REGLERTEKNIK

ÖVN F

1996

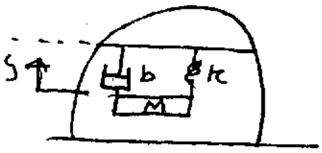
SIDOR: 31

PRIS: ~~10:-~~ 15 kr

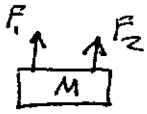
Räkneövning i reglerteknik

961005

Uppgift 3.2



Frilägg vikt



$$F_1 = B\cdot(\dot{x}-\dot{y})$$
$$F_2 = k(x-y)$$

$$M\cdot\ddot{y} = B(\dot{x}-\dot{y}) + k(x-y) \implies M\ddot{y} + B\dot{y} + ky = B\cdot\dot{x} + kx$$

Laplacetransformerar

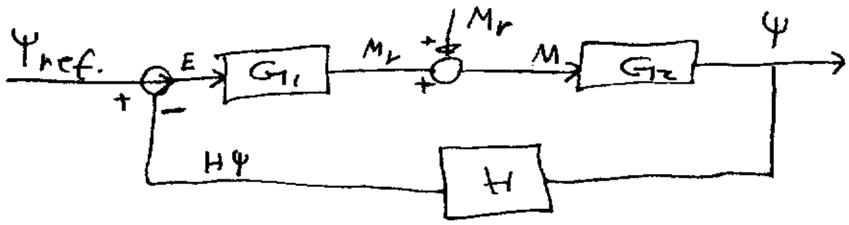
$$\implies M Y(s)\cdot s^3 + B\cdot s\cdot Y(s) + k\cdot Y(s) = B\cdot s\cdot X(s) + k\cdot X(s)$$

$$\implies (Ms^2 + B\cdot s + k)\cdot Y(s) = (Bs + k)\cdot X(s)$$

$$\implies Y(s) = \underbrace{\frac{Bs+k}{Ms^2+Bs+k}}_{G(s)}\cdot X(s)$$

Allmänt $G(s) = \frac{\text{utsignal}}{\text{insignal}}$

3.16



$$\Psi = (\Psi_{ref} - H\Psi)G_1 + M_v)G_2$$

$$\Psi = \frac{G_1G_2}{1+G_1G_2H}\Psi_{ref} + \frac{G_2}{1+G_1G_2H}M_v$$

$$G_2: \; 100\ddot{\Psi} + \dot{\Psi} = 0{,}1M \qquad \mathcal{L} \implies$$

$$\implies 100s^2\Psi + s\cdot\Psi = 0{,}1M$$

$$G_2 = \frac{\Psi}{M} = \frac{0{,}1}{100s^2+s}$$

$$G_1 = \frac{M_v}{E} = \frac{M_K}{R} \cdot \frac{R}{U} \cdot \frac{U}{E}$$

$$\frac{U}{E} = 0{,}5 \qquad \frac{R}{U} = \frac{0{,}1}{5s+1}$$

$$5\dot{r} + r = 0{,}1u$$

$$5s \cdot R + R = 0{,}1\,U$$

$$\Longrightarrow \begin{cases} G_1 = \dfrac{50}{5s+1} \\ H = \dfrac{\varphi_g}{\varphi} = 0{,}1 \end{cases}$$

$$\varphi_{ref} = 0 \Longrightarrow \varphi = \frac{G_2}{1 + G_1 G_2 H} \cdot M_v$$

$$\Longrightarrow \varphi(1 + G_1 G_2 H) = G_2 M_v$$

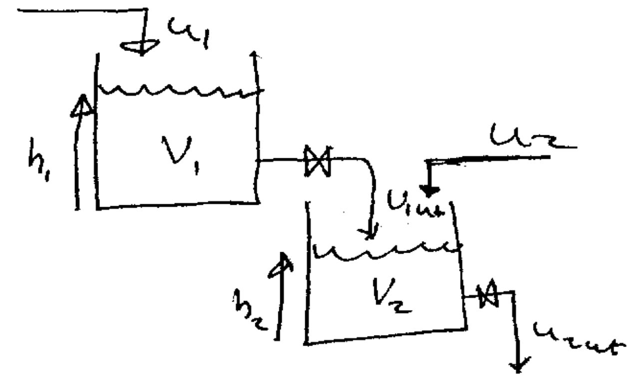
$$\Longrightarrow \varphi\left(1 + \frac{50 \cdot 0{,}1 \cdot 0{,}1}{s \cdot (100s+1)(5s+1)}\right) = \frac{0{,}1}{100s^2+s} M_v$$

$$\Longrightarrow (500s^3 + 105s^2 + s + 0{,}5)\,\varphi = (0{,}5s + 0{,}1) M_v$$

$$\Longrightarrow \mathcal{L}^{-1} \quad 500\dddot{\varphi}(t) + 105\ddot{\varphi}(t) + \dot{\varphi}(t) + 0{,}5\varphi(t) =$$

$$= 0{,}5\dot{M}_v(t) + 0{,}1 M_v$$

3.26



Tank1: $u_1 - u_{1ut} = \dot{V}_1$

$$\begin{cases} \dot{V}_i = A_i \cdot \dot{h}_i \\ U_{i\,ut} = \dfrac{h_i}{R_i} \end{cases}$$

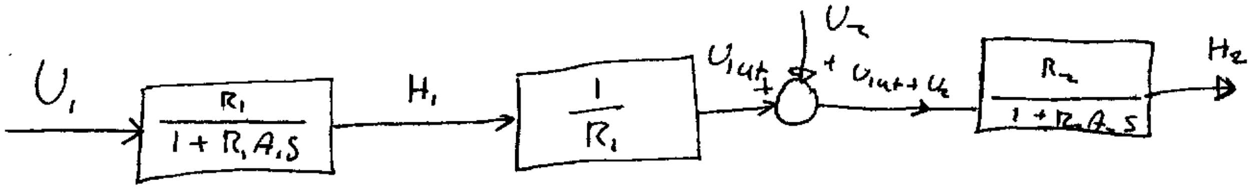
Tank2 $u_2 + u_{1ut} - u_{2ut} = \dot{V}_2$

$$\Longrightarrow \quad u_1 - \frac{h_1}{R_1} = A \cdot \dot{h}_1$$

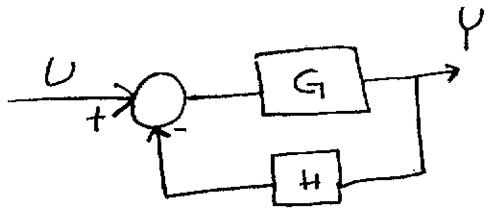
$$u_2 + u_{1ut} - \frac{h_2}{R_2} = A_2 \dot{h}_2$$

$$\overset{\mathcal{L}}{\Longrightarrow} \begin{cases} U_1 - \dfrac{1}{R_1} H_1 = A_1 \cdot S \cdot H_1 \\ U_2 + U_{1ut} - \dfrac{H_2}{R_2} = A_2 S H_2 \end{cases} \Longrightarrow \quad H_1 = \frac{U_1}{\frac{1}{R_1} + A_1 \cdot S} = \frac{R_1}{1 + R_1 A_1 S} \cdot U_1$$

$$H_2 = \frac{U_2 + U_{1ut}}{\frac{1}{R_2} + A_2 s} = \frac{R_2 (U_2 + U_{1ut})}{1 + A_2 R_2 \cdot S}$$



5.3.



$$Y = G(U - HY) \implies Y = \frac{G}{1+GH} U$$

Karaktäristisk ekvation $1 + GH = 0$

Syst. stabilt om rötterna till KE ligger i VHP.

Routh-Hurwitz stabilitetskriterium

$a_0 s^n + a_1 s^{n-1} \quad = 0$

genom korsmultipl.

$$\begin{array}{ccc} a_0 & a_2 & a_4 \\ a_1 & a_3 & a_5 \\ c_0 & c_1 & c_2 \\ d_0 & d_1 & d_2 \end{array}$$

$$c_0 = \frac{a_1 a_2 - a_3 \cdot a_0}{a_1}$$

$$c_1 = \frac{a_1 \cdot a_4 - a_5 \cdot a_0}{a_1}$$

$$d_0 = \frac{c_0 a_3 - c_1 a_1}{c_0}$$

Systemet är stabilt om alla koefficienter i vänstra kolumnen är > 0

$$G = \frac{6}{s(s+1)(s+a)} \implies G_{slutna} = [H=1] = \frac{G}{1+G}$$

$\implies$ KE : $\underline{\underline{1 + G = 0}} \implies s^3 + (1+a)s^2 + a \cdot s + 6 = 0$

$$\begin{array}{ccc} ① & ② & ③ \\ 1 & a & 0 \\ 1+a & 6 & 0 \\ a - \frac{6}{1+a} & 0 & 0 \\ 6 & 0 & \\ 0 & & \end{array}$$

$$c_0 = \frac{(1+a) \cdot a - 6 \cdot 1}{1+a} = a - \frac{6}{1+a}$$

$$c_1 = \frac{0 - 0}{1+a} = 0$$

$$d_0 = \frac{6\cancel{(a - \frac{6}{1+a})} - 0}{\cancel{a - \frac{6}{1+a}}} = 6$$

stabilt då alla koeff i $\textcircled{O} > 0$

$1+a > 0$ & $a - \frac{6}{1+a} > 0$ (fram tills enbart nollor uppstår)

$\Longrightarrow$ $a > -1$ $a^2 + a - 6 > 0$ $a > 2$ el $a < -3$

sgst. stabilt omm.

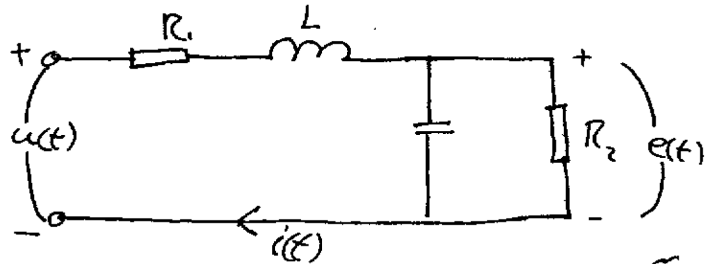
<u>Tillståndsmodeller</u> <u>96-11-12</u>

$$\dot{x} = Ax + Bu$$

tillstånd insignal (styrsignal)

$$y = Cx + Du$$

utsignal (mätvärde)

4.5 a) 

$u \equiv$ styrsignal

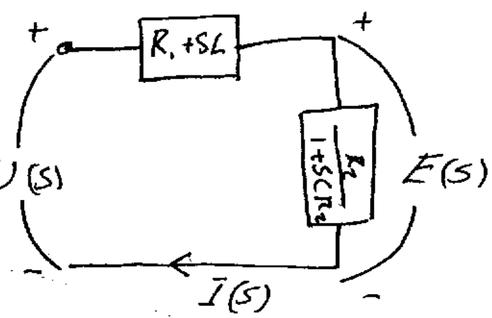
<u>ellära</u>

Tillstånd: $x_1 = e$ $\mathcal{L}$: $R \longrightarrow R$

$x_2 = i$ $L \longrightarrow sL$

styrsignal: $u = u$ $C \longrightarrow \frac{1}{sC}$

utsignal: $y = e$

$\mathcal{L} \Longrightarrow$ 

<u>Ω:s lag</u> $U = ZI$

① $E(s) = \frac{R_2}{1+sCR_2} I(s)$

② $U(s) = (R_1 + sL)\,I(s) + E(s)$

① och ② $\Rightarrow$ $\begin{cases} E(s) + CR_2\,s\,E(s) = R_2\,I(s) \\ U(s) = R_1 I(s) + L \cdot s I(s) + E(s) \end{cases}$

$\mathcal{L}^{-1} \Longrightarrow \begin{cases} e(t) + CR_2\,\dot{e}(t) = R\,i(t) \\ u(t) = R_1 i(t) + L \cdot \dot{i}(t) + e(t) \end{cases}$

$\Longrightarrow \begin{cases} \dot{e}(t) = -\frac{1}{CR_2} e(t) + \frac{1}{C} i(t) \\ \dot{i}(t) = -\frac{1}{L} e(t) - \frac{R_1}{L} i(t) + \frac{1}{L} u(t) \end{cases}$

$\Longrightarrow \begin{cases} \dot{x}_1 = -\frac{1}{CR_2} x_1 + \frac{1}{C} x_2 \\ \dot{x}_2 = -\frac{1}{L} x_1 - \frac{R_1}{L} x_2 + \frac{1}{L} u \end{cases}$

$\Longrightarrow \dot{x} = \begin{bmatrix} -\frac{1}{CR_2} & \frac{1}{C} \\ -\frac{1}{L} & -\frac{R_1}{L} \end{bmatrix} x + \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix} u$

$y = e(t) = x_1$

$y = \underbrace{(1,\ 0)}_{C}\, x$

4.18 a) $\frac{Y(s)}{U(s)} = G(s) = \frac{5s+23}{s^2+9s+20}$

skriv på diagonalform!

Partialbråksuppdela

$G(s) = \frac{3}{s+4} + \frac{2}{s+5}$

$Y(s) = \left(\frac{3}{s+4} + \frac{2}{s+5}\right) U(s)$

$\Rightarrow Y(s) = \frac{3\,U(s)}{s+4} + \frac{2\,U(s)}{s+5}$

diagonalform: $A = \begin{bmatrix} a_{11} & & 0 \\ & a_{22} & \\ 0 & & a_{33} \end{bmatrix}$

styrbar form $A = \begin{bmatrix} a_1 & a_2 & \cdots \\ 1 & & 0 \\ & 1 & \\ 0 & & 1 \end{bmatrix}$

observerbar form $A = \begin{bmatrix} a_1 & 1 & & 0 \\ a_2 & & 1 & \\ \vdots & 0 & & 1 \end{bmatrix}$

$$Y(s) = (1 \quad 1)\, X(s)$$

$$X_1(s) = \frac{3}{s+4} U(s) \Longrightarrow s X_1(s) + 4 X_1(s) = 3 U(s)$$

$$\mathcal{L}^{-1} \Longrightarrow \underline{\dot{X}_1(t) = -4 X_1(t) + 3 u(t)}$$

$$X_2(s) = \frac{2}{s+5} U(s) \Longrightarrow s X_2(s) + 5 X_2(s) = 2 U(s)$$

$$\mathcal{L}^{-1} \Longrightarrow \underline{\dot{X}_2(t) = -5 X_2(t) - 2 u(t)}$$

$$\dot{X} = \begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} -4 & 0 \\ 0 & -5 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} 3 \\ 2 \end{bmatrix} u$$

läs om olika ansättningar i boken

<u>Introduktion till uppgift b</u>

b)

$$\dot{X} = A X + B u$$

$$\Longrightarrow s \cdot X(s) - X(0) = A X(s) + B u(s)$$

$$\Longrightarrow (sI - A) X(s) = X(0) + B u(s)$$

$$\Longrightarrow X(s) \; (sI-A)^{-1} \cdot X(0) + (sI-A)^{-1} \cdot B \cdot U(s)$$

$$\xrightarrow{\mathcal{L}^{-1}} X(t) = \underbrace{\mathcal{L}^{-1}\{(sI-A)^{-1}\}}_{\phi(t) = \text{övergångsmatris}} X(0) + \int_0^t \phi(t-\tau) \cdot B \cdot u(\tau)\, d\tau$$

$$= e^{At}$$

$$\Longrightarrow X(t) = \phi(t) \cdot X_0 + \int_0^t \phi(t-\tau) B u(\tau)\, d\tau$$

4.18 b)

$$A = \begin{bmatrix} -4 & 0 \\ 0 & -5 \end{bmatrix}$$

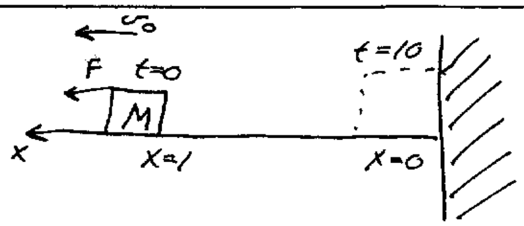
$$\phi = \mathcal{L}^{-1}\{(sI-A)^{-1}\}$$

$$(sI-A)^{-1} = \begin{bmatrix} s+4 & 0 \\ 0 & s+5 \end{bmatrix}^{-1} =$$

$$= \frac{1}{(s+4)(s+5)}\begin{bmatrix} s+5 & 0 \\ 0 & s+4 \end{bmatrix} = \begin{bmatrix} \frac{1}{s+4} & 0 \\ 0 & \frac{1}{s+5} \end{bmatrix}$$

$$\xrightarrow{\mathcal{L}^{-1}} \begin{bmatrix} e^{-4t} & 0 \\ 0 & e^{-5t} \end{bmatrix} = \phi(t)$$

4.20)



$v_0 = 1$ m/s

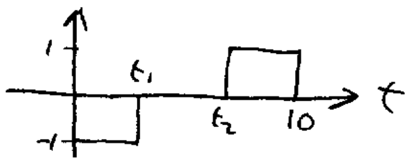
$M = 1$ kg

$$\begin{cases} x_1 = x \\ x_2 = \dot{x} = \dot{x}_1 \end{cases} \quad \text{Tillstånd}$$

Begynnelsevikor: $\begin{cases} x_1(0) = 1 \\ x_2(0) = \dot{x}_1(0) = 1 \end{cases}$

Slutvilkor: $\begin{cases} x_1(10) = 0 \\ x_2(10) = 0 \end{cases}$

Insignal: $u = F$



$F = ma$

$u = M \cdot \ddot{x} = M\dot{x}_2$

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \frac{u}{M} = u \end{cases}$$

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \overbrace{\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}}^{A} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \overset{B}{\begin{bmatrix} 0 \\ 1 \end{bmatrix}} u$$

$$x(t) = \phi(t)\cdot x(0) + \int_0^t \phi(t-\tau) B \cdot u(\tau) d\tau$$

$$\phi(t) = \mathcal{L}^{-1}\{(SI-A)^{-1}\} = \mathcal{L}^{-1}\left\{\begin{bmatrix} s & -1 \\ 0 & s \end{bmatrix}^{-1}\right\} = \mathcal{L}^{-1}\left\{\begin{bmatrix} s & 1 \\ 0 & s \end{bmatrix} \cdot \frac{1}{s^2}\right\} = \begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix}$$

$$x(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$x(10) = \begin{bmatrix} 1 & 10 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} + \int_0^{10} \begin{bmatrix} 1 & 10-\tau \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} u(\tau) d\tau$$

$$= \begin{bmatrix} 11 \\ 1 \end{bmatrix} + \int_0^{10} \begin{bmatrix} 10-\tau \\ 1 \end{bmatrix} u(\tau) d\tau = \begin{bmatrix} 11 \\ 1 \end{bmatrix} + \begin{bmatrix} 10\tau - \frac{10\tau^2}{2} \\ \tau \end{bmatrix}_0^{t_1} (-1) +$$

$$+ 0 + \begin{bmatrix} 10\tau - \frac{\tau^2}{2} \\ \tau \end{bmatrix}_{t_2}^{10} = \begin{bmatrix} 11 \\ 1 \end{bmatrix} - \begin{bmatrix} 10t_1 - \frac{t_1^2}{2} \\ t_1 \end{bmatrix} + \begin{bmatrix} 10\cdot 10 - \frac{10^2}{2} \\ 10 \end{bmatrix}$$

$$- \begin{bmatrix} 10t^2 - \frac{t_2^2}{2} \\ t_2 \end{bmatrix} =$$

$$= \begin{bmatrix} 11 - 10t_1 + \frac{t_1^2}{2} + 100 - 50 - 10t_2 + \frac{t_2^2}{2} \\ 1 - t_1 + 10 - t_2 \end{bmatrix}$$

$$= x(10) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

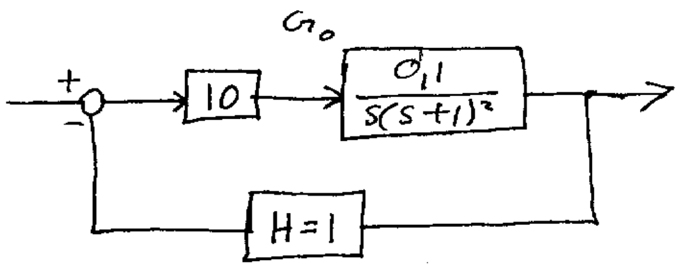
$$\Longrightarrow \begin{cases} 61 + \frac{t^2}{2} - 10t_1 + \frac{t_2}{2} - 10t^2 = 0 \\ 11 - t_1 - t_2 = 0 \end{cases}$$

$$\Longrightarrow \begin{cases} t_1 = 1{,}17\,s \\ t_2 = 9{,}83\,s \end{cases}$$

96-11-19

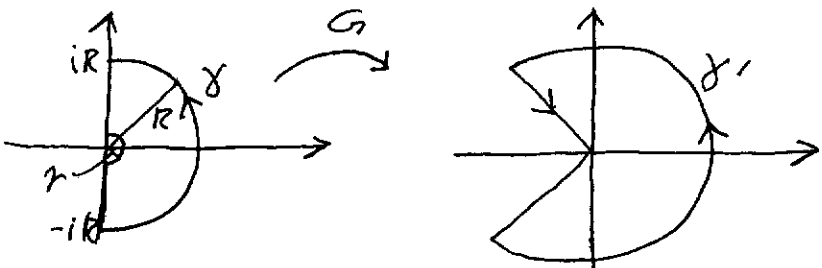
stabilitet

5.9



Kretsöverforming: $G = G_0 \cdot H = G_0 = \frac{1}{s(s+1)^2}$

Nyhvisht:



$R \to \infty$ $r \to \infty$ så täck hela hhp in

Förenklade nyqwist

$G(i\omega)$ då $\omega \to \infty$

Systemet stabilt om −1 ligger tv om kurvan.

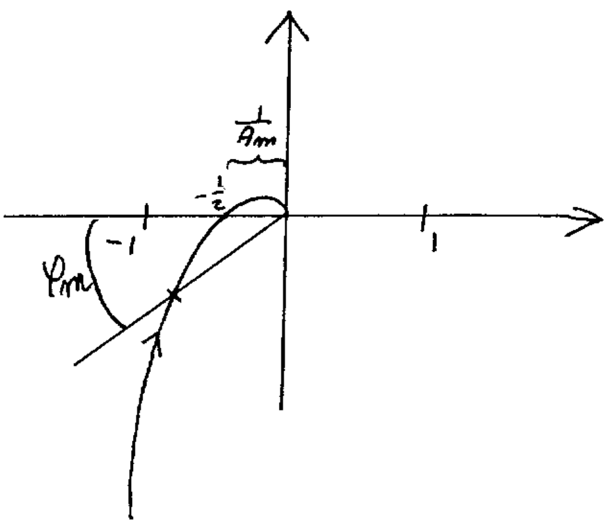
$$G(j\omega) = \frac{1}{j\omega(j\omega+1)^2} = -\frac{j}{\omega(-\omega^2+2j\omega+1)} = -\frac{j}{\omega}\cdot\frac{(1-\omega^2)-2j\omega}{(1-\omega^2)^2+4\omega^2} =$$

$$= \frac{-2\omega - j(1-\omega^2)}{\omega(1+\omega^2)^2}$$

$$\Longrightarrow Re(G(j\omega)) = -\frac{2}{(1+\omega^2)^2}$$

$$Im(G(j\omega)) = -\frac{1-\omega^2}{\omega(1+\omega^2)^2}$$

ω	Re	Im
0	−2	∞
1	−1/2	0
2	$-\frac{2}{25}$	0,06
∞	0	0



φ_m = fasmarginal
A_m = amplitudmarginal.

5.4

$$G = \frac{k}{s(1+Ts)(1+0{,}1s)} = \frac{k}{N}$$

$$A_m = 2 \implies \begin{cases} Im(G(j\omega_\pi)) = 0 \\ Re(G(j\omega_\pi)) = -\frac{1}{2} \end{cases}$$

Räkna på nämnaren: $N = j\omega_\pi(1 + 0{,}1j\omega_\pi + Tj\omega_\pi - 0{,}1T\omega_\pi^2)$

$$= \omega(j - 0{,}1\omega - T\omega - 0{,}1Tj\omega^2) = \omega(-\omega(0{,}1+T) + j(1-0{,}1T\omega^2))$$

$$\implies G = \frac{k}{\omega}\underbrace{\left(-(1-0{,}1T\omega^2)^2 - \omega^2(0{,}1+T)^2\right)}_{=N_2}{}^{-1} \cdot (\omega(0{,}1+T) +$$

$$+ j(1-0{,}1T\omega^2)) = \frac{k(0{,}1+T)}{N_2} + j\,\frac{k}{\omega}\,\frac{(1-0{,}1T\omega^2)}{N_2}$$

$$\implies Im(G(j\omega)) = \frac{k}{\omega}\cdot\frac{(1-0{,}1T\omega^2)}{N_2} = 0$$

$$\implies k(1-0{,}1T\omega^2) = 0$$

$$\omega = \sqrt{\frac{10}{T}}$$

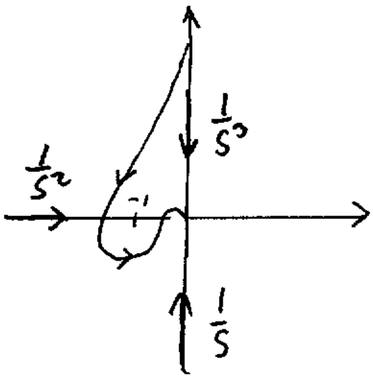
$$Re(G(j\omega)) = k\cdot\frac{0{,}1+T}{N_2} = -\frac{1}{2} \implies 2k(0{,}1+T) = -N_2$$

$$\implies \left[\sqrt{\frac{10}{T}} = \omega\right] = -N_2 = \frac{10}{T}(0{,}1+T)^2$$

$$\implies 2k = \frac{10}{T}(0{,}1+T)$$

$$\implies k = \frac{1}{2T} + 5$$

5.12



$$G(s) = \frac{1}{s} \Rightarrow G(j\omega) = \frac{1}{j\omega}$$

$$G(s) = \frac{1}{s^n} \Rightarrow G(j\omega) = \frac{1}{(j\omega)^n}$$

n=1:

$$Re(G) = 0$$

$$G(s) = \frac{1}{s} \Longleftrightarrow G(j\omega) = -\frac{j}{\omega}$$

$$\omega = 0 \Longrightarrow Im(G) \longrightarrow -\infty$$

$$\omega = \infty \Longrightarrow Im(G) \longrightarrow 0$$

n=2:

$$G(j\omega) = -\frac{1}{\omega^2} \Longrightarrow Im(G) = 0$$

$$\omega = 0 \Longrightarrow Re(G) \longrightarrow -\infty$$

$$\omega = \infty \Longrightarrow Im(G) \longrightarrow 0$$

n=3

$$G(j\omega) = j \cdot \frac{1}{\omega^3} \Longrightarrow Re(G) = 0$$

$$\omega = 0 \Longrightarrow Im(G) \longrightarrow \infty$$

$$\omega = \infty \Longrightarrow Im(G) \longrightarrow 0$$

∴ stabil om −1 ligger till vänster när man går längs kurvan

6.6 Insignal $u(t) = A \sin \omega t$

Utsignal $y(t) = A \underbrace{|G(j\omega)|}_{\text{amplitud-först.}} \sin(\omega t + \underbrace{\angle G(j\omega)}_{\text{fasvridning}})$

Bodediagram: $|G(j\omega)|$ mot ω

$\angle G(j\omega)$ mot ω

enhetsåterkoppling

$$\underline{G = G_0 \cdot H = G_0 \cdot 1 = G_0}$$

$$G_0 = G = k_1 \cdot G_1 \cdot k_2 \cdot G_2 = \frac{0{,}2 \cdot 3}{(0{,}1 + 0{,}2s)\cdot(s(1+s\cdot 0{,}5) + 2{,}0)} =$$

$$= \frac{3}{\underbrace{(1+2s)}_{G_a}\underbrace{\left(\left(\frac{s}{2}\right)^2 + \frac{s}{2} + 1\right)}_{G_b}}$$

Formelsamling?

Bodeform. 1:a gradsekvation $1+Ts$ vi har $T_a = 2$

2:a gradsekv. $1 + 2\xi Ts + (Ts)^2$

$$G_b = \left(\left(\frac{s}{2}\right)^2 + 2 \cdot \underbrace{\frac{1}{2}}_{\xi_b} \cdot \underbrace{\frac{1}{2}}_{T_b} \cdot s + 1\right) \Rightarrow \begin{cases} \xi_b = \frac{1}{2} \\ T_b = \frac{1}{2} \end{cases}$$

Bodediagram: log – log

y-axeln: $20 \log |G(j\omega)|$

$$|G(j\omega)| = \frac{3}{|G_a(j\omega)| \cdot |G_b(j\omega)|} \iff$$

$$|G(j\omega)|_{dB} = |3|_{dB} - |G_a(j\omega)|_{dB} - |G_b(j\omega)|_{dB}$$

$\angle G(j\omega) = \angle 3 - \angle G_a(j\omega) - \angle G_b(j\omega)$

$|3|_{dB} = 20 \lg 3 = 9{,}5$

$\angle 3 = 0$

<u>G_a:</u> $|G_a(j\omega)|_{dB} = |1+2j\omega|_{dB} = 20 \lg \sqrt{1+(2\omega)^2}$

$\angle G_a(j\omega) = \tan^{-1} 2\omega$

LF-assymptot: $2\omega \ll 1 \Longrightarrow |G_a(j\omega)|_{dB} = 0$

HF-assymptot: $2\omega \gg 1 \Longrightarrow |G_a(j\omega)|_{dB} = 20 \lg 2\omega$

<u>G_b:</u> $|G_b|_{dB} = |1 + 2\zeta j\omega T_b + (j\omega)^2 \cdot T_b^2| =$

$= 20 \lg \left((1+4(\zeta^2 - \tfrac{1}{2})\omega^2 T_b^2 + \omega^4 T_b^4 \right)^{1/2} =$

$= 20 \lg \left(1 - \tfrac{1}{4}\omega^2 + \tfrac{1}{16}\omega^4\right)^{1/2}$

låg frekvens: $\omega T_b \ll 1 \quad |G_b(j\omega)|_{dB} = 0$

Hög frekvens: $\omega T_b \gg 1 \quad |G_b(j\omega)|_{dB} = 40 \lg \omega T_b$

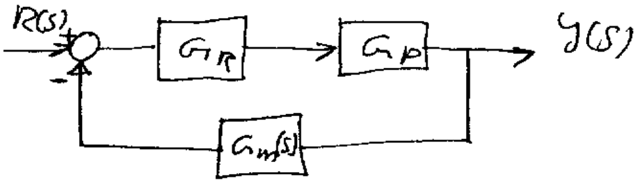
$\angle G_b(j\omega) = \arctan \dfrac{2\zeta\omega T_b}{1-\omega^2 T_b^2}$ $(+\pi$ om $\omega T_b > 1)$

<u>Resultat:</u> $|G|_{dB} = 9{,}5 - 20 \lg \sqrt{1+4\omega^2} - 20 \lg \sqrt{1 - \frac{\omega^2}{4} + \frac{\omega^4}{16}}$

$\angle G = 0 - \arctan(2\omega) - \arctan\left(\dfrac{\omega}{2\left(1-\frac{\omega^2}{4}\right)}\right)$

<u>Rita som övning själv in i Bode</u>

uppgift 8:8 — 96-11-26

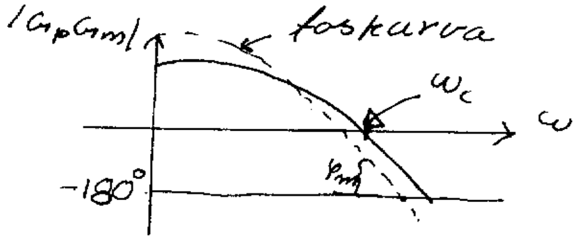


kretsöverföring: $G(s) = G_R(s) \cdot G_p(s) \cdot G_m(s)$

$$G_p(s) = \frac{10\,e^{-5s}}{1+50s} \qquad G_m(s) = \frac{1}{1+10s}$$

a) $G_R(s) = k$

söker k så att $\varphi_m = 40°$



φ_m är avståndet mellan $-180°$ och faskurvan vid ω_c

Def: $\varphi_m = 180° + \angle G(j\omega_c)$

$|G(j\omega_c)| = 1$

bode $\Longrightarrow$ $\omega_c = 0{,}083$

$$G(s) = \frac{k \cdot 10^{-5s}}{1+50s} \cdot \frac{1}{1+10s}$$

$$|G(j\omega_c)| = \frac{10k}{(1+2500\omega_c^2)^2(1+100\omega_c^2)^2} = 1$$

$\Longrightarrow [\omega_c = 0{,}083] \Longrightarrow k = 0{,}55$

b) PI-regulator: $k\left(1 + \frac{1}{sT_i}\right) = k \cdot \left(\frac{1+sT_i}{sT_i}\right)$

sökt T_i s.a φ_m: ej försämras mer än $5°$. k samma som i a) $(0{,}55)$

$|G(j\omega_c)| = |G_R(j\omega_c)| \cdot |G_p(j\omega_c) \cdot G_m(j\omega_c)| = 1$

$$|G_R(j\omega_c)| = k \cdot \frac{|1+sT_i|}{|sT_i|} \approx k \quad \text{Ty} \quad \underline{T_i \gg 1}$$

$$\Longrightarrow \underline{\underline{\omega_c = 0{,}083}}$$

$$\varphi_m = 180^\circ + \angle G(j\omega_c) = 180^\circ + \underbrace{\angle G_m(j\omega_c) G_p(j\omega_c)}_{40^\circ} + \angle G_R(j\omega_c)$$

$$\Longrightarrow \underline{\underline{G_R(j\omega_c) = -5^\circ}}$$

$$\angle G_R(j\omega_c) = -90^\circ + \arctan(T_i \omega_c) = -5^\circ$$

$$\Longrightarrow T_i = \frac{1}{\omega_c} \tan 85^\circ = \underline{\underline{138}}$$

8.7 $$G_p(s) = \frac{2e^{-2s}}{1+10s}$$

Dimensionera reg. s.a

a) stigtid $t_s < 2{,}8s$

b) $\varphi_m > 60^\circ$

c) Inget kvarstående fel $\Longrightarrow$ PI reg.

a) $t_s \approx \frac{1{,}4}{\omega_c}$ $\boxed{\text{formelsamling } \underline{\text{s. 20}}}$

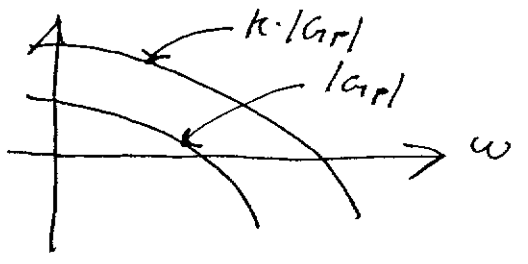
$t_s = 2{,}83 \Longrightarrow \underline{\underline{\omega_c = 0{,}5}}$

$$|G(j\omega_c)| = 1$$

$$|K \cdot G_p(j\omega_c)| = K \left| \frac{2e^{-2s}}{1+10j\omega_c} \right| = k \cdot \frac{2}{\sqrt{1+100 \cdot 0{,}5^2}}$$

$$\Longrightarrow k = 2{,}55$$

$$\Longrightarrow G_R(s) = k = 2{,}55$$



b) $\varphi_m = 180° + \angle G_0(j\omega_c) = 180° - \frac{2\omega_c}{\pi}\cdot 180° - \arctan(10\omega_c) =$

$= [\omega_c = 0{,}5] = 44°$

$\Longrightarrow$ Höj φ_m $17° + 10° = 27°$ "10° tumregel"
"17° 60−44+1"

$$G_{lead}(s) = \frac{1}{\sqrt{b}} \frac{1+T_d s}{1+\frac{T_d}{b}\cdot s}$$

$\xRightarrow{\text{diagr 8.1 övn bok}}$ $\underline{\underline{b = 3}}$

max faslyft i $\omega_c \Rightarrow \frac{\sqrt{b}}{T_d} = \omega_c \Longrightarrow T_d = \frac{\sqrt{3}}{0{,}5} = \underline{3{,}46}$

$$\Longrightarrow G_{lead}(s) = \frac{1}{\sqrt{3}} \frac{1+3{,}5\cdot s}{1+1{,}2\cdot s}$$

$$\Longrightarrow G_R(s) = k\cdot G_{lead}(s)$$

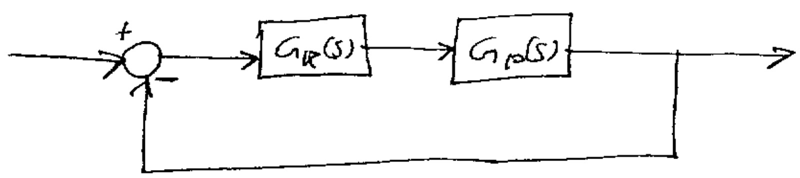
c) $G_{PI}(s) = k_i \cdot \frac{1+T_i\cdot s}{T_i\cdot s}$ ($k_i = 1$ i vårt fall)

tumregel
välj: $T_i = \frac{1}{0{,}2\omega_c} = 10$

$$\Longrightarrow G_{PI}(s) = \frac{1+10s}{10s}$$

$$\Longrightarrow \underline{\underline{G_R(s) = 2{,}55\cdot\frac{1}{\sqrt{3}}\cdot\frac{1+3{,}5\cdot s}{1+1{,}2\cdot s}\cdot\frac{1+10s}{10s}}}$$

8.13



$$G_R(s) = k\left(1+\frac{1}{T_i\cdot s}+T_d\cdot s\right)$$

Sökt: k, T_i, T_d m.h.a Ziegler-Nichols.

givet: Bode $\longrightarrow \frac{1}{A_m} = 0{,}35 \Longrightarrow$ förstärkning på $\frac{1}{0{,}35} = 2{,}8$ ggr

$\Longrightarrow k_0 = 2{,}8$ T_0 = svängn periodtid

$T = \frac{2\pi}{\omega_1} \Rightarrow T_0 = \frac{2\pi}{\omega_\pi} = \frac{2\pi}{1{,}8} = \underline{\underline{3{,}5}}$

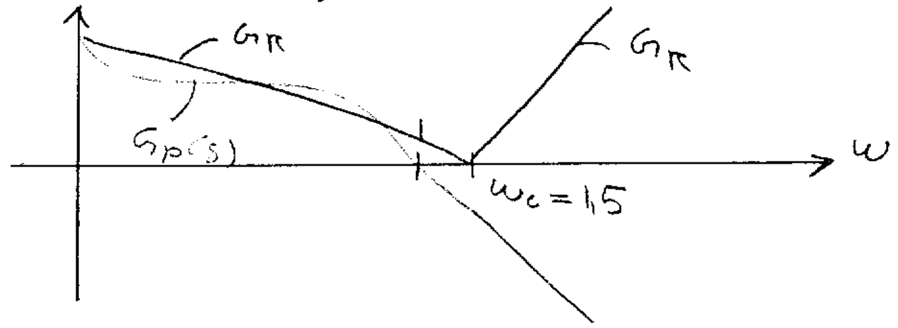
Tabell 8.1 $\longrightarrow$ $k = 0{,}6 k_0 = 1{,}71$

$T_i = \frac{T_0}{2} = 1{,}75$

$T_d = 0{,}125 \cdot T_0 = 0{,}44$

$\Longrightarrow G_R(s) = 1{,}71\left(1 + \frac{1}{1{,}75 s} + 0{,}44 s\right)$

b) kretsöverföring. $G(s) = G_R(s) \cdot G_P(s)$



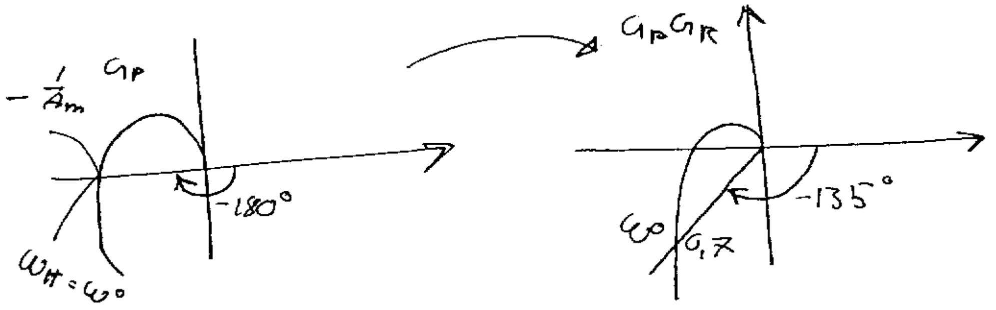
$G_R(s) = \frac{1}{s}(1 + 0{,}88 s)^2$

φ_m? $\angle G_P(1{,}5j) = -170°$

$\angle G_R(1{,}5j) = -90° + 2 \arctan(0{,}88 \cdot 1{,}5) = 15{,}71°$

$\Longrightarrow \varphi_m = 180° + \angle G_P + \angle G_R = 26°$

Alternativ metod att bestämma en regulator.



① $\begin{cases} |G_P(j\omega_0)| = 0{,}35 \\ \angle G_P(j\omega_0) = -180° \end{cases}$

② $\begin{cases} |G_P(j\omega^0) G_R(j\omega^0)| = 0{,}7 \\ \angle G_P G_R = -135° \end{cases}$

$G_R(s) = k \cdot \left(1 + \frac{1}{sT_i} + T_d s\right) = \underset{\text{tumregel}}{\left[T_i = 4T_d\right]} = k\left(\frac{1 + 4T_d s + 4T_d^2 s^2}{4 T_d s}\right) =$

$= k\left(\frac{(1 + 2T_d s)^2}{4 T_d s}\right)$

$|G_R(j\omega)| = k\left(\frac{1 + 4T_d^2 \omega^2}{4 T_d \omega}\right)$

$\angle G_R(j\omega) = -90^\circ + 2\arctan 2T_d\omega$

① & ② $\Longrightarrow$ $0{,}35\,|G_R(j\omega^\circ)| = 0{,}7$

$-180^\circ + \angle G_R(j\omega^\circ) = -135^\circ$

$\Longrightarrow$ $\angle G_R(j\omega^\circ) = -90^\circ + 2\arctan(2T_d\omega^\circ) = 45^\circ$

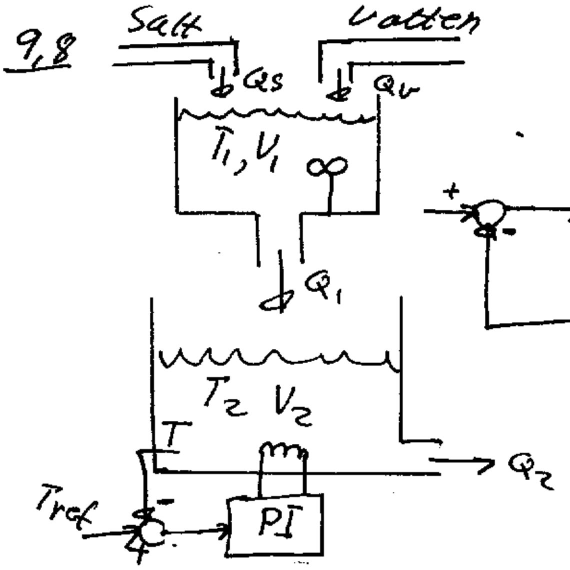
$\Longrightarrow$ $\underline{\underline{T_d = 0{,}71}}$

$\Longrightarrow$ $\underline{\underline{T_i = 4T_d = 2{,}84}}$

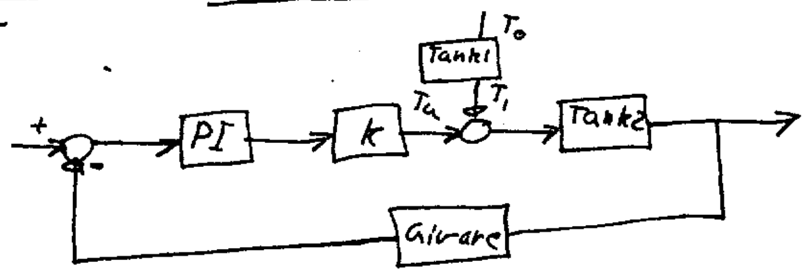
$$|G_R(j\omega^\circ)| = \frac{k(1+4T_d^2\omega^{\circ 2})}{4T_d\omega^\circ} = 2$$

$\Longrightarrow$ $k = 1{,}4$

9,8 96 12 03



Blockschema



$$\begin{cases} G_R = k_p\left(1+\frac{1}{sT_i}\right) \\ k_p = 2\cdot 10^3 \end{cases} \quad PI-reg$$

Givare: $\frac{1}{1+15s}$

Numeriska värden: $V_1 = 150\,l$, $V_2 = 300\,l$ $Q_1 = Q_2 = 0{,}75\,l/s$ $Q_s + Q_v = Q_0 = 0{,}75\,l/s$

Energibalans

Tank1: $\frac{d}{dt}\{\rho C_p V_1 T_1\} = \rho C_p (Q_0 T_0 - Q_1 T_1) \overset{\mathcal{L}}{\Longrightarrow} V_1 \cdot s \cdot T_1 = Q_0 T_0 - Q_1 T_1 \Longrightarrow T_1 = \underbrace{\frac{Q_0}{Q_1 + V_1 s}}_{G_1} T_0$

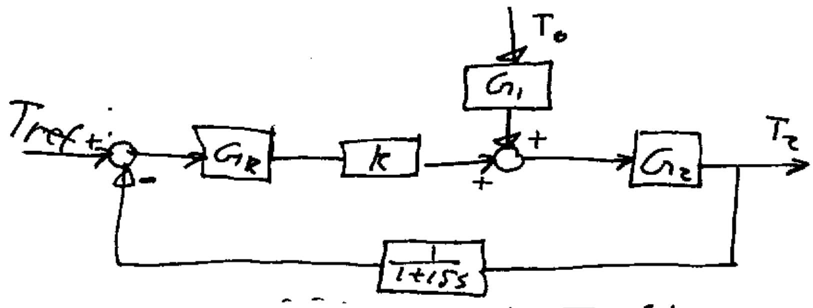
Tank2 $\frac{d}{dt}\{\rho C_p V_2 T_2\} = \rho C_p Q_1 T_1 - \rho C_p Q_2 T_2 + P \overset{\mathcal{L}}{\Longrightarrow} V_2 \cdot s \cdot T_2 = Q_1 T_1 - Q_2 T_2 + \frac{P}{\rho C_p} \Longrightarrow$

$$\Longrightarrow T_2 = \frac{Q_1 T_1 + \frac{P}{\rho C_p}}{V_2 s + Q_2} \overset{(Q_1 = Q_2)}{=} \frac{T_1 + \frac{P}{\rho C_p Q_1}}{\frac{V_2 s}{Q_1} + 1} = \frac{1}{V_2 s + 1}\left(T_1 + \underbrace{\frac{1}{\rho C_p Q_1}}_{k} P\right)$$

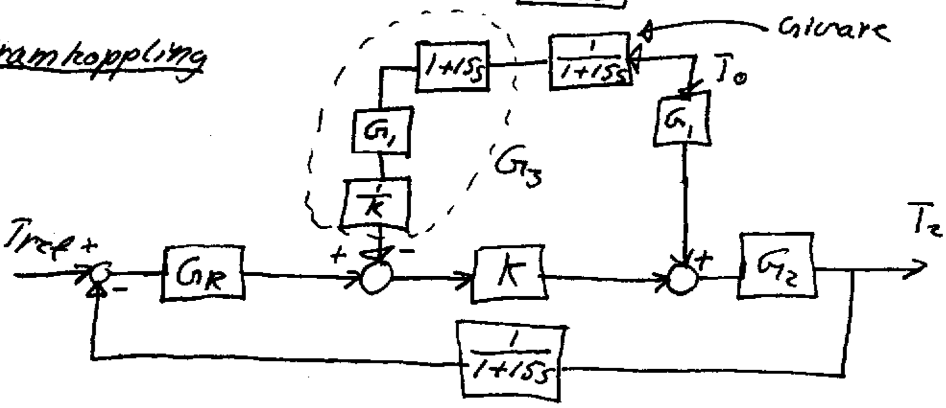
$$\Longrightarrow \boxed{G_1 = \frac{1}{200s+1}, \quad G_2 = \frac{1}{400s+1}}$$

$\rho = 1000\ \frac{kg}{m^3}$

$C_p = 4{,}19\ \frac{kJ}{kg\,K}$

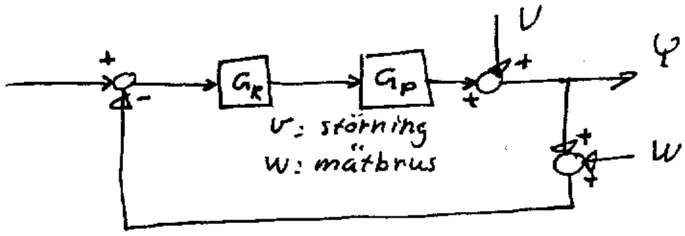


Framkoppling



$$G_3 = \frac{G_1}{k}(1+15s) = 3125 \cdot \frac{1+15s}{1+200s}$$

7.8



$G_p(s) = \frac{1}{s(s+1)}$

$G_r(\ ) = k$

tre fall $k = I, II, III$

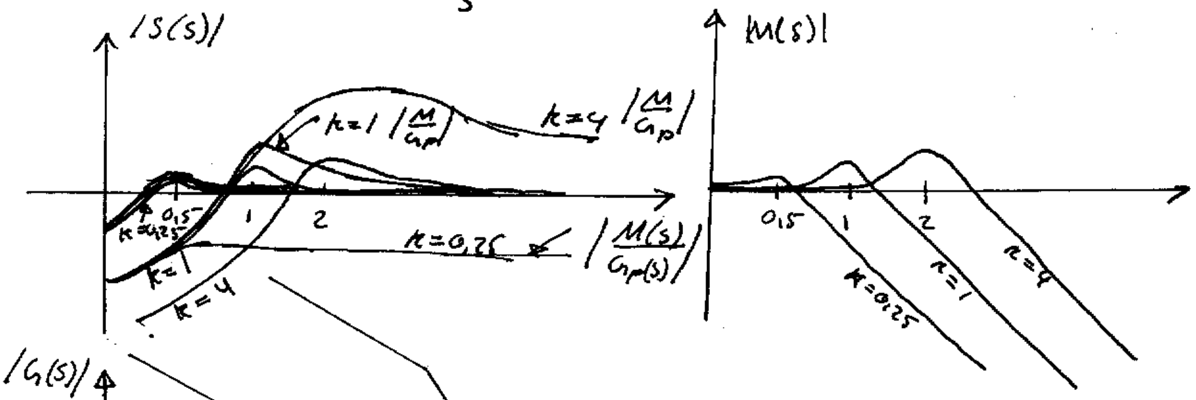
$I = 0{,}25$
$II = 1$
$III = 4$

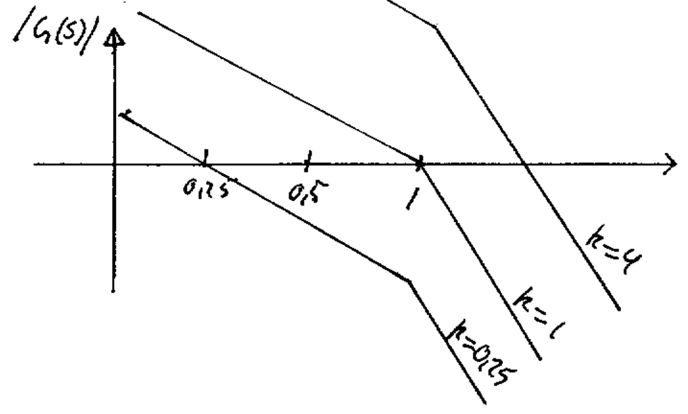
$$Y(s) = \underbrace{\frac{G(s)}{1+G(s)}}_{M(s)} R(s) + \underbrace{\frac{1}{1+G(s)}}_{S(s)} V(s) + \underbrace{\frac{-G(s)}{1+G(s)}}_{-M(s)} W(s)$$

$S(s)$: känslighetsfunktion
$M(s)$: komplementär känslighetsfunktion $(= 1 - S(s))$

$$S(s) = \frac{1}{1 + \frac{k}{s(s+1)}} = \frac{s(s+1)}{s^2+s+k} \qquad M(s) = \frac{\frac{k}{s(s+1)}}{1 + \frac{k}{s(s+1)}} = \frac{k}{s^2+s+k} =$$

$$= \frac{1}{\left(\frac{s}{\sqrt{k}}\right)^2 + 2\underbrace{\frac{1}{2\sqrt{k}}}_{\xi}\frac{s}{\sqrt{k}} + 1}$$





- ∵ ω_c ökar då k ökar
- ∵ φ_m [redacted] minskar då k ökar
- ∵ V kompenseras bättre då k ökar
  R följer — " —
  W komp. sämre då k ökar

$$\frac{U(s)}{R(s)} = \frac{M(s)}{G_p(s)} = k \cdot S(s) = k \cdot \frac{s(s+1)}{s^2+s+k} = \frac{s(s+1)}{\left(\frac{s}{\sqrt{k}}\right)^2 + \frac{s}{k} + 1} \Longrightarrow \text{LF assymptot: } s$$

c) KE $\quad 1 + G_R G_{rs} = 0 \Longrightarrow 1 + \frac{k}{s(s+1)} = 0 \Longrightarrow s^2 + s + k = 0 \Longrightarrow s = -\frac{1}{2} \pm \sqrt{\frac{1}{4} - k}$

$\Longrightarrow s_I = -\frac{1}{2} \qquad s_{II} = -\frac{1}{2} \pm \frac{\sqrt{3}}{2} j \qquad s_{III} = -\frac{1}{2} \pm \frac{\sqrt{15}}{2} j$

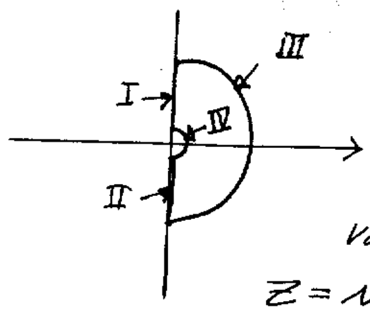
Dampning: $\xi = \frac{1}{2\sqrt{k}} = \begin{cases} 1 \\ 0{,}5 \\ 0{,}25 \end{cases}$ [redacted] ξ ökar $\Longrightarrow \varphi_m$ ökar

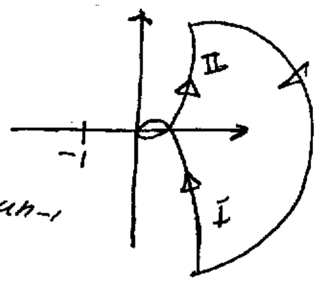
bandbredd $\quad \omega_b = \omega$ då $|M(s)| = -3$dB

ω_b ökar med k

5.13 b)

$G_p(s) = \frac{s+2}{s(2-s)}$ pol i HHP $\Rightarrow$ fullst Nyquist





Varv runt -1

$Z = N - P$

pol till $G_p(s)$ i HHP

I längs Im axeln $s = j\omega$, $\omega: 0 \to \infty$

$$G(j\omega) = \frac{j\omega+2}{j\omega(2-j\omega)} = \dots = \frac{4}{4+\omega^2} + j\frac{\omega^2-4}{\omega(4+\omega^2)}$$

I spegling

$$s \to re^{j\theta} \Rightarrow G(re^{j\theta}) = \underbrace{\frac{re^{j\theta}+2}{re^{j\theta}(2-re^{j\theta})}}_{A}$$

$A \xrightarrow[r\to\infty]{} 0$

Små r $G(re^{j\theta}) = \frac{1}{r}e^{-j\theta} = \frac{1}{r}(\cos\theta$

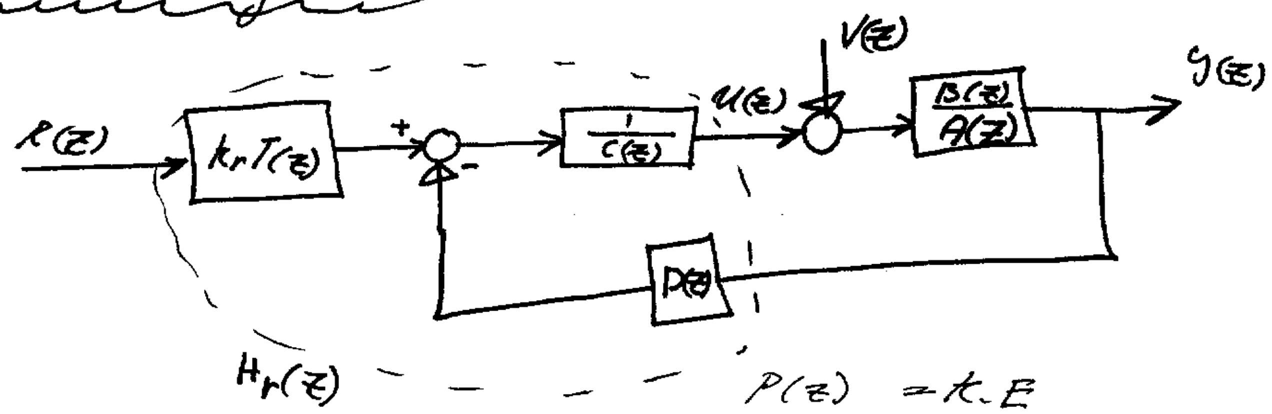
$\theta: -\frac{\pi}{2} \to \frac{\pi}{2}$

$Z = 0 + 1 = 1$

∴ Instabilt

Tidsdiskreta system

10.11)



$$Y(z) = \frac{B(z)}{A(z)} \cdot \left(V(z) + \frac{1}{C(z)} \left(R(z) \cdot k_r T(z) - D(z) \cdot Y(z) \right) \right)$$

$$\Longrightarrow \quad Y(z) = \frac{B(z)}{A(z)} \left(V(z) + \frac{k_r T(z) R(z)}{C(z)} \right) \cdot \frac{1}{1 + \frac{B(z)}{A(z)} \cdot \frac{1}{C(z)} \cdot D(z)}$$

$$= \frac{B(z)\left(C(z) V(z) + k_r T(z) R(z) \right)}{A(z) C(z) + B(z) D(z)}$$

a) $\Longrightarrow P(z) = A(z) C(z) + B(z) D(z)$

b) $$\frac{Y(z)}{R(z)} = [V(z) = 0] = \frac{k_r T(z) B(z)}{P(z)}$$

$$\frac{Y(z)}{V(z)} = \frac{B(z) \cdot C(z)}{P(z)}$$

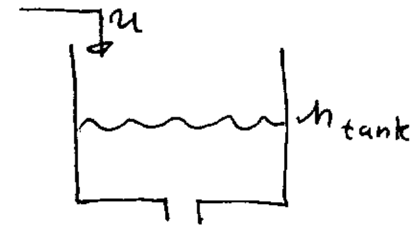
$$U(z) = \frac{1}{C(z)} \cdot \left(R(z) k_r \cdot T(z) - D(z) \cdot \frac{B(z)}{A(z)} \left(U(z) + V(z) \right) \right)$$

$$\Longrightarrow \quad U(z) = \frac{A(z) R(z) k_r T(z) - D(z) B(z) \cdot V(z)}{A(z) C(z) + B(z) D(z)}$$

$$\frac{U(z)}{R(z)} = \frac{A(z) T(z) k_r}{P(z)}$$

$$\frac{U(z)}{V(z)} = -\frac{D(z) \cdot R(z)}{P(z)}$$

uppgift 10.13



$$G(s) = \frac{H_{tank}(s)}{U(s)} = \frac{1}{2+10s}$$

$$H(z) = Z\left\{ \mathcal{L}^{-1}\left\{ G(s) \right\}_{t=k\cdot h} \right\}$$

Formelsamling 5.24 $\Longrightarrow$ $H(z) = \frac{0{,}1}{0{,}2} \cdot \frac{1-e^{-0{,}2h}}{z - e^{-0{,}2h}}$

olika samplingsintervall (h)

$h = 0{,}1 \longrightarrow H(z) = \frac{0{,}0099}{z - 0{,}980}$

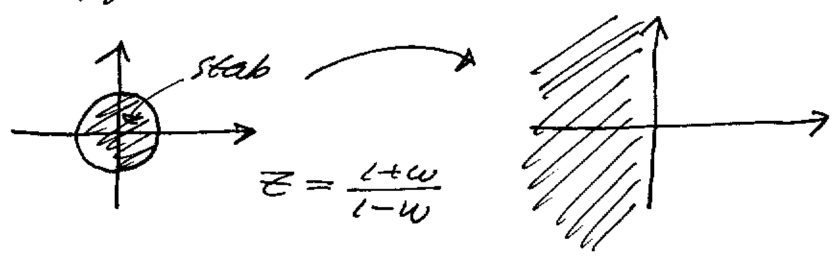
$h = 0{,}5 \Longrightarrow H(z) = \frac{0{,}048}{z - 0{,}905}$

---

uppgift 11.7 För vilka h är syst stabilt?

$$H_{sluten}(z) = \frac{1{,}2\left(1+0{,}4\cdot\frac{z^{-1}}{1-z^{-1}}\right)\left(\frac{hz^{-1}}{1-z^{-1}}\right)}{\left(1+1{,}2\left(1+0{,}4\frac{z^{-1}}{1-z^{-1}}\right)\frac{hz^{-1}}{1-z^{-1}}\right)} = \ldots = \frac{1{,}2hz^{-1}(1-0{,}6z)}{z^2+z(1{,}2h-2)+1-0{,}72h}$$

om syst är stabilt så skall polerna ligga inom enhetscirkeln



$$z = \frac{1+w}{1-w}$$

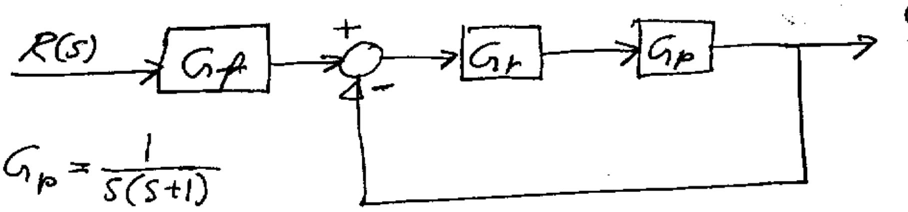
Möbiusavbildningar (avbilda nämnaren)

$$\Longrightarrow \frac{(1+w)^2}{(1-w)^2} + \frac{1+w}{1-w}(1{,}2h-2) + 1 - 0{,}72h$$

$$= \ldots = w^2(4-1{,}92h) + \underbrace{1{,}44h}_{>0}w + \underbrace{0{,}48h}_{>0} = 0$$

$4 - 1{,}92h > 0 \Longrightarrow h < \frac{4}{1{,}92} < 2{,}08$

7.13



$$G_p = \frac{1}{s(s+1)}$$

I $\quad G_r(s) = \frac{25(s+1)}{s+10}$, $\quad G_f = 1$

II $\quad G_r(s) = 1$, $\quad G_f(s) = \frac{25(s^2+s+1)}{s^2+10s+25}$

I $\quad S(s) = \frac{1}{1+G_p(s)\cdot G_r(s)} = \frac{1}{1+\frac{25\cancel{(s+1)}}{s+10}\cdot\frac{1}{s\cancel{(s+1)}}} = \frac{s(s+10)}{s^2+10s+25} = \frac{s(s+10)}{(s+5)^2}$

$$M(s) = \frac{G_pG_r}{1+G_pG_r} = \frac{25(s+1)}{s+10}\cdot\frac{1}{s(s+1)}\cdot\frac{s(s+10)}{(s+5)^2} = \frac{25}{(s+5)^2}$$

$$\frac{M(s)}{G_p(s)} = \frac{25s(s+1)}{(s+5)^2}$$

II

$$S(s) = \frac{s(s+1)}{s^2+s+1}$$

$$M(s) = \frac{1}{s^2+s+1}$$

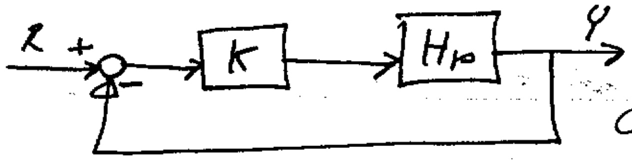
$$\frac{M(s)}{G_ps} = \frac{s(s+1)}{s^2+s+1}$$

Rita Bode!

Regulator 1:

- större styrsignal amplitud

$\Rightarrow$ kompenserar felet V bättre
men ω sämre

12,2 

96 12 10

$$G_p(s)=\frac{1}{1+s}\xrightarrow[\text{5.24}]{\text{F.S}} H_p(z)=\frac{1-e^{-h}}{z-e^{-h}}$$

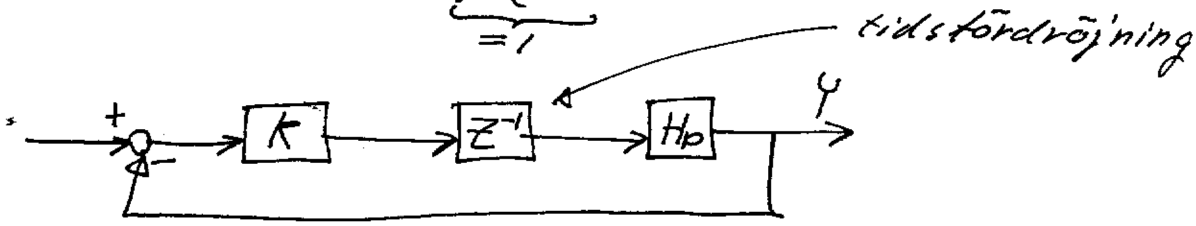
a) Snabb dator: $Y(z)=\dfrac{k\cdot H_p(z)}{1+k\cdot H_p(z)}$

$$\Longrightarrow \quad KE: \; 1+kH_p(z)=0 \Longrightarrow 1+k\cdot\frac{1-e^{-h}}{z-e^{-h}}=0$$

$$\Longrightarrow \quad z=e^{-h}-k(1-e^{-h})$$

stabilt då $|z|<1 \Longrightarrow \begin{cases} k(1-e^{-h})<1-e^{-h} \\ k(1-e^{-h})>-1-e^{-h} \end{cases}$

$$\Longrightarrow -\frac{1+e^{-h}}{1-e^{-h}}<k<\underbrace{\frac{1-e^{-h}}{1-e^{-h}}}_{=1}$$

b) 

$$KE. \quad 1+kz^{-1}H_p(z)=0 \Longrightarrow z^2-ze^{-h}+k(1-e^{-h})=0$$

Möbius: $z=\dfrac{1+w}{1-w} \Longrightarrow z^2-ze^{-h}+k(1-e^{-h})=$

$$=\left(\frac{1+w}{1-w}\right)^2-\frac{1+w}{1-w}e^{-h}+k(1-e^{-h})=0 \Longrightarrow \dots$$

$$\dots \Longrightarrow w^2\underbrace{(1+e^{-h}+k(1-e^{-h}))}_{a_0}+w\underbrace{(2-2k(1-e^{-h}))}_{a_1}+\underbrace{(1+k)(1-e^{-h})}_{a_2}$$

Routh - Hurwitz

w^2	a_0	a_2	$0 \dots$
w^1	a_1	$0 \dots$	
w^0	$\dfrac{a_1a_2-0}{a_1}$		

Stabilt då

$a_0>0 \quad a_1>0 \quad a_2>0$

$1+e^{-h}+k(1-e^{-h})>0$

$2(1-k)(1-e^{-h})>0$

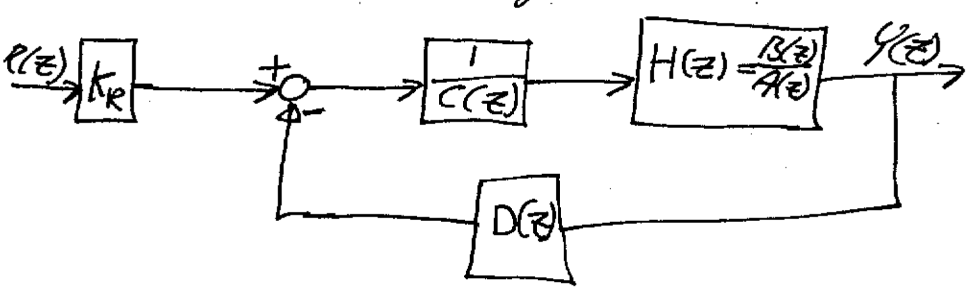
$(1+k)(1-e^{-h})>0$

$$\Longrightarrow \begin{cases} k>\dfrac{1+e^{-h}}{1-e^{-h}} \\ k<\dfrac{1}{1-e^{-h}} \\ k>-1 \end{cases} \Longrightarrow \quad -1<k<\frac{1}{1-e^{-h}}$$

c) a) $-10<K<1$

b) $5{,}52 > K > -1$

13.1 Dimensionering mha polplacering



söker $C(z)$ och $D(z)$ så att syst. får samtliga poler i $z = 0{,}5 \Longrightarrow \underline{P(z) = (1-0{,}5z^{-1})^{n_p}}$

Bestäm k_R s.a. ∄ kvarstående fel.

$$Y(z) = \frac{B(z)\cdot k_R \cdot R(z)}{\underbrace{A(z)\cdot C(z) + B(z)\cdot D(z)}_{P(z)}} = \frac{B(z)\cdot k_R}{P(z)}\cdot R(z)$$

Givet: $\begin{cases} A(z) = 1 + z^{-1} + 0{,}5z^{-2} \\ B(z) = 2z^{-1} + 3z^{-2} \end{cases}$

Ansätt $\begin{cases} C(z) = 1 + c_1 z^{-1} + \ldots c_{n_c} z^{-n_c} \\ D(z) = d_0 + d_1 z^{-1} + \ldots d_{n_d} z^{-n_d} \end{cases}$

välj $\begin{cases} n_c = n_b - 1 = 1 \\ n_d = n_a - 1 = 1 \\ n_p = n_a + n_b - 1 \end{cases} \Longrightarrow \begin{cases} C(z) = 1 + c_1 z^{-1} \\ D(z) = d_0 + d_1 z^{-1} \\ P(z) = (1-0{,}5z^{-1})^3 \end{cases}$

① $P(z) = (1 + 0{,}5z^{-2} + z^{-1})(1 + c_1 z^{-1}) + (2z^{-1} + 3z^{-2})(d_0 + d_1 z^{-1}) =$

$\ldots = 1 + (c_1 + 1 + 2d_0)z^{-1} + (c_1 + 0{,}5 + 2d_1 + 3d_0)z^{-2} +$

$+ (0{,}5c_1 + 3d_1)z^{-3}$

② $P(z) = 1 - 1{,}5z^{-1} + 0{,}75z^{-2} - 0{,}125z^{-3}$

① = ② $\Longrightarrow \begin{cases} c_1 + 2d_0 + 1 = -1{,}5 \\ c_1 + 0{,}5 + 2d_1 + 3d_0 = 0{,}75 \\ 0{,}5c_1 + 3d_1 = -0{,}125 \end{cases} \Longrightarrow \begin{matrix} c_1 = -4{,}9 \\ d_0 = 1{,}2 \\ d_1 = 0{,}775 \end{matrix}$

$$\Longrightarrow \begin{cases} C(z) = 1 - 4{,}4z^{-1} \\ D(z) = 1{,}2 + 0{,}\cancel{47}5z^{-1} \end{cases}$$

<u>Sök k_r!</u> $\quad Y(z) = \dfrac{B(z)\cdot k_R}{P(z)} \cdot R(z)$

enhetssteg: $\quad R(z) = \dfrac{z}{z-1}$

$$\lim_{k\to\infty} y(k) = \lim_{z\to 1} (z-1)\, Y(z) = \lim_{z\to 1} (z-1)\cdot \frac{B(z)\cdot k_r}{P(z)} \times R(z) =$$

$$= \lim_{z\to 1} \cancel{(z-1)}\cdot \frac{B(z)\cdot k_R}{P(z)} \cdot \frac{z}{\cancel{z-1}} = \frac{B(1)}{P(1)}\cdot k_K$$

om felet skall vara noll så gäller

$$\frac{B(1)}{P(1)}\cdot k_K = 1 \Longrightarrow k_R = \frac{P(1)}{B(1)} = \underline{\underline{0{,}025}}$$

<u>13.13</u> $\quad H_p(z) = \dfrac{z^{-1}(1-0{,}4z^{-1})}{(1-z^{-1})(1-0{,}8z^{-1})} = \dfrac{B(z)}{A(z)}$

a) $\quad P(z) = (1-0{,}5z^{-1})^2 \qquad \dfrac{Y(z)}{R(z)} = \dfrac{k_R B(z)}{P(z)}$

$$\frac{Y(1)}{R(1)} = 1 \Longrightarrow k_R = \frac{P(1)}{B(1)} = 2{,}5$$

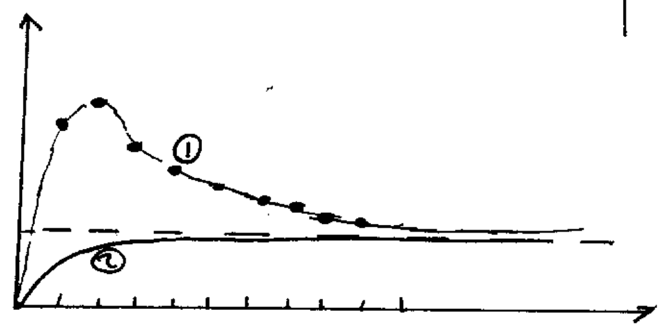
$$\Longrightarrow \quad Y(z) = \frac{2{,}5z^{-1} - 2{,}25z^{-2}}{1 - z^{-1} + 0{,}25z^{-2}} R(z)$$

$$Y(z) - Y(z)\cdot z^{-1} + 0{,}25\, Y(z)\cdot z^{-2} = 2{,}5z^{-1}\cdot R(z) - 2{,}25z^{-2} R(z)$$

$$\Longrightarrow y(k) - y(k-1) + 0{,}25\, y(k-2) = 2{,}5\, r(k-1) - 2{,}25\, r(k-2)$$

k	$y(k) =$	$y(k-1)$	$-0{,}25\,y(k-2)$	$+2{,}5\,r(k-1)$	$-2{,}25\,r(k-2)$
0	0	0	0	0	0
1	2,5	0	0	2,5	0
2	2,75	2,5	0	2,5	−2,25
3	2,37	2,75	−0,63	2,5	−2,25
4	1,93	2,37	−0,69	2,5	−2,25
5	1,7	1,93	−0,6	2,5	−2,25
6	1,52	1,7	−0,48	2,5	−2,25
7	1,39	1,52	−0,43	2,5	−2,25
8	1,29	1,39	−0,38	2,5	−2,25
9	1,19	1,29	−0,35	2,5	−2,25
∞	1			2,5	−2,25

stegsvar



b) förkorta bort $(1-0{,}9z^{-1})$

välj $P(z) = (1-0{,}9z^{-1})(1-0{,}6z^{-1})$

$$\Longrightarrow \quad \frac{Y(z)}{R(z)} = \frac{0{,}4\cdot z^{-1}\cancel{(1-0{,}9z^{-1})}}{\cancel{(1-0{,}9z^{-1})}(1-0{,}6z^{-1})}$$

$$k_R = \frac{P(1)}{B(1)} = \underline{\underline{0{,}4}}$$

tabellering kommer att ge kurva ② i fig ovan. (stabil utan översväng)

2.6 kvarstående fel: $\lim_{k\to\infty} e(k) = \lim_{z\to 1} (z-1)\, E(z)$

$$E(z) = R(z) - Y(z) = R(z)\left(1 - \frac{Y(z)}{R(z)}\right)$$

$$H(z) = \frac{k}{1-z^{-1}} \cdot \frac{1+0{,}5z^{-1}}{1-0{,}9z^{-1}}$$

$$\frac{Y(z)}{R(z)} = \frac{H(z)}{1+H(z)} \Longrightarrow 1 - \frac{Y(z)}{R(z)} = 1 - \frac{H(z)}{1+H(z)} = \frac{1}{1+H(z)} =$$

$$= \frac{1}{1 + \frac{k(1+0{,}5z^{-1})}{(1-z^{-1})(1-0{,}9z^{-1})}} = \frac{(1-z^{-1})(1-0{,}9z^{-1})}{(1-z^{-1})(1-0{,}9z^{-1}) + k(1+0{,}5z^{-1})}$$

$$R(z) = \frac{z}{z-1} R_0$$

$$\lim_{k\to\infty} e(k) = \lim_{z\to 1} (z-1)\cdot\frac{z}{z-1} R_0\left(1 - \frac{H(z)}{1+H(z)}\right) =$$

$$= \lim_{z\to 1} (z-1)\cdot\frac{z}{z-1} R_0 \left(\frac{(1-z^{-1})(1-0{,}9z^{-1})}{(1-z^{-1})(1-0{,}9z^{-1}) + k(1+0{,}5z^{-1})}\right) =$$

$$= \lim_{z\to 1} \frac{R_0 \cdot z \cdot (1-z^{-1})(1-0{,}9z^{-1})}{(1-z^{-1})(1-0{,}9z^{-1}) + k(1+0{,}5z^{-1})} = \underline{\underline{0}}$$

Inget kvarstående fel.

b) $r(kh) = v_0 h \Longrightarrow R(z) = \frac{z \cdot v_0 h}{(z-1)^2}$

$$\lim_{k\to\infty} e(kh) = \lim_{z\to 1} \frac{z \cdot v_0 \cdot h}{(z-1)^{\cancel{2}}} \cdot \cancel{(z-1)} \cdot \left(\frac{(1-z^{-1})(1-0{,}9z^{-1})}{(1-z^{-1})(1-0{,}9z^{-1}) + k(1+0{,}5z^{-1})}\right)$$

$$= \lim_{z\to 1} \frac{1}{\cancel{1-z^{-1}}} V_0 h \left(\frac{\cancel{(1-z^{-1})}(1-0{,}9z^{-1})}{(1-z^{-1})(1-0{,}9z^{-1}) + k(1+0{,}5z^{-1})}\right) =$$

$$= \frac{V_0 h \cdot 0{,}1}{0 + 1{,}5k} = \underline{\underline{\frac{V_0 h}{15k}}}$$