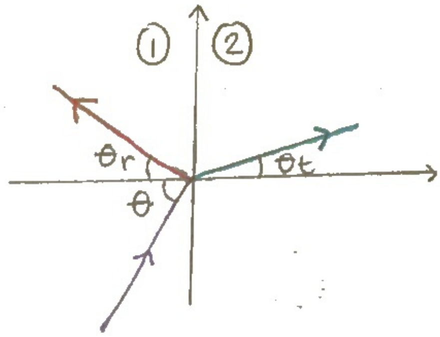
Föreläsning 4/12-13

Snells lag: $c_2 \sin\theta_i = c_1 \sin\theta_t$

Vad händer om $\varepsilon_1 > \varepsilon_2$
$c_1 < c_2$ $\quad (c = 1/\sqrt{\varepsilon\mu}$



Total reflektion 8.10.1

Antag $\mu_1 = \mu_2 = \mu_0$

Snells lag ger: $\sin\theta_t = \sqrt{\frac{\varepsilon_1}{\varepsilon_2}} \sin\theta_i$

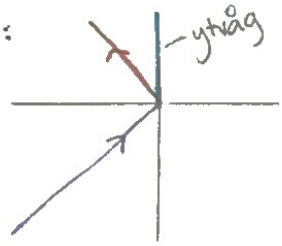
Fallet $\theta_t = \pi/2$

$\Rightarrow \theta_i = \arcsin\sqrt{\frac{\varepsilon_2}{\varepsilon_1}} = \theta_{\text{kritiska vinkeln}}$

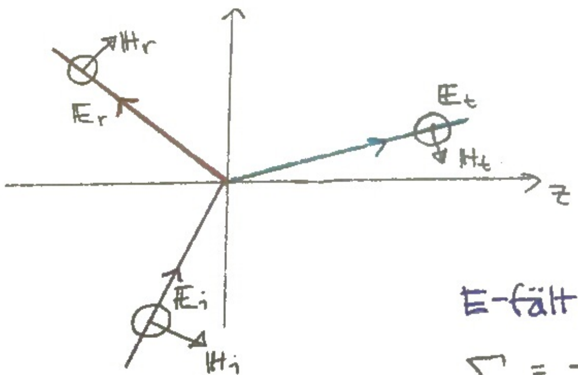
För $\theta_i > \theta_{\text{kritiska vinkeln}} = \theta_c$

$\Rightarrow \sin\theta_t \sqrt{\frac{\varepsilon_1}{\varepsilon_2}} > 1$ ingen reell lösning

Vi får en ytvåg:



Fresnels ekvationer 8.10.2, 8.10.3



E-fält vinkelrätt mot infallsplanet:

$$\Gamma_\perp = \left(\frac{E_{r0}}{E_{i0}}\right)_\perp = \frac{1/z_1 \cos\theta_i - 1/z_2 \cos\theta_t}{1/z_1 \cos\theta_i + 1/z_2 \cos\theta_t}$$

$$T_\perp = \left(\frac{E_{t0}}{E_{i0}}\right)_\perp = \frac{2/z_1 \cos\theta_i}{1/z_1 \cos\theta_i + 1/z_2 \cos\theta_t}$$

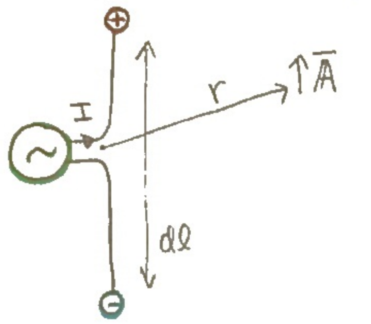
E-fält parallellt m. infallsplanet:

$$\Gamma_\parallel = \frac{-z_1 \cos\theta_i + z_2 \cos\theta_t}{z_1 \cos\theta_i + z_2 \cos\theta_t}$$

$$T_\parallel = \frac{2 z_2 \cos\theta_i}{z_1 \cos\theta_i + z_2 \cos\theta_t}$$

Brewstervinkeln:
kan hända att $Z_2 \cos\theta_t = Z_1 \cos\theta_i \implies \Gamma_{\parallel} = 0$.

Antenner och Hertzdipolen 11.1, 11.2



Approx. $d\ell \ll \lambda$
$d\ell \ll r$

Låt I vara konstant längs $d\ell$ (*)

Retarderad potential

$$\mathbb{A} = \frac{\mu_0}{4\pi} \int_{V_1} \frac{\mathbb{J}(\mathbb{r}_1) e^{-\frac{j\omega r}{c}}}{r} dv_1$$

$$(*) \implies \bar{A} = \frac{\mu_0}{4\pi} \frac{\bar{I}\, d\ell}{r} e^{-j\beta r} \hat{z}$$

$$\hat{z} = \hat{r}\cos\theta - \hat{\theta}\sin\theta$$

$$\begin{cases} A_r = A_z \cos\theta = \frac{\mu_0 I\, d\ell}{4\pi} \frac{e^{-j\beta r}}{r} \cos\theta \\ A_\theta = -A_z \sin\theta = -\frac{\mu_0 I\, d\ell}{4\pi} \frac{e^{-j\beta r}}{r} \sin\theta \\ A_\phi = 0 \end{cases}$$

$$\mathbb{H} = \frac{1}{\mu_0} \nabla \times \mathbb{A} = \hat{\Phi} \frac{1}{\mu_0 r}\left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial \theta}\right] = -\hat{\Phi} \frac{\bar{I}\, d\ell}{4\pi} \beta^2 \sin\theta \left[\frac{1}{j\beta r} + \frac{1}{(j\beta r)^2}\right] e^{-j\beta r}$$

$$\mathbb{E} = \frac{1}{j\omega\varepsilon_0} \nabla \times \mathbb{H} = \frac{1}{j\omega\varepsilon_0}\left[\hat{r} \frac{1}{r\sin\theta} \frac{\partial}{\partial\theta}\left(H_\phi \sin\theta - \hat{\theta} \frac{1}{r} \frac{\partial}{\partial r}(rH_\phi)\right)\right]$$

$$\bar{E} = -\frac{\bar{I}\, d\ell}{4\pi} Z_0 \beta^2 \left[\hat{r}\left(\frac{2}{(j\beta r)^3} + \frac{2}{(j\beta r)^2}\right)\cos\theta + \hat{\theta}\left(\frac{1}{(j\beta r)^3} + \frac{1}{(j\beta r)^2} + \frac{1}{j\beta r}\right)\sin\theta\right] e^{-j\beta r}$$

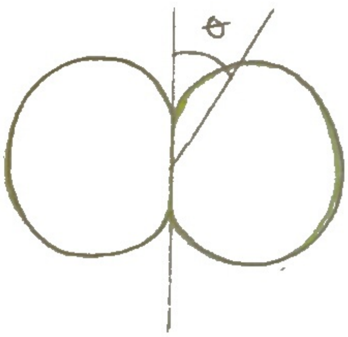
Approximera $r \gg \lambda$ (fjärrfält):

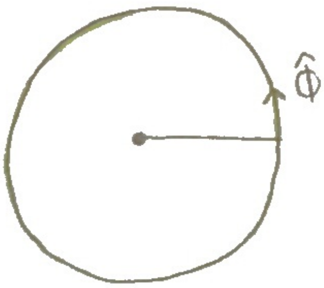
$r \gg 1/\beta = \lambda/2\pi \gg d\ell$

$$\bar{E}_\theta = i \frac{\bar{I} d\ell}{4\pi} \frac{e^{-i\beta r}}{r} Z_0 \beta \sin\theta$$

$$\bar{H}_\phi = i \frac{\bar{I} d\ell}{4\pi} \frac{e^{-i\beta r}}{r} \beta \sin\theta$$

Strålningsdiagram 11.3





θ-planet
E-plan

Φ-planet
H-plan

Strålningsresistans ex 11.3

$$S_{av} = \hat{r} \cdot \mathrm{Re}\, \tfrac{1}{2} \{ \bar{E} \times \bar{H}^* \}$$

$$P_{av} = \oint_S S_{av} \cdot dS = R_{rad} I_{eff}^2$$

yta

$$\bar{E} \times \bar{H}^* = \hat{\theta} Z_0 \frac{i\omega d\ell \bar{I} \sin\theta}{4\pi c r} e^{-i\beta r} \times \hat{\Phi} \frac{-i\omega d\ell \bar{I} \sin\theta}{4\pi c r} e^{i\beta r} =$$

$$= \hat{r} Z_0 \frac{\omega^2 d\ell^2 |\bar{I}|^2 \sin^2\theta}{16\pi^2 c^2 r^2}$$

$$\Rightarrow R_{rad} = 80\pi^2 \left(\frac{d\ell}{\lambda}\right)^2$$

Antennförstärkning 11.3

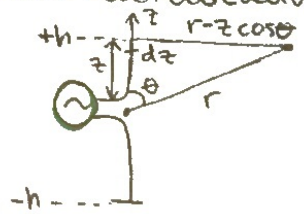
$$G_D(\theta, \phi) = \frac{S_{rad}(\theta, \phi)}{S_{isotrop}} = \frac{S_{rad}(\theta, \phi)}{P_{av}/4\pi r}$$

För Hertzdipol: $G_D(\theta, \phi) = 3/2 \cdot \sin^2\theta$

Direktivitet: $D = \max(G_D) \Longrightarrow D = 1{,}5$ för dipol

Dipolantenner 11.4

Med antennlängd $l \sim \lambda$



Hertzdipolbidrag:

$$d\mathbb{E}_{rad} = \hat{\theta}\, Z_0 \frac{i\omega\, dz\, \bar{I}(z) \sin\theta}{4\pi c r} e^{-i\beta(r - z\cos\theta)}$$

Antag $\bar{I}(z) = I_0 \sin\{\beta(h - |z|)\}$

$$\bar{E}_z = \int_{-h}^{h} d\mathbb{E}_{rad}$$

Halvvågsantenn 11.4.1

$$E_\theta = Z_0 H_\varphi = \ldots = i\frac{60\bar{I}_0}{r} e^{-i\beta r} \left\{ \frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta} \right\}$$

$$S_{av} = \mathrm{Re}\,\frac{1}{2}\left(E_{rad} \times \mathbb{H}^*_{rad}\right) = \frac{15 I_0^2}{\pi r^2}\left(\frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta}\right)^2$$

$$P_{av} = \int_S S_{av}\, dS = R_{rad}\frac{I_0^2}{2} = 36{,}5\, I_0^2$$

Strålningsresistans: $R_{rad} = 73{,}1\ \Omega$

Direktivitet: $D = 1{,}64$