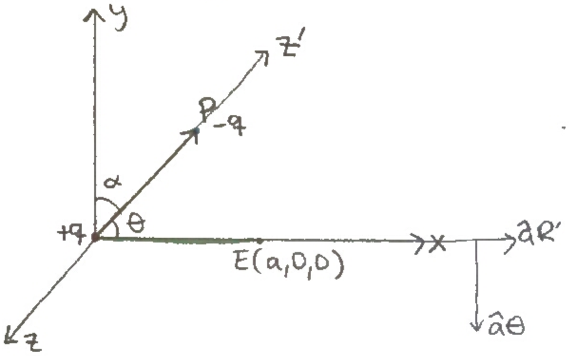
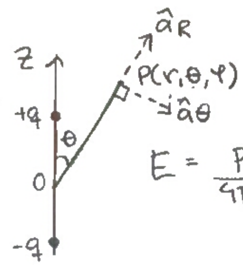
Storgruppsövning 12/11-13

Electric dipole and dielectric material

2.12
A point dipole is located at the origin. The dipole moment P, lies in the x-y-plane. Fin $E(a,0,0)$





se 3.31 i boken

$$E = \frac{P}{4\pi\varepsilon_0 R^3}(\hat{a}_R 2\cos\theta + \hat{a}_\theta \sin\theta)$$

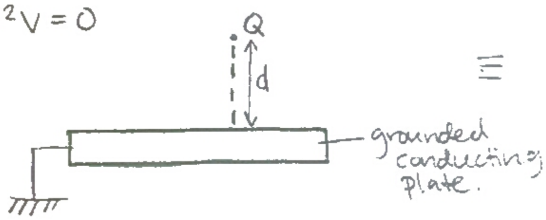
for this problem: $\hat{a}_{R'} = \hat{a}_x$, $\hat{a}_{\theta'} = -\hat{a}_y$, $\theta = \frac{\pi}{2} - \alpha$, $R = a$

$$\Rightarrow E = \frac{P}{4\pi\varepsilon_0 a^3}\left(\hat{a}_x 2\cos\left(\frac{\pi}{2}-\alpha\right) - \hat{a}_y \sin\left(\frac{\pi}{2}-\alpha\right)\right) =$$

$$= \frac{P}{4\pi\varepsilon_0 a^3}(\hat{a}_x 2\sin\alpha - \hat{a}_y\cos\alpha)$$

Method of images

$\nabla^2 V = 0$



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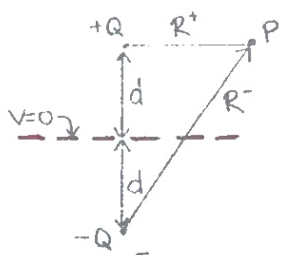
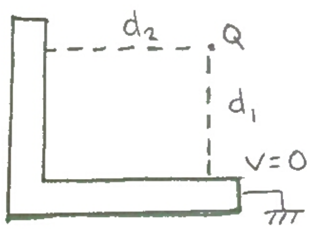


$$V_p = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{R^+} - \frac{1}{R^-}\right)$$



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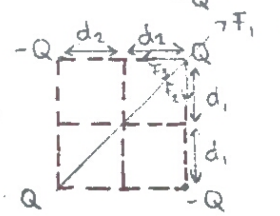

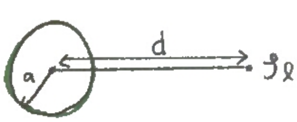

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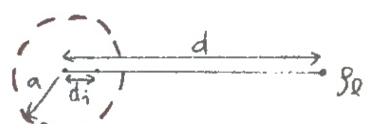
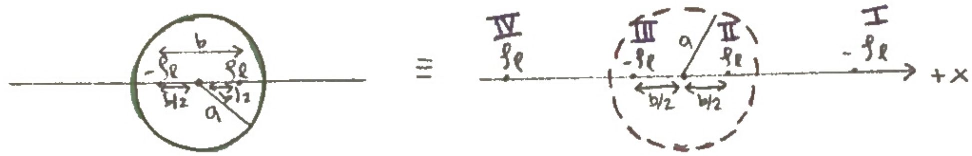


$$d_i = \frac{a^2}{d}, \quad \rho_i = -\rho_\ell$$

5.7 speglling i cylinderyta:
inside a long metal tube (radius a), we have 2 thin metal wires passing through cylinder.



$d_i = \frac{2a^2}{b}$ $\qquad d_i = \frac{a^2}{b/2} = \frac{2a^2}{b}$

$\mathbf{E} = \hat{a}_r \frac{\rho_\ell}{2\pi\varepsilon_0 r}$

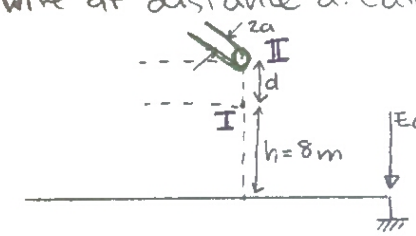
$$\mathbf{E}^{II} = -\underbrace{\frac{-\rho_\ell}{2\pi\varepsilon_0\left(\frac{2a^2}{b}-\frac{b}{2}\right)}}_{I} + \underbrace{\frac{-\rho_\ell}{2\pi\varepsilon_0 b}}_{III} + \underbrace{\frac{\rho_\ell}{2\pi\varepsilon_0\left(\frac{2a^2}{b}+\frac{b}{2}\right)}}_{IV} = 0 \qquad \frac{2a^2}{b} - \frac{b}{2}$$

$$\mathbf{E}_x(x=b/2) = 0 \Rightarrow \frac{\rho_\ell}{2\pi\varepsilon_0}\left(\frac{1}{\frac{2a^2}{b}-\frac{b}{2}} + \frac{1}{\frac{2a^2}{b}+\frac{b}{2}} - \frac{1}{b}\right) = 0$$

$$\Rightarrow 16a^2b^2 - 16a^4 + b^4 = 0 \Rightarrow b = \pm\sqrt{4(\sqrt{5}-2)}\, a$$

5.5 Spegling i plan yta: $\qquad$ * in E-field

we have an isolated ~~uncharged~~ grounded thin wire at h = 8 m from ground* We add a protective wire of radius a in parallel with wire at distance d. Calculate the relative change in wire potential



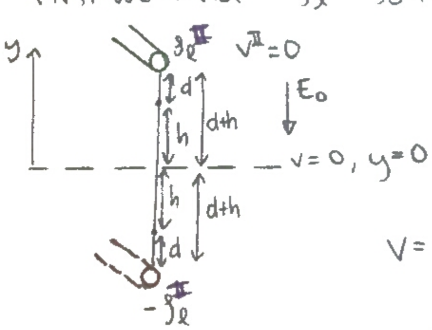
relative change in wire potential $= \frac{V^I - V_0}{V_0} = ?$

V_0: potential without protective wire.

$$\begin{pmatrix} V^{II} = 0, & \rho_\ell^{II} \neq 0 \\ V^I \neq 0, & \rho_\ell^I = 0 \end{pmatrix}$$

a) $d = 1\text{m}$, $\frac{V^I - V_0}{V_0} = ?$

first we find ρ_ℓ^{II} so that $V^{II} = 0$



$V(y=h+d) = V^{II} = 0$
$V(y=h+d) = E_0(h+d) + \ldots$

The electric potential at distance r from a line charge ρ_ℓ is given as:

$$V = -\int_{r_0}^{r} E_r\, dr = -\frac{\rho_\ell}{2\pi\varepsilon_0}\int_{r_0}^{r}\frac{1}{r}\, dr = \frac{\rho_\ell}{2\pi\varepsilon_0}\ln\left(\frac{r_0}{r}\right) \qquad V_{r_0} = 0 \text{ reference point.}$$

$$V = \frac{\rho_\ell}{2\pi\varepsilon_0}\ln\left(\frac{r_0}{r^+}\right) - \frac{\rho_\ell}{2\pi\varepsilon_0}\ln\left(\frac{r_0}{r^-}\right) = \frac{\rho_\ell}{2\pi\varepsilon_0}\ln\left(\frac{r^-}{r^+}\right)$$

$$V(y=h+d) = E_0(h+d) + \frac{\rho_\ell^{II}}{2\pi\varepsilon_0}\ln\left(\frac{2(h+d)}{a}\right) = 0$$

$$\Longrightarrow \rho_\ell^{II} = \frac{-2\pi\varepsilon_0 E_0(h+d)}{\ln\left(\frac{2(h+d)}{a}\right)}$$

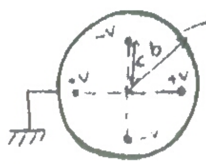
$$V^I = V(y=h) = E_0 h + \frac{\rho_\ell^{II}}{2\pi\varepsilon_0}\ln\left(\frac{2h+d}{d}\right)$$, substitute ρ_ℓ^{II}

$$V^I = \underbrace{E_0 h}_{V_0} - \frac{E_0(h+d)}{\ln\left(\frac{2(h+d)}{a}\right)}\ln\left(\frac{2h+d}{d}\right)$$

$$\Longrightarrow \frac{V^I - V_0}{V_0} = -\left(\frac{h+d}{h}\right)\cdot\frac{\ln\left(\frac{2h+d}{d}\right)}{\ln\left(\frac{2(h+d)}{a}\right)}$$

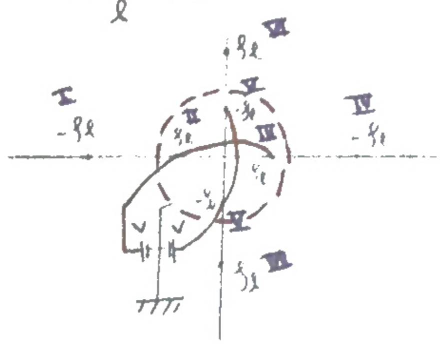
6.12

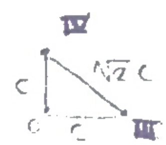
Speglings: cylinderyta, radius of wires $= a$.



cross section of a cable (four-conductor)

Find the $\frac{C}{\ell}$ for two-conductor-cable.





$$V = \frac{\rho_\ell}{2\pi\varepsilon_0}\ln\left(\frac{r^-}{r^+}\right)$$

$$\frac{C}{\ell} = \frac{Q/V_{12}}{\ell} = \frac{Q/\ell}{V_{12}} = \frac{\rho_\ell}{2V^{III}} = \frac{\rho_\ell}{V^{III} - (-V^{III})}$$

$$= V^{III} = \frac{\rho_\ell}{2\pi\varepsilon_0}\left[\ln\left(\frac{b^2/c + c}{2c}\right) + \ln\left(\frac{b^2/c - c}{a}\right) + \ln\left(\frac{\sqrt{2}c}{\sqrt{\left(\frac{b^2}{c}\right)^2 + c^2}}\right)^2\right] =$$

$$= \frac{\rho_\ell}{2\pi\varepsilon_0}\ln\left[\frac{c(b^4 - c^4)}{a(b^4 + c^4)}\right]$$

$$\frac{C}{\ell} = \frac{\rho_\ell}{2V^{III}} = 2\pi\varepsilon_0 / \ln\left[\frac{c(b^4 - c^4)}{a(b^4 + c^4)}\right]$$