Storgruppsövning 13/11-13

Steady state currents

Conduction current: caused by motion of conduction electrons & holes
Convection current: caused by motion of electrons and ions.

Current density: J (A/m^2) , $J = Nqu$ ← velocity of charged carriers

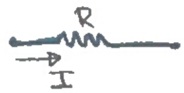
(N: number of charge carrier per unit charge)

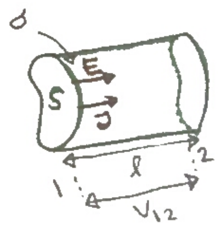
$J = \rho u$, $J = \sigma E$

(ρu: convection current density; σ: conductivity)

$I = \int_S J \cdot dS$ (A)

Ohm's law: $V_{12} = RI$





$$\left.\begin{array}{l} V_{12} = E\ell \Rightarrow E = V_{12}/\ell \\ I = \int_S J\, dS = JS \Rightarrow \underset{\sigma E}{J} = I/S \end{array}\right\} J = \sigma E = \sigma \frac{V_{12}}{\ell} = \frac{I}{S}$$

$$\Rightarrow \frac{V_{12}}{I} = \left(\frac{\ell}{\sigma S}\right) = R$$ – resistance for this conductor with cross section S.

$$\begin{cases} R_{seri.} = R_1 + R_2 + \dots \\ \frac{1}{R_{11}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots = G_{11} = G_1 + G_2 + \dots \end{cases}$$

Equation of continuity: $\nabla \cdot J = \frac{-\partial \rho}{\partial t}$ (A/m^3)

Steady state current, DC current $\Rightarrow \frac{\partial \rho}{\partial t} = 0 \Rightarrow \nabla \cdot J = 0$

$\Rightarrow \oint_S J\, dS = 0 \Rightarrow \sum_j I_j = 0$ Kirchoff's current law

6.1
Stationär strömning

$\begin{cases} \sigma(R) \\ \varepsilon(R) \end{cases}$ varying in a medium, if a dc current pass $\Rightarrow$ we have a charge distribution (ρ).

Find a relation between σ & $\varepsilon \Rightarrow \rho = 0$.

forts. →

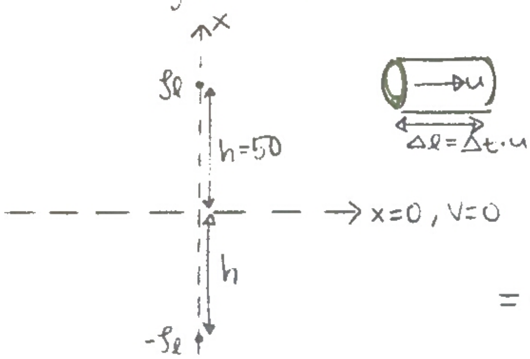
$$\begin{cases} J = \sigma E \\ D = \varepsilon E \end{cases} \qquad \begin{cases} \nabla\cdot D = \rho \\ \nabla\cdot J = -\frac{\partial \rho}{\partial t} \end{cases} \xrightarrow{\text{dc current}} \begin{cases} \nabla\cdot D = \rho \\ \nabla\cdot J = 0 \end{cases}$$

$$\Rightarrow \begin{cases} \nabla\cdot(\varepsilon E) = \rho \\ \nabla\cdot(\sigma E) = 0 \end{cases} \xrightarrow[\varepsilon = \sigma\alpha \text{ (const.)}]{\text{assume}} \nabla\cdot(\alpha\sigma E) = \alpha \underbrace{\nabla\cdot(\sigma E)}_{=0} = \alpha\cdot 0 = \rho \Rightarrow \rho = 0$$

if $\varepsilon = \alpha\sigma$.

6.2

Dust charged particles are emitting from a chimney h=50m from ground. Wind velocity: 5m/s, they make a horizontal cylindrical charged cloud. (ρ_l)
Current: 100 μA, ground plane is a perfect conductor.
Find E on ground!



E of ρ_l ⟶ $E = \frac{\rho_l}{2\pi\varepsilon_0 r}\hat{r}$

In case of convection current:

$J = \rho u$, $i = \frac{\Delta Q}{\Delta t} = \frac{\rho_l \Delta l}{\Delta t} =$

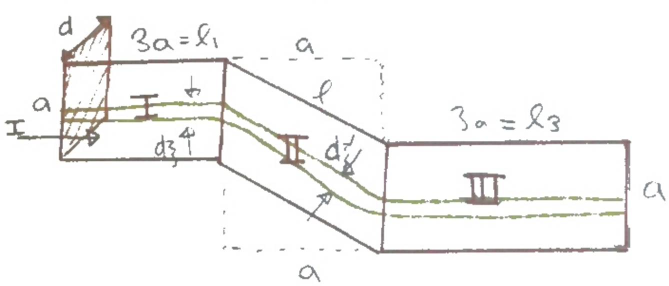
$= \frac{\rho_l u \cdot \Delta t}{\Delta t} = \rho_l u \Rightarrow \rho_l = \frac{i}{u}$

$$E_x(x=0) = \frac{\rho_l}{2\pi\varepsilon_0 h}(-\hat{x}) + \frac{-\rho_l}{2\pi\varepsilon_0 h}(\hat{x}) = \frac{-\rho_l}{\pi\varepsilon_0 h} = \frac{-i}{\pi\varepsilon_0 h u}\hat{x}$$

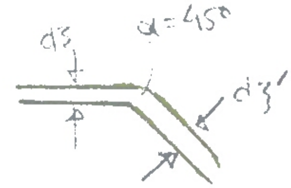
$$E_x(x=0) = \frac{-100\cdot 10^{-6}}{\pi\cdot 8{,}85\cdot 10^{-12}\cdot 50\cdot 5} = 14\left(\frac{kV}{m}\right)$$

6.11

Resistansberäkning direkt
use two approximation methods to find a lower and upper limit for the resistance between 2 electrons.



R_{min}?, R_{max}?



Upper bound: use non-physical current tubes ⟶ R_{max}

$d\xi' = d\xi \sin\alpha = d\xi \frac{1}{\sqrt{2}}$

$R = R^{I} + R^{II} + R^{III}$

$R^{I} = R^{III} = \frac{3a}{\sigma \cdot ad} = \frac{3}{\sigma d} = 3s \quad \left(s = \frac{1}{\sigma d}\right)$

$l = \sqrt{2}a \; , \; R^{II} = \frac{l}{\sigma S}$

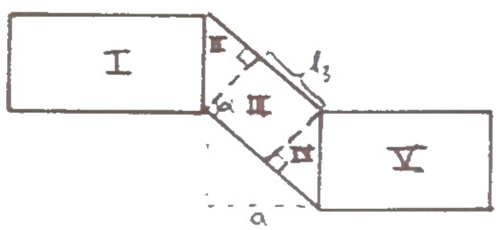
$S = d\,d\xi' \; , \quad S = \frac{d}{\sqrt{2}}\, d\xi$

$dG^{II} = \frac{\sigma S}{l} = \frac{\sigma}{\sqrt{2}a}\left(\frac{d}{\sqrt{2}}\, d\xi\right) \Longrightarrow G^{II} = \int_{\xi=0}^{a} dG^{II} = \int_0^a \frac{\sigma d}{a \cdot 2}\, d\xi = \frac{\sigma a d}{2a} = \frac{\sigma d}{2}$

$R^{II} = \frac{2}{\sigma d} = 2s$

$R_{max} = 2R^{I} + R^{II} = 2 \cdot 3s + 2s = 8s = \frac{8}{\sigma d}$

lower limit: use constant potential surface on dashed line



assume: $R^{II} = R^{IV} = 0 \quad (\sigma = \infty)$

$R_{min} = R^{I} + R^{III} + R^{V}$

$R^{I} = R^{V} = \frac{l}{\sigma S} = 3s$

$l_3 = a\sin\alpha = a\sin 45° = a/\sqrt{2}$

$h = a/\sqrt{2}$

$\Longrightarrow R^{III} = \frac{l_3}{\sigma S} = \frac{a/\sqrt{2}}{\sigma d\, a/\sqrt{2}} = \frac{1}{\sigma d} = s$

$R_{tot} = R_{min} = 2 \cdot 3s + s = 7s$

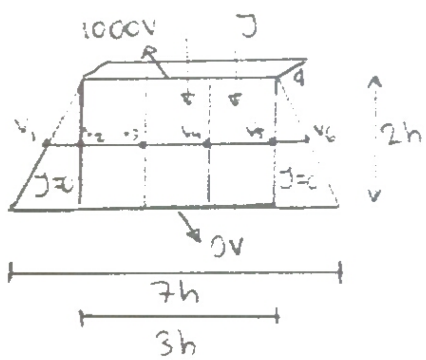
$\Longrightarrow \quad 7s < R < 8s$

6.20

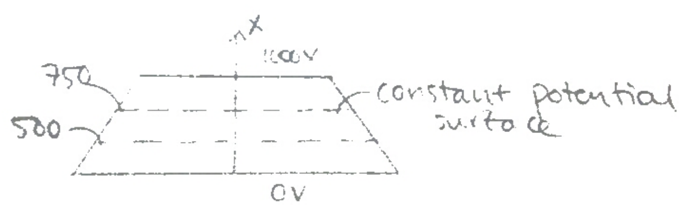
Numerisk beräkning

On a thin sheet two electrodes are fastened. $(\sigma = 5\ S/m, d = 0{,}1mm)$

Find the upper and lower resistance R!



$R^{upper} = \frac{l}{\sigma S_{min}} = \frac{2h}{\sigma d 3h} = \frac{2}{3\sigma d}$



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if $\varepsilon = \alpha \sigma$.

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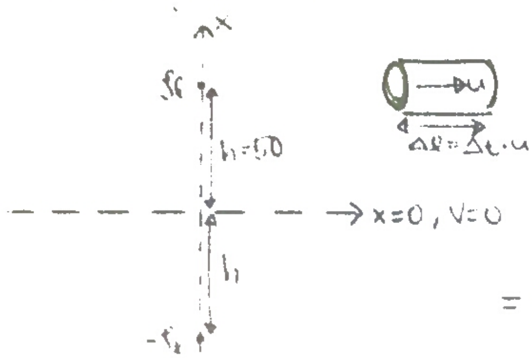

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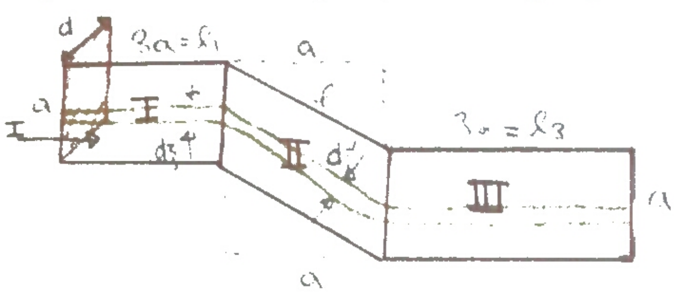
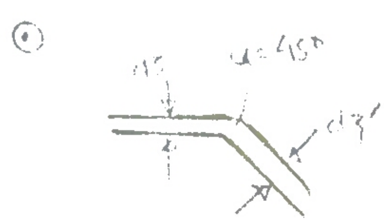


Rmin? , Rmax?



Upper bound: use non-physical current tubes ⟶ R_{max}

$$dz' = dz \sin\alpha = dz \frac{1}{\sqrt{2}}$$