Exam in Course Radar Systems and Applications (RRY080)

15th January 2008, 14:00-18:00.

Additional materials: Beta, Physics Handbook, Approved electronic calculator, Dictionary.

Gustaf Sandberg (tel: 5652) will visit around 15:00 and 17:00.

Read through all questions before this time to check that you have understood the questions.

Answers may only be given in **English**.

Each question carries a total of ten points. Where a question consists of different parts, the number of points for each part of the question is indicated in parentheses. **In most cases it should be possible to answer later parts of the question even if you cannot answer one part.**

Grades will be awarded approximately as <20 = Fail, 20-29 = 3, 30-39 = 4, ≥40 = 5
(exact boundaries may be adjusted later).

Review of corrected exam papers will take place on the two following occasions:

Monday the 26th of January, 10:00-11:45
Monday the 2nd of February, 10:00-11:45

Clearly show answers (numbers and dimensions!) to each part of question just after the corresponding solution.

Begin each question on a new sheet of paper!

1. Basic terms & principles

a) An oscilloscope is used to capture both the transmitted and received signals in a radar system. The peak-to-peak voltages of the sinusoidal signals are measured to 100 V and 10^{-6} V, respectively. Compute the ratio in dB between the measured voltages. (2p)

b) What is the maximum free-space RCS of a dihedral corner reflector composed of two orthogonal square metallic plates, each with side length a >> c/f (c = speed of light, f = frequency). (2p)

c) Compute the pulse length τ required for achieving a radial velocity resolution of 1 m/s with the following waveform (f_0 = 25 GHz):

$$s(t) = \begin{cases} \cos(2\pi f_0 t) & \text{for } |t| \le \tau/2 \\ 0 & \text{otherwise} \end{cases}$$ (3p)

d) Compute the distance to the radar horizon for a marine navigation radar located 10 m above sea level. Assume that the Earth radius is R_E = 6378 km and take into account refraction by air in normal atmospheric conditions. (3p)

2. Noise figure and system noise temperature

a) Show that, if we for a radar system define the noise figure $F_2 = (S/N)_{in}/(S/N)_{out}$ and if $T_{antenna} = T_{radar}$, then $T_{receiver} = (F_2 - 1)T_{antenna}$. T stands for temperature, S is the signal power and N is the noise power. Subscript "in" stands for input, and "out" stands for output. (3p)

b) Show that the noise figure $F = (S/N)_{in}/(S/N)_{out} = (N_{out}/N_{in})(S_{in}/S_{out}) = (N_{out}/N_{in})(1/G_{LNA})$ if $T_{antenna} = T_{radar} = T_0$. G_{LNA} is the gain of the low noise amplifier. (3p)

Consider a sensitive radar observing targets against deep space ($T_{antenna}$ = 3K), with an LNA cooled with liquid helium to $T_{receiver}$ = 4.3K. Assume that the losses L_{radar} = 0 dB. Use a standard temperature T_0 = 290 K.

c) What is the noise figure in decibel? (2p)

d) Calculate the system temperature T_{system}? (2p)

3. Matched Filter and Ambiguity Function

The complex envelope of a linear-frequency modulated (LFM) signal is given by

$$u(t) = \frac{1}{\sqrt{T}} \exp\left(j\pi k t^2\right) \text{ for } |t| \le T/2 \text{, zero otherwise}$$

where $k = \pm B/T$ is the frequency-modulation rate.

a) Prove that the energy of this complex envelope is equal to unity. (1p)

b) Under what conditions does B represent the pulse bandwidth? (1p)

c) Write down an expression for the complex envelope of the impulse response h(t) for the linear filter which is matched to this signal. (1p)

d) Write down the general expression for the maximum signal-to-noise ratio (SNR_{max}) attainable at the output of a linear filter, that is the expression for the signal-to-noise ratio (SNR) at the output of the matched filter. (1p)

e) Show that maximum SNR at the output of the matched filter is $SNR_{\max} = CR \cdot SNR_{in}$, where CR is the compression ratio and SNR_{in} is the SNR at the input of the matched filter. (**Hint:** Assume for simplicity that the spectrum of the complex envelope of the signal is concentrated within frequency band from -B/2 to B/2, where B is the bandwidth of the signal and equals zero outside of this frequency band and amplitude A of the signal is constant. Note also that the average SNR at the input of the matched filter can be written as $SNR_{in} = P_{sin}/P_{nin} = A^2/(2P_{nin})$, where $P_{sin} = A^2/2$ is the average signal power at the input of the filter and the input noise power P_{nin} within the signal bandwidth (as well as within the bandwidth of the matched filter) can be represented as $P_{nin} = (N_o/2)\cdot B$ since the power spectral density of the input noise is constant.) (2p)

f) Write down the general expression (by the definition) for the ambiguity function. (1p)

g) Prove the following fundamental property of the ambiguity function (Property 1):

- At point τ = 0, ν = 0 the ambiguity function is equal to unity, that is $|\chi(0,0)| = 1$.
- For all other points on the (τ, ν) plane the ambiguity function cannot be greater than $|\chi(0,0)|$ and generally is less, that is $0 \le |\chi(\tau,\nu)| \le |\chi(0,0)| = 1$.

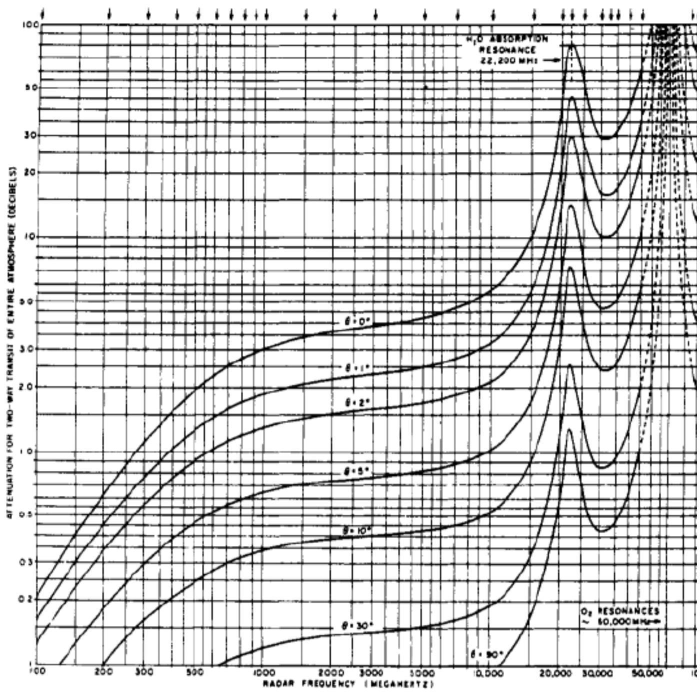
This property states that the ambiguity function can not be higher than at the origin, where it is normalized to unity by normalizing with the signal energy $\int_{-\infty}^{+\infty} |u(t)|^2 dt = 1$. (**Hint:** Apply the Schwarz inequality to $|\chi(\tau,\nu)|^2$, that is for any two complex functions f(x) and g(x) the following inequality is true: $\left|\int_{-\infty}^{+\infty} f(x) g(x) dx\right|^2 \le \int_{-\infty}^{+\infty} |f(x)|^2 dx \int_{-\infty}^{+\infty} |g(x)|^2 dx$. The equality holds if and only if $f(x) = \alpha g^*(x)$, where α is an arbitrary complex constant.) (3p)

4. Radar system design

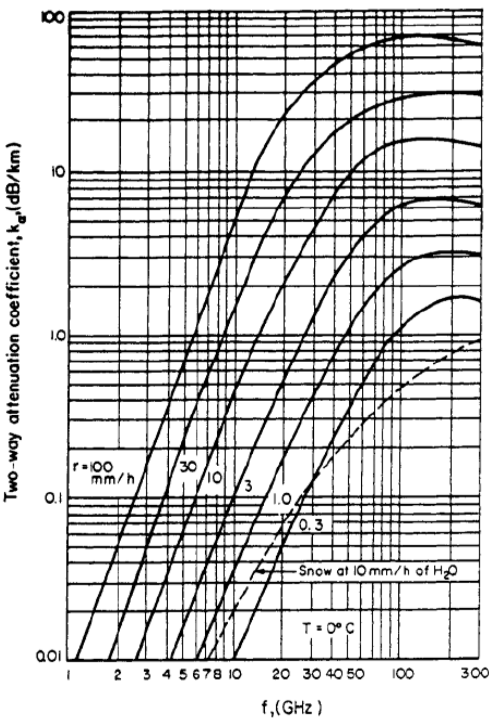
You are designing a Space Surveillance Radar, i.e. a ground-based radar pointing towards space. The system is designed for “24-hour surveillance” which requires reliable detection at all times.

Your task is to make a selection of frequency based on the following assumptions: One of the modes requires that a metallic sphere of radius 0.1 m shall be detected at range R = 35786 km (geostationary orbit) with 90% probability of detection and a false alarm rate of 10^{-6}. For this mode, assume that the antenna is pointing in the zenith direction and that coherent integration is used to increase the signal-to-noise ratio and achieve the required detection performance. Assume maximum duty cycle 40% of the transmitter, 1 dB receiver noise figure and that the antenna is a parabolic reflector with diameter d = 34 m and efficiency η = 50%. Assume peak transmitted power of 1 MW, loss factor of 1 dB (excluding tropospheric propagation loss – see below), and filter mismatch factor C_B = 1 dB. Consider alternative designs at K_u-band (15 GHz) and K_a-band (35 GHz). Determine the following parameters for the two frequency bands:

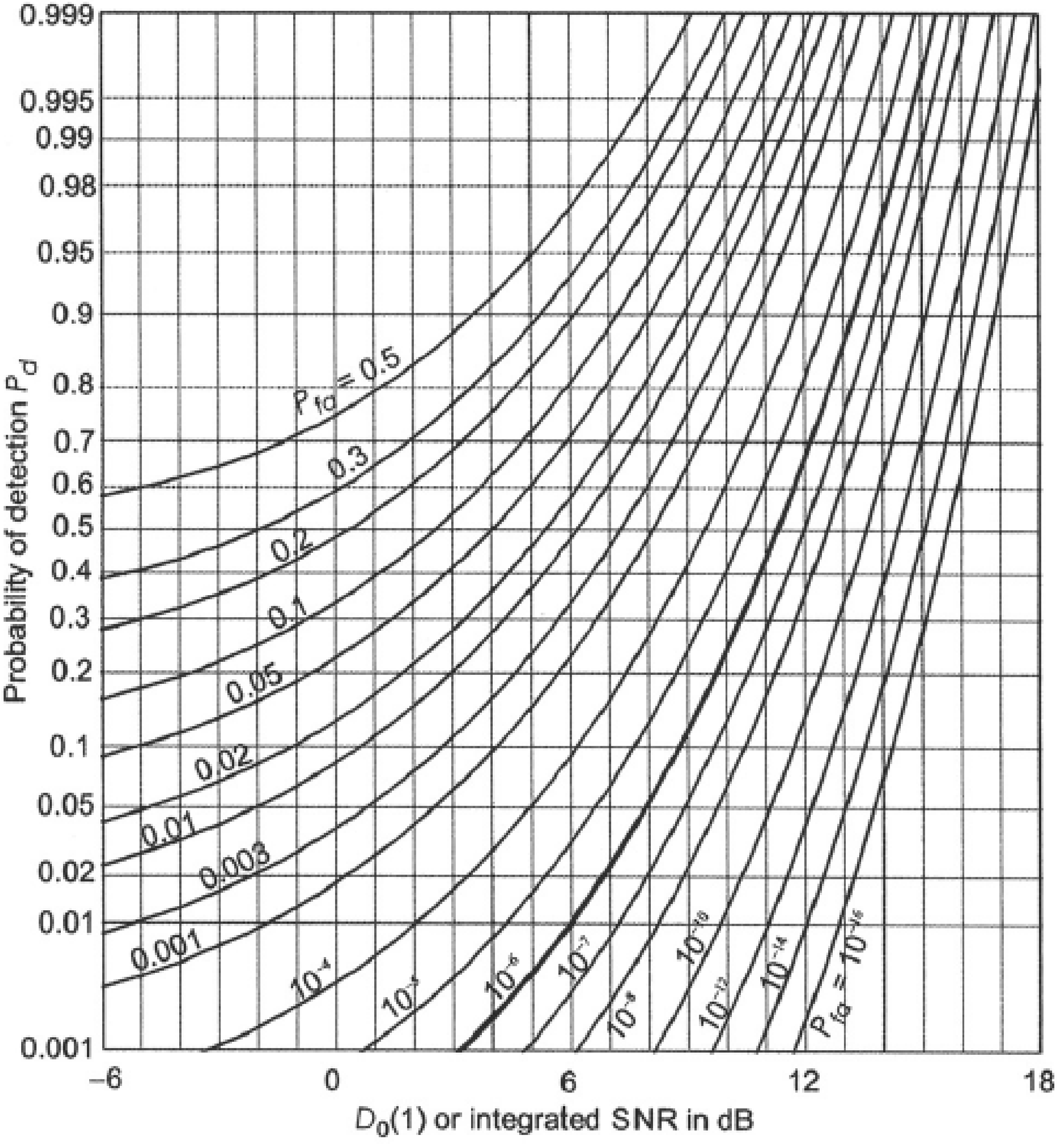
a) Required signal-to-noise ratio after coherent integration. (1p)
b) One-way antenna gain. (2p)
c) System noise temperature. (2p)
d) Tropospheric propagation losses due to a worst-case weather at the location of interest with a 3 km vertical rain-cell and 30 mm/hour. (2p)
e) Required coherent integration time (dwell time). (2p)
f) Which frequency (K_u- or K_a-band) would you choose? Motivate your answer. (1p)



Attenuation by entire troposphere (dB)



Rain attenuation (dB/km)



Detectability factor for a steady target
[David K. Barton, 2004, Radar System Analysis and Modeling, p. 45]

5. Synthetic Aperture Radar

You are designing an airborne SAR for detecting boats with radar properties according to the table below. Two different pulse lengths are selectable with different range resolution.

Your task is to select the mode which has the best detectability of point scatterers in the presence of noise and ocean clutter. To compare the different choices of radar pulse length, consider a test target with radar-cross section (RCS) 10 m^2 in a background of ocean with backscattering coefficient assumed to be $\sigma^o = -23$ dB. Assume the imaging is done at a slant range distance of 50 km, and a depression angle of 30^o below the horizontal. Aircraft speed is v = 300 m/s.

Assume spotlight SAR (antenna pointing towards fixed point on the ground) centered around broadside, i.e. coherent integration of Doppler frequencies centered on zero Doppler frequency.

a) Determine the required pulse-repetition frequency? (1p)

Consider first using the longer pulse length (mode 1, range resolution 0.3 m after compression).

b) What is the required coherent integration time when the azimuth resolution δ_{cr} is the same as that in range (i.e. 0.3 m)? (1p)

The signal-to-clutter ratio (SCR) is defined as the ratio of the maximum signal strength (in this case from the test target), compared to the average radar cross-section of the clutter.

c) For the given resolution in range and azimuth (0.3 m x 0.3 m) calculate the SCR. (2p)
d) Calculate the clutter-to-noise-ratio (CNR) in Mode 1, and hence give the signal-to-noise ratio (SNR) for the test target. (**Hint:** $T_{sys}=FT_0$) (2p)

Using the shorter pulse length (mode 2, range resolution 0.4 m after compression), a suggestion is to use a mode with an azimuth resolution of 0.2 m.

e) Determine the required coherent integration time? (1p)
f) Calculate SCR and SNR. (2p)
g) Would you recommend Mode 1 or 2 for detecting boats? Motivate your answer. (1p)

Radar Properties

Peak transmit power, P_{peak}	1 kW
Wavelength, λ	3 cm
Antenna gain (one-way, boresight), G	36 dB
Antenna azimuth beamwidth, β_a	4.3°
Noise figure, F	8 dB
System losses, L	2 dB
Boltzmann constant, k_B	1.38×10^{-23} J/K
Reference temperature, T_0	290 K
Maximum allowed PRF to avoid range ambiguities	1.5 kHZ

Properties for different transmit pulse modes (linear FM)

	Mode 1	Mode 2
Pulse length (uncompressed)	50 µs	1 µs
Range resolution (compressed pulse)	0.3 m	0.4 m

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15th January 2009.

Solutions:

1. Basic terms & principles

a) $\dfrac{8\pi a^4}{\lambda^2}$

b) 160 dB

c) $\tau = \dfrac{\lambda}{2\Delta v_r} = 0.006$ s

d) $\sqrt{2(4/3)R_E h} = 13$ km

2. Noise figure and system noise temperature

a) $$\mathrm{F_2} = \frac{(\mathrm{S/N})_{\mathrm{in}}}{(\mathrm{S/N})_{\mathrm{out}}} = \frac{\frac{\mathrm{T_{ant}}}{\mathrm{L}}+\mathrm{T_{radar}}\left(1-\frac{1}{\mathrm{L}}\right)+\mathrm{T_{recv}}}{\frac{\mathrm{T_{ant}}}{\mathrm{L}}+\mathrm{T_{radar}}\left(1-\frac{1}{\mathrm{L}}\right)} = \{\mathrm{T_{ant}} = \mathrm{T_{radar}}\} = \frac{\mathrm{T_{ant}}+\mathrm{T_{recv}}}{\mathrm{T_{ant}}} = 1 + \frac{\mathrm{T_{recv}}}{\mathrm{T_{ant}}}$$

From this we get $T_{recv} = T_{ant}(F_2\text{-}1)$

b) The noise figure F is defined in terms of the standard temp, T_0=290K [Sullivan, page 37]:
$T_{recv} = (F-1)T_0$, F>1
This can be re-written as $F = 1 + T_{recv}/T_0$

$$\frac{(S/N)_{in}}{(S/N)_{out}} = \frac{N_{out}}{N_{in}}\frac{S_{in}}{S_{out}} = \frac{N_{out}}{N_{in}}\frac{1}{G_{LNA}} = \frac{kB\left(\frac{T_{ant}}{L}+T_{radar}\left(1-\frac{1}{L}\right) + T_{recv}\right)G_{LNA}}{kB\left(\frac{T_{ant}}{L}+T_{radar}\left(1-\frac{1}{L}\right)\right)}\frac{1}{G_{LNA}} =$$

$$= \{T_{ant} = T_{radar} = T_0\} = \frac{\frac{T_0}{L}+T_0\left(1-\frac{1}{L}\right) + T_{recv}}{\frac{T_0}{L}+T_0\left(1-\frac{1}{L}\right)} = \frac{T_0 + T_{recv}}{T_0} = \frac{T_{recv}}{T_0} + 1 =$$

$$= F$$

c) $T_{recv} = 4.3$ K $\qquad T_0 = 290$ K

$T_{recv} = (F-1)T_0$

$F = 1 + T_{recv}/T_0 = 1 + 4.3/290 = 1.01483$

$F_{dB} = 10\log F = 0.06392$

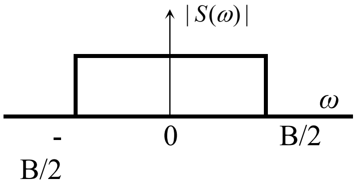
d) $T_{ant} = 3$ K $\qquad L = 1$

$$T_{sys} = \frac{\mathrm{T_{ant}}}{\mathrm{L}} + \mathrm{T_{radar}}\left(1-\frac{1}{\mathrm{L}}\right) + \mathrm{T_{recv}} = 3 + 0 + 4.3 = 7.3\ \mathrm{K}$$

3. Matched filter and ambiguity function

(a) $$E_u = \int_{-T/2}^{T/2} |u(t)|^2 \, dt = \int_{-T/2}^{T/2} \left| \frac{1}{\sqrt{T}} \exp(j\pi k t^2) \right|^2 dt = \frac{1}{T} \int_{-T/2}^{T/2} dt = \frac{1}{T} \cdot T = 1$$

b) $BT >> 1$ since under this condition the module of the LFM signal spectrum $|S(\omega)|$ is well approximated by a rectangular function of width B (see figure)



c)

$$h(t) = \mathrm{K} u^*(t_m - t) = \mathrm{K} \frac{1}{\sqrt{T}} \exp[-j\pi k (t_m - t)^2] \text{ for } |t| \le T/2, \quad \text{zero otherwise}$$

K is an arbitrary complex constant and $t_m \ge T$

d) $$\mathrm{SNR}_{\max} = \frac{2\mathrm{E}_s}{\mathrm{N}_0} = \frac{\mathrm{E}_u}{\mathrm{N}_0} \text{ since } \mathrm{E_u} = 2\mathrm{E_s}, \; s(t) = \mathrm{Re}\left\{ u(t) e^{j2\pi f_0 t} \right\}$$

e)

$$\mathrm{SNR}_{\max} = \frac{2\mathrm{E}_s}{\mathrm{N}_0} = \frac{\mathrm{E}_u}{\mathrm{N}_0} \text{ since } \mathrm{E_u} = 2\mathrm{E_s}, \; s(t) = \mathrm{Re}\left\{ u(t) e^{j2\pi f_0 t} \right\}$$

$$\mathrm{E_s} = \int_{-\infty}^{\infty} |s(t)|^2 \, dt, \; \mathrm{E_u} = \int_{-\infty}^{\infty} |u(t)|^2 \, dt, \; \mathrm{E_u} = \int_{-T/2}^{T/2} \mathrm{A}^2 dt = \mathrm{A}^2\mathrm{T}, \; \mathrm{N_0} = 2\mathrm{P_{nin}}/\mathrm{B}$$

$$\mathrm{SNR}_{\max} = \frac{\mathrm{A}^2 \mathrm{T}}{2\mathrm{P_{nin}}/\mathrm{B}} = \mathrm{BT} \cdot \frac{\mathrm{A}^2}{2\mathrm{P_{nin}}} = \mathrm{CR} \cdot \mathrm{SNR_{in}}$$

f) $$\chi(\tau, \nu) = \int_{-\infty}^{\infty} u(t) u^*(t-\tau) \exp(j2\pi\nu t) dt$$

g) Applying the Schwarz inequality to the ambiguity function squared yields:

$$\begin{aligned} |\chi(\tau,\nu)|^2 &= \left| \int_{-\infty}^{\infty} u(t) u^*(t-\tau) \exp(j2\pi\nu t) dt \right|^2 \le \int_{-\infty}^{\infty} |u(t)|^2 \, dt \int_{-\infty}^{\infty} |u^*(t-\tau) \exp(j2\pi\nu t)|^2 \, dt \\ &\le \int_{-\infty}^{\infty} |u(t)|^2 \, dt \int_{-\infty}^{\infty} |u^*(t-\tau)|^2 \, dt = E \cdot E = 1 \cdot 1 = 1 \end{aligned} \tag{1}$$

Therefore, $|\chi(\tau,\nu)|^2 \le 1$ (2)

From (2) since $0 \le |\chi(\tau,\ \nu)|^2 \le 1$ then $0 \le |\chi(\tau,\ \nu)| \le 1$ as well.

Equality i.e., $|\chi(\tau,\ \nu)|^2 = 1$ will replace the inequality in (1) when the functions in the two integrals [second expression in (1)] are conjugates of each other

$$u(t) = [u^*(t-\tau)\exp(j2\pi\nu t)]^* = u(t-\tau)\exp(-j2\pi\nu t),$$

which obviously takes place when $\tau = 0$ and $\nu = 0$. Thus, we conclude that

$$0 \le |\chi(\tau,\ \nu)|^2 \le |\chi(0,\ 0)|^2 = 1$$

and

$$0 \le |\chi(\tau,\ \nu)| \le |\chi(0,\ 0)| = 1\ .$$

4. Radar system design

a) (from graph) 13.2 dB

b) $G = \eta\left(\frac{\pi d}{\lambda}\right)^2$ = 71.5 dB (Ku-band); = 78.9 dB (Ka-band)

c) $T_{sys} = (F-1)T_0$ = 75 K

d) (from graphs: clear air and rain)
0.16 + 3 x 2.5 dB = 7.7 dB (Ku-band); 0.45 + 3 x 15.2 dB = 46.0 dB (Ka-band)

e) t_{dwell} = 0.6 s (Ku-band); = $8 \cdot 10^2$ s (Ka-band)

f) Select Ku-band. Besides reducing integration time during rainy conditions, it also relaxes antenna smoothness requirement.

5. Synthetic Aperture Radar

a) PRF = 4 * 300 * sin(4.3°/2) / 0.03 Hz = 1.5 kHz

b) t_{dwell} = 0.03 * 50000 / (2 * 0.3 * 300) = 8.3 s

c) SCR = 10 / (0.005 x 0.3 x 0.3 / cos30°) = 42.8 dB

d) Use expression for CNR in formula sheet and insert values gives CNR = 9.7 dB and thus SNR = 44.1 + 9.7 dB = 53.8 dB

e) t_{dwell} = 0.03 * 50000 / (2 * 0.2 * 300) = 12.5 s

f) (see 5b and 5c) SCR = 44.6 dB, CNR = -6.1 dB and thus SNR = 38.5 dB

g) Mode 1 and 2 have almost the same resolution and therefore similar SCR. The SNR, however, is much higher for Mode 1 (53.8 dB) compared to Mode 2 (38.5 dB). Select Mode 1 since it gives superior detection performance, in particular for boats with small radar-cross section.