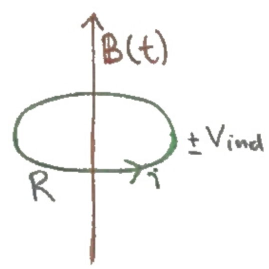
Föreläsning 26/11-13

Induktion

Faradays induktionslag: $\nabla \times E = -\frac{\partial B}{\partial t}$ (postulat)



$$\oint E\,dl = -\int \frac{\partial B}{\partial t}\,dS$$

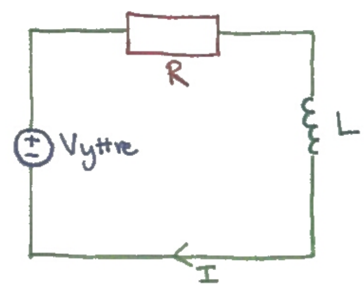
$$V_{ind} = -\frac{\partial \Phi}{\partial t}$$

$$\Phi = \Phi_{totalt} = \Phi_{yttre} + \Phi_{eget}$$

$$V_{ind} - RI = 0$$

I en krets: $\Phi_{eget} = LI$

$$V_{ind} = \frac{R}{L}\Phi_{eget} = \frac{-\partial \Phi_{totalt}}{\partial t}$$



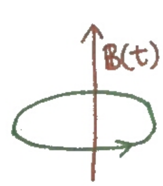
$$\frac{-\partial(\Phi_{yttre} + \Phi_{eget})}{\partial t} = \frac{R}{L}\Phi_{eget} = RI$$

$$\boxed{V_{yttre} = \frac{-\partial \Phi_{yttre}}{\partial t} = RI + \frac{\partial (LI)}{\partial t} = RI + L\frac{\partial I}{\partial t}}$$

Tre fall

1. Fix slinga i tidsvarierande fält
2. Ledare i rörelse i statiskt fält
3. Rörlig ledare i tidsvarierande fält.

1.



$$V_{ind} = \int \nabla \times E\,dS = \oint E \cdot dl = -\int \frac{\partial B}{\partial t}\,dS = \left\{ \begin{array}{l} \text{statisk} \\ \text{slinga} \end{array} \right\} =$$
$$= -\frac{\partial}{\partial t}\int B\,dS = -\frac{\partial \Phi}{\partial t}$$

Studera postulatet: $\nabla \times E = \frac{-\partial B}{\partial t}$, $B = \nabla \times A$, $\nabla \times E = -\frac{\partial}{\partial t}(\nabla \times A)$

$\nabla \times E = -\nabla \times \frac{\partial A}{\partial t}$, $\nabla \times \left(E + \frac{\partial A}{\partial t}\right) = 0$ forts. →

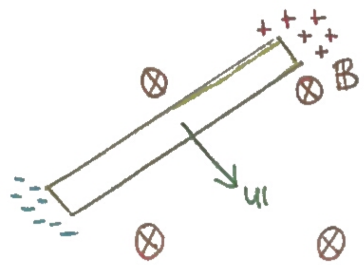
Inför potential: $\mathbb{E} + \frac{\partial \mathbb{A}}{\partial t} = -\nabla V$

$\mathbb{E} = -\nabla V - \frac{\partial \mathbb{A}}{\partial t}$ (annat sätt att skriva det på).

laddningar

tidsvarierande strömmar

(2.)



Kraft: $\mathbb{F}_m = q(\mathbb{u} \times \mathbb{B})$

Laddningar i vila på ledaren:

$-\nabla V + \frac{\mathbb{F}_m}{q} = 0$

$\Longrightarrow \nabla V = \mathbb{u} \times \mathbb{B}$

(ex 7.3 hemma)

I labbsystemet: $V_2 - V_1 = \int_1^2 \nabla V d\ell = \int_1^2 \mathbb{u} \times \mathbb{B} d\ell$

(3.) Kraft på laddning p.g.a $\mathbb{E}$ & $\mathbb{B}$-fält: $\mathbb{F} = q(\mathbb{E} + \mathbb{u} \times \mathbb{B}) =$

$= \left\{ \begin{matrix} \text{en observatör} \\ \text{som åkar med } q \end{matrix} \right\} = q(\mathbb{E}' + \mathbb{O} \times \mathbb{B}') = q\mathbb{E}'$

$\Longrightarrow \mathbb{E}' = \mathbb{E} + \mathbb{u} \times \mathbb{B}$ $\quad \mathbb{E}'$ i rörligt system

P.s.s $\mathbb{B}' = \mathbb{B} - \frac{1}{c^2}(\mathbb{u} \times \mathbb{E}$

$V_{ind} = \oint_c \mathbb{E}' d\ell = \oint_c \mathbb{E} d\ell + \oint_c \mathbb{u} \times \mathbb{B} d\ell = \int_S \frac{\partial \mathbb{B}}{\partial t} dS + \oint_c \mathbb{u} \times \mathbb{B} d\ell =$

$= V_{ind}^{trans} + V_{ind}^{rörelse} = -\frac{\partial \Phi}{\partial t}$

<u>Maxwells ekvationen kap 7.3</u>

$\nabla \times \mathbb{E} = -\frac{\partial \mathbb{B}}{\partial t}$ Faraday

$\nabla \times \mathbb{H} = \mathbb{J}$ Ampere

$\nabla \cdot \mathbb{D} = \rho$ Gauss

$\nabla \cdot \mathbb{B} = 0$

$\nabla \cdot \mathbb{J} + \frac{\partial \rho}{\partial t} = 0$ Kont. ekv.

Är de konsistenta?

$$\underbrace{\nabla\cdot(\nabla\times\mathbb{H})}_{=0} \neq \underbrace{\nabla\cdot\mathbb{J}}_{=-\frac{\partial\rho}{\partial t}}$$

Vi behöver: $\nabla\cdot(\nabla\times\mathbb{H}) = \nabla\cdot\mathbb{J} + \frac{\partial\rho}{\partial t} \equiv 0$

Men $\nabla\cdot\mathbb{D} = \rho$

$$\nabla\cdot(\nabla\times\mathbb{H}) = \nabla\cdot\mathbb{J} + \frac{\partial(\nabla\cdot\mathbb{D})}{\partial t} = \nabla\cdot\left(\mathbb{J} + \frac{\partial\mathbb{D}}{\partial t}\right)$$

$$\Longrightarrow \nabla\times\mathbb{H} = \mathbb{J} + \frac{\partial\mathbb{D}}{\partial t}$$ förskjutningsström

(På integralform 7.3.1)

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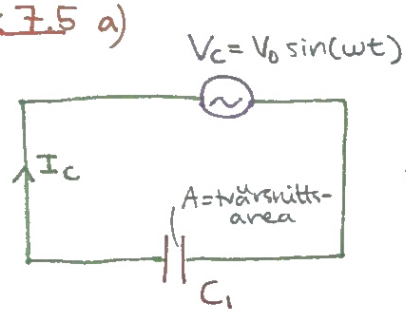
ex 7.5 a)



$$I_c = C_1\frac{\partial V}{\partial t} = \omega C_1 V_0\cos(\omega t)$$

Fält mellan plattorna:

$$\mathbb{D} = \varepsilon\mathbb{E} = \frac{V_0\sin(\omega t)}{d}\cdot\varepsilon$$

Förskjutningsström: $I_D = \int_A \frac{\partial\mathbb{D}}{\partial t}\,dS = \varepsilon\frac{A}{d}V_0\omega\cos(\omega t) =$

$= \omega C_1 V_0\cos(\omega t) = I_c$

b) hemma.