Förelåsning 3/9-13 em

Skalära fält: en funktion från $\mathbb{R}^3$ till $\mathbb{R}$
$\phi(\mathbb{r})$ eller $\phi(\mathbb{r}, t)$
En skalär ska vara oberoende av val av koordinatsystem.

Vektorfält: $\mathbb{F}(\mathbb{r})$ eller $\mathbb{F}(\mathbb{r}, t)$
Vektorkomponenterna ändras med hjälp av ortogonal matris.

Hur deriverar man fält?

$$\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \longrightarrow \nabla = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}$$

$\nabla \phi$ ~ vektor ~ gradient
$\nabla \cdot \mathbb{F}$ ~ skalär ~ divergens
$\nabla \times \mathbb{F}$ ~ vektor ~ rotation

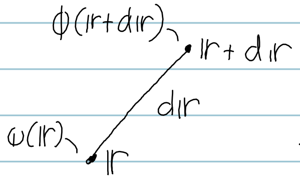
$$\nabla \phi = \hat{x}\frac{\partial \phi}{\partial x} + \hat{y}\frac{\partial \phi}{\partial y} + \hat{z}\frac{\partial \phi}{\partial z}$$

$$\nabla \cdot \mathbb{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$$

$$\nabla \times \mathbb{F} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ F_x & F_y & F_z \end{vmatrix} = \hat{x}\left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right) + \ldots$$

Laplaceoperatorn: $\Delta \phi \equiv \nabla \cdot \nabla \phi = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\phi$

Uppgift: Visa att $\nabla \times \nabla \phi = 0$!



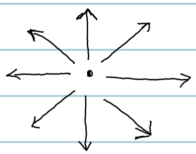
$$d\phi = \underbrace{\phi(\mathbf{r}+d\mathbf{r})}_{\phi(\mathbf{r}) + dx\frac{\partial \phi}{\partial x} + dy\frac{\partial \phi}{\partial y} + dz\frac{\partial \phi}{\partial z}} - \phi(\mathbf{r}) = \star$$

$$\star = (dx, dy, dz) \cdot \left(\frac{\partial \phi}{\partial x}, \frac{\partial \phi}{\partial y}, \frac{\partial \phi}{\partial z}\right) = d\mathbf{r} \cdot \nabla\phi$$

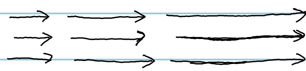
Riktningsderivata:
Hur mycket ändras ϕ per l.e när man flyttar sig i en riktning som ges av enhetsvektorn $\mathbf{n}$.
$|\mathbf{n}| = 1$

Gradienten av ϕ pekar åt det håll i vilket ϕ ökar fortast.

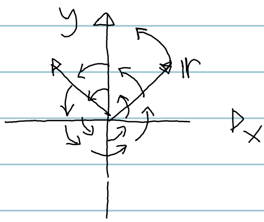
Divergens



"Det går isär"



Rotation



"Det snurrar"

Fältlinjer: Kurvor som överallt pekar åt samma håll som fältet.

ex uppgift 5

$F = F_0 \left(\frac{x}{a}\hat{x} + \hat{y} \right)$

$\frac{\partial x}{\partial t} = f(t) F_0 \frac{x}{a}$, $\frac{\partial y}{\partial t} = f(t) F_0$

$\frac{\partial x}{\cancel{\partial t}} \Big/ \frac{\partial y}{\cancel{\partial t}} = \frac{x}{a}$ $\quad \frac{\partial x}{\partial y} = \frac{x}{a}$ $\quad x(y) = C e^{y/a}$