Storgruppsövning 19/11-13

Static magnetic fields, kap 6.

$\begin{cases} F_e = qE & \text{, electric force on } q \text{ in } E \\ F_m = q\,V \times B & \text{, magnetic force on moving charge } q \text{ in } B \end{cases}$

velocity

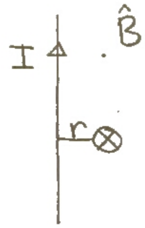
magnetic flux density

$F = F_e + F_m = q(E + V \times B)$ total electromagnetic force.

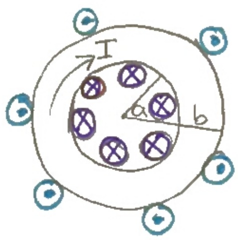
Fundamental postulates of magnetostatic (free space):

$\nabla \cdot B = 0 \Longrightarrow \oint_S B\,ds = 0$ law of conservation of magnetic flux

$\nabla \times B = \mu_0 J \Longrightarrow \oint_C B\,dl = \mu_0 I$ Ampere's law



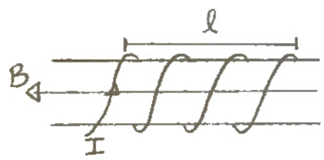
$\hat{B} = \hat{\Phi}\,\dfrac{\mu_0 I}{2\pi r}$



$B = \hat{\Phi}\,\dfrac{\mu_0 N I}{2\pi r}$

$a < r < b$

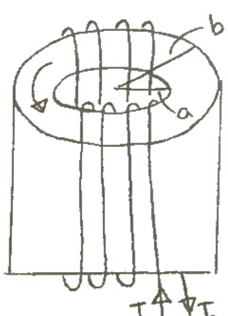
a toroidal-coil

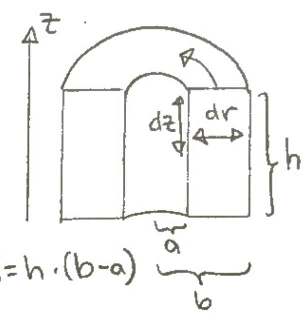


$B = \mu_0 n I$, $n = \dfrac{N}{l}$ (number of turns per length)

P 6.14

A circular toroid with rectangular cross section. Find the total magnetic flux through its cross section.





$S = h \cdot (b-a)$

$\Phi = ? = \int_S B \cdot dS$

$\int B_\phi \cdot dl = \mu_0 N I$

$\Rightarrow B_\phi = \dfrac{\mu_0 N I}{2\pi r}$, $a<r<b$

$\Phi = \int B_\phi \cdot \underbrace{dS}_{dz\,dr} = (*)$

$(*) = \dfrac{\mu_0 N I}{2\pi} \int_0^h \int_a^b \dfrac{1}{r}\,dz\,dr = \dfrac{\mu_0 N I h}{2\pi} \ln\left(\dfrac{b}{a}\right)$

for its.

Find the percentage of error if: the flux is found by multiplying the $\underbrace{\text{cross-section}}_{S}$ area by flux density at the $\underbrace{\text{mean radius.}}$

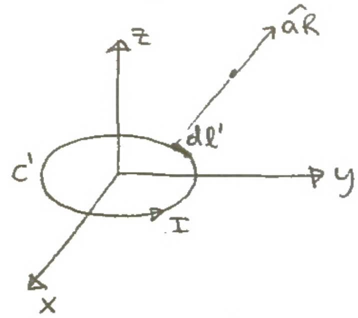
$$B_\phi\left(r=\frac{a+b}{2}\right)=\frac{\mu_0 NI}{2\pi\left(\frac{a+b}{\cancel{2}}\right)}=\frac{\mu_0 NI}{\pi(a+b)}$$

$$\Longrightarrow \phi' = B_\phi\left(r=\frac{a+b}{2}\right)\cdot h(b-a) \Longrightarrow \phi' = \frac{\mu_0 NIh}{\pi}\left(\frac{b-a}{b+a}\right)$$

$$\%\ \text{error} = \frac{\phi'-\phi}{\phi} = \left[\frac{2(b-a)}{(b+a)\ln(b/a)}-1\right]\cdot 100$$

The Biot-Savart law:

Gives the magnetic field of a current carrying circuit.



Vector magnetic potential (A)

$$B=\nabla\times A \xrightarrow[\substack{\text{assume}\\ \nabla\cdot A=0}]{} \nabla^2 A = -\mu_0 J \Longrightarrow A=\frac{\mu_0}{4\pi}\int_{V'}\frac{J}{R}\,dV'$$

For a closed circuit $A=\frac{\mu_0 I}{4\pi}\oint_{C'}\frac{dl'}{R} \Longrightarrow B=\frac{\mu_0 I}{4\pi}\int\frac{dl'\times a_R}{R^2}$

$B=\oint_{C'} dB \quad , \; dB=\frac{\mu_0 I}{4\pi}\left(\frac{dl'\times \mathbb{R}}{R^3}\right)$

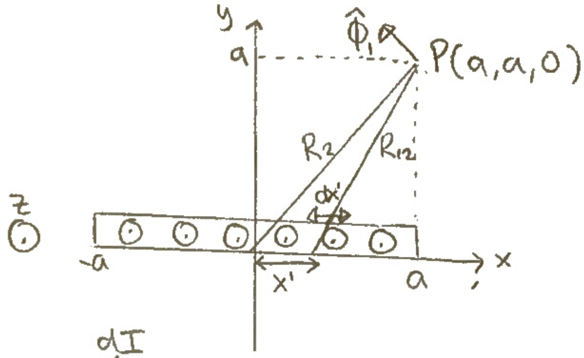
unit vector from source point to the field point

7.2

Biot-Savart law, a very long flat metal strip of width $2a$, located in x-z-plane. The current is distributed uniformly. Find the magnitude and direction of B at point P.

$B(P)=?$

The current is dist. uniformly $\Rightarrow I=I_0\hat{z} \Longrightarrow \left(J=\frac{I_0}{2a}\hat{z}\right)$



$$dB = \frac{\mu_0 \overbrace{J_s\, dx'}^{dI}}{2\pi R_{12}}\,\hat{\Phi}_1$$

$$\begin{cases} \mathbb{R}_{12} = \mathbb{R}_2 - \mathbb{R}_1 = \hat{x}a + \hat{y}a - x'\hat{x} = \hat{x}(a-x') + \hat{y}a \\ R_{12} = \sqrt{(a-x')^2 + a^2} \end{cases}$$

$$\hat{\Phi}_1 = \frac{d\mathbf{l}' \times \mathbb{R}_{12}}{R_{12}} = \frac{\hat{z} \times \mathbb{R}_{12}}{R_{12}} = \frac{\hat{y}(a-x') - \hat{x}(a)}{\sqrt{(a-x)^2 + a^2}}$$

$$dB = \frac{\mu_0 (I_0/2a)}{2\pi}\,\frac{-\hat{x}a + \hat{y}(a-x')}{(a-x')^2 + a^2}\,dx' \Longrightarrow B = \int_{x'=-a}^{a} dB$$

$$\Longrightarrow dB = \hat{x}\,dB_x + \hat{y}\,dB_y \Longrightarrow B_x = \int_{x'=-a}^{a} \frac{-\mu_0 I_0}{4\pi}\,\frac{dx'}{(a-x')^2 + a^2} =$$

$$= \left\{ \begin{array}{l} a - x' = \xi,\ d\xi = -dx' \\ x' = -a \Longrightarrow \xi = 2a \\ x' = a \Longrightarrow \xi = 0 \end{array} \right\} = B_x = \int_{\xi=2a}^{0} \frac{\mu_0 I_0}{4\pi a}\,\frac{d\xi}{\xi^2 + a^2} =$$

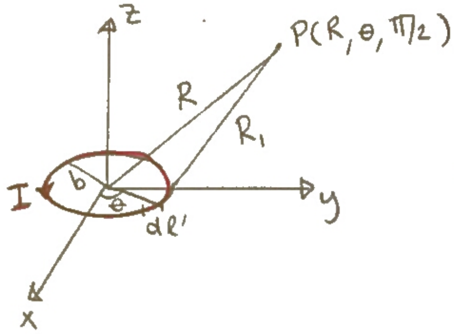
$$= \frac{\mu_0 I_0}{4\pi}\left[\frac{1}{a}\arctan\left(\frac{\xi}{a}\right)\right]_{2a}^{0} = \boxed{\frac{-\mu_0 I_0}{4\pi a}\arctan(2)}$$

$$B_y = \frac{\mu_0 I_0}{4\pi a}\int_{x'=-a}^{a} \frac{(a-x')}{(a-x')^2 + a^2}\,dx' = \{\text{p.s.s}\} = \frac{-\mu_0 I_0}{4\pi a}\int_{2a}^{a} \frac{\xi}{\xi^2 + a^2}\,d\xi =$$

$$= \frac{-\mu_0 I_0}{4\pi a}\left[\frac{1}{2}\ln(\xi^2 + a^2)\right]_{2a}^{0} = \frac{\mu_0 I_0}{8\pi a}\ln\left(\frac{4a^2 + a^2}{a^2}\right)\ \boxed{\frac{\mu_0 I_0}{8\pi a}\ln(5)}$$

$$\Longrightarrow B(P) = B_x + B_y = \frac{\mu_0 I_0}{4\pi a}\left(-\hat{x}\arctan(2) + \hat{y}\,\frac{\ln(5)}{2}\right)$$

The magnetic dipole: a small circular loop carries a current I.



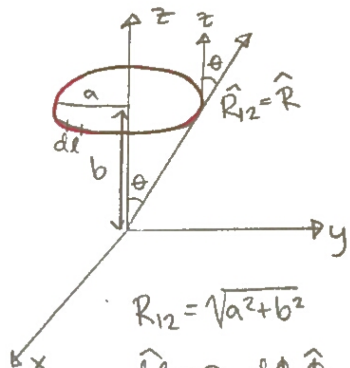
magnetic dipole moment

$$A = \mu_0 \frac{\mathbb{m} \times \hat{a}_R}{4\pi R^2}, \quad \mathbb{m} = \hat{a}_z \pi b^2 I = \hat{a}_z S I = \hat{a}_z m$$

$$\mathbb{B} = \frac{\mu_0 m}{4\pi R^3}\left(\hat{a}_R 2\cos\theta + \hat{a}_\theta \sin\theta\right)$$

7.10

A magnetic dipole $\mathbb{m} = \hat{z} m$ is in origin.
Find the $\underbrace{\text{magnetic flux}}_{\Phi}$ through the ring.



$$\Phi = \int_S B \cdot dS = ? = \int_S (\nabla \times A) \cdot dS = \oint_C A \cdot dl$$

$$A = \mu_0 \frac{\mathbb{m} \times \mathbb{R}_{12}}{4\pi R_{12}^2} = \frac{\mu_0 \hat{z} m \times \hat{R}}{4\pi (a^2+b^2)} =$$

$$= \frac{\mu_0}{4\pi(a^2+b^2)} m \underbrace{\sin\theta \hat{\Phi}}_{\hat{z} \times \hat{R}}$$

$$R_{12} = \sqrt{a^2+b^2}$$

$$\hat{dl} = a \cdot d\Phi \, \hat{\Phi}$$

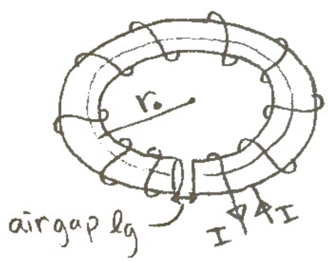
$$\sin\theta = \frac{a}{\sqrt{a^2+b^2}}$$

$$\Phi = \oint_C A\,dl = \int_{\Phi=0}^{2\pi} \frac{\mu_0 m}{4\pi(a^2+b^2)} \cdot \frac{a}{\sqrt{a^2+b^2}} \hat{\Phi} (\hat{\Phi} a \, d\Phi) =$$

$$= \frac{\mu_0 m a^2}{2(a^2+b^2)^{3/2}}$$

ex 6.10 i boken

Magnetic circuits, N-turns of wire around ferromatic Toroidal-coil. Find B and H both in core and in air-gap.



Permeability: μ
mean radius: r_0
cross-section of coil: $a << r_0$

forts →

Neglecting fringing and leakage: $\mathbb{B}_f = \mathbb{B}_g = \hat{\Phi} B_f$

$$\longrightarrow \begin{cases} \mathbb{H}_f = \hat{\Phi}\, \dfrac{B_f}{\mu} & \text{magnetic flux intensity in coil} \\ \mathbb{H}_g = \hat{\Phi}\, \dfrac{B_f}{\mu_0} & \text{magnetic flux intensity in airgap} \end{cases}$$

Ampere's law:

$$\oint \mathbb{H} \cdot dl = NI_0 \longrightarrow \frac{B_f}{\mu}(2\pi r_0 - l_g) + \frac{B_f}{\mu_0} \cdot l_g = NI_0$$

$$\longrightarrow B_f = \hat{\Phi}\, \frac{\mu \mu_0 N I_0}{\mu_0 (2\pi r_0 - l_g) + \mu l_g}$$

Assume B constant $\Longrightarrow$ $\Phi = BS = \dfrac{NI_0}{\dfrac{2\pi r_0 - l_g}{S} + \dfrac{l_g}{S\mu_0}}$

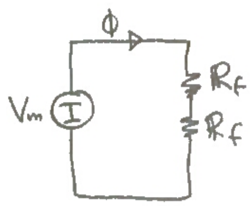
magneto motive force.

$$\phi = \frac{V_m}{\mathbb{R}_f + \mathbb{R}_g}$$

— reluctance

$$\mathbb{R}_f = \frac{2\pi r_0 - l_g}{\mu S}, \quad \mathbb{R}_g = \frac{l_g}{S \mu_0}$$

$$I = \frac{V_e}{R_f + R_g}$$

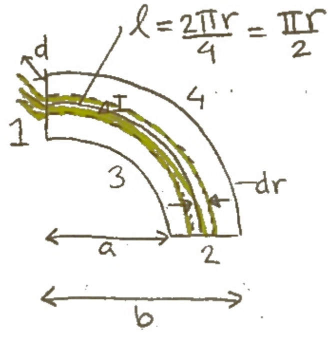


Storgruppsövning 19/11-13 em.

Resistansberäkningar, övre och undre gräns

6.16

quarter disk, $\left(\begin{array}{l} \sigma = \text{conductivity} \\ d = \text{thickness} \end{array} \right)$



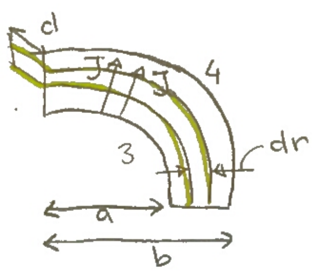
a) Calculate R_{12} (between electrodes 1 & 2)

$$dG = \frac{\sigma S}{l} = \frac{\sigma(dr \cdot d)}{\pi r/2} = \frac{2\sigma d}{\pi r} dr$$

$$G_{12} = \int_{r=a}^{b} dG = \int_a^b \frac{2\sigma d}{\pi r} dr = \frac{2\sigma d}{\pi} \ln\left(\frac{b}{a}\right)$$

$$\Longrightarrow R_{12} = \frac{1}{G_{12}} = \frac{\pi}{2\sigma d \ln(b/a)}$$ parallellkoppling

b)



$$dR = \frac{l}{\sigma S} = \frac{dr}{\sigma \cdot \pi r d/2} = \frac{2}{\sigma \pi r d} dr$$

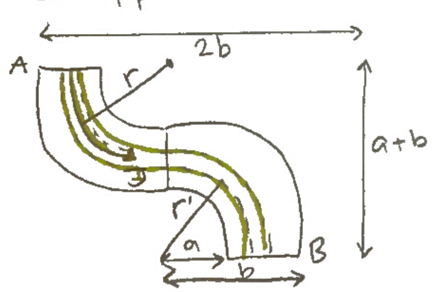
$$S = \frac{\pi}{2} r d$$

$$R_{34} = \int_{r=a}^{b} dR = \int_a^b \frac{2}{\sigma \pi r d} dr = \frac{2}{\sigma \pi d} \ln(b/a)$$ seriekoppling

$$R_{12} \cdot R_{34} = \frac{\pi}{2\sigma d \ln(b/a)} \cdot \frac{2 \ln(b/a)}{\sigma \pi d} = \left(\frac{1}{\sigma d}\right)^2$$

6.17

A thin plate, has thickness $d = 0{,}1$ mm and conductivity σ.
Find upper and lower bounds for R_{AB} if $2a = b$.



- Upper bound: assume current tubes.

$$dG = \frac{\sigma S}{l}, \quad \begin{cases} l = \frac{\pi}{2} r + \frac{\pi}{2} \underbrace{(a+b-r)}_{r'} \\ S = d \cdot dr \end{cases}$$

forts. →

Storgruppsövning 19/11-13 em.

Resistansberäkningar, övre och undre gräns

6.16

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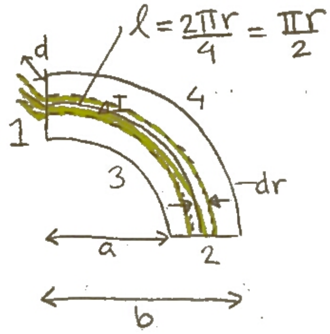

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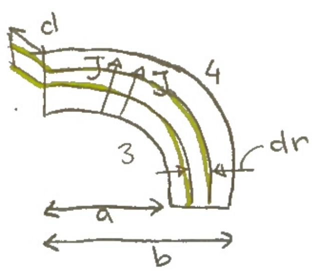
b)



$$dR = \frac{l}{\sigma S} = \frac{dr}{\sigma \cdot \pi r d/2} = \frac{2}{\sigma \pi r d}\, dr$$

$$S = \frac{\pi}{2} r d$$

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$$R_{12} \cdot R_{34} = \frac{\pi}{2\sigma d \ln(b/a)} \cdot \frac{2\ln(b/a)}{\sigma \pi d} = \left(\frac{1}{\sigma d}\right)^2$$

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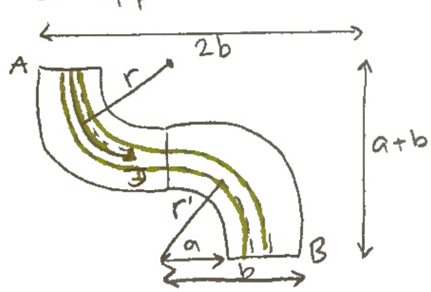
A thin plate, has thickness $d = 0{,}1\,\text{mm}$ and conductivity σ.
Find upper and lower bounds for R_{AB} if $2a = b$.



- Upper bound: assume current tubes.

$$dG = \frac{\sigma S}{l}, \quad \begin{cases} l = \frac{\pi}{2} r + \frac{\pi}{2} \underbrace{(a+b-r)}_{r'} \\ S = d \cdot dr \end{cases}$$

forts. →

$$\Longrightarrow dG = \frac{\sigma d\, 2}{\pi(a+b)}\, dr$$

$$\Longrightarrow G = \int_{r=a}^{b} dG = \int_a^b \frac{2\sigma d}{\pi(a+b)}\, dr = \frac{3\sigma d(b-a)}{\pi(a+b)} = \{b = 2a\} = \frac{2\sigma d}{3}$$

$$\Longrightarrow R = \frac{3\pi}{2\sigma d} = 0{,}471\ \Omega$$

lower bound: assume an equipotential surface as real-line

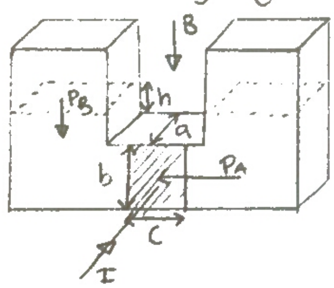
$$R^{l} = \underset{\substack{\uparrow \\ \text{from } 6.16}}{2R_{12}} = \cancel{2}\cdot\frac{\pi}{\cancel{2}\,\sigma d \ln(b/a)} = \frac{\pi}{\sigma d \ln(2)} = 0{,}453\ \Omega$$

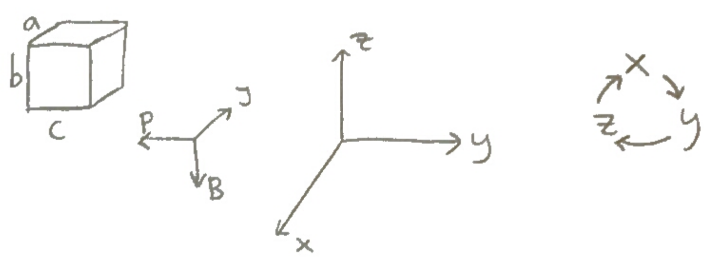
$$\Longrightarrow 0{,}453 < \text{actual } R_{AB} < 0{,}471$$

7.8

(u-tube) in the figure is filled with a conductor fluid, mass density η.

Find height difference in two legs.





$$J_o = \frac{I}{bc}$$

$$\begin{cases} \mathbb{J} = -\hat{x}\, J_o \\ \mathbb{B} = -B_o\, \hat{z} \end{cases}$$

$$F_m = q\mathbb{V}\times\mathbb{B} = \rho\Delta V\, \mathbb{V}\times\mathbb{B} \Longrightarrow \frac{\mathbb{F}}{\Delta V} = \rho\, \mathbb{V}\times\mathbb{B}$$

$$\rho = q/\Delta V \Longrightarrow q = \rho\Delta V$$

$\mathbb{J} = \rho\mathbb{V}$ convection current

$$\Longrightarrow \frac{\mathbb{F}}{\Delta V} = \mathbb{J}\times\mathbb{B} = -J_o B_o\, \hat{y}\ \left(\frac{N}{m^3}\right) \qquad \left((-\hat{x}\, J_o)\times(-B_o\,\hat{z})\right)$$

$F_A = \frac{F}{\Delta V}\cdot abc = J_o B_o\, abc$ (the force in cubic region)

$P_A = \frac{F_A}{ab}$ (the force acts on the shaded area, cause the pressure P_A)

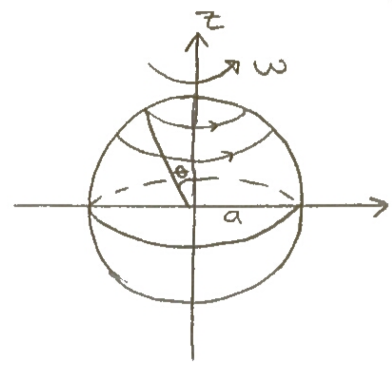
$$P_A = \frac{J_o B_o\, abc}{ab} = J_o B_o c$$

$$P_B = \eta g h = J_0 B_0 C \Longrightarrow h = \frac{J_0 B_0 C}{\eta g}$$

mass density

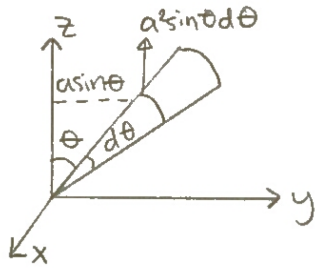
7.7

Biot-Savart law, a metal sphere of radius a with charge Q, distributed uniformly
- angular velocity ω
- Find B in center!



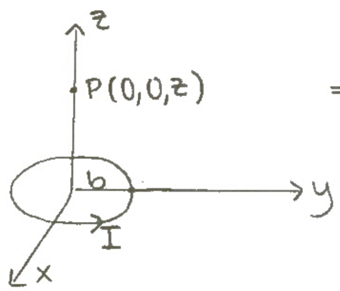
$$\omega = \frac{d\phi}{dt} \quad , \quad \rho_s = \frac{Q}{4\pi a^2}$$

$$dq = \rho_s ds = \rho_s ds_R = \rho_s a^2 \sin\theta \, d\theta \, d\phi$$



$$di = \frac{dq}{dt} = \rho_s a^2 \sin\theta \, d\theta \frac{d\phi}{dt} =$$

$$= \rho_s a^2 \sin\theta \, \omega \, d\theta$$



$$\Longrightarrow \mathbb{B}(P) = \hat{z} \frac{\mu_0 I b^2}{2(z^2+b^2)^{3/2}} \qquad \left\{ \begin{array}{l} I = di \\ b = a\sin\theta \\ z = a\cos\theta \\ z^2+b^2 = a^2 \end{array} \right\}$$

$$\Longrightarrow d\mathbb{B} = \hat{z} \frac{\mu_0 di (a\sin\theta)^2}{2a^3} =$$

$$= \hat{z} \frac{\mu_0 Q \omega}{8\pi a} \sin^3\theta \, d\theta$$

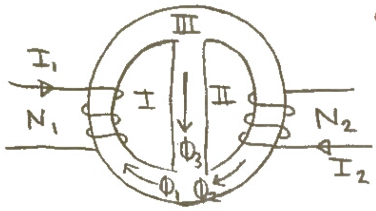
$$\mathbb{B} = \int_{\theta=0}^{\pi} d\mathbb{B} = \hat{z} \frac{\mu_0 Q \omega}{6\pi a}$$

8.5

Rules: $\sum_j N_j I_j = \sum R_k \phi_k$ around a close path in a magnetic circuit.

$\sum_j \phi_j = 0$ in a junction

Iron ring ($a = 7.5\,cm$) and ($A_1 = 1.2\,cm^2$)
($A_3 = 0.8\,cm^2$) $\begin{pmatrix} N_1 = 160 \text{ turns} \\ N_2 = 120 \text{ turns} \end{pmatrix}$ $I_1 = I_2 = 2\,mA$



a) Calculate the flux through the bridge (ϕ_3)

$$\begin{cases} \text{I}: R_1\phi_1 + R_3\phi_3 = N_1 I_1 \\ \text{II}: R_2\phi_2 - R_3\phi_3 = N_2 I_2 \\ \text{III}: \phi_1 = \phi_2 + \phi_3 \end{cases}$$

$$R_1 = R_2 = \frac{l_1}{\mu A_1} = \frac{\pi a}{\mu A_1}$$

$$R_3 = \frac{l_3}{\mu A_3} = \frac{2a}{\mu A_3}$$

In matrix-form:

$$\begin{bmatrix} R_1 + R_3 & -R_3 \\ -R_3 & R_1 + R_3 \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} = \begin{bmatrix} N_1 I_1 \\ N_2 I_2 \end{bmatrix} \Longrightarrow \phi_3 = 2{,}639 \cdot 10^{-9}\ (wb)$$

b) we assume $I_2 = 0$, Find I_1 in order to have $\phi_3 = 60 \cdot 10^{-6}\,wb$
use magnetication chart for iron

$$\phi_3 = 60 \cdot 10^{-6} \Longrightarrow \phi_3 = B_3 A_3 \Longrightarrow B_3 = \frac{\phi_3}{A_3} = \frac{60 \cdot 10^{-6}}{0{,}8 \cdot 10^{-4}} = 0.75\ (T)$$

use chart for iron: $H_3 = 4200\ (A/m)$

$$\oint H \cdot dl = I \xrightarrow[\text{in loop II}]{} H_3 \cdot 2a - H_2 \cdot \pi a = 0 \Longrightarrow H_2 = 2800\ (A/m)$$

use chart $\Longrightarrow B_2 = 0.65\ (T)$, $B_2 = \phi_2 / A_2 \Longrightarrow \phi_2 = 78 \cdot 10^{-6}\,wb$

in junction III: $\phi_1 = \phi_2 + \phi_3 \Longrightarrow \phi_1 = 138 \cdot 10^{-6}\,wb$

$\Longrightarrow B_1 = \frac{\phi_1}{A_1} = 1{,}15\ (T)$, chart $\Longrightarrow H_1 = 20000\ (A/m)$

forts $\rightarrow$

loop I $\Longrightarrow \oint H \cdot dl = I \Longrightarrow H_1 \cdot \pi a + H_3 \cdot 2a = NI_1$

$$\Longrightarrow I_1 = \frac{H_1 \cdot \pi a + H_3 \cdot 2a}{N} = 33{,}6\,A$$