Föreläsning 3/12-13

Grupphastighet 8.4

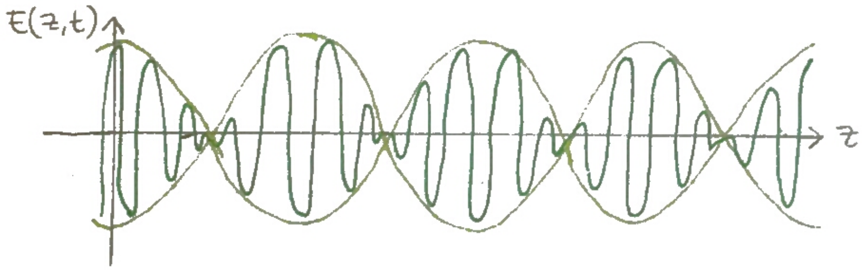
Fashastighet: $V_{fas} = \frac{\omega}{\beta}$

Våglängd: $\lambda = \frac{2\pi}{\beta}$

Betrakta två vågor med olika frekvens:
$\omega_0 + \Delta\omega$ och $\omega_0 - \Delta\omega$
$\beta_0 + \Delta\beta$ och $\beta_0 - \Delta\beta_0$

$$E(z,t) = E_0 \cos[(\omega_0+\Delta\omega)t - (\beta_0+\Delta\beta)z] + E_0 \cos[(\omega_0-\Delta\omega)t - (\beta_0-\Delta\beta)z]$$

$$= \ldots = 2E_0 \cos(\Delta\omega t - \Delta\beta z)\cos(\omega_0 t - \beta_0 z)$$



Grupphastighet: $\Delta\omega t - \Delta\beta z =$ konstant, $V_g = \frac{\partial z}{\partial t} = \frac{\Delta\omega}{\Delta\beta}$

Låt $D \rightarrow 0 \Rightarrow V_g = 1 \Big/ \frac{\partial\beta}{\partial\omega}$

Om $\beta \propto \omega \Rightarrow V_{grupp} = V_{fas}$

Poyntingvektorn 8.5

$$\nabla\cdot(\mathbb{E}\times\mathbb{H}) = \mathbb{H}(\nabla\times\mathbb{E}) - \mathbb{E}(\nabla\times\mathbb{H}) = \mathbb{H}\left(-\frac{\partial\mathbb{B}}{\partial t}\right) - \mathbb{E}\left(\mathbb{J} + \frac{\partial\mathbb{D}}{\partial t}\right) =$$

$$= \frac{1}{2}\frac{\partial}{\partial t}(\mathbb{H}\cdot\mathbb{B}) - \frac{1}{2}\frac{\partial}{\partial t}(\mathbb{E}\cdot\mathbb{D}) - \frac{\mathbb{J}\cdot\mathbb{J}}{\sigma}$$

Integrera över volym:

$$\underbrace{\int_{V'}\nabla\cdot(\mathbb{E}\times\mathbb{H})\,dV'}_{\text{effekt ut}} + \underbrace{\frac{\partial}{\partial t}\int_{V'} W_m + W_e \, dV'}_{\text{tidsändring av fältenergi}} + \underbrace{\int_{V'}\frac{\mathbb{J}\cdot\mathbb{J}}{\sigma}\,dV'}_{\text{Joules lag, ohmska förluster}} = 0$$

$$\int_{V'}\nabla\cdot(\mathbb{E}\times\mathbb{H})\,dV' = \int_S \mathbb{E}\times\mathbb{H}\, d\mathbb{S}$$

(se ex. 8.7 hemma)

$\mathbb{S} = \mathbb{E}\times\mathbb{H}$ Poyntingvektorn $[W/m^2]$

Komplexa poyntingvektorn

$$\left.\begin{aligned} \mathbb{E}(\mathbb{r}) &= E_{re}(\mathbb{r}) + iE_{im}(\mathbb{r}) \\ \mathbb{H}(\mathbb{r}) &= H_{re}(\mathbb{r}) + iH_{im}(\mathbb{r}) \end{aligned}\right\} \text{Komplexa fält}$$

$$\left.\begin{aligned} \mathbb{E}(\mathbb{r},t) &= E_{re}(\mathbb{r})\cos\omega t - E_{im}(\mathbb{r})\sin\omega t \\ \mathbb{H}(\mathbb{r},t) &= H_{re}(\mathbb{r})\cos\omega t - H_{im}(\mathbb{r})\sin\omega t \end{aligned}\right\} \text{Reell form}$$

$$\mathbb{S} = \mathbb{E}\times\mathbb{H} = (E_{re}\cos\omega t - E_{im}\sin\omega t)\times(H_{re}\cos\omega t - H_{im}\sin\omega t) =$$

$$= (E_{re}\times H_{re})\cos^2\omega t + (E_{im}\times H_{im})\sin^2\omega t -$$

$$- [(E_{im}\times H_{re}) + (E_{re}\times H_{im})]\underbrace{\sin\omega t\cos\omega t}_{\text{tidsm.}=0}$$

Tidsmedelvärde:

$$S_{av} = \frac{1}{T}\int_0^T S(r,t)\,dt = \boxed{\frac{1}{2}\left[E_{re}\times H_{re} + E_{im}\times H_{im}\right]}$$

helt antal perioder

Tidsmv. av poyntingvektorn

Komplexa fält:

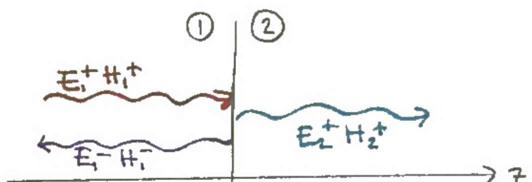
$$\frac{1}{2}\mathrm{Re}\{\mathbb{E}\times\mathbb{H}^*\} = \frac{1}{2}\mathrm{Re}\{(\mathbb{E}_{re}+i\mathbb{E}_{im})\times(\mathbb{H}_{re}-i\mathbb{H}_{im})\} =$$

$$= \frac{1}{2}\{(\mathbb{E}_{re}\times\mathbb{H}_{re}) + (\mathbb{E}_{im}\times\mathbb{H}_{im}) + i(\mathbb{E}_{im}\times\mathbb{H}_{re} - \mathbb{E}_{re}\times\mathbb{H}_{im})\} =$$

$$= \boxed{\frac{1}{2}\left(\mathbb{E}_{re}\times\mathbb{H}_{re} + \mathbb{E}_{im}\times\mathbb{H}_{im}\right)}$$

Tidsmv. av komplexa poyntingvektorn.

Reflektion och transmission 8.8, 8.6



$$\gamma_1 = \sqrt{i\omega\mu_1(i\omega\varepsilon_1+\sigma_1)} \qquad \gamma_2 = \sqrt{i\omega\mu_2(i\omega\varepsilon_2+\sigma_2)}$$

$$Z_1 = \sqrt{\frac{i\omega\mu_1}{i\omega\varepsilon_1+\sigma_1}} \qquad Z_2 = \sqrt{\frac{i\omega\mu_2}{i\omega\varepsilon_2+\sigma_2}}$$

forts. →

Antag plana vågor som propagerar i z-led.
E-fältet polariserat i $\hat{x}$-led
H ———— '' ———— $\hat{y}$-led

$$\bar{E}_1^+ = \hat{x}\bar{E}_{10}^+ e^{-\gamma_1 z} \qquad \bar{H}_1^+ = \hat{y}(\bar{E}_{10}^+/Z_1)e^{-\gamma_1 z}$$

$$\bar{E}_1^- = \hat{x}\bar{E}_{10}^- e^{+\gamma_1 z} \qquad \bar{H}_1^- = -\hat{y}(\bar{E}_{10}^-/Z_1)e^{\gamma_1 z}$$

$$\bar{E}_2^+ = \hat{x}\bar{E}_{20}^+ e^{-\gamma_2 z} \qquad \bar{H}_2^+ = \hat{y}(\bar{E}_{20}^+/Z_2)e^{-\gamma_2 z}$$

Randvillkor ger: $E_{1tang} = E_{2tang}$

$H_{1tang} = H_{2tang}$ (om inga fria strömmar)

Om gränsytan vid $z = 0$

$$\Longrightarrow \bar{E}_{10}^+ + \bar{E}_{10}^- = \bar{E}_{20}^+$$

$$\frac{\bar{E}_{10}^+}{Z_1} - \frac{\bar{E}_{10}^-}{Z_1} = \frac{\bar{E}_{20}^+}{Z_2}$$

Eliminera:

$$\bar{E}_{10}^- = \frac{Z_2 - Z_1}{Z_2 + Z_1}\bar{E}_{10}^+ \qquad \Gamma = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

$$\bar{E}_{20}^+ = \frac{2Z_2}{Z_2 + Z_1}\bar{E}_{10}^+ \qquad \tau = \frac{2Z_2}{Z_2 + Z_1}$$

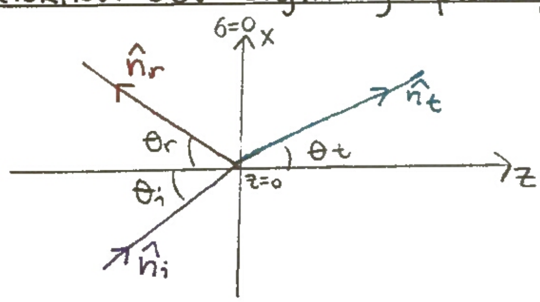
Kan visa att $1 + \Gamma = \tau$

För effekt reflekterande effekt $R = \frac{|S_r|}{|S_i|} = \frac{|\bar{E}_r|^2}{|\bar{E}_i|^2} = |\Gamma|^2$

transmitterande effekt $T = \frac{|S_t|}{|S_i|} = \frac{|S_i| - |S_r|}{|S_i|} = 1 - \frac{|S_r|}{|S_i|} = 1 - R$

$\Longrightarrow R + T = 1$ energiprincipen gäller än

Reflektion och brytning i plan gränsyta 8.10



Ansätt $\bar{E}_i(\mathbb{R}) = \bar{E}_{i0} e^{-j\beta_1 \hat{n}_i \mathbb{R}}$

$\bar{E}_r(\mathbb{R}) = \bar{E}_{r0} e^{-j\beta_1 \hat{n}_r \mathbb{R}}$

$\bar{E}_t(\mathbb{R}) = \bar{E}_{t0} e^{-j\beta_2 \hat{n}_t \mathbb{R}}$

forts. →

$\hat{n}_i = (\sin\theta_i, 0, \cos\theta_i) \Longrightarrow \hat{n}_i \mathbb{R} = x\sin\theta_i + z\cos\theta_i$

$\hat{n}_t = (\sin\theta_t, 0, \cos\theta_t) \Longrightarrow \hat{n}_t \mathbb{R} = x\sin\theta_t + z\cos\theta_t$

$\hat{n}_r = (\sin\theta_r, 0, -\cos\theta_r) \Longrightarrow \hat{n}_r \mathbb{R} = x\sin\theta_r - z\cos\theta_r$

Vid $z=0$ gäller: $(\bar{E}_i + \bar{E}_r)_{tang} = (\bar{E}_t)_{tang}$

$(\bar{H}_i + \bar{H}_r)_{tang} = (\bar{H}_t)_{tang}$

$$\bar{E}_{i0tang}\, e^{-i\beta_1 x \sin\theta_i} + \bar{E}_{r0tang}\, e^{-i\beta_1 x \sin\theta_r} = \bar{E}_{t0tang}\, e^{-i\beta_2 x \sin\theta_t}$$

$(\beta_1 = \omega/c_1)$

Uppfyllt om: $\frac{\omega}{c_1} x\sin\theta_i = \frac{\omega}{c_1} x\sin\theta_r = \frac{\omega}{c_2} x\sin\theta_t$

$$\boxed{\begin{aligned} &\text{Gäller om } \theta_i = \theta_r \\ &c_2\sin\theta_i = c_1\sin\theta_t \end{aligned}}$$

Snells lag