Storgruppsövning 3/12-13

The relationship between the induced emf and the rate of change of flux linkage is known as Faraday's law

$$\nabla \times \mathbb{E} = -\frac{\partial \mathbb{B}}{\partial t} \xrightarrow[\text{integral}]{\text{surface}} \begin{cases} \underbrace{\oint_C \mathbb{E} \cdot d\ell}_{v} = \underbrace{-\int_S \frac{\partial \mathbb{B}}{\partial t}\, dS}_{-\partial \phi / \partial t} \\ v = -\partial \Phi / \partial t \end{cases}$$

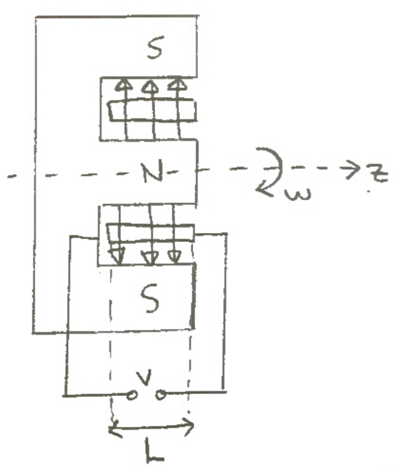
Stationary circuit in $\mathbb{B}(t)$
Moving ——— " ———

Moving conductor in a static magnetic field $\mathbb{B}_0$:

$$V_{21} = \int_1^2 (\underbrace{u}_{\text{circuit velocity}} \times \mathbb{B})\, d\ell \quad , \quad V = \oint_C (u \times \mathbb{B}) \cdot d\ell$$

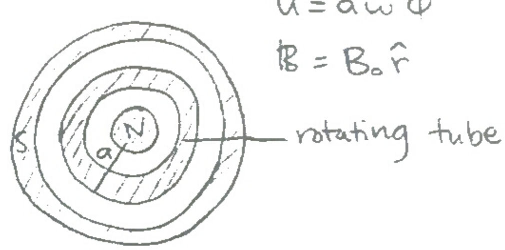
10.4

A tube is rotating in a pemanent magnet.
$\Phi = 0{,}25$ wb , $V = 10$ V
How many $\underbrace{\text{rotations per minute}}_{N\,(\text{turns}/\text{minute})}$, tube has to give $V = 10V$?



$\hat{u} = a\omega\,\hat{\Phi}$

$\mathbb{B} = B_0\,\hat{r}$



$$V_{12} = \int_1^2 (u \times \mathbb{B})\, d\ell = \int_1^2 (a\omega\hat{\Phi}) \times (B_0 \hat{r}) \cdot \hat{z}\, dz = \int_1^2 -\hat{z}\,(a\omega B_0)\,\hat{z}\, dz =$$

$$= -a\omega B_0 L$$

$$\Phi = \int_S B \cdot dS = 2\pi a L B_0$$ flux passing through cross sectional tube

$$V_{12} = -a\omega L B_0 = \frac{-\cancel{a}\omega \cancel{L}\Phi}{2\pi \cancel{a} \cancel{L}} = \frac{-\omega\Phi}{2\pi} \Longrightarrow \frac{\omega}{2\pi} = \frac{-V_{12}}{\Phi}$$

$$N = (\omega/2\pi) \cdot 60 = V_{12}/\Phi \cdot 60 = 10/0.25 \cdot 60 = 2400$$

Time harmonic $\mathbb{E}$-field: $\mathbb{E}(x,y,z,t) = \mathrm{Re}\{\underbrace{\bar{E}(x,y,z)}_{\text{vector phasor}} e^{j\omega t}\}$

Magnetic force on a current carrying circuit:

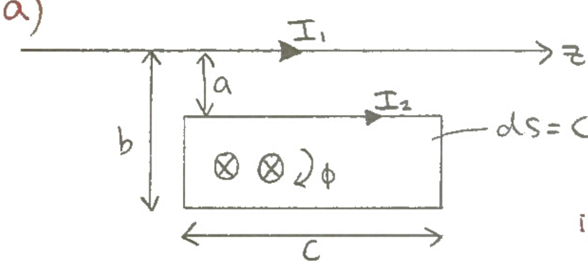
$$\mathbb{F}_m = I \oint_C d\ell \times \mathbb{B}$$

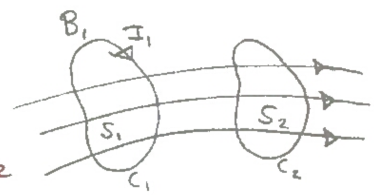
external magnetic field.

10.6

We have a rectangular loop with resistance R and inductance L located near a long wire with current I_1, $I_1 = I_0 \cos(\omega t)$. Find the mutual inductance (L_{12})!

a)





mutual inductance

$$L_{12} = \frac{\Lambda_{12}}{I_1} = \frac{N_2 \Phi_{12}}{I_1}$$

flux linkage

$$\mathbb{B} = \frac{\mu_0 I_1}{2\pi r}\hat{\Phi}$$

$$\Phi_{12} = \int_S \mathbb{B} \cdot ds = \int_a^b \frac{\mu_0 I_1}{2\pi r}\hat{\Phi} \cdot c\,\hat{\Phi}\,dr =$$

$$= \frac{\mu_0 I_1 c}{2\pi}\left[\ln r\right]_a^b = \frac{\mu_0 I_1 c}{2\pi}\ln\left(\frac{b}{a}\right)$$

$$L_{12} = \frac{\Phi_{12}}{I_1} = \frac{\mu_0 c}{2\pi}\ln\left(\frac{b}{a}\right)$$

b) Find current I_2!

$$V = \frac{-\partial \Phi}{\partial t} = \frac{-\partial}{\partial t}\left(\frac{\mu_0 \overset{I_0\cos(\omega t)}{I_1} c}{2\pi}\ln\left(\frac{b}{a}\right)\right) = \frac{I_0 \mu_0 c \omega}{2\pi}\ln\left(\frac{b}{a}\right)\underbrace{\sin(\omega t)}_{\cos(\omega t - \pi/2)}$$

$$V = \mathrm{Re}\{\bar{V}_1\, e^{j(\omega t - \pi/2)}\} = \mathrm{Re}\{\underbrace{\bar{V}_1\, e^{-j\pi/2}}_{\bar{V}_2} e^{j\omega t}\}$$

$$\Rightarrow \bar{V}_2 = \frac{I_0 \mu_0 c \omega}{2\pi}\ln\left(\frac{b}{a}\right)e^{j\pi/2}$$

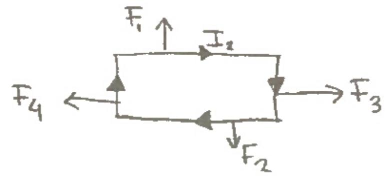
$$\bar{I}_2 = \frac{\bar{V}_2}{R + j\omega L} = \frac{\bar{V}_2 (R - j\omega L)}{R^2 + \omega^2 L^2} = \frac{I_0 \mu_0 c \omega \ln\left(\frac{b}{a}\right)}{2\pi(R^2 + \omega^2 L^2)}(R - j\omega L)e^{-j\pi/2}$$

forts. →

$$I_2 = Re\{\bar{I}_2 \cdot e^{i\omega t}\} = \frac{I_0 \mu_0 c\, \omega \ln(\frac{b}{a})}{2\pi(R^2+\omega^2L^2)}\left[R\sin\omega t - \omega L\cos(\omega t)\right]$$

c) Calculate the magnetic force on the loop!

$$F_m = I \oint_c dl \times B$$



$$B = \frac{\mu_0 I_1}{2\pi r}\hat{\phi}$$

$$F_m = \frac{I_2 c \mu_0 I_1}{2\pi b}\hat{r} - \frac{I_2 c \mu_0 I_1}{2\pi a}\hat{r}$$

At short sides the forces cancel eachother.

$$F_m = \frac{\mu_0 c I_1 I_2}{2\pi}\left(\frac{1}{b} - \frac{1}{a}\right)\hat{r} =$$

$$= \hat{r}\,\frac{\mu_0 c}{2\pi}\left(\frac{1}{b} - \frac{1}{a}\right)\underbrace{I_0\cos(\omega t)}_{I_1}\,\underbrace{\frac{I_0\mu_0 c\,\omega\ln(\frac{b}{a})}{2\pi(R^2+\omega^2L^2)}\left[R\sin(\omega t) - \omega L\cos(\omega t)\right]}_{I_2}$$

d) Calculate time average force $\langle F_m \rangle$!

$$\langle F_m \rangle = \frac{1}{T}\int_0^T F_m(t)\,dt =$$

$$= \frac{1}{\cancel{T}_{\frac{2\pi}{\omega}}}\,\hat{r}\,\underbrace{\left(\frac{\mu_0 c I_0}{2\pi}\right)^2\left(\frac{1}{b} - \frac{1}{a}\right)\frac{\omega\ln(\frac{b}{a})}{R^2+\omega^2L^2}}_{\alpha}\int_0^T\left[R\sin(\omega t) - \omega L\cos^2(\omega t)\right]dt\left(\frac{\omega}{\omega}\right) =$$

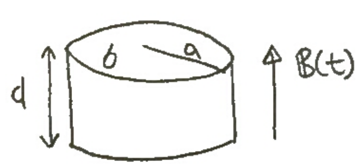
$$= \hat{r}\,\alpha\,\frac{\cancel{\omega}}{2\pi}\int_{\omega t=0}^{2\pi}\left[R\sin(\omega t) - \omega L\cos^2(\omega t)\right]\frac{d(\omega t)}{\cancel{\omega}} =$$

$$= \hat{r}\,\alpha\,\frac{1}{2\pi}\left[R\cdot 0 - \omega L\pi\right] = -\hat{r}\,\alpha\,\frac{\omega L}{2}$$

$$\left(\int_{\omega t=0}^{2\pi}\underbrace{\cos^2(\omega t)}_{\frac{1+\cos(2\omega t)}{2}}d(\omega t) = \left[\frac{\omega t}{2} + \frac{1}{4}\sin(2\omega t)\right]_0^{2\pi} = \pi\right)$$

12.22

A circular disk located in time varying uniform magn. field.



$$\begin{cases} B(t) = \hat{z}B_0\cos(\omega t) \\ \omega = 2\pi\cdot 10^3 \end{cases}$$

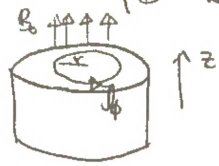
$$\begin{cases} a = 3\,cm \\ d = 0{,}1\,mm \\ \sigma = 10^7\,s/m \end{cases}$$

a) Calculate induced eddy currents!

$$\mathbb{J}_\phi(r) = \sigma \mathbb{E}_\phi(r)$$

$$\nabla \times \mathbb{E} = -\frac{\partial \mathbb{B}}{\partial t} \Longrightarrow \text{phasor form } \nabla \times \bar{\mathbb{E}} = -i\omega \bar{\mathbb{B}}$$

$$\xRightarrow{\text{surface integral}} \oint_c \mathbb{E}\cdot d\ell = -i\omega \int_s \mathbb{B}\, dS\ , \quad \begin{cases} \bar{\mathbb{E}} = \bar{\mathbb{E}}_\phi(r)\hat{\Phi} \\ \bar{\mathbb{B}} = \mathbb{B}\hat{z} = B_0\hat{z} \end{cases}$$



$$\Longrightarrow \mathbb{E}_\phi(r)\cdot 2\pi r = -i\omega B_0 \pi r^2$$

$$\Longrightarrow \bar{\mathbb{E}}_\phi(r) = \frac{-i\omega B_0 r}{2}$$

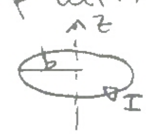
$$\bar{\mathbb{J}}_\phi(r) = \sigma \bar{\mathbb{E}}_\phi(r) = \frac{-i\sigma\omega B_0 r}{2}$$

$$\mathbb{J}_\phi(r,t) = \mathrm{Re}\{\bar{\mathbb{J}}_\phi(r)e^{i\omega t}\} = \frac{\sigma B_0 \omega r}{2}\sin(\omega t)$$

b) Calculate $B(0,t)$ caused by eddy current at center.
$d \ll a \Longrightarrow$ neglect the thickness of the plate.
Consider the current as many current loops.

Magnetic field at the center of a circular loop with current I:

$$\mathbb{B} = \hat{z}\frac{\mu_0 I b^2}{2(\cancel{z^2}+b^2)^{3/2}} = \hat{z}\frac{\mu_0 I}{2b}$$



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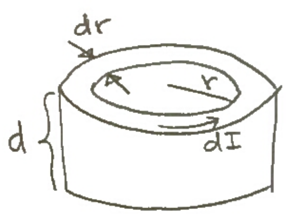

$$d\mathbb{B} = \frac{\mu_0 dI}{2r}\hat{z}$$

$$dI = \mathbb{J}_\phi \cdot d \cdot dr$$

$$\text{Total } \mathbb{B} = \int_{r=0}^{a} d\mathbb{B} = \int \frac{\mu_0 \mathbb{J}_\phi d}{2r}\hat{z}\, dr$$

$$\mathbb{B} = \int_0^a \frac{\mu_0 \sigma \omega B_0 r d}{4r}\sin(\omega t)\hat{z}\, dr = \frac{\mu_0 \sigma \omega B_0 d a}{4}\sin(\omega t)\hat{z}$$

$$\mathbb{B} = \frac{4\pi\cdot 10^{-7}\cdot 10^{4}\cdot 2\pi\cdot 10^{5}\cdot 0{,}1\cdot 10^{-3}\cdot 3\cdot 10^{-2}\cdot B_0 \sin(\omega t)}{4}\hat{z} =$$

$$= 0{,}0592\, B_0 \sin(\omega t)\,\hat{z}$$