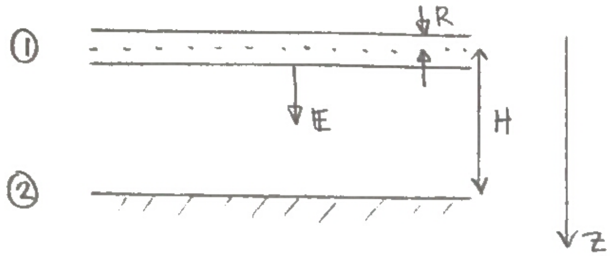
Storgruppsövning 22/11-13

Genomgång av tenta 23/8-13

1.

A phone line is suspended 6 m above the ground. Calculate the capacitance per unit length. (assume the line is very long and has $d = 1\,mm$).



$C = \dfrac{Q}{\Delta V}$

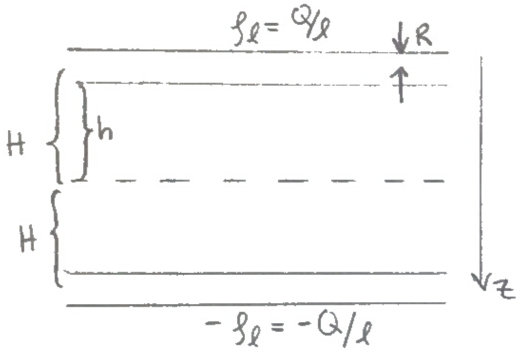
$\rho_\ell = Q/\ell$

$|\Delta V| = \int E\,d\ell$

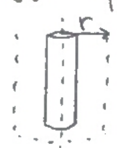
Find E

use image method !!!

use Gauss law to find E



Use Gauss law:



$\oint E\,ds = \dfrac{\bar{Q}}{\varepsilon_0}$

$\Rightarrow E = \dfrac{Q}{2\pi\varepsilon_0 r \ell} = \dfrac{\rho_\ell \ell}{\ell\, 2\pi\varepsilon_0 r}$

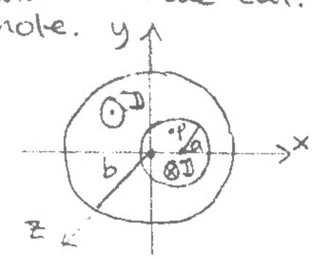
$$E(h) = \frac{\rho_\ell}{2\pi\varepsilon_0}\left(\frac{1}{H-h} + \frac{1}{H+h}\right) \qquad 0 \le h < H-R$$

$$|\Delta V| = \int_0^{H-R} \frac{Q}{2\pi\varepsilon_0}\left(\frac{1}{H-h} + \frac{1}{H+h}\right)dh = \frac{Q}{2\pi\varepsilon_0}\left(\ln(2H-R) - \ln R\right)$$

$$C = \frac{Q}{\Delta V} = \frac{Q}{\frac{Q}{2\pi\varepsilon_0}\left(\ln(2H-R) - \ln R\right)} = \frac{2\pi\varepsilon_0}{\ln(2H-R) - \ln R} = 5{,}5\ \frac{pF}{m}$$

2.

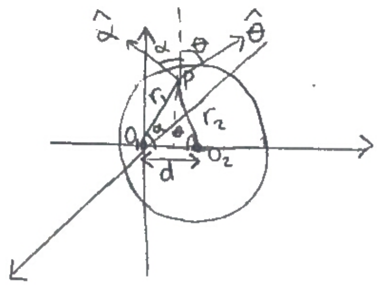
In a very long straight cylinder there is a cylindrical hole-cut. Find the magnitude and direction of $\mathbb{B}$ in the hole. (assume current is uniformly distributed).



The $\mathbb{B}$-field at point P in the cavity is the superposition of 2 $\mathbb{B}$-field:

$\mathbb{B}_1$ = produced by $J = J_0\hat{z}$ in a cylinder with $r = b$

$\mathbb{B}_2$ = —— " —— $J = -J_0\hat{z}$ —— " —— $r = a$



$$\mathbb{B}_1 = \frac{\mu_0 I_1}{2\pi r_1}\hat{\alpha} = \frac{\mu_0 J_0 \pi r_1^2}{2\pi r_1}\hat{\alpha} \quad (I_1 = J_0 \pi r_1^2$$

$$\mathbb{B}_2 = \frac{\mu_0 I_2}{2\pi r_2}\hat{\theta} = \frac{\mu_0 J_0 \pi r_2^2}{2\pi r_2^2} \quad (I_2 = J_0 \pi r_2^2)$$

$$\hat{\alpha} = \cos\alpha\,\hat{y} - \sin\alpha\,\hat{x}$$

$$\hat{\theta} = \cos\theta\,\hat{y} + \sin\theta\,\hat{x}$$

$$\mathbb{B} = \mathbb{B}_1 + \mathbb{B}_2 = \frac{\mu_0 J_0}{2}\left[r_1\cos\alpha\,\hat{y} - \cancel{r_1\sin\alpha\,\hat{x}} + r_2\cos\theta\,\hat{y} + \underbrace{\cancel{r_2\sin\theta\,\hat{x}}}_{= r_1\sin\alpha}\right]$$

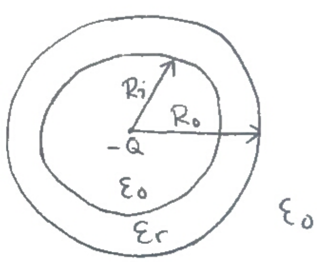
$$r_2\cos\theta = d - r_1\cos\alpha$$

$$\Rightarrow \mathbb{B} = \frac{\mu_0 J_0}{2}\left[r_1\cos\alpha\,\hat{y} + (d - r_1\cos\alpha)\hat{y}\right] = \frac{\mu_0 J_0}{2} d\,\hat{y}$$

Dugga 19/11-11

1.

A charge $-Q$ located in center of a dielectric sphere with permittivity ε_r. Find E, V, D and P as function or radius.



1. Find E and D by Gauss law
2. Find P by $P = D - \varepsilon_0 E = \varepsilon_0(\varepsilon_r - 1)E$
3. Find V by integration of E.

$R > R_o$:

$$E_1 = \frac{-Q}{4\pi\varepsilon_0 R^2} \qquad D_1 = \varepsilon_0 E_1 = \frac{-Q}{4\pi R^2} \qquad V_1 = \frac{-Q}{4\pi\varepsilon_0 R} \qquad P = 0$$

$R_i < R < R_o$:

$$E_2 = \frac{-Q}{4\pi\varepsilon_0\varepsilon_r R^2} \qquad D_2 = \varepsilon_0\varepsilon_r E = \frac{-Q}{4\pi R^2}$$

$$V_2 = -\int_\infty^{R_o} E_1\,dR - \int_{R_o}^{R} E_2\,dR = V_1(R = R_o) + \frac{Q}{4\pi\varepsilon_0\varepsilon_r}\int_{R_o}^{R}\frac{1}{R^2}\,dR =$$

$$= \frac{-Q}{4\pi\varepsilon_0}\left[\left(1-\frac{1}{\varepsilon_r}\right)\frac{1}{R_0}+\frac{1}{\varepsilon_r R}\right]$$

$$P_2 = (1 - 1/\varepsilon_r)\frac{-Q}{4\pi R^2}$$

$R < R_i$:

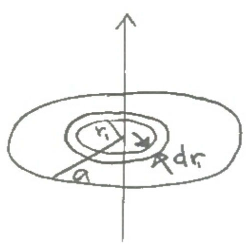
$$E_3 = \frac{-Q}{4\pi\varepsilon_0 R^2} \qquad D_3 = \frac{-Q}{4\pi R^2}$$

$$V_3 = V_2(R=R_i) - \int_{R_i}^{R} E_3\, dR = \frac{-Q}{4\pi\varepsilon_0}\left[\left(1-\frac{1}{\varepsilon_r}\right)\frac{1}{R_0} - \left(1-\frac{1}{\varepsilon_r}\right)\frac{1}{R_i} + \frac{1}{R}\right]$$

$$P_3 = 0$$

2.

En tunn cirkulär metallskiva (radien=a) befinner sig i vakuum. $\rho_s(r) = \frac{Q}{2\pi a\sqrt{a^2-r^2}}$ Beräkna kapacitansen till ∞ hos skivan.



$$\begin{cases} C = \frac{Q}{\Delta V} \\ V(\infty) = 0 \end{cases}$$

$$dq = \rho_s(r)\, 2\pi r_1 dr_1 = \frac{Q}{2\pi a\sqrt{a^2-r^2}}\, 2\pi r_1 dr_1$$

$\mathbb{R}_1 = r_1\hat{r}$ source point

$\mathbb{R}_2 = 0$ field point $\Longrightarrow \begin{cases} \mathbb{R}_{12} = \mathbb{R}_2 - \mathbb{R}_1 = -r_1\hat{r} \\ |\mathbb{R}_{12}| = r \end{cases}$

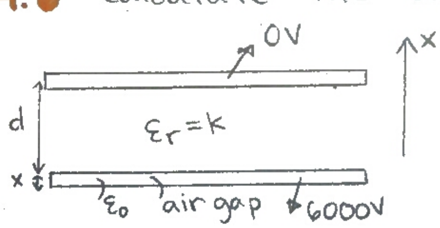
$$V(\mathbb{R}_2) = \int_S \frac{dq}{4\pi\varepsilon_0 R_{12}} = \int_{r_1=0}^{a} \frac{Q}{4\pi\varepsilon_0 a}\left[\frac{dr_1}{\sqrt{a^2-r_1^2}}\right] = \frac{Q}{4\pi\varepsilon_0 a}\left[\arcsin\left(\frac{r_1}{a}\right)\right]_0^a =$$

$$= \frac{Q}{4\pi\varepsilon_0 a}\left(\underbrace{\arcsin(1)}_{=\pi/2} - \underbrace{\arcsin(0)}_{=0}\right) = \frac{Q}{8\varepsilon_0 a}$$

$$\Longrightarrow C = \frac{Q}{\Delta V} = 8\varepsilon_0 a$$

Exempelsamling

4.6 Calculate the force per unit area of the workpiece.



$F_v = \nabla W_e$ fixed voltage system

$$= \frac{\partial}{\partial x} W_e = \frac{\partial}{\partial x}\left(\frac{1}{2}CV^2\right) = \frac{1}{2}V^2\frac{\partial}{\partial x}(C)$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\begin{cases} C_1 = \varepsilon_0 k \dfrac{S}{d} \\ C_2 = \varepsilon_0 \dfrac{S}{x} \end{cases} \Longrightarrow C = \frac{C_1 C_2}{C_1 + C_2} = \frac{\varepsilon_0 k S}{xk + d}$$

$$F_v = \frac{1}{2} V^2 \frac{\partial}{\partial x}\left(\frac{\varepsilon_0 k S}{xk+d}\right) = \frac{1}{2} V^2 \varepsilon_0 k S x \frac{-k}{(\underbrace{\cancel{xk}}_{x \text{ small}}+d)^2}$$

$$\frac{F_v}{S} = \frac{1}{2} V^2 \varepsilon_0 k^2 / d^2 \quad \left[\frac{N}{m^2}\right]$$