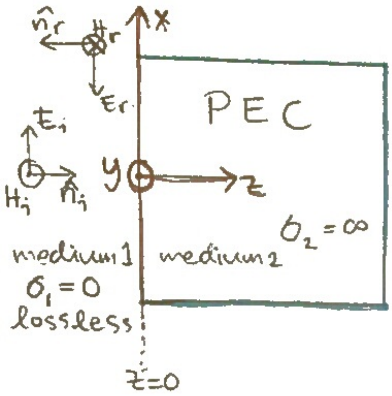
Storgruppsövning 4/12-13

Normal incidence on conductor (plane wave)



$E_2 = H_2 = 0$ in medium 2

incident wave (E_i, H_i)

$E_i(z) = \hat{x} E_{i0} e^{-i\beta_1 z}$ — phase constant of medium 1.

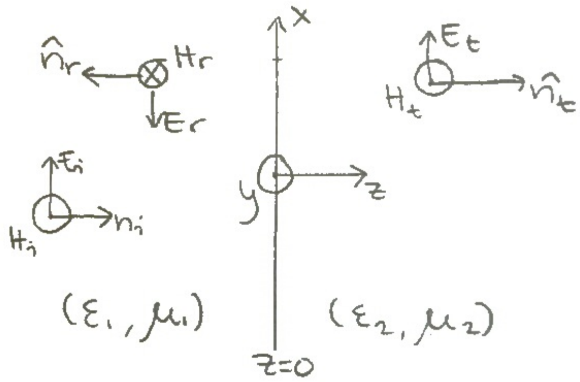
$$H_i(z) = \frac{1}{\eta_1} \hat{n}_i \times E_i = \frac{E_{i0}}{\eta_1} \hat{y} e^{-i\beta_1 z}$$

$$E_1 = E_i + E_r = \hat{x}\left(E_{i0} e^{-i\beta_1 z} + E_{r0} e^{-i\beta_1 z}\right) = -\hat{x}\, 2i E_{i0} \sin \beta_1 z$$

by writing the B.C at interface $E_1(0) = E_2(0) = 0 \Rightarrow (E_{r0} = -E_{i0})$

$$\begin{cases} H_r(z) = \dfrac{1}{\eta_1} \hat{n}_r \times E_r(z) \\ H_1 = H_i(z) + H_r(z) \end{cases}$$

Normal incidence on a dielectric boundary



$$\begin{cases} \bar{E}_i(z) = \hat{x} E_{i0} e^{-i\beta_1 z} \\ \bar{H}_i(z) = \hat{y} \dfrac{E_{i0}}{\eta_1} e^{-i\beta_1 z} \end{cases} \qquad \eta_1 = \sqrt{\frac{\mu_1}{\varepsilon_1}}$$

η_1 — intrinsic impedance

$$\begin{cases} \bar{E}_r(z) = \hat{x} E_{r0} e^{i\beta_1 z} \\ \bar{H}_r(z) = -\hat{y} \dfrac{E_{r0}}{\eta_1} e^{i\beta_1 z} \end{cases}$$

$$\begin{cases} \bar{E}_t(z) = \hat{x} E_{t0} e^{-\beta_2 z} \\ \bar{H}_t(z) = \hat{y} \dfrac{E_{t0}}{\eta_2} e^{-i\beta_2 z} \end{cases}$$

B.C $\Longrightarrow$
$$\bar{E}_i(0) + \bar{E}_r(0) = \bar{E}_t(0)$$
$$\bar{H}_i(0) + \bar{H}_r(0) = \bar{H}_t(0)$$

$$\Longrightarrow \Gamma = \frac{E_{r0}}{E_{i0}} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}, \quad \tau = \frac{E_{t0}}{E_{i0}} = \frac{2\eta_2}{\eta_1 + \eta_2}, \quad 1 + \Gamma = \tau$$

Plane wave in lossy media:

$\nabla^2 E - \gamma^2 E = 0$ source free wave equation

$\Rightarrow \bar{E} = \hat{x} E_0 e^{-\gamma z}$

$\gamma = \alpha + j\beta$

$\eta_c = Z_c = \sqrt{\dfrac{\mu}{\varepsilon_c}}$ intrinsic impedance

Good conductors:

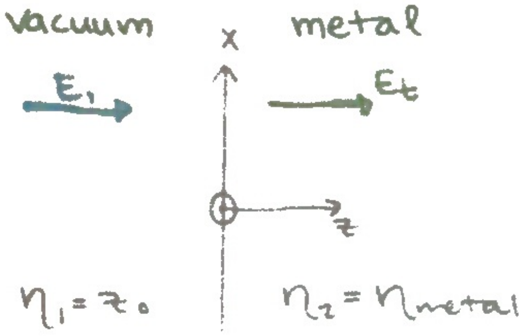
$\sigma >> \omega\varepsilon$, $\gamma = \alpha + j\beta = (1+j)/\delta$ penetration coeff.

$Z_c = Z_0 \sqrt{\dfrac{\omega \mu_r \varepsilon_0}{2\sigma}}(1+j)$

13.3

A plane wave in vacuum has normal incidence upon a conductive metal plate ($\delta = 1mm$)
- instantaneous E at boundary: ($E = \hat{x}\, 100\sqrt{2}\sin\omega t \cdot 10^{-6}$)
- % 99 of the incident power is reflected ($|\Gamma|^2 = 0{,}99$)

Find the instantaneous E-field at z inside metal



$$\begin{cases} E_x(z=0^-, t) = 10^{-4}\sqrt{2}\,\overbrace{\sin\omega t}^{\cos(\omega t - \pi/2)} \\ \bar{E}_x(z=0^-) = 10^{-4}\sqrt{2}\,(-j) \end{cases}$$

$$\begin{cases} \text{in metal: } \bar{E}_x(z) = \tau \bar{E}_0 e^{-\gamma z} \quad ; \quad \gamma = (1+j)/\delta \\ \eta_2 = Z_c = Z_0 \underbrace{\sqrt{\dfrac{\omega \mu_r \varepsilon_0}{2\sigma}}}_{a}(1+j) = a(1+j) Z_0 \quad (a <<) \end{cases}$$

$$|\Gamma|^2 = 0{,}99 = \left|\frac{Z_c - Z_0}{Z_c + Z_0}\right|^2 = \left|\frac{a(1+j)-1}{a(1+j)+1}\right|^2 = \frac{(a-1)^2 + a^2}{(a+1)^2 + a^2} = \frac{2a^2 - 2a + 1}{2a^2 + 2a + 1}$$

$$\Rightarrow \begin{cases} a_1 = 0{,}0025 = 1/400 \\ a_2 = 198{,}99 \quad (a<<1) \end{cases}$$

$$\tau = \frac{2\eta_2}{\eta_1 + \eta_2} = \frac{2a(1+j)}{\underbrace{\cancel{a(1+j)}}_{a<<1}+1} \approx 2a(1+j) = 2\sqrt{2}\, a\, e^{j\pi/4}$$

$$\bar{E}_T = T\bar{E}_0 e^{-(1+i)\delta} = \frac{e^{i\pi/4}}{100\sqrt{2}} \left(10^{-4}\sqrt{2}(-i)\right) e^{-10^3 z} e^{-10^3 i z} =$$

$$= 10^{6}\, e^{-i\pi/4}\, e^{-10^3 z}\, e^{-i10^3 z}$$

$$E_{Tx}(z,t) = \mathrm{Re}\left\{\bar{E}_{Tx}(z)\, e^{i\omega t}\right\} = 10^{-6} e^{-10^3 z} \cos(\omega t - 10^3 z - \pi/4)$$

For a conducting media $(\sigma \neq 0)$, $\mathbb{J} = \sigma \mathbb{E}$

$$\nabla \times \mathbb{H} = \mathbb{J} + i\omega \mathbb{D} = \sigma \mathbb{E} + i\omega\varepsilon \mathbb{E} = i\omega \underbrace{\left(\varepsilon + \frac{\sigma}{i\omega}\right)}_{\varepsilon_c \text{ - complex permittivity}} \mathbb{E}$$

$$\varepsilon_c = \varepsilon - i\frac{\sigma}{\omega}$$

1.3.5

Planear linearly polarized wave, $(\lambda = 30\,cm)$, in vacuum normal incidence on water surface.

Calculate reflected power coef. : $|\Gamma|^2$ water: $\begin{cases} \sigma = 5 \ S/m \\ \varepsilon_r = 80 \end{cases}$

$$\omega\varepsilon = \omega\varepsilon_0\varepsilon_r = 2\pi f \varepsilon_0 \varepsilon_r = 2\pi \cdot 10^9 \cdot \frac{1}{36\pi} \cdot 10^{-9} \cdot 80 = 4{,}45$$

$$f = \frac{c}{\lambda} = \frac{3\cdot 10^8}{30\cdot 10^{-2}} = 1\,GHz \qquad \Longrightarrow \quad \cancel{\sigma \gg \omega\varepsilon} \quad \cancel{\sigma \ll \omega\varepsilon}$$

$$Z_2 = \eta_2 = \sqrt{\frac{\mu}{\varepsilon_c}} = \sqrt{\frac{\mu}{\varepsilon_0\left(\varepsilon_r - \frac{i\sigma}{\omega\varepsilon_0}\right)}} = \overbrace{\sqrt{\frac{\mu}{\varepsilon_0}}}^{Z_0}\sqrt{\frac{1}{\varepsilon_r - i\frac{\sigma}{\omega\varepsilon_0}}}$$

$$Z_2 = Z_0\sqrt{\frac{1}{8 - i90}}$$ intrinsic impedance of water, $\begin{cases} \Gamma = \dfrac{Z_2 - Z_0}{Z_2 + Z_0} \Longrightarrow |\Gamma| = 0{,}847 \\ |\Gamma|^2 = 0{,}717 \end{cases}$

Poynting vector:

is a power density of an electromagnetic field.

$$\bar{\mathbb{P}} = \mathbb{E} \times \mathbb{H} \quad \left[\frac{W}{m^2}\right]$$

instantaneous power density: $\mathbb{P}(z,t) = \mathbb{E}(z,t) \times \mathbb{H}(z,t)$

average power density: $\mathbb{P}_{av} = \frac{1}{T}\int_0^T \mathbb{P}(z,t)\, dT$

$$\mathbb{P}_{av} = \frac{1}{2}\mathrm{Re}\left\{\mathbb{E}(z) \times \mathbb{H}(z)^*\right\}$$

11.10

Linearly polarized plane wave, propagating through a lossless dielectric, $\varepsilon_r = 2.5$ $\mu_r = 1$, the wave has power density of 0.2 $[\frac{W}{m^2}]$. Find the peak values of E and H!

$$\begin{cases} \mathbb{E} = \mathbb{E}_0 e^{-i\mathbb{k}\cdot R} \\ \mathbb{H} = \frac{1}{\eta} \hat{n} \times \mathbb{E} \end{cases}$$

$\begin{cases} \mathbb{k} \text{ - wave number vector} \\ \hat{n} \text{ - direction of propagation} \end{cases}$

$$\begin{cases} \bar{\mathbb{E}} = \bar{\mathbb{E}}_0 e^{-ikz} \hat{x} \\ \mathbb{H} = \frac{\bar{\mathbb{E}}_0}{\eta} e^{-ikz} \hat{y} \end{cases}$$

we assume E_x, H_y travelling in z direction.

$$P_{av} = \frac{1}{2} \mathrm{Re}\{\mathbb{E} \times \mathbb{H}^*\} = \frac{1}{2} \mathrm{Re}\left\{ (\mathbb{E}_0 e^{-ikz} \hat{x}) \times \left(\frac{\bar{\mathbb{E}}_0^*}{\eta} e^{-ikz} \hat{y} \right) \right\} =$$

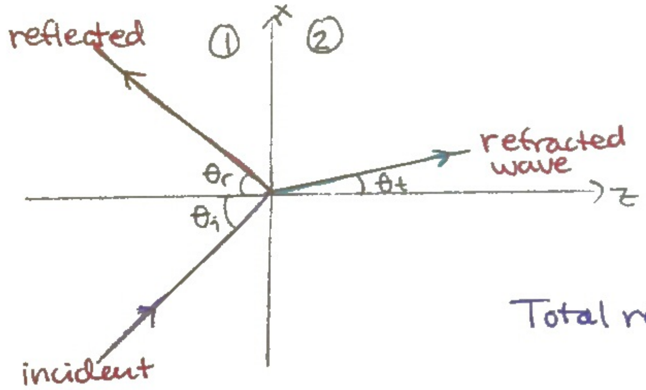
$$= \frac{1}{2\eta} |\bar{\mathbb{E}}_0|^2 \hat{z} \implies |\bar{\mathbb{E}}_0|^2 = P_{av} \cdot 2\eta = 0{,}2 \cdot 2\eta$$

$$\eta = z_c = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0 \mu_r}{\varepsilon_0 \varepsilon_r}} = z_0 \sqrt{\frac{\mu_r}{\varepsilon_r}} = 120\pi \sqrt{\frac{1}{2{,}5}} \approx 238{,}43$$

$$|\mathbb{E}_0| = \sqrt{0{,}4 \cdot 238{,}43} \approx 9{,}77$$

$$|\mathbb{H}_0| = \frac{|\mathbb{E}_0|}{\eta} \approx 0{,}041$$

Oblique incidence at a plane dielectric boundary



Snell's law of refraction:

$$\theta_i = \theta_r$$

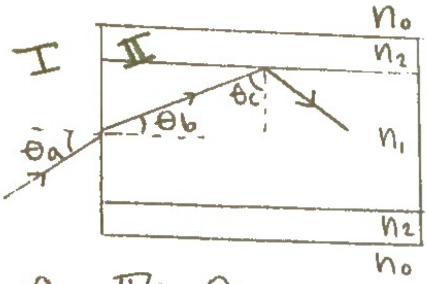
$$\frac{\sin\theta_t}{\sin\theta_i} = \frac{n_1}{n_2} = \frac{\beta_1}{\beta_2} = \sqrt{\frac{\varepsilon_1}{\varepsilon_2}}$$

$$n = \frac{c}{u_p} = \sqrt{\varepsilon_r \mu_r}$$

Total reflection: $\theta_i = \theta_c \implies \theta_t = \pi/2$

P8.41

Find the maximum θ_a so that the ray will be trapped inside.



We need total reflection at II

(I) $n_0 \sin\theta_a = n_1 \sin\theta_b$ refraction

(II) $n_1 \sin\theta_c = n_2 \sin(\pi/2)$ total reflection

$\theta_c = \pi/2 - \theta_b$

$$\underbrace{n_1 \sin\left(\frac{\pi}{2} - \theta_b\right)}_{\cos\theta_b} = n_2 \Rightarrow \cos\theta_b = \frac{n_2}{n_1} \Rightarrow \underbrace{\sin\theta_b = \sqrt{1 - \left(\frac{n_2}{n_1}\right)^2}}_{\text{substitute in I}}$$

$$\Rightarrow n_0 \sin\theta_a = n_1 \sqrt{1 - \left(\frac{n_2}{n_1}\right)^2} = \sqrt{n_1^2 - n_2^2}$$

$$\Rightarrow \sin\theta_a = \frac{\sqrt{n_1^2 - n_2^2}}{n_0} \Rightarrow \theta_a = \arcsin\left(\frac{\sqrt{n_1^2 - n_2^2}}{n_0}\right), \quad \theta_i \le \theta_a$$