INSTRUCTOR'S MANUAL

To Accompany

ENGINEERING MECHANICS - DYNAMICS

Volume 2

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USE OF THE INSTRUCTOR'S MANUAL

The problem solution portion of this manual has been prepared for the instructor who wishes to occasionally refer to the authors' method of solution or who wishes to check the answer of his (her) solution with the result obtained by the authors. In the interest of space and the associated cost of educational materials, the solutions are very concise. Because the problem solution material is not intended for posting of solutions or classroom presentation, the authors request that it not be used for these purposes.

In the transparency master section there are approximately 65 solved problems selected to illustrate typical applications. These problems are different from and in addition to those in the textbook. Instructors who have adopted the textbook are granted permission to reproduce these masters for classroom use.

8/1

$$k = \frac{W}{\delta_{st}} = \frac{3(9.81)}{0.04} = \underline{736 \text{ N/m}}$$

$$k = 736 \frac{\text{N}}{\text{m}} \left(\frac{1 \text{ lb/in.}}{175.13 \text{ N/m}} \right) = \underline{4.20 \text{ lb/in.}}$$

$$k = 4.20 \frac{\text{lb}}{\text{in.}} \left(\frac{12 \text{ in.}}{\text{ft}} \right) = \underline{50.4 \text{ lb/ft}}$$

8/2

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{24(12)}{2}} = \underline{12 \text{ rad/sec}}$$

$$f_n = \left(12\, \frac{\text{rad}}{\text{sec}}\right)\left(\frac{1 \text{ cycle}}{2\pi \text{ rad}}\right) = \underline{\frac{6}{\pi} \text{ Hz}}$$

8/3

$$x = x_0 \cos \omega_n t + \frac{\dot{x}_0 \xrightarrow{} 0}{\omega_n} \sin \omega_n t$$

$$= \underline{2 \cos 12t \text{ in.}}$$

8/4

$$x = C \sin(\omega_n t + \psi)$$

$$C = \left[x_0^2 + \left(\frac{\dot{x}_0}{\omega_n}\right)^2\right]^{1/2} = \left[(-2)^2 + \left(\frac{7}{12}\right)^2\right]^{1/2} = \underline{2.08 \text{ in.}}$$

$$\psi = \tan^{-1}\left(\frac{x_0 \omega_n}{\dot{x}_0}\right) = \tan^{-1}\left(\frac{(-2)(12)}{7}\right) = -1.287 \text{ rad}$$

$$\therefore \quad \underline{x = 2.08 \sin(12t - 1.287) \text{ in.}}$$

8/5

$$\delta_{st} = \frac{W}{k} = \frac{4(9.81)}{144} = \underline{0.273 \text{ m}}$$

$$\omega_n = \sqrt{k/m} = \sqrt{144/4} = 6 \text{ rad/s}$$

$$\tau = \frac{2\pi}{\omega_n} = \frac{2\pi}{6} = \underline{\frac{\pi}{3} \text{ s}}$$

$$\dot{x}_{max} = C\omega_n = 0.1(6) = \underline{0.6 \text{ m/s}} = v_{max}$$

8/6

$$\omega_n = \sqrt{k/m} = \sqrt{144/4} = 6 \text{ rad/s}$$

$$y = y_0 \cos \omega_n t + \frac{\dot{y}_0}{\omega_n} \sin \omega_n t$$

$$= 0.1 \cos 6t \quad (\text{m})$$

$$v = \dot{y} = -0.6 \sin 6t \quad (\text{m/s})$$

$$a = \ddot{y} = -3.6 \cos 6t \quad (\text{m/s}^2)$$

When $t = 3$ s:

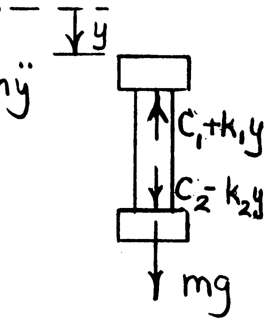
$$y = 0.1 \cos(6 \cdot 3) = \underline{0.0660 \text{ m}} \quad (\text{below eq.})$$

$$v = -0.6 \sin(6 \cdot 3) = \underline{0.451 \text{ m/s}} \quad (\text{down})$$

$$a_{max} = \underline{3.6 \text{ m/s}^2}$$

8/7

Equil. pos.



$\Sigma F_y = m\ddot{y}$: $C_2 - k_2y + mg - C_1 - k_1y = m\ddot{y}$

At equilibrium, $C_2 + mg - C_1 = 0$

So $-(k_1+k_2)y = m\ddot{y}$

$$\ddot{y} + \frac{k_1+k_2}{m}y = 0$$

$$\omega_n = \sqrt{\frac{k_1+k_2}{m}} = \sqrt{\frac{3600+1800}{2.5}} = 46.5 \text{ rad/s}$$

$$f_n = \frac{\omega_n}{2\pi} = \frac{46.5}{2\pi} = \underline{7.40 \text{ Hz}}$$

8/8

$$\omega_n = \sqrt{\frac{2k}{m}} = \sqrt{\frac{2(180{,}000)}{100}} = 60 \text{ rad/s}$$

$$x = x_0 \cos \omega_n t + \frac{\dot{x}_0}{\omega_n} \sin \omega_n t$$

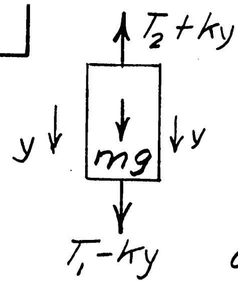
$$= 0 + \frac{0.5}{60} \sin 60t = 8.33(10^{-3}) \sin 60t$$

$$\dot{x} = 60(8.33)(10^{-3}) \cos 60t = 0.5 \cos 60t$$

$$\ddot{x} = -60(0.5) \sin 60t = -30 \sin 60t$$

$$a_{max} = 30 \text{ m/s}^2$$

8/9



For equil. position,

$$T_1 + mg - T_2 = 0 \qquad \text{-------}(1)$$

$\Sigma F_y = ma_y$;

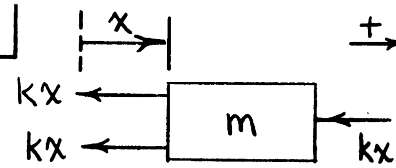
$$T_1 - ky + mg - T_2 - ky = m\ddot{y}$$

Combine with (1) & get

$$0 - 2ky = m\ddot{y} \quad \text{or} \quad \ddot{y} + \frac{2k}{m}y = 0$$

$$f_n = \frac{\omega_n}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{2k}{m}} = \frac{1}{2\pi}\sqrt{\frac{2(3000)}{10}} = \underline{3.90 \text{ Hz}}$$

8/10



$\xrightarrow{+} \Sigma F_x = m\ddot{x}$:

$-3kx = m\ddot{x}$

$\ddot{x} + \frac{3k}{m} x = 0$

$$\omega_n = \sqrt{\frac{3k}{m}} = \sqrt{\frac{3(0.5)(12)}{4/16 / 32.2}} = 48.1 \text{ rad/sec}$$

$$\tau = \frac{2\pi}{\omega_n} = \frac{2\pi}{48.1} = \underline{0.1305 \text{ sec}}$$

$$x = x_0 \cos\omega_n t + \frac{\dot{x}_0}{\omega_n} \sin\omega_n t$$
$$= 0.1 \cos(48.1 \cdot 2) + \frac{0.5}{48.1} \sin(48.1 \cdot 2)$$
$$= \underline{-0.0371 \text{ in.}}$$

$$\dot{x} = -x_0\omega_n \sin\omega_n t + \dot{x}_0 \cos\omega_n t$$
$$= -0.1(48.1)\sin(48.1 \cdot 2) + 0.5 \cos(48.1 \cdot 2)$$
$$= \underline{-4.5 \text{ in./sec}}$$

8/11

$$\omega_n = \sqrt{\frac{4k}{m}} = \sqrt{\frac{4(17{,}500)}{1000}} = 8.37 \ \frac{\text{rad}}{\text{sec}}$$

$$f_n = \frac{\omega_n}{2\pi} = \frac{8.37}{2\pi} = \underline{1.332 \text{ Hz}}$$

We have assumed the unsprung mass (wheels, axles, etc.) to be a small fraction of the total car mass.

8/12 An astronaut can attach himself or herself to a suitable spring anchored to the orbiter. Upon excitation of simple harmonic motion, one can measure the period, which is indicative of the spring constant (known) and the astronaut mass (the quantity to be measured).

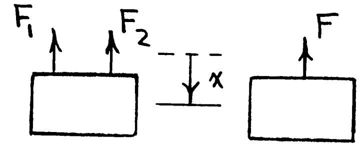
8/13 (a) From Eq. 8/3 frequency $f_n = \frac{\omega_n}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{3k}{m}}$

$3 = \frac{1}{2\pi}\sqrt{\frac{3k}{4000}}$, $k = 474 \times 10^3$ N/m or $\underline{k = 474 \text{ kN/m}}$

(b) For $m = 4000 + 40\,000 = 44\,000$ kg,

$$f_n = \frac{1}{2\pi}\sqrt{\frac{3(474 \times 10^3)}{44 \times 10^3}} = \underline{0.905 \text{ Hz}}$$

8/14



(a) $F = F_1 + F_2$

$$kx = k_1 x + k_2 x$$

$$\underline{k = k_1 + k_2}$$

(b) $F = F_1 = F_2$

$$x_1 = \frac{F_1}{k_1}, \quad x_2 = \frac{F_2}{k_2}, \quad x = \frac{F}{k}$$

From $x = x_1 + x_2$, we have $\dfrac{F}{k} = \dfrac{F_1}{k_1} + \dfrac{F_2}{k_2}$

or $\dfrac{F}{k} = \dfrac{F}{k_1} + \dfrac{F}{k_2}$. Thus $\underline{\dfrac{1}{k} = \dfrac{1}{k_1} + \dfrac{1}{k_2}}$

8/15

$$f_n = \frac{\omega_n}{2\pi} = \frac{1}{2\pi}\sqrt{k/m_{Tot}}$$

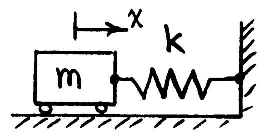
$$\frac{1}{0.75} = \frac{1}{2\pi}\sqrt{600/(m+6)}, \quad \underline{m = 2.55 \text{ kg}}$$

$$\omega_n = \sqrt{k/m_{Tot}} = \sqrt{600/(6+2.55)} = 8.38 \text{ rad/s}$$

$$a_{max} = \omega_n^2 C = 8.38^2(0.050) = 3.51 \text{ m/s}^2$$

$$a_{max} = \mu_s g: \quad 3.51 = \mu_s (9.81), \quad \underline{\mu_s = 0.358}$$

8/16 Equivalent system :



$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{(3000)(12)}{2500/32.2}} = 21.5 \text{ rad/sec}$$

$$x = \cancelto{0}{x_o} \cos \omega_n t + \frac{\dot{x}_o}{\omega_n} \sin \omega_n t$$

$$= \frac{5(5280/3600)}{21.5} \sin 21.5t = 0.341 \sin 21.5t$$

$x_{max} = 0.341$ ft or $\underline{4.09 \text{ in.}}$

$$v = (0.341)(21.5) \cos 21.5t = \underline{7.33 \cos 21.5t}$$

$\underline{(\text{in ft/sec})}$

or $\underline{v = 88.0 \cos 21.5t \text{ in./sec}}$

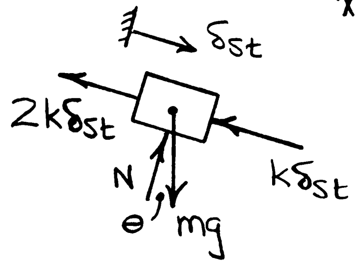
8/17

$$k = \frac{W}{\delta_{st}} = \frac{120}{0.9} = 133.3 \text{ lb/in.}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{(133.3)(12)}{120/32.2}} = 20.7 \text{ rad/sec}$$

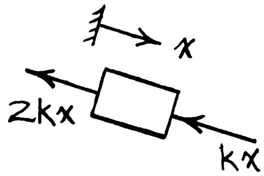
$$f_n = \frac{\omega_n}{2\pi} = \underline{3.30 \text{ Hz}}$$

8/18.



$$\Sigma F_x = 0: \; -3k\delta_{st} + mg\sin\theta = 0$$

$$\delta_{st} = \frac{mg\sin\theta}{3k}$$



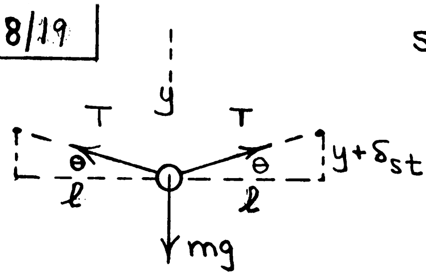
$$\Sigma F_x = m\ddot{x}: \; -3kx = m\ddot{x}$$

$$\ddot{x} + \frac{3k}{m}x = 0$$

$$\tau = \frac{2\pi}{\omega_n} = \frac{2\pi}{\sqrt{\frac{3k}{m}}} = 2\pi\sqrt{\frac{m}{3k}}$$

8/19

$$\sin\theta \cong \theta \cong \frac{y + \delta_{st}}{\ell}$$



δ_{st} = static deflection
y = dynamic deflection

$$\Sigma F_y = m\ddot{y} : \quad -2T\sin\theta + mg = m\ddot{y}$$

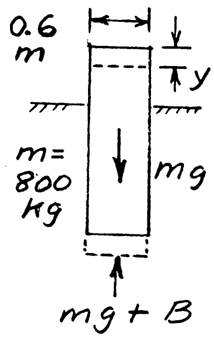
$$-2T\left(\frac{y+\delta_{st}}{\ell}\right) + mg = m\ddot{y}$$

$$-2T\frac{y}{\ell} \cancel{- 2T\frac{\delta_{st}}{\ell}} \cancel{+ mg} = m\ddot{y}$$

$$\ddot{y} + \left(\frac{2T}{m\ell}\right)y = 0 \,, \quad \underline{\omega_n = \sqrt{\frac{2T}{m\ell}}}$$

Although done above, the inclusion of the forces $+mg$ and $-2T\frac{\delta_{st}}{\ell}$, which sum to zero, is not necessary.

8/20 B = added buoyancy due to deflection y below equil. position



$$B = \text{Vol.} \times \text{density} \times g$$

$$= \frac{\pi d^2}{4} y \rho g = \frac{\pi (0.6^2)}{4} y (1.03 \times 10^3)(9.81)$$

$$= 2857 y \quad \text{N}$$

$$\Sigma F_y = m a_y; \quad mg - (mg + B) = m\ddot{y}$$

$$\ddot{y} + \frac{2857}{800} y = 0$$

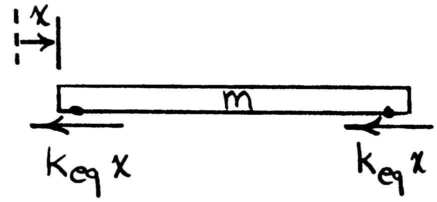
$$\omega_n = \sqrt{\frac{2857}{800}} = 1.890 \frac{\text{rad}}{\text{sec}}, \quad f_n = \frac{\omega_n}{2\pi} = \frac{1.890}{2\pi}$$

$$= \underline{0.301 \text{ Hz}}$$

8/21 For one upright $P = \left(\frac{12EI}{L^3}\right)\delta = k_{eq}\delta$

So $k_{eq} = \frac{12EI}{L^3}$.

FBD of top mass:
(dynamic forces only)



$$\Sigma F_x = m\ddot{x} : \quad -2k_{eq}x = m\ddot{x}$$

$$\ddot{x} + \frac{2k_{eq}}{m}x = 0 \quad \text{or} \quad \ddot{x} + \frac{24EI}{mL^3}x = 0$$

$$\omega_n = \sqrt{\frac{24EI}{mL^3}} = \underline{2\sqrt{\frac{6EI}{mL^3}}}$$

8/22 The velocity of the putty after dropping 2 m is

$$v = \sqrt{2gh} = \sqrt{2(9.81)(2)} = 6.26 \text{ m/s}$$

The additional static deflection due to the 3-kg mass is

$$\delta_{st} = \frac{W}{4k} = \frac{3(9.81)}{4(800)} = 9.20(10^{-3}) \text{ m}$$

Velocity after impact : $\dot{x}_o = \dfrac{3(6.26)}{31} = 0.606$ m/s

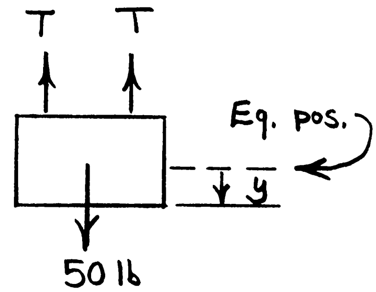
Natural frequency of motion : $\omega_n = \sqrt{\dfrac{4k}{m}} = 10.16 \dfrac{\text{rad}}{\text{s}}$

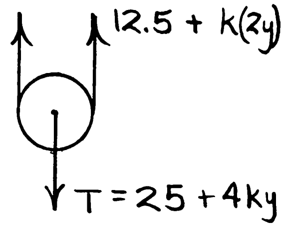
$$x = \delta_{st} + x_o \cos\omega_n t + \frac{\dot{x}_o}{\omega_n} \sin \omega_n t$$

$$= 9.20(10^{-3}) - 9.20(10^{-3}) \cos 10.16t + 59.7(10^{-3}) \sin 10.16t$$

$$= 9.20(10^{-3})(1 - \cos 10.16t) + 59.7(10^{-3}) \sin 10.16t \text{ m}$$

8/23





$$T = 25 + 4ky = 25 + 4(6 \times 12)y = 25 + 288y$$

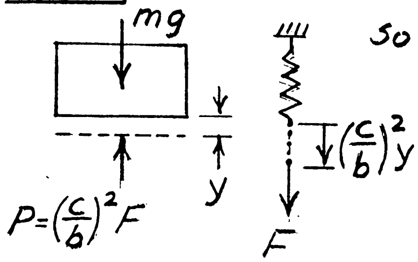
$$\Sigma F_y = m\ddot{y} : \quad 50 - 2(25 + 288y) = \frac{50}{32.2}\ddot{y}$$

$$\ddot{y} + 371y = 0$$

$$\omega_n = \sqrt{371} = 19.26 \text{ rad/sec}$$

$$f_n = \frac{19.26}{2\pi} = \underline{3.07 \text{ Hz}}$$

8/24 Equil. value of spring force is $\left(\frac{b}{c}\right)^2 mg$



so $F = \left(\frac{b}{c}\right)^2 mg + k\left(\frac{c}{b}\right)^2 y$

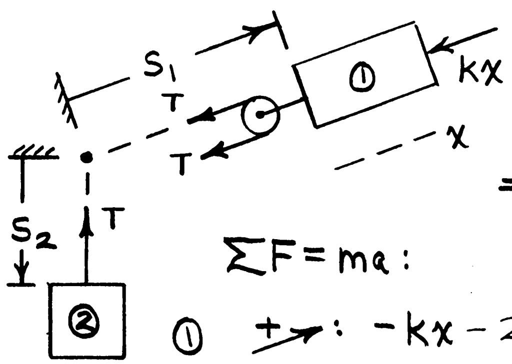
& $P = \left(\frac{c}{b}\right)^2\left[\left(\frac{b}{c}\right)^2 mg + k\left(\frac{c}{b}\right)^2 y\right]$

$= mg + k\left(\frac{c}{b}\right)^4 y$

$\Sigma F = ma_y;\quad mg - \left(mg + k\left[\frac{c}{b}\right]^4 y\right) = m\ddot{y}$

$\underline{\ddot{y} + \frac{k}{m}\left(\frac{c}{b}\right)^4 y = 0}$, $\tau = \frac{2\pi}{\omega_n} = \underline{2\pi\left(\frac{b}{c}\right)^2\sqrt{\frac{m}{k}}}$

8/25 Dynamic forces only, with s_1 and s_2 being inertial displacements and x is the displacement of mass 1 from equil.



$2s_1 + s_2 + \text{Constant} = L$

$\Rightarrow 2a_1 + a_2 = 0 \qquad (1)$

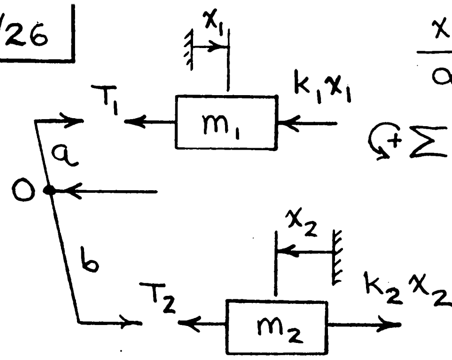
$\Sigma F = ma$:

① $+\nearrow$: $-kx - 2T = ma_1 = m\ddot{x} \qquad (2)$

② $+\downarrow$: $-T = ma_2 \qquad (3)$

Elimination of T and a_2 from Eqs. (1)-(3) gives $\ddot{x} + \dfrac{k}{5m}x = 0 \Rightarrow \underline{\omega_n = \sqrt{\dfrac{k}{5m}}}$

8/26



$\frac{x_1}{a} = \frac{x_2}{b}$, $x_2 = \frac{b}{a} x_1$

$\circlearrowleft + \Sigma M_0 = 0: \; -T_1 a + T_2 b = 0$

$T_2 = \frac{a}{b} T_1$

$\Sigma F_x = m\ddot{x} \; : \; -T_1 - k_1 x_1 = m_1 \ddot{x}_1$ (a)

$T_2 - k_2 x_2 = m_2 \ddot{x}_2$

Second eq. : $\frac{a}{b} T_1 - k_2 \left(\frac{b}{a} x_1\right) = m_2 \frac{b}{a} \ddot{x}_1$ (b)

Solve (b) for T_1 and substitute into (a):

$\left[m_1 + \frac{b^2}{a^2} m_2\right] \ddot{x}_1 + \left[k_1 + \frac{b^2}{a^2} k_2\right] x_1 = 0$

For $k_1 = k_2 = k$, $m_1 = m_2 = m$: $m\ddot{x}_1 + kx_1 = 0$, $\omega_n' = \sqrt{k/m}$

$\underline{8/27}$ $\omega_n = \sqrt{k/m} = \sqrt{392/2} = 14 \text{ rad/s}$

$$\zeta = \frac{c}{2m\omega_n} = \frac{42}{2(2)(14)} = \underline{0.75}$$

8/28

$$\tau_d = \frac{2\pi}{\omega_d} = \frac{2\pi}{\omega_n\sqrt{1-\zeta^2}}$$

$$= \frac{2\pi}{\omega_n\sqrt{1-\left(\frac{c}{2m\omega_n}\right)^2}} = \frac{2\pi}{\sqrt{k/m}\sqrt{1-\frac{c^2}{4m^2}\frac{m}{k}}}$$

$$= \frac{2\pi}{\sqrt{\frac{k}{m}-\frac{c^2}{4m^2}}}$$

Substitute numbers and solve for c to obtain $\underline{c = 38.0 \text{ N}\cdot\text{s/m}.}$

$\underline{8/29}$

$$\omega_d = \omega_n\sqrt{1-\zeta^2}$$

$$0.9\,\omega_n = \omega_n\sqrt{1-\zeta^2}\quad , \quad \underline{\zeta = 0.436}$$

8/30

$$\tau_d = 1.25\,\tau$$

$$\frac{2\pi}{\omega_n\sqrt{1-\zeta^2}} = 1.25\frac{2\pi}{\omega_n}\ , \quad \underline{\zeta = 0.6}$$

8/31

$$\omega_n = \sqrt{k/m} = \sqrt{\frac{200(12)}{80/32.2}} = 31.1 \text{ rad/sec}$$

$$\zeta \xrightarrow{} 1 = \frac{c}{2m\omega_n}, \quad c = 2m\omega_n = 2\left(\frac{80}{32.2}\right)(31.1)$$

$$= \underline{154.4 \text{ lb-sec/ft}}$$

8/32 $\frac{x_1}{x_2} = 4$, so log decrement δ is

$$\delta = \ln \frac{x_1}{x_2} = \ln 4 = 1.386$$

Viscous damping factor $\zeta = \frac{\delta}{\sqrt{4\pi^2 + \delta^2}}$

$$= \frac{1.386}{\sqrt{4\pi^2 + 1.386^2}} = \underline{0.215}$$

Natural frequency $\omega_n = \frac{2\pi}{\tau_n} = \frac{2\pi}{0.75} = 8.38 \frac{\text{rad}}{\text{s}}$

From $\omega_n = \sqrt{\frac{k}{m}}$, $k = m\omega_n^2 = 2.5(8.38)^2 = \underline{175.5 \frac{\text{N}}{\text{m}}}$

Damping ratio $\zeta = \frac{c}{2m\omega_n}$

So $c = 2m\omega_n\zeta = 2(2.5)(8.38)(0.215) = \underline{9.02 \frac{\text{N}\cdot\text{s}}{\text{m}}}$

8/33

$$\frac{x_0}{x_N} = \frac{Ce^{-\zeta\omega_n t_0}}{Ce^{-\zeta\omega_n(t_0+N\tau_d)}} = e^{\zeta\omega_n N\tau_d}$$

Define $\underline{\delta_N = \ln \frac{x_0}{x_N}} = \zeta\omega_n N\tau_d$, $\tau_d = \dfrac{2\pi}{\omega_n\sqrt{1-\zeta^2}}$

so $\delta_N = \zeta\cancel{\omega_n} N \dfrac{2\pi}{\cancel{\omega_n}\sqrt{1-\zeta^2}} = \dfrac{2\pi N\zeta}{\sqrt{1-\zeta^2}}$

Solve for ζ and get $\underline{\zeta = \dfrac{\delta_N}{\sqrt{(2\pi N)^2+\delta_N^2}}}$

$\underline{8/34}$ $\omega_n = \sqrt{392/2} = 14$ rad/s

$\zeta = \frac{c}{2m\omega_n} = \frac{42}{2(2)(14)} = 0.75$ (underdamped)

$\omega_d = \omega_n\sqrt{1-\zeta^2} = 14\sqrt{1-0.75^2} = 9.26$ rad/s

$$x = (A_3 \cos \omega_d t + A_4 \sin \omega_d t)\, e^{-\zeta\omega_n t}$$
$$= (A_3 \cos 9.26t + A_4 \sin 9.26t)\, e^{-10.5t}$$

Initial conditions: $x_0 = A_3$

$$\dot{x} = -10.5(A_3 \cos 9.26t + A_4 \sin 9.26t)\, e^{-10.5t}$$
$$+ 9.26(-A_3 \sin 9.26t + A_4 \cos 9.26t)\, e^{-10.5t}$$

$0 = -10.5 A_3 + 9.26 A_4$, $A_4 = 1.134 A_3 = 1.134 x_0$

So $\underline{x = x_0 (\cos 9.26t + 1.134 \sin 9.26t)\, e^{-10.5t}}$

8/35 | Eq. 8/11 : $x = (A_3 \cos\omega_d t + A_4 \sin\omega_d t)e^{-\zeta\omega_n t}$

At time $t = 0$: $x_0 = A_3$

$$\dot{x} = -\zeta\omega_n (A_3 \cos\omega_d t + A_4 \sin\omega_d t)e^{-\zeta\omega_n t}$$
$$+ \omega_d(-A_3 \sin\omega_d t + A_4 \cos\omega_d t)e^{-\zeta\omega_n t}$$

At time $t = 0$: $\dot{x}_0 = -\zeta\omega_n A_3 + \omega_d A_4 = 0$

$$A_4 = x_0\omega_n\zeta / \omega_d$$

Thus $x = x_0\left[\cos\omega_d t + \frac{\zeta\omega_n}{\omega_d}\sin\omega_d t\right]e^{-\zeta\omega_n t}$

At time $t = \tau_d = 2\pi/(\omega_n\sqrt{1-\zeta^2})$

$$x_{\tau_d} = x_0[1+0]e^{-\zeta\omega_n\left(\frac{2\pi}{\omega_n\sqrt{1-\zeta^2}}\right)}$$

$$\frac{x_0}{2} = x_0 e^{-2\pi\zeta/\sqrt{1-\zeta^2}}$$

$$\ln\left(\tfrac{1}{2}\right) = -\frac{2\pi\zeta}{\sqrt{1-\zeta^2}}, \quad \underline{\zeta = 0.1097}$$

8/36 | The 20° ramp is irrelevant.

The equation of motion is derived to be

$$\ddot{x} + \frac{2c}{m}\dot{x} + \frac{k}{m}x = 0.$$

$$\omega_n = \sqrt{k/m} = \sqrt{\frac{15(12)}{40/32.2}} = 12.04 \text{ rad/sec}$$

$$2\zeta\omega_n = \frac{2c}{m}, \qquad c = \zeta m \omega_n$$

(a) $\zeta = 0.5$: $\quad c = (0.5)\left(\frac{40}{32.2}\right)(12.04) = \underline{7.48 \ \frac{\text{lb-sec}}{\text{ft}}}$

(b) $\zeta = 1.5$: $\quad c = (1.5)\left(\frac{40}{32.2}\right)(12.04) = \underline{22.4 \ \frac{\text{lb-sec}}{\text{ft}}}$

8/37 Combined $c = 2m\omega_n\zeta$ where $\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{3\times474\times10^3}{4000}}$
$= 18.85$ rad/s

and $\zeta = \dfrac{\delta}{\sqrt{(2\pi)^2+\delta^2}}$

But the logarithmic decrement $\delta = \ln\left(\frac{x_1}{x_2}\right) = \ln 4 = 1.386$

so the viscous damping factor $\zeta = 1.386/\sqrt{(2\pi)^2+1.386^2}$
$= 0.215$

Thus combined $c = 2(4000)(18.85)(0.215) = 32.5\times10^3$

& for each damper $c = 32.5\times10^3/2 = \underline{16.24\times10^3 \text{ N·s/m}}$

8/38 $\omega_n = \sqrt{k/m} = \sqrt{108/3} = 6$ rad/s

$$\zeta = \frac{c}{2m\omega_n} = \frac{18}{2(3)(6)} = 0.5$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2} = 6\sqrt{1-0.5^2} = 5.196 \text{ rad/s}$$

$$x = (A_1 \cos\omega_d t + A_2 \sin\omega_d t)\, e^{-\zeta\omega_n t}$$

$$x(t=0) = A_1 = x_0$$

$$\dot{x} = -\zeta\omega_n (A_1 \cos\omega_d t + A_2 \sin\omega_d t)\, e^{-\zeta\omega_n t}$$
$$+ \omega_d(-A_1 \sin\omega_d t + A_2 \cos\omega_d t)\, e^{-\zeta\omega_n t}$$

$$\dot{x}(t=0) = -\zeta\omega_n A_1 + A_2\omega_d = 0$$

$$A_2 = \zeta\omega_n A_1/\omega_d = 0.5(6)x_0/5.196 = 0.577\, x_0$$

So $x = x_0[\cos 5.196t + 0.577 \sin 5.196t]\, e^{-3t}$

and $x(t = \frac{\tau_d}{2}) = x(t = 0.605) = \underline{-0.1630\, x_0}$

8/39 $\quad x = (A_1 + A_2 t)e^{-\omega_n t}$

$$x(t=0) = A_1 = x_0$$

$$\dot{x} = A_2 e^{-\omega_n t} - \omega_n (A_1 + A_2 t)e^{-\omega_n t}$$

$$\dot{x}(t=0) = A_2 - \omega_n A_1 = \dot{x}_0$$

$$A_2 = \dot{x}_0 + \omega_n x_0$$

So $\quad x = [x_0 + (\dot{x}_0 + \omega_n x_0)t]e^{-\omega_n t}$

For x to become negative with $x_0 > 0$,

$\dot{x}_0 + \omega_n x_0 < 0$, $\dot{x}_0 < -\omega_n x_0$ or $\underline{(\dot{x}_0)_c = -\omega_n x_0}$

8/40 $\omega_n = \sqrt{k/m} = \sqrt{\dfrac{12}{96.6/32.2}} = 2$ rad/sec

(a) $\zeta = \dfrac{c}{2m\omega_n} = \dfrac{12}{2(3)(2)} = 1$ (Critical damping)

$x = (A_1 + A_2 t)e^{-\omega_n t}$

Consideration of initial conditions yields

$x = (0.5 + t)\,e^{-2t}$, $x(t=0.5) = 0.368$ ft $\underline{(4.42 \text{ in.})}$

(b) $\zeta = \dfrac{c}{2m\omega_n} = \dfrac{18}{2(3)(2)} = 1.5$ (Overdamped)

$x = A_1 e^{\lambda_1 t} + A_2 e^{\lambda_2 t}$

Determine A_1 and A_2 in usual fashion:

$x = \left(\dfrac{\lambda_2 x_0}{\lambda_2 - \lambda_1}\right) e^{\lambda_1 t} + \left(\dfrac{\lambda_1 x_0}{\lambda_1 - \lambda_2}\right) e^{\lambda_2 t}$, where

$\lambda_{1,2} = \omega_n\left[-\zeta \pm \sqrt{\zeta^2 - 1}\right] = -0.7639, -5.236$

$x = 0.585\, e^{-0.764t} - 0.085\, e^{-5.24t}$

$x(t=0.5) = \underline{0.393 \text{ ft } (4.72 \text{ in.})}$

$\boxed{8/41.}$ $\omega_n = \sqrt{k/m} = \sqrt{\frac{150(12)}{1610/32.2}} = 6 \text{ rad/s}$

$\zeta = \frac{c}{2m\omega_n} = \frac{600}{2\frac{1610}{32.2}6} = 1$ (critically damped)

Conservation of linear momentum during firing:

x--- $0 = -\frac{10}{32.2}(800\cos 20^\circ) + \frac{1610}{32.2}\dot{x}_0$

$\dot{x}_0 = 4.67 \text{ ft/sec}$

$x = (A_1 + A_2 t)e^{-\omega_n t}$

$\dot{x} = -\omega_n(A_1 + A_2 t)e^{-\omega_n t} + A_2 e^{-\omega_n t}$

I.C.: $0 = A_1$ $\quad \dot{x}_0 = -\omega_n A_1 + A_2$

So $x = \dot{x}_0 t e^{-\omega_n t} = 4.67 t e^{-6t}$ (in ft)

$\dot{x} = -28.0 t e^{-6t} + 4.67 e^{-6t} = 0$ for x_{max}

$t = \frac{1}{6}$ sec

$x_{max} = x\big|_{t=\frac{1}{6}\text{ sec}} = 4.67\left(\frac{1}{6}\right)e^{-1} = \underline{0.286 \text{ ft}}$

8/42

$$\zeta = \frac{\delta}{\sqrt{(2\pi)^2+\delta^2}} \text{ where } \delta = \ln\left(\frac{x_1}{x_2}\right) = \ln\frac{3}{1/2} = 1.792$$

$$\zeta = \frac{1.792}{\sqrt{(2\pi)^2+1.792^2}} = \underline{0.274}$$

$c = 2m\omega_n\zeta$ where equivalent m for each absorber is

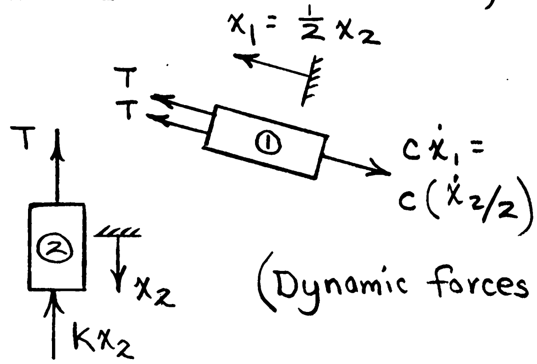
$$\frac{1}{2}\left(\frac{1}{2}\,\frac{3400}{32.2}\right) = 26.4 \text{ lb-sec}^2/\text{ft}$$

$$k = F/\delta_{st} = 100/\tfrac{3}{12} = 400 \text{ lb/ft (for both)}$$

$$\omega_n = \sqrt{k/m} = \sqrt{400/52.8} = 2.75 \text{ rad/sec}$$

so for each shock, $c = 2(26.4)(2.75)(0.274) = \underline{39.9 \text{ lb-sec/ft}}$

8/43 From constraint, $v_2 = 2v_1$, $a_2 = 2a_1$



(Dynamic forces only)

②: $\Sigma F_x = m\ddot{x}$: $-T - kx_2 = m_2\ddot{x}_2$

①: $\Sigma F_x = m\ddot{x}$: $2T - \frac{1}{2}c\dot{x}_2 = m_1\ddot{x}_1 = \frac{1}{2}m_1\ddot{x}_2$

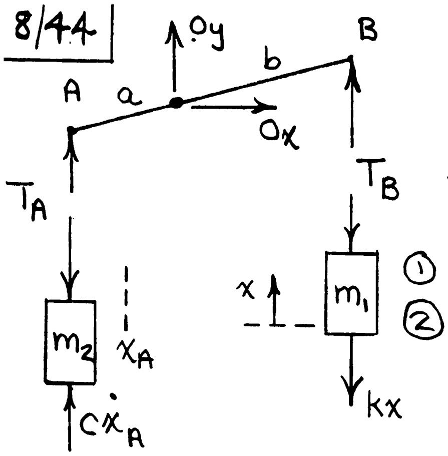
Eliminate T: $[m_1 + 4m_2]\ddot{x}_2 + c\dot{x}_2 + 4kx_2 = 0$

$$\omega_n = \sqrt{\frac{4k}{m_1 + 4m_2}} = 2\sqrt{\frac{k}{m_1 + 4m_2}}$$

$$2\zeta\omega_n = \frac{c}{m_1 + 4m_2}, \quad \zeta = \frac{c}{2(m_1 + 4m_2)} \cdot \frac{1}{2}\sqrt{\frac{m_1 + 4m_2}{k}}$$

$$= \frac{c}{4\sqrt{k(m_1 + 4m_2)}}$$

8/44



$$\frac{x}{b} = \frac{x_A}{a}, \quad \dot{x}_A = \left(\frac{a}{b}\right)\dot{x}$$

$$\ddot{x}_A = \left(\frac{a}{b}\right)\ddot{x}$$

$\Sigma F_x = m\ddot{x}$:

$$① \quad -kx - T_B = m_1\ddot{x}$$

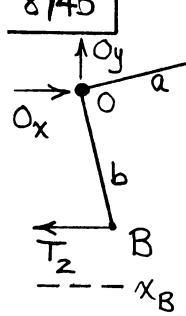
$$② \quad T_A - c\dot{x}_A = m_2\ddot{x}_A$$

$\Sigma M_O = 0$: $T_B b - T_A a = 0$

Elimination of T_B from Eq. ① yields

$$\left[m_1 + \frac{a^2}{b^2}m_2\right]\ddot{x} + \frac{a^2}{b^2}c\dot{x} + kx = 0$$

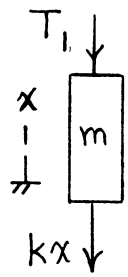
8/45



$$\frac{x}{a} = \frac{x_B}{b}\ , \quad T_2 = c\dot{x}_B = c\,\frac{b}{a}\,\dot{x}$$

$$\circlearrowleft \Sigma M_O = 0: \quad T_1 a - T_2 b = 0$$

$$T_1 = \frac{b}{a}\,T_2 = \frac{b}{a}\left(c\,\frac{b}{a}\,\dot{x}\right) = c\,\frac{b^2}{a^2}\,\dot{x}$$



$$\Sigma F_x = m\ddot{x}: \quad -T_1 - kx = m\ddot{x}$$

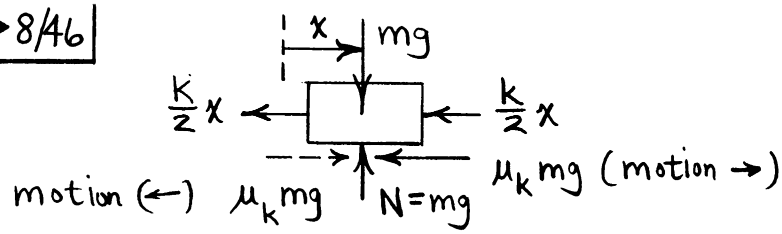
$$-c\,\frac{b^2}{a^2}\,\dot{x} - kx = m\ddot{x}$$

$$\underline{\ddot{x} + \frac{b^2}{a^2}\,\frac{c}{m}\,\dot{x} + \frac{k}{m}\,x = 0}$$

$$2\zeta\omega_n = \frac{b^2}{a^2}\,\frac{c}{m}$$

$$\zeta = \frac{b^2}{a^2}\,\frac{c}{m}\,\frac{1}{2\sqrt{\frac{k}{m}}} = \underline{\frac{1}{2}\,\frac{b^2}{a^2}\,\frac{c}{\sqrt{km}}}$$

▶ 8/46



$$\Sigma F_x = m\ddot{x}: \quad -kx \pm \mu_k mg = m\ddot{x}$$

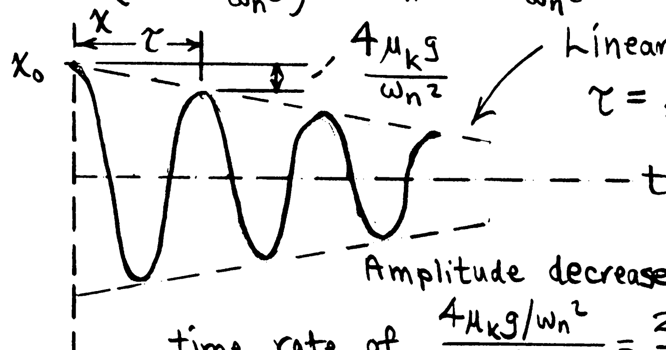
$$\ddot{x} + \omega_n^2 x = \mp \mu_k g$$

Solution: $x = A\cos\omega_n t + B\sin\omega_n t \mp \dfrac{\mu_k g}{\omega_n^2}$

Initial conditions for first half-cycle $\left.\begin{matrix} x = x_0 \\ \dot{x} = 0 \end{matrix}\right\}$ @ $t=0$

Results: $A = x_0 - \dfrac{\mu_k g}{\omega_n^2}$, $B = 0$

So $x = \left(x_0 - \dfrac{\mu_k g}{\omega_n^2}\right)\cos\omega_n t + \dfrac{\mu_k g}{\omega_n^2}$ for first $\frac{1}{2}$ cycle



Linear envelope

$\tau = \dfrac{2\pi}{\omega_n} = 2\pi\sqrt{\dfrac{m}{k}}$

Amplitude decreases at constant time rate of $\dfrac{4\mu_k g/\omega_n^2}{2\pi/\omega_n} = \underline{\dfrac{2\mu_k g}{\pi}\sqrt{\dfrac{m}{k}}}$

8/47 $\omega_n = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{100,000}{10}} = 100 \text{ rad/s}$

$$\zeta = \frac{c}{2m\omega_n} = \frac{500}{2(10)(100)} = 0.25 \text{ for (a)}$$

$$X = \frac{F_0/k}{\left\{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\zeta\frac{\omega}{\omega_n}\right]^2\right\}^{1/2}} = \frac{1000/100,000}{\left\{\left[1-1.2^2\right]^2+\left[2(0.25)(1.2)\right]^2\right\}}$$

$$= \frac{1000/100\,000}{\left\{\left[1-1.2^2\right]^2+\left[2(0.25)(1.2)\right]^2\right\}^{1/2}} = \underline{1.344(10^{-2}) \text{ m}}$$

(b) With $\zeta = 0$, $\underline{X = 2.27(10^{-2}) \text{ m}}$

8/48

$$(M)_{\frac{\omega}{\omega_n}=1} = 8(M)_{\frac{\omega}{\omega_n}=2}$$

$$\frac{1}{\{[1-1^2]^2+[2\zeta(1)]^2\}^{1/2}} = \frac{8}{\{[1-2^2]^2+[2\zeta(2)]^2\}^{1/2}}$$

Square both sides and solve for ζ to obtain $\underline{\zeta = 0.1936}$

8/49 $\omega_n = \sqrt{k/m} = \sqrt{6(12)/\frac{64.4}{32.2}} = 6$ rad/sec

$$X = \frac{F_0/k}{1-(\omega/\omega_n)^2} = \frac{5/6(12)}{1-\frac{\omega^2}{6^2}}$$

Because $|X| < \frac{3}{12}$, we set $X < \frac{3}{12}$ & $X > -\frac{3}{12}$

and obtain $\underline{\omega > 6.78 \text{ rad/sec}}$ & $\underline{\omega < 5.10 \text{ rad/sec}}$

8/50 $\omega_n = 6$ rad/sec (from Prob. 8/49)

$$X = \frac{F_o/k}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\zeta\frac{\omega}{\omega_n}\right]^2}}$$

$$\boxed{\zeta = \frac{c}{2m\omega_n} = \frac{2.4}{2(2)6} = 0.1}$$

$$= \frac{5/6(12)}{\sqrt{\left[1-\left(\frac{\omega}{6}\right)^2\right]^2 + \left[2(0.1)\frac{\omega}{6}\right]^2}} < \frac{3}{12}$$

Square both sides to obtain a quadratic in ω^2. Solution : $\omega < 5.32$ rad/sec

$\underline{\omega > 6.50 \text{ rad/sec}}$

As expected, damping allows a wider range of ω than when $\zeta = 0$ (Prob. 8/49).

8/51 $\omega_n = 6$ rad/sec (from Prob. 8/49)

$$X = \frac{F_o/k}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\zeta\frac{\omega}{\omega_n}\right]^2}}$$

$$= \frac{5/6(12)}{\sqrt{\left[1-\left(\frac{6}{6}\right)^2\right]^2 + \left[2\zeta\frac{6}{6}\right]^2}} = \frac{3}{12}$$

$$\zeta = 0.1389$$

$$\zeta = \frac{c}{2m\omega_n}, \quad c = 2\zeta m\omega_n = 2(0.1389)(2)(6)$$

$$= \underline{3.33 \text{ lb-sec/ft}}$$

8/52 $\omega_n = \sqrt{2k/m} = \sqrt{\dfrac{2(200)}{100/32.2}} = 11.35 \text{ rad/sec}$

(a) $\zeta = 0$: $X = \left| \dfrac{F_0/k_{eff}}{1 - \left(\frac{\omega}{\omega_n}\right)^2} \right|$

$$= \left| \frac{75/(2\cdot 200)}{1 - \left(\frac{15}{11.35}\right)^2} \right| = \underline{0.251 \text{ ft}}$$

(b) $\zeta \neq 0$: $\zeta = \dfrac{c}{2m\omega_n} = \dfrac{60}{2\left(\frac{100}{32.2}\right)(11.35)}$

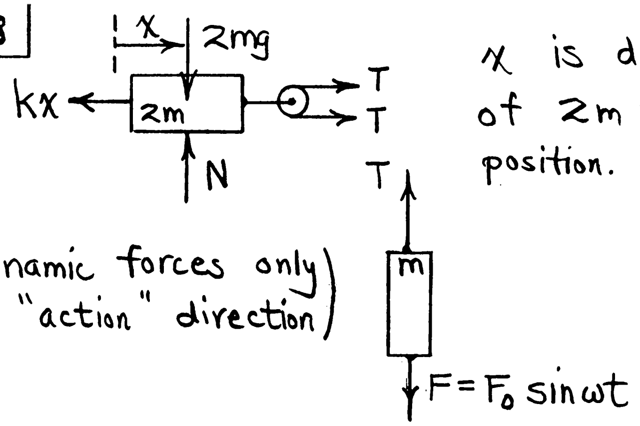
$$\zeta = 0.851$$

$$X = \frac{F_0/k_{eff}}{\left\{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\zeta\frac{\omega}{\omega_n}\right]^2\right\}^{1/2}}$$

$$= \frac{75/(2\cdot 200)}{\left\{\left[1 - \left(\frac{15}{11.35}\right)^2\right]^2 + \left[2(0.851)\left(\frac{15}{11.35}\right)\right]^2\right\}^{1/2}} = \underline{0.0791 \text{ ft}}$$

$$\delta_{st} = \frac{W}{k_{eff}} = \frac{100}{2(200)} = \underline{0.25 \text{ ft}}$$

8/53



x is displacement of $2m$ from equil. position.

(Dynamic forces only in "action" direction)

$\Sigma F = ma$:

$2m \overset{+}{\rightarrow}$: $\quad -kx + 2T = 2m\ddot{x}$

$m \downarrow +$: $\quad F_0 \sin\omega t - T = m(2\ddot{x})$

Eliminate T: $\quad \ddot{x} + \frac{k}{6m}x = \frac{F_0}{3m}\sin\omega t$

$$\underline{\omega_c = \sqrt{\frac{k}{6m}}}$$

(It is assumed that the cable is always in tension.)

8/54

$$M = \frac{1}{\left\{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\zeta \frac{\omega}{\omega_n}\right]^2\right\}^{1/2}}$$

$$M_1 = \frac{1}{\left\{[1-1^2]^2 + [2(0.1)(1)]^2\right\}^{1/2}} = 5$$

$$M_1' = \frac{1}{\left\{[1-1^2]^2 + [2(0.2)(1)]^2\right\}^{1/2}} = 2.5$$

$$R_1 = \frac{M_1 - M_1'}{M_1}(100) = \underline{50\%}$$

$$M_2 = \frac{1}{\left\{[1-2^2]^2 + [2(0.1)(2)]^2\right\}^{1/2}} = 0.3304$$

$$M_2' = \frac{1}{\left\{[1-2^2]^2 + [2(0.2)(2)]^2\right\}^{1/2}} = 0.3221$$

$$R_2 = \frac{M_2 - M_2'}{M_2}(100) = \underline{2.52\%}$$

8/55 From Fig. 8/11, for $\zeta=0.2$ & $M \leq 2$, $(\omega/\omega_n)_1 \lesssim 0.75$ & $(\omega/\omega_n)_2 \gtrsim 1.1$. So $\underline{f \lesssim 4.5 \text{ Hz}}$ or $\underline{f \gtrsim 6.6 \text{ Hz}}$ for the limited amplitude.

From Eq. 8/23, with $M=2$ & $\zeta=0.2$,

$$2 = \frac{1}{\left\{(1-[\omega/\omega_n]^2)^2+(2\times 0.2)^2(\omega/\omega_n)^2\right\}^{1/2}}$$

Simplify & get $(\omega/\omega_n)^4 - 1.84(\omega/\omega_n)^2 + 0.75 = 0$

$$(\omega/\omega_n)^2 = \frac{1.84}{2} \pm \frac{1}{2}\sqrt{1.84^2 - 4(0.75)} = 0.92 \pm 0.310$$
$$= 1.230 \text{ or } 0.610$$

so $\omega/\omega_n = 1.109$ or 0.781, $\underline{f \leq 4.68 \text{ Hz or } f \geq 6.66 \text{ Hz}}$

8/56 The condition for the maxima is

$$\frac{dM}{d\left(\frac{\omega}{\omega_n}\right)} = \frac{d}{d\left(\frac{\omega}{\omega_n}\right)}\left[\frac{1}{\left\{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2+\left[2\zeta\frac{\omega}{\omega_n}\right]^2\right\}^{1/2}}\right] = 0$$

Differentiate to obtain $\frac{\omega}{\omega_n} = \underline{\sqrt{1-2\zeta^2}}$

8/57 Equivalent spring constant

$$k = P/\delta_{st} = \frac{2}{4\times10^{-3}} = 500 \text{ N/m}$$

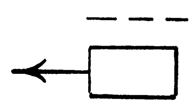
$$y_0 = \frac{b}{1-(\omega/\omega_n)^2} \quad \text{where } \omega_n^2 = \frac{k}{m} = \frac{500}{0.5} = 1000 \ (\tfrac{rad}{s})^2$$

$$\text{\& } \omega^2 = (4\times 2\pi)^2 = 632 \ (\tfrac{rad}{s})^2$$

Thus $$y_0 = \frac{3}{1-\frac{632}{1000}} = \underline{8.15 \text{ mm}}$$

8/58 Let x_m be the absolute cart displacement.

Then $x_m = x_B + x$, x = mass relative displacement $= (x_m - x_B)$



$k(x_m - x_B) = kx$

$$\Sigma F_x = ma_x : \; -k(x_m - x_B) = m(\ddot{x}_B + \ddot{x})$$

$$\ddot{x} + \frac{k}{m}x = b\omega^2 \sin\omega t$$

Assume $x = X \sin\omega t$ & obtain $X = \dfrac{b(\omega/\omega_n)^2}{1-(\omega/\omega_n)^2}$

Requirement : $|X| < 2b$

So $\dfrac{b(\omega/\omega_n)^2}{1-(\omega/\omega_n)^2} < 2b$ and $\dfrac{b(\omega/\omega_n)^2}{1-(\omega/\omega_n)^2} > -2b$

$\Downarrow$ $\Downarrow$

$\underline{\dfrac{\omega}{\omega_n} < \sqrt{2/3}}$ and $\underline{\dfrac{\omega}{\omega_n} > \sqrt{2}}$

See Fig. 8/14.

8/59 $\omega_n = \sqrt{k/m} = \sqrt{2(2.1)(10^3)/20} = 14.49$ rad/s

$$\zeta = \frac{c}{2m\omega_n} = \frac{2(58)}{2(20)(14.49)} = 0.200$$

$$M^2 = \frac{1}{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\zeta\frac{\omega}{\omega_n}\right]^2} \; ; \text{ let } r = \frac{\omega}{\omega_n} = \frac{\omega}{14.49}$$

$$2^2 = \frac{1}{[1-r^2]^2 + 4(0.200)^2 r^2}$$

$r^4 - 1.84r^2 + 0.75 = 0$, $r^2 = 0.610$ or $r^2 = 1.230$

Thus $\frac{\omega}{14.49} = r = \sqrt{0.610}$, $\omega = 11.32$ rad/s

or 108.1 rev/min

or $\frac{\omega}{14.49} = r = \sqrt{1.230}$, $\omega = 16.07$ rad/s

or 153.5 rev/min

Summary : $N \leq 108.1 \frac{\text{rev}}{\text{min}}$ or $N \geq 153.5 \frac{\text{rev}}{\text{min}}$

8/60 $W = k_{eq}\,\delta_{st}$; $k_{eq} = \frac{W}{\delta_{st}} = \frac{mg}{\delta_{st}}$

$\omega_n = \sqrt{k_{eq}/m} = \sqrt{\frac{mg/\delta_{st}}{m}} = \sqrt{g/\delta_{st}}$

For maximum response, $\omega = \omega_n = \sqrt{g/\delta_{st}}$

and $f = \frac{\omega}{2\pi} = \underline{\frac{1}{2\pi}\sqrt{\frac{g}{\delta_{st}}}}$

8/61

$$\frac{X}{b} = \left|\frac{1}{1-(\omega/\omega_n)^2}\right| = \frac{0.15}{0.10} = 1.5$$

$$\left.\begin{array}{ll} \text{For } \omega < \omega_n, & \dfrac{\omega}{\omega_n} = 0.577 \\ \text{For } \omega > \omega_n, & \dfrac{\omega}{\omega_n} = 1.291 \end{array}\right\}$$

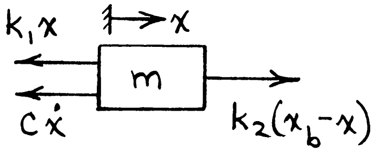
$$\omega_n = \sqrt{\frac{4k}{m}} = \sqrt{\frac{4(7200)}{43}} = 25.9 \text{ rad/s}$$

For $\omega < \omega_n$, $\omega = 0.577(25.9) = 14.94$ rad/s

For $\omega > \omega_n$, $\omega = 1.291(25.9) = 33.4$ rad/s

Thus prohibited range is $\underline{2.38 < f_n < 5.32 \text{ Hz}}$

8/62



$\Sigma F_x = m\ddot{x}$:

$$-k_1x - c\dot{x} + k_2(x_b - x) = m\ddot{x}$$

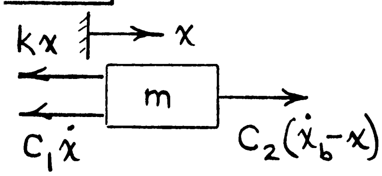
$$\underline{m\ddot{x} + c\dot{x} + (k_1+k_2)x} = k_2 x_b$$

$$= \underline{k_2 b \cos \omega t}$$

$$\underline{\omega_c = \sqrt{\frac{k_1+k_2}{m}}}$$

(It is assumed that the damping is light so that the forced response is a maximum at $(\omega/\omega_n) \cong 1$.)

8/63



$\Sigma F_x = m\ddot{x}$:

$$-kx - c_1\dot{x} + c_2(\dot{x}_b - x) = m\ddot{x}$$

$$\underline{m\ddot{x} + (c_1+c_2)\dot{x} + kx =} \ c_2\dot{x}_b$$

$$\underline{= -c_2 b\omega \sin \omega t}$$

$$\underline{\omega_c = \sqrt{\frac{k}{m}}}$$

$$2\zeta\omega_n = \frac{c_1+c_2}{m}, \quad \underline{\zeta = \frac{c_1+c_2}{2\sqrt{km}}}$$

(See assumption in solution of Prob. 8/62)

8/64 The maximum value of the force transmitted to the base, from Sample Problem 8/6, is $(F_{tr})_{max} = X\sqrt{k^2 + c^2\omega^2}$

$$= (F_0/k)M\sqrt{k^2 + (4\zeta^2 m^2 \omega_n^2)\omega^2}$$

$$= (F_0/k)M\sqrt{k^2 + \frac{4\zeta^2 m^2 \omega_n^2 \omega^2}{1} \cdot \frac{k^2/m^2}{\omega_n^4}}$$

$$= (F_0/k)Mk\sqrt{1 + \left(2\zeta\frac{\omega}{\omega_n}\right)^2}$$

$$= MF_0\sqrt{1 + \left(2\zeta\frac{\omega}{\omega_n}\right)^2}$$

Then transmission ratio T is

$$T = \frac{(F_{tr})_{max}}{F_0} = \underline{M\sqrt{1 + \left(2\zeta\frac{\omega}{\omega_n}\right)^2}}$$

(M = magnification factor)

$\underline{8/65}$ $F_0 = 2m_0 e\omega^2 = 2(1)(0.012)\left(1800\,\frac{2\pi}{60}\right)^2$
$= 853$ N

Force transmitted $kX = 1500$ N

But $kX = \left|\dfrac{F_0}{1-(\omega/\omega_n)^2}\right|$ or $1500 = \left|\dfrac{853}{1-(\omega/\omega_n)^2}\right|$

Solving, $\left(\dfrac{\omega}{\omega_n}\right)^2 = 1.568$ or 0.432

With $\omega_n^2 = k/m$, we obtain

$k = \dfrac{m\omega^2}{1.568}$ or $k = \dfrac{m\omega^2}{0.432}$

With $m = 10$ kg and $\omega = 1800\left(\frac{2\pi}{60}\right)$,
we obtain

$k = \underline{227 \text{ kN/m}}$ or $\underline{823 \text{ kN/m}}$

8/66 For seismic response,

$$\frac{X}{b} = \frac{(\omega/\omega_n)^2}{\sqrt{[1-(\frac{\omega}{\omega_n})^2]^2 + [2\zeta\omega/\omega_n]^2}}$$

$X = 2$ mm, $\omega = 3(2\pi) = 18.85$ rad/s

$\omega_n = \sqrt{k/m} = \sqrt{20/0.5} = 6.32$ rad/s

$\frac{\omega}{\omega_n} = 2.98$, $(\omega/\omega_n)^2 = 8.88$

$$\zeta = \frac{c}{2m\omega_n} = \frac{3}{2(0.5)(6.32)} = 0.474$$

So $$\frac{2}{b} = \frac{8.88}{\sqrt{[1-8.88]^2 + [2(0.474)(2.98)]^2}}$$

$\underline{b = 1.886 \text{ mm}}$

8/67 $\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{150}{0.008}} = 136.9$ rad/s

$\omega = 2\pi(5) = 31.4$ rad/s, $\frac{\omega}{\omega_n} = \frac{31.4}{136.9} = 0.229$

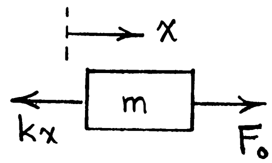
With $\frac{\omega}{\omega_n} << 1$ and $\zeta = 0.75$, the response is not sensitive to ω/ω_n & $M \cong 1$.

So $X \cong b\left(\frac{\omega}{\omega_n}\right)^2$. But $a_{max} = b\omega^2$, so that

$X \cong \frac{a_{max}}{\omega_n^2}$ or $a_{max} = X\omega_n^2 = 0.004\left(\frac{150}{0.008}\right)$

$= \underline{75.0 \text{ m/s}^2}$

8/68



$$\Sigma F = m\ddot{x}: \quad -kx + F_0 = m\ddot{x}$$

$$\ddot{x} + \frac{k}{m}x = \frac{F_0}{m}$$

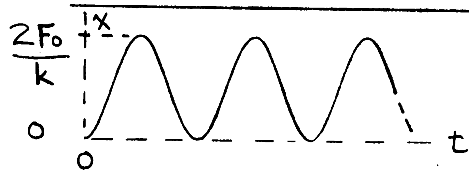
$$x = x_c + x_p$$

or $$x = \left(A_1 \cos \omega_n t + A_2 \sin \omega_n t\right) + \frac{F_0}{k}$$

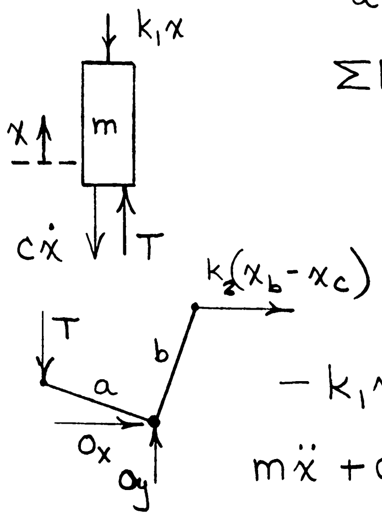
$$x(t=0) = A_1 + \frac{F_0}{k} = 0, \quad A_1 = -\frac{F_0}{k}$$

$$\dot{x}(t=0) = A_2 \omega_n = 0, \quad A_2 = 0$$

So $$x = \frac{F_0}{k}\left(1 - \cos \omega_n t\right)$$



8/69



$$\frac{x}{a} = \frac{x_c}{b}, \quad x_c = \frac{b}{a}x$$

$$\Sigma M_O = 0: \; Ta - k_2(x_b - x_c)b = 0$$

$$T = \frac{b}{a}k_2\left(x_b - \frac{b}{a}x\right)$$

$$\Sigma F_x = m\ddot{x}:$$

$$-k_1 x - c\dot{x} + T = m\ddot{x}$$

$$-k_1 x - c\dot{x} + k_2\frac{b}{a}x_b - k_2\frac{b^2}{a^2}x = m\ddot{x}$$

$$m\ddot{x} + c\dot{x} + \left(k_1 + k_2\frac{b^2}{a^2}\right)x = k_2\frac{b}{a}b_0\cos\omega t$$

8/70 For steady-state motion,

$$\frac{X}{b} = \frac{(\omega/\omega_n)^2}{\sqrt{[1-(\frac{\omega}{\omega_n})^2]^2 + [2\zeta\frac{\omega}{\omega_n}]^2}}$$

$$\frac{X}{b} = \frac{24}{18} = \frac{4}{3}; \quad \omega_n = \sqrt{k/m} = \sqrt{\frac{1500}{2}} = 27.4 \ \frac{\text{rad}}{\text{s}}$$

$$\omega = 5(2\pi) = 31.4 \text{ rad/s} \Rightarrow \frac{\omega}{\omega_n} = 1.147, \ \left(\frac{\omega}{\omega_n}\right)^2 = 1.316$$

So $$\left(\frac{4}{3}\right)^2 = \frac{1.316^2}{[1-1.316]^2 + [4(1.316)\zeta^2]}, \quad \zeta = 0.408$$

From $\zeta = \frac{c}{2m\omega_n}$, $c = 2\zeta m\omega_n = 2(0.408)(2)(27.4)$

$= \underline{44.6 \text{ N}\cdot\text{s/m}}$

▸8/71 Take road contour to be $x = b\sin\omega t$

Wavelength $L = v\tau = v\,\frac{2\pi}{\omega} \Rightarrow \omega = \frac{2\pi v}{L}$

Thus $x = b\sin\frac{2\pi v}{L}t$.

$$X = \left|\frac{b}{1-(\omega/\omega_n)^2}\right|. \quad \text{Set } b = 25 \text{ mm}$$

$$\omega = \frac{2\pi\left(\frac{25}{3.6}\right)}{1.2} = 36.4 \text{ rad/s}, \text{ and } \omega_n = \sqrt{k/m}$$

$$= \left(\frac{75(9.81)/0.003}{500}\right)^{1/2} = 22.1 \text{ rad/s} \quad \text{to}$$

obtain $\underline{X = 14.75 \text{ mm}}$

Critical speed: $\omega_c = \omega_n$

$$\frac{2\pi v_c}{L} = \sqrt{k/m} = 22.1$$

$$v_c = 4.23 \text{ m/s or } \underline{15.23 \frac{\text{km}}{\text{h}}}$$

▶8/72 Energy loss dE during motion dx is the negative of the work done by the friction force $c\dot{x}$ so

$dE = c\dot{x}\,dx = c\dot{x}(\dot{x}\,dt) = c\dot{x}^2 dt$, and the loss per cycle is

$$E = \int_0^\tau c\dot{x}^2 dt \quad \text{where } \tau = 2\pi/\omega$$

For the damped linear oscillator $x = X\sin(\omega t - \varphi)$, Eq. 8/20,

where $$X = \frac{F_0/k}{\{[1-(\omega/\omega_n)^2]^2 + [2\zeta\omega/\omega_n]^2\}^{1/2}}$$

$\dot{x} = X\omega\cos(\omega t - \varphi)$, $\dot{x}^2 = X^2\omega^2\cos^2(\omega t - \varphi)$

Thus

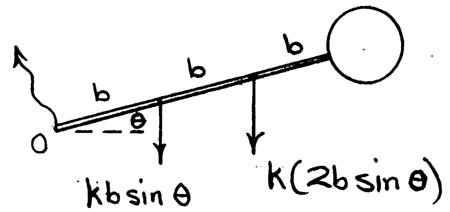
$$E = cX^2\omega^2 \int_0^{2\pi/\omega} \cos^2(\omega t - \varphi)\,dt = cX^2\omega^2\left[\frac{t}{2} + \frac{\sin 2(\omega t - \varphi)}{4\omega}\right]_0^{2\pi/\omega}$$

$$= cX^2\omega\pi$$

Power loss P = (Energy loss per cycle)(cycles per sec)

$$= cX^2\omega\pi \times \frac{\omega}{2\pi} = cX^2\frac{\omega^2}{2}, \quad \underline{P = cX^2\omega^2/2}$$

8/73 | Dynamic forces:

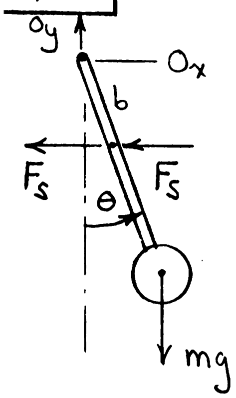


$$\circlearrowleft \Sigma M_O = I_O\ddot{\theta} : -(kb\sin\theta)(b\cos\theta) - k(2b\sin\theta)(2b\cos\theta)$$
$$= m(3b)^2\ddot{\theta}$$

For small θ: $\ddot{\theta} + \dfrac{5k}{9m}\theta = 0$

$\omega_n = \sqrt{\dfrac{5k}{9m}}$, $\tau = \dfrac{2\pi}{\omega_n} = \underline{6\pi\sqrt{\dfrac{m}{5k}}}$

8/74



Each spring force is $F_s = kb\sin\theta$

(Springs are assumed to remain horizontal)

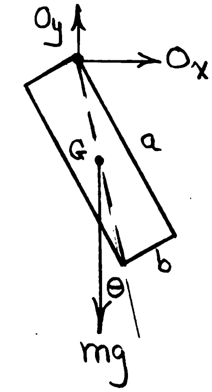
$$\Sigma M_O = I_O\ddot{\theta}: -mgl\sin\theta - 2F_s\, b\cos\theta = ml^2\ddot{\theta}$$

Simplify to obtain (for small θ)

$$\underline{\ddot{\theta} + \left(\frac{g}{l} + \frac{2kb^2}{ml^2}\right)\theta = 0}$$

$$\tau = \frac{2\pi}{\omega_n} = \underline{2\pi \Big/ \sqrt{\frac{g}{l} + \frac{2kb^2}{ml^2}}}$$

8/75



$$\sum M_O = I_O\ddot{\theta} : -mg\,\frac{\sqrt{a^2+b^2}}{2}\sin\theta$$

$$= \left[\frac{1}{12}m(a^2+b^2) + m\left(\left(\frac{a}{2}\right)^2+\left(\frac{b}{2}\right)^2\right)\right]\ddot{\theta}$$

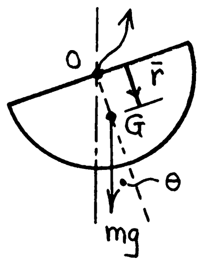
For small θ:

$$\ddot{\theta} + \frac{\frac{3}{2}g}{\sqrt{a^2+b^2}}\,\theta = 0$$

$$\underline{\omega_n = \frac{\sqrt{\frac{3g}{2}}}{\sqrt[4]{a^2+b^2}}}$$

8/76

$\bar{r} = \frac{4r}{3\pi}$, $I_O = \frac{1}{2}mr^2$



$\circlearrowleft + \sum M = I_O\ddot{\theta}: \; -mg\frac{4r}{3\pi}\sin\theta = \frac{1}{2}mr^2\ddot{\theta}$

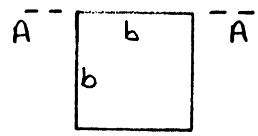
For small θ, $\sin\theta \cong \theta$, so

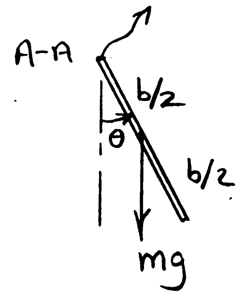
$$\ddot{\theta} + \frac{8g}{3\pi r}\theta = 0$$

$\omega_n = \sqrt{\frac{8g}{3\pi r}}$, $f_n = \frac{\omega_n}{2\pi} = \underline{\frac{1}{\pi}\sqrt{\frac{2g}{3\pi r}}}$

8/77

$$I_{A-A} = \frac{1}{12} m b^2 + m\left(\frac{b}{2}\right)^2 = \frac{1}{3} m b^2$$





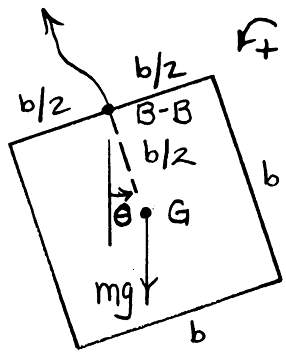
(Small θ)

$$\sum M_{A-A} = I_{A-A}\ddot{\theta}: \quad -mg\frac{b}{2}\theta = \frac{1}{3} m b^2 \ddot{\theta}$$

$$\ddot{\theta} + \frac{3g}{2b}\theta = 0, \quad \omega_n = \sqrt{\frac{3g}{2b}}$$

$$\tau = \frac{2\pi}{\omega_n} = \underline{2\pi\sqrt{\frac{2b}{3g}}}$$

8/78 $\quad I_{B-B} = \frac{1}{6}mb^2 + m\left(\frac{b}{2}\right)^2 = \frac{5}{12}mb^2$



$\circlearrowleft + \Sigma M_{B-B} = I_{B-B}\ddot{\theta}$: (small θ)

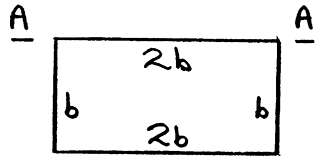
$$-mg\frac{b}{2}\theta = \frac{5}{12}mb^2\ddot{\theta}$$

$$\ddot{\theta} + \frac{6}{5}\frac{g}{b}\theta = 0$$

$$\omega_n = \sqrt{\frac{6g}{5b}}, \quad \tau = \frac{2\pi}{\omega_n} = \underline{2\pi\sqrt{\frac{5b}{6g}}}$$

(11.8 % higher than result of Prob. 8/77)

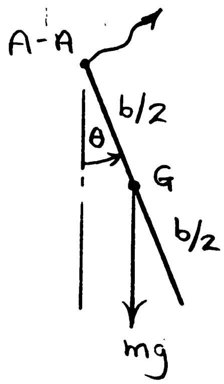
8/79 Let ρ = mass per unit length.



Body mass $m = 6\rho b$

$$I_{A\text{-}A} = 2\frac{1}{3}(\rho b)b^2 + 2\rho b(b^2)$$

$$= \frac{8}{3}\rho b^3\left(\frac{m}{6\rho b}\right) = \frac{4}{9}mb^2$$

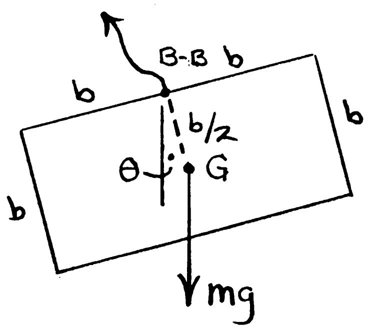


$$\circlearrowleft \Sigma M_{A\text{-}A} = I_{A\text{-}A}\ddot{\theta}: -mg\frac{b}{2}\theta = \frac{4}{9}mb^2\ddot{\theta}$$

$$\ddot{\theta} + \frac{9}{8}\frac{g}{b}\theta = 0, \quad \underline{\omega_n = \frac{3}{2}\sqrt{\frac{g}{2b}}}$$

8/80 Body mass $m = 6\rho b$

$$I_{B-B} = 2\tfrac{1}{3}(\rho b)b^2 + 2\left[\tfrac{1}{12}(\rho b)b^2 + \rho b\left(b^2 + \left(\tfrac{b}{2}\right)^2\right)\right]$$
$$+ \left[\tfrac{1}{12}(2\rho b)(2b)^2 + (2\rho b)\,b^2\right]$$
$$= 6\rho b^3\left(\frac{m}{6\rho b}\right) = mb^2$$

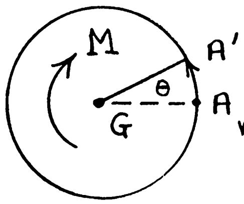


$$\curvearrowleft + \sum M_{B-B} = I_{B-B}\,\ddot{\theta}:$$
$$-mg\,\frac{b}{2}\,\theta = mb^2\ddot{\theta}$$
$$\ddot{\theta} + \frac{1}{2}\frac{g}{b}\theta = 0$$
$$\omega_n = \sqrt{\frac{g}{2b}}$$

(33.3 % lower than result for Prob. 8/79)

8/81

$M = k_T\theta$, where k_T is the torsional spring constant associated with the supporting wire.



$\circlearrowleft \Sigma M = \bar{I}\ddot{\theta}: \quad -k_T\theta = \bar{I}\ddot{\theta}$

$$\ddot{\theta} + \frac{k_T}{\bar{I}}\theta = 0$$

$$\tau = \frac{2\pi}{\omega_n} = 2\pi\left(\frac{\bar{I}}{k_T}\right)^{1/2}$$

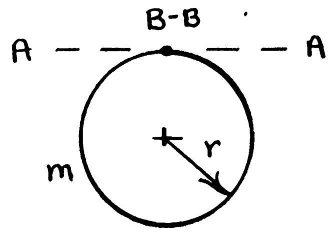
Initially, $\tau_1 = 2\pi\left(\frac{I}{k_T}\right)^{1/2}$ (a)

With weights, $\tau_2 = 2\pi\left(\frac{I + 2mr^2}{k_T}\right)^{1/2}$ (b)

Eliminate k_T from Eqs. (a) and (b) and solve for I as

$$\underline{I = \frac{2mr^2}{(\tau_2/\tau_1)^2 - 1}}$$

8/82



$$I_{A\text{-}A} = \tfrac{1}{2}mr^2 + mr^2 = \tfrac{3}{2}mr^2$$

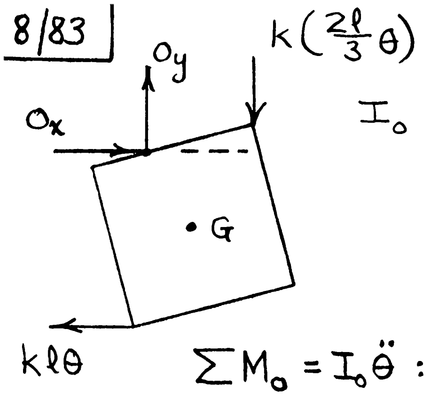
$$I_{B\text{-}B} = mr^2 + mr^2 = 2mr^2$$

Desired ratio $R = \dfrac{\tau_{B\text{-}B}}{\tau_{A\text{-}A}}$

or $R = \dfrac{\omega_{A\text{-}A}}{\omega_{B\text{-}B}}$

Each natural frequency is proportional to $\dfrac{1}{\sqrt{I}}$, so $R = \dfrac{1/\sqrt{I_{A\text{-}A}}}{1/\sqrt{I_{B\text{-}B}}} = \dfrac{\sqrt{2mr^2}}{\sqrt{\frac{3}{2}mr^2}} = \underline{\dfrac{2}{\sqrt{3}}}$

8/83



$$I_o = \bar{I} + md^2$$
$$= \frac{1}{6}ml^2 + m\left[\left(\frac{l}{2}\right)^2 + \left(\frac{l}{6}\right)^2\right]$$
$$= \frac{4}{9}ml^2$$

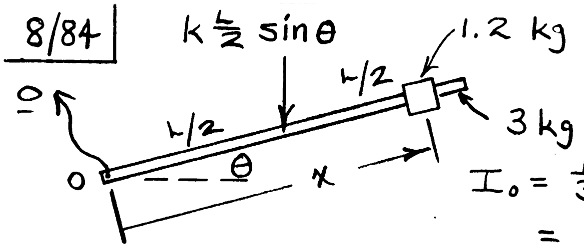
$\sum M_o = I_o\ddot{\theta}$: (for small θ)

$$-k\left(\frac{2l}{3}\theta\right)\left(\frac{2l}{3}\right) - (kl\theta)\,l = \frac{4}{9}ml^2\ddot{\theta}$$

$$\ddot{\theta} + \frac{13}{4}\frac{k}{m}\theta = 0$$

$$f_n = \frac{\omega_n}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{13k}{4m}} = \underline{\frac{1}{4\pi}\sqrt{\frac{13k}{m}}}$$

8/84



Let L be the 0.8-m rod length

$$I_O = \frac{1}{3}(3)L^2 + 1.2x^2$$
$$= L^2 + 1.2x^2$$

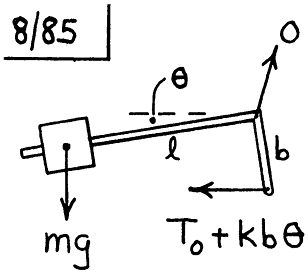
$$\curvearrowleft \Sigma M_O = I_O\ddot{\theta}: -k\frac{L}{2}\sin\theta\left(\frac{L}{2}\cos\theta\right) = (L^2 + 1.2x^2)\ddot{\theta}$$

$$\Rightarrow \quad \ddot{\theta} + \frac{kL^2}{4(L^2+1.2x^2)}\theta = 0$$

$$\tau = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{4(L^2+1.2x^2)}{kL^2}} = 4\pi\sqrt{\frac{L^2+1.2x^2}{kL^2}}$$

So $\quad 4\pi\sqrt{\dfrac{0.8^2+1.2x^2}{250\,(0.8)^2}} = 1, \quad \underline{x = 0.558\text{ m}}$

8/85



For equilibrium with $\theta = 0$,

$T_0 b = mgl$, $T_0 = \dfrac{mgl}{b}$

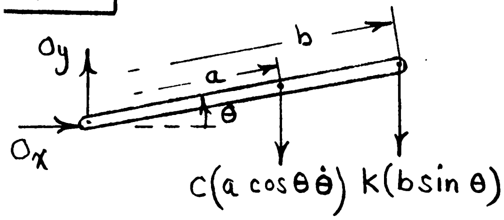
$\curvearrowleft + \Sigma M = I_O \ddot{\theta}$: $mgl\cos\theta$

$- \left(mg\frac{l}{b} + kb\theta\right)(b\cos\theta) = ml^2\ddot{\theta}$

$$\ddot{\theta} + \frac{kb^2}{ml^2}\theta = 0 \quad \text{for small } \theta.$$

$$\omega_n = \sqrt{\frac{kb^2}{ml^2}}, \quad f_n = \frac{\omega_n}{2\pi} = \underline{\frac{b}{2\pi l}\sqrt{k/m}}$$

8/86



$$\sum M_O = I_O\ddot\theta: \; -kb\sin\theta(b\cos\theta) - ca\cos\theta\,\dot\theta\,(a\cos\theta) = \frac{1}{3}mb^2\,\ddot\theta$$

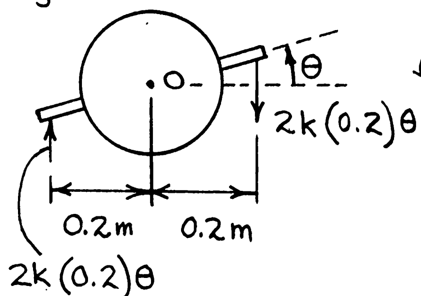
Small θ: $\quad \ddot\theta + \frac{3a^2}{b^2}\frac{c}{m}\dot\theta + \frac{3k}{m}\theta = 0$

$\omega_n = \sqrt{\frac{3k}{m}}$, $\quad 2\zeta\omega_n = \frac{3a^2}{b^2}\frac{c}{m}$, $\quad \underline{\zeta = \frac{1}{2}\frac{a^2}{b^2}c\sqrt{\frac{3}{km}}}$

For $\zeta = 1$, $\quad \underline{c_{cr} = \frac{2b^2}{a^2}\sqrt{\frac{km}{3}}}$

8/87

Dynamic forces:



$I_O = mk_O^2 = 43(0.1)^2$

$= 0.43\ \mathrm{kg\cdot m^2}$

$\circlearrowleft + \Sigma M_O = I_O\ddot{\theta}: \ -4k(0.2)^2\theta$

$= 0.43\ddot{\theta}$

$$\ddot{\theta} + \frac{0.16k}{0.43}\theta = 0$$

$$\omega_n = \left(\frac{0.16k}{0.43}\right)^{1/2} = 360\left(\frac{2\pi}{60}\right)$$

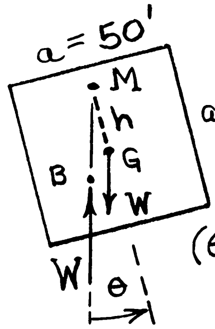
$$\underline{k = 3820\ \mathrm{N/m}}$$

8/88

$h = 3$ ft

$\bar{I} = \frac{1}{12} m (a^2 + a^2)$

$= \frac{1}{6} m a^2$



(θ small)

$\curvearrowleft + \Sigma M_G = \bar{I}\ddot{\theta}$:

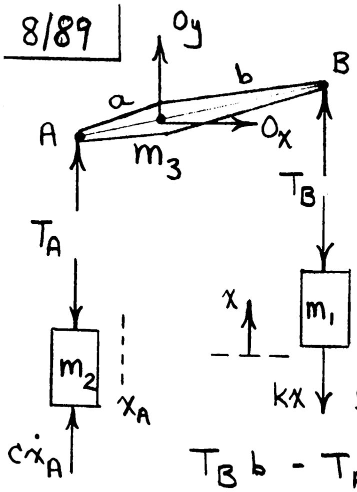
$-mgh\,\theta = \frac{1}{6} m a^2 \ddot{\theta}$

$\ddot{\theta} + \frac{6gh}{a^2}\theta = 0$

$\omega_n = \frac{1}{a}\sqrt{6gh}, \quad \tau = \frac{2\pi}{\omega_n} = \frac{2\pi a}{\sqrt{6gh}}$

$= \frac{2\pi(50)}{\sqrt{6(32.2)(3)}} = \underline{13.05 \text{ sec}}$

8/89



$$\frac{x}{b} = \frac{x_A}{a}, \quad x_A = \frac{a}{b}x, \quad \ddot{x}_A = \frac{a}{b}\ddot{x}$$

$$x = b\theta, \quad \ddot{x} = b\ddot{\theta}$$

$$\Sigma F_x = m\ddot{x}:$$

$$(1) \quad -kx - T_B = m_1\ddot{x}$$

$$(2) \quad T_A - c\dot{x}_A = m_2\ddot{x}_A$$

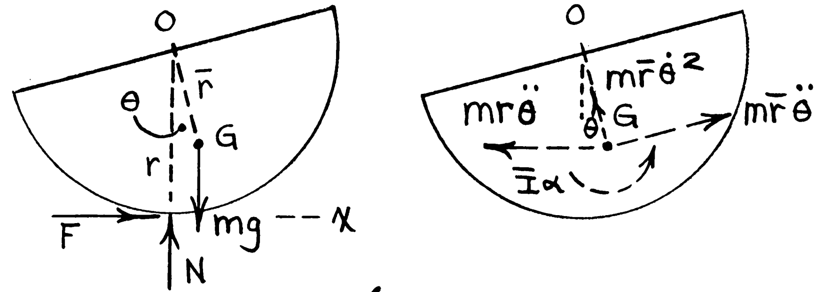
$$\circlearrowleft \Sigma M_O = I_O\ddot{\theta}:$$

$$T_B b - T_A a = m_3 k_O^2 \ddot{\theta}$$

Elimination of T_B from Eq. (1) yields

$$\left[m_1 + \frac{a^2}{b^2}m_2 + \frac{k_O^2}{b^2}m_3\right]\ddot{x} + \left[\frac{a^2}{b^2}c\right]\dot{x} + kx = 0$$

8/90



From Table D/3, $\bar{r} = \dfrac{4r}{3\pi}$, $I_0 = \frac{1}{2}mr^2$

$$\circlearrowleft + \Sigma M_0 = \bar{I}\ddot{\theta} + m\bar{r}^2\ddot{\theta} - mr\ddot{\theta}\,(\bar{r}\cos\theta)$$

$$= I_0\ddot{\theta} - m\left(\frac{4r^2}{3\pi}\right)\ddot{\theta}\cos\theta$$

$$Fr - mg\left(\frac{4r}{3\pi}\right)\sin\theta = \left(\frac{1}{2}mr^2 - \frac{4}{3\pi}mr^2\cos\theta\right)\ddot{\theta}$$

$$F = \frac{4mg}{3\pi}\sin\theta + mr\left(\frac{1}{2} - \frac{4}{3\pi}\cos\theta\right)\ddot{\theta}$$

$$\Sigma F_x = m\bar{a}_x : \quad F = -mr\ddot{\theta} + m\left(\frac{4r}{3\pi}\right)\cos\theta\,\ddot{\theta} - m\bar{r}\,\dot{\theta}^2\sin\theta$$

Eliminate F and obtain

$$\left(\frac{3}{2} - \frac{8\cos\theta}{3\pi}\right)\ddot{\theta} + \frac{4g}{3\pi r}\sin\theta + \frac{4}{3\pi}\dot{\theta}\sin\theta = 0$$

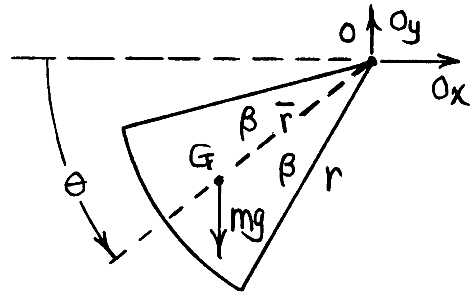
For small θ, $\cos\theta \to 1$, $\sin\theta \to \theta$:

$$\left(\frac{3}{2} - \frac{8}{3\pi}\right)\ddot{\theta} + \frac{4g}{3\pi r}\,\theta = 0$$

$$\omega_n = \sqrt{\frac{4g}{3\pi r}\Big/\left(\frac{3}{2} - \frac{8}{3\pi}\right)} = 0.807\sqrt{g/r}$$

$$\tau = \frac{2\pi}{\omega_n} = \frac{2\pi}{0.807\sqrt{g/r}} = \underline{7.78\sqrt{r/g}}$$

8/91 From Table D/3,



$$\bar{r} = \frac{2}{3}\frac{r\sin\beta}{\beta}$$

$$I_O = \frac{1}{2}mr^2$$

$$\Sigma M_O = I_O\ddot{\theta}:$$

$$mg\left(\frac{2}{3}\frac{r\sin\beta}{\beta}\cos\theta\right) = \frac{1}{2}mr^2\ddot{\theta}$$

$$\ddot{\theta} - K\cos\theta = 0, \text{ where } K = \frac{4}{3\beta}\frac{g}{r}\sin\beta$$

Let $\gamma = \frac{\pi}{2} - \theta$, so $\cos\theta = \sin\gamma$

For $\theta \cong \frac{\pi}{2}$, $\sin\gamma \cong \gamma$ and $\ddot{\gamma} = -\ddot{\theta}$

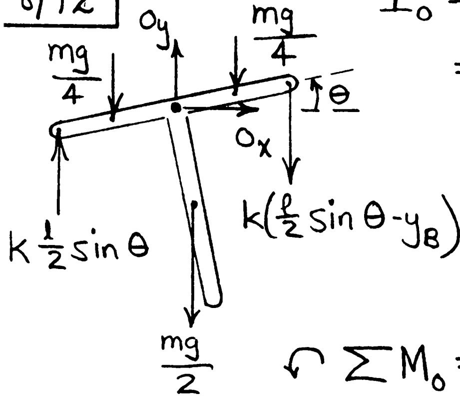
So $\ddot{\gamma} + K\gamma = 0$

$$\omega_n = \sqrt{K} = 2\sqrt{\frac{g\sin\beta}{3r\beta}}$$

$$\tau = \frac{2\pi}{\omega_n} = \pi\sqrt{\frac{3r\beta}{g\sin\beta}}$$

$\left(\text{or } \tau = \pi\sqrt{\frac{3r}{g}} \text{ for } \beta \text{ small}\right)$

8/92



$$I_O = \frac{1}{3}\frac{m}{2}l^2 + 2\left(\frac{1}{3}\frac{m}{4}\left(\frac{l}{2}\right)^2\right) = \frac{5}{24}ml^2$$

$$\curvearrowleft \Sigma M_O = I_O\ddot{\theta}:$$

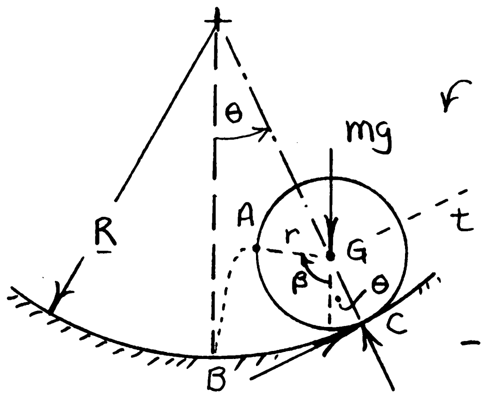
$$-\left(k\frac{l}{2}\sin\theta\right)\frac{l}{2}\cos\theta - k\left(\frac{l}{2}\sin\theta - y_B\right)\frac{l}{2}\cos\theta - \frac{mg}{2}\left(\frac{l}{2}\sin\theta\right) = \frac{5}{24}ml^2\ddot{\theta}$$

Small θ: $\ddot{\theta} + \left[\frac{12}{5}\frac{k}{m} + \frac{6}{5}\frac{g}{l}\right]\theta = \frac{12}{5}\frac{kb}{ml}\sin\omega t$

$$\underline{\omega_c = \sqrt{\frac{6}{5}\left(\frac{2k}{m} + \frac{g}{l}\right)}}$$

8/93 Arc AC = Arc BC : $r(\beta+\theta) = R\theta$

$$\beta = \left(\frac{R}{r}-1\right)\theta$$



$\sum M_G = \bar{I}\ddot{\beta}$:

$$Fr = \frac{1}{2}mr^2(-\ddot{\beta})$$

$\sum F_t = ma_t$:

$$-mg\sin\theta + F = mr\ddot{\beta}$$

Eliminate F, substitute $\ddot{\beta} = \left(\frac{R}{r}-1\right)\ddot{\theta}$, & assume small θ : $\ddot{\theta} + \frac{2g}{3(R-r)}\theta = 0$

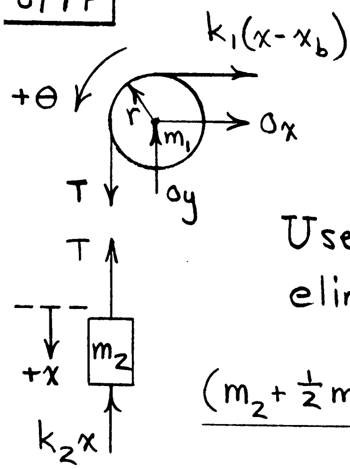
$$\omega_n = \sqrt{\frac{2g}{3(R-r)}}, \quad \tau = \frac{2\pi}{\omega_n} = \underline{2\pi\sqrt{\frac{3(R-r)}{2g}}}$$

Solution of differential eq.: $\theta = \theta_0 \sin\omega_n t$

$\dot{\theta} = \theta_0\omega_n \cos\omega_n t$, $\dot{\theta}_{max} = \theta_0\omega_n$.

$$\omega = \dot{\beta}_{max} = \left(\frac{R}{r}-1\right)\theta_0\omega_n = \underline{\frac{\theta_0}{r}\sqrt{2g(R-r)/3}}$$

8/94



$\sum M_o = I_o \ddot{\theta}$:

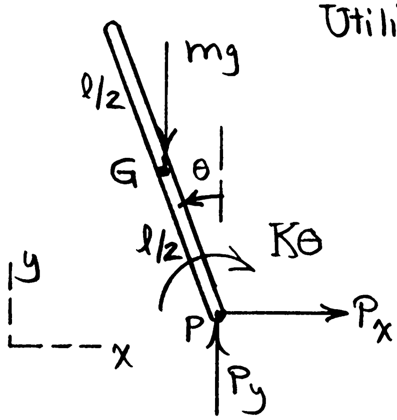
$$r\left(T - k_1(x - x_b)\right) = \frac{1}{2} m_1 r^2 \ddot{\theta}$$

$$\sum F_x = m\ddot{x} : \ -T - k_2 x = m_2 \ddot{x}$$

Use the constraint $x = r\ddot{\theta}$ and eliminate T to obtain

$$\underline{\left(m_2 + \tfrac{1}{2} m_1\right)\ddot{x} + (k_1 + k_2)\, x = k_1 b \cos \omega t}$$

8/95 $x_B = x_P = b\sin\omega t$



Utilize $\Sigma \underline{M}_P = \dot{\underline{H}}_P + \bar{\underline{\rho}} \times m\ddot{\underline{r}}_P$

$$\Sigma \underline{M}_P = \left(mg\frac{l}{2}\sin\theta - K\theta\right)\underline{k}$$

$$\dot{\underline{H}}_P = I_P\ddot{\theta}\underline{k} = \frac{1}{3}ml^2\ddot{\theta}\ \underline{k}$$

$$\bar{\underline{\rho}} = \frac{l}{2}\left[-\sin\theta\underline{i} + \cos\theta\underline{j}\right]$$

$$\ddot{\underline{r}}_P = -b\omega^2\sin\omega t\ \underline{i}$$

$$mg\frac{l}{2}\sin\theta - K\theta = \frac{1}{3}ml^2\ddot{\theta} - \left(-m\frac{l}{2}b\omega^2\cos\theta\sin\omega t\right)$$

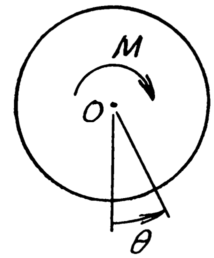
Small θ: $\ddot{\theta} + \left[\frac{3K}{ml^2} - \frac{3}{2}\frac{g}{l}\right]\theta = -\frac{3}{2}\frac{b}{l}\omega^2\sin\omega t$

Assume $\theta = \Theta\sin\omega t$ to obtain

$$\Theta = \frac{-\frac{3}{2}\frac{b}{l}\omega^2}{\omega_n^2 - \omega^2}, \quad \text{where} \quad \omega_n = \sqrt{\frac{3K}{ml^2} - \frac{3}{2}\frac{g}{l}}$$

8/96

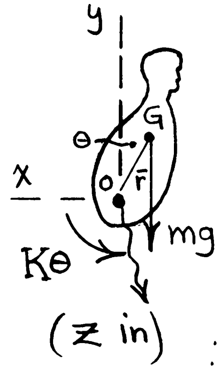
$\circlearrowright \Sigma M_O = I_O \alpha: \quad -\frac{JG}{L}\theta = I\ddot{\theta}$



$$\ddot{\theta} + \frac{JG}{IL}\theta = 0, \quad \omega_n = \sqrt{\frac{JG}{IL}}$$

$$\underline{f_n = \frac{\omega_n}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{JG}{IL}}}$$

8/97



$$\Sigma \underline{M}_O = \dot{\underline{H}}_O + \bar{\underline{\rho}} \times m\ddot{\underline{r}}_O$$

$$\Sigma \underline{M}_O = (mg\bar{r}\sin\theta - K\theta)\,\underline{k}$$

$$\dot{\underline{H}}_O = I_O\ddot{\theta}\,\underline{k} = mk_O^2\ddot{\theta}\,\underline{k}$$

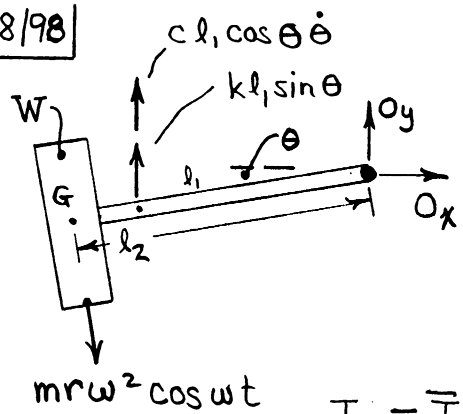
$$\bar{\underline{\rho}} = \bar{r}(-\sin\theta\,\underline{i} + \cos\theta\,\underline{j})$$

$$\ddot{\underline{r}}_O = a\underline{i}$$

$$\therefore \quad mg\bar{r}\sin\theta - K\theta = mk_O^2\ddot{\theta} - ma\bar{r}\cos\theta$$

Rearrange : $\underline{mk_O^2\ddot{\theta} + K\theta - m\bar{r}(g\sin\theta + a\cos\theta) = 0}$

▶ 8/98



View of wheel from left side :

(force on particle)

$$I_0 = \bar{I} + \frac{W}{g} l_2^{\,2} = 1 + \frac{100}{32.2}(3)^2$$

$$= 28.95 \text{ lb-sec}^2\text{-ft}$$

$\Sigma M_0 = I_0 \ddot\theta$:

$$-(k l_1 \sin\theta)(l_1 \cos\theta) - (c l_1 \dot\theta \cos\theta)(l_1 \cos\theta)$$

$$+ (mr\omega^2 \cos\omega t)\cos\theta\, l_2 = I_0 \ddot\theta$$

For small θ :

$$\ddot\theta + \frac{c l_1^{\,2}}{I_0}\dot\theta + \frac{k l_1^{\,2}}{I_0}\theta = \frac{mr\omega^2 l_2 \cos\omega t}{I_0}$$

$$\omega_n = \sqrt{\frac{k l_1^{\,2}}{I_0}} = \sqrt{\frac{(50)(12)\left(\frac{27}{12}\right)^2}{28.95}} = \underline{10.24 \ \frac{\text{rad}}{\text{sec}}}$$

$$v = r\omega_n = \frac{14}{12}(10.24) = \underline{11.95 \text{ ft/sec}}$$

$$2\zeta\omega_n = \frac{c l_1^{\,2}}{I_0}, \qquad \zeta = \frac{c l_1^{\,2}}{2 I_0 \omega_n}$$

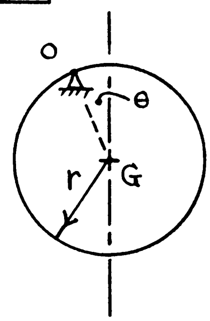
$$\zeta = \frac{(200)\left(\frac{27}{12}\right)^2}{2(10.24)(28.95)} = \underline{1.707}$$

8/99 Energy $E = T + V = 8\dot{x}^2 + 64x^2 = \text{constant}$

so $dE/dt = 16\dot{x}\ddot{x} + 128x\dot{x} = 0$, $\underline{\ddot{x} + 8x = 0}$

$\omega_n = \sqrt{8} = 2\sqrt{2}$ rad/sec, $\tau = \dfrac{2\pi}{\omega_n} = \dfrac{2\pi}{2\sqrt{2}} = \underline{2.22 \text{ sec}}$

8/100



$I_o = \bar{I} + md^2 = mr^2 + mr^2$
$= 2mr^2$

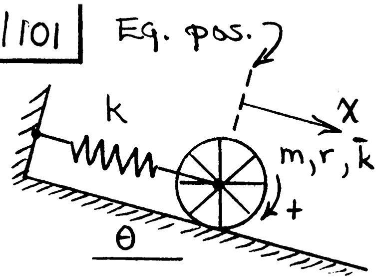
$E = T + V = \frac{1}{2} I_o \dot{\theta}^2 + mgr(1 - \cos\theta)$

Set $\frac{dE}{dt} = 0$ and assume small θ to obtain

$$\ddot{\theta} + \frac{g}{2r}\theta = 0$$

$$\tau = \frac{2\pi}{\omega_n} = \underline{2\pi\sqrt{\frac{2r}{g}}}$$

8/101



Choose $V = 0$ @ $x = 0$

Then $V = \frac{1}{2}kx^2$

$$T = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}\bar{I}\omega^2$$

$$= \frac{1}{2}m\dot{x}^2 + \frac{1}{2}(m\bar{k}^2)\left(\frac{\dot{x}}{r}\right)^2$$

$$= \frac{1}{2}m\left(1 + \frac{\bar{k}^2}{r^2}\right)\dot{x}^2$$

$$E = T + V = \frac{1}{2}m\left(1 + \frac{\bar{k}^2}{r^2}\right)\dot{x}^2 + \frac{1}{2}kx^2$$

$$\frac{dE}{dt} = m\left(1 + \frac{\bar{k}^2}{r^2}\right)\dot{x}\ddot{x} + kx\dot{x} = 0$$

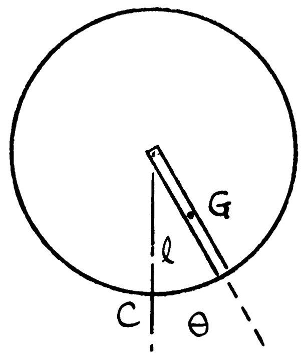
$$\ddot{x} + \frac{k}{m\left(1 + \frac{\bar{k}^2}{r^2}\right)}x = 0$$

$$\underline{\omega_n = \sqrt{\frac{k}{m(1 + \bar{k}^2/r^2)}}}$$

$$\begin{cases} \bar{k} = 0: & \underline{\omega_n = \sqrt{k/m}} \\ \bar{k} = r: & \underline{\omega_n = \sqrt{k/2m}} \end{cases}$$

$\underline{8/102}$ $V = 0$ @ $\theta = 0$; $V_{\theta_0} = V_{max} = mg\frac{l}{2}(1-\cos\theta_0)$

$$= mg\frac{l}{2}\left[1-\left(1-\frac{\theta_0^2}{2}+\cdots\right)\right]$$



$$(T_{max})_{\theta=0} = \frac{1}{2}I_C\omega^2$$

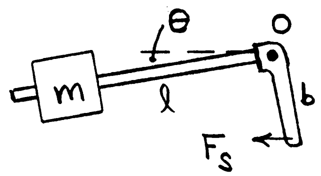
$$= \frac{1}{2}\left(\frac{1}{3}ml^2\right)(\theta_0\omega_n)^2$$

$$(\omega = \omega_{max} = \theta_0\omega_n)$$

$$V_{max} = T_{max} : \frac{mgl\theta_0^2}{4} = \frac{ml^2\theta_0^2\omega_n^2}{6}$$

$$\omega_n^2 = \frac{3g}{2l}, \quad \tau = \frac{2\pi}{\omega_n} = \underline{2\pi\sqrt{\frac{2l}{3g}}}$$

8/103 At equilibrium, $\Sigma M_O = 0$ to obtain



$$\delta_{st} = \frac{mgl}{kb}$$

$$E = T + V$$

$$= \frac{1}{2} m (l\dot{\theta})^2 + \frac{1}{2} k \left(\frac{mgl}{kb} + b \sin\theta \right)^2 - mgl \sin\theta$$

Set $\frac{dE}{dt} = 0$ and assume θ small

to obtain $\ddot{\theta} + \frac{kb^2}{ml^2}\theta = 0$

$$f_n = \frac{\omega_n}{2\pi} = \underline{\frac{1}{2\pi} \frac{b}{l} \sqrt{\frac{k}{m}}}$$

8/104 Let y be the downward displacement from the equilibrium position where $V = V_e + V_g$ is taken to be zero.

$$(T_{max})_{y=0} = (V_{max})_{y=y_{max}}$$

$$T_{max} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}(70)\dot{y}_{max}^2 + \frac{1}{2}(40)(0.2)^2\left(\frac{\dot{y}_{max}}{0.3}\right)^2$$

$$= 43.9\,\dot{y}_{max}^2$$

But $y = y_{max}\sin\omega_n t$, $\quad \dot{y}_{max} = y_{max}\,\omega_n$

So $T_{max} = 43.9\,y_{max}^2\,\omega_n^2$

$$V_{max} = \frac{1}{2}(2000)(2y_{max})^2 = 4000\,y_{max}^2$$

Thus $43.9\,y_{max}^2\,\omega_n^2 = 4000\,y_{max}^2$

$\omega_n = 9.55$ rad/s, $\quad f_n = \frac{\omega_n}{2\pi} = \underline{1.519\text{ Hz}}$

8/105 Let x be the displacement (downward) from the equilibrium position, where V is taken to be zero.

$$T = \frac{1}{2} m \dot{x}^2, \quad V = 2\left[\frac{1}{2} k (2x)^2\right]$$

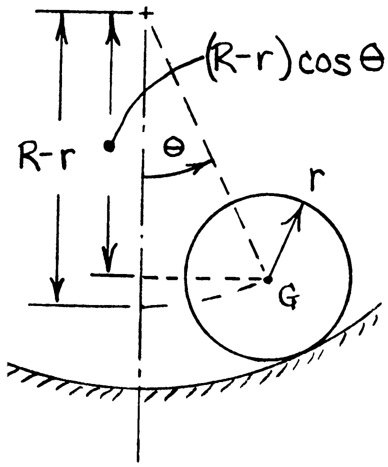
$$E = T + V = \frac{1}{2} m \dot{x}^2 + 4kx^2$$

$$\frac{dE}{dt} = m\dot{x}\ddot{x} + 8kx\dot{x} = 0, \quad \ddot{x} + \frac{8k}{m} x = 0$$

$$\omega_n = \sqrt{\frac{8k}{m}}, \quad \tau = \frac{2\pi}{\omega_n} = \pi\sqrt{\frac{m}{2k}}$$

Numbers: $\tau = \pi \sqrt{\dfrac{50/32.2}{2(6)(12)}} = \underline{0.326 \text{ sec}}$

8/106



$$E = T + V = \frac{1}{2} m v_G^2 + \frac{1}{2} \bar{I} \omega^2 + mg(R-r)(1-\cos\theta)$$

Now, $v_G = (R-r)\dot{\theta}$

$$\omega = \frac{v_G}{r} = \frac{(R-r)\dot{\theta}}{r}$$

Thus $E = \frac{3}{4} m (R-r)^2 \dot{\theta}^2 + mg(R-r)(1-\cos\theta)$

Set $\frac{dE}{dt} = 0$: $\ddot{\theta} + \frac{2}{3} \frac{g}{R-r} \sin\theta = 0$

For small θ, $\tau = \frac{2\pi}{\omega_n} = \underline{\pi\sqrt{\frac{6(R-r)}{g}}}$

8/107 For the bar, $I_O = \frac{1}{12} m_2 l^2 + m_2 \left(\frac{3}{10} l\right)^2$

$$= \frac{13}{75} m_2 l^2$$

Combined : $I_O = \frac{1}{2} m_1 \left(\frac{l}{5}\right)^2 + \frac{13}{75} m_2 l^2$

$$= \frac{1}{50} m_1 l^2 + \frac{13}{75} m_2 l^2$$

Let $\theta = 0$ be the equilibrium position shown & choose $V = 0$ @ $\theta = 0$; $V = \frac{1}{2} k \left(\frac{3l}{5}\theta\right)^2 = \frac{9}{50} k l^2 \theta^2$

$$E = T + V = \frac{1}{2} I_O \dot{\theta}^2 + \frac{9}{50} k l^2 \theta^2$$

$$= l^2 \left[\left(\frac{1}{100} m_1 + \frac{13}{150} m_2\right) \dot{\theta}^2 + \frac{9}{50} k \theta^2\right]$$

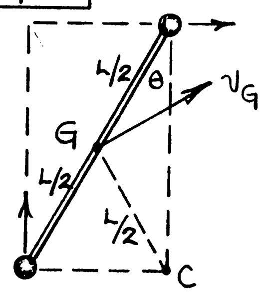
Set $\frac{dE}{dt} = 0$ to obtain

$$\ddot{\theta} + \frac{54k}{3m_1 + 26m_2} \theta = 0, \quad \omega_n = 3\sqrt{\frac{6k}{3m_1 + 26m_2}}$$

$$\theta = \theta_0 \sin \omega_n t, \quad \dot{\theta} = \theta_0 \omega_n \cos \omega_n t$$

$$\dot{\theta}_{max} = \omega = 3\theta_0 \sqrt{\frac{6k}{3m_1 + 26m_2}}$$

8/108



$v_G = \frac{L}{2}\dot{\theta}$ (C is the instantaneous center)

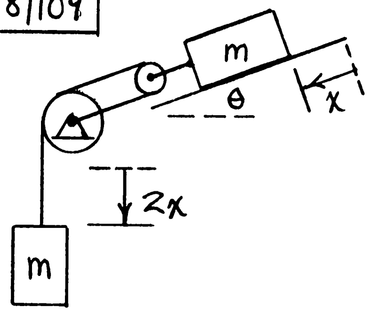
$$E = T + V = \frac{1}{2}mv_G^2 + \frac{1}{2}\left(\frac{mL^2}{12}\right)\dot{\theta}^2 + \frac{1}{2}k\left(\delta_{st} + L\sin\theta\right)^2 + \frac{1}{2}k\left(\delta_{st} - L\sin\theta\right)^2 - mg\frac{L}{2}\cos\theta$$

, where δ_{st} is the spring deflection at $\theta = 0$. Substitute $v_G = \frac{L}{2}\dot{\theta}$ into the expression for E, set $\frac{dE}{dt} = 0$ and assume θ is small to obtain

$$\ddot{\theta} + \left[\frac{6k}{m} + \frac{3g}{2L}\right]\theta = 0$$

So $$\underline{\underline{\omega_n = \sqrt{\frac{6k}{m} + \frac{3g}{2L}}}}$$

8/109



From an equilibrium analysis, the static spring stretch is

$$\delta_{st} = \frac{mg}{k}(2+\sin\theta)$$

$$E = T + V = \tfrac{1}{2}m\dot{x}^2 + \tfrac{1}{2}m(2\dot{x})^2 - mgx\sin\theta$$
$$- mg(2x) + \tfrac{1}{2}k\left[\frac{mg}{k}(2+\sin\theta) + x\right]^2$$
$$= \tfrac{5}{2}m\dot{x}^2 - mgx(2+\sin\theta) + \tfrac{1}{2}k\left[\frac{mg}{k}(2+\sin\theta)+x\right]^2$$

Set $\frac{dE}{dt} = 0$ to obtain $\ddot{x} + \frac{k}{5m}x = 0$

So $\underline{\omega_n = \sqrt{k/5m}}$

8/110 Let x denote the downward displacement from static equilibrium.

$$E = T + V = \text{constant}$$

$$T = \frac{1}{2}(140)\dot{x}^2 + 2\left(\frac{1}{2}80\dot{x}^2\right) + 2\left(\frac{1}{2}\right)\left(80(0.4)^2\right)\left(\frac{\dot{x}}{0.6}\right)^2$$

$$= 185.6\,\dot{x}^2$$

$$V = \frac{1}{2}k(2x)^2 = 2kx^2 = 8000x^2$$

Set $\frac{dE}{dt} = 0$: $\quad \ddot{x} + 43.1x = 0$

$$\tau = \frac{2\pi}{\omega_n} = \frac{2\pi}{\sqrt{43.1}} = \underline{0.957 \text{ s}}$$

8/111 $E = T + V = 2\left(\frac{1}{2} M v_0^2\right) + 2\left(\frac{1}{2}\bar{I}\dot{\theta}^2\right)$

$$+\frac{1}{2} m v_A^2 + mgr_0(1-\cos\theta)$$

But $v_0 = r\dot{\theta}$ and $\bar{I} = \frac{1}{2}Mr^2$. Also, from kinematics, $v_A^2 = \dot{\theta}^2(r^2+r_0^2) - 2rr_0\dot{\theta}^2\cos\theta$

Thus $E = \left[\frac{3}{2}Mr^2 + \frac{1}{2}m(r^2+r_0^2 - 2rr_0\cos\theta)\right]\dot{\theta}^2$

$$+ mgr_0[1-\cos\theta] = \text{constant}$$

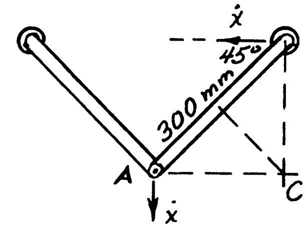
Set $\frac{dE}{dt} = 0$: $2\left[\frac{3}{2}Mr^2 + \frac{1}{2}m(r^2+r_0^2 - 2rr_0\cos\theta\right]\ddot{\theta}$

$$+\dot{\theta}^2[mrr_0\sin\theta] + mgr_0\sin\theta = 0$$

Small θ, $\dot{\theta}$: $\ddot{\theta} + \left[\frac{mgr_0}{3Mr^2 + m(r^2+r_0^2-2rr_0)}\right]\theta = 0$

$$f_n = \frac{1}{2\pi}\left[\frac{mgr_0}{3Mr^2 + m(r-r_0)^2}\right]^{1/2}$$

8/112 Take $V = V_g + V_e = 0$ at equilibrium position, where spring tension is $2(W/2) = W$



so for downward displacement x_0 from equilibrium position,

$$V_{max} = \Delta V_e + \Delta V_g = \left(W x_0 + \frac{1}{2} k x_0^2\right) + \left(-2W\frac{x_0}{2}\right)$$

$$= \frac{1}{2} k x_0^2 = \frac{1}{2} 1050\, x_0^2$$

$$T_{max} = 2\left(\frac{1}{2} I_c \omega^2\right) = I_c \left(\frac{\dot{x}_{max}}{0.300/\sqrt{2}}\right)^2$$

where $I_c = I_A = \frac{1}{3} m l^2 = \frac{1}{3} 1.5 \times 0.300^2 = 0.045\ kg \cdot m^2$

$$T_{max} = 0.045 \frac{2\dot{x}_{max}^2}{0.09} = \dot{x}_{max}^2, \text{ at } x = 0$$

But $\dot{x}_{max} = x_0 \omega_n$ for harmonic motion

so $T_{max} = V_{max}$ gives $x_0^2 \omega_n^2 = 525\, x_0^2$

$$\omega_n = \sqrt{525} = 22.9 \text{ rad/s},$$

$$f_n = \frac{\omega_n}{2\pi} = \frac{22.9}{2\pi} = \underline{3.65 \text{ Hz}}$$

8/113 Let $V = 0$ at $\theta = 0$, which is position of static equilibrium.

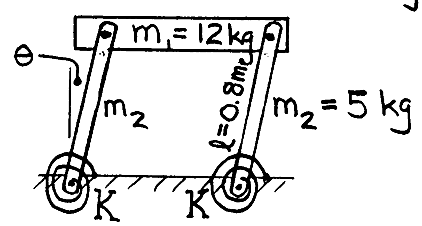
$$E = T + V = \frac{1}{2} I \dot{\theta}^2 + mg \frac{b}{2}(1 - \cos\theta)\sin\alpha$$
$$= \frac{1}{2}\left(\frac{1}{3} m b^2\right)\dot{\theta}^2 + mg \frac{b}{2}(1 - \cos\theta)\sin\alpha$$
$$= \frac{1}{6} m b^2 \dot{\theta}^2 + mg \frac{b}{2}(1 - \cos\theta)\sin\alpha$$

Set $\frac{dE}{dt} = 0$: $\frac{1}{3} m b^2 \dot{\theta}\ddot{\theta} + mg\frac{b}{2}\sin\theta \sin\alpha = 0$

Small θ: $\ddot{\theta} + \left(\frac{3g}{2b}\sin\alpha\right)\theta = 0$

$$\omega_n = \sqrt{\frac{3g\sin\alpha}{2b}}, \quad \tau = \frac{2\pi}{\omega_n} = \underline{2\pi\sqrt{\frac{2b}{3g\sin\alpha}}}$$

8/114



(θ small)

$$V_g = V_{g_1} + V_{g_2}$$
$$= -m_1 g \ell (1-\cos\theta)$$
$$- 2m_2 g \frac{\ell}{2}(1-\cos\theta)$$
$$= -(m_1+m_2) g \ell (1-\cos\theta)$$
$$\cong -(m_1+m_2) g \ell \frac{\theta^2}{2}$$

$$V_e = 2\left(\tfrac{1}{2}K\theta^2\right) = K\theta^2$$

$$V_{max} = -(m_1+m_2) g \ell \frac{\theta_{max}^2}{2} + K\theta_{max}^2$$
$$= \left[K - \frac{m_1+m_2}{2} g \ell\right]\theta_{max}^2$$
$$= \left[500 - \frac{12+5}{2}(9.81)(0.8)\right]\theta_{max}^2 = 433.3\ \theta_{max}^2$$

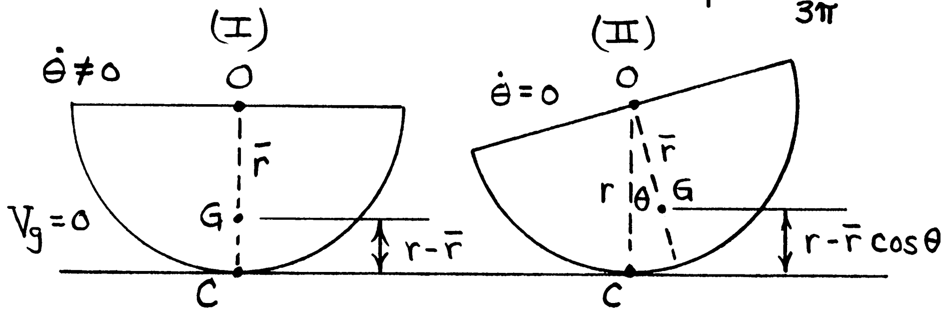
$$T = \tfrac{1}{2}m_1 v_1^2 + 2\left(\tfrac{1}{2}I\dot\theta^2\right) = \tfrac{1}{2}m_1(\ell\dot\theta)^2 + 2\left(\tfrac{1}{2}\cdot\tfrac{1}{3}m_2\ell^2\dot\theta^2\right)$$
$$= \left(\tfrac{1}{2}m_1 + \tfrac{1}{3}m_2\right)\ell^2\dot\theta^2$$

$$T_{max} = \left(\frac{12}{2} + \frac{5}{3}\right)(0.8)^2\dot\theta_{max}^2 = 4.907\dot\theta_{max}^2$$
$$= 4.907(\theta_{max}\omega_n)^2 = 4.907\omega_n^2\theta_{max}^2$$

Set $T_{max} = V_{max}$ & obtain $\omega_n = 9.40$ rad/s, $\underline{f_n = 1.496 \text{ Hz}}$

8/115

$\bar{r} = \frac{4r}{3\pi}$



In position II, $(V_g)_{max} = mg\left[(r - \bar{r}\cos\theta) - (r - \bar{r})\right]$

$$= mg\left(\frac{4r}{3\pi}\right)(1 - \cos\theta)$$

In position I, $T_{max} = \frac{1}{2} I_C \dot{\theta}^2$

$I_C = \bar{I} + m(r - \bar{r})^2 = I_O - m\bar{r}^2 + m(r - \bar{r})^2$

$= \frac{1}{2}mr^2 + mr^2 - 2mr\bar{r} = \left(\frac{3}{2} - \frac{8}{3\pi}\right)mr^2$

$T_{max} = (V_g)_{max}: \quad mg\frac{4r}{3\pi}(1 - \cos\theta) = \frac{1}{2}\left(\frac{3}{2} - \frac{8}{3\pi}\right)mr^2\dot{\theta}^2$

For small θ, replace $\cos\theta$ by $1 - \frac{\theta^2}{2}$

For harmonic oscillation, $\dot{\theta}_{max} = \theta\omega_n$

So $\frac{4g}{3\pi}\left[1 - \left(1 - \frac{\theta^2}{2}\right)\right] = \left(\frac{3}{4} - \frac{4}{3\pi}\right)r\theta^2\omega_n^2$

$\omega_n = 0.807\sqrt{g/r}$, $\quad \tau = \frac{2\pi}{\omega_n} = \underline{7.78\sqrt{r/g}}$

8/116 Let y_o = amplitude of vertical deflection of frame & body

$\frac{12}{18}y_o = \frac{2}{3}y_o$ = corresponding spring deflection

ΔV = change in $V_e + V_g$ due only to y_o so

$$\Delta V = 2\left(\frac{1}{2}\right)(270)\left(\frac{2}{3}y_o\right)^2 = 120\,y_o^2 \text{ in.-lb}$$

$$\Delta T = T_{max} = \frac{1}{2}\,\frac{1800}{32.2\times12}\,\dot{y}_{max}^2 \quad \text{but } \dot{y}_{max} = y_o\omega_n$$

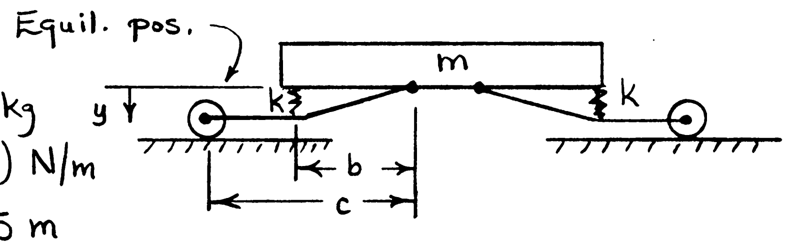
so $T_{max} = 2.33\,y_o^2\omega_n^2$

Thus with $T_{max} = \Delta V$, $2.33y_o^2\omega_n^2 = 120y_o^2$

$$\& \ \omega_n^2 = \frac{120}{2.33}, \quad \omega_n = 7.18 \frac{\text{rad}}{\text{sec}}, \quad f_n = \frac{\omega_n}{2\pi} = \frac{7.18}{2\pi}$$

$$= \underline{1.142 \text{ Hz}}$$

8/117



$m = 24$ kg

$k = 9(10^3)$ N/m

$b = 0.15$ m

$c = 0.25$ m

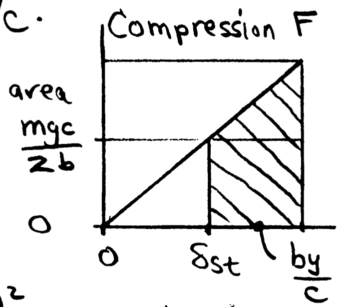
Let y = maximum downward deflection of m

Static spring deflection $\delta_{st} = \dfrac{F_{st}}{k}$, where

$F_{st} = \dfrac{mg}{2}\dfrac{c}{b}$ (from equilibrium); so $\delta_{st} = \dfrac{mgc}{2kb}$

Added spring deflection is by/c.

For each spring, ΔV_e = hatched area



$\Delta V_e = \frac{1}{2}k\left(\frac{by}{c}\right)^2 + \frac{mg}{2}\frac{c}{b}\left(\frac{by}{c}\right)$

For both, $\Delta V_e = k\dfrac{b^2y^2}{c^2} + mgy$

$\Delta V_g = -mgy$

$V_{max} = k\dfrac{b^2y^2}{c^2} + mgy - mgy = k\dfrac{b^2y^2}{c^2}$; $T_{max} = \frac{1}{2}m(\omega_n y)^2$

$V_{max} = T_{max}$: $\frac{1}{2}m\omega_n^2 y^2 = k\dfrac{b^2y^2}{c^2}$, $\omega_n = \dfrac{2kb^2}{mc^2}$

$\omega_n = \left[\dfrac{2(9)(10^3)(0.15)^2}{24(0.25)^2}\right]^{1/2} = 16.43\ \dfrac{\text{rad}}{s}$, $f_n = \dfrac{\omega_n}{2\pi} = \underline{2.62\text{ Hz}}$

8/118 $V_{max} = T_{max}$

$V_{max} = mgh = mgl(1-\cos\beta_o) = mgl(1-[1-\frac{\beta_o^2}{2!}+\dots]) = \frac{1}{2}mgl\beta_o^2$

But $l\beta_o \approx b\theta_o$ so $V_{max} = \frac{1}{2}mg\frac{b^2\theta_o^2}{l}$ where

θ_o = max. angular twist

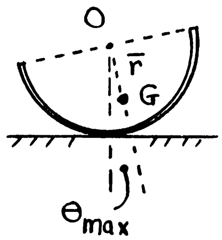
$T_{max} = \frac{1}{2}I_o\dot{\theta}^2_{max}$ where $\dot{\theta} = \theta_o\omega_n\cos\omega_n t$ & $\dot{\theta}^2_{max} = \theta_o^2\omega_n^2$

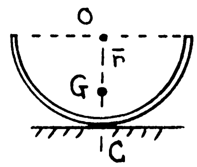
& $T_{max} = \frac{1}{2}(\frac{1}{12}m[2b]^2)\theta_o^2\omega_n^2 = \frac{1}{6}mb^2\theta_o^2\omega_n^2$

Thus $\frac{1}{2}mg\frac{b^2\theta_o^2}{l} = \frac{1}{6}mb^2\theta_o^2\omega_n^2$, $\omega_n = \sqrt{3g/l}$

so $\underline{\tau = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{l}{3g}}}$

8/119





$$V_{max} = mg\bar{r}(1-\cos\theta_{max})$$
$$= mg\frac{2r}{\pi}(1-\cos\theta_{max})$$

For θ small, $\cos\theta \cong 1 - \theta^2/2$

Thus $V_{max} = mgr\,\theta_{max}^2/\pi$

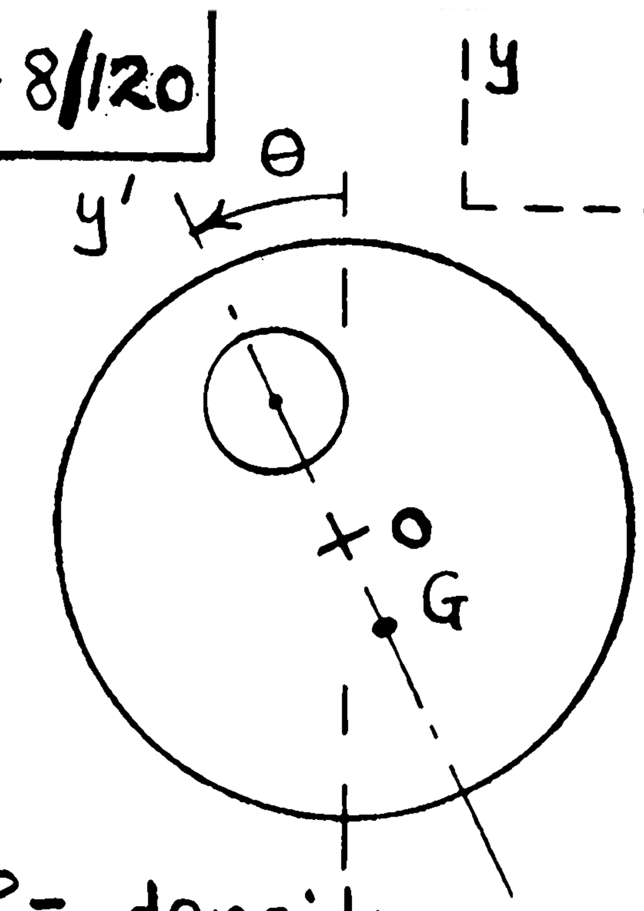
$$T = \frac{1}{2}I_C\omega^2 = \frac{1}{2}\left[\bar{I} + m(r-\bar{r})^2\right]\omega^2$$
$$= \frac{1}{2}\left[I_O - m\bar{r}^2 + m(r-\bar{r})^2\right]\omega^2 = mr^2\left(1-\frac{2}{\pi}\right)\omega^2$$

$$T_{max} = mr^2\left(1-\frac{2}{\pi}\right)(\omega_n\theta_{max})^2$$

$$T_{max} = V_{max}: \quad mr^2\left(1-\frac{2}{\pi}\right)\omega_n^2\theta_{max}^2 = mgr\,\theta_{max}^2/\pi$$

$$\omega_n^2 = \frac{g}{\pi r(1-2/\pi)}, \quad \tau = \frac{2\pi}{\omega_n} = \underline{2\pi\sqrt{\frac{(\pi-2)r}{g}}}$$

▶ 8/120



ρ = density
t = length

$$\overline{Y}' = \frac{\Sigma \bar{y}' A}{\Sigma A} = \frac{-\frac{R}{2}(\pi)\left(\frac{R}{4}\right)^2}{\pi R^2 - \pi\left(\frac{R}{4}\right)^2}$$

$$= -\frac{R}{30}, \quad \text{So } \overline{OG} = R/30$$

$$I_o = I_{whole} - I_{hole}$$

$$= \frac{1}{2} m_1 R^2 - \left[\frac{1}{2} m_2 \left(\frac{R}{4}\right)^2 + m_2 \left(\frac{R}{2}\right)^2\right]$$

$$= \frac{247}{512} \pi t \rho R^4$$

But $m = \pi \rho t \left[\pi R^2 - \pi\left(\frac{R}{4}\right)^2\right] = \frac{15}{16}\pi \rho t R^2$

$$\therefore I_o = \frac{247}{512} \pi t \rho R^4 \left(\frac{m}{\frac{15}{16}\pi \rho t R^2}\right) = \frac{247}{480} m R^2$$

$$\bar{I} = I_o - m\left(\overline{OG}^2\right) = \frac{247}{480} m R^2 - m\left(\frac{R}{30}\right)^2$$

$$= 0.5135\, m R^2$$

$$\underline{v}_G = \underline{v}_o + \underline{\omega} \times \underline{r}_{G/O} = -R\omega \underline{i} + \omega \underline{k} \times \frac{R}{30}\left[\sin\theta \underline{i} - \cos\theta \underline{j}\right]$$

$$= \omega\left[\left(\frac{R}{30}\cos\theta - R\right)\underline{i} + \left(\frac{R}{30}\sin\theta\right)\underline{j}\right]$$

$$v_G^2 = \omega^2\left[1.001 R^2 - \frac{1}{15} R^2 \cos\theta\right]$$

Now, $\quad E = T + V$

$$E = \frac{1}{2}\bar{I}\dot{\theta}^2 + \frac{1}{2} m v_G^2 + mg\frac{R}{30}(1 - \cos\theta)$$

$$E = \frac{1}{2}\left(0.5135\, mR^2\right)\dot{\theta}^2 + \frac{1}{2} m\left[\dot{\theta}^2\left(1.001 R^2 - \frac{1}{15}R^2\cos\theta\right)\right]$$
$$+ mg\frac{R}{30}(1 - \cos\theta)$$

$$\frac{dE}{dt} = \left(0.5135\, \cancel{m} R^2\right)\cancel{\dot{\theta}}\ddot{\theta} + \cancel{m}\cancel{\dot{\theta}}\ddot{\theta}\left(1.001 R^2 - \frac{1}{15}R^2\cos\theta\right)$$
$$+ \frac{1}{2}\cancel{m}\dot{\theta}^2\left(\frac{1}{15}R^2\sin\theta\,\cancel{\dot{\theta}}\right) + \cancel{m}g\frac{R}{30}\sin\theta\,\cancel{\dot{\theta}} = 0$$

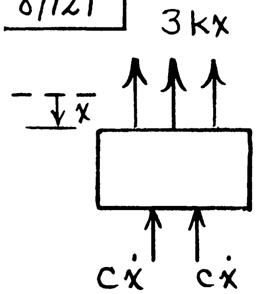
We now assume small θ:

$$1.448 R^2\ddot{\theta} + 0.0333\, gR\theta = 0$$

$$\ddot{\theta} + 0.0230\frac{g}{R}\theta = 0$$

$$\tau = \frac{2\pi}{\omega_n} = \underline{41.4\sqrt{\frac{R}{g}}}$$

8/121



$$\Sigma F_x = m\ddot{x} : -3kx - 2c\dot{x} = m\ddot{x}$$

$$\ddot{x} + \frac{2c}{m}\dot{x} + \frac{3k}{m}x = 0$$

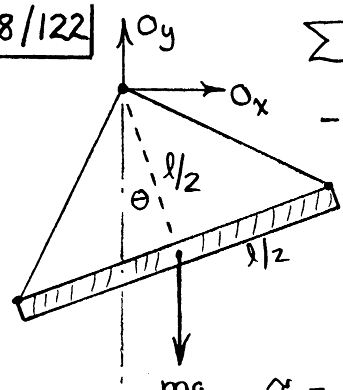
$$\omega_n = \sqrt{\frac{3k}{m}} = \sqrt{\frac{3(70000)}{100}}$$

$$= 45.8 \text{ rad/s}$$

$$2\zeta\omega_n = \frac{2c}{m}, \quad c = \overset{\nearrow 1}{\zeta} m\omega_n = 100(45.8)$$

$$= \underline{4580 \text{ N·s/m}}$$

8/122



$$\Sigma M_O = I_O \ddot{\theta} :$$

$$-mg\frac{l}{2}\sin\theta = \left[\frac{1}{12}ml^2 + m\left(\frac{l}{2}\right)^2\right]\ddot{\theta}$$

For small θ,

$$\ddot{\theta} + \frac{3g}{2l}\theta = 0$$

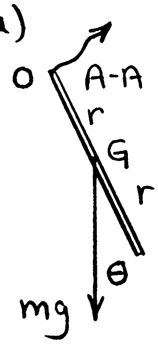
$$\tau = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{2l}{3g}} = \underline{7.33\text{ s}}$$

8/123

$$I_{A\text{-}A} = \frac{1}{4}mr^2 + mr^2 = \frac{5}{4}mr^2$$

$$I_{B\text{-}B} = \frac{1}{2}mr^2 + mr^2 = \frac{3}{2}mr^2$$

(a)

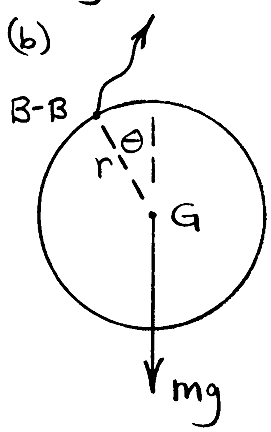


$$\circlearrowleft + \ \Sigma M_O = I_O\ddot{\theta}: \ -mgr\sin\theta = \frac{5}{4}mr^2\ddot{\theta}$$

Small θ: $\ddot{\theta} + \frac{4g}{5r}\theta = 0$

$$\underline{\omega_n = 2\sqrt{\frac{g}{5r}}}$$

(b)



$$\circlearrowleft \ \Sigma M_O = I_O\ddot{\theta}: \ -mgr\sin\theta = \frac{3}{2}mr^2\ddot{\theta}$$

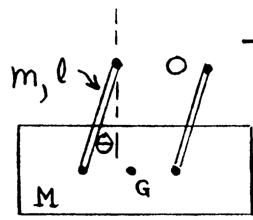
Small θ: $\ddot{\theta} + \frac{2}{3}\frac{g}{r}\theta = 0$

$$\underline{\omega_n = \sqrt{\frac{2g}{3r}}}$$

8/124

$$E = T + V = \frac{1}{2} M v_G^2 + Mgl(1-\cos\theta)$$

$$+ 2\left(\frac{1}{2} I_{rod_O} \dot{\theta}^2\right) + 2mg\frac{l}{2}(1-\cos\theta)$$

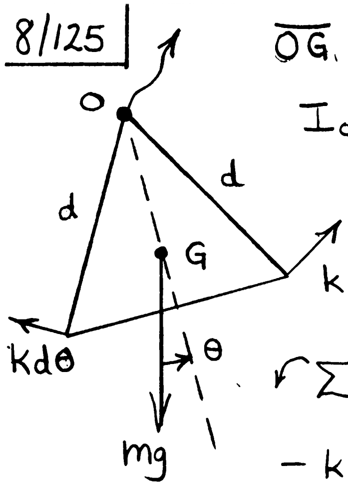


With $I_{rod_O} = \frac{1}{3} m l^2$,

set $\frac{dE}{dt} = 0$ to obtain

$$\ddot{\theta} + \frac{g}{l}\left(\frac{M+m}{M+\frac{2m}{3}}\right)\theta = 0, \quad \omega_n = \sqrt{\frac{g}{l}\left(\frac{M+m}{M+\frac{2m}{3}}\right)}$$

8/125



$$\overline{OG} = \frac{2}{3}\frac{\sqrt{3}}{2}d = \frac{\sqrt{3}}{3}d$$

$$I_O = 2\frac{1}{3}\left(\frac{m}{3}\right)d^2 + \left[\frac{1}{12}\frac{m}{3}d^2 + \frac{m}{3}\left(\frac{d\sqrt{3}}{2}\right)^2\right]$$

$$= \frac{1}{2}md^2$$

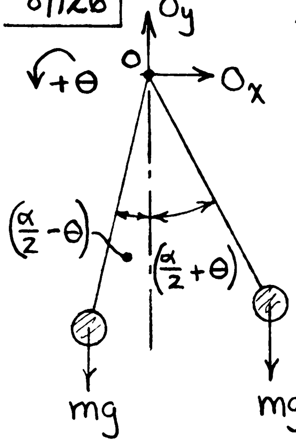
$$\curvearrowleft \Sigma M_O = I_O\ddot{\theta} : -mg\frac{\sqrt{3}}{3}d\theta$$

$$-kd\theta(d) + k(b\sin\omega t - d\theta)d = \frac{1}{2}md^2\ddot{\theta}$$

$$\ddot{\theta} + \left[\frac{mgd/\sqrt{3} + 2kd^2}{\frac{1}{2}md^2}\right]\theta = -\frac{2kb}{md}\sin\omega t$$

$$\underline{\omega_c = \omega_n = \sqrt{\frac{2g}{\sqrt{3}d} + 4\frac{k}{m}}}$$

8/126



$\sum M_O = I_O \ddot{\theta}$:

$$-mgl\sin\left(\frac{\alpha}{2}+\theta\right) + mgl\sin\left(\frac{\alpha}{2}-\theta\right) = 2ml^2\ddot{\theta} \qquad (a)$$

$$\sin\left(\frac{\alpha}{2}+\theta\right) = \sin\frac{\alpha}{2}\cos\theta + \cos\frac{\alpha}{2}\sin\theta$$

(for θ small) $= \sin\frac{\alpha}{2} + \theta\cos\frac{\alpha}{2}$

Similarly,

$$\sin\left(\frac{\alpha}{2}-\theta\right) = \sin\frac{\alpha}{2} - \theta\cos\frac{\alpha}{2}$$

The equation of motion (a) becomes

$$\ddot{\theta} + \left(\frac{g}{l}\cos\frac{\alpha}{2}\right)\theta = 0$$

$$\tau = \frac{2\pi}{\omega_n} = \underline{2\pi\sqrt{\frac{l}{g\cos(\alpha/2)}}}$$

For $\alpha \to 0$, $\underline{\tau \to 2\pi\sqrt{l/g}}$; For $\alpha \to 180°$, $\underline{\tau \to \infty}$

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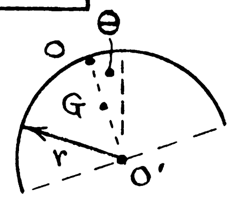

From Appendix D, $\overline{O'G} = 2r/\pi$

$$\bar{I} = I_{O'} - m(\overline{O'G})^2$$
$$= mr^2 - m\left(\frac{2r}{\pi}\right)^2 = mr^2\left(1 - \frac{4}{\pi^2}\right)$$
$$I_O = \bar{I} + m(\overline{OG}^2)$$
$$= mr^2\left(2 - \frac{4}{\pi}\right)$$

$$E = T + V = \frac{1}{2} I_O \dot{\theta}^2 + mg\,\overline{OG}\,(1 - \cos\theta)$$
$$= \frac{1}{2} mr^2\left(2 - \frac{4}{\pi}\right)\dot{\theta}^2 + mgr\,\frac{\pi - 2}{\pi}(1 - \cos\theta)$$

Set $\frac{dE}{dt} = 0$ and assume small θ:

$$\ddot{\theta} + \frac{g}{2r}\theta = 0, \quad \underline{f_n = \frac{1}{2\pi}\sqrt{\frac{g}{2r}}}$$

(Same result as for full circular shape!)

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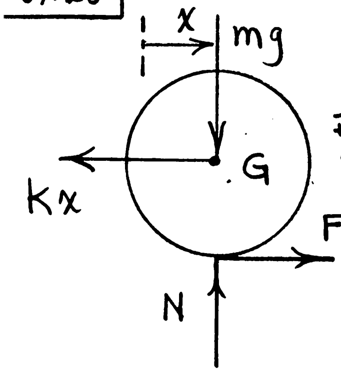

$$\Sigma F_y = 0 \Rightarrow N = mg$$

$$\Sigma F_x = m\ddot{x} : \quad F - kx = m\ddot{x} \qquad (1)$$

$$\Sigma M_G = \bar{I}\ddot{\theta} : \quad -Fr = \tfrac{1}{2}mr^2\ddot{\theta} \qquad (2)$$

$$\text{Constraint} : \quad \ddot{x} = r\ddot{\theta} \qquad (3)$$

Combine (1), (2), & (3):

$$\ddot{x} + \frac{2k}{3m}x = 0$$

$$x = x_0 \sin \omega_n t, \quad \ddot{x} = -x_0\omega_n^2 \sin\omega_n t, \quad \ddot{x}_{max} = x_0\omega_n^2$$

$$\ddot{\theta}_{max} = \frac{\ddot{x}_{max}}{r} = \frac{x_0\omega_n^2}{r}$$

Eq. (2): $\quad |F_{max}| r = \frac{1}{2}mr^2\ddot{\theta}_{max}$

$$\mu_s mg\, r = \frac{1}{2}mr^2 \frac{x_0\omega_n^2}{r}$$

$$x_0 = \frac{2\mu_s g}{\omega_n^2} = \frac{2\mu_s g}{2k/3m} = \underline{\frac{3\mu_s mg}{k}}$$

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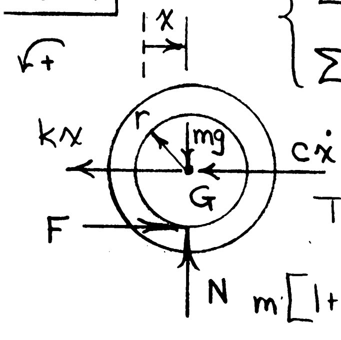

$$\begin{cases} \Sigma F_x = m\ddot{x} : -kx - c\dot{x} + F = m\ddot{x} \\ \Sigma M_G = \bar{I}\alpha : \quad Fr = mk_G^2\,\alpha \end{cases}$$

Roll with no slip : $\ddot{x} = -r\alpha$

The x-equation reduces to

$$m\left[1 + \frac{k_G^2}{r^2}\right]\ddot{x} + c\dot{x} + kx = 0$$

$$\omega_n = \sqrt{\frac{k}{m\,(1 + k_G^2/r^2)}} = \sqrt{\frac{15(12)}{\frac{20}{32.2}\left(1 + \frac{5.5^2}{6^2}\right)}} = 12.55\ \frac{\text{rad}}{\text{sec}}$$

$$\zeta = \frac{c}{2\omega_n m\left(1 + \frac{k_G^2}{r^2}\right)} = \frac{2}{2(12.55)\left(\frac{20}{32.2}\right)\left(1 + \frac{5.5^2}{6^2}\right)}$$

$\underline{\zeta = 0.0697}$

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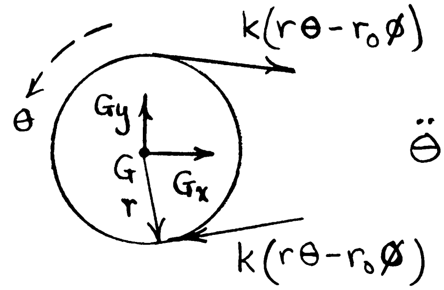


$\Sigma M_G = \bar{I}\ddot{\theta}$:

$$-2k(r\theta - r_0\phi)r = m\bar{k}^2\ddot{\theta}$$

$$\ddot{\theta} + \frac{2kr^2}{m\bar{k}^2}\theta = \frac{2krr_0\phi_0}{m\bar{k}^2}\cos\omega t$$

Assume $\theta = \theta_{max}\cos\omega t$, substitute, and solve for $\theta_{MAX} = \phi_0 \dfrac{r_0/r}{1-\left(\frac{\omega}{\omega_n}\right)^2}$,

where $\omega_n = \dfrac{r}{\bar{k}}\sqrt{\dfrac{2k}{m}}$

8/131 Equivalent spring constant is $k = F/\delta = 3/0.6 = 5$ lb/in.

From Eq. 8/19

$$X = \frac{\delta_{st}}{1-(\omega/\omega_n)^2} \quad \text{where } \delta_{st} = 0.30 \text{ in.}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{5}{5/(32.2 \times 12)}} = \sqrt{386}$$

$$= 19.66 \text{ rad/sec}$$

$$\omega = 2(2\pi) = 4\pi \text{ rad/sec}$$

$$(\omega/\omega_n)^2 = (4\pi/19.66)^2 = 0.409, \quad X = \frac{0.30}{1-0.409} = \underline{0.507 \text{ in.}}$$

8/132 For seismic instruments,

$$\frac{X}{b} = \frac{(\omega/\omega_n)^2}{\left\{\left[1-(\omega/\omega_n)^2\right]^2 + \left[2\zeta\frac{\omega}{\omega_n}\right]^2\right\}^{1/2}}$$

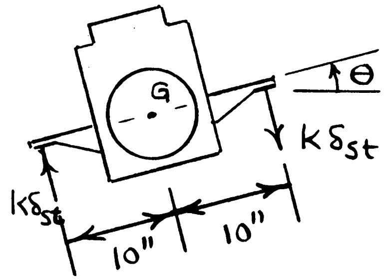
For $X = 0.75$ mm, $\zeta = 0.5$, $\frac{\omega}{\omega_n} = \frac{\frac{180}{60}(3)}{3} = 3$,

solve for $b = \delta_0 = \underline{0.712 \text{ mm}}$

8/133 Vertical vibration :

$$(f_n)_y = \frac{\omega_n}{2\pi} = \frac{1}{2\pi}\sqrt{k/m} = \frac{1}{2\pi}\sqrt{\frac{2(600)(12)}{480/32.2}} = \underline{4.95 \text{ Hz}}$$

Rotation about G :



Force changes for position of static equilibrium.

$k\delta_{st} = 600(10\theta)$
$= 6000\,\theta$ lb
(θ small)

$$\sum M_G = I_G\ddot{\theta} : -2(6000\,\theta)\left(\frac{10}{12}\right) = \frac{48.0}{32.2}\left(\frac{4.60}{12}\right)^2\ddot{\theta}$$

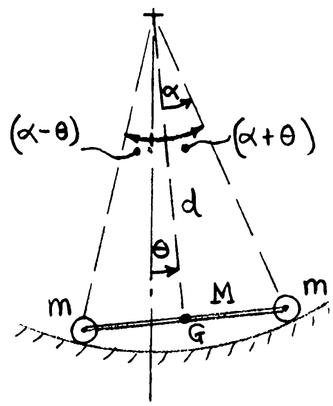
$$\ddot{\theta} + 4570\,\theta = 0, \quad \omega_n = 67.6 \text{ rad/sec}$$

$$(f_n)_\theta = \omega_n/2\pi = \underline{10.75 \text{ Hz}}$$

Critical speed : $N = \omega_n = 10.75(60) = \underline{645 \frac{\text{rev}}{\text{min}}}$

8/134 $\alpha = \sin^{-1}\left(\frac{l}{2R}\right)$, $d = \frac{l}{2\tan\alpha} = \frac{l}{2l/\sqrt{4R^2-l^2}}$

$= \sqrt{4R^2-l^2}\,/2$



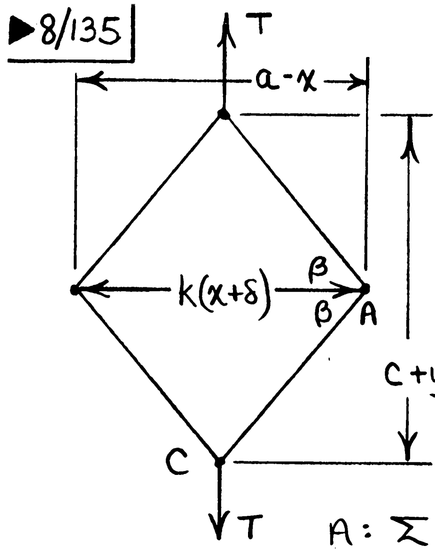
$$E = T + V = 2\left(\tfrac{1}{2}mv^2\right) + \tfrac{1}{2}Mv_G^2 + \tfrac{1}{2}\bar{I}\dot{\theta}^2 + mgR[\cos\alpha - \cos(\alpha+\theta)] + mgR[\cos\alpha - \cos(\alpha-\theta)] + Mgd[1-\cos\theta]$$

Set $\frac{dE}{dt} = 0$, substitute $v = R\dot{\theta}$, $v_G = d\dot{\theta}$, $\cos\alpha = \sqrt{4R^2-l^2}/2R$, the above expression for d, and assume small θ to obtain

$$\left[2mR^2 + MR^2 - \tfrac{1}{6}Ml^2\right]\ddot{\theta} + \left[g\sqrt{4R^2-l^2}\left(m+\tfrac{M}{2}\right)\right]\theta = 0$$

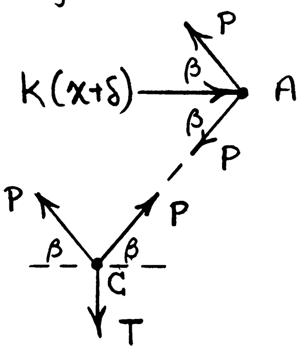
$$\tau = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{2mR^2 + MR^2 - \frac{1}{6}Ml^2}{g\sqrt{4R^2-l^2}\left(m+\frac{M}{2}\right)}}$$

▶8/135



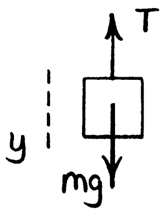
$x = y = 0$ @ equilibrium

Spring compressed δ at eq.



$A: \Sigma F_x = 0: \; k(x+\delta) - 2P\cos\beta = 0$

$C: \Sigma F_y = 0: \; -2P\sin\beta + T = 0$



mass: $\Sigma F_y = ma_y: \; mg - T = m\ddot{y}$

So $m\ddot{y} = mg - 2P\sin\beta$

$= mg - k(x+\delta)\tan\beta$

$= mg - k(x+\delta)\dfrac{c+y}{a-x}$

$\Rightarrow (a-x)\ddot{y} + \frac{k}{m}(x+\delta)(c+y) = (a-x)g$ (1)

Now, $(a-x)^2 + (c+y)^2 = (2b)^2 = a^2 + c^2$

$a^2 - 2ax + x^2 + c^2 + 2cy + y^2 = a^2 + c^2$

$\Rightarrow$ $ax = cy$, to first order. (2)

Expand (1) :

$$a\ddot{y} - x\ddot{y} + \frac{k}{m}cx + \frac{k}{m}c\delta + \frac{k}{m}xy + \frac{k}{m}\delta y = ag - xg$$

$\ddot{y} = y = 0$ when $x = 0$: $\quad \frac{k}{m}c\delta = ag \qquad (3)$

With (2) and (3)

$$a\ddot{y} - \frac{c}{a}y\ddot{y} + \frac{k}{m}c\frac{c}{a}y + \frac{k}{m}\frac{c}{a}yy + \frac{k}{m}\delta y = -\frac{c}{a}yg$$

Neglect HOT :

$$a\ddot{y} + \frac{k}{m}\frac{c^2}{a}y + \frac{k}{m}\delta y + \frac{c}{a}yg = 0$$

Use (2) : $\quad a\ddot{y} + \frac{k}{m}\frac{c^2}{a}y + \frac{ag}{c}y + \frac{cg}{a}y = 0$

$$\ddot{y} + \left[\left(\frac{c}{a}\right)^2\frac{k}{m} + \left(1 + \left(\frac{c}{a}\right)^2\right)\frac{g}{c}\right]y = 0$$

With $\quad a = c = b\sqrt{2}$:

$$\ddot{y} + \left[\frac{k}{m} + \sqrt{2}\,\frac{g}{b}\right]y = 0$$

$$\underline{\omega_n = \sqrt{\frac{k}{m} + \sqrt{2}\,\frac{g}{b}}}$$

***8/136**

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{100}{2}} = 7.071 \text{ rad/s}$$

$$\zeta = \frac{c}{2m\omega_n} = \frac{50}{2(2)(7.071)} = 1.768$$

$$\lambda_{1,2} = \omega_n\left[-\zeta \pm \sqrt{\zeta^2-1}\right], \quad \begin{cases} \lambda_1 = -2.192 \\ \lambda_2 = -22.81 \end{cases}$$

$$x = A_1 e^{-2.192t} + A_2 e^{-22.81t}$$

Determine A_1 and A_2 via initial conditions:

$$x = -0.1319 e^{-2.192t} + 0.2319 e^{-22.81t}$$

For $x = -0.05$ m, we have

$$f(t) = -0.1319 e^{-2.192t} + 0.2319 e^{-22.81t} + 0.05 = 0$$

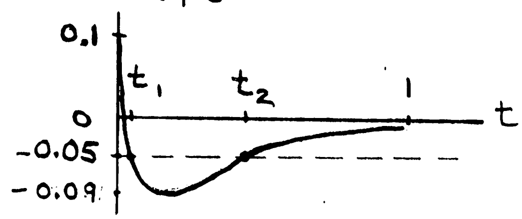
$$f'(t) = 0.2892 e^{-2.192t} - 5.289 e^{-22.81t}$$

Newton's method:

$t_1 = \underline{0.0544 \text{ s}}$

or

$t_2 = \underline{0.442 \text{ s}}$



*8/137 $\omega_n = \sqrt{k/m} = \sqrt{1280/10} = 10.39$ rad/s

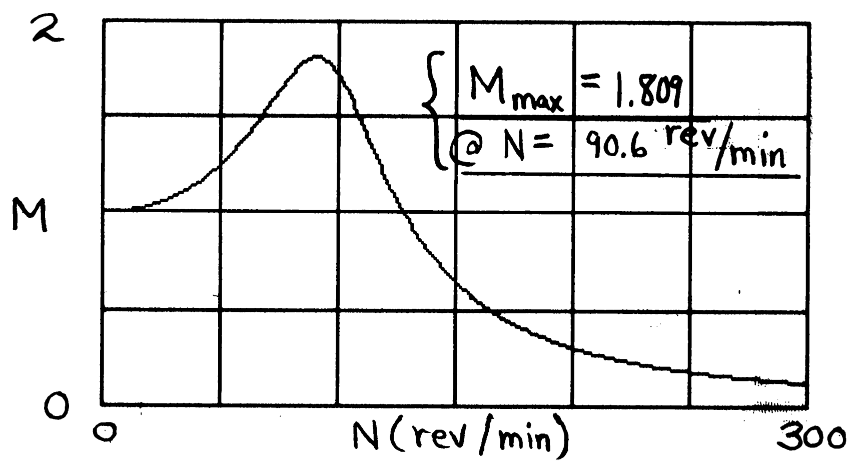
$N_n = 10.39\left(\frac{60}{2\pi}\right) = 99.2$ rev/min

$c = \frac{F}{\dot{x}} = \frac{30}{0.5} = 60$ N·s/m; $\zeta = \frac{c}{2m\omega_n} = \frac{60}{2(10)(10.39)} = \underline{0.289}$

$$M = \frac{1}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\zeta\frac{\omega}{\omega_n}\right]^2}}, \text{ where } \frac{\omega}{\omega_n} = \frac{N}{99.2}$$

$$= \frac{1}{\sqrt{\left[1-\left(\frac{N}{99.2}\right)^2\right]^2 + \left[2(0.289)\frac{N}{99.2}\right]^2}}$$

$$= \frac{1}{\sqrt{\left[1-\left(\frac{N}{99.2}\right)^2\right]^2 + \left[0.00582N\right]^2}}$$



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$$\omega_n = \sqrt{k/m} = \sqrt{\frac{100(12)}{50/32.2}} = 27.8 \text{ rad/sec}$$

$$\zeta = \frac{c}{2m\omega_n} = \frac{18}{2\left(\frac{50}{32.2}\right)(27.8)} = 0.208$$

$$\frac{\omega}{\omega_n} = \frac{60}{27.8} = 2.158, \quad \omega_d = 27.8\sqrt{1-0.208^2} = 27.2 \text{ rad/sec}$$

$$X = \frac{160/1200}{\left\{\left[1-2.158^2\right]^2 + \left[2(0.208)(2.158)\right]^2\right\}^{1/2}} = 0.03539 \text{ ft}$$

$$\phi = \tan^{-1}\left[\frac{2(.208)(2.158)}{1-2.158^2}\right] = 2.90 \text{ rad}$$

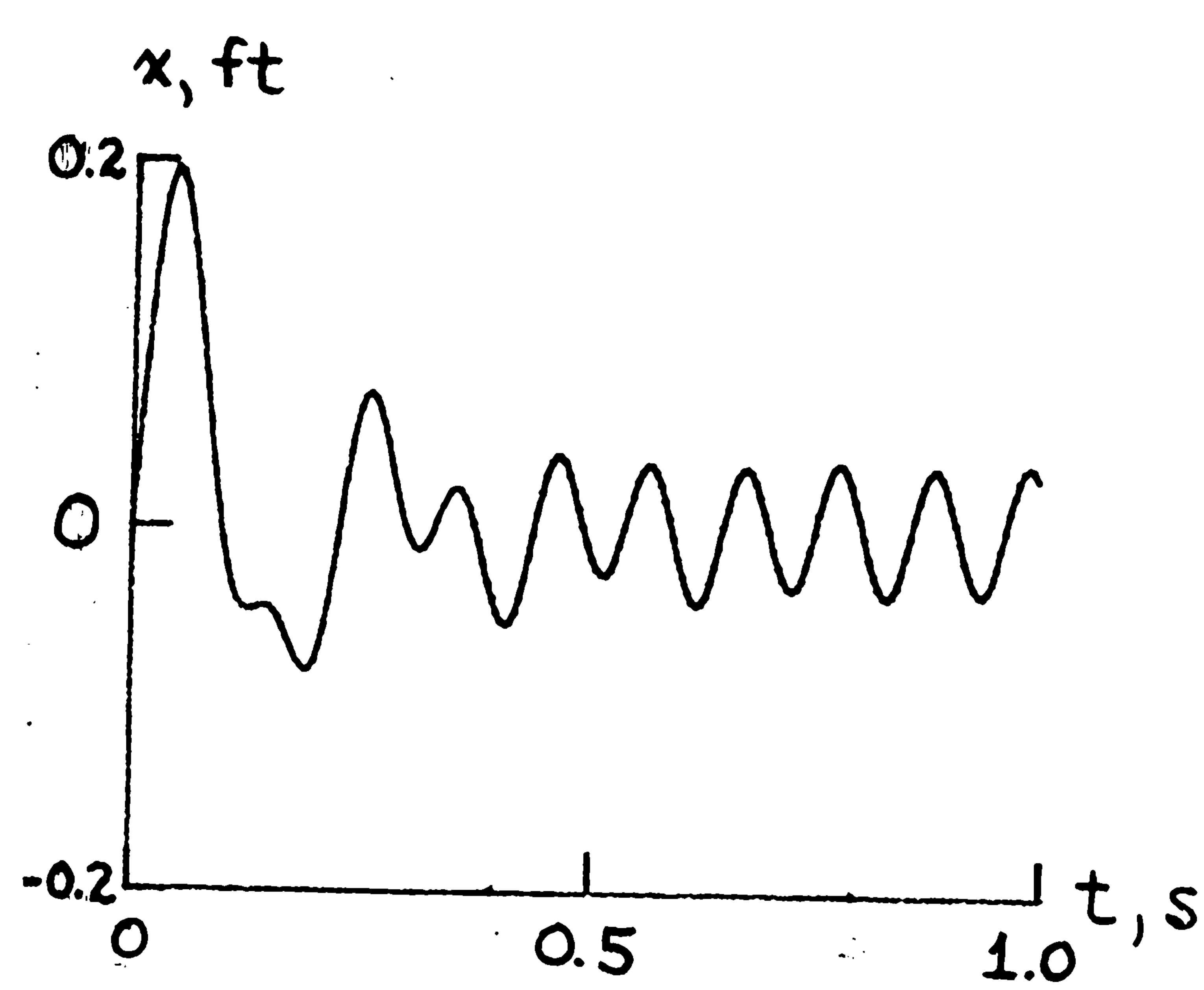
So $x = Ce^{-\zeta\omega_n t}\cos(\omega_d t - \psi) + X\cos(\omega t - \phi)$

$$= Ce^{-5.796t}\cos(27.2t - \psi) + 0.0354\cos(60t - 2.9)$$

Determine C and ψ : $\begin{cases} C = 0.212 \text{ ft} \\ \psi = 1.408 \text{ rad} \end{cases}$

$x_{max} = 0.1955$ ft @ $t = 0.046$ sec

$x_{min} = -0.079$ ft @ $t = 0.192$ sec



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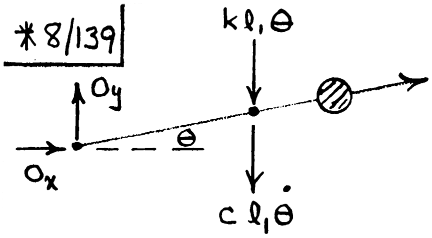


Eq. 4/13: (with P=0)

$$\Sigma \underline{M}_O = \dot{\underline{H}}_{O_r} + \bar{\underline{\rho}} \times m\underline{a}_O$$

For small θ,

$$-kl_1^2\theta - cl_1^2\dot\theta = ml_2^2\ddot\theta + ml_2\ddot y_B$$

or $$\ddot\theta + \frac{cl_1^2}{ml_2^2}\dot\theta + \frac{kl_1^2}{ml_2^2}\theta = \frac{b}{l_2}\omega^2\sin\omega t$$

Steady-state amplitude:

$$\Theta = Mb\left(\frac{\omega}{\omega_n}\right)^2\frac{1}{l_2}\,,$$ where M = magnification factor

Pen amplitude $= l_3\Theta = Mb\left(\frac{\omega}{\omega_n}\right)^2\frac{l_3}{l_2} = A$

Set up computer program to determine range of k for which $A \le 1.5b$. Note that $\omega_n = \frac{l_1}{l_2}\sqrt{\frac{k}{m}}$, $2\zeta\omega_n = \frac{cl_1^2}{ml_2^2}$ or

$$\zeta = \frac{cl_1}{2l_2}\sqrt{\frac{1}{km}}.$$

Answer: $\underline{0 < k < 1.895 \frac{lb}{ft}}$

8/140 Eq. 8/9: $\ddot{y} + 2\zeta\omega_n\dot{y} + \omega_n^2 y = 0$

Solution: $y = A_1 e^{\lambda_1 t} + A_2 e^{\lambda_2 t}$

where $\lambda_{1,2} = \omega_n \left(-\zeta \pm \sqrt{\zeta^2 - 1}\right)$

$\omega_n = \sqrt{k/m} = \sqrt{800/4} = 14.14$ rad/s; $\zeta = \dfrac{c}{2m\omega_n}$

(a) $\zeta = \dfrac{124}{2(4)(14.14)} = 1.096$; (b) $\zeta = \dfrac{80}{2(4)(14.14)} = 0.707$

(a) $\zeta > 1$ (overdamped)

$\lambda_1 = 14.14\left(-1.096 + \sqrt{1.096^2 - 1}\right) = -9.16 \text{ s}^{-1}$

$\lambda_2 = 14.14\left(-1.096 - \sqrt{1.096^2 - 1}\right) = -21.8 \text{ s}^{-1}$

Initial condition considerations:

$$\left.\begin{array}{l} y_0 = 0.1 = A_1 + A_2 \\ \dot{y}_0 = 0 = A_1\lambda_1 + A_2\lambda_2 \end{array}\right\} \Rightarrow \begin{array}{l} A_1 = 0.1722 \text{ m} \\ A_2 = -0.0722 \text{ m} \end{array}$$

Solution: $\underline{y = 0.1722e^{-9.16t} - 0.0722e^{-21.8t} \text{ m}}$

(b) $\zeta < 1$ (underdamped)

Eq. (8/12): $y = Ce^{-\zeta\omega_n t}\sin[\omega_d t + \psi]$

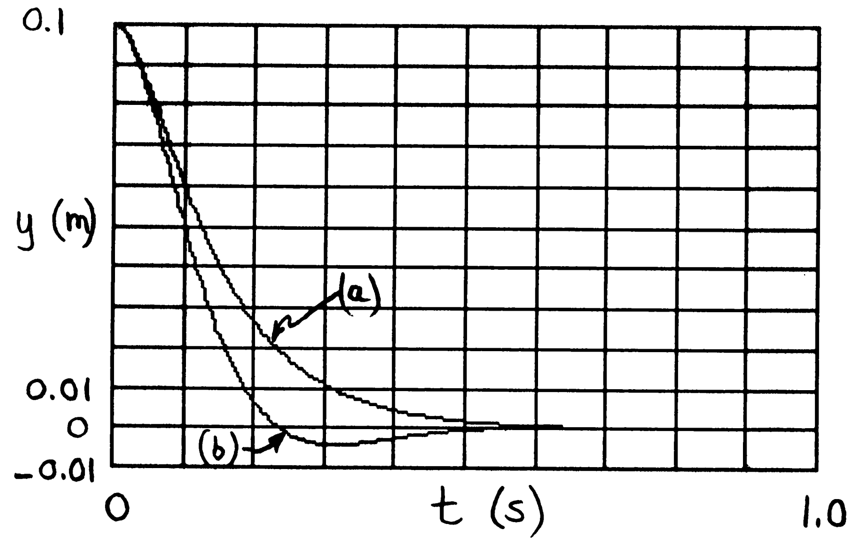
$\omega_d = \omega_n\sqrt{1-\zeta^2} = 14.14\sqrt{1-0.707^2} = 10$ rad/s

Initial condition considerations:

$$\left.\begin{aligned} y_o &= 0.1 = C\sin\psi \\ \dot{y}_o &= 0 = -\zeta\omega_n C\sin\psi + C\omega_d\cos\psi \end{aligned}\right\} \begin{aligned} C &= 0.1414 \text{ m} \\ \psi &= 0.785 \text{ rad} \end{aligned}$$

Solution: $y = 0.1414e^{-0.707(14.14)t}\sin[10t + 0.785]$

$= \underline{0.1414e^{-10t}\sin[10t + 0.785] \text{ m}}$



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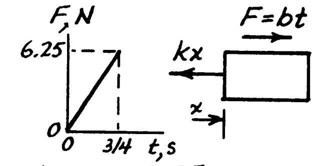
$\Sigma F_x = m\ddot{x}:\ bt - kx = m\ddot{x},$

$\ddot{x} + \frac{k}{m}x = \frac{bt}{m}$



Sol. is $x = x_c + x_p$ where

$x_c = C_1 \sin\omega_n t + C_2 \cos\omega_n t,\ x_p = C_3 t$ with $C_3 = \frac{b}{k}$

$b = \frac{6.25}{3/4} = 8.33$ N/s

so $x = C_1 \sin\omega_n t + C_2 \cos\omega_n t + \frac{b}{k}t$

$k = 90$ N/m

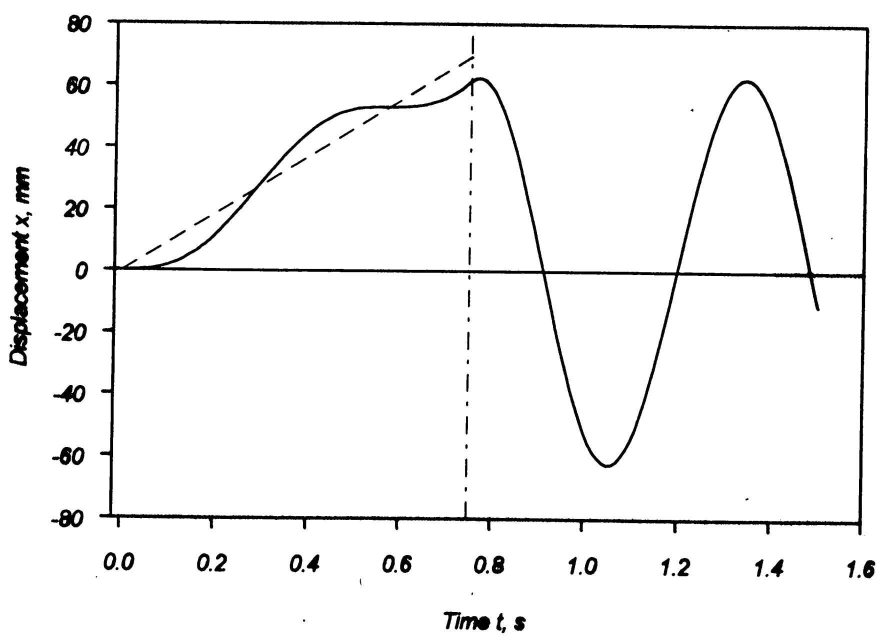
When $t = 0$, $\dot{x} = 0$ & $x = 0$ giving $C_1 = -\frac{b}{\omega_n k}$, $C_2 = 0$

$b/k = \frac{8.33}{90} = 0.0926$ m/s

& $x = -\frac{b}{\omega_n k}\sin\omega_n t + \frac{b}{k}t = \frac{b}{k}\left(t - \frac{1}{\omega_n}\sin\omega_n t\right)$

where $\omega_n = \sqrt{k/m} = \sqrt{90/0.75} = 10.95$ rad/s, $\frac{1}{\omega_n} = 0.0913$ s

Thus $\underline{x = 0.0926\,(t - 0.0913 \sin 10.95t)\ \text{m}}$ for first 3/4 s



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$I = \int F\,dt = m\dot{x}$, $8 = 4\dot{x}$, $\dot{x} = 2$ m/s at $t \approx 0$

After impulse, oscillator obeys Eq. 8/9 with $\zeta = 0.1 < 1$ so underdamped with solution given by Eq. 8/12

$$x = Ce^{-\zeta\omega_n t}\sin(\omega_d t + \psi), \quad C, \psi \text{ constants}$$

$$\dot{x} = -C\zeta\omega_n e^{-\zeta\omega_n t}\sin(\omega_d t + \psi) + Ce^{-\zeta\omega_n t}\omega_d\cos(\omega_d t + \psi)$$

where $\omega_n = \sqrt{k/m} = \sqrt{200/4} = 7.07$ rad/s,

$$\omega_d = \omega_n\sqrt{1-\zeta^2} = 7.07\sqrt{1-0.1^2} = 7.04 \text{ rad/s}$$

When $t = 0$, $x = 0$, so $0 = C\sin\psi$, $\psi = 0$

" ", $\dot{x} = 2$ m/s, so $2 = -C(0.1)(7.07)(0) + C \times 7.04$, $C = 0.284$ m

Thus $\underline{x = 0.284\,e^{-0.707t}\sin 7.04t}$