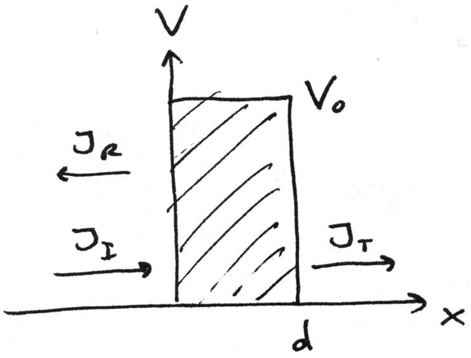
KVANTRÖV 4

Anders

VI 6



metall | oxid | metall

Infallande elektron med kinetisk energi

$E = 2{,}0 \text{ eV}$

$V_0 = 5{,}0 \text{ eV}$

$d = 2{,}0 \text{ nm}$

Sannolikhetsströmtäthet

Sökt: Tunnlingssannolikhet $T = \dfrac{J_T}{J_I}$

TOSE: $\psi''(x) + \dfrac{2m(E-V)}{\hbar^2}\psi(x) = 0$

$x<0,\ V=0:\quad \psi_1(x) = Ae^{ik_1x} + Be^{-ik_1x}$ $\qquad k_1 = \dfrac{\sqrt{2mE}}{\hbar}$

$0<x<d,\ V=V_0:\quad \psi_2(x) = C'e^{ik_2x} + D'e^{-ik_2x}$ $\qquad k_2 = \dfrac{\sqrt{2m(E-V_0)}}{\hbar}$

$x>d,\ V=0:\quad \psi_3(x) = Ee^{ik_1x} + \cancel{Fe^{-ik_1x}}$

Eftersom $V_0 > E$ följer vi konventionen och skriver:

$$k_2 = \frac{\sqrt{2m(E-V_0)}}{\hbar} = \frac{\sqrt{2m(V_0-E)(-1)}}{\hbar} = \frac{i\sqrt{2m(V_0-E)}}{\hbar}$$

$$\varkappa \equiv -ik_2 = \frac{\sqrt{2m(V_0-E)}}{\hbar}$$ och:

$$\psi_2(x) = Ce^{\varkappa x} + De^{-\varkappa x}$$

J för planvåg: Från IV 5, $J = \dfrac{\hbar k}{m}|\psi|^2$

Här: $J_I = \dfrac{\hbar k_1}{m}|Ae^{ik_1x}|^2 = \dfrac{\hbar k_1}{m}|A|^2$

$J_R = \dfrac{\hbar k_1}{m}|Be^{-ik_1x}|^2 = \dfrac{\hbar k_1}{m}|B|^2$

$J_T = \dfrac{\hbar k_1}{m}|Ee^{ik_1x}|^2 = \dfrac{\hbar k_1}{m}|E|^2$

...

$$\ldots \Rightarrow \quad T = \frac{J_T}{J_I} = \frac{|E|^2}{|A|^2} = \left|\frac{E}{A}\right|^2$$

<u>Skarvningsproblem</u>

OBS först att den exp. växande termen $Ce^{\kappa x}$ är reflektionen från den avtagande vågfkn i oxiden

Den kommer alltså att vara mycket liten vid $x=0$.
Vi får vid $x=0$:

$$\left.\begin{array}{l} \psi_1(0) = \psi_2(0) \Rightarrow A+B = D \\ \psi_1'(0) = \psi_2'(0) \Rightarrow ik_1(A-B) = \kappa D \end{array}\right\} \Rightarrow \begin{array}{l} ik_1(A-D+A) = \kappa D \\ \Leftrightarrow \frac{D}{A}(\kappa + ik_1) = 2ik_1 \end{array}$$

och: $\boxed{\frac{D}{A} = \frac{2ik_1}{-\kappa+ik_1}}$ $(*)$

Vid $x=d$ fås:

$$\left.\begin{array}{l} \psi_2(d) = \psi_3(d) \Rightarrow Ce^{\kappa d} + De^{-\kappa d} = Ee^{ik_1 d} \\ \psi_2'(d) = \psi_3'(d) \Rightarrow \kappa\left(Ce^{\kappa d} + (-De^{-\kappa d})\right) = ik_1 E e^{ik_1 d} \end{array}\right\}$$

$$\Rightarrow \kappa\left(-De^{-\kappa d} + Ee^{ik_1 d} - De^{-\kappa d}\right) = ik_1 E e^{ik_1 d} \Leftrightarrow$$

$$\Leftrightarrow Ee^{ik_1 d}(ik_1 - \kappa) = -2\kappa De^{-\kappa d}$$

$$\Leftrightarrow Ee^{ik_1 d} = -\frac{2\kappa}{ik_1-\kappa} De^{-\kappa d} \overset{(*)}{=} \frac{2\kappa}{\kappa - ik_1}\left(\frac{2ik_1}{\kappa+ik_1}\right)Ae^{-\kappa d} =$$

$$= \frac{4ik_1\kappa}{\kappa^2+k_1^2}Ae^{-\kappa d} \Leftrightarrow E = A\frac{4ik_1\kappa}{\kappa^2+k_1^2}e^{-\kappa d - ik_1 d}$$

$$\Rightarrow T = \left|\frac{E}{A}\right|^2 = \left|\frac{4ik_1\kappa}{\kappa^2+k_1^2}e^{-\kappa d}\right|^2 = \underbrace{\frac{16k_1^2\kappa^2}{(\kappa^2+k_1^2)^2}}_{\text{"förfaktor"}} \underbrace{e^{-2\kappa d}}_{\text{"barriär-penetreringsfa..."}}$$

... Här: $d = 2\cdot 10^{-9}\,m$

$E = 2\,eV$

$V_0 = 5\,eV$

$m = 9{,}1\cdot 10^{-31}\,kg$

$\hbar = 1{,}05\cdot 10^{-34}\,Js$

$$\Rightarrow \quad k_1 = \frac{\sqrt{2mE}}{\hbar} = 7{,}2\cdot 10^{9}\,m^{-1}$$

$$\varkappa = \frac{\sqrt{2m(V_0-E)}}{\hbar} = 8{,}9\cdot 10^{9}\,m^{-1}$$

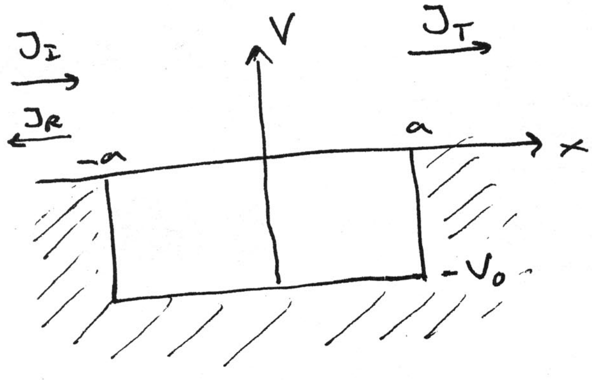
$\Rightarrow$ ~~förfaktor~~: $e^{-2\varkappa d} = 4\cdot 10^{-16}$ (barriär-penetrerings-faktor)

~~barriärpenetreringsfaktor~~: $\dfrac{16\,k_1^2\varkappa^2}{(\varkappa^2+k_1^2)^2} = 3{,}8$ (förfaktor)

$\Rightarrow$ $T = 10^{-15}$

VI 7

$$V(x) = \begin{cases} 0 & |x|>a \\ -V_0 & |x|<a \end{cases} \qquad (V_0>0)$$



Sökt: E för transmissionsresonans ($J_T = J_I$)

TOSE: $\psi''(x) + k^2\psi = 0$, $k = \dfrac{\sqrt{2m(E-V)}}{\hbar}$

$x<-a,\ V=0:\ \psi_1(x) = Ae^{ik_1x} + Be^{-ik_1x}$ $\qquad k_1 = \dfrac{\sqrt{2mE}}{\hbar}$

$|x|<a,\ V=V_0:\ \psi_2(x) = Ce^{ik_2x} + De^{-ik_2x}$ $\qquad k_2 = \dfrac{\sqrt{2m(E+V_0)}}{\hbar}$

$x>a,\ V=0:\ \psi_3(x) = Ee^{ik_1x}$

Nu: skarva som i VI 6:

$\psi_1(-a) = \psi_2(-a)$

$\psi_1'(-a) = \psi_2'(-a)$

$\psi_2(a) = \psi_3(a)$, $\psi_2'(a) = \psi_3'(a)$