Exam in Course Radar Systems and Applications (RRY080)

24th October 2009, 8:30am-12:30pm, Halls at "Väg och vatten"

Additional materials: Beta and Physics Handbook (or equivalent), Dictionary and Electronic calculator (it is free-of-choice but no computers are allowed)

Leif Eriksson (tel: 031 772 4856) and Anatoliy Kononov
will visit around 9:30am and 11:00am

Read through all exam questions before this time to check that you have understood the questions

Answers must be given in English

Clearly show answers (numbers and dimensions!) to each part of each question just after the corresponding solution

Begin each question on a new sheet of paper!

Each question carries a total of ten points. Where a question consists of different parts (subquestions), the number of points for each subquestion is indicated in parentheses. **In most cases it should be possible to answer later parts of the question even if you cannot answer one part.**

Grades will be awarded approximately as <20 = Fail, 20-29 = 3, 30-39 = 4, ≥40 = 5 (exact boundaries may be adjusted later).

Review of corrected exam papers will take place on the two following occasions:

Monday, 9th of November at 10:00-11:45
Thursday, 12th of November at 10:00-11:45

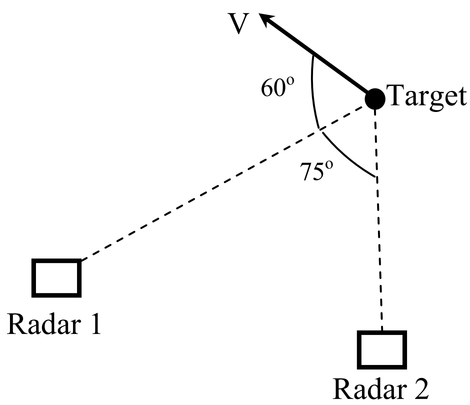
1. Basic Terms and Principles

a) A radar transmits pulses having rectangular envelope (amplitude) of duration T (pulsewidth) with pulse repetition interval $T_r = 10^{-3}$ s. The peak power of pulses is $P_{peak} = 10$ kW.
What is the condition for acceptable pulsewidth T if the average transmitted power P_{avg} must satisfy the condition $P_{avg} \le P_{avgmax} = 200$ W? (2p)

b) What is the area A m^2 of a metallic flat plate that yields the maximum radar cross section σ = 20 dBsm at mid X-band (f = 10 GHz)? (2p)

c) Compute the expected dynamic range $D_{SNR}= SNR_{max}/SNR_{min}$ (in decibels) for the signal-to-noise ratio (SNR) at the input to radar receiver if the range between radar and a given target is supposed to vary from a minimum of R_{min} = 2 km to a maximum of R_{max} = 100 km assuming free space measurements and other conditions being equal. (2p)

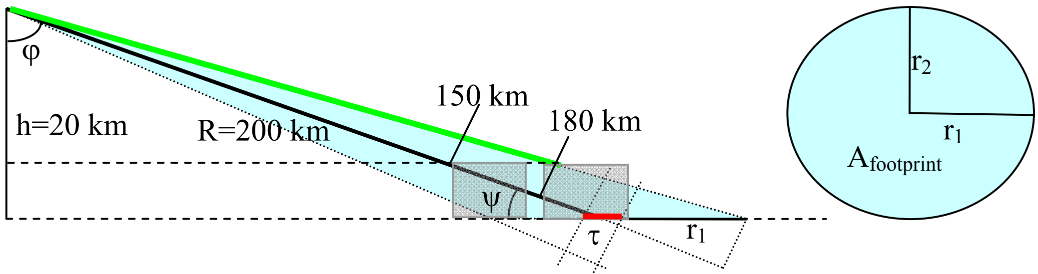
d) What target speed is measured by radar 1 and radar 2 (see figure below), respectively, if the magnitude of the velocity vector is |V| = 100 m/s? For each of the two radars, explain whether the target is coming towards or away (with respect to a radar) and compute the corresponding Doppler shifts if the radars operate at C-band (f = 6 GHz)? (2p)



e) What is the compression ratio CR for a linear frequency-modulated (LFM) pulse with the pulsewidth $T = 5\cdot10^{-6}$ s if it provides the range resolution δR = 15 m? (2p)

2. Attenuation and Backscatter Interference due to Rainstorm

An airborne radar has h = 20 km, vertical lobe width $\theta = 2^{\circ}$, horizontal lobe width $\phi = 4^{\circ}$ and pulsewidth τ = 100 ns. While observing terrain with σ^0 = -25 dB at a slant range R = 200 km, it encounters a large rainstorm with rainfall rate of 30 mm/h located between R_{storm} and R_{storm} + 25 km, and height 5 km. You should assume that the terrain is horizontal and flat, and may use the fact that the grazing angle is small.



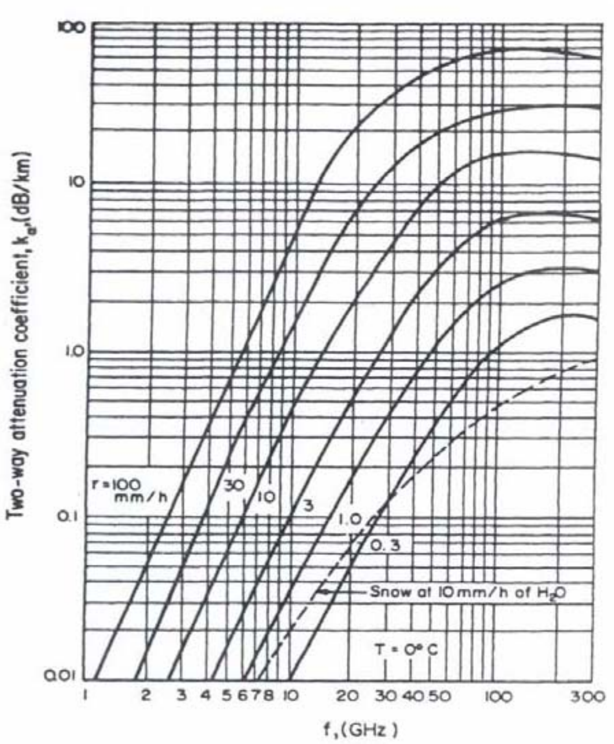
a) Estimate the attenuation (dB) due to the rain for f = 10 GHz (X-band) and 16 GHz (Ku-band); and for R_{storm} = 150 km and 180 km. (2p)

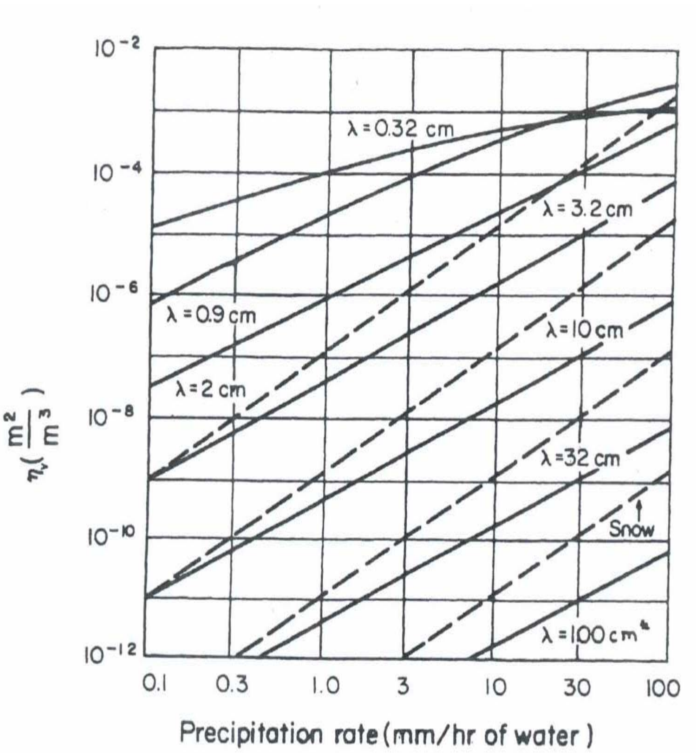
b) What is the radar cross section of terrain? (3p)

c) Estimate the ratio of received power from the terrain and from the rain at R_{storm} = 150 km and 180 km for X-band and Ku-band. (4p)

d) Would you, the designer, have chosen X-band or Ku-band for the radar? (1p)

Hint: The radar cross section of rain is given by $\sigma_{rain} = \eta_V V$, where V is the illuminated volume of rain in the echo (similar to the relation between σ and σ^0)





3. Ambiguity Function

a) The complex envelope of a linear frequency-modulated (LFM) signal is given by $u(t) = \frac{1}{\sqrt{T}} \exp\left(j\pi k t^2\right)$ for $|t| \le T/2$, zero otherwise, where $k = B/T$ is the frequency-modulation rate , B is the spectrum bandwidth (frequency sweep of the LFM signal), and $B \cdot T >> 1$.

- Write down an expression for the complex envelope of the impulse response $h(t)$ for the linear filter which is matched to this signal.
- Write down the formulas for the resolutions in time delay and Doppler for radar that uses this LFM signal. Draw a contour plot of the LFM ambiguity function at -3.92 dB level and indicate in the plot the segments that determine the radar resolution in time delay and Doppler. (3p)

b) Write down the definition for the complex ambiguity function $\chi(\tau,\nu)$ in terms of the signal's complex envelope $u(t)$. Prove that the complex ambiguity function can be presented in terms of the Fourier transform of $u(t)$ as

$\chi(\tau,\nu) = \int_{-\infty}^{\infty} U^*(f)U(f-\nu)e^{j2\pi f\tau}df$, where $U(f)$ is the Fourier transform of $u(t)$. (2p)

Hint: Use the definition of the ambiguity function in terms of the complex envelope, the inverse Fourier transform $u(t) = \int_{-\infty}^{\infty} U(f)e^{j2\pi f t}df$, and then the sifting property of the delta function $\delta(x)$:

$\int_{-\infty}^{\infty} f(x)\delta(x-x_0)dx = f(x_0)$

c) Prove that all the signals that have identical envelope $|u(t)|$, where $u(t) = |u(t)|\exp[j\varphi(t)]$ is the complex envelope of a signal and the phase modulation function $\varphi(t)$ can be different for different signals, have identical zero-delay cut of the ambiguity function $\chi(0,\nu)$. In other words this means that $\chi(0,\nu)$ depends only on the envelope $|u(t)|$ and does not depend on the phase modulation $\varphi(t)$ of a signal (1p)

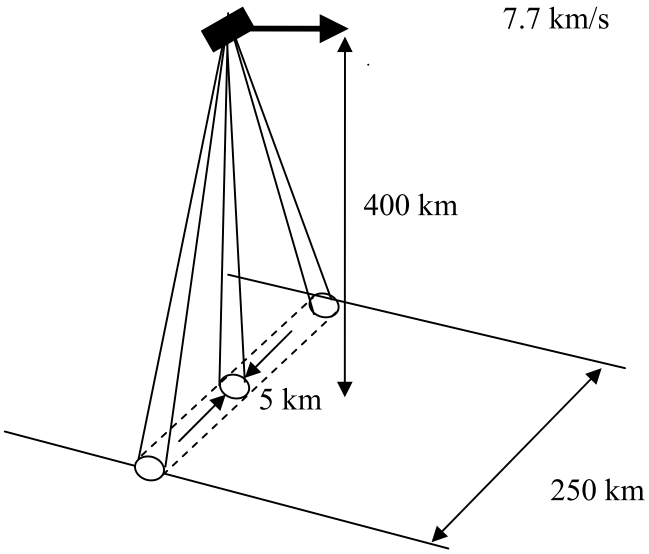
d) Prove that all the signals that have identical module of spectrum $|U(f)|$, where $U(f) = \int_{-\infty}^{\infty} u(t)e^{-j2\pi ft}dt$ is the Fourier transform of the complex envelope $u(t)$, have identical zero-Doppler cut of the ambiguity function $\chi(\tau,0)$ (1p)

e) Plot a zero-delay cut $|\chi(0,\nu)|$ over $-1.5/T_r \le \nu \le 1.5/T_r$ (T_r is the pulse repetition interval) for the ambiguity function of a coherent pulse train that consists of N = 6 constant-frequency pulses with rectangular envelope (amplitude). The pulsewidth is T and $T << T_r$. Mark in the plot positions of zeros and positions of the ambiguity peaks.

- Indicate in the plot the segment that determines the Doppler resolution (at -3.92 dB level)
- Write down a formula for the radar resolution in Doppler frequency assuming that radar employs the specified pulse train (3p)

4. Spaceborne Rain Radar

You are set to perform a system design for spaceborne rain radar. The platform will fly in a circular orbit at an altitude of 400 km above the ground surface and moving with a velocity of 7.7 km/s. Consider a design at Ku-band, i.e. 15 GHz, and an antenna which is electrically scanning a swath width of 250 km in the across-track direction around the nadir direction. The design requirement is to perform 3D-mapping of rain and unambiguously resolve cells from the ground surface to 20 km altitude with a range resolution of 30 m and with a horizontal resolution of 5 km as indicated in the figure. Assume horizontal and flat ground, i.e. neglect the geometrical effects of the spherical earth.



a) Determine the required bandwidth of the radar signal. (1p)

b) Determine the antenna size in the across-track dimension to fulfill the horizontal resolution requirement. (2p)

c) Assume a transmitted pulse length of 40 μs, and determine the maximum PRF for the nadir-pointing beam position to avoid range ambiguities. Select the maximum PRF so that the swath return does not coincide with a transmission event when the receiver is blocked. (3p)

d) Determine the along-track antenna size to avoid Doppler ambiguities using the selected PRF. (1p)

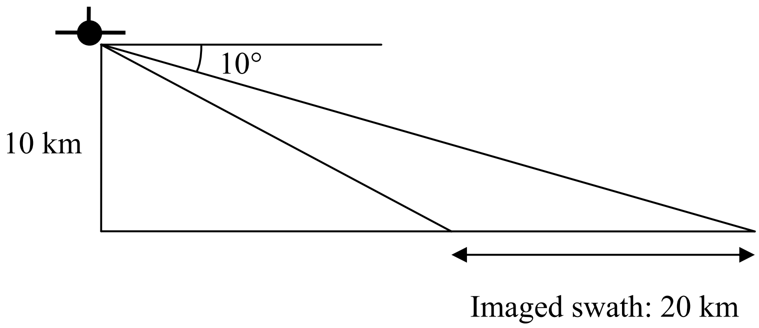
e) Determine the required transmitted peak power assuming that the signal-to-noise ratio is 10 dB after (lossless) pulse compression for a rain rate of 1 mm/hour. Assume that the rain cell is located in the nadir direction at 3 km above ground and completely fills the resolution volume. Set system noise temperature = 900 K, system losses =2 dB (including antenna beamshape loss), and antenna aperture efficiency = 50%. (3p)

Hint: Use the graph provided in problem 2 to estimate the rain backscatter

5. Airborne Synthetic Aperture Radar

You are designing a radar to be installed on a patrol aircraft. The platform operates at 10 km altitude with 250 m/s speed. The antenna is pointed at 10° depression angle, and zero Doppler frequency. System requirements and parameters are summarized in the table, and the imaging geometry is depicted in the figure Assume clear air conditions and horizontal ground in your design computations.

Centre frequency, f_c	10 GHz
Antenna gain (one-way, at boresight), G	32 dB
System noise temperature, Ts	2000 K
System losses (including antenna beamshape loss), L_s	5 dB
Boltzmann's constant, k_B	1.38×10^{-23} J/K
Transmitted pulse	Linear frequency modulated
Range resolution (single look)	1 m
Azimuth resolution (single look)	1 m
SNR (σ = 1 m^2 point target at swath far edge)	> 15 dB
CNR (σ^o = -30 dB clutter at swath far edge)	> 0 dB



a) Define the transmitted pulse parameters and write the time function for a single pulse which starts transmission at t = 0. Use the longest possible pulse which avoids receiving the swath echo at the same time as the pulse is being transmitted. (1p)

b) Determine the average transmitted power which fulfils both the signal-to-noise ratio (SNR) and clutter-to-noise (SCR) ratio requirements. (2p)

c) Compute the Doppler bandwidth corresponding to the single-look azimuth resolution and the minimum PRF. (2p)

d) Determine maximum PRF and duty factor which fulfils the requirements using the (longest) pulse while avoiding range ambiguities. Neglect the potential problem due to the nadir echo, i.e. assume that it can be sufficiently suppressed by the antenna pattern. Assume also that the transmit-receive switching time is negligible. (2p)

e) Determine the minimum peak transmitted power. (1p)

f) In this design, the required peak transmitted power can be significantly reduced by increasing the PRF from its minimum to maximum value. What is the main problem of using the maximum instead of the minimum PRF? Describe a method to reduce the problem. (2p)

Formulas which may be used at examination of the course Radar Systems and Applications (equation and page numbers refer to the course book by Sullivan)

Radar equation

$$SNR = \frac{P_{peak} G^2 \lambda^2 \sigma \tau}{(4\pi)^3 R^4 k_B T_s C_B L} \qquad (1.23)$$

$$SNR = \frac{P_{avg} G^2 \lambda^2 \sigma t_{dwell}}{(4\pi)^3 R^4 k_B T_s C_B L} \qquad (1.24)$$

Antennas

$$A_e = \frac{G\lambda^2}{4\pi} \qquad (1.18)$$

$$A_e = A\eta \qquad (p. 15)$$

$$R_{far} = \frac{2L^2}{\lambda} \qquad (p. 43)$$

System noise

$$k_B T_{sys} = k_B \left[T_{ant} / L_{radar} + T_{radar} \left(1 - 1/L_{radar}\right) + T_{rcvr} \right] \qquad (2.10)$$

$$T_{rcvr} = (F-1)T_0 \quad ,F > 1 \qquad (2.11)$$

Radar Cross Section

$$\sigma = \lim_{R \to \infty} 4\pi R^2 \frac{|\mathbf{E_s}|^2}{|\mathbf{E_i}|^2} \qquad (3.1)$$

$$\sigma = 4\pi \frac{A^2}{\lambda^2}$$ (Broadside RCS of large metallic flat plate, p. 69-70)

Radar clutter

$$\sigma_c = \sigma^0 A \qquad (p. 87)$$

$$\sigma_c = \eta V \qquad (p. 88)$$

Envelope detection of a single pulse

$$P_D \approx \frac{1}{2}\left[erfc\left(\sqrt{\ln\left(\frac{1}{P_{FA}}\right)} - \sqrt{SNR + \frac{1}{2}} \right) \right]$$ (4.9) corrected

Matched filter

$$h(t) = Ku^*(t_m - t) \quad \Leftrightarrow \quad H(\omega) = KU^*(\omega)\exp(-j\omega t_m) \qquad (4.32)$$

Ambiguity function

$$|\chi(\tau,\nu)| = \left| \int_{-\infty}^{+\infty} u(t)\, u^*(t-\tau) \exp(j2\pi\nu t)\, dt \right| \quad \text{(p. 116)}$$

Doppler frequency (this formula takes into account the sign of Doppler frequency)

$$f_d = -\frac{2v}{\lambda} = -\frac{2 \cdot (dR/dt)}{\lambda} \quad (1.32)$$

Maximum unambiguous velocity interval

$$\Delta v_u = \frac{f_R \lambda}{2} \quad \text{(p. 22)}$$

Inverse SAR

$$\delta_{rpn} = \frac{c}{2B} \quad (6.4)$$

$$\delta_{crpn} = \frac{\lambda}{2\Delta\phi} \quad (6.5)$$

SAR

$$\delta_r \approx \frac{c}{2B} \quad \text{(p. 216)}$$

$$\delta_{cr} \approx \frac{\lambda}{2\Delta\theta} \quad (7.1)$$

$$CNR = \frac{P_{avg} G^2 \lambda^3 \sigma^0 \delta_r}{2(4\pi)^3 R^3 k_B T_{sys} L V \cos\psi} \quad (7.46)$$

Solutions to problems of exam 2009-10-24

1. Basic Terms and Principles

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a) Using the relationship between the average and peak powers (for rectangular pulses) yields

$$P_{avg} = P_{peak}\frac{T}{T_r} \le P_{avg\max} \rightarrow T \le T_r \frac{P_{avg\max}}{P_{peak}}$$

$$T \le 10^{-3}\frac{200}{10\cdot 10^3} = 20\cdot 10^{-6}\, s$$

Answer: $T \le 20\cdot 10^{-6}\, s$

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b) For a flat plate of an area A the maximum radar cross section (in direction of a normal to plate) is given by: $\sigma = 4\pi\frac{A^2}{\lambda^2}$, $A >> \lambda^2$. Hence, $A = \lambda\sqrt{\frac{\sigma}{4\pi}}$, where $\lambda = \frac{c}{\mathrm{f}}$ (c = $3\cdot 10^8$ m/s).

For the radar cross section one can write $\sigma[\mathrm{dBsm}] = 10\log\left(\frac{\sigma[\mathrm{m}^2]}{1[\mathrm{m}^2]}\right) \rightarrow \sigma[\mathrm{m}^2] = 10^{0.1\cdot\sigma[\mathrm{dBsm}]}$.

Thus, we have

$$\lambda = \frac{3\cdot 10^8}{10\cdot 10^9} = 0.03\,\mathrm{m},\ \sigma = 10^{0.1\cdot 20} = 100\,\mathrm{m}^2,\ A = 0.03\sqrt{\frac{100}{4\pi}} = 0.0846\,\mathrm{m}^2 >> \lambda^2 = 0.0009\,\mathrm{m}^2$$

Answer: $A = 0.0846\,\mathrm{m}^2$

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c) Using free space radar equation (of any form) yields

$$D_{SNR} = \frac{SNR_{\max}}{SNR_{\min}} = \frac{R_{\max}^4}{R_{\min}^4} = \left(\frac{R_{\max}}{R_{\min}}\right)^4$$

$$D_{SNR}[\mathrm{dB}] = 10\cdot\log D_{SNR} = 10\cdot\log\left(R_{\max}/R_{\min}\right)^4 = 40\cdot\log\left(R_{\max}/R_{\min}\right)$$

$$D_{SNR}[\mathrm{dB}] = 40\cdot\log\frac{100}{2} = 40\cdot(\log 100 - \log 2) \approx 40\cdot(2-0.3) = 68\,\mathrm{dB}$$

Answer: $D_{SNR}[\mathrm{dB}] \approx 68\,\mathrm{dB}$

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d) From figure on the next page for the magnitude of the projections V_{r1} and V_{r2} of the vector V as seen by the radars 1 and 2 one can write, respectively

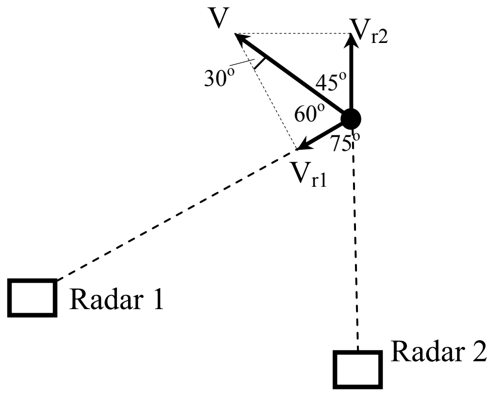
$|V_{r1}| = |V|\cos 60^o = |V|\sin 30^o = 100\cdot 0.5 = 50$ m/s

$|V_{r2}| = |V|\cos 45^o = 100/\sqrt{2} = 70.7$ m/s

Radar 1 sees the target as an inbound object: the direction of vector V_{r1} is toward the radar 1 so the range between the target and radar 1 decreases, hence, dR/dt < 0 and the projection V_{r1} is negative so the sign of the Doppler frequency f_{d1} as measured by the radar 1 is positive since

$$\mathrm{f_d} = -\frac{2\cdot \mathrm{V_r}}{\lambda} = -\frac{2\cdot(\mathrm{dR/dt})}{\lambda} = -\frac{2\cdot(\pm|\mathrm{V_r}|)}{\lambda}$$

Radar 2 sees the target as an outbound object: the direction of vector V_{r2} is outward the radar 2 so the range between the target and radar 2 increases, hence, dR/dt > 0 and the projection V_{r2} is positive so the sign of the Doppler frequency f_{d1} as measured by the radar 2 is negative



The corresponding Doppler frequencies are

$$f_{d1} = -\frac{2 \cdot V_{r1}}{\lambda} = -\frac{2 \cdot (-|V_{r1}|)}{\lambda} = -\frac{2 \cdot (-50.0)}{0.05} = +2000\,\text{Hz}$$

$$f_{d2} = -\frac{2 \cdot V_{r2}}{\lambda} = -\frac{2 \cdot (+|V_{r2}|)}{0.05} = -\frac{2 \cdot (+70.7)}{0.05} = -2828\,\text{Hz}$$

Answer: $|V_{r1}|$ = 50 m/s, inbound target, $f_{d1} = +2000\,\text{Hz}$;
$|V_{r2}|$ = 70.7 m/s, outbound target, $f_{d2} = -2828\,\text{Hz}$

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e) Using the relationship between the spectrum bandwidth B and range resolution $\delta R = \frac{c}{2B}$

we have $B = \frac{c}{2\delta R} = \frac{3 \cdot 10^8}{2 \cdot 15} = 10^7\,\text{Hz} = 10\,\text{MHz}$ and $CR = BT = 10^7 \cdot 5 \cdot 10^{-6} = 50$

Answer: $CR = 50$.

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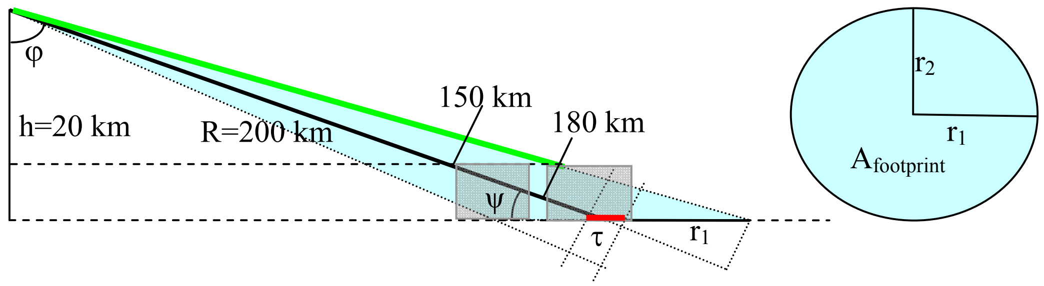
2. Attenuation and Backscatter Interference due to Rainstorm



Storm at 180 km will cause attenuation and backscatter that will interfere with ground backscatter at 200 km.
Storm at 150 km will cause attenuation but the backscattering from the storm will not interfere with the backscatter from the ground at 200 km.

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a) Attenuation of rain $L=\Delta R * k_a$

R_{storm}=150 km => ΔR=25 km, R_{storm}=180 km => ΔR =20 km.

k_{aKu}=5 dB/km $\qquad$ k_{aX}=1.3 dB/km

Frequency band	Rstorm=150	Rstorm=180
Ku-band	L=125 dB	L=100 dB
X-band	L=32.5	L=26 dB

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b) RCS of terrain σ_{ground}:

$$r_2 = R\sin\frac{\varphi}{2} \quad = 2\cdot 10^5 \sin\frac{4^o}{2} \quad = 6980m$$

$$A_{illu\min ated} = 2r_\tau r_2 = \quad \left\{ r_\tau = \frac{c\tau}{2\cos\psi} \right\} \quad = 2\frac{3\cdot 10^8 \times 100\cdot 10^{-9}}{2\cdot\cos\left(\arcsin(0.1)\right)} r_2 \quad = 30.15\cdot 6980 = 2.104\cdot 10^5$$

$$\sigma_{ground} = \sigma^0 A_{illu\min ated} \quad = 10^{-2.5}\cdot 2.104\cdot 10^5 = 665.3m^2$$

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c) Ratio of received power from terrain and rain:

$$P_{rcvd} = \sigma\frac{\lambda^2 G^2}{(4\pi)^3 R^4 L} P_{trnsm}$$

$$\sigma_{rain} = \eta V = \eta\frac{\pi R^2}{4}\frac{\theta}{2}\varphi\frac{c\tau}{2} = \eta\frac{\pi R^2}{4}\cdot\frac{\pi}{180}\cdot\frac{4\pi}{180}\cdot\frac{3\cdot 10^8\cdot 1\cdot 10^{-7}}{2} = 1.44\cdot 10^{-2}\cdot\eta R^2 = 5.742\cdot 10^8\eta$$

$$B = \frac{P_{r_{rain}}}{P_{r_{ground}}} \quad = \quad \frac{\sigma_{rain}}{\sigma_{ground}} = \quad \frac{5.742\cdot 10^8\cdot\eta}{665.3} = 8.632\cdot 10^5\eta$$

$$B_{Ku} \quad = \left\{\eta = 10^{-4}\right\} \quad = 86.32 = 19.4dB$$

$$B_X \quad = \left\{\eta = 10^{-5}\right\} \quad = 8.632 = 9.4dB$$

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d) X-band is less sensitive to rain.

3. Ambiguity Function

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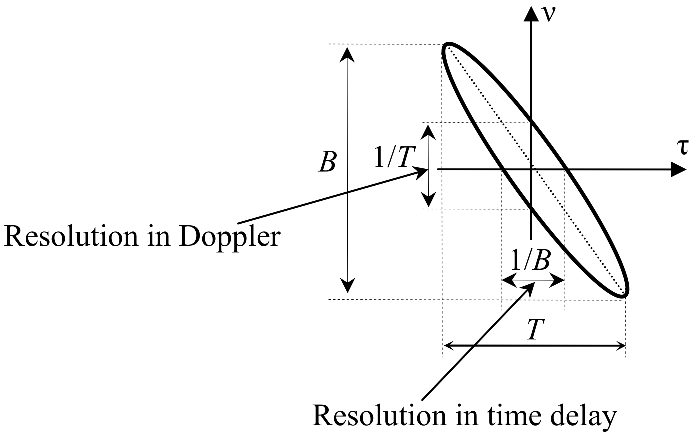
a) The impulse response of the matched filter for a signal with complex envelope $u(t)$ is given by $h(t) = Ku^*(t_m - t),\ t_m \ge T$. Then, the impulse response of the filter matched to the specified LFM signal

$$h(t) = \frac{1}{\sqrt{T}} \exp\left[-j\pi k(T-t)^2\right]$$

Contour plot at – 3.92 dB level for the LFM ambiguity function



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b) The definition of the ambiguity function in terms of the complex envelope $u(t)$

$$\chi(\tau,\nu) = \int_{-\infty}^{\infty} u(t)u^*(t-\tau)\,\mathrm{e}^{j2\pi\nu t}\,dt \quad (*)$$

The relationship between the complex envelope $u(t)$ and its Fourier transform $U(f)$ (spectrum)

$$U(f) = \int_{-\infty}^{\infty} u(t)e^{-j2\pi f t}dt \quad (1)$$

$$u(t) = \int_{-\infty}^{\infty} U(f)e^{j2\pi f t}df \quad (2)$$

Substituting (2) into the definition (*) yields

$$\chi(\tau,\nu) = \int_{-\infty}^{\infty} u(t)u^*(t-\tau)\,\mathrm{e}^{j2\pi\nu t}\,dt$$

$$= \int_{-\infty}^{\infty} \underbrace{\int_{-\infty}^{\infty} U(f_1)e^{j2\pi f_1 t}df_1}_{u(t)} \underbrace{\int_{-\infty}^{\infty} U^*(f_2)e^{-j2\pi f_2(t-\tau)}df_2}_{u^*(t-\tau)}\,\mathrm{e}^{j2\pi\nu t}\,dt$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} U(f_1)U^*(f_2)\ \underbrace{\int_{-\infty}^{\infty} e^{j2\pi(f_1-f_2+\nu)t}dt}_{\delta(f_1-f_2+\nu)}\ df_1\,\mathrm{e}^{j2\pi f_2\tau}\,df_2$$

$$= \int_{-\infty}^{\infty} U^*(f_2)\int_{-\infty}^{\infty} U(f_1)\delta[f_1-(f_2-\nu)]df_1\ e^{j2\pi f_2\tau}df_2 \quad (3)$$

Then by using the fundamental property of the delta-function (sifting property)

$$\int_{-\infty}^{\infty} f(x)\delta(x-x_0)dx = f(x_0)$$

we get from formula (3) $\chi(\tau,\nu)=\int_{-\infty}^{\infty} U^*(f_2)U(f_2-\nu)e^{j2\pi f_2\tau}df_2$ and, finally, by changing $f_2 \to f$

$$\chi(\tau,\nu)=\int_{-\infty}^{\infty} U^*(f)U(f-\nu)e^{j2\pi f\tau}df$$

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c) From the definition $\chi(\tau,\nu)=\int_{-\infty}^{\infty} u(t)u^*(t-\tau)\,\mathrm{e}^{j2\pi\nu t}\,dt$ we have

$\chi(0,\nu)=\chi(\tau,\nu)\big|_{\tau=0}=\int_{-\infty}^{\infty} u(t)u^*(t)\,\mathrm{e}^{j2\pi\nu t}\,dt=\int_{-\infty}^{\infty} |u(t)|^2\,\mathrm{e}^{j2\pi\nu t}\,dt$

It is clear that $\chi(0,\nu)$ is the Fourier transform of the envelope squared $|u(t)|^2$ and does not depend on the phase modulation $\varphi(t)$ of a signal

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d) Using $\chi(\tau,\nu)=\int_{-\infty}^{\infty} U^*(f)U(f-\nu)e^{j2\pi f\tau}df$ yields

$\chi(\tau,0)=\chi(\tau,\nu)\big|_{\nu=0}=\int_{-\infty}^{\infty} U^*(f)U(f)e^{j2\pi f\tau}df=\int_{-\infty}^{\infty} |U(f)|^2\,e^{j2\pi f\tau}df$

It is clear that $\chi(\tau,0)$ is the Fourier transform of the module of the spectrum squared $|U(f)|^2$

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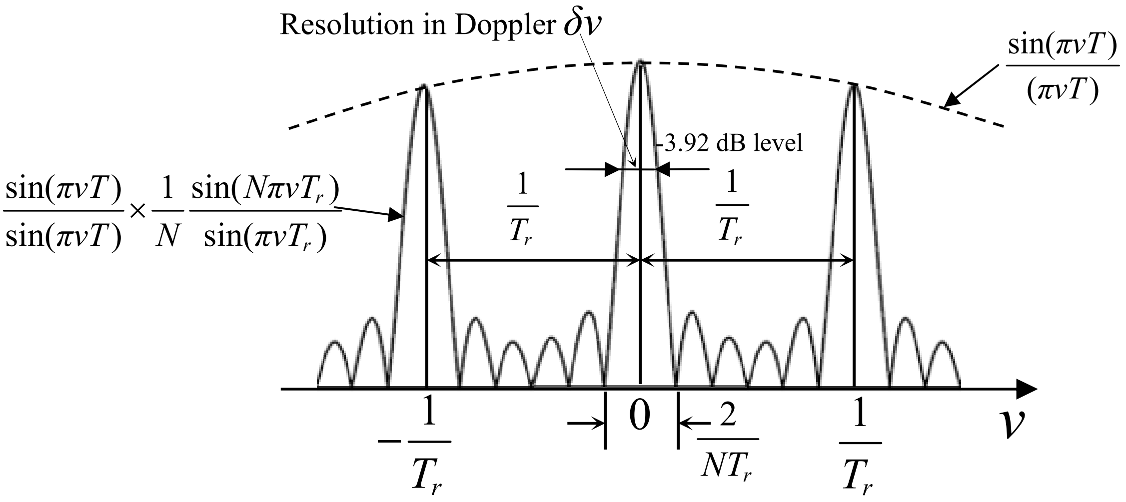
e)



Resolution in Doppler $\delta\nu=\frac{1}{NT_r}=\frac{1}{6T_r}$

4. Spaceborne Rain Radar

4a) $B = \frac{c}{2\delta_r} = \frac{2.9979x10^8}{2 \cdot 30} Hz = 5.0MHz$

4b) $D_1 = \frac{\lambda R}{\delta_{hor}} = \frac{0.02 \cdot 400}{5} = 1.6m$

4c) $PRF_{\max} = \frac{c}{2R_u} = \frac{2.9979x10^5}{2(20 + 2 \cdot 40 \cdot 0.15)} = 4684Hz$

Select $PRF'_{\max} = \frac{12 \cdot 2.9979x10^5}{2(400 + 40 \cdot 0.15)} = 4430Hz$

4d) $D_2 = \frac{2v}{PRF} = \frac{2 \cdot 7700}{4430} = 3.5m$

4e) $G = \frac{4\pi A_e}{\lambda^2}\eta = \frac{4\pi(2.8)(1.6)}{(0.02)^2}0.5 = 48.5dB \qquad V = 30 \cdot 5000 \cdot \frac{0.02}{3.5} \cdot 397000 = 3.40x10^8 m^3$

Volume backscattering coefficient for 1 mm/h from graph: $\eta_v = 10^{-6}\, m^2/m^3$

$$P_{peak} = SNR\frac{(4\pi)^3 R^4 k_B T_s L}{G^2\lambda^2\eta_v V\tau} = 10\frac{(4\pi)^3(397x10^3)^4(1.38x10^{-23})(900)(10^{0.2})}{(10^{4.85})^2(0.02)^2(10^{-6})(3.40x10^8)(40x10^{-6})} = 0.36kW$$

4f) $t_{dwell} = \frac{0.02 \cdot 400}{3.5 \cdot 7.700}\frac{5}{250}s = 0.059s \quad P_{peak} = SNR\frac{(4\pi)^3 R^4 k_B T_s L}{G^2\lambda^2\eta_v V\tau(PRF \cdot t_{dwell})} = \frac{0.36kW}{26} = 14W$

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5. Airborne Synthetic Aperture Radar

5a) $s(t) = \cos\left(2\pi ft + \pi Kt^2\right)$ for $0 < t < \tau$; otherwise s(t) =0

with $\tau = \frac{2R_{\min}}{c} = \frac{2 \cdot 38.1}{3x10^5} s = 254 \mu s$; $B = \frac{c}{2\delta_r} = \frac{3x10^8}{2} Hz = 150 MHz$; $f = 9.925 GHz$

5b) The clutter defines the average power since the clutter radar cross section within a resolution cell is 10^{-3} m^2 which is less than the corresponding requirement for the point target ($10^{-1.5}$ m^2).

$$P_{avg} = CNR \frac{2(4\pi)^3 R^3 k_B T_{sys} L V \cos\psi}{G^2 \lambda^3 \sigma^0 \delta_r} = \frac{2(4\pi)^3 \left(56.7x10^3\right)^3 k_B (2000)\left(10^{0.5}\right)(250)\cos 10^o}{\left(10^{3.2}\right)^2 (0.03)^3 \left(10^{-3.5}\right)(1)} W = 725 W$$

5c) $PRF_{\min} = B_D = \frac{v}{\delta_a} = \frac{250}{1} Hz = 250 Hz$

5d) $PRF_{\max} = \frac{c}{2R_u} = \frac{3x10^5}{2\left(\frac{10}{\tan 10^o} + 254 \cdot 0.15\right)} Hz = 1582 Hz$

5e) $P_{peak} = \frac{P_{avg}}{PRF_{\max}\tau} = \frac{725}{0.40} W = 1.8 kW$

5f) The main drawback is the increased data rate due to the higher PRF. It can be mitigated by implementing a Doppler low-pass filter.
