Storgruppsövning 26/11-13

Time varying fields and Maxwell's equation kap 7

Fundamental rule for electromagnetic induction:

Faraday's law

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \xrightarrow[\text{integral}]{\text{surface}} \oint_C \mathbf{E}\, d\ell = -\underbrace{\int_S \frac{\partial \mathbf{B}}{\partial t}\, ds}_{=\Phi}$$

- Stationary circuit in a time-varying $\underbrace{\text{magnetic field}}_{B(t)}$:

$V = \oint_C \mathbf{E} \cdot d\ell$ induced emf in circuit

$\phi = \int_S \mathbf{B} \cdot ds$ magnetic flux

$V = -\frac{d\phi}{dt}$ negative rate of increasing magnetic flux

- Moving conductor in a $\underbrace{\text{time-varying magnetic field}}_{B(t)}$:

$V' = \frac{-d\Phi}{dt}$ emf induced in circuit.

- Moving conductor in a static magnetic field:

$\frac{\mathbf{E}_m}{q} = \mathbf{u} \times \mathbf{B}$ induced electric field

$V_{12} = \int_1^2 (\mathbf{u} \times \mathbf{B}) \cdot d\ell$ $\quad V_{12} = \oint_C (\mathbf{u} \times \mathbf{B}) \cdot d\ell$ when moving conductor is a part of closed path C.

P 7.7

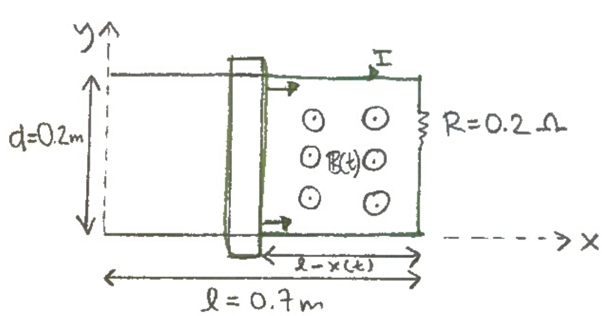
A conducting bar oscillates over tub parallell-conducting rails in a varying magnetic field.

$\mathbf{B}(t) = \hat{a}_z\, 5 \cos(\omega t)$

Position of bas: $x(t) = \underbrace{0.35}_{\ell/2}(1-\cos(\omega t))$

Find I!

$V = \frac{-d\Phi}{dt}$



$$\phi(t) = \int_S \mathbf{B} \cdot ds = B_z(t) \cdot (\ell - x(t)) \cdot d = \frac{B_0 \ell d}{2} \cos(\omega t)(1+\cos(\omega t))$$

$$\phi(t) = \frac{B_0 \ell d}{2}\left(\cos(\omega t) + \cos^2(\omega t)\right)$$

fel i riktning / tecken

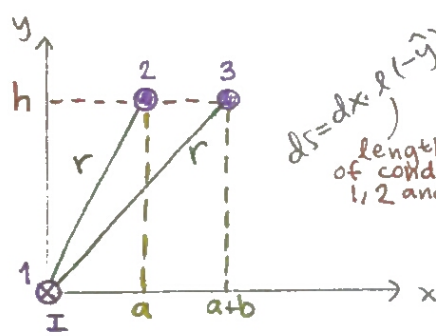
$$V = -\frac{d\phi}{dt} = \frac{B_0 \ell d}{2}\,\omega \sin(\omega t)\left(1 + 2\cos(\omega t)\right)$$

$V = (-)Ri$ induce current opposes the charge of the magnetic flux (ϕ).

$$I = -\frac{V}{R} = -\frac{B_0 \ell d}{2R}\,\omega\sin(\omega t)(1+2\cos(\omega t)) = -1{,}75\cdot 10^{-3}\,\omega\sin(\omega t)(1+2\cos(\omega t))$$

10.2

Three very long parallell conductor current $I = I_0\cos(\omega t)$ in conductor 1. Find induced voltage between 2 and 3.



$d\mathbf{s} = dx\cdot \ell(-\hat{y})$

length of conductor 1, 2 and 3.

$$V_{23} = \frac{-d\phi}{dt}$$

$$\phi = \int_S \mathbf{B}\, d\mathbf{s}, \quad \mathbf{B} = -\hat{\phi}\,\frac{\mu_0 I}{2\pi r}$$

magnetic field by current I

$$\mathbf{r} = x\hat{x} + h\hat{y}, \quad r = \sqrt{x^2+h^2}$$

$$\hat{\phi} = \hat{z}\times\hat{r} = \hat{z}\times\frac{\mathbf{r}}{r} = \hat{z}\times\frac{x\hat{x}+h\hat{y}}{\sqrt{x^2+h^2}} = \frac{x\hat{y}-h\hat{x}}{\sqrt{x^2+h^2}}$$

$$\phi = \int_a^{a+b} -\left(\frac{x\hat{y}-h\hat{x}}{\sqrt{x^2+h^2}}\right)\frac{\mu_0 I}{2\pi\sqrt{x^2+h^2}}\cdot \ell\, dx(-\hat{y})$$

$$\phi = \frac{\mu_0 I \ell}{2\pi}\int_a^{a+b}\frac{x}{x^2+h^2}\,dx = \frac{\mu_0 I \ell}{2\pi}\left[\frac{1}{2}\ln(x^2+h^2)\right] =$$

$$= \frac{\mu_0 I \ell}{2\pi}\ln\left(\frac{(a+b)^2+h^2}{a^2+h^2}\right)$$

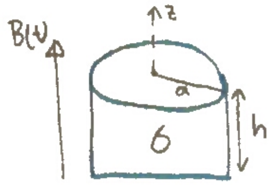
$$V = -d\phi/dt = -\frac{d}{dt}\left[\frac{\mu_0 I_0 \ell\cos(\omega t)}{4\pi}\ln\left(\frac{(a+b)^2+h^2}{a^2+h^2}\right)\right] =$$

$$= \frac{\mu_0 I_0 \ell\,\omega\sin(\omega t)}{4\pi}\left(\ln\left(\frac{(a+b)^2+h^2}{a^2+h^2}\right)\right)$$

$$\frac{V}{\ell} = \frac{\mu_0 I_0\,\omega\sin(\omega t)}{4\pi}\cdot\ln\left(\frac{(a+b)^2+h^2}{a^2+h^2}\right)$$

10.10

A thin conducting disk is in $B(t)$.
$B(t) = B_0 \cos(\omega t)$, conductivity σ.
Find the average power dissipation in disk.



$$P = \int_V \sigma E^2 \, dV$$

$$\oint_C E \, dl = -\frac{d\Phi}{dt} \xrightarrow[\text{contour with radius } r]{\text{assume a}} 2\pi r E_\phi =$$

$$= \frac{-d}{dt} \underbrace{\int_s B \cdot ds}_{\Phi} = -\frac{d}{dt}\left[B_0 \cos(\omega t) \pi r^2\right]$$

$$2\cancel{\pi} \cancel{r} E_\phi = B_0 \, \omega \sin(\omega t) \cancel{\pi} r^{\cancel{2}}$$

$$\Rightarrow E_\phi = \frac{B_0 \omega \sin(\omega t) r}{2}$$

$$P = \int_{r=0}^{a} \sigma \left(\frac{B_0 \omega \sin(\omega t) r}{2}\right)^2 \overbrace{2\pi h r dr}^{=dV} = \frac{\sigma B_0^2 \omega^2 \sin^2(\omega t) \pi h}{2} \int_0^a r^3 dr =$$

$$= \frac{\sigma B_0^2 \omega^2 \sin^2(\omega t) \pi h}{8} \cdot a^4$$

$$\bar{P} = \underset{\text{dissipation}}{\text{average power}} = \frac{1}{T}\int_0^T P(t)\,dt = \frac{\sigma B_0^2 \omega^2 \pi h a^4}{8T} \underbrace{\int_0^T \sin^2(\omega t)\,dt}_{T/2} =$$

$$= \frac{\sigma B_0^2 \pi h a^4 \omega}{16}$$

11.2

Use Ohm's law and Maxwell's equations for $\nabla \cdot D$ and $\nabla \times H$ and derive differential equation for $\rho(t)$ and solve this equation.

$$\left.\begin{cases} \nabla \cdot \mathbb{D} = \rho \quad \text{free charge density} \\ \nabla \times \mathbb{E} = \dfrac{-\partial \mathbb{B}}{\partial t} \\ \nabla \cdot \mathbb{B} = 0 \\ \nabla \times \mathbb{H} = \mathbb{J} + \dfrac{\partial \mathbb{D}}{\partial t} \end{cases}\right\} \text{Maxwell's equations}$$

($\mathbb{J}$: free charge density)

Ohm's law: $\mathbb{J} = \sigma \mathbb{E}$

forts. →

$$\nabla \cdot (\nabla \times \mathbb{H}) = 0 \Longrightarrow \nabla \cdot \left(\mathbb{J} + \frac{\partial \mathbb{D}}{\partial t} \right) = 0$$

$$\Longrightarrow \nabla \cdot \underbrace{\mathbb{J}}_{\sigma \mathbb{E}} + \frac{\partial}{\partial t} \big(\underbrace{\nabla \cdot \mathbb{D}}_{\rho} \big) = 0$$

$$\Longrightarrow \nabla \cdot (\sigma \mathbb{E}) + \frac{\partial}{\partial t} \rho = 0 \Longrightarrow \frac{\sigma}{\varepsilon} \underbrace{\nabla \cdot (\overbrace{\varepsilon \mathbb{E}}^{\mathbb{D}})}_{\rho} + \frac{\partial \rho}{\partial t} = 0$$

$$\Longrightarrow \frac{\sigma}{\varepsilon} \rho + \frac{\partial \rho}{\partial t} = 0 \Longrightarrow \rho = \rho_0 \, e^{-\frac{\sigma}{\varepsilon} t}$$