}? (2p)

4. Target detection

Consider an air surveillance radar with an antenna that rotates at a constant angular velocity $\Omega = 0.2\pi$ radians per second with an azimuth beamwidth of $\theta = 0.576$ degree and a pulse repetition frequency of PRF = 1 kHz. The radar employs noncoherent pulse integration of *n* pulses received on each scan and a binary integration technique by combining the results of detections from N consecutive scans within an observation interval of $T_o = 40$ seconds (it is assumed that the range between the radar and target remains constant during the observation interval). The target fluctuation model obeys Swerling-2 case.

a) Calculate the number of pulses *n* received by the radar from the target on a single scan (one scan means that the radar antenna completes 360^o sweep) (1p)

b) Calculate the number of consecutive scans N available to the radar to detect the target within the specified observation interval T_o. (1p)

c) Assume the first threshold (i.e. the detection threshold on each scan) was set to yield $P_{FA} = 10^{-4}$. What is the single-pulse signal-to-noise ratio SNR_{req} required to ensure the detection probability $P_D = 0.7$ on each scan? (4p)

d) With $P_{FA} = 10^{-4}$ and $P_D = 0.7$, compute the cumulative probabilities of detection $P_{CD} = P_D(2, N)$ and false alarm $P_{CFA} = P_{FA}(2, N)$ for binary detection procedure based on the "2 of N" decision rule. Compare $P_D(2, N)$ with P_D and $P_{FA}(2, N)$ with P_{FA} and explain the result. (4p)

Auxiliary Formulas

$$L_f(n_e) = \left[L_f(1)\right]^{1/n_e} \qquad (4.15)$$

$$L_f(n_e)[\text{dB}] = \frac{1}{n_e}\cdot\left(10\cdot\lg L_f(1)\right) = \frac{L_f(1)[\text{dB}]}{n_e}$$

$$n_e = \begin{cases} 1 & \text{for Swerling-1} \\ n & \text{for Swerling-2} \\ 2 & \text{for Swerling-3} \\ 2n & \text{for Swerling-4} \end{cases}$$

$n_e \to \infty$ for nonfluctuating model (Swerling - 0 or 5) and $L_f(1) = 1$ or $L_f(1)[\text{dB}] = 0$

$$SNR_{\text{req}} = D_e(n, n_e) = \frac{D_0(1)L_i(n)L_f(n_e)}{n} \qquad (4.16)$$

Binomial Distribution

For a random experiment that consists of N trials and satisfies the following conditions:

(1) Each trial results in only two possible outcomes (“1” or “0”)
(2) The trials are independent (the outcome of one trial has no effect on outcomes from any other trial)
(3) The probability of success in each trial, denoted as p, remains constant.

For such a random experiment, the random variable K that equals the number of trials that result in a success (the number of trials with outcomes that are “1”) is a so called *binomial random variable* or random variable that obeys a *binomial distribution*. The probability mass function of K is

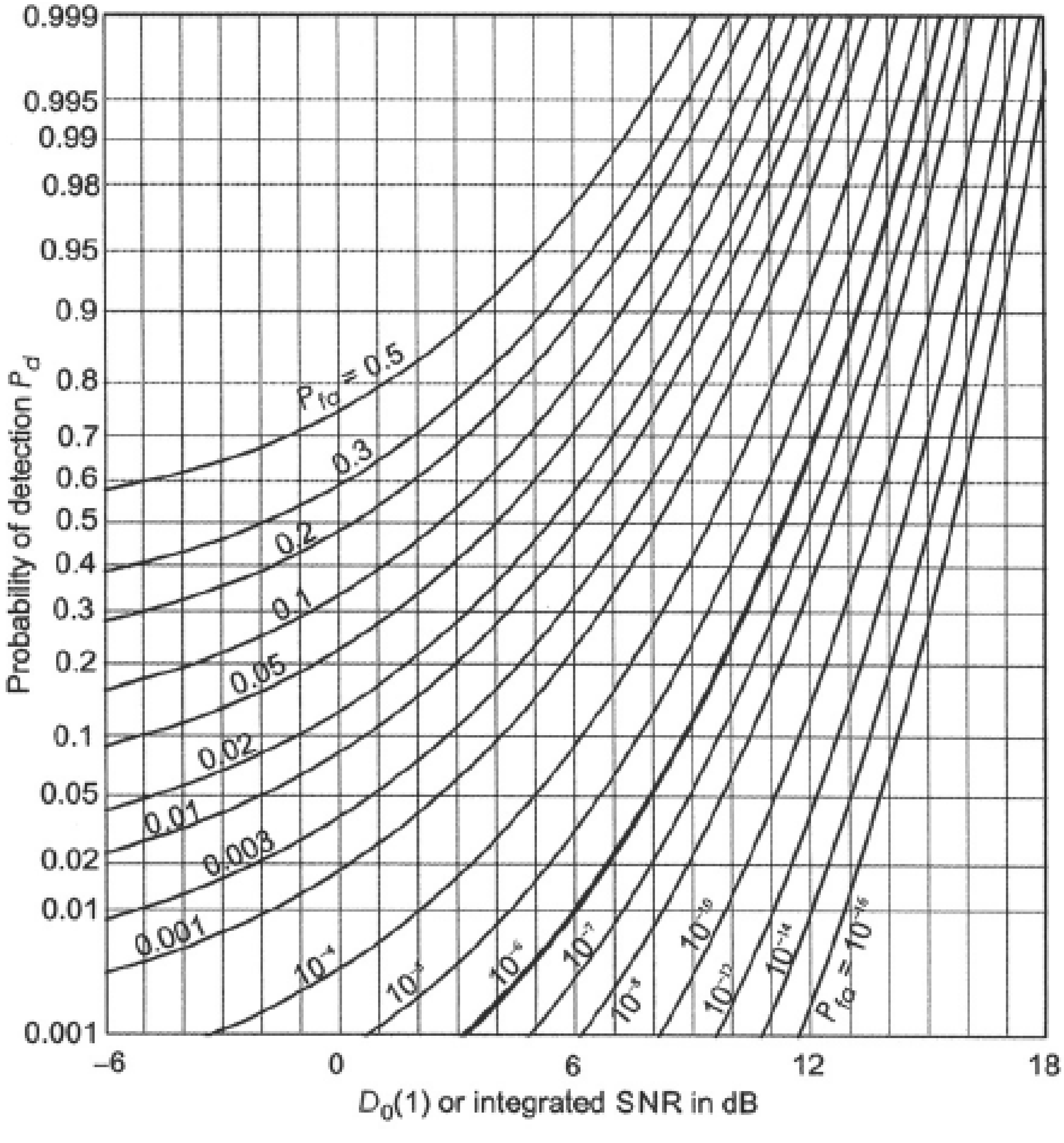
$$f(k) = P'(k, N) = \binom{N}{k} p^k (1-p)^{N-k},\ k = 0, 1, 2, ..., N$$

$$\text{where } \binom{N}{k} = \frac{N!}{k!(N-k)!},\ \text{ factorial } m! = 1\cdot 2\cdot 3\cdot ...\cdot (m-1)\cdot m \qquad (*)$$

Equation (1) defines the probability of exactly k “successes” (number of trials with outcomes “1”) in N independent trials, when the probability of success on each trial is p. The probability of at least M successes (that is the probability that k will be equal to M, or to M+1, …, or to N) for a binomial random variable is given by

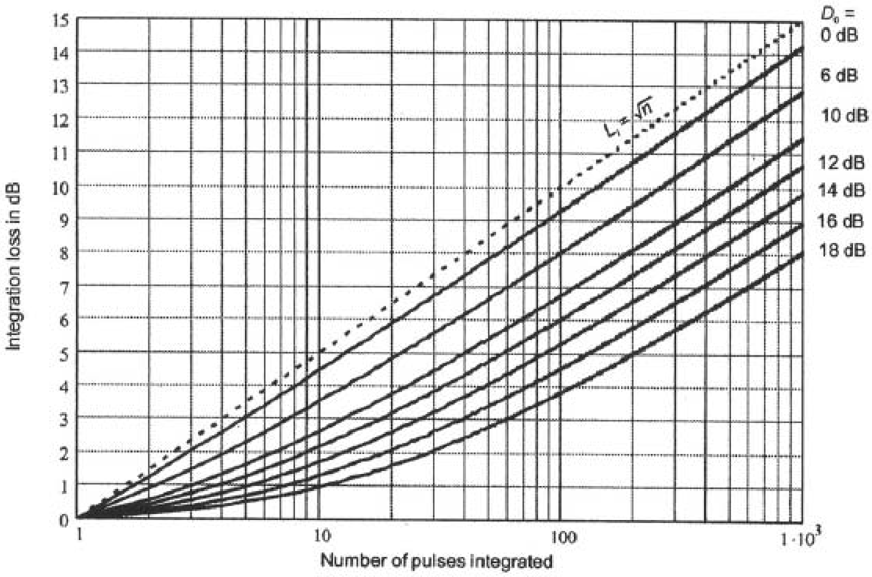
$$P(M \le k \le N, N) = P(M, N) = \sum_{k=M}^{N} \binom{N}{k} p^k (1-p)^{N-k} \qquad (**)$$

FIGURE 1



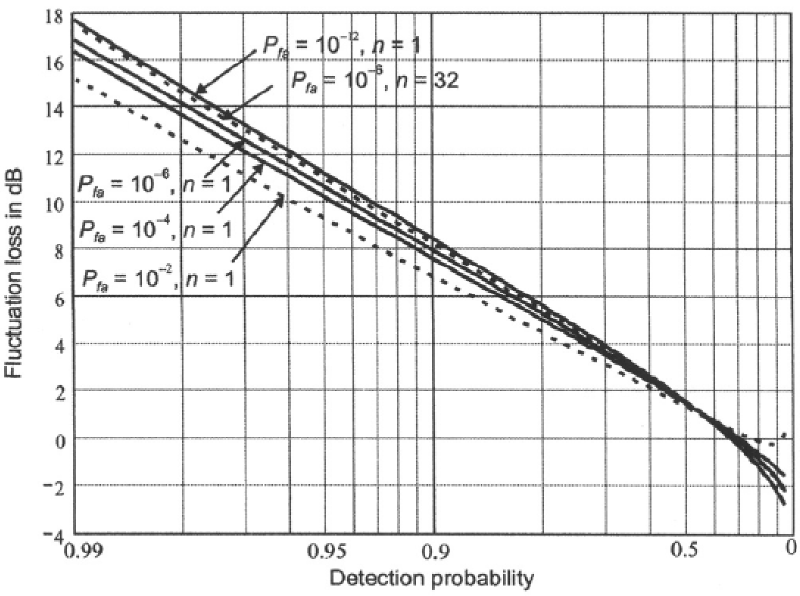
Detectability factor for a steady target
[David K. Barton, 2004, Radar System Analysis and Modeling, p. 45]

FIGURE 2



Integration loss versus number of pulses integrated after envelope detection for different values of output detectability factor $D_0(1)$
[David K. Barton, 2004, Radar System Analysis and Modeling, p. 52]

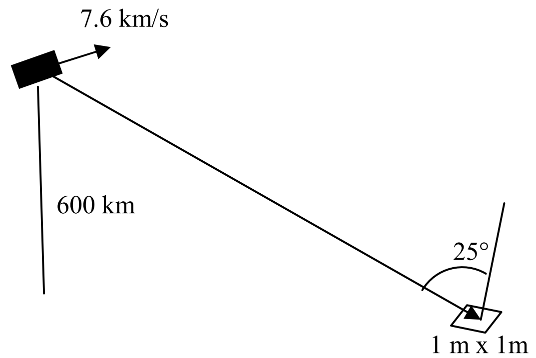
FIGURE 3



Fluctuation losses for case 1 target (Swerling-1 model)
[David K. Barton, 2004, Radar System Analysis and Modeling, p. 61]

5. Spaceborne Synthetic Aperture Radar

You are to perform a system design for a spaceborne SAR. The platform will fly in a circular orbit at an altitude of 600 km above the Earth's surface with an in-orbit velocity of 7.6 km/s. Consider a design at X-band, i.e. 10 GHz, which has a fixed antenna with area 3 m^2 pointing towards zero Doppler frequency. The required single-look image resolution is 1 m x 1 m in ground range and azimuth projected on a horizontal surface. The incidence angle at the center of the imaged swath is 25°. Assume that the Earth's surface is spherical with radius 6378 km.



a) Determine the required bandwidth of the radar signal. (1p)

b) Determine and define the antenna size in azimuth (in-orbit direction, cross-range) which matches the azimuth resolution requirement. (1p)

c) Determine the required average transmitted power to achieve a clutter-to-noise ratio (CNR) of 10 dB for σ° = -10 dB at mid-swath. Assume system noise temperature 1000 K, system losses 2 dB, and antenna aperture efficiency 50%. (4p)

d) Determine the lowest of the allowable PRF intervals which avoids range and Doppler ambiguities as well as interference from the nadir echo and transmit pulses. (4p)

Formulas which may be used at examination of the course Radar Systems and Applications (equation and page numbers refer to the course book by Sullivan)

Radar equation

$$SNR = \frac{P_{peak} G^2 \lambda^2 \sigma \tau}{(4\pi)^3 R^4 k_B T_s C_B L} \qquad (1.23)$$

$$SNR = \frac{P_{avg} G^2 \lambda^2 \sigma t_{dwell}}{(4\pi)^3 R^4 k_B T_s C_B L} \qquad (1.24)$$

Antennas

$$A_e = \frac{G\lambda^2}{4\pi} \qquad (1.18)$$

$$A_e = A\eta \qquad \text{(p. 15)}$$

$$R_{far} = \frac{2L^2}{\lambda} \qquad \text{(p. 43)}$$

System noise

$$k_B T_{sys} = k_B \left[T_{ant} / L_{radar} + T_{radar} \left(1 - 1/L_{radar}\right) + T_{rcvr} \right] \qquad (2.10)$$

$$T_{rcvr} = (F-1) T_0 \quad , F > 1 \qquad (2.11)$$

Radar Cross Section

$$\sigma = \lim_{R \to \infty} 4\pi R^2 \frac{|\mathbf{E_s}|^2}{|\mathbf{E_i}|^2} \qquad (3.1)$$

$$\sigma = 4\pi \frac{A^2}{\lambda^2}$$

(Broadside RCS of large metallic flat plate, p. 69-70)

Radar clutter

$$\sigma_c = \sigma^0 A \qquad \text{(p. 87)}$$

$$\sigma_c = \eta V \qquad \text{(p. 88)}$$

Envelope detection of targets embedded in noise (single-pulse decision)

a) Nonfluctuating (steady) target

$$P_D \approx \frac{1}{2} \left[erfc \left(\sqrt{\ln\left(\frac{1}{P_{FA}}\right)} - \sqrt{SNR + \frac{1}{2}} \right) \right]$$

(4.9) corrected

b) Fluctuating target

$$P_D = P_{FA}^{\;1/(1+SNR)} \qquad (4.13)$$

Matched filter

$$h(t) = Ku^*(t_m - t) \quad \Leftrightarrow \quad H(\omega) = KU^*(\omega)\exp(-j\omega t_m) \qquad (4.32)$$

Ambiguity function

$$\left|\chi(\tau,\nu)\right| = \left|\int_{-\infty}^{+\infty} u(t)u^*(t-\tau)\exp(j2\pi\nu t)\,dt\right| \qquad \text{(p. 116)}$$

Doppler frequency (this formula takes into account the sign of Doppler frequency)

$$f_d = -\frac{2v}{\lambda} = -\frac{2\cdot(dR/dt)}{\lambda} \qquad (1.32)$$

Maximum unambiguous velocity interval

$$\Delta v_u = \frac{f_R\lambda}{2} \qquad \text{(p. 22)}$$

Inverse SAR

$$\delta_{rpn} = \frac{c}{2B} \qquad (6.4)$$

$$\delta_{crpn} = \frac{\lambda}{2\Delta\phi} \qquad (6.5)$$

SAR

$$\delta_r \approx \frac{c}{2B} \qquad \text{(p. 216)}$$

$$\delta_{cr} \approx \frac{\lambda}{2\Delta\theta} \qquad (7.1)$$

$$CNR = \frac{P_{avg}G^2\lambda^3\sigma^0\delta_r}{2(4\pi)^3R^3k_BT_{sys}LV\cos\psi} \qquad (7.46)$$

Solutions to problems of exam 2010-08-25

1. Basic Terms and Principles

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a) $\frac{\gamma\tau}{2\pi} = 100\,\mathrm{MHz}$

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b) $\frac{\gamma\tau^2}{2\pi} = 1000$

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c) $\frac{8\pi a^4}{\lambda^2}$

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d) $1\,\mathrm{kW}$

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e) $4\cdot 10^{-4}\,\frac{W}{\mathrm{m}^2}$

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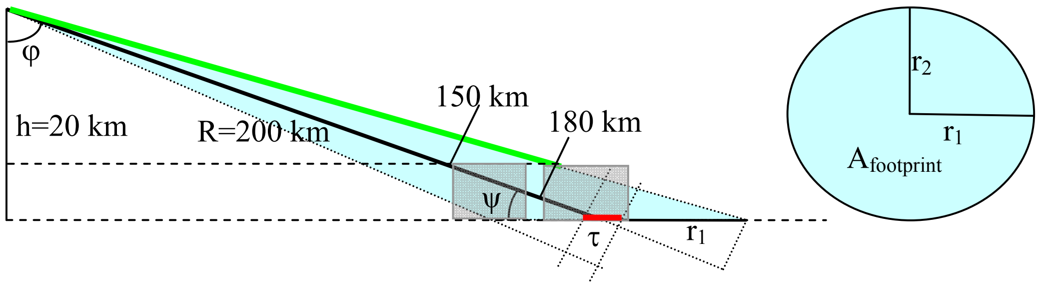
2. Attenuation and Backscatter Interference due to Rainstorm



Storm at 180 km will cause attenuation and backscatter that will interfere with ground backscatter at 200 km.
Storm at 150 km will cause attenuation but the backscattering from the storm will not interfere with the backscatter from the ground at 200 km.

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a) Attenuation of rain $L=\Delta R * k_a$

R_{storm}=150 km => ΔR=25 km, R_{storm}=180 km => ΔR =20 km.

k_{aKu}=5 dB/km $\quad$ k_{aX}=1.3 dB/km

Frequency band	Rstorm=150	Rstorm=180
Ku-band	L=125 dB	L=100 dB
X-band	L=32.5	L=26 dB

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b) RCS of terrain σ_{ground}:

$$r_2 = R\sin\frac{\varphi}{2} = 2\cdot 10^5 \sin\frac{4^o}{2} = 6980m$$

$$A_{illu\min ated} = 2r_\tau r_2 = \left\{ r_\tau = \frac{c\tau}{2\cos\psi} \right\} = 2\frac{3\cdot 10^8 \times 100\cdot 10^{-9}}{2\cdot\cos\left(\arcsin\left(0.1\right)\right)} r_2 = 30.15\cdot 6980 = 2.104\cdot 10^5$$

$$\sigma_{ground} = \sigma^0 A_{illu\min ated} = 10^{-2.5}\cdot 2.104\cdot 10^5 = 665.3m^2$$

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c) Ratio of received power from terrain and rain:

$$P_{rcvd} = \sigma\frac{\lambda^2 G^2}{\left(4\pi\right)^3 R^4 L} P_{trnsm}$$

$$\sigma_{rain} = \eta V = \eta\frac{\pi R^2}{4}\frac{\theta}{2}\varphi\frac{c\tau}{2} = \eta\frac{\pi R^2}{4}\cdot\frac{\pi}{180}\cdot\frac{4\pi}{180}\cdot\frac{3\cdot 10^8\cdot 1\cdot 10^{-7}}{2} = 1.44\cdot 10^{-2}\cdot\eta R^2 = 5.742\cdot 10^8\eta$$

$$B = \frac{P_{r_{rain}}}{P_{r_{ground}}} = \frac{\sigma_{rain}}{\sigma_{ground}} = \frac{5.742\cdot 10^8\cdot\eta}{665.3} = 8.632\cdot 10^5\eta$$

$$B_{Ku} = \left\{\eta = 10^{-4}\right\} = 86.32 = 19.4dB$$

$$B_X = \left\{\eta = 10^{-5}\right\} = 8.632 = 9.4dB$$

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d) X-band is less sensitive to rain.

3. Noise figure and system noise temperature

$$F_2 = \frac{\left(S/N\right)_{in}}{\left(S/N\right)_{out}} = \frac{\frac{T_{ant}}{L} + T_{radar}\left(1-\frac{1}{L}\right) + T_{recv}}{\frac{T_{ant}}{L} + T_{radar}\left(1-\frac{1}{L}\right)} = \{T_{ant} = T_{radar}\} = \frac{T_{ant} + T_{recv}}{T_{ant}} = 1 + \frac{T_{recv}}{T_{ant}}$$

From this we get $T_{recv} = T_{ant}(F_2-1)$

The noise figure F is defined in terms of the standard temp, T_0=290K [Sullivan, page 37]:
$T_{recv} = (F-1)T_0$, F>1
This can be re-written as $F = 1 + T_{recv}/T_0$

$$T_{ant} = T_r$$

$T_{recv} = 4.3$ K $\qquad$ $T_0 = 290$ K

$T_{recv} = (F-1)T_0$

$F = 1 + T_{recv}/T_0 = 1 + 4.3/290 = 1.01483$

$F_{dB} = 10\log F = 0.06392$

$T_{ant} = 3$ K $\qquad$ L = 1

$$T_{sys} = \frac{T_{ant}}{L} + T_{radar}\left(1-\frac{1}{L}\right) + T_{recv} = 3 + 0 + 4.3 = 7.3\ \mathrm{K}$$

4. Target detection

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a) Compute the number of pulses *n* received by the radar from the target on a single scan

$$n = \frac{\theta}{\Omega} PRF$$

$$\theta = 0.576^{\circ} \quad = 0.576\frac{\pi}{180}\,\text{radian} \qquad \Rightarrow \quad n = \frac{0.576\frac{\pi}{180}}{0.2\pi}1000 = \frac{0.576\cdot 1000}{0.2\cdot 180}$$

$$\Omega = 0.2\pi, \quad PRF = 1\,\text{kHz} = 1000\,\text{Hz} \qquad = 16$$

$\underline{n = 16}$

b) Calculate the number of consecutive scans N available to the radar to detect the target within the specified observation interval T_o.

The rotation period of the radar antenna (i.e. time needed to complete 360° sweep)

$$T_{rot} = \frac{2\pi}{\Omega} = \frac{2\pi}{0.2\pi} = 10 \text{ sec}$$

The number of successive scans available to the radar to detect the target is

$$N = \frac{T_o}{T_{rot}} = \frac{40}{10} = 4$$

$\underline{N = 4}$

c)

Step 1: Determine the detectability factor $D_0(1)$ [this corresponds to envelope detection of a single steady (i.e. nonfluctuating) pulse, see Roger J. Sullivan, Radar Foundations, p. 106] from Figure 2.5 [David K. Barton, 2004, Radar System Analysis and Modeling, p. 45] : $D_0(1)$ = 10.45 dB (see dashed blue lines in Fig. 1 below)

Step 2: Determine the integration loss for noncoherent integration of n = 16 pulses $L_i(\text{n})$ = $L_i(16)$ from Figure 2.8 [David K. Barton, 2004, Radar System Analysis and Modeling, p. 52]: for $D_0(1)$ = 10.45 dB we obtain $L_i(16) \approx 3.3$ dB (see dashed blue lines in Fig. 2)

Step 3: Determine the fluctuation loss $L_f(\text{n}_\text{e})$ for Swerling-2 model and number of pulses n = 16.

4.1 From Figure 2.10 [David K. Barton, 2004, Radar System Analysis and Modeling, p. 61] (see dashed blue lines in Fig. 3 below) we get $L_f(1)$ for Swerling-1: $L_f(1)$ = 3.5 dB

4.2. Then, using equation (4.15) [Sullivan, Radar Foundations, p. 109] yields:

$$L_f(n_e) = \left[L_f(1)\right]^{1/n_e}, \; L_f(n_e)[\text{dB}] = 10\lg L_f(n_e), \; L_f(n_e)[\text{dB}] = \frac{1}{n_e}\cdot\left(10\cdot\lg L_f(1)\right) = \frac{L_f(1)[\text{dB}]}{n_e}$$

Since given target fluctuation model is Swerling-2 case we have $n_e = n = 16$ and

$$L_f(16) = 3.5/16 = 0.22 \text{ dB}$$

Step 4. Compute the required signal-to-noise ration SNR_{req}.

To compute this value equation (4.16) [Roger J. Sullivan, Radar Foundations, p. 109] is used. It is handy to use a dB-form of the equation (4.16). Using eq. (4.16) yields:

$$SNR_{\text{req}} = D_e(n, n_e) = \frac{D_0(1)L_i(n)L_f(n_e)}{n}$$

$$SNR_{\text{req}}[\text{dB}] = D_0(1)[\text{dB}] + L_i(n)[\text{dB}] + L_f(n_e)[\text{dB}] - n[\text{dB}]$$

$$n[\text{dB}] = 10\lg n \;\rightarrow\; 10\lg 16 = 12.04\text{dB}$$

Thus, $\underline{\text{SNR}_{\text{req}}[\text{dB}] = 10.45 + 3.3 + 0.22 - 12.04 = 1.93 \text{ dB}}$

$\underline{\text{SNR}_{\text{req}} = 10^{0.1\text{SNRreq[dB]}} = 10^{0.193} = 1.56}$

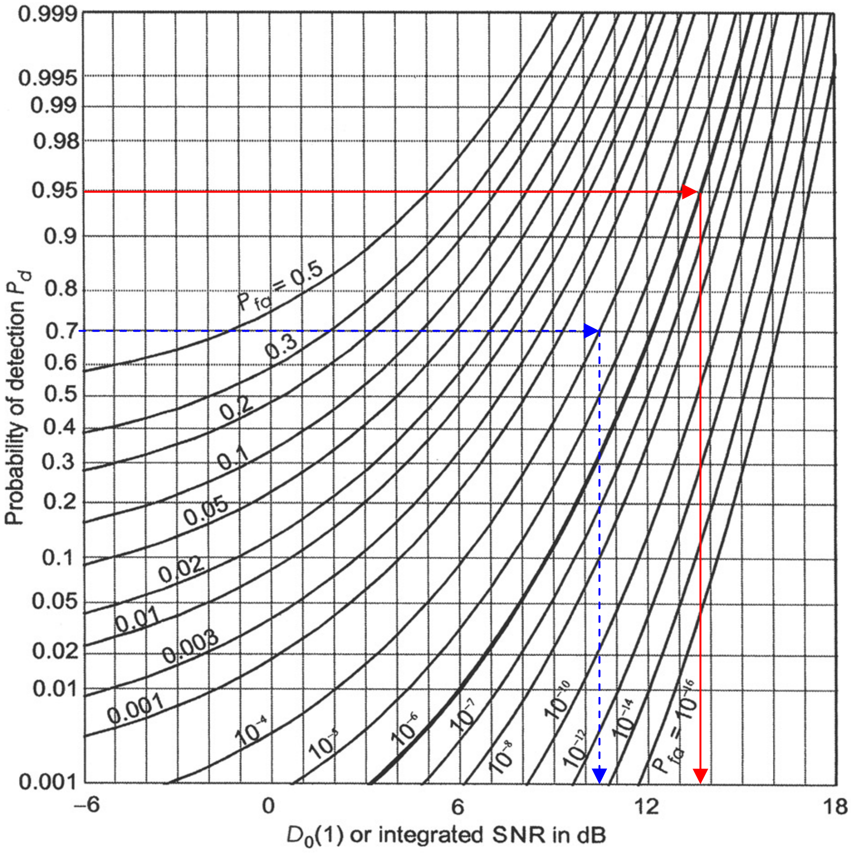

Fig. 1 Finding the detectability factor $D_0(1)$

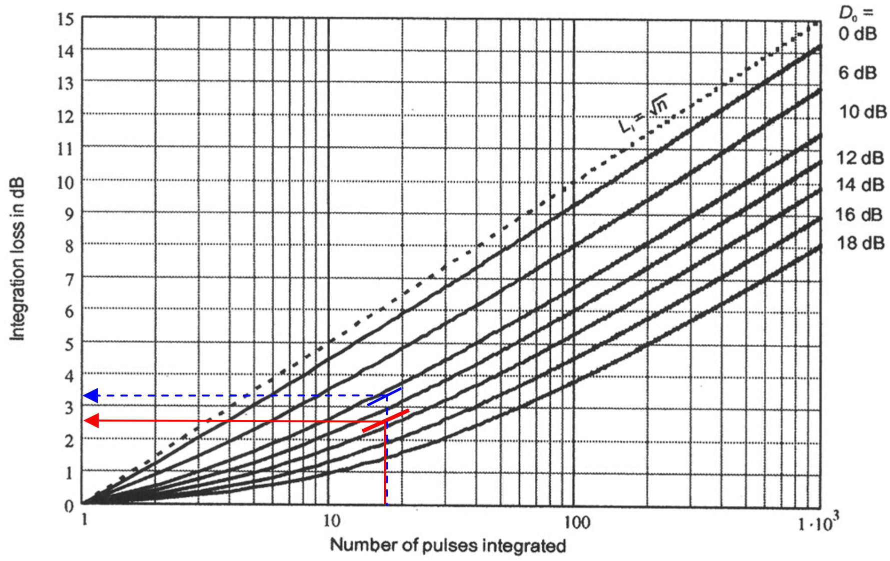

Fig. 2 Finding the integration loss L_i(n)

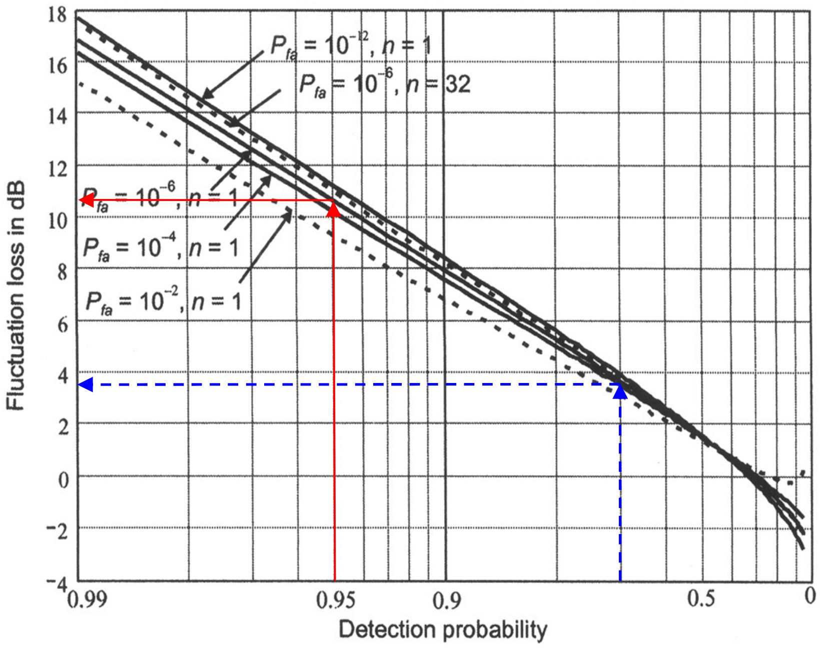

Fig. 3 Finding the fluctuation loss $L_f(1)$ for case 1 target (Swerling-1 model)

d) Using equations $P_D(M,N)=\sum_{k=M}^{N}\binom{N}{k}P_D^k(1-P_D)^{N-k}$ and $P_{FA}(M,N)=\sum_{k=M}^{N}\binom{N}{k}P_{FA}^k(1-P_{FA})^{N-k}$

we have for the "2 of 4" decision rule

$$
\begin{aligned}
P_D(2,4) &= \sum_{k=2}^{4}\binom{4}{k}0.7^k(1-0.7)^{4-k} \\
&= \binom{4}{2}0.7^2\,0.3^2+\binom{4}{3}0.7^3\,0.3^1+\binom{4}{4}0.7^4\,0.3^0 \\
&= 6\cdot 0.7^2\cdot 0.3^2+4\cdot 0.7^3\cdot 0.3^1+1\cdot 0.7^4\cdot 0.3^0 \\
&= 0.2646+0.4116+0.2401 \\
&= 0.9163
\end{aligned}
$$

$$\binom{4}{2}=\frac{4!}{2!(4-2)!}=\frac{4!}{2!\,2!}=\frac{1\cdot 2\cdot 3\cdot 4}{(1\cdot 2)\cdot(1\cdot 2)}=3\cdot 2=6$$

$$\binom{4}{3}=\frac{4!}{3!(4-3)!}=\frac{4!}{3!\,1!}=\frac{1\cdot 2\cdot 3\cdot 4}{(1\cdot 2\cdot 3)\cdot(1)}=4$$

$$\binom{4}{4}=\frac{4!}{4!(4-4)!}=\frac{4!}{4!\;0!}=\frac{4!}{4!\cdot 1}=1$$

$$
\begin{aligned}
P_{FA}(2,4) &= \sum_{k=2}^{4}\binom{4}{k}0.0001^k(1-0.0001)^{4-k} \\
&= \binom{4}{2}0.0001^2\,0.9999^2+\binom{4}{3}0.0001^3\,0.9999^1+\binom{4}{4}0.0001^4\,0.9999^0 \\
&= 6\cdot 0.0001^2\,0.9999^2+4\cdot 0.0001^3\,0.9999^1+1\cdot 0.0001^4\,0.9999^0 \\
&= 5.9992\cdot 10^{-8}\approx 6\cdot 10^{-8}
\end{aligned}
$$

Thus, we have
$$\underline{P_D(2,4)\;=0.9163\;>\;P_D=0.7}$$
$$\underline{P_{FA}(2,4)=6\cdot 10^{-8}<<P_{FA}=10^{-4}}$$

Explanation: It is known from the theory of binary integration that the "2 of 4" rule provides acceptable false alarm reduction for small p (in particular for $P_{FA}=10^{-4}$) and detection improvement for large values of p (in particular for $P_D = 0.7$). Generally speaking, the "2 of 4" rule is the best choice for N = 4.

5. Spaceborne Synthetic Aperture Radar

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a) $B = \dfrac{c}{2\delta_r \cos\psi} = \dfrac{c}{2\delta_r \cos 65^o} = 0.35\,\text{GHz}$

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b) $\dfrac{\lambda}{\Delta\vartheta} = 2\delta_{cr} = 2\,m$

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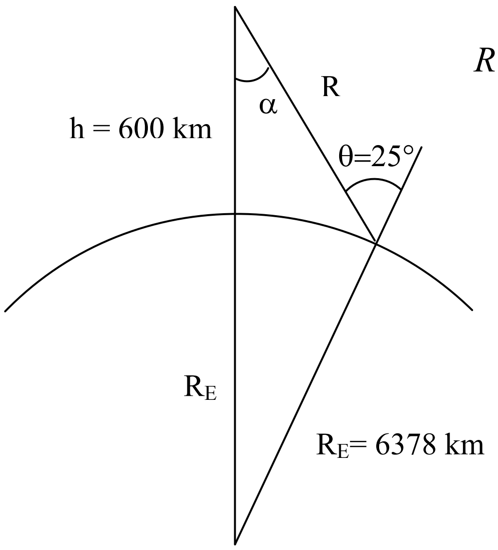
c)



$$R^2 = R_E^2 + \left(R_E + h\right)^2 - 2R_E\left(R_E + h\right)\cos\left(\theta - \alpha\right)$$

$$\frac{\sin\theta}{R_E + h} = \frac{\sin\alpha}{R_E}$$

$$\Rightarrow R = 656\,km$$

Antenna gain is $G = \dfrac{4\pi\eta A}{\lambda^2} = 2.09x10^4$

$$P_{avg} = CNR\frac{2(4\pi)^3 R^3 k_B T_{sys} LV\cos\psi}{G^2\lambda^3\sigma^0\delta_r} =$$

$$= 10\frac{2(4\pi)^3\left(656x10^3\right)^3 k_B(1000)\left(10^{0.2}\right)(7600)\cos 65^o}{\left(2.09x10^4\right)^2(0.03)^3\left(10^{-1}\right)(1)}W = 0.67\,kW$$

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d) $PRF_{\min} = \dfrac{2v}{w_{ant}} = 7600\,Hz$. Max PRF in lowest interval: 7622 Hz (defined by nadir echo, see fig).