7/2

$\underline{v}_P$ & $\underline{\omega}$ are perpendicular so that $\underline{\omega}\cdot\underline{v}=0$

$$\underline{\omega}=\frac{600\times 2\pi}{60}\,\frac{8\underline{i}+12\underline{j}+4\underline{k}}{\sqrt{8^2+12^2+4^2}}\text{ rad/sec},\quad \underline{v}=12\underline{i}-6\underline{j}+v_z\underline{k}$$

So $(8\underline{i}+12\underline{j}+4\underline{k})\cdot(12\underline{i}-6\underline{j}+v_z\underline{k})=0$

$96-72+4v_z=0,\quad \underline{v_z=-6\text{ ft/sec}}$

$v=\sqrt{12^2+(-6)^2+(-6)^2}=14.70\text{ ft/sec}$

$R=v/\omega=14.70/(20\pi)=0.234\text{ ft}$ or $\underline{R=2.81\text{ in.}}$

$a_p=a_n=r\omega^2=0.234(20\pi)^2=923\text{ ft/sec}^2$

or $\underline{a_p=11{,}080\text{ in./sec}^2}$

7/9

$\underline{\alpha}=\underline{\Omega}\times\underline{\omega}=0.6\underline{k}\times 2\underline{j}=\underline{-1.2\underline{i}\text{ rad/sec}^2}$

$\underline{a}_P=\dot{\underline{\omega}}\times\underline{r}+\underline{\omega}\times(\underline{\omega}\times\underline{r}),\quad \underline{\omega}=\underline{\Omega}+\underline{\omega}_0$

$\dot{\underline{\omega}}=\underline{\alpha}=-1.2\underline{i}\text{ rad/sec}^2$

$\underline{r}=34\underline{j}+20\underline{k}$ in. (for $\beta=90°$)

Carry out algebra to obtain

$\underline{\underline{a}_P=35.8\underline{j}-80\underline{k}\text{ in./sec}^2}$

7/13

$$\Omega = 4 \times 2\pi = 8\pi \text{ rad/s}$$

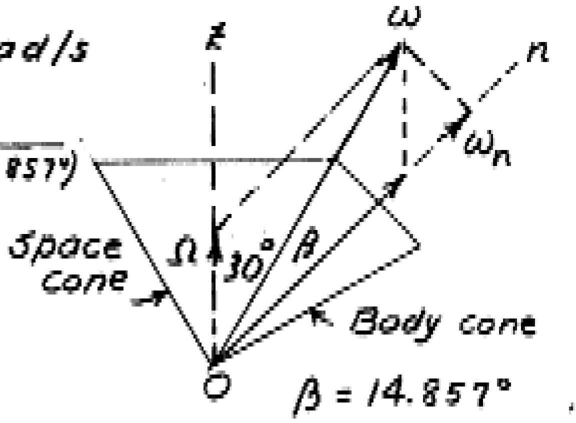
$$\frac{8\pi}{\sin 14.857^\circ} = \frac{\omega}{\sin(180^\circ - 30^\circ - 14.857^\circ)}$$

$$\omega = 8\pi \frac{\sin 135.143^\circ}{\sin 14.857}$$

$$= \underline{69.1 \text{ rad/s}}$$

$$\omega_n = 69.1 \cos 14.857^\circ$$

$$= \underline{66.8 \text{ rad/s}}$$



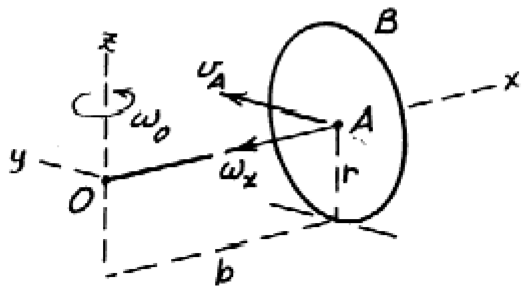
7/18

$$v_A = b\omega_0$$

$$\underline{\omega} = (-v_A/r)\underline{i} + \omega_0 \underline{k}$$

$$\underline{\omega} = -\frac{b\omega_0}{r}\underline{i} + \omega_0 \underline{k}$$

$$\underline{\underline{\omega} = \omega_0\left(-\frac{b}{r}\underline{i} + \underline{k}\right)}$$

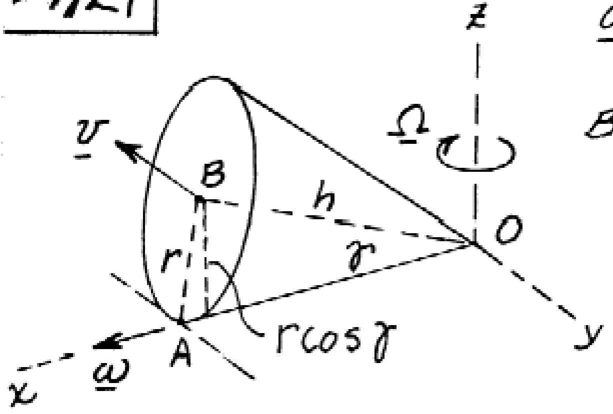


$$\underline{\alpha} = \dot{\underline{\omega}} = \omega_0\left(-\frac{b}{r}\dot{\underline{i}}\right) + \underline{0} \quad \text{where } \dot{\underline{i}} = \underline{\omega_z} \times \underline{i} = \omega_0 \underline{j}$$

so

$$\underline{\alpha} = \omega_0\left(-\frac{b}{r}\omega_0 \underline{j}\right), \quad \underline{\underline{\alpha} = -\frac{b}{r}\omega_0^2 \underline{j}}$$

▸ 7/27



$$\underline{\omega} = \frac{v}{r\cos\gamma}\,\underline{i}$$

But $\cos\gamma = \dfrac{h}{\sqrt{r^2+h^2}}$

$$\underline{\omega} = \frac{v\sqrt{r^2+h^2}}{rh}\,\underline{i}$$

$$= v\sqrt{\frac{1}{h^2}+\frac{1}{r^2}}\,\underline{i}$$

$\omega = \text{const}$ so $\underline{\alpha} = \underline{\Omega}\times\underline{\omega}$

$$\underline{\Omega} = -\frac{v}{h\cos\gamma}\,\underline{k}$$

so $\underline{\alpha} = -\dfrac{v}{h\cos\gamma}\underline{k} \times \dfrac{v}{r\cos\gamma}\underline{i} = -\dfrac{v^2}{hr\cos^2\gamma}\,\underline{j}$

$$\underline{\alpha} = -\frac{v^2}{h^2}\left(\frac{r}{h}+\frac{h}{r}\right)\underline{j}$$

7/30 $\underline{\omega} = \underline{\Omega} + \underline{p} = 4\underline{i} + 10\underline{k}$, $\underline{\omega} = \sqrt{4^2+10^2} = 10.77\ \frac{rad}{s}$

$\underline{\alpha} = \underline{\Omega}\times\underline{p} = 4\underline{i}\times 10\underline{k} = -40\underline{j}$ rad/s^2

7/36 $\underline{v}_A = \underline{v}_B + \underline{\omega}\times\underline{r}_{A/B}$

$$\underline{a}_A = \underline{a}_B + \dot{\underline{\omega}}\times\underline{r}_{A/B} + \underline{\omega}\times(\underline{\omega}\times\underline{r}_{A/B})$$

$\underline{\omega} = 1.4\underline{i} + 1.2\underline{j}$ rad/sec ; $\dot{\underline{\omega}} = 2\underline{i} + 3\underline{j}$ rad/sec^2

$\underline{r}_{A/B} = 5\underline{i}$ ft , $\underline{v}_B = 3.2\underline{j}$ ft/sec , $\underline{a}_B = 4\underline{j}$ ft/sec^2

Substitution and simplification yield

$\underline{v}_A = 3.2\underline{j} - 6\underline{k}$ ft/sec $\Rightarrow$ $v_A = 6.8$ ft/sec

$\underline{a}_A = -7.2\underline{i} + 12.4\underline{j} - 15\underline{k}$ ft/sec^2 $\Rightarrow$ $a_A = 20.8$ ft/sec^2

7/47 | $\underline{\Omega}$ = angular velocity of axes x-y-z

$\underline{\omega}$ = " " " simulator $= \underline{\Omega} + \underline{p}$

Let N = angular velocity of frame = 0.2 rad/s const.

$p = 0.9$ rad/s const., $\dot{\beta} = 0.15$ rad/s const.

$$\underline{\Omega} = \underline{i}\dot{\beta} + \underline{j}N\cos\beta - \underline{k}N\sin\beta \;;\; \underline{p} = p\underline{k}$$

$$\underline{\omega}_{\beta=0} = 0.15\underline{i} + 0.2\underline{j} + 0.9\underline{k} \text{ rad/s}$$

From Eq. 7/7, $\underline{\alpha} = \left(\frac{d\underline{\omega}}{dt}\right)_{XYZ} = \left(\frac{d\underline{\omega}}{dt}\right)_{xyz} + \underline{\Omega}\times\underline{\omega}$

$$\underline{\alpha} = (\underline{0} - \underline{j}N\dot{\beta}\sin\beta - \underline{k}N\dot{\beta}\cos\beta + \underline{0}) + \underline{\Omega}\times(\underline{\Omega}+\underline{p})$$

where $\underline{\Omega}\times(\underline{\Omega}+\underline{p}) = \underline{\Omega}\times\underline{p} = (\underline{i}\dot{\beta} + \underline{j}N\cos\beta - \underline{k}N\sin\beta)\times p\underline{k}$

$$= \underline{i}Np\cos\beta - \underline{j}p\dot{\beta}$$

so $\underline{\alpha}_{\beta=0} = \underline{i}Np - \underline{j}p\dot{\beta} - \underline{k}N\dot{\beta}$

$$= 0.2(0.9)\underline{i} - 0.9(0.15)\underline{j} - 0.2(0.15\underline{k}) \text{ rad/s}^2$$

$$= 0.18\underline{i} - 0.135\underline{j} - 0.030\underline{k} \text{ rad/s}^2$$

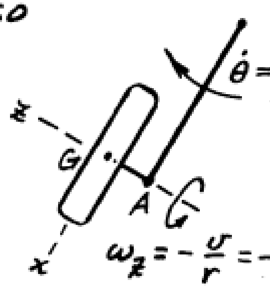
7/55 | x-y-z are principal axes so

$$\underline{H} = I_{xx}\omega_x\underline{i} + I_{yy}\omega_y\underline{j} + I_{zz}\omega_z\underline{k}$$

$$I_{zz} = mk^2$$
$$= 45(0.370)^2 = 6.16 \text{ kg}\cdot\text{m}^2$$

$I_{xx} + I_{yy} = I_{zz}$ & $I_{xx} = I_{yy}$

so $I_{yy} = \frac{1}{2}I_{zz} = 3.08 \text{ kg}\cdot\text{m}^2$



$\dot{\theta} = \frac{30}{180}\pi = 0.524$ rad/s $= -\omega_y$

$\omega_z = -\frac{v}{r} = -\frac{200(10^3)/3600}{0.920/2} = -120.8$ rad/s

$\omega_x = 0$

About G, $\underline{H}_G = \underline{0} + 3.08(-0.524)\underline{j} + 6.16(-120.8)\underline{k}$

$$\underline{H}_G = -1.613\underline{j} - 744\underline{k} \text{ kg}\cdot\text{m}^2/\text{s}$$

About A, $I_{yy} = \bar{I}_{yy} + md^2 = 3.08 + 45(0.215)^2 = 5.16 \text{ kg}\cdot\text{m}^2$

$$\underline{H}_A = \underline{0} + 5.16(-0.524)\underline{j} + 6.16(-120.8)\underline{k}$$

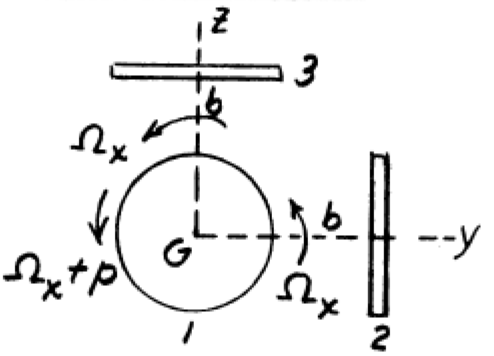
$$\underline{H}_A = -2.70\underline{j} - 744\underline{k} \text{ kg}\cdot\text{m}^2/\text{s}$$

7/61 About G,

$H_{x_1} = I(\Omega_x + p)$

$H_{x_2} = (\frac{I}{2} + mb^2)\Omega_x$

$H_{x_3} = (\frac{I}{2} + mb^2)\Omega_x$



So $H_x = I(\Omega_x + p) + (I + 2mb^2)\Omega_x$

$\quad = Ip + 2(I + mb^2)\Omega_x$

similarly

$H_y = Ip + 2(I + mb^2)\Omega_y$

$H_z = Ip + 2(I + mb^2)\Omega_z$

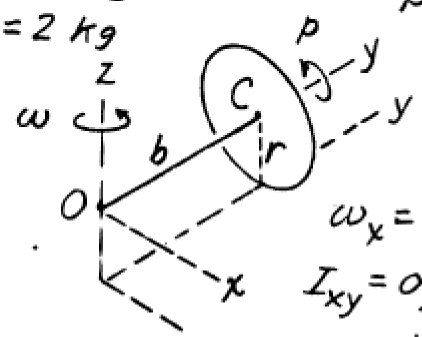
Thus $\underline{H}_G = \underline{Ip(\underline{i} + \underline{j} + \underline{k}) + 2(I + mb^2)\underline{\Omega}}$

where $\underline{\Omega} = \Omega_x \underline{i} + \Omega_y \underline{j} + \Omega_z \underline{k}$

7/70 $r = 100$ mm $\quad \omega = 4\pi$ rad/s

$b = 200$ mm $\quad p = \frac{v_C}{r} = \frac{b}{r}\omega = 8\pi$ rad/s

$m = 2$ kg



Eq. 7/11 holds for point O as a fixed point on axis of disk

$\omega_x = 0,\ \omega_y = -p = -8\pi$ rad/s, $\omega_z = \omega = 4\pi \frac{\text{rad}}{\text{s}}$

$I_{xy} = 0;\ I_{yy} = \frac{1}{2}mr^2 = \frac{1}{2}(2)(0.1)^2 = 0.01$ kg·m²

$I_{yz} = 0,\ I_{xz} = 0,\ I_{zz} = \frac{1}{4}mr^2 + mb^2 = 2(\frac{1}{4}\overline{0.1}^2 + \overline{0.2}^2)$

$\quad = 0.085$ kg·m²

So $\underline{H}_O = \underline{j} I_{yy}\omega_y + \underline{k} I_{zz}\omega_z = \underline{j}(-\frac{1}{2}mr^2 p) + \underline{k}(\frac{1}{4}mr^2 + mb^2)\omega$

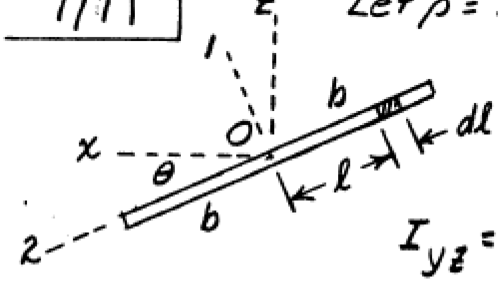
$\quad = mr^2\omega\left(-\frac{1}{2}\frac{b}{r}\underline{j} + [\frac{1}{4} + \frac{b^2}{r^2}]\underline{k}\right)$

$\quad = 2(0.1)^2 4\pi\left(-\frac{1}{2}2\underline{j} + [\frac{1}{4} + 4]\underline{k}\right) = \underline{0.251(-\underline{j} + 4.25\underline{k})\ \text{N·m·s}}$

$T = \frac{1}{2}\underline{\omega}\cdot\underline{H}_O = \frac{1}{2}(-8\pi\underline{j} + 4\pi\underline{k})\cdot 0.251(-\underline{j} + 4.25\underline{k})$

$\quad = 3.15 + 6.71 = \underline{9.87\ \text{J}}$

7/71



Let ρ = mass per unit of panel area

$$I_{xz} = \int_{-b}^{b} (-l\cos\theta)(l\sin\theta)2c\rho\, dl$$

$$= -\frac{2}{3}\rho c b^3 \sin 2\theta \quad \text{for 2 panels}$$

$$I_{yz} = I_{xy} = 0 \quad \text{by symmetry}$$

$$I_{zz} = \bar{I}_{zz} + md^2 \quad \text{for each panel}$$

For total, $$I_{zz} = 2\left\{\frac{2bc\rho}{12}\left[c^2 + (2b\cos\theta)^2\right] + 2bc\rho\left[a + \frac{c}{2}\right]^2\right\}$$

$$= 4bc\rho\left\{\frac{c^2}{3} + \frac{b^2}{3}\cos^2\theta + a^2 + ac\right\}$$

$$\underline{H}_O = -I_{xz}\omega_z \underline{i} + I_{zz}\omega_z \underline{k}, \quad m = 4bc\rho \quad (\text{total})$$

$$\underline{H}_O = \frac{m}{6}b^2\omega \sin 2\theta\, \underline{i} + m\omega\left\{\frac{c^2}{3} + \frac{b^2}{3}\cos^2\theta + a^2 + ac\right\}\underline{k}$$

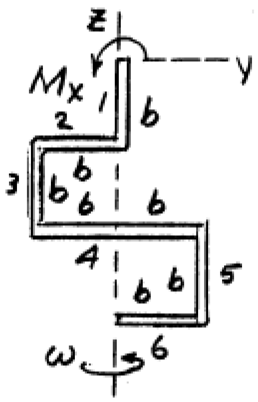
By symmetry, principal axes are O-1, O-2, O-y

$I_1 = m\left\{\frac{c^2 + b^2}{3} + a^2 + ac\right\}$ (max)

$I_2 = m\left\{\frac{1}{3}c^2 + a^2 + ac\right\}$ (intermediate)

$I_3 = \frac{1}{3}mb^2$ (minimum)

7/76



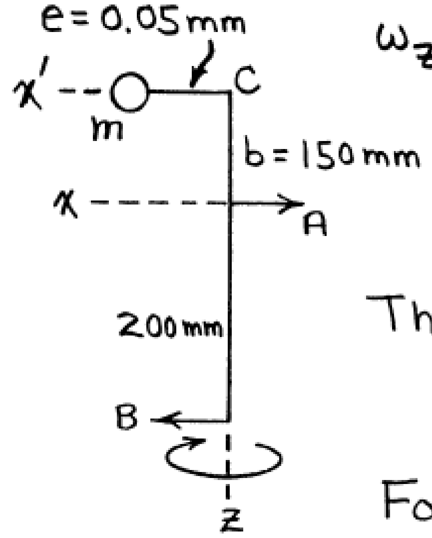
	I_{yz}
①	0
②	$\rho b(-\frac{b}{2})(-b) = +\frac{1}{2}\rho b^3$
③	$\rho b(-b)(-\frac{3b}{2}) = \frac{3}{2}\rho b^3$
④	0
⑤	$\rho b(b)(-\frac{5}{2}b) = -\frac{5}{2}\rho b^3$
⑥	$\rho b(\frac{b}{2})(-3b) = -\frac{3}{2}\rho b^3$

Total $I_{yz} = \rho b^3\left(\frac{1}{2} + \frac{3}{2} - \frac{5}{2} - \frac{3}{2}\right) = -2\rho b^3$

From Eq. 7/23 $\Sigma M_x = I_{yz}\omega_z^2, \quad \dot{\omega}_z = 0$

$$M = M_x = -2\rho b^3\omega^2$$

7/77



$$\Sigma M_y = -I_{yz}\dot{\omega}_z - I_{xz}\omega_z^2 \,, \quad \dot{\omega}_z = 0$$

$$\omega_z = \omega = 10{,}000\left(\frac{2\pi}{60}\right) = 1047 \ \frac{\text{rad}}{\text{sec}}$$

$$I_{xz} = -mbe = -6(0.15)(50)(10^{-6})$$

$$= -45(10^{-6}) \ \text{kg}\cdot\text{m}^2$$

Thus $B(0.20) = 45(10^{-6})(1047)$

$$= \underline{247 \text{ N}}$$

For origin of coordinates x'-y'-z at C, $\Sigma M_{y'} = 0$, since $I_{x'z} = 0$

Thus $0.35B - 0.15A = 0$, $A = \dfrac{0.35}{0.15}(247) = \underline{576 \text{ N}}$

7/79

$\Sigma M_{z_1} = I_{z_1}\alpha$ where I_{z_1} is given by Eq. B/10 with $l = \cos\theta$, $m = 0$, $n = \sin\theta$

$$I_{xy} = I_{xz} = I_{yz} = 0$$

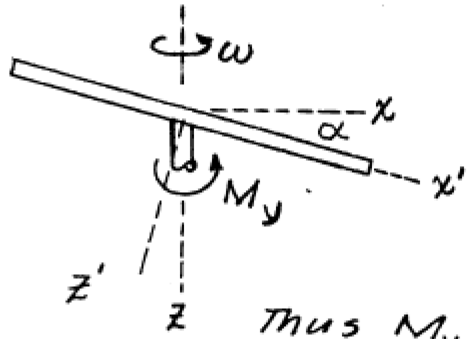
Thus $I_{z_1} = I_{xx}l^2 + I_{yy}m^2 + I_{zz}n^2 + 0$

$$= I_0\cos^2\theta + 0 + I\sin^2\theta$$

so $M = (I_0\cos^2\theta + I\sin^2\theta)\alpha$

$$\alpha = \underline{\frac{M}{I_0\cos^2\theta + I\sin^2\theta}}$$

7/87 $\Sigma M_y = -I_{xz}\,\omega_z^2$; $I_{xz} = \int (x'\cos\alpha)(x'\sin\alpha)\,dm$



$$= \frac{\sin 2\alpha}{2} I_{yy}$$

where $I_{yy} = \frac{1}{4} mr^2$

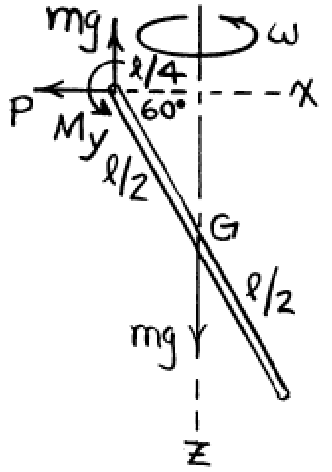
so $I_{xz} = \frac{1}{8} mr^2 \sin 2\alpha$

Thus $\underline{M}_y = (-\frac{1}{8} mr^2 \sin 2\alpha)\omega^2 \underline{j}$

But moment on shaft is

$$\underline{M} = (\tfrac{1}{8} mr^2\omega^2 \sin 2\alpha)\underline{j}$$

7/91 $\Sigma F_x = m\bar{a}_x$: $P = 0$



$$\Sigma M_y = -I_{xz}\,\omega_z^2$$

$$I_{xz} = \int xz\,dm = \int_{-\ell/4}^{\ell/4} x\sqrt{3}\left(\frac{\ell}{4}+x\right)\rho\,dx$$

where ρ = mass/(x-comp. of length)

$$I_{xz} = \frac{\sqrt{3}}{48} m\ell^2, \text{ where } m = \frac{\rho\ell}{2}$$

So $M_y - mg\frac{\ell}{4} = -\frac{\sqrt{3}}{48} m\ell^2\omega^2$

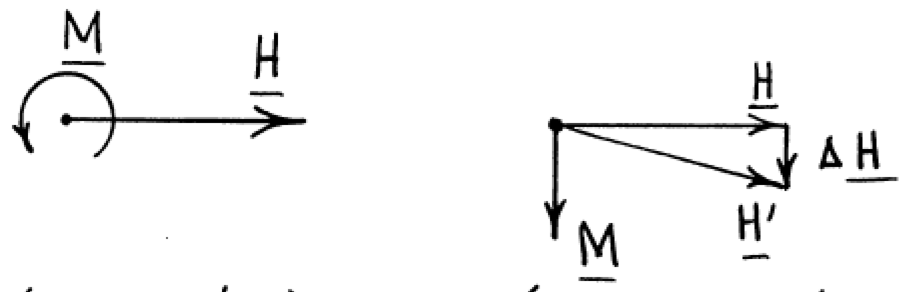
& for $M_y = 0$, $\omega = 2\sqrt{\frac{\sqrt{3}\,g}{\ell}}$

7/95 $\underline{M} = I\underline{\Omega} \times \underline{p}$: $-M\underline{i} = I\underline{\Omega} \times p\underline{j}$

$\underline{\Omega}$ is in $+\underline{k}$ direction

So precession is CCW when viewed from above.

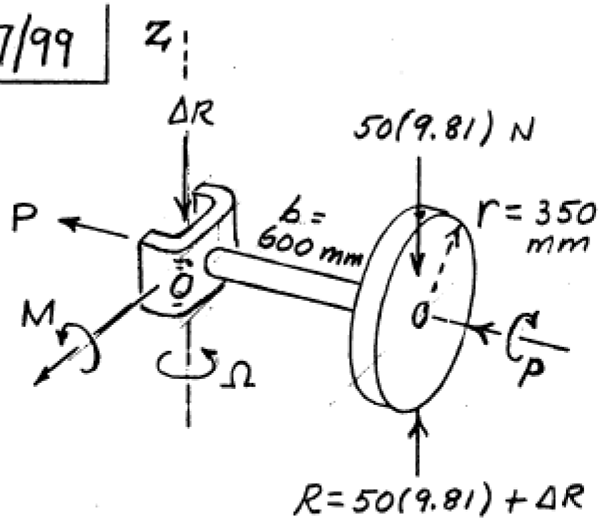
7/96



(Side view) (Overhead view)

$\underline{M}$ is the moment exerted on the handle by the student; $\underline{H}$ is the wheel angular momentum. From $\underline{M} = \dot{\underline{H}} \cong \frac{\Delta \underline{H}}{\Delta t}$, we see that $\Delta\underline{H}$ is in the same direction as $\underline{M}$. $\underline{H}'$ is the new angular momentum. The student will sense a tendency of the wheel to <u>rotate to her right</u>.

7/99



$$\Omega = \frac{48 \times 2\pi}{60} = 5.03 \text{ rad/s}$$

$$I = \frac{1}{2} m r^2 = \frac{1}{2}(50)(0.350)^2 = 3.06 \text{ kg}\cdot\text{m}^2$$

$$p = \frac{v}{r} = \frac{b\Omega}{r} = \frac{600}{350} 5.03 = 8.62 \text{ rad/s}$$

$$M = I\Omega p = 3.06(5.03)(8.62) = 132.6 \text{ N}\cdot\text{m}$$

$$M = \Delta R(b), \quad \Delta R = \frac{132.6}{0.600} = 221 \text{ N}$$

Thus $R = 50(9.81) + 221 = \underline{712 \text{ N}}$

7/103 $\Omega = \frac{10}{180}\pi = 0.1745\ \frac{rad}{sec}$

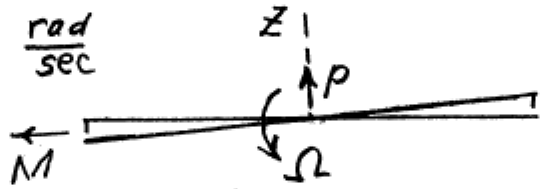
$p = \frac{500}{60}2\pi = 52.4\ rad/sec$

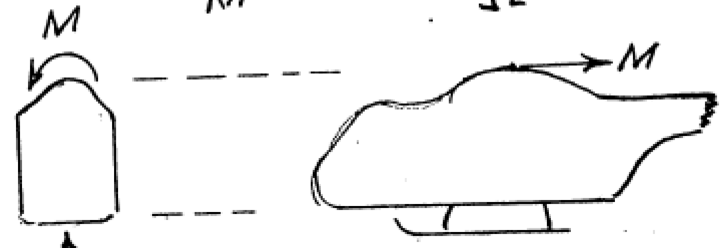
$I = \frac{140}{32.2}10^2 = 435$ lb-ft-sec^2

$M = I\Omega p$

$= 435(0.1745)52.4$

$= \underline{3970 \text{ lb-ft}}$





As viewed by passenger looking forward

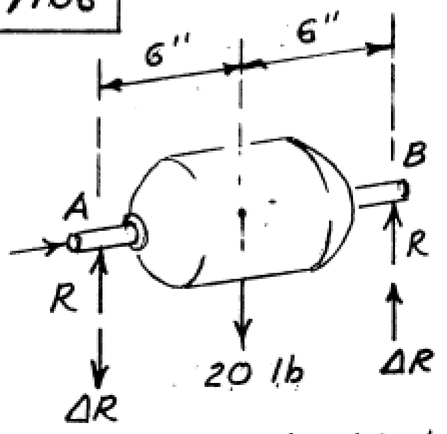
Conclusion: <u>CCW deflection</u>

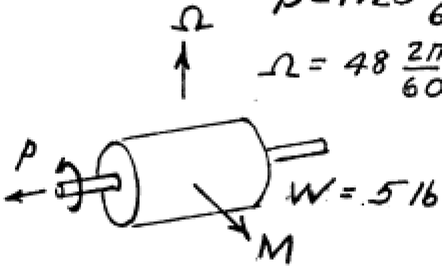
7/107 From Eq. 7/30 with θ small so that $\cos\theta \approx 1$, the precessional rate is

$$\dot\psi = \frac{Ip}{I_0 - I} = \frac{p}{(I_0/I) - 1} = \frac{3}{\frac{1}{2} - 1} = \underline{-6 \text{ rev/min}}$$

Where the minus sign indicates <u>retrograde precession</u>

7/108





$p = 1725\frac{2\pi}{60} = 180.6\ \frac{rad}{sec}$

$\Omega = 48\frac{2\pi}{60} = 5.03\ \frac{rad}{sec}$

static reactions

$R = \frac{1}{2}20 = 10$ lb

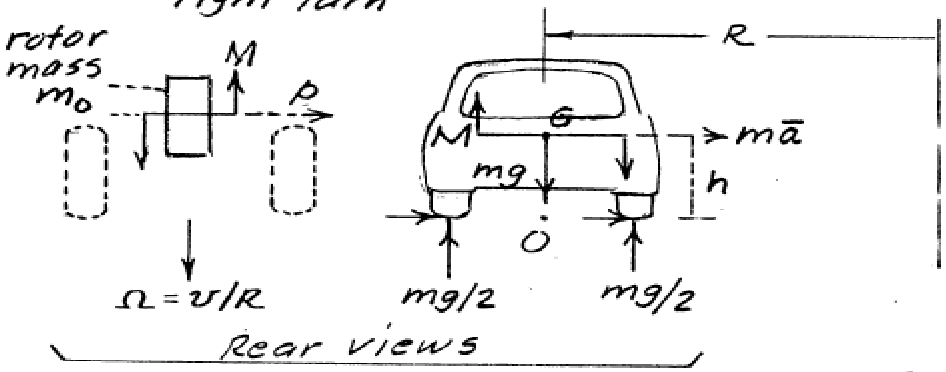
$M = I\Omega p$; $2(\Delta R)(6/12) = \frac{5}{32.2}\left(\frac{1.5}{12}\right)^2(5.03)(180.6)$

$\Delta R = 2.20$ lb

$R_A = 10 - 2.20 = \underline{7.80 \text{ lb}}$

$R_B = 10 + 2.20 = \underline{12.20 \text{ lb}}$

7/113 Assume right turn

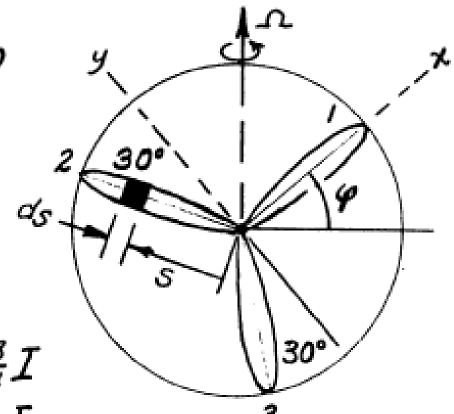


$m\bar{a} = mv^2/R$; $\Sigma M_O = m\bar{a}h$ so $M = mv^2h/R$

$M = I\Omega p$; $\dfrac{mv^2h}{R} = m_o k^2 \dfrac{v}{R} p$

$$\underline{p = \frac{m}{m_o}\frac{vh}{k^2}}$$ opposite direction to rotation of wheels

7/122 By symmetry $I_{xy} = I_{xz} = I_{yz} = 0$ for whole propeller.



Let $\rho = f(s)$ be mass per unit length so $dm = \rho\,ds = f(s)\,ds$

Blade 1: $I_{xx} = 0$, $I_{yy} = \int f(s)\,ds \times s^2 = I$

Blades 2 & 3: $I_{xx} = \int f(s)\,ds\,(s\cos 30°)^2 = \frac{3}{4}I$

$I_{yy} = \int f(s)\,ds\,(s\sin 30°)^2 = \frac{1}{4}I$

Thus for the three blades

$I_{xx} = 0 + 2(\frac{3}{4}I) = \frac{3}{2}I$

$I_{yy} = I + 2(\frac{1}{4}I) = \frac{3}{2}I$

$I_{zz} = 3I$

$\omega_x = \Omega\sin\varphi$, $\dot{\omega}_x = \Omega p\cos\varphi$

$\omega_y = \Omega\cos\varphi$, $\dot{\omega}_y = -\Omega p\sin\varphi$

$\omega_z = \dot{\varphi} = p$, $\dot{\omega}_z = 0$

From Eq. 7/21 $M_x = I_{xx}\dot{\omega}_x - (I_{yy} - I_{zz})\omega_y\omega_z$

$= \frac{3}{2}I\Omega p\cos\varphi - (\frac{3}{2}I - 3I)\Omega p\cos\varphi = \underline{3I\Omega p\cos\varphi}$

$M_y = I_{yy}\dot{\omega}_y - (I_{zz} - I_{xx})\omega_z\omega_x$

$= \frac{3}{2}I(-\Omega p\sin\theta) - (3I - \frac{3}{2}I)\Omega p\sin\varphi = \underline{-3I\Omega p\sin\varphi}$

acting on hub; reaction on shaft has opposite signs.

The magnitude of M is $M = \sqrt{M_x^2 + M_y^2} = \underline{3I\Omega p}$