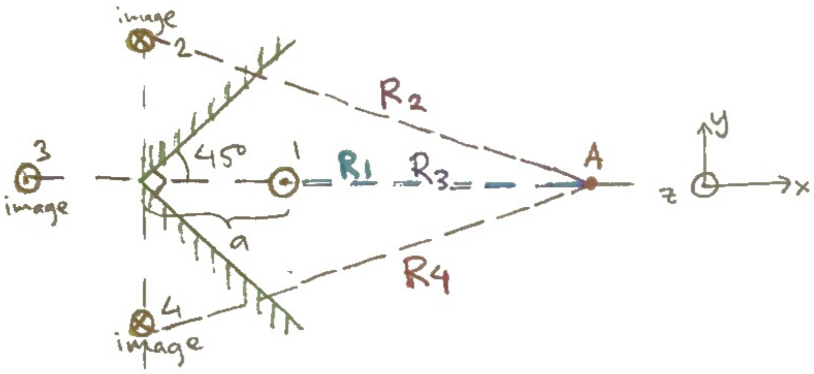
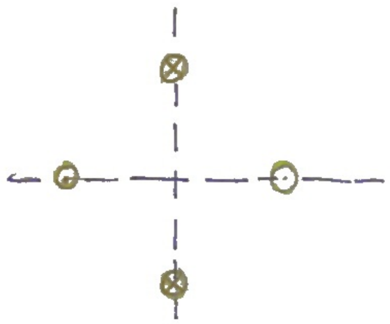
Storgruppsövning 6/12-13

16.1
A dipole is in front of two very long, perpendicular conductive plate. Field is observed at very long distance at $\theta = \pi/2$.
Find distance {a} such that the E-field in point {A} is maximised.



hint: use image method.



for field for each dipole antenna is:

$$\bar{E} = \bar{E}_0 F(\theta, \phi)\, e^{-i\beta R}/R$$

$$\bar{E}_\theta = i\,\frac{I\,d\ell}{4\pi}\left(\frac{e^{-i\beta R}}{R}\right)\eta\,\beta\sin\theta$$

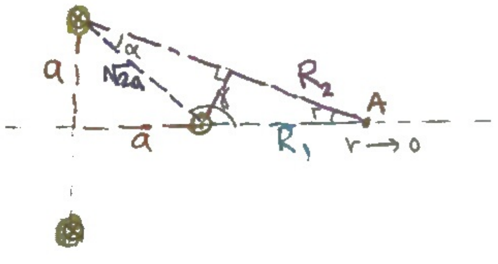
total field at point A, is the summation of the fields of 4 dipoles.

$F(\theta,\phi)$ and $1/R$ can be approximated the same for all antennas, but for the phase term we need more accurate approximation.

$$\bar{E}_{tot} = \bar{E}_1 + \bar{E}_2 + \bar{E}_3 + \bar{E}_4 = \frac{\bar{E}}{R_0} F(\theta,\phi)\, e^{-i\beta R_1} - \frac{\bar{E}}{R_0} F(\theta,\phi) e^{-i\beta R_2} +$$

$$+ \frac{\bar{E}}{R_0} F(\theta,\phi)\, e^{-i\beta R_3} - \frac{\bar{E}}{R_0} F(\theta,\phi)\, e^{-i\beta R_4}$$

$$\bar{E}_{tot} = \frac{\bar{E}}{R_0} F(\theta,\phi)\left[e^{-i\beta R_1} - e^{-i\beta R_2} + e^{-i\beta R_3} - e^{-i\beta R_4}\right]$$



$R_4 \approx R_1 + a\sqrt{2}\cos\alpha \quad (\alpha \approx 45°)$
$R_4 \approx R_1 + a$

Use cosine line in triangel:

$$R_4^2 = R_1^2 + (a\sqrt{2})^2 - 2R_1 a\sqrt{2}\cos(\gamma) = R_1^2 + 2aR_1 + 2a^2 =$$
$$= (R_1+a)^2 + a^2$$

$$\begin{cases} R_1 \gg a \Rightarrow R_4^2 \approx (R_1+a)^2 \Rightarrow R_4 \approx R_1 + a \\ R_4 = R_2 \\ R_3 = 2a + R_1 \end{cases}$$

$$\bar{E}_{tot} = \frac{\bar{E}_0}{R_0} F(\theta,\phi)\left[e^{-i\beta R_0} - e^{-i\beta(R_0+a)} + e^{-i\beta(R_0+2a)} - e^{-i\beta(R_0+a)}\right] =$$

$$= \frac{\bar{E}_0}{R_0} F(\theta,\phi)\left[e^{-i\beta R_0}\left(1 - 2e^{-i\beta a} + e^{-2i\beta a}\right)\right] =$$

$$= \frac{\bar{E}_0}{R_0} F(\theta,\phi)\left[e^{-i\beta R_0}\left(1 - e^{-i\beta a}\right)^2\right]$$

$$\Rightarrow \left|\bar{E}_{tot}\right| = \frac{|\bar{E}_0|}{R_0}\, |F(\theta,\phi)|\, \left|(1-e^{-i\beta a})\right|^2$$

$\bar{E}_{tot}$ is maximised when $|(1-e^{-i\beta a})|^2$ is maximised.

$$|(1-e^{-i\beta a})|^2 = |1 - \cos\beta a + i\sin\beta a|^2 = (1-\cos\beta a)^2 + \sin^2\beta a =$$

$$= 2 - 2\cos\beta a = 2(1-\cos\beta a)$$

when $\cos\beta a = -1$, $|\bar{E}_{tot}|$ has the max value.

$$\cos\beta a = -1 \Rightarrow \beta a = (2n+1)\pi \Rightarrow a = (2n+1)\pi/\beta$$

$$\beta = \frac{2\pi}{\lambda} \Rightarrow a = (n + \tfrac{1}{2})\lambda \qquad n = 0, 1, 2 \ldots$$

$$|\bar{E}_{tot}| = \frac{4|E_0|}{R} F(\theta,\phi) \quad \text{for } \cos\beta a = -1$$

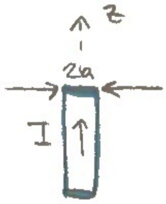
P 11.5 a)

Very thin center- half-ware dipole lying along z-axis.

Current dist. : $\begin{cases} I = I_0 \cos\beta z \\ \beta = \frac{\omega}{c} = \frac{2\pi}{\lambda} \end{cases}$

$l = \frac{\lambda}{2}$

$S = \pi a^2$



Find the charge distribution on dipole.

$\nabla \cdot \bar{J} + i\omega\rho = 0$ continuity equation

Only z depending for both current and charge.

$$I = J_z(z)\cdot\pi a^2 = I_0\cos\beta z \Rightarrow J_z(z) = \frac{I_0\cos\beta z}{\pi a^2} \quad \left[\frac{A}{m^2}\right]$$

$$\nabla \cdot J_z(z) = \frac{\partial J_z}{\partial z} = -i\omega \rho \implies \rho = \frac{-1}{i\omega} \frac{\partial J_z}{\partial z}$$

$$\rho_\ell = \pi a^2 \cdot \rho =$$

volume charge dist.

line charge dist.

$$= \pi a^2 \cdot \frac{-1}{i\omega} \frac{\partial J_z}{\partial z} = \pi a^2 \cdot \frac{-1}{i\omega} \cdot \frac{\partial \left(\frac{I_0 \cos \beta z}{\pi a^2} \right)}{\partial z} =$$

$$= \frac{-1}{i\omega} \frac{\partial (I_0 \cos \beta z)}{\partial z} = \frac{-1}{i\omega} \left(-I_0 \beta \sin \beta z \right) = \frac{\beta}{i\omega} I_0 \sin \beta z = \left\{ \beta = \frac{\omega}{c} \right\} =$$

$$= \frac{(\omega / c)}{i\omega} I_0 \sin \beta z = -i \frac{I_0}{c} \sin \beta z$$