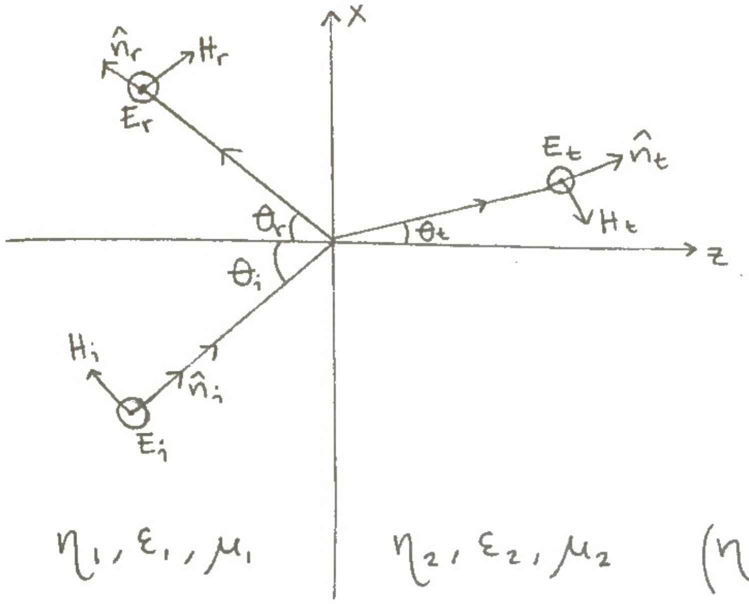
Storgruppsövning 10/12-13

Oblique incidence at a plane dielectric boundary.

Perpendicular polarization

E-field is perpendicular to the plane of incidence.



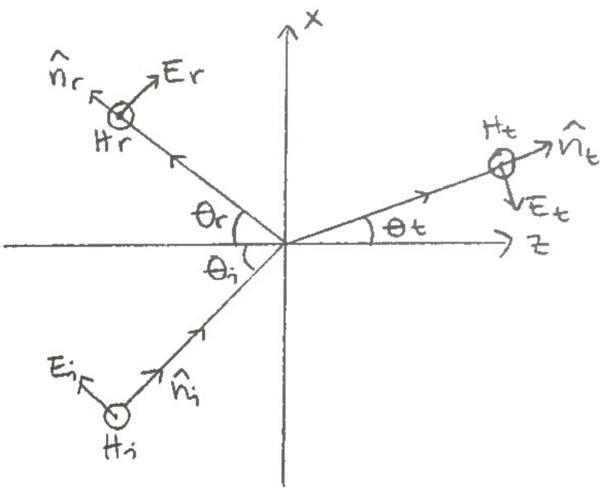
$$\left(\eta_2 = \sqrt{\frac{\mu_2}{\varepsilon_2}}\right)$$

$$\begin{cases} \Gamma_\perp = \dfrac{E_{r0}}{E_{i0}} = \dfrac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t} \\ \tau_\perp = \dfrac{E_{t0}}{E_{i0}} = \dfrac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i + \eta_1\cos\theta_t} \end{cases}$$

$$1 + \Gamma_\perp = \tau_\perp$$

Parallell polarization

E-field is lying in the plane of incidence.



$$\begin{cases} \Gamma_{\parallel} = \dfrac{E_{r0}}{E_{i0}} = \dfrac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i} \\ \tau_{\parallel} = \dfrac{E_{t0}}{E_{i0}} = \dfrac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i + \eta_1\cos\theta_t} \end{cases}$$

$$1 + \Gamma_{\parallel} = \tau_{\parallel}$$

No reflection $\Longleftrightarrow$ θ_i = Brewster angle = θ_B

$\Gamma_\perp = 0 \Longrightarrow \eta_2 \cos\theta_i = \eta_2 \cos\theta_t$

$\cos\theta_t = \sqrt{1 - \sin^2\theta_t} \xrightarrow[\text{law}]{\text{Snell's}} \cos\theta_t = \sqrt{1 - \left(\frac{\eta_1}{\eta_2}\right)^2 \sin^2\theta_t}$

$\Longrightarrow \sin\theta_i = \sin\theta_{B\perp} = \frac{1 - \mu_1\varepsilon_2/\mu_2\varepsilon_1}{1 - \left(\frac{\mu_1}{\mu_2}\right)^2}$

if $\mu_1 = \mu_2 \Longrightarrow \theta_B$ does not exist.

$\Gamma_{\parallel} = 0 \Longrightarrow \begin{cases} \eta_2\cos\theta_t = \eta_1\cos\theta_i \\ \cos\theta_t = \sqrt{1-\sin^2\theta_t} \end{cases} \Longrightarrow \sin\theta_i = \sin\theta_{B\parallel} =$

$= \frac{1 - \mu_2\varepsilon_1/\mu_1\varepsilon_2}{1 - \left(\frac{\varepsilon_1}{\varepsilon_2}\right)^2}$

$\mu_2 = \mu_1 \Longrightarrow \sin\theta_{B\parallel} = \frac{1}{\sqrt{1 + \frac{\varepsilon_1}{\varepsilon_2}}} \Longrightarrow \tan\theta_{B\parallel} = \frac{\eta_2}{\eta_1}$

12.12

Circular cross section wire.
Radius $a = 0{,}1\,mm$
$\sigma = 5 \cdot 10^6\ S/m$
$\mu_r = 100$.
Calculate the ratio of the resistance at $f_1 = 50\,Hz$ & $f_2 = 10\,MHz$

Skin depth for good conductor, $\delta = \sqrt{\frac{1}{\pi f \mu \sigma}}$ $\left(\sigma\omega\varepsilon \gg 1\right)$

$\begin{cases} f_1 = 50\,Hz \Longrightarrow \delta_1 = 3{,}2 \cdot 10^{-3} \\ f_2 = 10 \cdot 10 \Longrightarrow \delta_2 = 7 \cdot 11 \cdot 10^{-6} \end{cases}$ $(a = 0{,}1\,mm)$

$\begin{cases} \delta_1 \gg a \Longrightarrow \text{current dist. on the whole cross-section.} \\ \delta_2 \ll a \Longrightarrow \text{— '' — on a thin layer} \end{cases}$

$R_1 = \frac{l}{\sigma S_1} = \frac{l}{\sigma \pi a^2}$

$R_2 = \frac{l}{\sigma S_2} = \frac{l}{\sigma 2\pi a \delta_2}$

$\frac{R_1}{R_2} = \frac{S_2}{S_1} = \frac{2\delta_2}{a} \simeq 0{,}142\ldots$

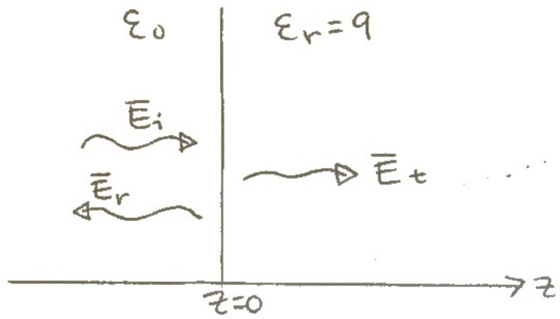
The resistance at 50Hz is approx 14% of that for 10 MHz.

13.7

A plane wave in vacuum incident normally to a flat surface at $z=0$, of a lossless dielectric, $\varepsilon_r = 9$.

$\mathbb{E} = \hat{x}\,10\cos(\omega t - \beta z)$, $f = 300\,\text{MHz}$

Find the $\underbrace{\text{location}}_{z}$ of $\underbrace{\text{max for E-field in vacuum.}}_{\max(E_{tot}),}$



$\bar{\mathbb{E}}_i = \hat{x}\,10 e^{-i\beta z}$, $\bar{\mathbb{E}}_r = \Gamma \bar{\mathbb{E}}_i = \Gamma \hat{x}\,10 e^{+i\beta z}$

$\bar{\mathbb{E}}_{tot} = \bar{\mathbb{E}}_i + \bar{\mathbb{E}}_r$

$$\Gamma = \text{refl. coef.} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\sqrt{\frac{\mu_0}{\varepsilon_0 \varepsilon_r}} - \sqrt{\frac{\mu_0}{\varepsilon_0}}}{\sqrt{\frac{\mu_0}{\varepsilon_0 \varepsilon_r}} + \sqrt{\frac{\mu_0}{\varepsilon_0}}} = \frac{1/\sqrt{\varepsilon_r} - 1}{1/\sqrt{\varepsilon_r} + 1} = \frac{1/3 - 1}{1/3 + 1} = -\frac{1}{2}$$

$$\Longrightarrow \bar{E}_r = -5\hat{x} e^{+i\beta z}$$

$$\bar{E}_{tot,1} = \hat{x}(10 e^{-i\beta z} - 5 e^{i\beta z}) = \hat{x}(10 - 5e^{2i\beta z}) e^{-i\beta z}$$

$$\Longrightarrow |\bar{\mathbb{E}}_{tot,1}| = |10 - 5e^{2i\beta z}| = |\underbrace{10 - 5\cos(2\beta z)}_{Re} - \underbrace{5 i \sin(2\beta z)}_{Im}|$$

$$|\bar{\mathbb{E}}_{tot,1}| = \sqrt{(10 - 5\cos(2\beta z))^2 + 25\sin^2(2\beta z)} = \sqrt{125 - 100\underbrace{\cos(2\beta z)}_{-1 \Rightarrow \max}}$$

$$\cos(2\beta z) = -1 \Longrightarrow |\bar{\mathbb{E}}_{tot,1}|_{max} = \sqrt{225} = 15$$

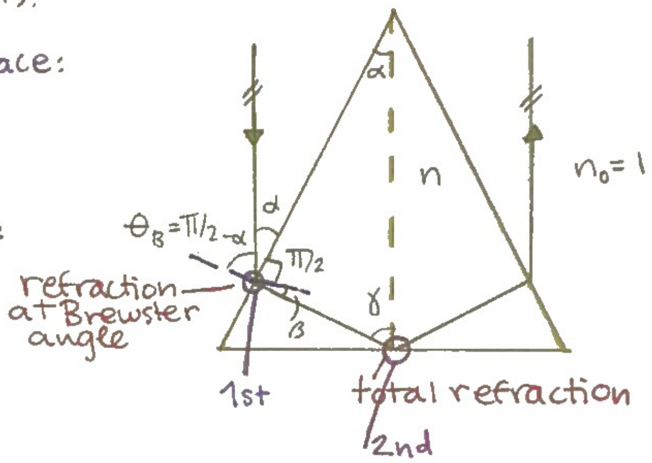
$$2\beta z_{max} = -(2n+1)\pi \Longrightarrow z_{max} = \frac{-(2n+1)\pi}{2\beta}$$

$$\beta = \frac{\omega}{c} = \frac{2\pi \cdot 3 \cdot 10^8}{3 \cdot 10^8} = 2\pi \Longrightarrow z_{max} = \frac{-(2n+1)\cancel{\pi}}{2 \cdot 2\cancel{\pi}} = \frac{-(2n+1)}{4}$$

$(n = 0, 1, 2, \dots)$

13.14

A light beam is broken and totally reflected in a lossless prism. Refraction is at Brewster angle. Return wave is parallell to incident wave.
Find a suitable range for (n).



① Brewster angle at 1st interface:
$\tan\theta_B = \frac{n}{n_0} = n$
$\tan(\pi/2 - \alpha) = n$

② Snell's law of refr., 1st interf.:
$\sin(\pi/2 - \alpha) = n \sin\beta$

③ Total reflection, 2nd interf.:
$\gamma > \theta_c$

④ Snell's law, 2nd interf.:
$\sin\theta_c \cdot n = \sin\pi/2 \cdot 1$
$\sin\theta_c = 1/n$

⑤ In triangle:
$\alpha + (\pi/2 + \beta) + \gamma = \pi$

Suppose $\gamma = \theta_c$ $\quad (\tan\alpha = 1/n)$
① $\sin(\pi/2 - \alpha) = n\cos(\pi/2 - \alpha) \Longrightarrow \cos\alpha = n\sin\alpha$
② $\cos\alpha = n\sin\beta$
$\Longrightarrow \alpha = \beta$

④, ⑤ : $\sin\gamma = \sin\theta_c = \sin(\pi/2 - (\alpha+\beta)) = 1/n$
$\Longrightarrow \cos(\alpha+\beta) = 1/n \Longrightarrow \cos 2\alpha = 1/n$
$\cos(2\alpha) = \frac{1 - \tan^2\alpha}{1 + \tan^2\alpha} = 1/n \Longrightarrow \{\tan(\alpha) = 1/n\} \Longrightarrow \frac{1 - (1/n)^2}{1 + (1/n)^2} = 1/n$

$\Longrightarrow n^3 - n^2 - n - 1 = 0$

$\Longrightarrow n_c \approx 1{,}839$

If n increase $\Longrightarrow$ from ① $\alpha = \beta$ will decrease $\Longrightarrow \gamma$ will decrease
from ④ θ_c will decrease

So if $n \geq n_c \Longrightarrow \gamma \geq \theta_c \Longrightarrow$ total refraction at 2nd interface