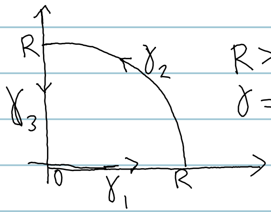
Storgruppsövning 4/10-13

3.1.2

Bestäm # nollställen till $f(z)=z^4-3z^2+3$ i 1:a kvadranten, Q_1.

Lösn:



$R>>1$

$\gamma=\gamma_1+\gamma_2+\gamma_3$

γ_1: $z=x$, $0\leq x\leq R$

$f(x)=x^4-3x^2+3=(x^2-\frac{3}{2})^2+\frac{3}{4}>0$ då $0\leq x\leq R$

$0\xrightarrow{x}R \Rightarrow \arg f(x)\equiv 0$

γ_2: $z=Re^{i\theta}$, $0\leq\theta\leq\pi/2$

$f(Re^{i\theta})=R^4e^{4i\theta}-3R^2e^{2i\theta}+3\simeq R^4e^{4i\theta}$ då $R>>1$

$\Rightarrow \arg(f(Re^{i\theta}))\approx\arg(R^4e^{4i\theta})=4\theta$

$0\xrightarrow{\theta}\pi/2 \Rightarrow 0\xrightarrow{\arg f(Re^{i\theta})}2\pi$

γ_3: $z=iy$, $R\geq y\geq 0$

$f(iy)=(iy)^4-3(iy)^2+3=y^4+3y^2+3>0$ då $R\geq y\geq 0$

$R\xrightarrow{y}0 \Rightarrow \arg(f(iy))\equiv 0$

f har inga poler!

Arg. principen $\Rightarrow N(f;Q_1)=\{R>>1\}=N(f;\gamma)=1$

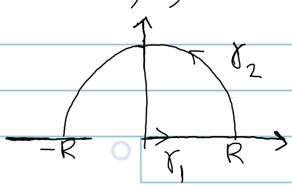
3.1.8

Bestäm # nollst. till $f(z)=2z^4-2iz^3+z^2+2iz-1$ i det ÖHP, $\mathcal{U}$.

Lösn:

$R>>1$

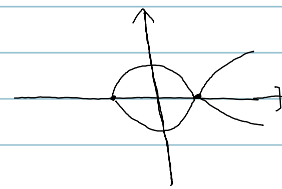
$\gamma=\gamma_1+\gamma_2$



γ_1: $z = x$, $-R \leq x \leq R$, $f(x) = 2x^4 - 2ix^3 + x^2 + 2ix - 1$

$$\begin{cases} \mathrm{Re}(f(x)) = 2x^4 + x^2 - 1 \\ \mathrm{Im}(f(x)) = -2x^3 + 2x = 2x(1-x^2) \end{cases}$$

$\mathrm{Im} f(x) = 0 \Rightarrow x = -1, 0, 1$



$-R \leq x \leq -1 \Rightarrow \begin{cases} \mathrm{Re}(f(x)) > 0 \\ \mathrm{Im}(f(x)) \geq 0 \end{cases}$

$-1 \leq x \leq 0 \Rightarrow \begin{cases} 2 \xrightarrow{\mathrm{Re} f(x)} -1 \\ \mathrm{Im} f(x) \leq 0 \end{cases}$

$0 \leq x \leq 1 \Rightarrow \begin{cases} -1 \xrightarrow{\mathrm{Re} f(x)} 2 \\ \mathrm{Im} f(x) \geq 0 \end{cases}$

$1 \leq x \leq R \Rightarrow \begin{cases} \mathrm{Re} f(x) > 0 \\ \mathrm{Im} f(x) \leq 0 \end{cases}$

$\Rightarrow -R \xrightarrow{x} R \Rightarrow 0 \xrightarrow[\arg f(x)]{} -2\pi$

γ_2: $z = Re^{i\theta}$, $0 \leq \theta \leq \pi$

$f(Re^{i\theta}) \simeq R^4 e^{4i\theta}$ då $R >> 1$

$\Rightarrow \arg(f(Re^{i\theta})) \simeq \arg(R^4 e^{4i\theta}) = 4\theta$

$0 \xrightarrow{\theta} \pi \Rightarrow 0 \xrightarrow[\arg(f(Re^{i\theta}))]{} 4\pi$

Sammantaget

$\left.\begin{array}{l} \gamma_1 \text{ - ett varv medurs} \\ \gamma_2 \text{ - två varv moturs} \end{array}\right\} \Rightarrow \gamma$ - ett varv moturs

f har inga poler

Arg.pr. $\Rightarrow N(f; \mathcal{U}) = \{R >> 1\} = N(f; \gamma) = 2 - 1 = 1$

Rouchés sats

$f, g \in A(D)$, γ "normal" kurva i D

$|f \pm g| < |f|$ på γ

$\Rightarrow N(f; \gamma) = N(g; \gamma)$

3.1.15

Bestäm # nollst. till $f(z) = 4z^3 - 12z^2 + 2z + 10$ i området $D = \{z \in \mathbb{C};\ \frac{1}{2} < |z-1| < 2\}$

Lösn: Låt $w = z-1 \Rightarrow D = \{w \in \mathbb{C};\ \frac{1}{2} < |w| < 2\}$

$z = w+1$

$g(w) = f(w+1) = 4(w+1)^3 - 12(w+1)^2 + 2(w+1) + 10 =$

$= 4w^3 - 10w + 4$

Använd nu Rouché till att bestämma

$N(g; \{\frac{1}{2} < |w| < 2\})$

$|w| = 1/2$: $-10w$ störst till belopp

$|g(w) - (-10w)| = |4w^3 + 4| \leq 4|w|^3 + 4 = 4 \cdot \frac{1}{8} + \frac{8}{2} = 9/2$

$|-10w| = 10 \cdot \frac{1}{2} = \frac{10}{2} > \frac{9}{2}$

$\Rightarrow |g(w) + 10w| < |10w|$ på $|w| = 1/2$

$\Rightarrow N(g; |w| = 1/2) = N(10w; |w| = 1/2) = 1$

$|w| = 2$: $4w^3$ störst till belopp på $|w| = 2$

$|g(w) - 4w^3| = |-10w + 4| \leq 10|w| + 4 = 24$

$|4w^3| = 4|w|^3 = 4 \cdot 8 = 32 > 27$

Rouché $\Rightarrow N(g; |w| = 2) = N(4w^3; |w| = 2) = 3$

$\therefore N(f; D) = N(g; \frac{1}{2} < |w| < 2) = 3 - 1 = 2$