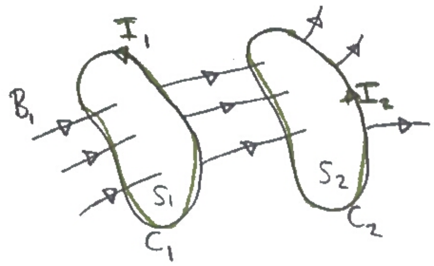
Storgruppsövning 20/11-13

Self inductance and mutual inductance



Current I_1 in $C_1 \Rightarrow B_1 \Rightarrow$ pass through S_2

from Biot-Savart's law:

$B_1 \propto I_1 \Rightarrow \phi_{12} = \int_{S_2} B_1\, dS \Rightarrow \phi_{12} = \ell_{12} I_1$

$\begin{cases} \ell_{12}: \text{mutual inductance between } C_1 \,\&\, C_2. \\ \ell_{11}: \text{self inductance of loop } C_1. \end{cases}$

flux linkage

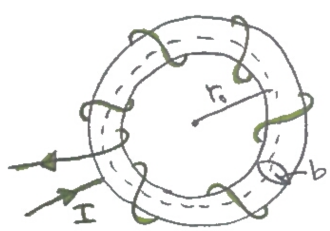
$\ell_{12} = \Lambda_{12}/I_1$

$\ell_{11} = \Lambda_{11}/I_1$

$\begin{cases} N_1 \text{ turns in } C_1 \Rightarrow \Lambda_{11} = N_1 \phi_{11} \\ N_2 \text{ turns in } C_2 \Rightarrow \Lambda_{12} = N_2 \phi_{12} \end{cases}$

P 6.35

Find the self-inductance of a toroidal coil, N-turns of wire, mean-radius $= r_0$, circular cross-section with $r = b$, $b \ll r_0$.



Use Ampere's law: $\oint_\ell B \cdot d\ell = \mu_0 I_{in}$

$\Rightarrow B_\phi 2\pi r = \mu_0 N I \Rightarrow B_\phi = \frac{\mu_0 N I}{2\pi r_0}$

$\phi = \int_S B \cdot dS = B_\phi \cdot S$ (cause $b \ll r_0$)

$\phi = B_\phi \pi b^2 \longrightarrow \phi = \frac{\mu_0 N I}{2\cancel{\pi} r_0} \cdot \cancel{\pi} b^2 = \frac{\mu_0 N I b^2}{2 r_0}$

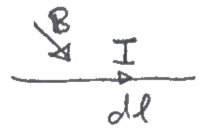
$\ell_{11} = \frac{\Lambda_{11}}{I_1} = \frac{N\phi}{I_1} = \frac{\mu_0 N^2 \cancel{I} b^2}{\cancel{I} 2 r_0} = \frac{\mu_0 N^2 b}{2 r_0}$

To find a self-inductance:

1) choose an appropriate coordinate system
2) assume a current
3) find B by Biot-Savart or by Ampere's law. (symmetry)
4) find the flux $\phi = \int_S B \cdot dS$
5) find flux linkage ($\Lambda = N\phi$)
6) find $\ell = \Lambda / I$

Magnetic force on a current-carrying conductor

$F_m = I\, d\ell \times B$ (N)



Magnetic force on a closed circuit with current I, in magnetic field B.

$$F_m = I \oint_C d\ell \times B \quad \text{(N)}$$

When we have 2 circuit carrying I_1 & I_2: the force F_{21} on circuit C_1:

$$F_{21} = I_1 \oint_{C_1} d\ell_1 \times B_{21}$$

caused by current I_2 in C_2.

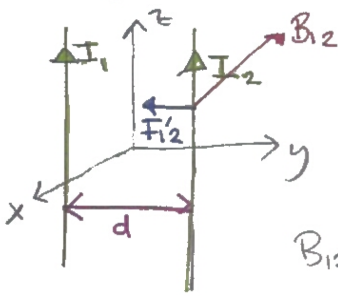
By Biot-Savart law: $B_{21} = \frac{\mu_0 I}{4\pi} \oint_{C_2} \frac{d\ell_2 \times \hat{a}_{R21}}{R_{21}^2}$

$$F_{21} = \frac{\mu_0 I_1 I_2}{4\pi} \oint_{C_1} \oint_{C_2} \frac{d\ell_1 \times (d\ell_2 \times \hat{a}_{R21})}{R_{21}^2}$$

Ampere's law of force between two current-carrying circuit.

example

force per unit length of two long parallell wire I_1 & I_2.



$F'_{12} = I_2 (\hat{a}_z \times B_{12})$ force per unit length on wire 2.

magnetic field at wire 2 from current I_1.

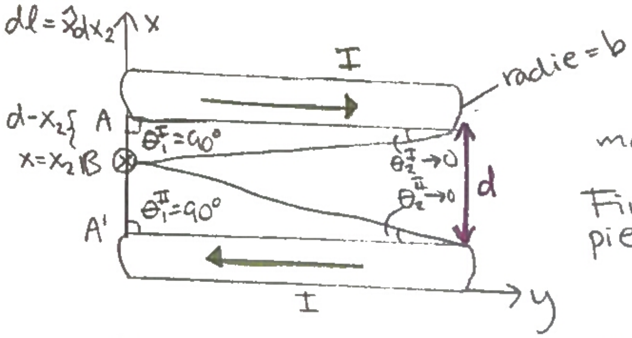
$B_{12} = -\hat{a}_x \frac{\mu_0 I_1}{2\pi d} \Rightarrow$ substitute in F'_{12}.

$$F'_{12} = -\hat{a}_y \frac{\mu_0 I_1 I_2}{2\pi d}$$

$$F'_{21} = -F'_{12}.$$

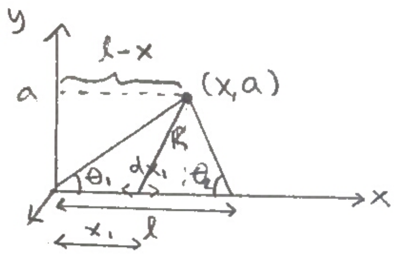
P6.46

The bar AA' in (Fig 6.53) connecting two very long parallell lines with current I.



Find direction and magnitude of F_m on AA'!

First we find B caused by a piece of line current.



$$dB = \frac{\mu_0 I}{4\pi}\left(\frac{dl' \times \mathbb{R}}{R^2}\right)$$

$$\begin{cases} \mathbb{R} = \hat{a}_x (x - x_1) + a\,\hat{a}_y \\ R = \sqrt{(x-x_1)^2 + a^2} \\ dl' = \hat{a}_x\, dx_1 \end{cases}$$

$$\Longrightarrow \quad dl' \times \mathbb{R} = \hat{z}\, a\, dx_1$$

$$dB = \hat{z}\frac{\mu_0 I}{4\pi}\frac{a}{((x-x_1)^2+a^2)^{3/2}}\,dx_1$$

$$B = \int_{x_1=0}^{l} dB = \hat{z}\frac{\mu_0 I a}{4\pi}\int_0^l \frac{dx_1}{((x-x_1)^2+a^2)^{3/2}} = \left\{ \begin{aligned} &\int [(x-a)^2+b^2]^{-3/2}\, dx = \\ &= \frac{x-a}{b^2\sqrt{(x-a)^2+b^2}}, \quad a = x_1,\ b^2 = a^2 \end{aligned} \right\} =$$

$$= \hat{z}\frac{\mu_0 I \cancel{a}}{4\pi}\left[\frac{1}{a^{\cancel{2}}}\frac{x_1 - x}{\sqrt{(x-x_1)^2+a^2}}\right]_{x_1=0}^{l} = \hat{z}\frac{\mu_0 I}{4\pi a}\left[\underbrace{\frac{l-x}{\sqrt{(l-x)^2+a^2}}}_{\cos\theta_2} + \underbrace{\frac{x}{\sqrt{x^2+a^2}}}_{\cos\theta_1}\right] =$$

$$= \hat{z}\frac{\mu_0 I}{4\pi a}(\cos\theta_2 + \cos\theta_1)$$

$$B(x = x_2, y = 0, z = 0) = \frac{\mu_0 I}{4\pi(d-x_2)}(0+1)\hat{z} + \frac{\mu_0 I}{4\pi x_2}(0+1)\hat{z} =$$

$$= \hat{z}\frac{\mu_0 I}{4\pi}\left(\frac{1}{d-x_2} + \frac{1}{x_2}\right)$$

$$F_m = \int_l I\, dl \times B, \quad F_m = \int_{x_2=b}^{d-b} I(\hat{x}\,dx_2) \times \left(\frac{\mu_0 I}{4\pi}\left(\frac{1}{d-x_2} + \frac{1}{x_2}\right)\right)\hat{z}$$

forts $\rightarrow$

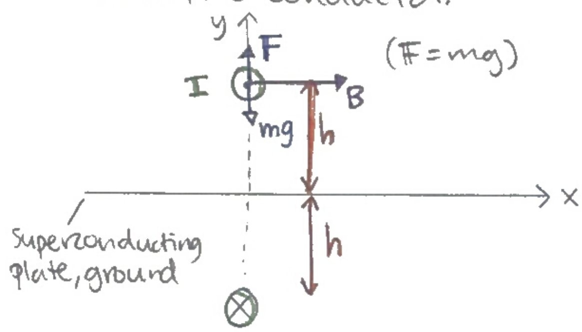
$$\mathbb{F}_m = \hat{y}\int_{x_2=b}^{d-b} -\frac{\mu_0 I^2}{4\pi}\left(\frac{1}{x_2}+\frac{1}{d-x_2}\right)dx_2$$

$$\mathbb{F}_m = -\hat{y}\frac{\mu_0 I^2}{4\pi}\Big[\ln(x_2)-\ln(d-x_2)\Big]_b^{d-b} = -\hat{y}\frac{\mu_0 I^2}{4\pi}\left[\ln\left(\frac{d-b}{b}\right)-\ln\left(\frac{b}{d-b}\right)\right]=$$

$$= -\hat{y}\frac{\mu_0 I^2}{2\pi}\ln\left(\frac{(d-b)}{b}\right)$$

7.21

long linear conductor; circular cross-section ($r=a$) float over a long superconductor plate. Find h and I_{min} to lift the conductor.



hint: use image method.

Find the magnetic flux density due to the image current at ($y=h$)

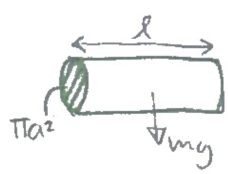
(Assume ℓ length of the wire)

Use Ampere's law: $\mathbb{B} = \frac{\mu_0 I}{2\pi(2h)}\hat{x}$

The force on length ℓ: $\mathbb{F}_m = \int_\ell I\, d\ell \times \mathbb{B} = \int_{z=-\ell/2}^{\ell/2} I(\hat{z}dz)\times\left(\frac{\mu_0 I}{4\pi h}\hat{x}\right)=$

$= \frac{\mu_0 I^2}{4\pi h}\hat{y}\int_{-\ell/2}^{\ell/2} dz = \frac{\mu_0 I^2 \ell}{4\pi h}\hat{y}$ magnetic force at $y=h$

$F_m = mg$



$$\frac{\mu_0 I^2 \ell}{4\pi h} = \eta V g = \eta(\pi a^2 \ell)g$$

$$h = \frac{\mu_0 I^2}{4\pi\eta\pi a^2 g}$$

in order to lift the wire, $h \geq a$:

$$\Rightarrow I^2_{min} = \frac{4\pi a^2 g\eta}{\mu_0}\cdot \underset{a}{\overset{\shortparallel}{h}} \Rightarrow I_{min} = 2\pi a\sqrt{\frac{g\eta a}{\mu_0}}$$