Föreläsning 10/12-13

Elektrodynamik - repetition

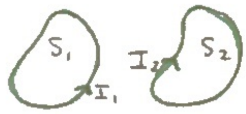
Postulaten:

Reell	Komplex
$\nabla \times \mathbb{E} = -\frac{\partial \mathbb{B}}{\partial t}$	$\nabla \times \bar{\mathbb{E}} = -i\omega \bar{\mathbb{B}}$
$\nabla \times \mathbb{H} = \mathbb{J} + \frac{\partial \mathbb{D}}{\partial t}$	$\nabla \times \bar{\mathbb{H}} = \bar{\mathbb{J}} + i\omega \bar{\mathbb{D}}$
$\nabla \cdot \mathbb{D} = \rho$	$\nabla \cdot \bar{\mathbb{D}} = \rho$
$\nabla \cdot \mathbb{B} = 0$	$\nabla \cdot \bar{\mathbb{B}} = 0$

Kont. ekv.: $\nabla \cdot \mathbb{J} + \frac{\partial \rho}{\partial t} = 0$

$\mathbb{F} = q(\mathbb{E} + u \times \mathbb{B})$ Brentzkraft

Självinduktans: $\Phi = LI$, $\mathcal{E}_{ind} = -L\frac{\partial I}{\partial t}$



Ömsesidig induktans: $\Phi_{12} = \int_{S_2} \mathbb{B}_1 \cdot d\mathbb{S}_2 = L_{12} i_1$

Beräkningsgång

1. Antag I_1
2. Beräkna $\mathbb{B}_1$
3. Beräkna Φ_{12}
4. Beräkna Λ_{12}
5. Induktansen Λ_{12}/I_1

Induktion: $V_{ind} = -\frac{\partial \Phi}{\partial t}$

Lentz lag: inducerade spänningar motverkar förändringar i pålagt fält.

Retarderade potentialer: $\mathbb{A}(\mathbb{r}_2, t) = \frac{\mu_0}{4\pi} \int_{V_1} \frac{\mathbb{J}(\mathbb{r}_1, t - r_{12}/c)}{r_{12}} dV'$

$$V(\mathbb{r}_2, t) = \frac{1}{4\pi\varepsilon_0} \int_{V_1} \frac{\rho(\mathbb{r}_1, t - r_{12}/c)}{r_{12}} dV'$$

Komplexa fält: $\mathbb{E}(\mathbb{r}, t) = \mathrm{Re}\{\bar{\mathbb{E}}(r) e^{i\omega t}$

Komplexa vågekv.: $\nabla^2 \bar{\mathbb{E}} - \gamma^2 \mathbb{E} = 0$, $\gamma = \alpha + i\beta = \sqrt{i\omega\mu(i\omega\varepsilon + \sigma)}$

Specialfall: $\sigma/\varepsilon\omega \gg 1$, $\sigma/\varepsilon\omega \ll 1$

Plan våg: $\bar{\mathbb{E}}(\mathbb{R}) = \bar{\mathbb{E}}(0)e^{-\gamma\hat{k}\mathbb{R}}$, $\hat{k}\cdot\bar{\mathbb{E}} = 0$ $(\mathbb{E}\perp\hat{k})$

$\mathrm{Re}\{\mathbb{E}\cdot\bar{\mathbb{H}}^*\} = 0$

Vågimpedans: $Z = i\omega\mu/\gamma$

Relation mellan $\mathbb{E}$ och $\mathbb{H}$: $\bar{\mathbb{H}}(\mathbb{R}) = \hat{k}\times\frac{\mathbb{E}(\mathbb{R})}{Z}$

Fashastighet: $V_{fas} = \omega/\beta$

Grupphastighet: $V_{grupp} = 1/\frac{\partial\beta}{\partial\omega}$

Poyntingvektorn: $\mathbb{S} = \mathbb{E}\times\mathbb{H}$

Tidsmv.: $S_{av} = \frac{1}{2}\mathrm{Re}\{\bar{\mathbb{E}}\times\bar{\mathbb{H}}\}$

Reflektion och transmission:

Vinkelrätt infall: $\Gamma = \frac{Z_2 - Z_1}{Z_2 + Z_1}$, $T = \frac{2Z_2}{Z_2+Z_1}$

Snells lag: $\theta_i = \theta_r$, $C_2\sin\theta_i = C_1\sin\theta_t$

Totalreflektion: $\theta_{kritisk} = \arcsin(C_1/C_2)$

Fresnels ekv.: $\Gamma_\perp = \frac{(1/Z_1)\cdot\cos\theta_i - (1/Z_2)\cdot\cos\theta_t}{(1/Z_1)\cos\theta_i + (1/Z_2)\cos\theta_t}$

$T_\perp = \frac{(2/Z_1)\cos\theta_i}{(1/Z_1)\cos\theta_i + (1/Z_2)\cos\theta_t}$

$\Gamma_\parallel = \frac{-Z_1\cos\theta_i + Z_2\cos\theta_t}{Z_1\cos\theta_i + Z_2\cos\theta_t}$

$T_\parallel = \frac{2Z_2\cos\theta_i}{Z_1\cos\theta_i + Z_2\cos\theta_t}$

Hertzdipol: $\bar{\mathbb{E}}_{rad} = \frac{\hat{\theta}Z_0 i\omega\, dl\, \bar{I}\sin\theta}{4\pi c r}e^{-i\beta r}$

$\bar{\mathbb{H}}_{rad} = \frac{\hat{\varphi}\, i\omega\, dl\, \bar{I}\sin\theta}{4\pi c r}e^{-i\beta r}$

Dipolantenn: $E_z = \int_{-h}^{h} d\mathbb{E}_{rad}$