INSTRUCTOR'S MANUAL

To Accompany

ENGINEERING MECHANICS - DYNAMICS

Volume 2

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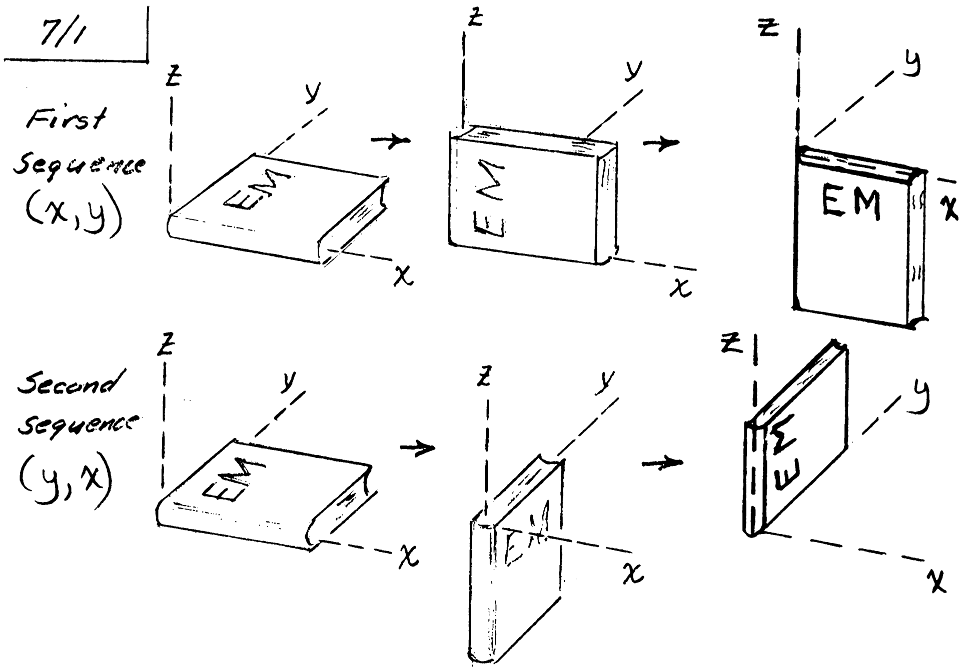
USE OF THE INSTRUCTOR'S MANUAL

The problem solution portion of this manual has been prepared for the instructor who wishes to occasionally refer to the authors' method of solution or who wishes to check the answer of his (her) solution with the result obtained by the authors. In the interest of space and the associated cost of educational materials, the solutions are very concise. Because the problem solution material is not intended for posting of solutions or classroom presentation, the authors request that it not be used for these purposes.

In the transparency master section there are approximately 65 solved problems selected to illustrate typical applications. These problems are different from and in addition to those in the textbook. Instructors who have adopted the textbook are granted permission to reproduce these masters for classroom use.

7/1



Final positions are different so finite rotations cannot be added as proper vectors

7/2

$\underline{v}_p$ & $\underline{\omega}$ are perpendicular so that $\underline{\omega}\cdot\underline{v} = 0$

$$\underline{\omega} = \frac{600\times 2\pi}{60}\,\frac{8\underline{i}+12\underline{j}+4\underline{k}}{\sqrt{8^2+12^2+4^2}}\text{ rad/sec},\quad \underline{v} = 12\underline{i} - 6\underline{j} + v_z\underline{k}$$

So $(8\underline{i}+12\underline{j}+4\underline{k})\cdot(12\underline{i}-6\underline{j}+v_z\underline{k}) = 0$

$96 - 72 + 4v_z = 0,\quad \underline{v_z = -6 \text{ ft/sec}}$

$v = \sqrt{12^2+(-6)^2+(-6)^2} = 14.70$ ft/sec

$R = v/\omega = 14.70/(20\pi) = 0.234$ ft or $\underline{R = 2.81 \text{ in.}}$

$a_p = a_n = r\omega^2 = 0.234(20\pi)^2 = 923$ ft/sec^2

or $\underline{a_p = 11{,}080 \text{ in./sec}^2}$

7/3

$\underline{a} = \dot{\underline{\omega}} \times \underline{r} + \underline{\omega} \times (\underline{\omega} \times \underline{r}), \quad \underline{r} = \vec{OC}, \quad \dot{\underline{\omega}} = \underline{0}$

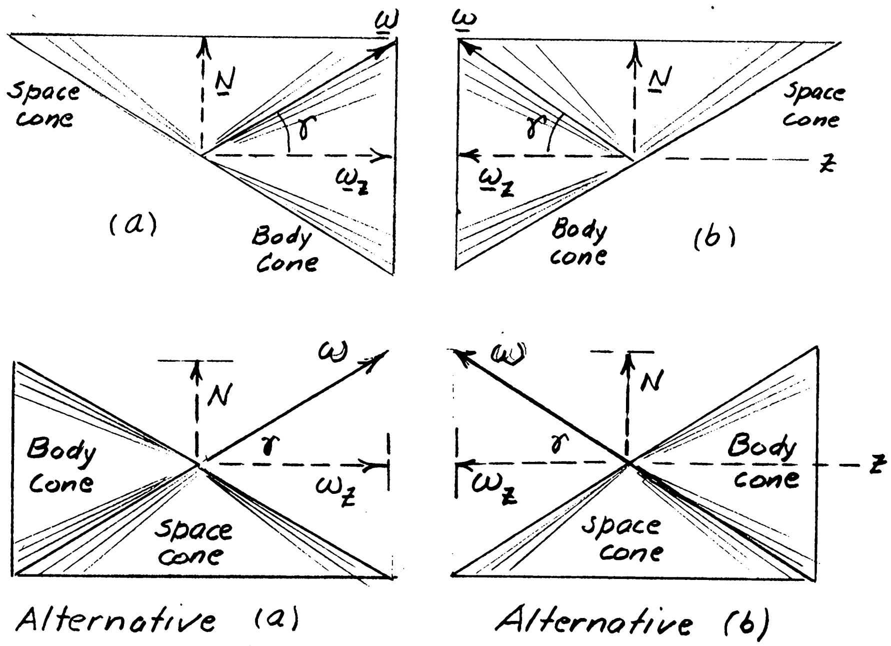
$\underline{r} = 10(2\underline{i} + 0\underline{j} + 8\underline{k})$ mm, $\underline{\omega} = 30(3\underline{i} + 2\underline{j} + 6\underline{k})$ rad/s

$$\underline{v} = \underline{\omega} \times \underline{r} = 300 \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 3 & 2 & 6 \\ 2 & 0 & 8 \end{vmatrix} = 300(16\underline{i} - 12\underline{j} - 4\underline{k})\ \frac{\text{mm}}{\text{s}}$$

$$\underline{a} = \underline{\omega} \times \underline{v} = 30(300) \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 3 & 2 & 6 \\ 16 & -12 & -4 \end{vmatrix} = 9000\left(64\underline{i} + 108\underline{j} - 68\underline{k}\right)\ \text{mm/s}^2$$

$$a = 9\sqrt{64^2 + 108^2 + (-68)^2} = 9\sqrt{20384} = \underline{1285\ \text{m/s}^2}$$

7/4 $\tan \gamma = \dfrac{N}{\omega_z} = \dfrac{10}{15} = 0.667, \quad \gamma = 33.7°$



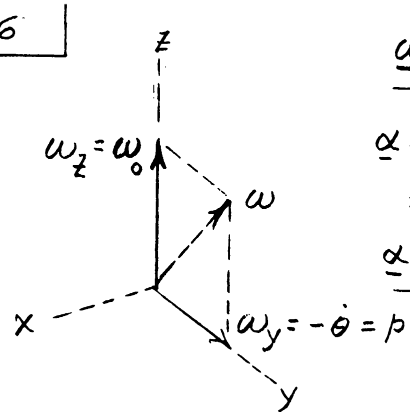
7/5 $\underline{v}_A = \underline{\omega} \times \underline{r} = (-4\underline{j} - 3\underline{k}) \times (0.5\underline{i} + 1.2\underline{j} + 1.1\underline{k})$

$= \underline{-0.8\underline{i} - 1.5\underline{j} + 2\underline{k} \ \text{m/s}}$

The rim speed of any point B is

$v_B = \sqrt{0.8^2 + 1.5^2 + 2^2} = \underline{2.62 \text{ m/s}}$

7/6



$$\underline{\omega} = p\underline{j} + \omega_0 \underline{k}$$

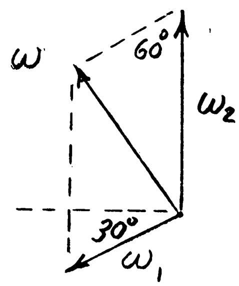
$$\underline{\alpha} = \underline{\omega}_z \times \underline{\omega} = \underline{\omega}_z \times \underline{\omega}_y$$

$$= \omega_0 \underline{k} \times p\underline{j}$$

$$\underline{\alpha} = -p\omega_0 \underline{i}$$

7/7

$$\omega_1 = \frac{2\pi N_1}{60} = \frac{2\pi(200)}{60} = 20.9 \text{ rad/s}$$



$\omega = 40$ rad/s

Law of cosines $\omega^2 = \omega_1^2 + \omega_2^2 - 2\omega_1\omega_2 \cos 60°$

$$40^2 = 20.9^2 + \omega_2^2 - 2(20.9)\omega_2(0.5)$$

$$\omega_2^2 - 20.9\omega_2 - 1161 = 0$$

$$\omega_2 = \frac{20.9}{2} \pm \frac{1}{2}\sqrt{20.9^2 + 4(1161)}$$

$$= 10.47 \pm 35.65 \quad , \ \omega_2 = 46.1 \text{ rad/s}$$

$$N_2 = \frac{46.1}{2\pi} 60 = \underline{440 \text{ rev/min}}$$

7/8

$$\underline{\omega} = (-\sin\theta\,\underline{i} + \cos\theta\,\underline{k})\omega \quad , \quad \underline{\alpha} = \underline{0}$$

$$\underline{r} = d\underline{i} + \ell\underline{j} - h\underline{k}$$

$\underline{v} = \underline{\omega} \times \underline{r}$ gives

$$\underline{v} = \omega\left[-\ell\cos\theta\,\underline{i} + (d\cos\theta - h\sin\theta)\underline{j} - \ell\sin\theta\,\underline{k}\right]$$

$\underline{a} = \underline{\alpha} \times \underline{r} + \underline{\omega} \times (\underline{\omega} \times \underline{r})$ gives

$$\underline{a} = \omega^2\left[(h\sin\theta\cos\theta - d\cos^2\theta)\underline{i} - \ell\underline{j} + (h\sin^2\theta - d\cos\theta\sin\theta)\underline{k}\right]$$

7/9 $\underline{\alpha} = \underline{\Omega} \times \underline{\omega} = 0.6\,\underline{k} \times 2\,\underline{j} = \underline{-1.2\,\underline{i}\ \text{rad/sec}^2}$

$\underline{a}_p = \underline{\dot{\omega}} \times \underline{r} + \underline{\omega} \times (\underline{\omega} \times \underline{r})$, $\quad \underline{\omega} = \underline{\Omega} + \underline{\omega}_0$

$\underline{\dot{\omega}} = \underline{\alpha} = -1.2\,\underline{i}$ rad/sec^2

$\underline{r} = 34\,\underline{j} + 20\,\underline{k}$ in. (for $\beta = 90°$)

Carry out algebra to obtain

$\underline{\underline{a}_p = 35.8\,\underline{j} - 80\,\underline{k}\ \text{in./sec}^2}$

7/10

$$\underline{\alpha} = \underline{\omega}_x \times \underline{\omega}_z = -\dot{\gamma}\underline{i} \times \omega_0 \underline{k} = -3\pi\underline{i} \times 4\pi\underline{k}$$

$$= \underline{12\pi^2 \underline{j} \text{ rad/sec}^2}$$

$$\underline{r} = 5\underline{j} + 10\underline{k} \text{ in.}$$

$$\underline{v} = \underline{\omega} \times \underline{r} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ -3\pi & 0 & 4\pi \\ 0 & 5 & 10 \end{vmatrix} = \underline{5\pi(-4\underline{i} + 6\underline{j} - 3\underline{k}) \text{ in./sec}}$$

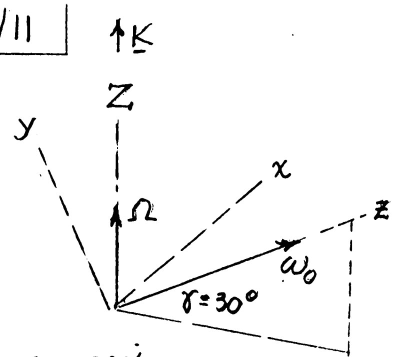
$$\underline{a} = \dot{\underline{\omega}} \times \underline{r} + \underline{\omega} \times (\underline{\omega} \times \underline{r}) = \underline{\alpha} \times \underline{r} + \underline{\omega} \times \underline{v}$$

$$= 12\pi^2 \underline{j} \times (5\underline{j} + 10\underline{k}) + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ -3\pi & 0 & 4\pi \\ -4 & 6 & -3 \end{vmatrix} 5\pi$$

$$= 120\pi^2 \underline{i} - 120\pi^2 \underline{i} - 125\pi^2 \underline{j} - 90\pi^2 \underline{k}$$

$$= \underline{-5\pi^2(25\underline{j} + 18\underline{k}) \text{ in./sec}^2}$$

7/11



$\Omega = 30 \times 2\pi/60$
$= \pi$ rad/sec

$\omega_0 = \alpha_0 t$

when $t = 2$ sec, $\omega_0 = \frac{3000(2\pi)}{60}$
$= 100\pi$ rad/s

so $\alpha_0 = 100\pi/2 = 50\pi$ rad/s^2

$\omega_0 = 50\pi t$

$\underline{\omega}_0 = 50\pi t \underline{k}$ rad/s

$\underline{\alpha} = \dot{\underline{\omega}}_0 = 50\pi \underline{k} + 50\pi t \dot{\underline{k}}$

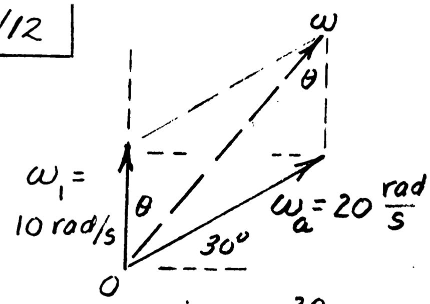
But $\dot{\underline{k}} = \underline{\Omega} \times \underline{k}$
$= \pi \underline{K} \times \underline{k}$
$= (\sqrt{3}/2)\pi \underline{i}$

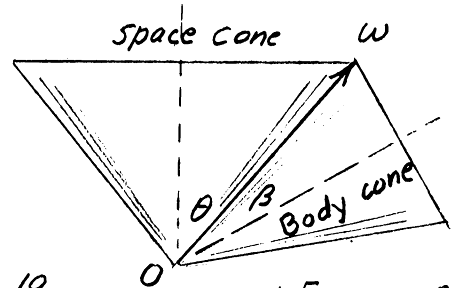
Thus for $t = 1/3$ s,

$$\underline{\alpha} = 50\pi \underline{k} + 50\pi \left(\tfrac{1}{3}\right)(\sqrt{3}/2)\pi \underline{i}$$
$$= 50\pi \left(\frac{\pi}{2\sqrt{3}} \underline{i} + \underline{k}\right) \text{ rad/s}^2$$

(Note: Total angular velocity is $\underline{\omega} = \underline{\Omega} + \underline{\omega}_0$
& $\underline{\alpha} = \dot{\underline{\omega}} = \dot{\underline{\Omega}} + \dot{\underline{\omega}}_0 = \underline{0} + \dot{\underline{\omega}}_0$)

7/12

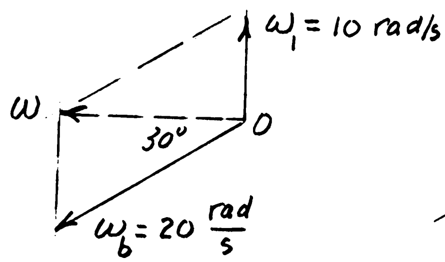


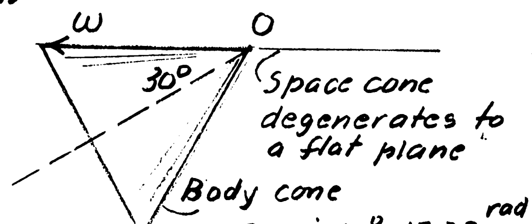


Law of sines $\dfrac{20}{\sin\theta} = \dfrac{10}{\sin(60-\theta)}$, $\theta = \tan^{-1}\dfrac{\sqrt{3}}{2} = 40.9^\circ$

$\omega = \sqrt{(20\cos 30^\circ)^2 + (20\sin 30^\circ + 10)^2}$, $\beta = 60^\circ - \theta = 19.1^\circ$

$\underline{\omega = 26.5 \text{ rad/s}}$





$\omega = 20\sin 60^\circ = \underline{17.32 \dfrac{\text{rad}}{\text{s}}}$

7/13

$\Omega = 4 \times 2\pi = 8\pi$ rad/s

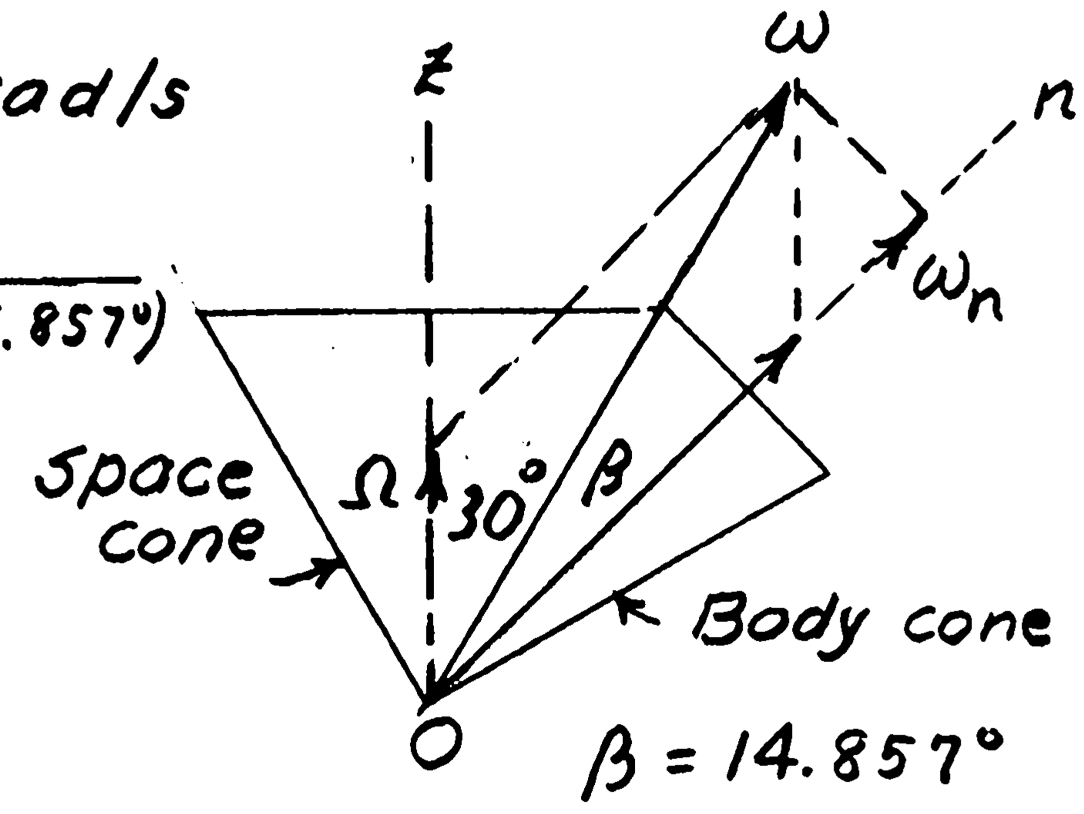
$$\frac{8\pi}{\sin 14.857°} = \frac{\omega}{\sin(180° - 30° - 14.857°)}$$

$$\omega = 8\pi \frac{\sin 135.143°}{\sin 14.857}$$

$$= \underline{69.1 \text{ rad/s}}$$

$$\omega_n = 69.1 \cos 14.857°$$

$$= \underline{66.8 \text{ rad/s}}$$



7/14 From Prob. 7/13 $\omega = 69.1$ rad/s

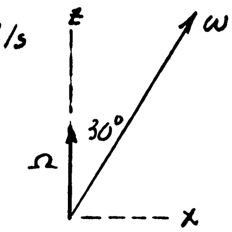
$\underline{\omega} = 69.1(\underline{i}\sin 30° + \underline{k}\cos 30°)$ rad/s

$\underline{\Omega} = 8\pi\underline{k}$ rad/s

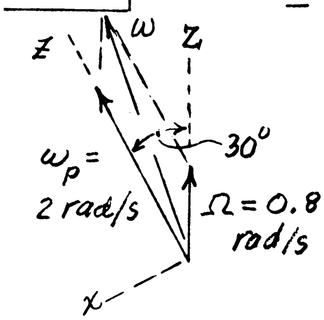
$\underline{\alpha} = \underline{\Omega} \times \underline{\omega} = 8\pi\underline{k} \times 69.1(\underline{i}\sin 30° + \underline{k}\cos 30°)$

$= 1737(0.5\underline{i} + \underline{0})$

$\underline{\alpha} = 869\underline{j}$ rad/s^2



7/15



$$\underline{\omega} = \underline{\omega}_p + \underline{\Omega}$$
$$= 2\underline{k} + 0.8\cos 30°\,\underline{k} - 0.8\sin 30°\,\underline{i}$$
$$= \underline{-0.4\underline{i} + 2.69\underline{k}\ \text{rad/s}}$$

$$\underline{\alpha} = \underline{\Omega} \times \underline{\omega}_p$$
$$= 0.8(-0.5\underline{i} + 0.866\underline{k}) \times 2\underline{k}$$
$$= 1.6(0.5\underline{j} + 0)$$
$$\underline{\underline{\alpha} = 0.8\underline{j}\ \text{rad/s}^2}$$

7/16

$\underline{\omega} = \underline{\omega}_1 + \underline{\omega}_2 = 2\underline{k} + 1.5\underline{i}$

$\omega = \sqrt{2^2 + 1.5^2} = \underline{2.5 \text{ rad/s}}$

$\underline{\alpha} = \underline{\omega}_1 \times \underline{\omega}_2 = 2\underline{k} \times 1.5\underline{i} = \underline{3\underline{j} \text{ rad/s}^2}$

7/17

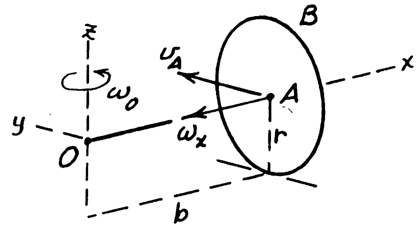
$$\underline{\omega} = \underline{\omega}_1 + \underline{\omega}_s$$
$$= 2\underline{k} + 0.8(\underline{j}\cos 30^\circ + \underline{k}\sin 30^\circ)$$
$$\underline{\underline{\omega} = 0.693\underline{j} + 2.40\underline{k} \text{ rad/s}}$$

$$\underline{\alpha} = \underline{\omega}_1 \times \underline{\omega}_s = 2\underline{k} \times 0.8(\underline{j}\cos 30^\circ + \underline{k}\sin 30^\circ)$$
$$\underline{\underline{\alpha} = -1.386\underline{i} \text{ rad/s}^2}$$

7/18

$v_A = b\omega_0$

$\underline{\omega} = (-v_A/r)\underline{i} + \omega_0\underline{k}$



$\underline{\omega} = -\frac{b\omega_0}{r}\underline{i} + \omega_0\underline{k}$

$\underline{\omega} = \omega_0\left(-\frac{b}{r}\underline{i} + \underline{k}\right)$

$\underline{\alpha} = \dot{\underline{\omega}} = \omega_0\left(-\frac{b}{r}\dot{\underline{i}}\right) + \underline{0}$ where $\dot{\underline{i}} = \underline{\omega}_z \times \underline{i} = \omega_0\underline{j}$

so

$\underline{\alpha} = \omega_0\left(-\frac{b}{r}\omega_0\underline{j}\right)$, $\quad \underline{\alpha} = -\frac{b}{r}\omega_0^2\underline{j}$

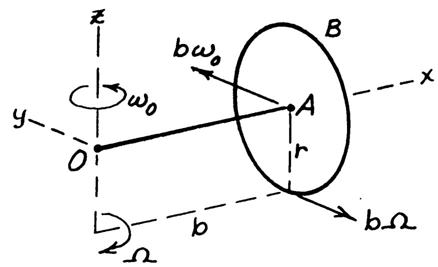
7/19

$$\underline{\omega}_x = \frac{-(b\omega_0 + b\Omega)\underline{i}}{r}$$

$$\underline{\omega} = \underline{\omega}_x + \underline{\omega}_z = \underline{-\frac{b}{r}(\omega_0 + \Omega)\underline{i} + \omega_0 \underline{k}}$$

$$\underline{\alpha} = \underline{\dot{\omega}} = -\frac{b}{r}(\omega_0 + \Omega)\omega_0 \underline{j} + \underline{0}$$

$$= \underline{-\frac{b}{r}\omega_0(\omega_0 + \Omega)\underline{j}}$$



7/20

$\underline{r} = \vec{OB} = -120 \sin 30° \underline{i} + 120 \cos 30° \underline{j} + 200 \underline{k}$ mm

$= -60\underline{i} + 103.9\underline{j} + 200\underline{k}$ mm

$\underline{\omega} = \underline{\omega}_x + \underline{\omega}_z = 10\underline{i} + 20\underline{k}$ rad/s

$\underline{v} = \underline{\omega} \times \underline{r} = 10(\underline{i} + 2\underline{k}) \times (-60\underline{i} + 103.9\underline{j} + 200\underline{k})$

$= 10(-208\underline{i} - 320\underline{j} + 103.9\underline{k})$

$v = 10\sqrt{208^2 + 320^2 + 103.9^2} = 3950$ mm/s

or $\underline{v = 3.95 \text{ m/s}}$

$a = \dot{\omega} \times r + \omega \times (\omega \times r)$

where $\underline{\dot{\omega}} = \underline{\alpha} = \underline{\omega}_x \times \underline{\omega} = \underline{\omega}_x \times \underline{\omega}_z = 10\underline{i} \times 20\underline{k} = -200\underline{j} \ \frac{\text{rad}}{s^2}$

$\underline{\dot{\omega}} \times \underline{r} = -200\underline{j} \times (-60\underline{i} + 103.9\underline{j} + 200\underline{k})$

$= -4000(10\underline{i} + 3\underline{k})$ mm/s^2

$\underline{\omega} \times (\underline{\omega} \times \underline{r}) = \underline{\omega} \times \underline{v} = 10(\underline{i} + 2\underline{k}) \times 10(-208\underline{i} - 320\underline{j} + 103.9\underline{k})$

$= 100(640\underline{i} - 520\underline{j} - 320\underline{k})$

$\underline{a} = 24.0\underline{i} - 52.0\underline{j} - 44.0\underline{k}$ m/s^2

$a = \sqrt{24.0^2 + 52.0^2 + 44.0^2} = \underline{72.2 \text{ m/s}^2}$

7/21 $\theta = \frac{\pi}{6}\sin 4\pi t$ where $\theta_0 = \frac{\pi}{6}$ rad, $\Omega = \pi$ rad/s, $\dot{\beta} = \Omega = \pi$ rad/s

$\underline{\omega} = \underline{\omega}_{OA} = -\dot{\theta}\underline{j} + \dot{\beta}\underline{k}$, $\underline{\alpha}_{OA} = \dot{\underline{\omega}}_{OA} = -\ddot{\theta}\underline{j} - \dot{\theta}\dot{\underline{j}} + \ddot{\beta}\underline{k} + \dot{\beta}\dot{\underline{k}}$, $\ddot{\beta} = 0$, $\dot{\underline{k}} = \underline{0}$

$\dot{\underline{j}} = -\Omega\underline{i} = -\pi\underline{i}$, $\dot{\theta} = \frac{2\pi^2}{3}\cos 4\pi t$, $\ddot{\theta} = -\frac{8\pi^3}{3}\sin 4\pi t$

$\underline{\omega} = \underline{\omega}_{OA} = -\frac{2\pi^2}{3}\cos 4\pi t(\underline{j}) + \pi\underline{k}$

$\underline{\alpha} = \underline{\alpha}_{OA} = \frac{8\pi^3}{3}\sin 4\pi t(\underline{j}) + \frac{2\pi^3}{3}\cos 4\pi t(\underline{i})$

(a) $t = \frac{1}{2}$ s, $\underline{\omega} = -\frac{2\pi^2}{3}\cos 2\pi(\underline{j}) + \pi\underline{k}$, $\underline{\underline{\omega = \pi\left(-\frac{2\pi}{3}\underline{j} + \underline{k}\right)}}$

$\underline{\alpha} = \frac{8\pi^3}{3}\sin 2\pi(\underline{j}) + \frac{2\pi^3}{3}\cos 2\pi(\underline{i})$, $\underline{\underline{\alpha = \frac{2\pi^3}{3}\underline{i}}}$

(b) $t = \frac{1}{8}$ s, $\underline{\omega} = -\frac{2\pi^2}{3}\cos\frac{\pi}{2}(\underline{j}) + \pi\underline{k}$, $\underline{\underline{\omega = \pi\underline{k}}}$

$\underline{\alpha} = \frac{8\pi^3}{3}\sin\frac{\pi}{2}(\underline{j}) + \frac{2\pi^3}{3}\cos\frac{\pi}{2}(\underline{i})$, $\underline{\underline{\alpha = \frac{8\pi^3}{3}\underline{j}}}$

7/22 $\underline{r}_A = \underline{r} = 0.220\,(\underline{i}\cos\theta + \underline{k}\sin\theta)$ m, $\Omega = \pi$ rad/s

(a) $t = \frac{1}{2}$ s, $\sin 4\pi t = 0$, $\cos\theta = \cos(\theta_0 \sin 4\pi \tfrac{1}{2}) = \cos 0 = 1$

$$\sin\theta = \sin(\theta_0 \sin 4\pi \tfrac{1}{2}) = \sin 0 = 0$$

$$\underline{r} = 0.220(\underline{i} + 0\underline{k}) = 0.220\underline{i}$$

$$\underline{v} = \underline{\omega} \times \underline{r} = \pi(-\tfrac{2\pi}{3}\underline{j} + \underline{k}) \times 0.220\underline{i}, \quad \underline{v} = 0.220\pi(\underline{j} + \tfrac{2\pi}{3}\underline{k}) \text{ m/s}$$

or $\underline{v} = 0.691\underline{j} + 1.448\underline{k}$ m/s

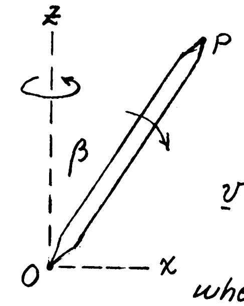
$$\underline{a} = \underline{\alpha} \times \underline{r} + \underline{\omega} \times \underline{v}$$

$$= \tfrac{2\pi^3}{3}\underline{i} \times 0.220\underline{i} + \pi(-\tfrac{2\pi}{3}\underline{j} + \underline{k}) \times 0.220\pi(\underline{j} + \tfrac{2\pi}{3}\underline{k})$$

$$= \underline{0} + 0.220\pi^2\left(-[\tfrac{2\pi}{3}]^2\underline{i} - \underline{i}\right) = -0.220\pi^2(1 + [\tfrac{2\pi}{3}]^2)\underline{i}$$

or $\underline{a} = -11.70\underline{i}$ m/s^2

7/23 $\overline{OP} = 24\ m$, $\dot{\beta} = 0.10$ rad/s const., $\beta = 30°$



$$\underline{r} = \overrightarrow{OP} = (24 \sin 30°)\underline{i} + (24 \cos 30°)\underline{k}$$
$$= 12\underline{i} + 20.78\underline{k}\ \ m$$

$$\underline{\omega} = \frac{2(2\pi)}{60}\underline{k} + 0.10\underline{j} = 0.209\underline{k} + 0.10\underline{j}\ \ \frac{rad}{s}$$

$$\underline{v} = \underline{\omega} \times \underline{r} = (0.209\underline{k} + 0.10\underline{j}) \times (12\underline{i} + 20.78\underline{k})$$
$$= 2.078\underline{i} + 2.513\underline{j} - 1.2\underline{k}\ \ m/s$$

where $v = |\underline{v}| = \sqrt{(2.078)^2 + (2.513)^2 + (-1.2)^2} = \underline{3.48\ \frac{m}{s}}$

$$\underline{a} = \dot{\underline{\omega}} \times \underline{r} + \underline{\omega} \times (\underline{\omega} \times \underline{r}) = \underline{\alpha} \times \underline{r} + \underline{\omega} \times \underline{v}$$

$$\underline{\alpha} = \dot{\underline{\omega}} = \underline{\omega}_z \times \underline{\omega}_y = 0.209\underline{k} \times 0.10\underline{j} = -0.0209\underline{i}\ \ rad/s^2$$

$$\dot{\underline{\omega}} \times \underline{r} = \underline{\alpha} \times \underline{r} = -0.0209\underline{i} \times (12\underline{i} + 20.78\underline{k}) = 0.435\underline{j}\ \ m/s^2$$

$$\underline{\omega} \times \underline{v} = (0.209\underline{k} \times 0.10\underline{j}) \times (2.078\underline{i} + 2.513\underline{j} - 1.2\underline{k})$$
$$= -0.646\underline{i} + 0.435\underline{j} - 0.208\underline{k}\ \ m/s^2$$

$$\underline{a} = -0.646\underline{i} + 0.870\underline{j} - 0.208\underline{k}\ \ m/s^2$$

$$a = |\underline{a}| = \sqrt{(-0.646)^2 + (0.870)^2 + (-0.208)^2} = \underline{1.104\ m/s^2}$$

7/24 (a) $\underline{\alpha} = \dot{\theta}\underline{i} \times \underline{\omega}_0 = 2\underline{i} \times (-4\underline{j} - 3\underline{k})$

$= \underline{6\underline{j} - 8\underline{k} \text{ rad/s}^2}$

$\underline{a}_A = \underline{\alpha} \times \underline{r} + \underline{\omega} \times (\underline{\omega} \times \underline{r})$

With $\underline{\alpha} = 6\underline{j} - 8\underline{k}$ rad/s², $\underline{r} = 0.5\underline{i} + 1.2\underline{j} + 1.1\underline{k}$ m, and $\underline{\omega} = \dot{\theta}\underline{i} + \underline{\omega}_0 = 2\underline{i} - 4\underline{j} - 3\underline{k}$ rad/s, we obtain $\underline{a}_A = -12.5\underline{i} - 10.4\underline{j} - 13.6\underline{k}$ m/s²

$a_A = \sqrt{12.5^2 + 10.4^2 + 13.6^2} = \underline{21.2 \text{ m/s}^2}$

(b) $\underline{\alpha} = \underline{\Omega} \times \underline{\omega}_0 = 2\underline{k} \times (-4\underline{j} - 3\underline{k})$

$= \underline{8\underline{i} \text{ rad/s}^2}$

$\underline{a}_A = \underline{\alpha} \times \underline{r} + \underline{\omega} \times (\underline{\omega} \times \underline{r})$

With $\underline{\alpha} = 8\underline{i}$ rad/s², $r = 0.5\underline{i} + 1.2\underline{j} + 1.1\underline{k}$ m, and $\underline{\omega} = \underline{\Omega} + \underline{\omega}_0 = 2\underline{k} + (-4\underline{j} - 3\underline{k}) = -4\underline{j} - \underline{k}$ rad/s, we obtain $\underline{a}_A = -8.5\underline{i} - 5.6\underline{j} - 3.2\underline{k}$ m/s²

$a_A = \sqrt{8.5^2 + 5.6^2 + 3.2^2} = \underline{10.67 \text{ m/s}^2}$

7/25

$$\underline{\omega} = \Omega \underline{k} + \dot{\gamma} \underline{i} - \omega_0 \cos\gamma \underline{j} - \omega_0 \sin\gamma \underline{k}$$

$$\underline{\alpha} = \dot{\underline{\omega}} = \Omega \dot{\underline{k}} + \dot{\gamma} \dot{\underline{i}} + \omega_0 \dot{\gamma} \sin\gamma \underline{j} - \omega_0 \cos\gamma \dot{\underline{j}}$$
$$- \omega_0 \dot{\gamma} \cos\gamma \underline{k} - \omega_0 \sin\gamma \dot{\underline{k}}$$

where $\Omega = 4$ rad/s const.

$\omega_0 = 3$ rad/s " $\gamma = 30°$

$\dot{\gamma} = -\pi/4$ rad/s "

& $\dot{\underline{i}} = \underline{\Omega} \times \underline{i} = \Omega \underline{k} \times \underline{i} = \Omega \underline{j}$; $\dot{\underline{j}} = \underline{\Omega} \times \underline{j} = \Omega \underline{k} \times \underline{j} = -\Omega \underline{i}$; $\dot{\underline{k}} = \Omega \underline{k} \times \underline{k} = \underline{0}$

so $\underline{\alpha} = \underline{0} + \dot{\gamma} \Omega \underline{j} + \omega_0 \dot{\gamma} \sin\gamma \underline{j} + \omega_0 \Omega \cos\gamma \underline{i} - \omega_0 \dot{\gamma} \cos\gamma \underline{k} + 0$

$$= \omega_0 \Omega \cos\gamma \underline{i} + \dot{\gamma}(\Omega + \omega_0 \sin\gamma) \underline{j} - \omega_0 \dot{\gamma} \cos\gamma \underline{k}$$

$$= 3(4)(0.866) \underline{i} - \frac{\pi}{4}(4 + 3 \times 0.5) \underline{j} + 3(\pi/4)(0.866) \underline{k}$$

$$= 10.392 \underline{i} - 4.320 \underline{j} + 2.040 \underline{k} \quad \text{rad/s}^2$$

$$\alpha = |\underline{\alpha}| = \sqrt{(10.392)^2 + (4.320)^2 + (2.040)^2} = \underline{11.44 \text{ rad/s}^2}$$

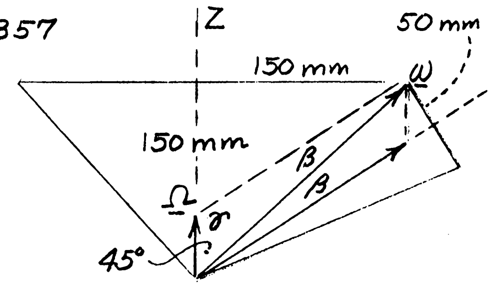
$$\underline{\omega} = -\frac{\pi}{4} \underline{i} - 3(0.866) \underline{j} + (4 - 3 \times 0.5) \underline{k}$$

$$= \underline{-0.785 \underline{i} - 2.60 \underline{j} + 2.5 \underline{k} \quad \text{rad/s}}$$

▶7/26

$$\sin\beta = \frac{50}{150\sqrt{2}} = 0.2357$$

$$\beta = 13.63^\circ$$



$$\Omega = \frac{2\pi}{4} = \pi/2 \text{ rad/s}$$

Law of sines

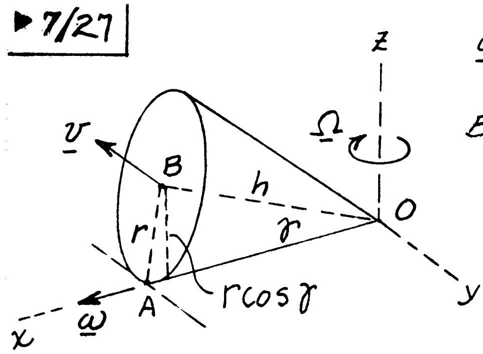
$$\frac{\omega}{\sin\gamma} = \frac{\Omega}{\sin\beta}, \quad \omega = \Omega\frac{\sin\gamma}{\sin\beta}$$

$$|\underline{\alpha}| = |\underline{\Omega}\times\underline{\omega}| = \Omega\omega\sin 45^\circ = \Omega^2\frac{\sin\gamma}{\sin\beta}\sin 45^\circ$$

$$\sin\gamma = \sin(180 - 45 - 13.63) = 0.8539$$

so $\alpha = \left(\frac{\pi}{2}\right)^2 \frac{0.8539}{0.2357} 0.7071 = \underline{6.32 \text{ rad/s}^2}$

▸7/27



$$\underline{\omega} = \frac{v}{r\cos\gamma}\,\underline{i}$$

But $\cos\gamma = \dfrac{h}{\sqrt{r^2+h^2}}$

$$\underline{\omega} = \frac{v\sqrt{r^2+h^2}}{rh}\,\underline{i}$$

$$= v\sqrt{\frac{1}{h^2}+\frac{1}{r^2}}\,\underline{i}$$

$\omega = \text{const}$ so $\underline{\alpha} = \underline{\Omega}\times\underline{\omega}$

$$\underline{\Omega} = -\frac{v}{h\cos\gamma}\,\underline{k}$$

so $\underline{\alpha} = -\dfrac{v}{h\cos\gamma}\,\underline{k} \times \dfrac{v}{r\cos\gamma}\,\underline{i} = -\dfrac{v^2}{hr\cos^2\gamma}\,\underline{j}$

$$\underline{\alpha} = -\frac{v^2}{h^2}\left(\frac{r}{h}+\frac{h}{r}\right)\underline{j}$$

7/28 For $t=0$ $\theta=0$ and position vector of B is $\underline{r} = 4\underline{i} - 8\underline{k}$ in.

$$\omega_x = -\dot{\theta} = -\frac{\pi}{6} 3\pi \cos 3\pi t = -\frac{\pi^2}{2} \text{ rad/sec for } t=0$$

$\omega_z = 2\pi$ rad/sec

$$\underline{\omega} = \omega_x \underline{i} + \omega_z \underline{k} = -\frac{\pi^2}{2}\underline{i} + 2\pi\underline{k} \text{ rad/sec for } t=0$$

$$\underline{v} = \underline{\omega} \times \underline{r} = \left(-\frac{\pi^2}{2}\underline{i} + 2\pi\underline{k}\right) \times (4\underline{i} - 8\underline{k}) = -4\pi^2\underline{j} + 8\pi\underline{j} = 4\pi(2-\pi)\underline{j} \text{ in./sec}$$

or $\underline{\underline{v} = -14.35\underline{j} \text{ in./sec}}$

$$\underline{a} = \dot{\underline{\omega}} \times r + \underline{\omega} \times (\underline{\omega} \times \underline{r})$$

$$\dot{\underline{\omega}} = \dot{\omega}_x \underline{i} + \omega_x \dot{\underline{i}} + \dot{\omega}_z \underline{k} + \omega_z \dot{\underline{k}} = +\frac{\pi^2}{2}(3\pi)\sin 3\pi t\,\underline{i} - \frac{\pi^2}{2}\cos 3\pi t\,(\omega_z \underline{j}) + \underline{0} + \underline{0}$$

$$\dot{\underline{\omega}}_{t=0} = \underline{0} - \frac{\pi^2}{2} 2\pi \underline{j} = -\pi^3 \underline{j}, \quad \underline{\alpha} = \dot{\underline{\omega}} = -\pi^3 \underline{j} = \underline{-31.0\underline{j} \text{ rad/sec}^2}$$

so $\underline{a} = -\pi^3 \underline{j} \times (4\underline{i} - 8\underline{k}) + \left(-\frac{\pi^2}{2}\underline{i} + 2\pi\underline{k}\right) \times 4\pi(2-\pi)\underline{j}$

$$= 16\pi^2(\pi-1)\underline{i} + 2\pi^4 \underline{k} \text{ in./sec}^2$$

$\underline{\underline{a} = 338\underline{i} + 194.8\underline{k} \text{ in./sec}^2}$

7/29 | Angular velocity of rotor is

$\underline{\omega} = p\underline{k} - q\underline{i}$, $\underline{\alpha} = \dot{\underline{\omega}} = p\dot{\underline{k}} - q\dot{\underline{i}} = \underline{\Omega} \times (p\underline{k} - q\underline{i})$

where $\underline{\Omega}$ = angular velocity of axes $= -q\underline{i}$

Thus $\underline{\alpha} = -q\underline{i} \times (p\underline{k} - q\underline{i}) = \underline{pq\underline{j}}$

or from Eq. 7/7, $\underline{\alpha} = \left(\frac{d\omega}{dt}\right)_{XYZ} = \underline{0} + \underline{\Omega} \times \underline{\omega}$

$= -q\underline{i} \times (p\underline{k} - q\underline{i}) = \underline{pq\underline{j}}$

7/30 $\underline{\omega} = \underline{\Omega} + \underline{p} = 4\underline{i} + 10\underline{k}$, $\omega = \sqrt{4^2+10^2} = \underline{10.77 \frac{rad}{s}}$

$\underline{\alpha} = \underline{\Omega} \times \underline{p} = 4\underline{i} \times 10\underline{k} = \underline{-40\underline{j} \text{ rad/s}^2}$

7/31 Angular velocity of x-y-z axes is $\mathbf{\Omega} = 4\mathbf{i}$ rad/s

$\mathbf{v}_A = \mathbf{v}_C + \mathbf{\Omega} \times \mathbf{r}_{A/C} + \mathbf{v}_{rel}$

$\mathbf{v}_C = 0.4(4)(-\mathbf{j}) = -1.6\mathbf{j}$ m/s

$\mathbf{\Omega} \times \mathbf{r}_{A/C} = 4\mathbf{i} \times 0.3\mathbf{j} = 1.2\mathbf{k}$ m/s

$\mathbf{v}_{rel} = 0.3(10)(-\mathbf{i}) = -3\mathbf{i}$ m/s

So $\mathbf{v}_A = -1.6\mathbf{j} + 1.2\mathbf{k} - 3\mathbf{i}$, $\underline{\mathbf{v}_A = -3\mathbf{i} - 1.6\mathbf{j} + 1.2\mathbf{k} \text{ m/s}}$

$\mathbf{a}_A = \mathbf{a}_C + \dot{\mathbf{\Omega}} \times \mathbf{r}_{A/C} + \mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r}_{A/C}) + 2\mathbf{\Omega} \times \mathbf{v}_{rel} + \mathbf{a}_{rel}$

$\mathbf{a}_C = 0.4(4^2)(-\mathbf{k}) = -6.4\mathbf{k}$ m/s^2, $\dot{\mathbf{\Omega}} = \mathbf{0}$

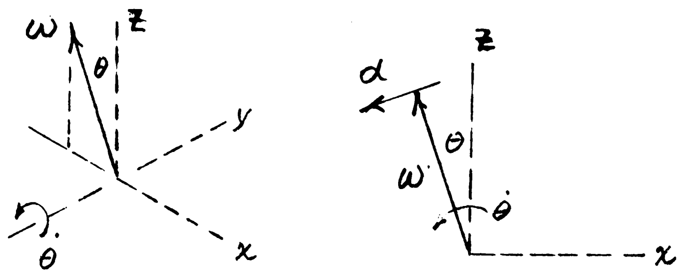
$\mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r}_{A/C}) = 4\mathbf{i} \times 1.2\mathbf{k} = -4.8\mathbf{j}$ m/s^2

$2\mathbf{\Omega} \times \mathbf{v}_{rel} = 2(4\mathbf{i}) \times (-3\mathbf{i}) = \mathbf{0}$

$\mathbf{a}_{rel} = 0.3(10^2)(-\mathbf{j}) = -30\mathbf{j}$ m/s^2

So $\mathbf{a}_A = -6.4\mathbf{k} - 4.8\mathbf{j} - 30\mathbf{j}$, $\underline{\mathbf{a}_A = -34.8\mathbf{j} - 6.4\mathbf{k}}$ m/s^2

7/32



$$\omega = \frac{2\pi N}{60} = \frac{2\pi(360)}{60} = 12\pi \text{ rad/s}$$

$$\underline{\alpha} = -\dot{\theta}\,\underline{j} \times \underline{\omega} = -0.2\underline{j} \times 12\pi(-\sin\theta\,\underline{i} + \cos\theta\,\underline{k})$$

$$= 2.4\pi\,(-0.5k - 0.866\underline{i})$$

$$= \underline{-1.2\pi\,(\sqrt{3}\,\underline{i} + \underline{k}) \text{ rad/s}^2}$$

7/33 The angular velocity $\underline{\omega}$ of the plate is $\underline{\omega} = \dot{\varphi}\underline{k} + \dot{\theta}\underline{i}$.

In x-y-z $\quad \underline{\alpha}_{xyz} = \left(\frac{d\underline{\omega}}{dt}\right)_{xyz} = \underline{\ddot{\varphi}\underline{k} + \underline{0}} \qquad (\ddot{\theta} = 0 \ \& \ \dot{\underline{i}} = \underline{0})$

So by Eq. 7/7 $\underline{\alpha}_{XYZ} = \left(\frac{d\underline{\omega}}{dt}\right)_{XYZ} = \ddot{\varphi}\underline{k} + \dot{\theta}\underline{i} \times (\dot{\varphi}\underline{k} + \dot{\theta}\underline{i}) = \underline{\ddot{\varphi}\underline{k} - \dot{\theta}\dot{\varphi}\underline{j}}$

or, by straight differentiation,

$$\underline{\alpha}_{XYZ} = \left(\frac{d\underline{\omega}}{dt}\right)_{XYZ} = \ddot{\varphi}\underline{k} + \dot{\varphi}\dot{\underline{k}} + \underline{0} \text{ where } \ddot{\theta} = 0 \ \& \ \dot{\underline{i}} = \underline{0}$$

But $\dot{\underline{k}} = \dot{\theta}\underline{i} \times \underline{k} = -\dot{\theta}\underline{j}$ so $\underline{\alpha}_{XYZ} = \ddot{\varphi}\underline{k} + \dot{\varphi}(-\dot{\theta}\underline{j}) = \underline{\ddot{\varphi}\underline{k} - \dot{\theta}\dot{\varphi}\underline{j}}$

7/34

In Eq. 7/6,

$\underline{v}_B = -R\dot{\theta}\underline{j}$, $\underline{\Omega} = \dot{\theta}\underline{i}$, $\underline{r}_{A/B} = \frac{b}{\sqrt{2}}\underline{i}$ (for $\varphi = 0$), $\underline{v}_{rel} = \frac{b}{\sqrt{2}}\dot{\varphi}\underline{j}$ (for $\varphi = 0$)

$\underline{v}_A = \underline{v}_B + \underline{\Omega}\times\underline{r}_{A/B} + \underline{v}_{rel}$, $\underline{v}_A = -R\dot{\theta}\underline{j} + \dot{\theta}\underline{i}\times\frac{b}{\sqrt{2}}\underline{i} + \frac{b}{\sqrt{2}}\dot{\varphi}\underline{j}$,

$$\underline{\underline{v}_A = \left(\frac{b}{\sqrt{2}}\dot{\varphi} - R\dot{\theta}\right)\underline{j}}$$

$\underline{a}_{rel} = -\frac{b}{\sqrt{2}}\dot{\varphi}^2\underline{i}$, $\underline{a}_B = -R\dot{\theta}^2\underline{k}$, $\dot{\underline{\Omega}} = \ddot{\theta}\underline{i} = \underline{0}$

$$\underline{a}_A = \underline{a}_B + \dot{\underline{\Omega}}\times\underline{r}_{A/B} + \underline{\Omega}\times(\underline{\Omega}\times\underline{r}_{A/B}) + 2\underline{\Omega}\times\underline{v}_{rel} + \underline{a}_{rel}$$

$$\underline{a}_A = -R\dot{\theta}^2\underline{k} + \underline{0} + \dot{\theta}\underline{i}\times\left(\dot{\theta}\underline{i}\times\frac{b}{\sqrt{2}}\underline{i}\right) + 2\dot{\theta}\underline{i}\times\frac{b}{\sqrt{2}}\dot{\varphi}\underline{j} - \frac{b}{\sqrt{2}}\dot{\varphi}^2\underline{i}$$

$$= -R^2\dot{\theta}^2\underline{k} + b\sqrt{2}\,\dot{\theta}\dot{\varphi}\underline{k} - \frac{b}{\sqrt{2}}\dot{\varphi}^2\underline{i},\quad \underline{\underline{a}_A = (-R\dot{\theta} + b\sqrt{2}\dot{\varphi})\dot{\theta}\underline{k} - \frac{b}{\sqrt{2}}\dot{\varphi}^2\underline{i}}$$

or $\underline{a}_A = -\frac{b}{\sqrt{2}}\dot{\varphi}^2\underline{i} - (R\dot{\theta} - b\sqrt{2}\dot{\varphi})\dot{\theta}\underline{k}$

7/35 Angular velocity of OA is $\underline{\omega} = -\dot{\beta}\underline{i} + p\sin\beta\underline{j} + (p\cos\beta + \Omega)\underline{k}$

Eq. 7/7a, $[\ ] = \underline{\omega}$, $\left(\frac{d[\ ]}{dt}\right)_{XYZ} = \left(\frac{d[\ ]}{dt}\right)_{xyz} + \underline{\Omega} \times [\ ]$

$$\left(\frac{d\omega}{dt}\right)_{xyz} = \underline{0} + p\dot{\beta}\cos\beta\underline{j} + (-p\dot{\beta}\sin\beta + 0)\underline{k}$$

$$\underline{\Omega} \times \underline{\omega} = \Omega\underline{k} \times (-\dot{\beta}\underline{i} + p\sin\beta\underline{j} + [p\cos\beta + \Omega]\underline{k})$$

$$= -\Omega\dot{\beta}\underline{j} - \Omega p\sin\beta\underline{i}$$

so $\underline{\alpha} = (p\dot{\beta}\cos\beta - \Omega\dot{\beta})\underline{j} - \Omega p\sin\beta\underline{i} - p\dot{\beta}\sin\beta\underline{k}$

$$\underline{\underline{\alpha} = -\Omega p\sin\beta\underline{i} + \dot{\beta}(p\cos\beta - \Omega)\underline{j} - p\dot{\beta}\sin\beta\underline{k}}$$

7/36 $\underline{v}_A = \underline{v}_B + \underline{\omega} \times \underline{r}_{A/B}$

$$\underline{a}_A = \underline{a}_B + \underline{\dot{\omega}} \times \underline{r}_{A/B} + \underline{\omega} \times (\underline{\omega} \times \underline{r}_{A/B})$$

$\underline{\omega} = 1.4\underline{i} + 1.2\underline{j}$ rad/sec ; $\underline{\dot{\omega}} = 2\underline{i} + 3\underline{j}$ rad/sec^2

$\underline{r}_{A/B} = 5\underline{i}$ ft , $\underline{v}_B = 3.2\underline{j}$ ft/sec , $\underline{a}_B = 4\underline{j}$ ft/sec^2

Substitution and simplification yield

$\underline{v}_A = 3.2\underline{j} - 6\underline{k}$ ft/sec $\Rightarrow$ $\underline{v_A = 6.8 \text{ ft/sec}}$

$\underline{a}_A = -7.2\underline{i} + 12.4\underline{j} - 15\underline{k}$ ft/sec^2 $\Rightarrow$ $\underline{a_A = 20.8 \text{ ft/sec}^2}$

7/37 Sol. I $\quad X^2+Y^2+Z^2=L^2$

$$X\dot{X}+Y\dot{Y}+0=0,\quad Z=\text{const.},\ L=\text{const.}$$

$$\dot{Y}=v_A=-\frac{X}{Y}\dot{X}=-\frac{0.3}{0.2}4=-6\ \text{m/s}\quad(-y\text{-dir.})$$

Sol. II $\quad \underline{v}_A=\underline{v}_B+\underline{\omega}\times\underline{r}_{A/B}\ ,\quad \underline{\omega}\cdot\underline{r}_{A/B}=0$ taking $\omega\perp AB$

$$v_A\underline{j}=4\underline{i}+\begin{vmatrix}\underline{i} & \underline{j} & \underline{k}\\ \omega_x & \omega_y & \omega_z\\ -0.3 & 0.2 & 0.6\end{vmatrix}$$

$$(\underline{i}\omega_x+\underline{j}\omega_y+\underline{k}\omega_z)\cdot(-0.3\underline{i}+0.2\underline{j}+0.6\underline{k})=0$$

Expand, equate coefficients & get

$$0.6\omega_y-0.2\omega_z=-4\qquad(1)$$

$$-0.6\omega_x-0.3\omega_z=v_A\qquad(2)$$

$$0.2\omega_x+0.3\omega_y=0\qquad(3)$$

$$-0.3\omega_x+0.2\omega_y+0.6\omega_z=0\qquad(4)$$

Solve simultaneously & get

$\omega_x=7.35$ rad/s, $\omega_y=-4.90$ rad/s, $\omega_z=5.31$ rad/s

$\underline{v}_A=-6\underline{j}$ m/s

7/38 | Angular velocity of axes is $\underline{\Omega} = p\underline{k}$

" " " A " $\underline{\omega} = \underline{\Omega} - \dot{\beta}\underline{i}$

$$\underline{\alpha} = \dot{\underline{\omega}} = \dot{\underline{\Omega}} - \ddot{\beta}\underline{i} - \dot{\beta}\dot{\underline{i}} = \dot{\underline{\Omega}} - \ddot{\beta}\underline{i} - \dot{\beta}\,\underline{\Omega}\times\underline{i}$$

$$= 0 - \ddot{\beta}\underline{i} - \dot{\beta}p\underline{j}$$

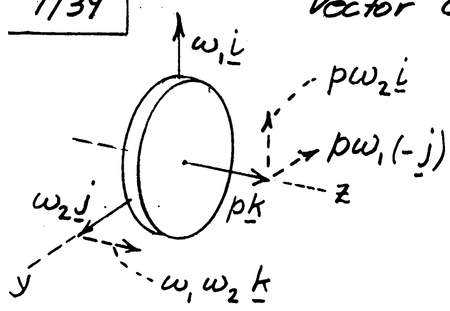
(a) before; $\dot{\beta}\,d\dot{\beta} = \ddot{\beta}\,d\beta$, $\ddot{\beta} = \dot{\beta}\dfrac{d\dot{\beta}}{d\beta} = \left(2\dfrac{2\pi}{360}\right)\dfrac{2}{18}$

$= 0.00388$ rad/s^2

$$\underline{\alpha} = -0.00388\underline{i} - \frac{2\pi}{180}\frac{1}{10}\underline{j} = \underline{-(3.88\underline{i} + 3.49\underline{j})10^{-3}\ \frac{\text{rad}}{\text{s}^2}}$$

(b) after; $\ddot{\beta} = 0$, $\underline{\alpha} = \underline{-3.49(10^{-3})\underline{j}\ \text{rad/s}^2}$

7/39



Vector $\omega_1 \underline{i}$ does not change orientation so $\frac{d}{dt}(\underline{i}\omega_1) = 0$

Accel. components give

$$\underline{\alpha} = p\omega_2 \underline{i} - p\omega_1 \underline{j} + \omega_1 \omega_2 \underline{k}$$

7/40 Let γ = angle between AB & y-axis

Angular velocity of AB is $\underline{\omega} = -\dot{\gamma}\underline{i} + \Omega\underline{k}$

So $\underline{\alpha} = \dot{\underline{\omega}} = -\ddot{\gamma}\underline{i} - \dot{\gamma}\dot{\underline{i}} + \underline{0}$

But $z = \ell \sin\gamma$, $v_A = \dot{z} = \ell\dot{\gamma}\cos\gamma$

& $\dot{v}_A = 0 = -\ell\dot{\gamma}^2\sin\gamma + \ell\ddot{\gamma}\cos\gamma$

So $\dot{\gamma} = \dfrac{v_A}{\ell\cos\gamma} = \dfrac{8}{5(4/5)} = 2 \text{ rad/sec}$

$\ddot{\gamma} = \dot{\gamma}^2\tan\gamma = 2^2(3/4) = 3 \text{ rad/sec}^2$

Also $\dot{\underline{i}} = \Omega\underline{k}\times\underline{i} = \Omega\underline{j} = 2\underline{j}$ rad/sec

Thus $\underline{\alpha} = -3\underline{i} - 2(2\underline{j}) = \underline{-3\underline{i} - 4\underline{j} \text{ rad/sec}^2}$

7/41 Precession is steady so $\underline{\alpha} = \underline{\Omega} \times \underline{p}$

$\underline{\alpha} = 4\pi\underline{k} \times 10\pi\underline{j} = \underline{-40\pi^2\underline{i}\ \ \text{rad/s}^2}$

$\underline{a}_A = \underline{a}_O + \dot{\underline{\Omega}} \times \underline{r}_{A/O} + \underline{\Omega} \times (\underline{\Omega} \times \underline{r}_{A/O}) + 2\underline{\Omega} \times \underline{v}_{rel} + \underline{a}_{rel}$

$\underline{a}_O = \underline{\Omega} \times (\underline{\Omega} \times \underline{r}_O) = -r_O\Omega^2\underline{i} = -0.3(4\pi)^2\underline{i} = -4.8\pi^2\underline{i}\ \text{m/s}^2$

$\dot{\underline{\Omega}} = 0;\ \underline{\Omega} \times \underline{r}_{A/O} = 4\pi\underline{k} \times 0.1\underline{k} = \underline{0}$

$\underline{v}_{rel} = \underline{p} \times \underline{r}_{A/O} = 10\pi\underline{j} \times 0.1\underline{k} = \pi\underline{i}\ \text{m/s}$

$2\underline{\Omega} \times \underline{v}_{rel} = 2(4\pi\underline{k}) \times \pi\underline{i} = 8\pi^2\underline{j}\ \text{m/s}^2$

$\underline{a}_{rel} = \underline{p} \times (\underline{p} \times \underline{r}_{A/O}) = -0.1(10\pi)^2\underline{k} = -10\pi^2\underline{k}\ \text{m/s}^2$

$\underline{a}_A = -4.8\pi^2\underline{i} + 8\pi^2\underline{j} - 10\pi^2\underline{k}$

$\quad = \underline{2\pi^2(-2.4\underline{i} + 4\underline{j} - 5\underline{k})\ \ \text{m/s}^2}$

7/42 | From Eqs. 7/6

$$\underline{v}_A = \underline{v}_O + \underline{\Omega} \times \underline{r}_{A/O} + \underline{v}_{rel}$$

$$\underline{v}_O = -R\Omega \underline{i},\ \underline{\Omega} = \Omega \underline{k},\ \underline{r}_{A/O} = b\sin\beta \underline{j} + b\cos\beta \underline{k},\ \underline{v}_{rel} = b\dot{\beta}(\cos\beta \underline{j} - \sin\beta \underline{k})$$

$$\underline{v}_A = -R\Omega \underline{i} + \Omega \underline{k} \times b(\sin\beta \underline{j} + \cos\beta \underline{k}) + b\dot{\beta}(\cos\beta \underline{j} - \sin\beta \underline{k})$$

$$\underline{\underline{v}_A = -\Omega(R + b\sin\beta)\underline{i} + b\dot{\beta}\cos\beta \underline{j} - b\dot{\beta}\sin\beta \underline{k}}$$

$$\underline{a}_A = \underline{a}_O + \dot{\underline{\Omega}} \times \underline{r}_{A/O} + \underline{\Omega} \times (\underline{\Omega} \times \underline{r}_{A/O}) + 2\underline{\Omega} \times \underline{v}_{rel} + \underline{a}_{rel}$$

$$\underline{a}_O = -R\Omega^2 \underline{j},\ \dot{\underline{\Omega}} = \underline{0},\ \underline{\Omega} \times (\underline{\Omega} \times \underline{r}_{A/O}) = \Omega \underline{k} \times (\Omega \underline{k} \times b[\sin\beta \underline{j} + \cos\beta \underline{k}])$$

$$2\underline{\Omega} \times \underline{v}_{rel} = 2\Omega \underline{k} \times b\dot{\beta}(\cos\beta \underline{j} - \sin\beta \underline{k}),\ \underline{a}_{rel} = b\dot{\beta}^2(\sin\beta \underline{j} + \cos\beta \underline{k})$$

Combine, collect terms, & get

$$\underline{\underline{a}_A = -2b\Omega\dot{\beta}\cos\beta \underline{i} - (\Omega^2[R + b\sin\beta] + b\dot{\beta}^2\sin\beta)\underline{j} - b\dot{\beta}^2\cos\beta \underline{k}}$$

7/43 $\underline{\Omega}$ = angular velocity of axes x-y-z $= \frac{2\pi N}{60}\underline{j} = \pi \underline{j}\ \frac{rad}{s}$

$\underline{v} = \underline{v}_A = \underline{v}_B + \underline{\Omega} \times \underline{r}_{A/B} + \underline{v}_{rel}$

where $\underline{v}_B = \pi \underline{j} \times \underline{r}_{OB} = \pi \underline{j} \times (-0.18\underline{i} + 0.1\underline{k}) = \pi(0.1\underline{i} + 0.18\underline{k})$ m/s

$\underline{\Omega} \times \underline{r}_{A/B} = \pi \underline{j} \times 0.1\underline{i} = -0.1\pi \underline{k}$ m/s

$\underline{v}_{rel} = p\underline{k} \times \underline{r}_{A/B} = \frac{240(2\pi)}{60}\underline{k} \times 0.1\underline{i} = 0.8\pi \underline{j}$ m/s

Collect terms & get $\underline{v} = \pi(0.1\underline{i} + 0.8\underline{j} + 0.08\underline{k})$ m/s

$\underline{a} = \underline{a}_A = \underline{a}_B + \dot{\underline{\Omega}} \times \underline{r}_{A/B} + \underline{\Omega} \times (\underline{\Omega} \times \underline{r}_{A/B}) + 2\underline{\Omega} \times \underline{v}_{rel} + \underline{a}_{rel}$, $\dot{\underline{\Omega}} = \underline{0}$

where $\underline{a}_B = \underline{\Omega} \times (\underline{\Omega} \times \underline{r}_{B/O}) = \pi \underline{j} \times (\pi \underline{j} \times [-0.18\underline{i} + 0.1\underline{k}])$

$= \pi^2(0.18\underline{i} - 0.1\underline{k})$ m/s²

$\underline{\Omega} \times (\underline{\Omega} \times \underline{r}_{A/B}) = \pi \underline{j} \times (-0.1\pi \underline{k}) = -0.1\pi^2 \underline{i}$ m/s²

$2\underline{\Omega} \times \underline{v}_{rel} = 2\pi \underline{j} \times 0.8\pi \underline{j} = \underline{0}$

$\underline{a}_{rel} = \dot{p}\underline{k} \times \underline{r}_{A/B} + \underline{r}_{A/B}\, p^2(-\underline{i}) = \underline{0} - (8\pi)^2 0.1\underline{i} = -6.4\pi^2 \underline{i}\ \frac{m}{s^2}$

Collect terms & get

$\underline{a} = -0.1\pi^2\underline{i} - 6.4\pi^2\underline{i} + 0.18\pi^2\underline{i} - 0.1\pi^2\underline{k}$

$\underline{a} = -\pi^2(6.32\underline{i} + 0.1\underline{k})$ m/s²

7/44 Angular velocity of drum is

$\underline{\omega} = (-p\cos\theta)\underline{i} + \dot{\theta}\underline{j} + (p\sin\theta + \Omega)\underline{k}$ From Eq. 7/7

$\underline{\alpha} = \dot{\underline{\omega}} = (p\dot{\theta}\sin\theta)\underline{i} + (p\dot{\theta}\cos\theta)\underline{k} + \underline{\Omega}\times\underline{\omega}$

But angular velocity of axes is $\underline{\Omega} = \Omega\underline{k}$, so

$$\underline{\alpha} = (p\dot{\theta}\sin\theta)\underline{i} + (p\dot{\theta}\cos\theta)\underline{k} + \Omega\underline{k}\times[(-p\cos\theta)\underline{i} + \dot{\theta}\underline{j} + (p\sin\theta + \Omega)\underline{k}]$$

$$= (p\dot{\theta}\sin\theta)\underline{i} + (p\dot{\theta}\cos\theta)\underline{k} - (p\Omega\cos\theta)\underline{j} - \Omega\dot{\theta}\underline{i}$$

$$= \underline{\dot{\theta}(p\sin\theta - \Omega)\underline{i} - (p\Omega\cos\theta)\underline{j} + (p\dot{\theta}\cos\theta)\underline{k}}$$

7/45 Angular velocity of axes $\underline{\Omega} = \Omega \underline{k}$

" " " panels $\underline{\omega} = -\dot{\theta}\underline{j} + \Omega \underline{k}$

$\dot{\underline{\omega}} = -\dot{\theta}\dot{\underline{j}} + \Omega\dot{\underline{k}} = -\dot{\theta}(\underline{\Omega} \times \underline{j}) + \Omega(\underline{\Omega} \times \underline{k}) = \underline{\Omega} \times \underline{\omega} = \Omega\dot{\theta}\underline{i}$

$= \frac{1}{2}\frac{1}{4}\underline{i} = \underline{\frac{1}{8}\underline{i} \text{ rad/sec}^2}$

$\underline{a}_A = \underline{a}_0 + \dot{\underline{\Omega}} \times \underline{r}_{A/0} + \underline{\Omega} \times (\underline{\Omega} \times \underline{r}_{A/0}) + 2\underline{\Omega} \times \underline{v}_{rel} + \underline{a}_{rel}$

$\underline{a}_0 = \underline{0}; \quad \underline{\Omega} \times \underline{r}_{A/0} = \frac{1}{2}\underline{k} \times (-\underline{i} + 8\underline{j} + \sqrt{3}\underline{k}) = -\frac{1}{2}\underline{j} - 4\underline{i} \ \frac{ft}{sec}$

$\underline{\Omega} \times (\underline{\Omega} \times \underline{r}_{A/0}) = \frac{1}{2}\underline{k} \times (-\frac{1}{2}\underline{j} - 4\underline{i}) = \frac{1}{4}\underline{i} - 2\underline{j} \ \text{ft/sec}^2$

$2\underline{\Omega} \times \underline{v}_{rel} = 2(\frac{1}{2}\underline{k}) \times (-\frac{\sqrt{3}}{4}\underline{i} - \frac{1}{4}\underline{k}) = -\frac{\sqrt{3}}{4}\underline{j} \ \text{ft/sec}^2$

$\underline{a}_{rel} = 2(\frac{1}{4})^2(\frac{1}{2}\underline{i} - \frac{\sqrt{3}}{2}\underline{k}) = \frac{1}{16}\underline{i} - \frac{\sqrt{3}}{16}\underline{k} \ \text{ft/sec}^2$

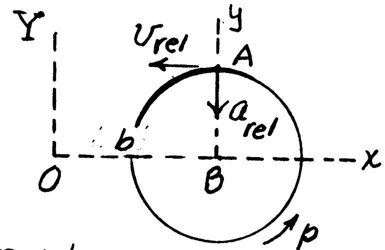
$\underline{a}_A = (\frac{1}{4} + \frac{1}{16})\underline{i} + (-2 - \frac{\sqrt{3}}{4})\underline{j} - \frac{\sqrt{3}}{16}\underline{k}$

$= \underline{0.313\underline{i} - 2.43\underline{j} - 0.1083\underline{k} \ \text{ft/sec}^2}$

with $a_A = 2.45 \ \text{ft/sec}^2$

7/46 Angular velocity of x-y-z axes is

$$\underline{\Omega} = -\omega_1 \underline{i} + \omega_2 \underline{J}$$



$$\underline{v} = \underline{v}_A = \underline{v}_B + \underline{\Omega} \times \underline{r}_{A/B} + \underline{v}_{rel}$$

where $\underline{v}_B = b\omega_2(-\underline{k}) = -b\omega_2 \underline{k}$

$$\underline{\Omega} \times \underline{r}_{A/B} = (-\omega_1 \underline{i} + \omega_2 \underline{J}) \times r\underline{j} = -r\omega_1 \underline{k}$$

$$\underline{v}_{rel} = -rp\underline{i}$$

Thus $\underline{v} = -b\omega_2 \underline{k} - r\omega_1 \underline{k} - rp\underline{i} = \underline{-rp\underline{i} - (r\omega_1 + b\omega_2)\underline{k}}$

$$\underline{a} = \underline{a}_A = \underline{a}_B + \dot{\underline{\Omega}} \times \underline{r}_{A/B} + \underline{\Omega} \times (\underline{\Omega} \times \underline{r}_{A/B}) + 2\underline{\Omega} \times \underline{v}_{rel} + \underline{a}_{rel}$$

where $\underline{a}_B = -b\omega_2^2 \underline{i}$

$$\dot{\underline{\Omega}} = -\omega_1 \dot{\underline{i}} + \omega_2 \dot{\underline{J}} = -\omega_1 \underline{\Omega} \times \underline{i} = \omega_1 \omega_2 \underline{k}$$

$$\underline{\Omega} \times (\underline{\Omega} \times \underline{r}_{A/B}) = (-\omega_1 \underline{i} + \omega_2 \underline{J}) \times (-r\omega_1 \underline{k}) = -r\omega_1(\omega_1 \underline{j} + \omega_2 \underline{i})$$

$$2\underline{\Omega} \times \underline{v}_{rel} = 2(-\omega_1 \underline{i} + \omega_2 \underline{J}) \times (-rp\underline{i}) = 2rp\omega_2 \underline{k}$$

$$\underline{a}_{rel} = -rp^2 \underline{j}, \quad \dot{\underline{\Omega}} \times \underline{r}_{A/B} = \omega_1 \omega_2 \underline{k} \times r\underline{j} = -r\omega_1 \omega_2 \underline{i}$$

Substitute, combine & get

$$\underline{\underline{a} = -\omega_2(b\omega_2 + 2r\omega_1)\underline{i} - r(\omega_1^2 + p^2)\underline{j} + 2rp\omega_2 \underline{k}}$$

7/47 | $\underline{\Omega}$ = angular velocity of axes x-y-z

$\underline{\omega}$ = " " " simulator $= \underline{\Omega} + \underline{p}$

Let N = angular velocity of frame $= 0.2$ rad/s const.

$p = 0.9$ rad/s const., $\dot{\beta} = 0.15$ rad/s const.

$\underline{\Omega} = \underline{i}\dot{\beta} + \underline{j}N\cos\beta - \underline{k}N\sin\beta$; $\underline{p} = p\underline{k}$

$$\underline{\omega}_{\beta=0} = 0.15\underline{i} + 0.2\underline{j} + 0.9\underline{k} \text{ rad/s}$$

From Eq. 7/7, $\underline{\alpha} = \left(\frac{d\omega}{dt}\right)_{XYZ} = \left(\frac{d\omega}{dt}\right)_{xyz} + \underline{\Omega}\times\underline{\omega}$

$\alpha = (\underline{0} - \underline{j}N\dot{\beta}\sin\beta - \underline{k}N\dot{\beta}\cos\beta + \underline{0}) + \underline{\Omega}\times(\underline{\Omega} + \underline{p})$

where $\underline{\Omega}\times(\underline{\Omega} + \underline{p}) = \underline{\Omega}\times\underline{p} = (\underline{i}\dot{\beta} + \underline{j}N\cos\beta - \underline{k}N\sin\beta)\times p\underline{k}$

$= \underline{i}Np\cos\beta - \underline{j}p\dot{\beta}$

so $\underline{\alpha}_{\beta=0} = \underline{i}Np - \underline{j}p\dot{\beta} - \underline{k}N\dot{\beta}$

$= 0.2(0.9)\underline{i} - 0.9(0.15)\underline{j} - 0.2(0.15\underline{k})$ rad/s^2

$$= 0.18\underline{i} - 0.135\underline{j} - 0.030\underline{k} \text{ rad/s}^2$$

▸ 7/48 From Sample Problem 7/2

$\Omega = 2\pi$ rad/sec, $\omega_y = \sqrt{3}\pi$ rad/sec, $\omega_z = 5\pi$ rad/sec, $\omega_0 = 4\pi \frac{\text{rad}}{\text{sec}}$

Also $\omega_x = -\dot{\gamma} = -3\pi$ rad/sec

In general $\underline{\omega} = (-\dot{\gamma}\underline{i} + \Omega\cos\gamma\underline{j} + [\omega_0 + \Omega\sin\gamma]\underline{k})$

For $\gamma = 30°$, $\underline{\omega} = \pi(-3\underline{i} + \sqrt{3}\underline{j} + 5\underline{k})$ rad/sec

From Eq. 7/7 $\quad \underline{\alpha} = [d\underline{\omega}/dt]_{XYZ} = [d\underline{\omega}/dt]_{xyz} + \underline{\omega}_{axes} \times \underline{\omega}$

But $[d\underline{\omega}/dt]_{xyz} = (\underline{0} - \Omega\dot{\gamma}\sin\gamma\underline{j} + \Omega\dot{\gamma}\cos\gamma\underline{k})$

$= 6\pi^2(-\frac{1}{2}\underline{j} + \frac{\sqrt{3}}{2}\underline{k}) = 3\pi^2(-\underline{j} + \sqrt{3}\underline{k})$ rad/sec^2

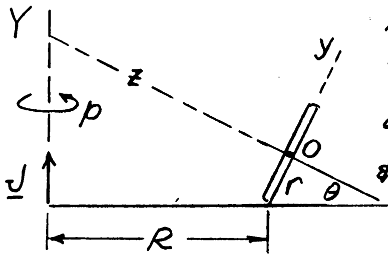
$\underline{\omega}_{axes} = \underline{\omega} - \omega_0\underline{k}$ & $\underline{\omega}_{axes} \times \underline{\omega} = (\underline{\omega} - \omega_0\underline{k}) \times \underline{\omega} = -\omega_0\underline{k} \times \underline{\omega}$

so $\underline{\omega}_{axes} \times \underline{\omega} = -4\pi\underline{k} \times \pi(-3\underline{i} + \sqrt{3}\underline{j} + 5\underline{k}) = 4\pi^2(\sqrt{3}\underline{i} + 3\underline{j}) \frac{\text{rad}}{\text{sec}^2}$

Thus $\underline{\alpha} = 3\pi^2(-\underline{j} + \sqrt{3}\underline{k}) + 4\pi^2(\sqrt{3}\underline{i} + 3\underline{j})$

$= \pi^2(4\sqrt{3}\underline{i} + 9\underline{j} + 3\sqrt{3}\underline{k})$ rad/sec^2

▶ 7/49



$\underline{\omega} = \underline{J}p + \frac{Rp}{r}\underline{k} = (p\cos\theta)\underline{j} + (p\sin\theta + \frac{Rp}{r})\underline{k}$

Angle $d\phi$ measured in x-y-z turned by wheel in time dt is

$d\phi = \frac{R(p\,dt)}{r}$ so $\dot{\phi} = \frac{Rp}{r}$

& $\underline{\omega} = p\left[\underline{j}\cos\theta + \underline{k}\left(\sin\theta + \frac{R}{r}\right)\right]$

Angular velocity of axes is $\underline{\Omega} = \underline{J}p$ so

$\underline{\omega} = \underline{\Omega} + (\frac{Rp}{r})\underline{k}$; Now use $\left(\frac{d[\,]}{dt}\right)_{XYZ} = \left(\frac{d[\,]}{dt}\right)_{xyz} + \underline{\Omega} \times [\,]$

Noting $\underline{\Omega}$ is constant in XYZ & xyz.

Thus $\underline{\alpha} = \left(\frac{d\underline{\omega}}{dt}\right)_{XYZ} = 0 + \underline{\Omega} \times \left[\underline{\Omega} \times \frac{Rp}{r}\underline{k}\right] = \underline{\Omega} \times \frac{Rp}{r}\underline{k}$

$\alpha = \left[(p\cos\theta)\underline{j} + (p\sin\theta)\underline{k}\right] \times \frac{Rp}{r}\underline{k}$, $\underline{\alpha} = (\frac{Rp^2}{r}\cos\theta)\underline{i}$

or merely $\underline{\alpha} = \dot{\underline{\omega}} = \underline{0} + \frac{Rp}{r}\dot{\underline{k}} = \frac{Rp}{r}(\underline{\Omega} \times \underline{k})$, etc.

▶ 7/50 | Angular vel. of x-y-z is $\boldsymbol{\Omega} = \mathbf{i}\, q \sin\theta - \mathbf{j}\dot{\theta} + \mathbf{k}\, q \cos\theta$

$$\mathbf{a} = \mathbf{a}_A = \mathbf{a}_O + \dot{\boldsymbol{\Omega}} \times \mathbf{r}_{A/O} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}_{A/O}) + 2\boldsymbol{\Omega} \times \mathbf{v}_{rel} + \mathbf{a}_{rel}$$

where $\mathbf{a}_O = -6(8^2)\mathbf{j} = -384\mathbf{j}$ in./sec^2; $\mathbf{r}_{A/O} = 6\mathbf{i} - 4\mathbf{k}$ in.

$$\dot{\boldsymbol{\Omega}} = \boldsymbol{\Omega} \times \boldsymbol{\Omega} + \mathbf{i}\, q\dot{\theta}\cos\theta + \mathbf{0} - \mathbf{k}\, q\dot{\theta}\sin\theta \quad \text{by Eq. 7/7a}$$

$$= q\dot{\theta}(\mathbf{i}\cos\theta - \mathbf{k}\sin\theta) = 8(\sqrt{3}\,\mathbf{i} - \mathbf{k}) \text{ rad/sec}^2$$

$$\dot{\boldsymbol{\Omega}} \times \mathbf{r}_{A/O} = 8(\sqrt{3}\,\mathbf{i} - \mathbf{k}) \times (6\mathbf{i} - 4\mathbf{k}) = 16(2\sqrt{3} - 3)\mathbf{j} = 7.43\mathbf{j} \text{ in./sec}^2$$

$$\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}_{A/O}) = -423\mathbf{i} + 7.43\mathbf{j} + 246\mathbf{k} \text{ in./sec}^2$$

$$2\boldsymbol{\Omega} \times \mathbf{v}_{rel} = 2(4\mathbf{i} - 2\mathbf{j} + 4\sqrt{3}\,\mathbf{k}) \times (+4[30]\mathbf{j}) = 960(-\sqrt{3}\,\mathbf{i} + \mathbf{k}) \ \frac{\text{in.}}{\text{sec}^2}$$

$$\mathbf{a}_{rel} = 4(30)^2\mathbf{k} = 3600\mathbf{k} \text{ in./sec}^2$$

Combine & get $\underline{\mathbf{a} = -2090\mathbf{i} - 369\mathbf{j} + 4810\mathbf{k} \text{ in./sec}^2}$

$$\boldsymbol{\omega} = \boldsymbol{\Omega} + p\mathbf{i}, \quad \boldsymbol{\alpha} = \dot{\boldsymbol{\omega}} = \dot{\boldsymbol{\Omega}} + p\dot{\mathbf{i}} = \dot{\boldsymbol{\Omega}} + p\boldsymbol{\Omega} \times \mathbf{i}$$

$$= 8(\sqrt{3}\,\mathbf{i} - \mathbf{k}) + 30(4\mathbf{i} - 2\mathbf{j} + 4\sqrt{3}\,\mathbf{k}) \times \mathbf{i}$$

$$\underline{\boldsymbol{\alpha} = 8\sqrt{3}\,\mathbf{i} + 120\sqrt{3}\,\mathbf{j} + 52\mathbf{k} \text{ rad/sec}^2}$$

▶ 7/51 | Angular velocity of axes $= \underline{\Omega}$

" " " rotor $= \underline{\omega} = \underline{\Omega} + p\underline{k}$

where $p = 100(2\pi)/60 = 10\pi/3$ rad/s

$\underline{\Omega} = -\dot{\gamma}\underline{i} + \underline{j}\,\omega_1 \cos\gamma + \underline{k}\,\omega_1 \sin\gamma\ ,\quad \omega_1 = \frac{2\pi}{60}20 = \frac{2\pi}{3}\frac{\text{rad}}{\text{s}}$

$\underline{\alpha} = \left(\frac{d\underline{\omega}}{dt}\right)_{XYZ} = \left(\frac{d\underline{\omega}}{dt}\right)_{xyz} + \underline{\Omega}\times\underline{\omega} \qquad (\text{Eq. } 8/7)$

$\left(\frac{d\omega}{dt}\right)_{xyz} = \left(\frac{d\Omega}{dt}\right)_{xyz} + 0 = 0 - \underline{j}\,\dot{\gamma}\omega_1 \sin\gamma + \underline{k}\,\dot{\gamma}\omega_1\cos\gamma$

$\underline{\Omega}\times\underline{\omega} = \underline{\Omega}\times(\underline{\Omega} + p\underline{k}) = \underline{\Omega}\times p\underline{k} = \dot{\gamma}p\underline{j} + p\omega_1\cos\gamma\,\underline{i}$

$\underline{\alpha} = (\dot{\gamma}p - \dot{\gamma}\omega_1\sin\gamma)\,\underline{j} + p\omega_1\cos\gamma\,\underline{i} + \dot{\gamma}\omega_1\cos\gamma\,\underline{k}$

substitute $\dot{\gamma} = 4$ rad/s, $p = 10\pi/3$ rad/s, $\omega_1 = 2\pi/3$ rad/s

& get

$\underline{\alpha} = \left(4\,\frac{10\pi}{3} - 4\,\frac{2\pi}{3}\,\frac{1}{2}\right)\underline{j} + \frac{10\pi}{3}\,\frac{2\pi}{3}\,\frac{\sqrt{3}}{2}\,\underline{i} + 4\,\frac{2\pi}{3}\,\frac{\sqrt{3}}{2}\,\underline{k}$

$\quad = 12\pi\underline{j} + \frac{10\pi^2}{3\sqrt{3}}\,\underline{i} + \frac{4\pi}{\sqrt{3}}\,\underline{k} = 18.99\underline{i} + 37.70\underline{j} + 7.25\underline{k}\ \frac{\text{rad}}{\text{s}^2}$

$\alpha = \sqrt{18.99^2 + 37.70^2 + 7.25^2} = \underline{42.8 \text{ rad/s}^2}$

▸ 7/52 Attach origin of translating axes to B with x-y-z parallel to X-Y-Z

$\underline{v}_A = \underline{v}_B + \underline{\omega} \times \underline{r}_{A/B}$ & note that $\underline{\omega}\cdot\underline{J}\times(\underline{r}_{A/B}\times\underline{J}) = 0$

where $\underline{j} = \underline{J}$ = unit vector in Y-dir.

$\underline{v}_A = v_A\underline{j}$, $\underline{v}_B = r\omega_0(-\underline{i}) = -0.080(4)\underline{i} = -0.32\underline{i}$ m/s

y-coord. of A is $\sqrt{\overline{0.300}^2 - \overline{0.100}^2 - \overline{0.200}^2} = 0.200$ m

$\underline{r}_{A/B} = -0.1\underline{i} + 0.2\underline{j} + 0.2\underline{k}$ m ;

Thus $v_A\underline{j} = -0.32\underline{i} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \omega_x & \omega_y & \omega_z \\ -0.1 & 0.2 & 0.2 \end{vmatrix}$

Expand & equate coefficients to get

$$0.2\omega_y - 0.2\omega_z = 0.32 \quad \text{-----(1)}$$

$$0.2\omega_x + 0.1\omega_z = -v_A \quad \text{----(2)}$$

$$0.2\omega_x + 0.1\omega_y = 0 \quad \text{----(3)}$$

Also $(\underline{i}\omega_x + \underline{j}\omega_y + \underline{k}\omega_z)\cdot\underline{j}\times[(-0.1\underline{i} + 0.2\underline{j} + 0.2\underline{k})\times\underline{j}] = 0$

which gives $\omega_x - 2\omega_z = 0$ ------(4)

Solve (1), (2), (3), (4) & get $\omega_x = -0.64$, $\omega_y = 1.28$, $\omega_z = -0.32$ rad/s

$\underline{v}_A = \underline{0.160\underline{j}\text{ m/s}}$, $\underline{\underline{\omega} = 0.32(-2\underline{i} + 4\underline{j} - \underline{k})\text{ rad/s}}$

7/53 With $\omega_x = \omega_y = 0$, $\omega_z = -\omega$,

$H_{O_x} = -I_{xz}\omega_z$, $H_{O_y} = -I_{yz}\omega_z$, $H_{O_z} = I_{zz}\omega_z$

$I_{xz} = mb^2$, $I_{yz} = 2mb^2$, $I_{zz} = 2mb^2$

$\underline{H}_O = -mb^2(-\omega)\underline{i} - 2mb^2(-\omega)\underline{j} + 2mb^2(-\omega)\underline{k}$

$\underline{H}_O = mb^2\omega(\underline{i} + 2\underline{j} - 2\underline{k})$, $\underline{H_O = 3mb\omega^2}$

$\underline{G} = \Sigma m_i \underline{v}_i = mb\omega(\underline{i} - \underline{j})$, $\underline{G = mb\omega\sqrt{2}}$

7/54 With $\omega_x = \omega_y = 0$, $\omega_z = -\omega$,

$H_{O_x} = -I_{xz}\omega_z$, $H_{O_y} = -I_{yz}\omega_z$, $H_{O_z} = I_{zz}\omega_z$

$I_{xz} = mb^2$, $I_{yz} = 2mb^2$, $I_{zz} = \frac{1}{6}ml^2 + 2mb^2$

$$\underset{-}{H}_O = -mb^2(-\omega)\underset{-}{i} - 2mb^2(-\omega)\underset{-}{j} + \left(\frac{1}{6}ml^2 + 2mb^2\right)(-\omega)\underset{-}{k}$$

$$\underline{\underset{-}{H}_o = mb^2\omega\left(\underset{-}{i} + 2\underset{-}{j} - \left[\frac{1}{6}\left(\frac{l}{b}\right)^2 + 2\right]\underset{-}{k}\right)}$$

7/55 x-y-z are principal axes so

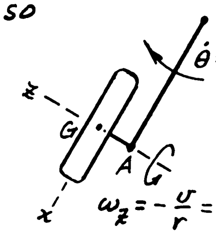
$\underline{H} = I_{xx}\omega_x \underline{i} + I_{yy}\omega_y \underline{j} + I_{zz}\omega_z \underline{k}$

$I_{zz} = mk^2$

$= 45(0.370)^2 = 6.16\ \text{kg}\cdot\text{m}^2$

$I_{xx} + I_{yy} = I_{zz}$ & $I_{xx} = I_{yy}$

so $I_{yy} = \frac{1}{2} I_{zz} = 3.08\ \text{kg}\cdot\text{m}^2$



$\dot{\theta} = \frac{30}{180}\pi = 0.524$ rad/s $= -\omega_y$

$\omega_z = -\frac{v}{r} = -\frac{200(10^3)/3600}{0.920/2} = -120.8$ rad/s

$\omega_x = 0$

About G, $\underline{H}_G = \underline{0} + 3.08(-0.524)\underline{j} + 6.16(-120.8)\underline{k}$

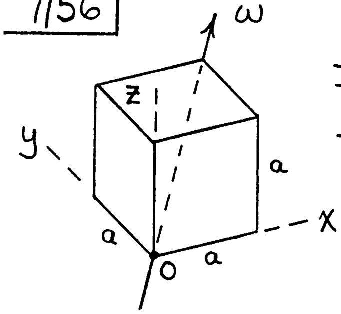
$$\underline{H}_G = -1.613\underline{j} - 744\underline{k}\ \text{kg}\cdot\text{m}^2/\text{s}$$

About A, $I_{yy} = \bar{I}_{yy} + md^2 = 3.08 + 45(0.215)^2 = 5.16\ \text{kg}\cdot\text{m}^2$

$$\underline{H}_A = \underline{0} + 5.16(-0.524)\underline{j} + 6.16(-120.8)\underline{k}$$

$$\underline{H}_A = -2.70\underline{j} - 744\underline{k}\ \text{kg}\cdot\text{m}^2/\text{s}$$

7/56



$$\omega_x = \omega_y = \omega_z = \omega/\sqrt{3}$$

$$I_{xx} = I_{yy} = I_{zz} = \frac{2}{3}ma^2$$

$$I_{xy} = I_{xz} = I_{yz} = \frac{1}{4}ma^2$$

$$H_x = H_y = H_z$$

$$= \frac{2}{3}ma^2\frac{\omega}{\sqrt{3}} - 2\left(\frac{1}{4}\right)ma^2\frac{\omega}{\sqrt{3}}$$

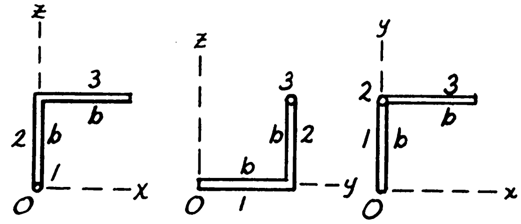
$$= \frac{ma^2\omega}{6\sqrt{3}}$$

$$\underline{H} = \frac{ma^2\omega}{6\sqrt{3}}(\underline{i}+\underline{j}+\underline{k}), \quad H = \frac{ma^2\omega}{6}$$

7/57 With $\omega_x = \omega_y = 0$,

Eq. 7/11 gives

$\underline{H}_O = (-I_{xz}\underline{i} - I_{yz}\underline{j} + I_{zz}\underline{k})\omega$



Part	I_{xz}	I_{yz}	I_{zz}
1	0	0	$\frac{1}{3}\rho b^3$
2	0	$\frac{1}{2}\rho b^3$	ρb^3
3	$\frac{1}{2}\rho b^3$	ρb^3	$\frac{4}{3}\rho b^3$
Totals	$\frac{1}{2}\rho b^3$	$\frac{3}{2}\rho b^3$	$\frac{8}{3}\rho b^3$

$$\left\{ (I_{zz})_3 = \frac{1}{12}\rho b\, b^2 + \rho b\left(b^2 + \left[\frac{b}{2}\right]^2\right) = \frac{4}{3}\rho b^3 \right\}$$

so $\underline{H}_O = \rho b^3\left(-\frac{1}{2}\underline{i} - \frac{3}{2}\underline{j} + \frac{8}{3}\underline{k}\right)\omega$

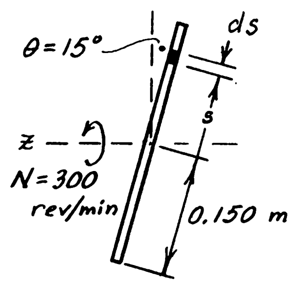
$T = \frac{1}{2}\underline{\omega}\cdot\underline{H}_O = \frac{1}{2}\omega\cdot\frac{8}{3}\rho b^3\omega$, $\quad T = \frac{4}{3}\rho b^3\omega^2$

7/58

With $\omega_x = \omega_y = 0$,

Eq. 7/11 gives

$\underline{H}_O = -I_{xz}\,\omega_z\,\underline{i} - I_{yz}\,\omega_z\,\underline{j} + I_{zz}\,\omega_z\,\underline{k}$



Mass per unit of s is $\dfrac{12}{0.300} = 40\ \dfrac{kg}{m}$

$I_{yz} = 2\int_0^{0.150} (s\cos 15°)(-s\sin 15°)\,40\,ds$

$\quad = -40\sin 30° \times \dfrac{s^3}{3}\Big|_0^{0.150}$

$\quad = -0.0225\ kg\cdot m^2$

$\omega_z = \dfrac{300\times 2\pi}{60} = 31.4\ rad/s$

$I_{zz} = \dfrac{1}{12}12\{0.240^2 + (0.300\cos 15°)^2\} = 0.1416\ kg\cdot m^2$

By symmetry $I_{xz} = 0$

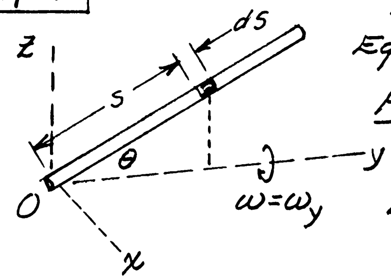
$\underline{H}_O = \underline{0} - 0.0225(31.4)\underline{j} + 0.1416(31.4)\underline{k}$

$\underline{\underline{H}_O = -0.707\underline{j} + 4.45\underline{k}\ \ kg\cdot m^2/s}$

$T = \frac{1}{2}\underline{\omega}\cdot\underline{H}_O = \frac{1}{2}(31.4\underline{k})\cdot(-0.707\underline{j} + 4.45\underline{k})$

$\underline{T = 69.9\ J}$

7/59



$\omega_x = \omega_z = 0, \quad \omega_y = \omega$, so

Eq. 7/11 gives

$$\underline{H} = (-\underline{i}\,I_{xy} + \underline{j}\,I_{yy} - \underline{k}\,I_{yz})\omega$$

But $I_{xy} = 0$

$$I_{yy} = \frac{1}{3}m(\ell \sin\theta)^2$$

& $I_{yz} = \int yz\,dm = \int_0^\ell (s\cos\theta)(s\sin\theta)\,\rho\,ds$

where ρ = mass per unit length

so $I_{yz} = \rho \sin\theta\cos\theta \frac{\ell^3}{3} = \frac{1}{3}m\ell^2 \sin\theta\cos\theta$

& $\underline{H} = \left[\underline{i}(0) + \underline{j}\,\frac{1}{3}m\ell^2\sin^2\theta - \underline{k}\,\frac{1}{3}m\ell^2\sin\theta\cos\theta\right]\omega$

$$= \underline{\frac{1}{3}m\ell^2\omega\sin\theta\,(\underline{j}\sin\theta - \underline{k}\cos\theta)}$$

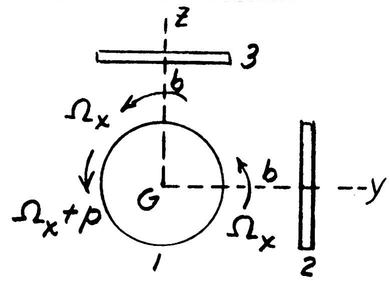
7/60 $\omega_x = \omega_y = 0$, $\omega_z = \omega$

$I_{xz} = 0$, $I_{yz} = 0 + m\left(\frac{4r}{3\pi}\right)\left(c + \frac{b}{2}\right)$, $I_{zz} = \frac{1}{2}mr^2$

So $\underline{H} = -I_{yz}\omega_z \underline{j} + I_{zz}\omega_z \underline{k}$

$$\underline{H} = mr\omega\left[-\frac{2(2c+b)}{3\pi}\underline{j} + \frac{r}{2}\underline{k}\right]$$

7/61 | About G,



$$H_{x_1} = I(\Omega_x + p)$$

$$H_{x_2} = \left(\frac{I}{2} + mb^2\right)\Omega_x$$

$$H_{x_3} = \left(\frac{I}{2} + mb^2\right)\Omega_x$$

So $H_x = I(\Omega_x + p) + (I + 2mb^2)\Omega_x$

$$= Ip + 2(I + mb^2)\Omega_x$$

similarly

$$H_y = Ip + 2(I + mb^2)\Omega_y$$

$$H_z = Ip + 2(I + mb^2)\Omega_z$$

Thus $\underline{H}_G = \underline{Ip(\underline{i} + \underline{j} + \underline{k}) + 2(I + mb^2)\underline{\Omega}}$

where $\underline{\Omega} = \Omega_x \underline{i} + \Omega_y \underline{j} + \Omega_z \underline{k}$

7/62 $\quad \underline{H}_O = \underline{\bar{H}} + \underline{\bar{r}} \times \underline{G}$; $\omega_x = \omega$, $\omega_y = p$, $\omega_z = 0$

where $\bar{H}_x = \left(\frac{1}{12} m b^2 + \frac{1}{4} m r^2\right)\omega$

$\bar{H}_y = \frac{1}{2} m r^2 p$, $\bar{H}_z = 0$

$\underline{\bar{r}} = -\frac{b}{2}\underline{j} + h\underline{k}$, $\underline{G} = -mh\omega\underline{j} - m\frac{b}{2}\omega\underline{k}$

$\underline{\bar{r}} \times \underline{G} = \left(-\frac{b}{2}\underline{j} + h\underline{k}\right) \times (-m\omega)\left(h\underline{j} + \frac{b}{2}\underline{k}\right)$

$= \frac{mb^2}{4}\omega\underline{i} + mh^2\omega\underline{i} = m\omega\left(h^2 + \frac{b^2}{4}\right)\underline{i}$

Thus $\underline{H}_O = m\omega\left(\frac{b^2}{12} + \frac{r^2}{4} + h^2 + \frac{b^2}{4}\right)\underline{i} + \frac{1}{2} m r^2 p \underline{j}$

$$\underline{H}_O = \left(\frac{b^2}{3} + \frac{r^2}{4} + h^2\right) m\omega\underline{i} + \frac{1}{2} m r^2 p \underline{j}$$

7/63

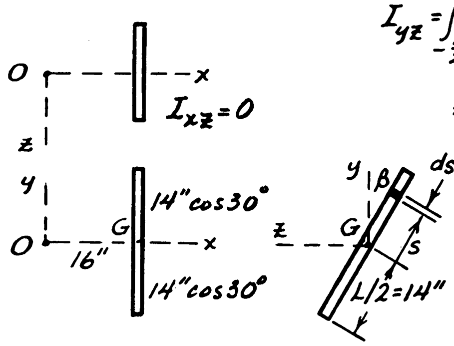
From Eq. 7/11 with $\omega_x = \omega_y = 0$,

$$\underline{H}_O = -I_{xz}\,\omega_z\,\underline{i} - I_{yz}\,\omega_z\,\underline{j} + I_{zz}\,\omega_z\,\underline{k}$$

$$I_{yz} = \int_{-\frac{L}{2}}^{\frac{L}{2}} (s\cos\beta)(-s\sin\beta)\rho\, ds$$

where ρ = mass/unit length

$$= -\rho \sin\beta\cos\beta \left.\frac{s^3}{3}\right|_{-L/2}^{L/2} = -\rho\frac{L^3}{24}\sin 2\beta$$



$$= -\frac{6.20/32.2}{28/12}\frac{(28/12)^3}{24}\sin 60^\circ$$

$$= -0.0378 \text{ lb-ft-sec}^2$$

$$I_{zz} = \bar{I}_O = \frac{1}{12}mL^2 + md^2$$

$$= \frac{6.20}{32.2}\left[\left(\frac{28\cos 30^\circ}{12}\right)^2\frac{1}{12} + \left(\frac{16}{12}\right)^2\right]$$

$$= 0.408 \text{ lb-ft-sec}^2$$

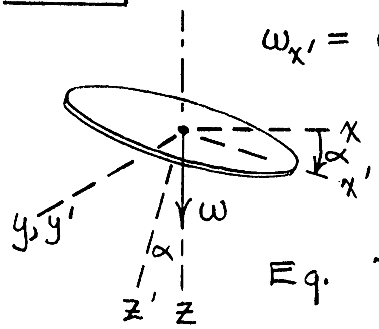
$$\underline{H}_O = (-I_{xz}\underline{i} - I_{yz}\underline{j} + I_{zz}\underline{k})\omega_z = (\underline{0} - [-0.0378]\underline{j} + 0.408\underline{k})\frac{600\times 2\pi}{60}$$

$$\underline{\underline{H}_O = 2.38\underline{j} + 25.6\,\underline{k} \text{ lb-ft-sec}}$$

From Eq. 7/18 $T = \frac{1}{2}\underline{\omega}\cdot\underline{H}_O = \frac{1}{2}\omega_z\underline{k}\cdot\underline{H}_O$

$$= \frac{1}{2}\frac{600\times 2\pi}{60}\times 25.6 = \underline{805 \text{ ft-lb}}$$

7/64 Introduce axes x'-y'-z' as shown.



$$\omega_{x'} = \omega \sin\alpha, \quad \omega_{y'} = 0, \quad \omega_{z'} = \omega\cos\alpha$$

$$I_{x'x'} = I_{y'y'} = \frac{1}{4}mr^2$$

$$I_{z'z'} = \frac{1}{2}mr^2$$

Eq. 7/11 yields

$$\underline{H} = \left(\frac{1}{4}mr^2\right)\omega\sin\alpha\,\underline{i}' + \left(\frac{1}{2}mr^2\right)\omega\cos\alpha\,\underline{k}'$$

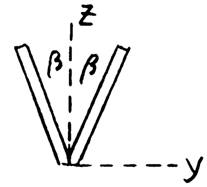
But
$$\begin{cases} \underline{i}' = \underline{i}\cos\alpha + \underline{k}\sin\alpha \\ \underline{k}' = -\underline{i}\sin\alpha + \underline{k}\cos\alpha \end{cases}$$

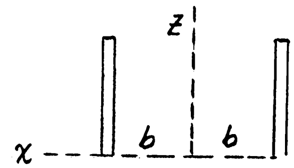
Thus
$$\underline{H} = \frac{1}{4}mr^2\omega\Big[(-\sin\alpha\cos\alpha)\underline{i} + (\sin^2\alpha + 2\cos^2\alpha)\underline{k}\Big]$$

$$\beta = \cos^{-1}\left(\frac{\underline{H}\cdot\underline{k}}{H}\right) = \underline{4.96^\circ} \quad \text{for } \alpha = 10^\circ$$

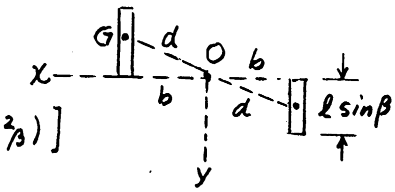
7/65 With $\omega_x = \omega_y = 0$, $\omega_z = \omega$, the components of $\underline{H}_O$ are $H_{O_x} = -I_{xz}\,\omega_z$, $H_{O_y} = -I_{yz}\,\omega_z$, $H_{O_z} = I_{zz}\,\omega_z$

By inspection
$I_{yz} = 0$, $I_{xz} = 0$





$$I_{zz} = 2(I_G + md^2)$$
$$= 2\left[\frac{1}{12}m(\ell\sin\beta)^2 + m\left(b^2 + \frac{\ell^2}{4}\sin^2\beta\right)\right]$$
$$= 2m\left[\frac{1}{3}\ell^2\sin^2\beta + b^2\right]$$



Thus $\underline{H}_O = 2m\left[\frac{1}{3}\ell^2\sin^2\beta + b^2\right]\omega\,\underline{k}$

7/66 | Let $\underline{\Omega}$ = angular velocity of x-y-z about z_0

For axes: $\Omega_x = -\Omega \sin\theta$, $\Omega_y = \dot{\theta} = 0$, $\Omega_z = \Omega \cos\theta$; $\Omega = 2\pi f$

Capsule: $\omega_x = -\Omega \sin\theta$, $\omega_y = 0$, $\omega_z = \Omega \cos\theta + p$

$H_{G_x} = I_{xx}\omega_x = mk'^2(-2\pi f \sin\theta)$, $H_{G_y} = I_{yy}\omega_y = 0$

$H_{G_z} = I_{zz}\omega_z = mk^2(2\pi f \cos\theta + p)$

$\underline{H}_G = 2\pi m f(-k'^2 \sin\theta\, \underline{i} + k^2 \cos\theta\, \underline{k}) + mk^2 p\, \underline{k}$

7/67 $\omega_x = -\omega_1$, $\omega_y = \omega_2$, $\omega_z = p$

Eq. 7/14, $\underline{H}_O = \underline{H}_B + \overrightarrow{OB} \times \underline{G}$, $\overrightarrow{OB} = b\underline{i}$, $\underline{G} = m\underline{v}_B = -mb\omega_2\underline{k}$

$$\overrightarrow{OB} \times \underline{G} = b\underline{i} \times (-mb\omega_2\underline{k}) = mb^2\omega_2\underline{j}$$

$I_{xx} = \frac{1}{4}mr^2$, $I_{yy} = \frac{1}{4}mr^2$, $I_{zz} = \frac{1}{2}mr^2$, $I_{xy} = I_{xz} = I_{yz} = 0$

Eq. 7/11, $\underline{H}_B = \frac{1}{4}mr^2(-\omega_1)\underline{i} + \frac{1}{4}mr^2\omega_2\underline{j} + \frac{1}{2}mr^2p\underline{k}$

so $\underline{H}_O = -\frac{1}{4}mr^2\omega_1\underline{i} + m\omega_2\left(b^2 + \frac{r^2}{4}\right)\underline{j} + \frac{1}{2}mr^2p\underline{k}$

$$= \frac{1}{4}mr^2\left\{-\omega_1\underline{i} + \left(1 + \frac{4b^2}{r^2}\right)\omega_2\underline{j} + 2p\underline{k}\right]$$

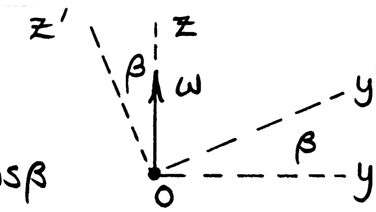
From Eq. 7/15 $T = \frac{1}{2}\bar{\underline{v}} \cdot m\bar{\underline{v}} + \frac{1}{2}\underline{\omega} \cdot \underline{H}_B$

so $T = \frac{1}{2}mb^2\omega_2^2 + \frac{1}{2}(-\omega_1\underline{i} + \omega_2\underline{j} + p\underline{k}) \cdot \left(-\frac{1}{4}mr^2\omega_1\underline{i} + \frac{1}{4}mr^2\omega_2\underline{j} + \frac{1}{2}mr^2p\underline{k}\right)$

$$= \frac{1}{2}mb^2\omega_2^2 + \frac{1}{8}mr^2(\omega_1^2 + \omega_2^2 + 2p^2)$$

$$= \frac{mr^2}{8}\left\{\omega_1^2 + \left(1 + \frac{4b^2}{r^2}\right)\omega_2^2 + 2p^2\right\}$$

7/68



Use principal axes $x'y'z'$

$\omega_{y'} = \omega \sin\beta$, $\omega_{z'} = \omega\cos\beta$

$I_{y'z'} = 0$, $I_{z'z'} = mr^2$, $I_{y'y'} = \frac{1}{2}mr^2$

$I_{x'y'} = I_{x'z'} = 0$

$\underline{H}_O = I_{y'y'}\omega_{y'}\underline{j}' + I_{z'z'}\omega_{z'}\underline{k}'$

But $\begin{cases} \underline{j}' = \underline{j}\cos\beta + \underline{k}\sin\beta \\ \underline{k}' = -\underline{j}\sin\beta + \underline{k}\cos\beta \end{cases}$

So $\underline{H}_o = \frac{1}{2}mr^2\omega\sin\beta(\underline{j}\cos\beta + \underline{k}\sin\beta)$

$\quad + mr^2\omega\cos\beta(-\underline{j}\sin\beta + \underline{k}\cos\beta)$

$= \underline{mr^2\omega\left[-\frac{1}{4}\sin 2\beta\,\underline{j} + (1-\frac{1}{2}\sin^2\beta)\underline{k}\right]}$

$T = \frac{1}{2}\underline{\omega}\cdot\underline{H}_o = \underline{\frac{1}{2}mr^2\omega^2(1-\frac{1}{2}\sin^2\beta)}$

7/69 x'-y'-z' are principal axes of inertia

so $\underline{H}_{O'} = \underline{i}\, I_{x'x'}\omega_x + \underline{j}\, I_{y'y'}\omega_y + \underline{k}\, I_{z'z'}\omega_z$

where $I_{x'x'} = I_{z'z'} = \frac{1}{4}mr^2$, $I_{y'y'} = \frac{1}{2}mr^2$

$\omega_x = \omega$, $\omega_y = p$, $\omega_z = 0$

so $\underline{H}_{O'} = \frac{1}{4}mr^2\omega\underline{i} + \frac{1}{2}mr^2 p\underline{j} = \frac{1}{2}mr^2\left(\frac{\omega}{2}\underline{i} + p\underline{j}\right)$

$$= \frac{1}{2}\frac{6}{32.2}\left(\frac{4}{12}\right)^2\left(\frac{10\pi}{2}\underline{i} + 40\pi\underline{j}\right) = \underline{0.1626(\underline{i} + 8\underline{j})}$$

lb-ft-sec

$T = \frac{1}{2}\underline{\omega}\cdot\underline{H}_{O'} + \frac{1}{2}\bar{\underline{v}}\cdot\underline{G} = \frac{1}{2}(\omega\underline{i} + p\underline{j})\cdot\frac{1}{2}mr^2\left(\frac{\omega}{2}\underline{i} + p\underline{j}\right)$

$+ \frac{1}{2}(-\bar{r}\omega\underline{j})\cdot(-m\bar{r}\omega\underline{j})$ where $\bar{r} = 10\underline{k}$ in.

$= \frac{1}{4}mr^2\left(\frac{1}{2}\omega^2 + p^2\right) + \frac{1}{2}m\bar{r}^2\omega^2$

$= \frac{1}{4}\frac{6}{32.2}\left(\frac{4}{12}\right)^2\left(\frac{1}{2}\overline{10\pi}^2 + \overline{40\pi}^2\right) + \frac{1}{2}\frac{6}{32.2}\left(\frac{10}{12}10\pi\right)^2$

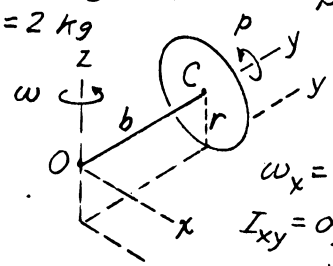
$= 84.29 + 63.85$

$= \underline{148.1 \text{ ft-lb}}$

7/70 $r = 100$ mm $\quad \omega = 4\pi$ rad/s

$b = 200$ mm $\quad p = \dfrac{v_C}{r} = \dfrac{b}{r}\omega = 8\pi$ rad/s

$m = 2$ kg



Eq. 7/11 holds for point O as a fixed point on axis of disk

$\omega_x = 0,\ \omega_y = -p = -8\pi$ rad/s, $\omega_z = \omega = 4\pi \frac{\text{rad}}{\text{s}}$

$I_{xy} = 0;\ I_{yy} = \frac{1}{2}mr^2 = \frac{1}{2}(2)(0.1)^2 = 0.01$ kg·m²

$I_{yz} = 0,\ I_{xz} = 0,\ I_{zz} = \frac{1}{4}mr^2 + mb^2 = 2\left(\frac{1}{4}\overline{0.1}^2 + \overline{0.2}^2\right)$

$= 0.085$ kg·m²

So $\underline{H}_O = \underline{j}I_{yy}\omega_y + \underline{k}I_{zz}\omega_z = \underline{j}\left(-\frac{1}{2}mr^2p\right) + \underline{k}\left(\frac{1}{4}mr^2 + mb^2\right)\omega$

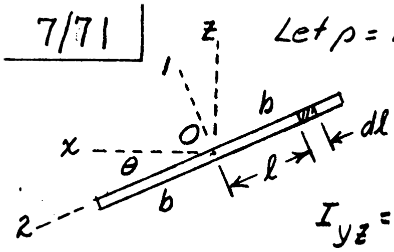
$= mr^2\omega\left(-\frac{1}{2}\frac{b}{r}\underline{j} + \left[\frac{1}{4} + \frac{b^2}{r^2}\right]\underline{k}\right)$

$= 2(0.1)^2 4\pi\left(-\frac{1}{2}2\underline{j} + \left[\frac{1}{4} + 4\right]\underline{k}\right) = 0.251(-\underline{j} + 4.25\underline{k})$ N·m·s

$T = \frac{1}{2}\underline{\omega}\cdot\underline{H}_O = \frac{1}{2}(-8\pi\underline{j} + 4\pi\underline{k})\cdot 0.251(-\underline{j} + 4.25\underline{k})$

$= 3.15 + 6.71 = \underline{9.87 \text{ J}}$

7/71



Let ρ = mass per unit of panel area

$$I_{xz} = \int_{-b}^{b} (-\ell\cos\theta)(\ell\sin\theta)2c\rho\,d\ell$$

$$= -\frac{2}{3}\rho c b^3 \sin 2\theta \quad \text{for 2 panels}$$

$I_{yz} = I_{xy} = 0$ by symmetry

$I_{zz} = \bar{I}_{zz} + md^2$ for each panel

For total, $$I_{zz} = 2\left\{\frac{2bc\rho}{12}\left[c^2 + (2b\cos\theta)^2\right] + 2bc\rho\left[a + \frac{c}{2}\right]^2\right\}$$

$$= 4bc\rho\left\{\frac{c^2}{3} + \frac{b^2}{3}\cos^2\theta + a^2 + ac\right\}$$

$\underline{H}_O = -I_{xz}\omega_z\underline{i} + I_{zz}\omega_z\underline{k}$, $m = 4bc\rho$ (total)

$$\underline{H}_O = \frac{m}{6}b^2\omega\sin 2\theta\,\underline{i} + m\omega\left\{\frac{c^2}{3} + \frac{b^2}{3}\cos^2\theta + a^2 + ac\right\}\underline{k}$$

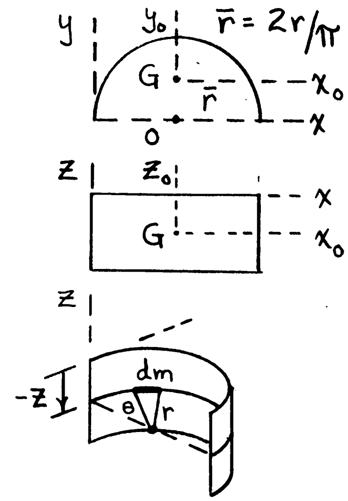
By symmetry, principal axes are O-1, O-2, O-y

$I_1 = m\left\{\frac{c^2 + b^2}{3} + a^2 + ac\right\}$ (max)

$I_2 = m\left\{\frac{1}{3}c^2 + a^2 + ac\right\}$ (intermediate)

$I_3 = \frac{1}{3}mb^2$ (minimum)

▶ 7/72 $\omega_x = \omega_y = 0, \quad \omega_z = \omega$



$H_x = -I_{xz}\omega_z, \quad H_y = -I_{yz}\omega_z$

$H_z = I_{zz}\omega_z$

$$I_{xz} = \bar{I}_{xz} + m d_x d_z$$
$$= 0 + mr(-b/2) = -\frac{mrb}{2}$$

$$dI_{yz} = (r\sin\theta)(-z)\rho r\,d\theta\,dz$$

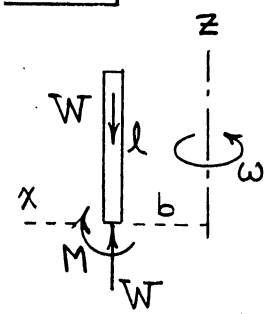
$$I_{yz} = -\rho r^2 \frac{z^2}{2}\Big]_0^{-b} (-\cos\theta)\Big]_0^{\pi}$$
$$= -\rho r^2 b^2 = -\frac{mrb}{\pi}$$

$$\left(\text{or more simply, } I_{yz} = \bar{I}_{yz} + m d_y d_z = 0 + m\left(\frac{2r}{\pi}\right)\left(-\frac{b}{2}\right) = -\frac{mrb}{\pi}\right)$$

$$I_{zz} = \bar{I}_{zz} + md^2 = (I_0 - m\bar{r}^2) + m(r^2 + \bar{r}^2)$$
$$= I_0 + mr^2 = 2mr^2$$

$$\underline{\underline{H} = mr\omega\left(\frac{b}{2}\underline{i} + \frac{b}{\pi}\underline{j} + 2r\underline{k}\right)}$$

7/73



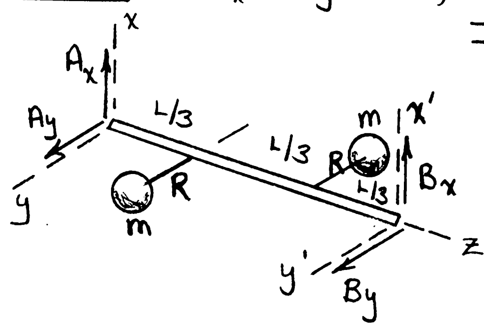
$$\sum M_y = -I_{xz}\,\omega_z^2 :$$

$$-M = -m\frac{bl}{2}\omega^2$$

$$M = \frac{mbl}{2}\omega^2$$

7/74

$\omega_x = \omega_y = 0, \quad \omega_z = \omega, \quad \dot{\omega}_z = 0$



$I_{yz} = m\frac{L}{3}R - m\frac{2L}{3}R$

$= -mLR/3$

$I_{xz} = 0$

$I_{y'z} = -mLR/3$

$I_{x'z} = 0$

x-y axes: $\Sigma M_x = I_{yz}\,\omega_z^2$ (from Eq. 7/23)

$-B_y L = -\frac{mLR}{3}\omega^2, \quad \underline{B_y = \frac{mR\omega^2}{3}}$

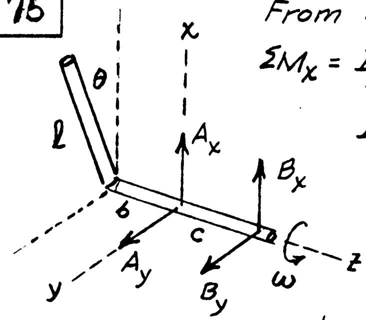
$\Sigma M_y = 0, \quad \underline{B_x = 0}$

x'-y' axes: $\Sigma M_{x'} = I_{y'z}\,\omega_z^2$

$A_y L = -\frac{mLR}{3}\omega^2, \quad \underline{A_y = -\frac{mR\omega^2}{3}}$

$\Sigma M_{y'} = 0, \quad \underline{A_x = 0}$

7/75



From Eqs. 7/23, with $\dot{\omega}_z = \dot{\omega} = 0$,

$\Sigma M_x = I_{yz}\,\omega_z^2$, $\Sigma M_y = -I_{xz}\,\omega_z^2$, $\Sigma M_z = 0$

$I_{yz} = -m\frac{bl}{2}\sin\theta$, $I_{xz} = -m\frac{bl}{2}\cos\theta$

So $-B_y c = -m\frac{bl}{2}\sin\theta\ (\omega^2)$

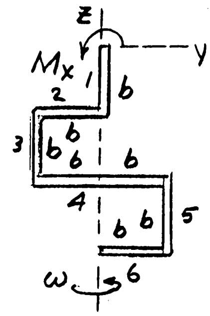
$B_y = \frac{mbl\omega^2}{2c}\sin\theta$

& $+B_x c = m\frac{bl}{2}\cos\theta\,(\omega^2)$

$B_x = \frac{mbl\omega^2}{2c}\sin\theta$

& $\underline{B} = \frac{mbl\omega^2}{2c}(\underline{i}\sin\theta + \underline{j}\cos\theta)$, $\underline{B = |\underline{B}| = \frac{mbl\omega^2}{2c}}$

7/76



I_{yz}

① 0

② $\rho b(-\frac{b}{2})(-b) = +\frac{1}{2}\rho b^3$

③ $\rho b(-b)(-\frac{3b}{2}) = \frac{3}{2}\rho b^3$

④ 0

⑤ $\rho b(b)(-\frac{5}{2}b) = -\frac{5}{2}\rho b^3$

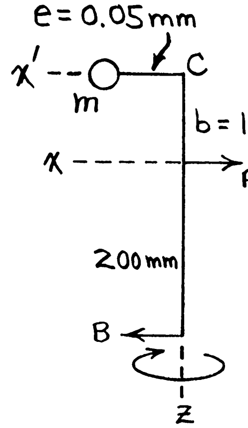
⑥ $\rho b(\frac{b}{2})(-3b) = -\frac{3}{2}\rho b^3$

Total $I_{yz} = \rho b^3(\frac{1}{2} + \frac{3}{2} - \frac{5}{2} - \frac{3}{2}) = -2\rho b^3$

From Eq. 7/23 $\Sigma M_x = I_{yz}\,\omega_z^2$, $\dot{\omega}_z = 0$

$M = M_x = -2\rho b^3 \omega^2$

7/77



$$\Sigma M_y = -I_{yz}\dot{\omega}_z - I_{xz}\omega_z^2, \quad \dot{\omega}_z = 0$$

$$\omega_z = \omega = 10{,}000\left(\frac{2\pi}{60}\right) = 1047 \ \frac{\text{rad}}{\text{sec}}$$

$$I_{xz} = -mbe = -6(0.15)(50)(10^{-6})$$

$$= -45(10^{-6}) \text{ kg}\cdot\text{m}^2$$

Thus $B(0.20) = 45(10^{-6})(1047)$

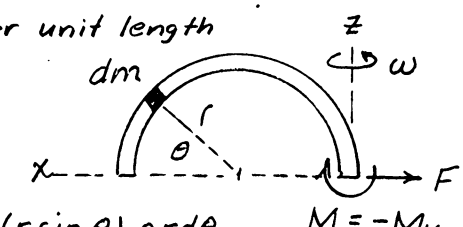
$$= \underline{247 \text{ N}}$$

For origin of coordinates x'-y'-z at C, $\Sigma M_{y'} = 0$, since $I_{x'z} = 0$

Thus $0.35B - 0.15A = 0$, $A = \frac{0.35}{0.15}(247) = \underline{576 \text{ N}}$

7/78 | Let ρ = mass per unit length

$$\Sigma M_y = -I_{xz}\omega_z^2$$



$$I_{xz} = \int xz\,dm = \int_0^{\pi} (r + r\cos\theta)(r\sin\theta)\,\rho r\,d\theta$$

$$= \rho r^3\left[-\cos\theta - \frac{1}{4}\cos 2\theta\right]_0^{\pi} = 2\rho r^3 = \frac{2}{\pi}mr^2$$

so $-M = -\frac{2}{\pi}mr^2\omega^2$, $\underline{M = \frac{2}{\pi}mr^2\omega^2}$

7/79 $\Sigma M_{z_1} = I_{z_1}\alpha$ where I_{z_1} is given by Eq. B/10

with $l = \cos\theta$, $m = 0$, $n = \sin\theta$

$I_{xy} = I_{xz} = I_{yz} = 0$

Thus $I_{z_1} = I_{xx}l^2 + I_{yy}m^2 + I_{zz}n^2 + 0$

$= I_0\cos^2\theta + 0 + I\sin^2\theta$

so $M = (I_0\cos^2\theta + I\sin^2\theta)\alpha$

$$\alpha = \frac{M}{I_0\cos^2\theta + I\sin^2\theta}$$

7/80

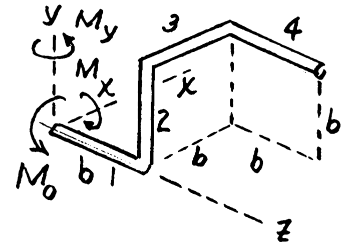
$$\Sigma M_{A_x} = I_{yz}\omega_z^2 \; ; \quad \Sigma M_{A_y} = -I_{xz}\,\omega_z^2$$

$$I_{yz} = (\rho b) b \frac{b}{2} + (\rho b) b b + (\rho b) b \frac{3b}{2} = 3\rho b^3$$

$$I_{xz} = (\rho b) \frac{b}{2} b + (\rho b) b \frac{3b}{2} = 2\rho b^3$$

$$M_x = 3\rho b^3\omega^2, \quad M_y = -2\rho b^3\omega^2, \quad M = \sqrt{M_x^2 + M_y^2} = \underline{\sqrt{13}\,\rho b^3\omega^2}$$

7/81 $I_{zz} = I_1 + I_2 + I_3 + I_4$



$I_1 = 0,\ I_2 = \frac{1}{3}\rho b^3$

$I_3 = \frac{1}{12}\rho b^3 + \rho b\left(\frac{b^2}{4} + b^2\right) = \frac{4}{3}\rho b^3$

$I_4 = \rho b(b^2 + b^2) = 2\rho b^3$

Thus $I_{zz} = \rho b^3\left(\frac{1}{3} + \frac{4}{3} + 2\right) = \frac{11}{3}\rho b^3$

Eq. 7/23 with $\omega = \omega_z = 0,\ \dot{\omega} = \dot{\omega}_z$

$\Sigma M_z = I_{zz}\dot{\omega}_z$: $M_0 = \frac{11}{3}\rho b^3 \dot{\omega}_z$, $\dot{\omega}_z = \frac{3M_0}{11\rho b^3}$

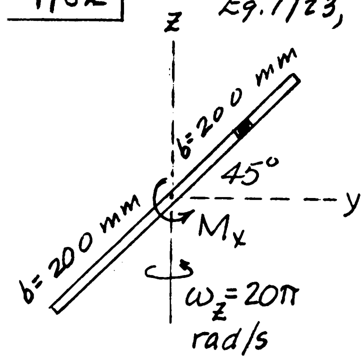
From sol. to Prob. 7/80 $I_{yz} = 3\rho b^3$, $I_{xz} = 2\rho b^3$

$\Sigma M_x = -I_{xz}\dot{\omega}_z$: $M_x = -2\rho b^3 \frac{3M_0}{11\rho b^3} = -\frac{6}{11}M_0$

$\Sigma M_y = -I_{yz}\dot{\omega}_z$: $M_y = -3\rho b^3 \frac{3M_0}{11\rho b^3} = -\frac{9}{11}M_0$

$M = \sqrt{M_x^2 + M_y^2} = \frac{M_0}{11}\sqrt{6^2 + 9^2} = \frac{3\sqrt{13}}{11}M_0$

7/82



Eq. 7/23, $\Sigma M_x = I_{yz}\,\omega_z^2$

$$I_{yz} = \int yz\,dm = \int y^2 \rho\,dl = \int_{-b/\sqrt{2}}^{b/\sqrt{2}} y^2 \rho\sqrt{2}\,dy$$

$$= \frac{\rho\sqrt{2}}{3}\left(\frac{b^3}{2\sqrt{2}} - \frac{-b^3}{2\sqrt{2}}\right) = \frac{mb^2}{6} = \frac{3(0.2)^2}{6} = 0.02 \text{ kg}\cdot\text{m}^2$$

$$M_x = 0.02\,(20\pi)^2$$
$$= 79.0 \text{ N}\cdot\text{m}$$

On plate, $\underline{M}_x = 79.0\,\underline{i}$ N·m

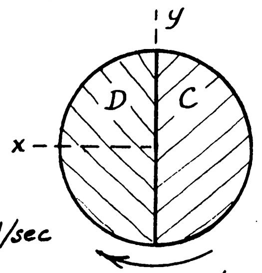
but acting on shaft, $\underline{M} = -79.0\,\underline{i}$ N·m

7/83

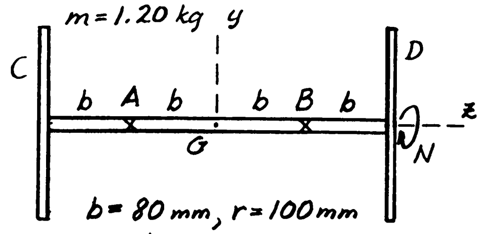
$\omega_x = \omega_y = 0,$

$\omega_z = \dfrac{1200 \times 2\pi}{60}$

$= 125.7$ rad/sec

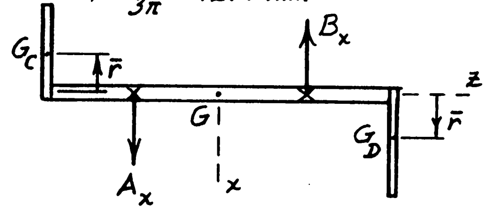


$N = 1200$ rev/min



$b = 80$ mm, $r = 100$ mm

$\bar{r} = \dfrac{4r}{3\pi} = 42.4$ mm



From Eqs. 7/23, about G,

$\Sigma M_x = I_{yz}\omega_z^2, \ \Sigma M_y = -I_{xz}\omega_z^2,$

$\Sigma M_z = 0$

$I_{yz} = 0, \ I_{xz} = \{0 + m(2b)(\bar{r})\} + \{0 + m(-2b)(-\bar{r})\} = 4mb\bar{r}$

$= 4(1.20)(0.080)(0.0424) = 0.01630 \ \text{kg}\cdot\text{m}^2$

$\Sigma F_x = 0$ so $A_x = B_x$

$\Sigma M_y = -A_x b - B_x b = -4mb\bar{r}\omega_z^2, \ A_x = B_x = 2mb\bar{r}\omega_z^2/b$

$A_x = B_x = \frac{1}{2}(0.01630)(125.7)^2/0.080$

$= 1608$ N

$\Sigma M_x = 0, \ A_y = B_y = 0$

$\underline{\underline{F}_A = 1608\,\underline{i} \text{ N}}, \ \underline{\underline{F}_B = -1608\,\underline{i} \text{ N}}$

7/84 With $\omega_x = \omega_y = \omega_z = \dot{\omega}_x = \dot{\omega}_y = 0$, $\dot{\omega}_z = 900 \text{ rad/s}^2$,

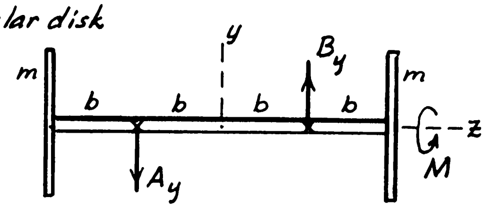
Eqs. 7/23 become

$\Sigma M_x = -I_{xz}\alpha$, $\Sigma M_y = -I_{yz}\alpha$, $\Sigma M_z = I_{zz}\alpha$

From the solution to Prob. 7/83 $I_{yz} = 0$, $I_{xz} = 0.01630 \text{ kg}\cdot\text{m}^2$

Also $I_{zz} = \frac{1}{2}(2m)r^2 = 1.20(0.100)^2 = 0.012 \text{ kg}\cdot\text{m}^2$

where m = mass of semicircular disk



$\Sigma F_y = 0$ so $A_y = B_y$

$\Sigma M_x = -0.080A_y - 0.080B_y$

$= -0.01630(900)$

$A_y = B_y = 91.7 \text{ N}$

so $\underline{\mathbf{F}_A = -91.7\mathbf{j} \text{ N}, \mathbf{F}_B = 91.7\mathbf{j} \text{ N}}$

$b = 80 \text{ mm}$

$m = 1.20 \text{ kg}$

$\alpha = \dot{\omega}_z = 900 \text{ rad/s}^2$

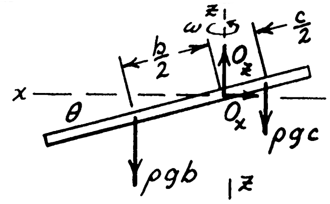
$M = \Sigma M_z = 0.012(900) = \underline{10.8 \text{ N}\cdot\text{m}}$

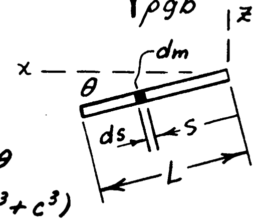
7/85 | $\omega_x = \omega_y = 0,\ \omega_z = \omega,$

$\dot\omega_x = \dot\omega_y = \dot\omega_z = 0,\ I_{yz} = 0$

so Eq. 7/23 becomes

$\Sigma M_y = -I_{xz}\,\omega_z^2$





$dI_{xz} = dm(s\cos\theta)(-s\sin\theta)$

$I_{xz} = -\sin\theta\cos\theta\,\rho\int_0^L s^2\,ds = -\rho\frac{L^3}{6}\sin 2\theta$

so for complete bar $I_{xz} = -\frac{\rho}{6}\sin 2\theta\,(b^3+c^3)$

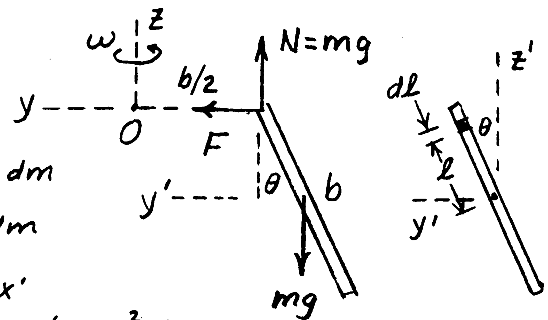
Thus $\rho g b\frac{b}{2}\cos\theta - \rho g c\frac{c}{2}\cos\theta = \frac{\rho}{3}\omega^2(b^3+c^3)\sin\theta\cos\theta$

$$\frac{g}{2}(b^2-c^2)\cancel{\cos\theta} = \frac{1}{3}(b^3+c^3)\omega^2\sin\theta\,\cancel{\cos\theta}$$

$$\sin\theta = \frac{3g}{2\omega^2}\frac{b^2-c^2}{b^3+c^3},\quad \underline{\theta = \sin^{-1}\frac{b^2-c^2}{b^3+c^3}\frac{3g}{2\omega^2}}$$

provided that $\omega^2 \geq \frac{3g}{2}\frac{b^2-c^2}{b^3+c^3}$; otherwise $\cos\theta = 0,\ \theta = 90^\circ$

7/86



$$I_{yz} = I_{y'z'} + m d_y d_z$$

$$I_{y'z'} = \int \ell \sin\theta \, \ell \cos\theta \, dm$$

$$= \sin\theta\cos\theta \int \ell^2 dm$$

$$= \sin\theta \cos\theta \, I_{x'x'}$$

$$= \frac{1}{2}\sin 2\theta \, \frac{1}{12} m b^2 = \frac{1}{24} m b^2 \sin 2\theta$$

$$I_{yz} = \frac{1}{24} m b^2 \sin 2\theta + m\left(-\frac{b}{2} - \frac{b}{2}\sin\theta\right)\left(-\frac{b}{2}\cos\theta\right)$$

$$= \frac{mb^2}{4}\left(\frac{2}{3}\sin 2\theta + \cos\theta\right)$$

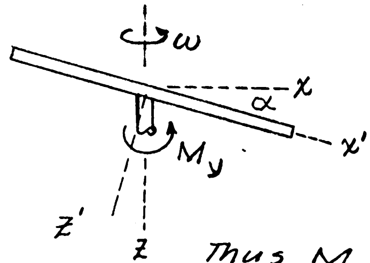
Eq. 7/23 $\Sigma M_x = 0 + I_{yz}\omega_z^2$

$$mg\left(\frac{b}{2} + \frac{b}{2}\sin\theta\right) - mg\frac{b}{2} = \frac{mb^2}{4}\left(\frac{2}{3}\sin 2\theta + \cos\theta\right)$$

$$g\tan\theta = b\left(\frac{2}{3}\sin\theta + \frac{1}{2}\right)\omega^2$$

$$\omega = \sqrt{\frac{1}{b}\,\frac{6g\tan\theta}{4\sin\theta + 3}}$$

7/87 $\Sigma M_y = -I_{xz}\,\omega_z^2\,;\quad I_{xz} = \int (x'\cos\alpha)(x'\sin\alpha)\,dm$



$$= \frac{\sin 2\alpha}{2} I_{yy}$$

where $I_{yy} = \frac{1}{4}mr^2$

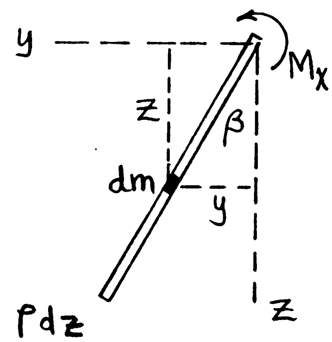
so $I_{xz} = \frac{1}{8}mr^2 \sin 2\alpha$

Thus $\underline{M}_y = (-\frac{1}{8}mr^2\sin 2\alpha)\omega^2\,\underline{j}$

But moment on shaft is

$$\underline{M} = (\tfrac{1}{8}mr^2\omega^2\sin 2\alpha)\,\underline{j}$$

7/88 For parallel-plane motion with $\dot{\omega}=0$ & $I_{xz}=0$, Eqs. 7/23 give



$$\Sigma M_x = I_{yz}\,\omega_z^2$$

$$I_{yz} = \int yz\,dm = \int_0^{b\cos\beta} z^2 \tan\beta\,\rho\,dz$$

where ρ = mass per unit of z-dimension

$$I_{yz} = \rho\tan\beta\,\frac{z^3}{3}\Big|_0^{b\cos\beta} = \frac{1}{6}mb^2\sin 2\beta$$

So $\underline{M_x = \frac{1}{6}mb^2\omega^2\sin 2\beta}$

(Moment due to weight is neglected.)

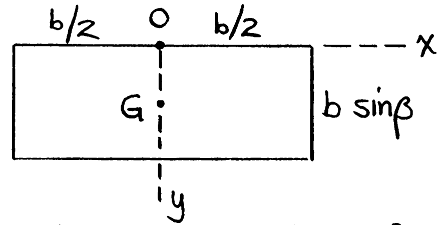
7/89 For parallel-plane motion with $\omega_z = 0$ and $I_{xz} = 0$, $\dot{\omega}_z = \alpha$, $I_{yz} = \frac{1}{6} mb^2 \sin 2\beta$ (from the solution to Prob. 7/88), Eqs. 7/23 give

$$\Sigma M_y = -I_{yz}\dot{\omega}_z, \qquad \underline{M_y = -\frac{1}{6} mb^2\alpha \sin 2\beta}$$

$$\Sigma M_z = I_{zz}\dot{\omega}_z$$

$$I_G = \frac{b(b\sin\beta)}{12}(b^2\sin^2\beta + b^2)\rho \quad \text{(from Table D3)}$$

(ρ = mass per unit of area projected onto x-y plane; $m = \rho b^2 \cos\beta$)

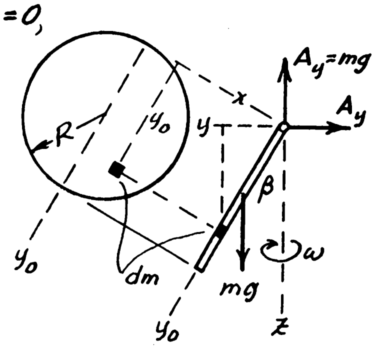


$$I_O = I_{zz} = \frac{b^2\sin\beta}{12}(b^2\sin^2\beta + b^2)\rho + b^2\sin\beta\left(\frac{b}{2}\sin\beta\right)^2\rho$$

$$= \frac{1}{12} mb^2 (1 + 4\sin^2\beta)$$

So $\underline{M_z = \frac{1}{12} mb^2\alpha (1 + 4\sin^2\beta)}$

7/90 From Eq. 7/23 with $\omega_z = \omega$, $\dot{\omega}_z = 0$,



$$\Sigma M_x = I_{yz}\,\omega^2$$

$$I_{yz} = \int yz\,dm = \int (y_0 \sin\beta)(y_0 \cos\beta)\,dm$$
$$= \sin\beta\cos\beta \int y_0^2\,dm$$
$$= \tfrac{1}{2}\sin 2\beta\, I_{xx}$$
$$= \tfrac{1}{2}\sin 2\beta\left(\tfrac{1}{4}mR^2 + mR^2\right)$$
$$= \tfrac{5}{8}mR^2 \sin 2\beta$$

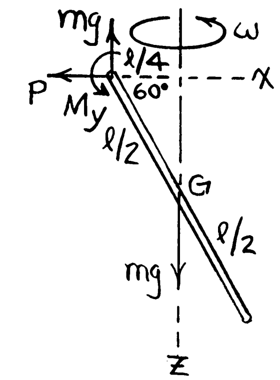
So $mgR\sin\beta = \left(\frac{5}{8}mR^2 \sin 2\beta\right)\omega^2$,

$$\sin\beta\left(g - \tfrac{5}{8}R\omega^2 \times 2\cos\beta\right) = 0,\quad \underline{\beta = \cos^{-1}\frac{4g}{5R\omega^2}\ \text{if}\ \omega^2 \ge \frac{4g}{5R}};$$

<u>otherwise $\beta = 0$</u>

7/91

$\Sigma F_x = m\bar{a}_x$: $P = 0$



$\Sigma M_y = -I_{xz}\,\omega_z^2$

$I_{xz} = \int xz\,dm = \int_{-\ell/4}^{\ell/4} x\sqrt{3}\left(\frac{\ell}{4}+x\right)\rho\,dx$

where ρ = mass/(x-comp. of length)

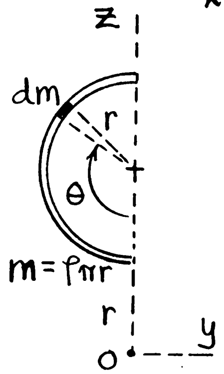
$I_{xz} = \frac{\sqrt{3}}{48}\,m\ell^2$, where $m = \frac{\rho\ell}{2}$

So $M_y - mg\frac{\ell}{4} = -\frac{\sqrt{3}}{48}\,m\ell^2\omega^2$

& for $M_y = 0$, $\underline{\omega = 2\sqrt{\frac{\sqrt{3}\,g}{\ell}}}$

7/92

$\omega_x = \omega_y = 0, \quad \omega_z = \omega, \quad \dot{\omega}_z = 0, \quad I_{xz} = 0$



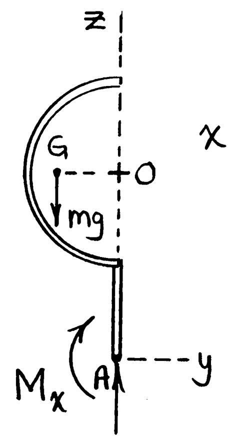
$$I_{yz} = \int yz\,dm = \int_0^{\pi} (-r\sin\theta)(2r - r\cos\theta)\,\rho r\,d\theta$$

$$= -\rho r^3 \int_0^{\pi} (2\sin\theta - \sin\theta\cos\theta)\,d\theta$$

$$= \rho r^3 \left[+2\cos\theta - \tfrac{1}{4}\cos 2\theta\right]\Big|_0^{\pi}$$

$$= -4\rho r^3 = -\frac{4mr^2}{\pi}$$

$$\overline{OG} = 2r/\pi$$



Eqs. 7/23:

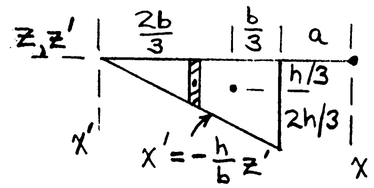
$$x: \quad mg\left(\frac{2r}{\pi}\right) - M_x = -\frac{4mr^2}{\pi}\omega^2$$

$$M_x = mg\left(\frac{2r}{\pi}\right) + \frac{4mr^2}{\pi}\omega^2$$

$$y: \quad M_y = 0 \quad ; \quad z: \quad M_z = 0$$

So $\underline{M = \dfrac{2mr}{\pi}\left(g + 2r\omega^2\right)}$

▶7/93



$$I_{x'z'} = \int x'_c z'_c \, dm$$

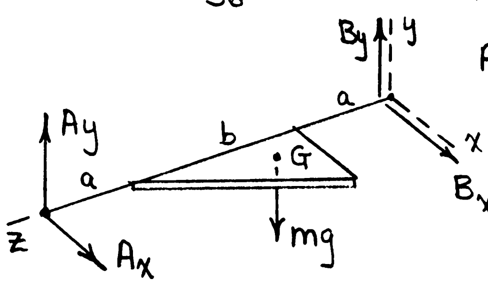
$$= \int \left(-\frac{h}{2b}z'\right)(z')\,\rho\,(x'\,dz')$$

$$= \int \left(-\frac{h}{2b}z'\right)(z')\,\rho\left(+\frac{h}{b}z'\,dz'\right)$$

$$I_{x'z'} = \frac{-h^2\rho}{2b^2}\int_0^{-b} z'^3\,dz' = -\frac{1}{4}mhb, \text{ since } m = \frac{\rho hb}{2}$$

$$\bar{I}_{x'z'} = I_{x'z'} - m\,d_{x'}d_{y'} = -\frac{1}{4}mhb - m\left(+\frac{h}{3}\right)\left(-\frac{2b}{3}\right)$$

$$= -\frac{1}{36}mhb.$$ Similarly, $I_{xz} = \frac{1}{12}mhb + \frac{1}{3}mha$



Also, $I_{zz} = \frac{1}{6}mh^2$, $I_{yz} = 0$

Eqs. 7/23: $\Sigma M_z = I_{zz}\dot{\omega}_z$

$$-mg\frac{h}{3} = \frac{1}{6}mh^2\dot{\omega}_z$$

$$\dot{\omega}_z = -2g/h$$

$$\Sigma M_y = 0 \Rightarrow A_x = 0$$

$\Sigma M_x = -I_{xz}\dot{\omega}_z$: $-A_y(2a+b) + mg\left(a + \frac{b}{3}\right) = -I_{xz}\dot{\omega}_z$

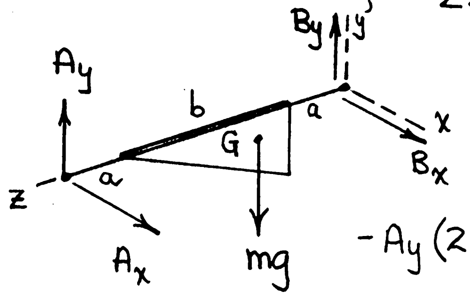
Simplifying, $A_y = A = \underline{mg/6}$

▸7/94

$$U = \Delta T + \Delta V_e + \Delta V_g$$

$$0 = \frac{1}{2} I_{zz}\,\omega_z^2 - mg(h/3)$$

From Prob. 7/93, $I_{zz} = \frac{1}{6} mh^2$, so $\omega_z = 2\sqrt{\frac{g}{h}}$



$\Sigma M_z = 0$, $\dot{\omega}_z = 0$

$\Sigma M_x = I_{yz}\,\omega_z^2$,

$$-A_y(2a+b) + mg\left(a+\frac{b}{3}\right) = -mh\left(\frac{b}{12}+\frac{a}{3}\right)4\frac{g}{h}$$

$$A_y = \frac{mg}{3}\left[\frac{7a+2b}{2a+b}\right]$$

$\Sigma M_y = 0$: $A_x(2a+b) = 0$, $A_x = 0$

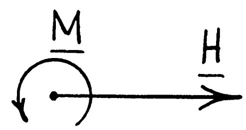
$$A = \sqrt{A_x^2 + A_y^2} = \underline{\frac{mg}{3}\left[\frac{7a+2b}{2a+b}\right]}$$

7/95 $\underline{M} = I\underline{\Omega} \times \underline{p}$: $-M\underline{i} = I\underline{\Omega} \times p\underline{j}$

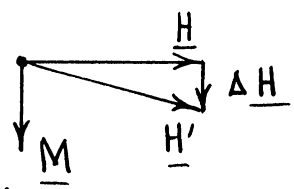
$\underline{\Omega}$ is in $+\underline{k}$ direction

So precession is CCW when viewed from above.

7/96



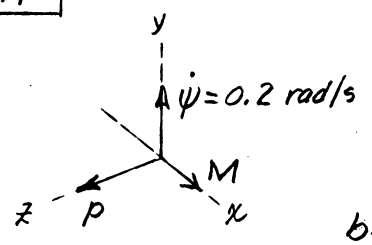
(Side view)



(Overhead view)

$\underline{M}$ is the moment exerted on the handle by the student ; $\underline{H}$ is the wheel angular momentum. From $\underline{M} = \dot{\underline{H}} \cong \frac{\Delta \underline{H}}{\Delta t}$, we see that $\Delta \underline{H}$ is in the same direction as $\underline{M}$. $\underline{H}'$ is the new angular momentum. The student will sense a tendency of the wheel to <u>rotate to her right</u>.

7/97



$M = I\dot{\psi}p$

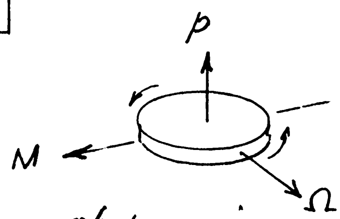
$0.8(9.81)(b - 0.180)$

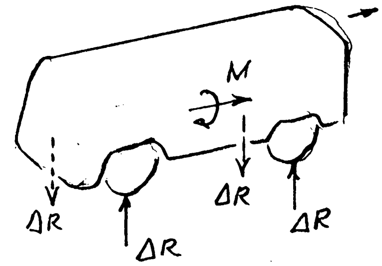
$= 2.2(0.06)^2(0.2)\,\dfrac{1725(2\pi)}{60}$

$b - 0.180 = 0.0364$

$b = 0.216$ m or $\underline{b = 216 \text{ mm}}$

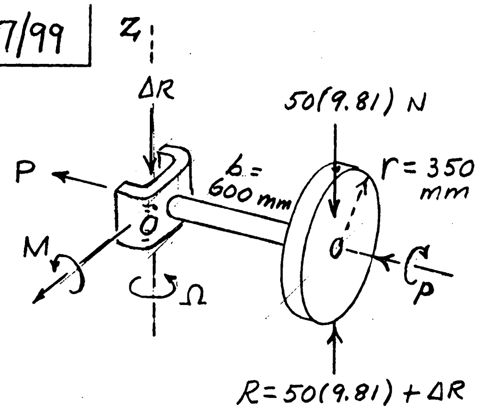
7/98





Because of precession Ω, gyroscopic moment on rotor points to the rear and reacting moment on bus is forward. Result is that the force under the right-hand tires is increased.

7/99



$\Omega = \dfrac{48 \times 2\pi}{60} = 5.03 \text{ rad/s}$

$I = \frac{1}{2} m r^2$

$= \frac{1}{2}(50)(0.350)^2$

$= 3.06 \text{ kg}\cdot\text{m}^2$

$p = \dfrac{v}{r} = \dfrac{b\Omega}{r} = \dfrac{600}{350} 5.03$

$= 8.62 \text{ rad/s}$

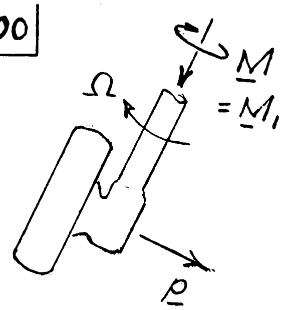
$M = I\Omega p = 3.06(5.03)(8.62)$

$= 132.6 \text{ N}\cdot\text{m}$

$M = \Delta R(b), \quad \Delta R = \dfrac{132.6}{0.600} = 221 \text{ N}$

Thus $R = 50(9.81) + 221 = \underline{712 \text{ N}}$

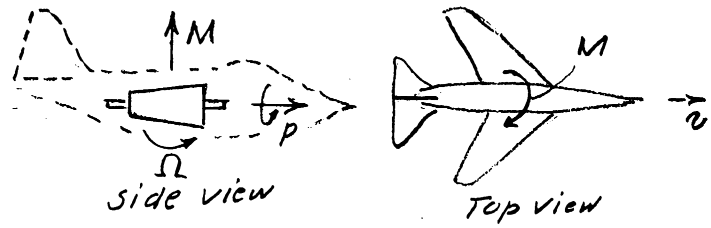
7/100



$M = I\Omega p$

$M = M_1 = mk^2\Omega\frac{v}{r}$

7/101



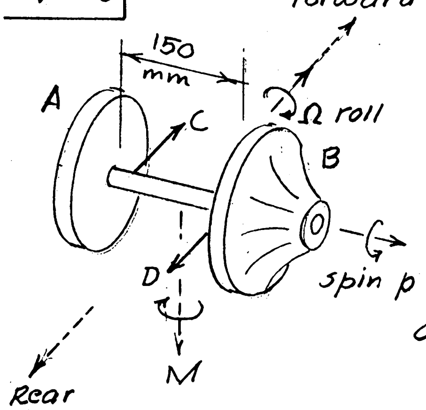
Pilot would apply <u>left rudder</u> to counter the clockwise (viewed from above) reaction to the gyroscopic moment

$$M = I\Omega p = 210(0.220)^2\left[\frac{1200(1000)}{3600}\Big/3800\right]\frac{18000\times 2\pi}{60}$$

$$= (10.16)(0.0877)(1885)$$

$$= \underline{1681\ \text{N}\cdot\text{m}}$$

7/102



$p = 20\,000 \frac{2\pi}{60} = 2094$ rad/s

$\Omega = 2$ rad/s

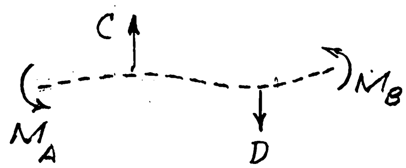
$I = 3.5(0.079)^2 + 2.4(0.071)^2$

$= 0.0339$ kg·m^2

$M = I\Omega p \quad (= M_A + M_B)$

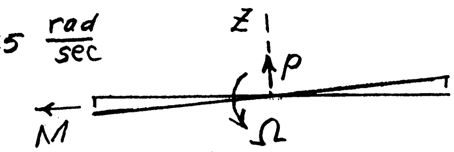
$0.15C = 0.0339(2)(2094)$

$\underline{C = D = 948 \text{ N}}$



7/103

$\Omega = \frac{10}{180}\pi = 0.1745 \ \frac{rad}{sec}$



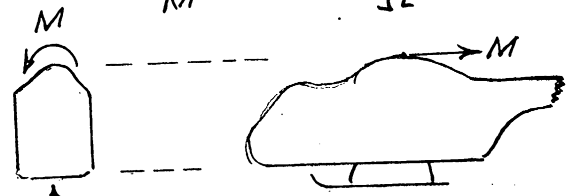
$p = \frac{500}{60} 2\pi = 52.4 \ rad/sec$

$I = \frac{140}{32.2} 10^2 = 435$ lb-ft-sec^2

$M = I\Omega p$

$= 435(0.1745)52.4$

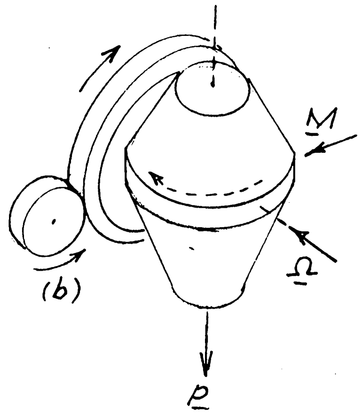
$= \underline{3970 \text{ lb-ft}}$

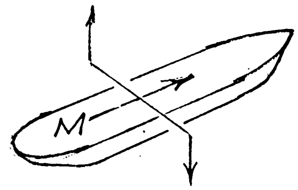


As viewed by passenger looking forward

Conclusion: <u>CCW deflection</u>

7/104

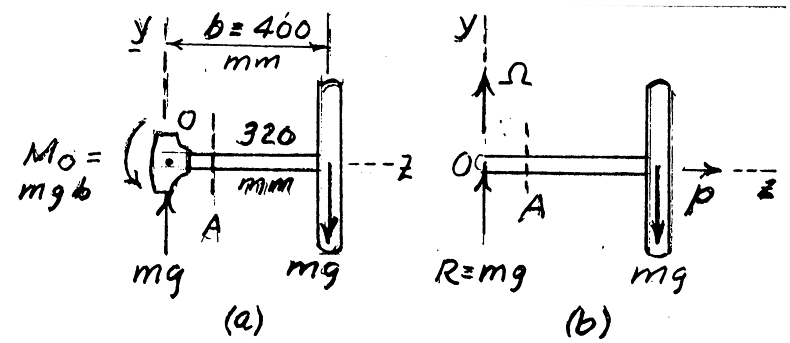




M needed on structure of ship to counteract roll to port (left). Reaction on gyro is opposite to M on ship. Proper directions of $\underline{p}$, $\underline{\Omega}$, $\underline{M}$ *shown - requiring rotation (b) of motor.*

$$M = I\Omega p = 80(1.45)^2\, 960\, \frac{2\pi}{60}\, 0.320 = \underline{5410 \text{ kN}\cdot\text{m}}$$

7/105



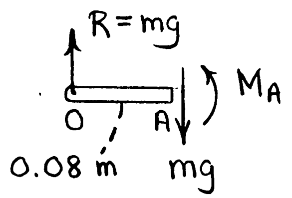
Case (a) $\Sigma M_x = 0$: so no precession

$M_A = 4(9.81)0.320 = \underline{12.56\ \text{N}\cdot\text{m}}$

Case (b) $\Sigma M_x = mgb = 4(9.81)\,0.4 = 15.70\ \text{N}\cdot\text{m}$

$\Sigma M_x = I_{zz}\Omega p: \; 15.70 = 4(0.12)^2 \Omega \dfrac{3600(2\pi)}{60}$

$\underline{\Omega = 0.723\ \text{rad/s}}$



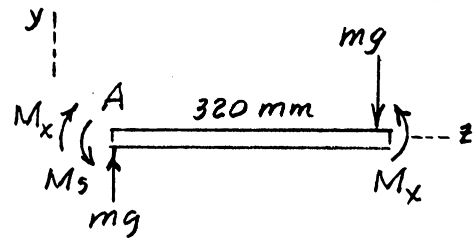
$\Sigma M_{A_x} = 0: \quad M_A = mg(0.08)$

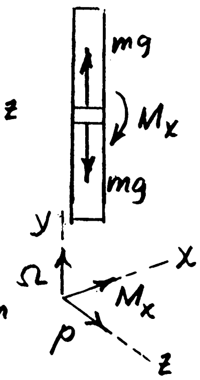
$M_A = 4(9.81)(0.08) = \underline{3.14\ \text{N}\cdot\text{m}}$

7/106

$mg = 4(9.81)$
$= 39.2\ N$

$\Omega = 2\ rad/s$
const





For rotor

$M_x = I_{zz}\,\Omega p = 4(0.12)^2\, 2\, \frac{3600(2\pi)}{60} = 43.4\ N\cdot m$

So $M_A = M_x - M_s = 43.4 - 39.2(0.320)$
$= \underline{30.9\ N\cdot m}$

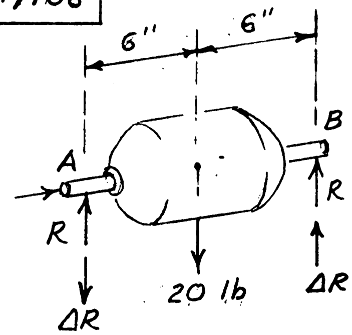
$\Sigma M_y = I_{yy}\,\dot{\Omega}$ but $\Omega = const.$ so $\dot{\Omega} = 0$ & $\underline{M_y = M_0 = 0}$

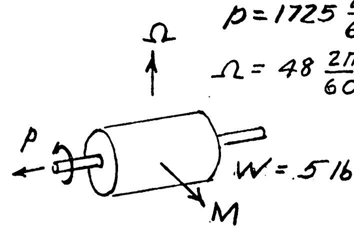
7/107 From Eq. 7/30 with θ small so that $\cos\theta \approx 1$, the precessional rate is

$$\dot{\psi} = \frac{Ip}{I_0 - I} = \frac{p}{(I_0/I) - 1} = \frac{3}{\frac{1}{2} - 1} = \underline{-6 \text{ rev/min}}$$

Where the minus sign indicates <u>retrograde precession</u>

7/108





$p = 1725 \frac{2\pi}{60} = 180.6 \frac{rad}{sec}$

$\Omega = 48 \frac{2\pi}{60} = 5.03 \frac{rad}{sec}$

static reactions

$R = \frac{1}{2}\,20 = 10\ lb$

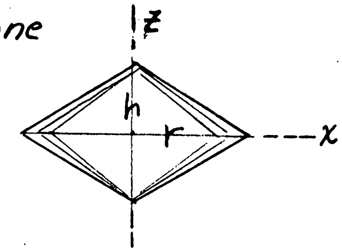
$M = I\Omega p;\quad 2(\Delta R)(6/12) = \frac{5}{32.2}\left(\frac{1.5}{12}\right)^2(5.03)(180.6)$

$\Delta R = 2.20\ lb$

$R_A = 10 - 2.20 = \underline{7.80\ lb}$

$R_B = 10 + 2.20 = \underline{12.20\ lb}$

7/109 Let m = mass of each cone



From Table D/4

$$I_0 = I_x = 2\left\{\frac{3}{20}mr^2 + \frac{1}{10}mh^2\right\}$$

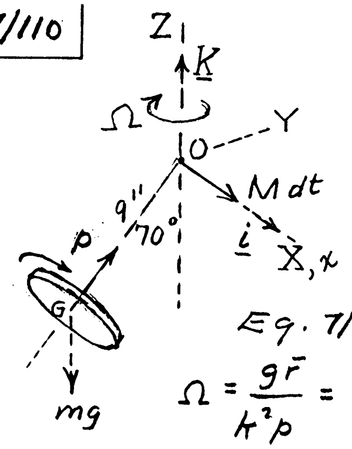
$$= \frac{mr^2}{10}\left(3 + 2\left[\frac{h}{r}\right]^2\right)$$

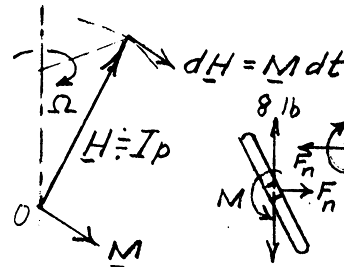
$$I = I_z = 2\,\frac{3}{10}mr^2$$

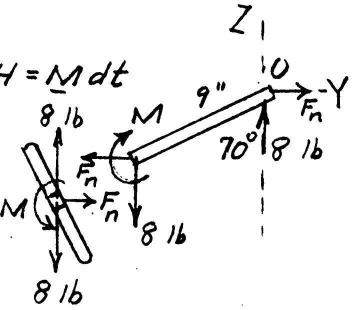
No wobble or precession if $I_0 = I$

so $\left(3 + 2\frac{h^2}{r^2}\right) = 6$, $\underline{h/r = \sqrt{3/2}}$

7/110







Eq. 7/25 applies

$$\Omega = \frac{g\bar{r}}{k^2 p} = \frac{(32.2\times 12)9}{3^2(3000\times 2\pi/60)}, \quad \underline{\boldsymbol{\Omega} = -1.23\mathbf{K}\ \frac{\text{rad}}{\text{sec}}}$$

Results are independent of 70°-angle. $\left(\text{or } \frac{1.23\times 60}{2\pi} = 11.75 \text{ rev/min}\right)$

$$M = I\Omega p \sin 70^\circ = mk^2\left(\frac{g\bar{r}}{k^2 p}\right)p \sin 70^\circ = mg\bar{r}\sin 70^\circ$$

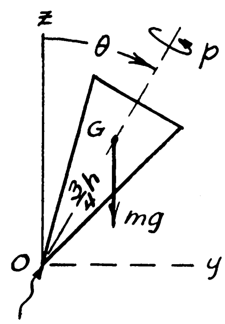
which agrees with static analysis of shaft where $\Sigma M_O = 0$ gives $M = 8\times 9 \sin 70^\circ$

$$\underline{\mathbf{M} = 67.7\mathbf{i}\ \text{lb-in.}}$$

7/111

$\underline{M}_O = mg\frac{3}{4}h\sin\theta\,(-\underline{i})$

so change in angular-momentum vector is in $-x$ direction and precession is designated by $\underline{\Omega\underline{k}}$. Eq. 7/25 gives the precession, so the period is



$\tau = 2\pi/\Omega$

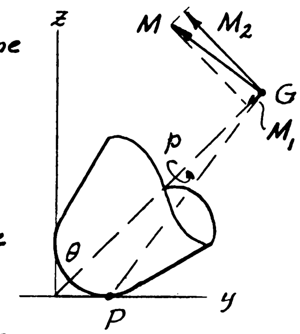
$\tau = 2\pi\Big/\left(\frac{g\bar{r}}{k^2 p}\right)$. For the solid cone, $\bar{r} = \frac{3}{4}h$

& from Table D/4, $I = \frac{3}{10}mr^2$ so $k^2 = \frac{3}{10}r^2$

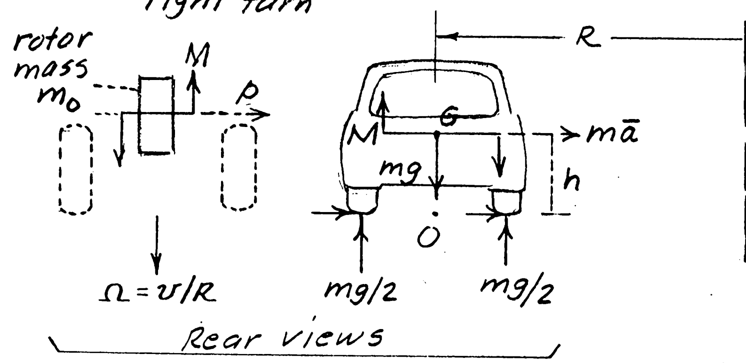
Thus

$$\tau = \frac{2\pi}{\dfrac{3gh/4}{\frac{3}{10}r^2 p}} = \underline{\frac{4\pi r^2 p}{5gh}}$$ independent of θ for large p.

7/112 For the given direction of spin p, the friction force acting on the cone at P will be in the $+x$-direction. This force produces a moment M about G, a small component of which, M_1, is along the spin axis and tends to reduce the spin. The other component M_2 causes a change in the principal angular momentum Ip in the direction of M_2, thus causing θ to decrease.



7/113 Assume right turn

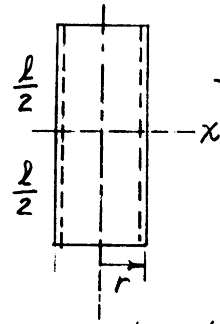


$m\bar{a} = mv^2/R$; $\Sigma M_O = m\bar{a}h$ so $M = mv^2h/R$

$M = I\Omega p$; $\dfrac{mv^2h}{R} = m_0 k^2 \dfrac{v}{R} p$

$$\underline{p = \frac{m}{m_0}\frac{vh}{k^2}}$$ opposite direction to rotation of wheels

7/114



$I = I_{zz} = mr^2$

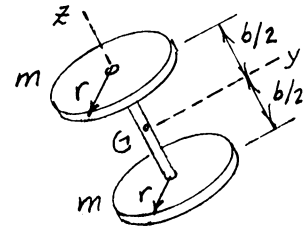
$I_0 = I_{xx} = \frac{1}{2}mr^2 + \frac{1}{12}ml^2$

$\frac{I_0}{I} = \frac{1}{2} + \frac{1}{12}\left(\frac{l}{r}\right)^2$

Direct precession if $I_0/I > 1$; $\frac{1}{2} + \frac{1}{12}\left(\frac{l}{r}\right)^2 > 1$, $\frac{l}{r} > \sqrt{6}$

Retrograde " if $I_0/I < 1$; $\frac{l}{r} < \sqrt{6}$

7/115



$I = I_{zz} = 2\left(\frac{1}{2}mr^2\right) = mr^2$

$I_0 = I_{yy} = 2\left(\frac{1}{4}mr^2 + m\left[\frac{b}{2}\right]^2\right)$

$= \frac{1}{2}mr^2 + \frac{1}{2}mb^2$

Precession is not possible when $I = I_0$ $(\theta = \beta = 0)$

So $\frac{1}{2}mr^2 + \frac{1}{2}mb^2 = mr^2$, $\underline{b = r}$

7/116 From Eq. 7/30 the frequency of precession is $f = \frac{\dot{\psi}}{2\pi} = \frac{1}{2\pi}\left|\frac{Ip}{(I_0 - I)\cos\theta}\right|$

with $\cos\theta \approx 1$; $\frac{p}{2\pi} = \frac{300}{60} = 5$ Hz ;

& with $\frac{I}{I_0 - I} = \frac{mr^2}{\frac{1}{2}mr^2 - mr^2} = -2$, (retrograde precession)

$f = |5(-2)| = \underline{10 \text{ Hz}}$

7/117 From Eq. 7/30,

$$\dot{\psi} = \frac{Ip}{(I_o - I)\cos\theta} = \frac{p}{[(I_o/I) - 1]\cos\theta}$$

where $I_o/I = \dfrac{\frac{1}{4}mr^2}{\frac{1}{2}mr^2} = \frac{1}{2}$, $p = \dfrac{300(2\pi)}{60} = 10\pi$ rad/s

$\tau = 2\pi/|\dot{\psi}|$ $\qquad \cos\theta = \cos 5° = 0.9962$

$$\tau = 2\pi \frac{|(1/2 - 1)|\,0.9962}{10\pi} = \underline{0.0996 \text{ s}}$$

Precession is <u>retrograde</u> since $I > I_o$

7/118 Case (a) $p = \frac{120 \times 2\pi}{60} = \underline{4\pi \text{ rad/s}}$

$\underline{\theta = \beta = 0 \quad , \dot{\psi} = 0}$

Case (b) $p = 4\pi \quad , \theta = 10^\circ, \quad I_0/I = 1/0.3$

From Eq. 7/30, the precessional rate is

$$\dot{\psi} = \frac{p}{\left(\frac{I_0}{I} - 1\right)\cos\theta} = \frac{4\pi}{\left(\frac{1}{0.3} - 1\right)\cos 10^\circ}$$

$$= \underline{5.47 \text{ rad/s}}$$

From Eq. 7/29,

$$\tan\beta = \frac{I}{I_0}\tan\theta = 0.3\tan 10^\circ, \quad \underline{\beta = 3.03^\circ}$$

Case (c) $\theta = \beta = 90^\circ, \ \underline{p = 0}$

$\underline{\dot{\psi} = 4\pi \text{ rad/s}}$

7/119 | I = moment of inertia about its longitudinal axis $= \frac{1}{12}m(a^2+a^2)$, $a = 4''$

I_0 = moment of inertia about transverse axis through O $= \frac{1}{12}m(a^2+\ell^2)$, $\ell = 8'' = 2a$

$I_0/I = \frac{1}{12}m(a^2+4a^2) \Big/ \frac{1}{6}ma^2 = 5/2$

Eq. 7/30 $\dot{\psi} = \dfrac{p}{\left(\frac{I_0}{I}-1\right)\cos\theta} = \dfrac{200}{\left(\frac{5}{2}-1\right)\cos 10^\circ} = 135.4$ rev/min

period of wobble $\tau = \dfrac{60}{135.4} = \underline{0.443 \text{ sec}}$

7/120 From Eq. 7/19, $M_x = \dot{H}_x - H_y \Omega_z - H_z \Omega_y$

Angular velocities of axes are

$$\Omega_x = \omega_x = \omega_0 \, , \; \Omega_y = \Omega_z = 0 \quad \text{so} \quad M_x = \dot{H}_x$$

But from Eq. 7/12

$$H_x = I_{xx}\omega_x - I_{xy}\omega_y - I_{xz}\omega_z$$

where $\omega_x = \omega_0 , \; \omega_y = 0 , \; \omega_z = \dot{\phi} = p$

$$I_{xx} = \frac{1}{12} m (l \sin\phi)^2 = \frac{1}{12} m l^2 \sin^2\phi$$

$$I_{xy} = \int xy\,dm = \frac{1}{24} m l^2 \sin 2\phi$$

$$I_{xz} = 0$$

Thus $M = M_x = \frac{d}{dt}\left(\frac{1}{12} m l^2 \sin^2\phi\right)\omega_x = \frac{1}{6} m l^2 \dot{\phi}\, \omega_x \sin\phi \cos\phi$

$$= \underline{\frac{1}{12} m l^2 p \omega_0 \sin 2\phi}$$

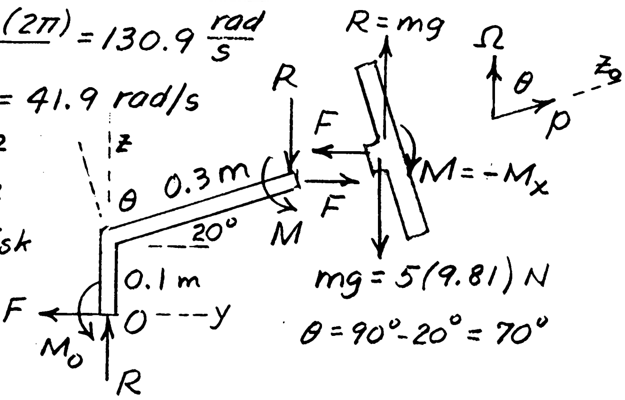
7/121 $p = \dfrac{1250(2\pi)}{60} = 130.9 \dfrac{rad}{s}$

$\Omega = \dot{\psi} = \dfrac{400(2\pi)}{60} = 41.9 \text{ rad/s}$

$I = mk^2 = 5(0.085)^2$

$= 0.0361 \text{ kg}\cdot\text{m}^2$

$I_0 = I/2$ for thin disk



$\theta = 90° - 20° = 70°$

Use notation of Fig. 7/20 & Eq. 7/20

$M = \dot{\psi}\sin\theta\left[I(\dot{\psi}\cos\theta + p) - I_0\dot{\psi}\cos\theta\right]$

$= 41.9 \sin 70°\left[0.0361(41.9\cos 70° + 130.9) - \dfrac{0.0361}{2}(41.9)\cos 70°\right] = 196.3 \text{ N}\cdot\text{m}$

$\underline{M} = \underline{-196.3\underline{i} \text{ N}\cdot\text{m}}$

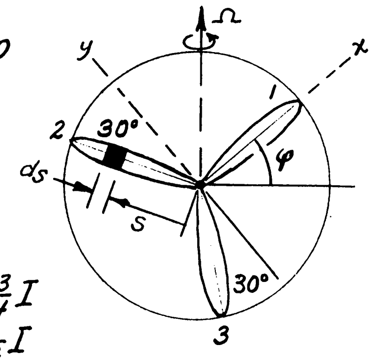
Also for disk $\Sigma F_y = m\bar{a}_y$: $-F = -5(0.3\cos 70°)(41.9)^2$,

$F = 2470 \text{ N}$

Shaft: $\Sigma M_0 \approx 0$: $M_0 + 196.3 - 2470(0.1 + 0.3\sin 20°) - 5(9.81)(0.3\cos 20°) = 0$

$M_0 = \underline{319 \text{ N}\cdot\text{m}}$

7/122



By symmetry $I_{xy} = I_{xz} = I_{yz} = 0$ for whole propeller.

Let $\rho = f(s)$ be mass per unit length so $dm = \rho\, ds = f(s)\, ds$

Blade 1: $I_{xx} = 0$, $I_{yy} = \int f(s)\, ds \times s^2 = I$

Blades 2 & 3: $I_{xx} = \int f(s)\, ds\, (s \cos 30°)^2 = \frac{3}{4} I$

$$I_{yy} = \int f(s)\, ds\, (s \sin 30°)^2 = \frac{1}{4} I$$

Thus for the three blades

$$I_{xx} = 0 + 2\left(\tfrac{3}{4} I\right) = \tfrac{3}{2} I$$
$$I_{yy} = I + 2\left(\tfrac{1}{4} I\right) = \tfrac{3}{2} I$$
$$I_{zz} = 3I$$

$\omega_x = \Omega \sin\varphi$, $\dot{\omega}_x = \Omega p \cos\varphi$

$\omega_y = \Omega \cos\varphi$, $\dot{\omega}_y = -\Omega p \sin\varphi$

$\omega_z = \dot{\varphi} = p$, $\dot{\omega}_z = 0$

From Eq. 7/21 $M_x = I_{xx} \dot{\omega}_x - (I_{yy} - I_{zz}) \omega_y \omega_z$

$$= \tfrac{3}{2} I \Omega p \cos\varphi - \left(\tfrac{3}{2} I - 3I\right) \Omega p \cos\varphi = \underline{3I\Omega p \cos\varphi}$$

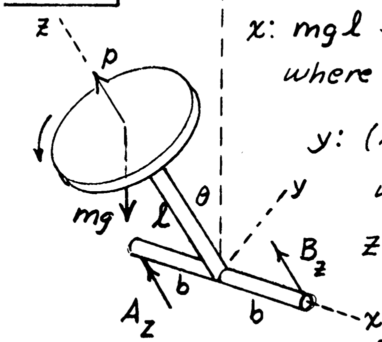
$$M_y = I_{yy} \dot{\omega}_y - (I_{zz} - I_{xx}) \omega_z \omega_x$$

$$= \tfrac{3}{2} I (-\Omega p \sin\theta) - \left(3I - \tfrac{3}{2} I\right) \Omega p \sin\varphi = \underline{-3I\Omega p \sin\varphi}$$

acting on hub; reaction on shaft has opposite signs.

The magnitude of M is $M = \sqrt{M_x^2 + M_y^2} = \underline{3I\Omega p}$

▸7/123



$\dot\psi = \ddot\psi = 0$; From moment Eqs. 7/26

x: $mg\ell \sin\theta = m(\frac{r^2}{4} + \ell^2)\ddot\theta$ .... (a)

where $I_o = I_{xx} = \frac{1}{4}mr^2 + m\ell^2$

y: $(A_z - B_z)b = -\frac{1}{2}mr^2\dot\theta p$ .... (b)

where $I = \frac{1}{2}mr^2$

z: $0 = I\dot p$ where $\omega_z = 0 + p$ .... (c)

From (c), $p = \text{const}$

From (a) with $\dot\theta d\dot\theta = \ddot\theta d\theta$, $\int_0^{\pi/2} g\ell \sin\theta \, d\theta = (\frac{r^2}{4} + \ell^2)\int_0^{\dot\theta} \dot\theta \, d\dot\theta$

which gives $\dot\theta^2 = 8g\ell/(r^2 + 4\ell^2)$

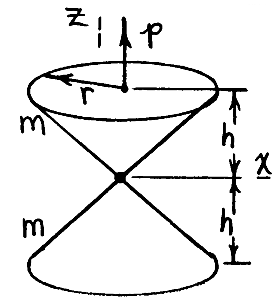
From (b) $-A_z + B_z = \frac{1}{2}m\frac{r^2}{b}\dot\theta p$ for $\theta = \pi/2$ ..... (d)

Also for $\theta = \pi/2$, $\Sigma F_z = m\bar a_z$; $-A_z - B_z = m\ell\dot\theta^2$ ...... (e)

Solve (d) & (e) & get

$$\left.\begin{aligned} A_z &= -\frac{m\dot\theta}{2}\left(\frac{r^2 p}{2b} + \ell\dot\theta\right) \\ B_z &= \frac{m\dot\theta}{2}\left(\frac{r^2 p}{2b} - \ell\dot\theta\right) \end{aligned}\right\} \text{where } \dot\theta = 2\sqrt{\frac{2g\ell}{r^2 + 4\ell^2}}$$

$\blacktriangleright$ **7/124** $\quad \dot{\psi} = \dfrac{Ip}{(I_0 - I)\cos\theta}$



(a) No precession if $I_0 = I$

From Table D4,

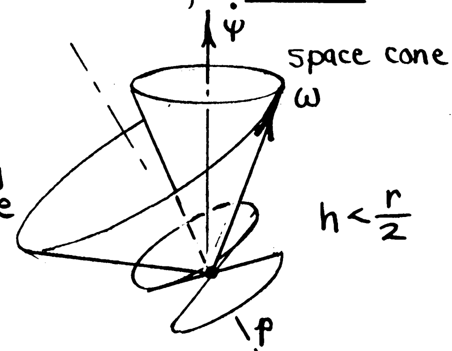
$$I = I_{zz} = 2\left(\frac{3}{10}mr^2\right) = \frac{3}{5}mr^2$$

$$I_0 = I_{xx} = 2\left(\frac{3}{20}mr^2 + \frac{3}{5}mh^2\right) = \frac{3}{10}mr^2 + \frac{6}{5}mh^2$$

$I = I_0: \quad \frac{3}{5}mr^2 = \frac{3}{10}mr^2 + \frac{6}{5}mh^2, \quad \underline{h = \frac{r}{2}}$

(b) For $h < \frac{r}{2}$, $I_0 < I$;

retrograde precession



(c) $h = r$, $I_0 = \frac{3}{10}mr^2 + \frac{6}{5}mr^2$

$$= \frac{3}{2}mr^2$$

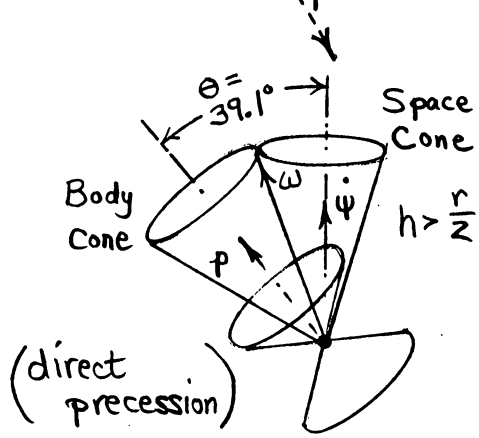
$$\frac{I}{I_0 - I} = \frac{3/5}{3/2 - 3/5} = \frac{2}{3}$$

$$p = 200\left(\frac{2\pi}{60}\right) = 20.9 \ \frac{\text{rad}}{\text{s}}$$

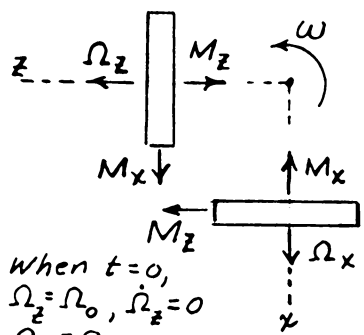
$$\theta = \cos^{-1}\left[\frac{I}{I_0 - I}\,\frac{p}{\dot{\psi}}\right]$$

$$= \cos^{-1}\left[\frac{2}{3}\,\frac{20.9}{18}\right]$$

$$= 39.1^\circ$$



►7/125 $\omega = \frac{2\pi}{\tau}$ = constant precessional rate about y-axis



When $t=0$,
$\Omega_z = \Omega_0$, $\dot{\Omega}_z = 0$
$\Omega_x = 0$

M_x = gyroscopic moment on z-wheel $= I\Omega_z\omega$

$-M_x$ = moment to accelerate x-wheel $= I\dot{\Omega}_x$

so $I\Omega_z\omega = -I\dot{\Omega}_x$, $\dot{\Omega}_x + \omega\Omega_z = 0$ (a)

M_z = gyroscopic moment on x-wheel $= -I\Omega_x\omega$

$-M_z$ = moment to accelerate z-wheel $= +I\dot{\Omega}_z$

so $I\Omega_x\omega = +I\dot{\Omega}_z$, $\dot{\Omega}_z - \omega\Omega_x = 0$ (b)

Combine (a) & (b) & get $\ddot{\Omega}_z + \omega^2\Omega_z = 0$ & $\ddot{\Omega}_x + \omega^2\Omega_x = 0$

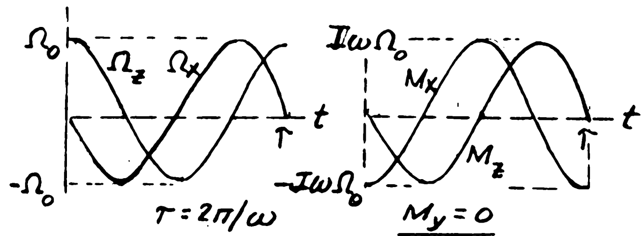
For given conditions at $t=0$, $\begin{cases} \Omega_x = -\Omega_0 \sin\omega t \\ \Omega_z = \Omega_0 \cos\omega t \end{cases}$

Thus motor torques on shafts are

$\underline{M_x = -I\omega\Omega_0 \cos\omega t}$

$\underline{M_z = -I\omega\Omega_0 \sin\omega t}$

$M = \sqrt{M_x^2 + M_z^2}$
$= I\omega\Omega_0$
constant



$\underline{M_y = 0}$

▶7/126 with $\omega_x = \Omega_x = -\omega_0 \cos\gamma \sin\beta$

$$\omega_y = \Omega_y = \omega_0 \sin\gamma + \dot{\beta}$$

$$\omega_z = \Omega_z + p = \omega_0 \cos\gamma \cos\beta + p$$

$$H_x = I_{xx}\omega_x = -I_0 \omega_0 \cos\gamma \sin\beta$$

$$H_y = I_{yy}\omega_y = I_0(\omega_0 \sin\gamma + \dot{\beta})$$

$$H_z = I_{zz}\omega_z = I(\omega_0 \cos\gamma \cos\beta + p)$$

Second of Eq. 7/19 becomes with $M_y = 0$,

$$0 = I_0(0 + \ddot{\beta}) - I(\omega_0 \cos\gamma \cos\beta + p)(-\omega_0 \cos\gamma \sin\beta)$$
$$- I_0 \omega_0 \cos\gamma \sin\beta(\omega_0 \cos\gamma \cos\beta)$$

Neglect ω_0^2 terms & replace $\sin\beta$ by β for small β

$$\ddot{\beta} + K^2\beta = 0 \quad \text{where} \quad K^2 = \frac{I}{I_0}\omega_0 p \cos\gamma$$

This is simple harmonic motion with period of

$$\tau = \frac{2\pi}{K} = 2\pi\sqrt{\frac{I_0}{I\omega_0 p \cos\gamma}}$$

. Thus gyro oscillates about north direction & with some damping will always point north.

7/127 | Angular velocity $\underline{\omega}$ and velocity $\underline{v}$ of point A are perpendicular.

Thus $\underline{\omega} \cdot \underline{v} = 0$

$$\underline{\omega} = \omega(300\,\underline{i} + 150\underline{j} + 300\underline{k})/\sqrt{300^2+150^2+300^2} = \frac{\omega}{3}(2\underline{i} + \underline{j} + 2\underline{k})$$

$$\underline{v} = 15\underline{i} - 20\underline{j} + v_z\underline{k} \ \text{m/s}$$

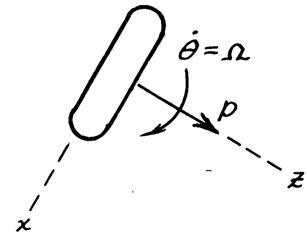
Thus $\frac{\omega}{3}(2\underline{i} + \underline{j} + 2\underline{k}) \cdot (15\underline{i} - 20\underline{j} + v_z\underline{k}) = 0$

$$30 - 20 + 2v_z = 0, \quad v_z = -5 \ \text{m/s}$$

$$v = \sqrt{15^2 + 20^2 + 5^2} = \underline{25.5 \ \text{m/s}}$$

$$v = \frac{d}{2}\omega, \quad d = \frac{2v}{\omega} = \frac{2(25.5)}{1720 \times 2\pi/60} = 0.283 \ \text{m or } \underline{d = 283 \ \text{mm}}$$

7/128



$$p = \frac{v}{r} = \frac{150(10^3)}{60^2 \times 0.560/2} = 148.8 \text{ rad/s}$$

$\underline{p} = 148.8\, \underline{k}$ rad/s

$$\Omega = \frac{30\pi}{180} = 0.524 \text{ rad/s}$$

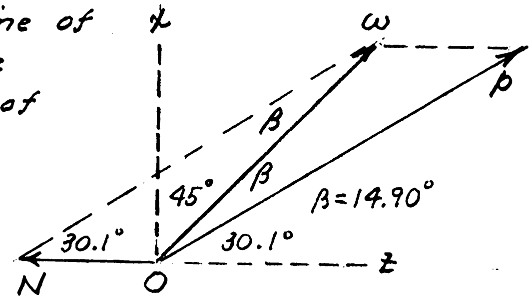
$\underline{\Omega} = 0.524\, \underline{j}$ rad/s

$\underline{\alpha} = \underline{\Omega} \times \underline{p} = 0.524\, \underline{j} \times 148.8\, \underline{k} = 77.9\, \underline{i}$ rad/s^2, $\underline{\underline{\alpha} = 77.9\, \underline{i} \text{ rad/s}^2}$

7/129 Angular velocity vector must be along the line of contact which is the instantaneous axis of zero velocity.

$N = 5 \times 2\pi = 31.4$ rad/s

Law of sines gives

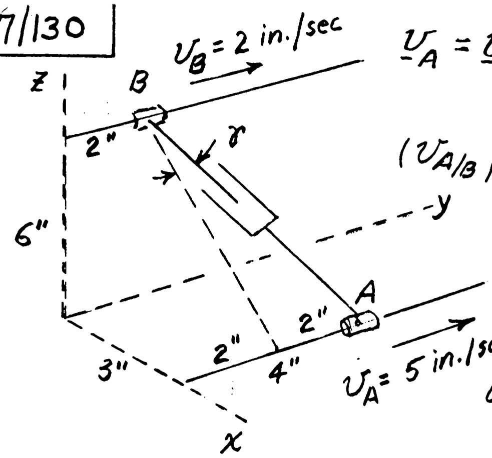


$$\omega/\sin 30.1° = 31.4/\sin 14.90°,\quad \omega = 31.4\,\frac{\sin 30.1°}{\sin 14.90°} = 61.3 \text{ rad/s}$$

$$\underline{\omega} = \frac{61.3}{\sqrt{2}}(\underline{i}+\underline{k}) = \underline{43.3(\underline{i}+\underline{k}) \text{ rad/s}}$$

$$\underline{\alpha} = \underline{N} \times \underline{\omega} = -31.4\underline{k} \times 43.3(\underline{i}+\underline{k}) = \underline{-1361\underline{j} \text{ rad/s}^2}$$

7/130



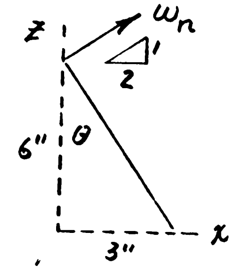
$\underline{v}_A = \underline{v}_B + \underline{v}_{A/B}$, $\underline{v}_{A/B} = (5-2)\underline{j}$

$= 3\underline{j}$ in./sec

$(v_{A/B})_{\substack{normal \\ to\ AB}} = 3\cos\gamma$

$= 3\,\dfrac{\sqrt{6^2+3^2}}{\sqrt{6^2+3^2+2^2}} = \dfrac{9\sqrt{5}}{7}$ in./sec

$\omega_n = \dfrac{9\sqrt{5}/7}{7} = \dfrac{9\sqrt{5}}{49}$ rad/sec

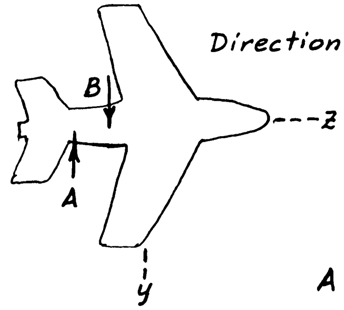


$\underline{\omega}_n = \dfrac{9\sqrt{5}}{49}(\underline{i}\cos\theta + \underline{k}\sin\theta)$

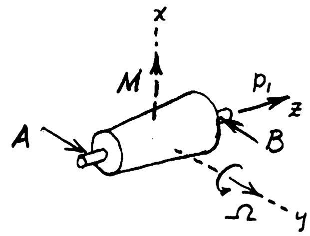
$= \dfrac{9\sqrt{5}}{49}\left(\dfrac{2}{\sqrt{5}}\underline{i} + \dfrac{1}{\sqrt{5}}\underline{k}\right)$

$\underline{\omega}_n = \dfrac{9}{49}(2\underline{i} + \underline{k})$ rad/sec

7/131



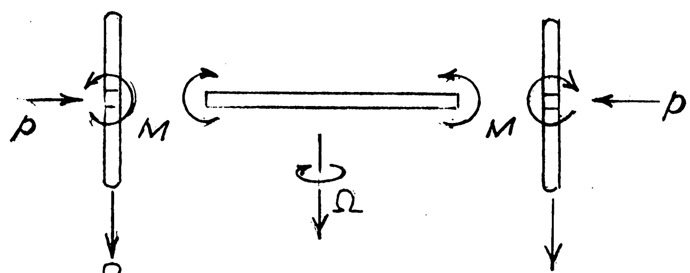
Direction of bearing forces acting on airframe



To satisfy $\underline{M} = I\underline{\Omega} \times \underline{p}$

p must be p_1

7/132



Case B

7/133 $\quad \underline{v}_A = \underline{v}_B + \underline{\omega}_n \times \underline{r}_{A/B}$

where $\underline{\omega}_n$ is perpendicular to AB

$\underline{r}_{A/B} = -0.1\underline{i} + 0.05\underline{j} + 0.1\underline{k}$ m

$\underline{v}_B = 0.5\underline{j}$ m/s

$\underline{v}_A = -0.1\omega\underline{i}$ m/s

Thus

$$-0.1\omega\underline{i} = 0.5\underline{j} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \omega_{n_x} & \omega_{n_y} & \omega_{n_z} \\ -0.1 & 0.05 & 0.1 \end{vmatrix}$$

Expand, equate like terms & get

$-0.1\omega = 0.1\omega_{n_y} - 0.05\omega_{n_z}$ (1)

$0 = 0.5 - 0.1\omega_{n_z} - 0.1\omega_{n_x}$ (2)

$0 = 0 + 0.05\omega_{n_x} + 0.1\omega_{n_y}$ (3)

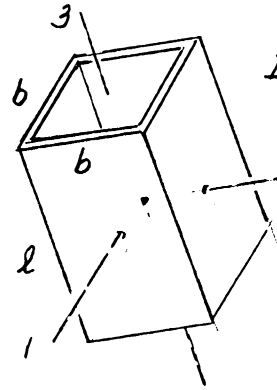
Also, $\omega_n \perp$ to AB so $\underline{\omega}_n \cdot \underline{r}_{AB} = 0$

so $-0.1\omega_{n_x} + 0.05\omega_{n_y} + 0.1\omega_{n_z} = 0$ (4)

Solve Eqs (1), (2), (3), (4) & get

$\underline{\omega = 2.5 \text{ rad/s}}$, $\quad \underline{\underline{\omega}_n = \frac{5}{9}(4\underline{i} - 2\underline{j} + 5\underline{k})}$ rad/s

7/134



Let m = mass of each of the four sides

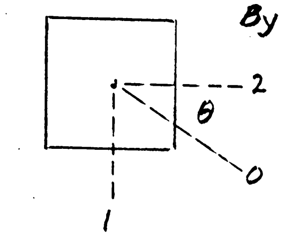
$$I_1 = I_2 = 2\left\{\frac{m}{12}(b^2+\ell^2) + \frac{1}{12}m\ell^2 + m\left(\frac{b}{2}\right)^2\right\}$$

$$= \frac{m}{3}(2b^2+\ell^2)$$

$$I_3 = 4\left\{\frac{m}{12}b^2 + m\left(\frac{b}{2}\right)^2\right\} = \frac{4}{3}mb^2$$

$$I_1 = I_3 \text{ if } \frac{m}{3}b^2(2+[\ell/b]^2) = \frac{4mb^2}{3}$$

$$\text{or } \ell/b = \sqrt{2}$$



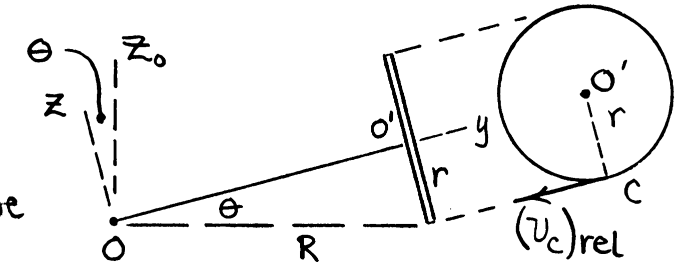
By Eq. B/10, $I_0 = I_1 = I_2 = \frac{m}{3}(2b^2+\ell^2)$

If $\underline{\ell > b\sqrt{2}}$, $I_0 > I_3$ <u>direct precession</u>

If $\underline{\ell < b\sqrt{2}}$, $I_0 < I_3$ <u>retrograde precession</u>

7/135

Let $\underline{\Omega}$ be the angular velocity of the axes xyz.



$$\underline{\Omega} = \frac{2\pi}{\tau}(\underline{j}\sin\theta + \underline{k}\cos\theta)$$

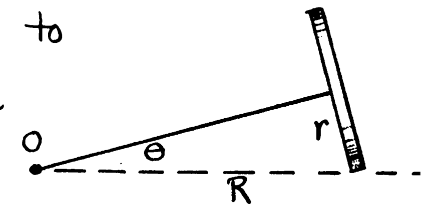
Relative to the xyz axes, O' is fixed and C moves with speed $(v_c)_{rel} = R\,\frac{2\pi}{\tau}$

So $\underline{\omega}_{rel} = \frac{(v_c)_{rel}}{r}(-\underline{j}) = -\frac{2\pi}{\tau}\frac{R}{r}\underline{j}$

Thus $\underline{\omega} = \frac{2\pi}{\tau}\left[-\frac{R}{r}\underline{j} + \underline{j}\sin\theta + \underline{k}\cos\theta\right]$

$$= \underline{\frac{2\pi}{\tau}\left[\left(-\frac{R}{r} + \frac{r}{R}\right)\underline{j} + \frac{\sqrt{R^2-r^2}}{R}\underline{k}\right]}$$

7/136 From the solution to Prob. 7/135, the absolute angular velocity of the disk is



$$\underline{\omega} = \frac{2\pi}{\tau}\left[\left(-\frac{R}{r} + \frac{r}{R}\right)\underline{j} + \frac{\sqrt{R^2-r^2}}{R}\underline{k}\right]$$

$\underline{\alpha} = \dot{\underline{\omega}}$; Need $\dot{\underline{j}} = \underline{\Omega} \times \underline{j} = \frac{2\pi}{\tau}(\underline{j}\sin\theta + \underline{k}\cos\theta) \times \underline{j}$

$$= -\frac{2\pi}{\tau}\cos\theta\,\underline{i} = \frac{2\pi}{\tau}\left(-\frac{\sqrt{R^2-r^2}}{R}\underline{i}\right)$$

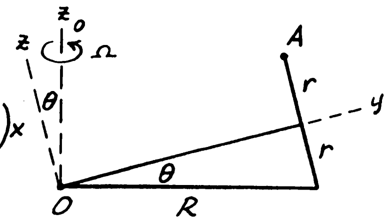
and $\dot{\underline{k}} = \underline{\Omega} \times \underline{k} = \frac{2\pi}{\tau}(\underline{j}\sin\theta + \underline{k}\cos\theta) \times \underline{k}$

$$= \frac{2\pi}{\tau}\sin\theta\,\underline{i} = \frac{2\pi}{\tau}\frac{r}{R}\underline{i}$$

So $\underline{\alpha} = \left(\frac{2\pi}{\tau}\right)^2\left\{\left[\frac{r}{R} - \frac{R}{r}\right]\left(-\frac{\sqrt{R^2-r^2}}{R}\right)\underline{i} + \frac{\sqrt{R^2-r^2}}{R}\frac{r}{R}\underline{i}\right\}$

$$= \left(\frac{2\pi}{\tau}\right)^2 \frac{\sqrt{R^2-r^2}}{r}\,\underline{i}$$

7/137 From Eq. 7/6



$$\underline{v}_A = \underline{v}_O + \underline{\Omega} \times \underline{r}_{A/O} + \underline{v}_{rel}$$

$$\underline{v}_O = \underline{0}, \quad \underline{\Omega} \times \underline{r}_{A/O} = \frac{2\pi}{\tau}\left(\frac{r}{R}\underline{j} + \frac{\sqrt{R^2-r^2}}{R}\underline{k}\right) \times \left(\sqrt{R^2-r^2}\,\underline{j} + r\underline{k}\right)$$

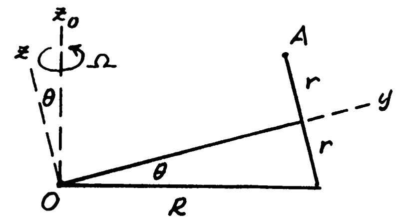
$$= \frac{2\pi}{\tau}\left(\frac{2r^2}{R} - R\right)\underline{i}$$

$$\underline{\Omega} = \frac{2\pi}{\tau}(\underline{j}\sin\theta + \underline{k}\cos\theta) = \frac{2\pi}{\tau}\left(\frac{r}{R}\underline{j} + \frac{\sqrt{R^2-r^2}}{R}\underline{k}\right)$$

$$\underline{v}_{rel} = -r\omega_{rel}\underline{i} = -r\left(\frac{R}{r}\frac{2\pi}{\tau}\right)\underline{i} = \frac{2\pi}{\tau}R\underline{i}$$

$$\underline{v}_A = \frac{2\pi}{\tau}\left[\frac{2r^2}{R} - R - R\right]\underline{i}, \quad \underline{\underline{v}_A = -\frac{4\pi}{\tau}\left(R - \frac{r^2}{R}\right)\underline{i}}$$

7/138 Using Eqs. 7/6



$$\mathbf{a}_A = \mathbf{a}_O + \dot{\boldsymbol{\Omega}} \times \mathbf{r}_{A/O} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}_{A/O}) + 2\boldsymbol{\Omega} \times \mathbf{v}_{rel} + \mathbf{a}_{rel}$$

$$\mathbf{a}_O = \mathbf{0}, \quad \dot{\boldsymbol{\Omega}} = \mathbf{0}$$

$$\boldsymbol{\Omega} = \frac{2\pi}{\tau}(\mathbf{j}\sin\theta + \mathbf{k}\cos\theta) = \frac{2\pi}{\tau}\left(\frac{r}{R}\mathbf{j} + \frac{\sqrt{R^2-r^2}}{R}\mathbf{k}\right)$$

$$\boldsymbol{\Omega} \times \mathbf{r}_{A/O} = \frac{2\pi}{\tau}\left(\frac{r}{R}\mathbf{j} + \frac{\sqrt{R^2-r^2}}{R}\mathbf{k}\right) \times \left(\sqrt{R^2-r^2}\,\mathbf{j} + r\mathbf{k}\right) = \frac{2\pi}{\tau}\left(\frac{2r^2}{R} - R\right)\mathbf{i}$$

$$\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}_{A/O}) = \left(\frac{2\pi}{\tau}\right)^2\left(\frac{r}{R}\mathbf{j} + \frac{\sqrt{R^2-r^2}}{R}\mathbf{k}\right) \times \left(\frac{2r^2}{R} - R\right)\mathbf{i}$$

$$= \left(\frac{2\pi}{\tau}\right)^2\left(\frac{2r^2}{R^2} - 1\right)\left(\sqrt{R^2-r^2}\,\mathbf{j} - r\mathbf{k}\right)$$

$$\mathbf{v}_{rel} = -r\omega_{rel}\,\mathbf{i} = -r\left(\frac{R}{r}\,\frac{2\pi}{\tau}\right)\mathbf{i} = -\frac{2\pi}{\tau}R\,\mathbf{i}$$

$$2\boldsymbol{\Omega} \times \mathbf{v}_{rel} = \frac{4\pi}{\tau}\left(\frac{r}{R}\mathbf{j} + \frac{\sqrt{R^2-r^2}}{R}\mathbf{k}\right) \times \left(-\frac{2\pi}{\tau}R\,\mathbf{i}\right) = -2\left(\frac{2\pi}{\tau}\right)^2\left(\sqrt{R^2-r^2}\,\mathbf{j} - r\mathbf{k}\right)$$

$$\mathbf{a}_{rel} = -r\omega_{rel}^2\,\mathbf{k} = -r\left(\frac{R}{r}\,\frac{2\pi}{\tau}\right)^2\mathbf{k} = -\left(\frac{2\pi}{\tau}\right)^2\frac{R^2}{r}\mathbf{k}$$

Substitute, simplify, & get

$$\mathbf{a}_A = \left(\frac{2\pi}{\tau}\right)^2\left[\sqrt{R^2-r^2}\left(\frac{2r^2}{R^2} - 3\right)\mathbf{j} + \left(3r - \frac{R^2}{r} - \frac{2r^3}{R^2}\right)\mathbf{k}\right]$$

7/139 $I_{zz} = mr^2, \; k = r = 0.060 \text{ m}$

$p = 10\,000\,(2\pi/60) = 1047 \text{ rad/s}$

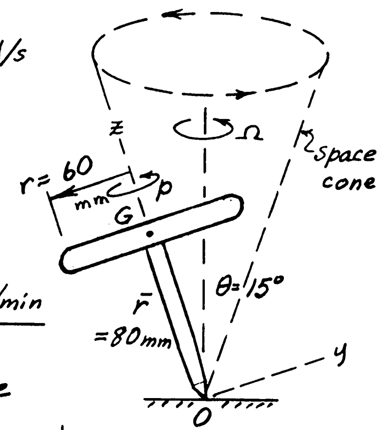
From Eq. 7/25,

$$\Omega \approx \frac{g\bar{r}}{k^2 p} = \frac{9.81(0.080)}{(0.060)^2(1047)}$$

$$= 0.208 \text{ rad/s}$$

$$N = \frac{\Omega}{2\pi} 60 = \frac{0.208}{2\pi} \times 60 = \underline{1.988 \text{ cycles/min}}$$

With $\Omega = \dot{\psi}$ very small, the body cone is too small to observe, so space cone is the only relatively apparent cone.

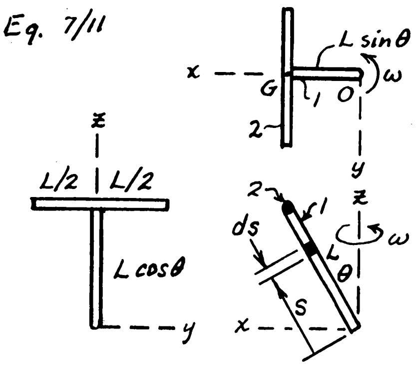


(Note direction of precession on diagram.)

7/140

$\omega_x = \omega_y = 0$, $\omega_z = \omega$ so from Eq. 7/11

$\underline{H}_O = (-I_{xz}\underline{i} - I_{yz}\underline{j} + I_{zz}\underline{k})\omega$



$[I_{xz}]$ For rod 1, mass per unit length is m/L, $dm = \frac{m}{L}ds$

$I_{xz} = \int xz\,dm = \int (s\sin\theta)(s\cos\theta)dm$

$= \frac{m\sin\theta\cos\theta}{L}\int_0^L s^2 ds$

$= \frac{1}{3}mL^2\sin\theta\cos\theta$

For rod 2, $I_{xz} = mL^2\sin\theta\cos\theta$, so total $I_{xz} = \frac{4}{3}mL^2\sin\theta\cos\theta$

$[I_{yz}]$ $I_{yz} = 0$ by symmetry

$[I_{zz}]$ For rod 2, $I_{zz} = I_G + md^2 = \frac{1}{12}mL^2 + m(L\sin\theta)^2 = mL^2(\frac{1}{12} + \sin^2\theta)$

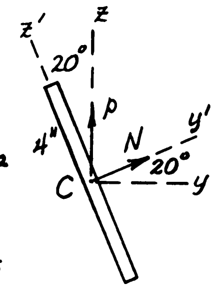
For rod 1, $I_{zz} = \frac{1}{3}m(L\sin\theta)^2 = \frac{1}{3}mL^2\sin^2\theta$

so total $I_{zz} = \frac{1}{3}mL^2(\frac{1}{4} + 4\sin^2\theta)$

Thus $\underline{\underline{H}_O = \frac{1}{3}mL^2\omega\left(-4\sin\theta\cos\theta\,\underline{i} + [\frac{1}{4} + 4\sin^2\theta]\underline{k}\right)}$

$T = \frac{1}{2}\underline{\omega}\cdot\underline{H}_O = \frac{1}{2}H_{O_z}\omega_z = \underline{\frac{1}{6}mL^2\omega^2(\frac{1}{4} + 4\sin^2\theta)}$

7/141 Eq. 7/14 becomes $\underline{H}_O = \underline{H}_C + \bar{\underline{r}} \times m\bar{\underline{v}}$, $\bar{\underline{r}} = \overrightarrow{OC}$, $\bar{\underline{v}} = \underline{v}_C$



For disk, $\omega_{y'} = \frac{300 \times 2\pi}{60} + \frac{60 \times 2\pi}{60} \sin 20° = 33.6$ rad/sec

$\omega_{z'} = \frac{60 \times 2\pi}{60} \cos 20° = 5.90$ rad/sec

$\omega_{x'} = 0$

$I_{y'y'} = \frac{1}{2} mr^2 = \frac{1}{2} \frac{8}{32.2} \left(\frac{4}{12}\right)^2 = 0.01380$ lb-ft-sec^2

$I_{z'z'} = \frac{1}{4} mr^2 = 0.00690$ lb-ft-sec^2

With $\omega_x = 0$ & principal axes x-y'-z', Eq. 7/13 gives

$\underline{H}_C = I_{y'y'}\omega_{y'}\underline{j}' + I_{z'z'}\omega_{z'}\underline{k}' = 0.01380(33.6)\underline{j}' + 0.00609(5.90)\underline{k}'$

$= 0.463\underline{j}' + 0.0407\underline{k}' = 0.421\underline{j} + 0.1967\underline{k}$

$\bar{\underline{r}} = \frac{10}{12}\underline{i} = 0.833\underline{i}$ ft

$\bar{\underline{v}} = p\underline{k} \times \bar{\underline{r}} = \frac{60 \times 2\pi}{60}\underline{k} \times 0.833\underline{i} = 5.24\underline{j}$ ft/sec

$\bar{\underline{r}} \times m\bar{\underline{v}} = 0.833\underline{i} \times \frac{8}{32.2}(5.24\underline{j}) = 1.084\,\underline{k}$ lb-ft-sec

$\underline{H}_o = 0.421\underline{j} + 0.1967\underline{k} + 1.084\underline{k} = \underline{0.421\underline{j} + 1.281\underline{k}}$ lb-ft-sec

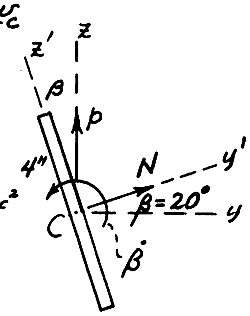
$T = \frac{1}{2}\bar{\underline{v}} \cdot \underline{G} + \frac{1}{2}\underline{\omega} \cdot \underline{H}_G$ $\quad$ ($G = C$ here)

$= \frac{1}{2}\, 5.24\underline{j} \cdot \frac{8}{32.2}(5.24\underline{j}) + \frac{1}{2}(29.5\underline{j} + 17.03\underline{k}) \cdot (0.421\underline{j} + 0.1967\underline{k})$

$= \underline{11.30 \text{ ft-lb}}$

7/142

Eq. 7/14 becomes $\underline{H}_O = \underline{H}_C + \bar{\underline{r}} \times m\bar{\underline{v}}$, $\bar{\underline{r}} = \overline{OC}$, $\bar{\underline{v}} = \underline{v}_C$



For disk, $\omega_x = \dot{\beta} = \frac{120 \times 2\pi}{60} = 12.57$ rad/sec

$$\omega_{y'} = \frac{300 \times 2\pi}{60} + \frac{60 \times 2\pi}{60} \sin 20° = 33.6 \text{ rad/sec}$$

$$\omega_{z'} = \frac{60 \times 2\pi}{60} \cos 20° = 5.90 \text{ rad/sec}$$

$$I_{xx} = I_{z'z'} = \frac{1}{4} mr^2 = \frac{1}{4} \frac{8}{32.2} \left(\frac{4}{12}\right)^2 = 0.00690 \text{ lb-ft-sec}^2$$

$$I_{y'y'} = \frac{1}{2} mr^2 = 0.01380 \text{ lb-ft-sec}^2$$

For principal axes x-y'-z' Eq. 7/13 gives

$$\underline{H}_C = I_{xx}\omega_x \underline{i} + I_{y'y'}\omega_{y'}\underline{j}' + I_{z'z'}\omega_{z'}\underline{k}'$$

$$= 0.00690(12.57)\underline{i} + 0.01380(33.8)\underline{j}' + 0.00690(5.90)\underline{k}'$$

$$\underline{H}_C = 0.0867\underline{i} + 0.463\underline{j}' + 0.0407\underline{k}'$$

$$= 0.0867\underline{i} + 0.421\underline{j} + 0.1967\underline{k} \text{ lb-ft-sec}$$

$$\bar{\underline{r}} = \frac{10}{12}\underline{i} = 0.833\underline{i} \text{ ft}$$

$$\bar{\underline{v}} = p\underline{k} \times \bar{\underline{r}} = \frac{60 \times 2\pi}{60}\underline{k} \times 0.833\underline{i} = 5.24\underline{j} \text{ ft/sec}$$

$$\bar{\underline{r}} \times m\bar{\underline{v}} = 0.833\underline{i} \times \frac{8}{32.2}(5.24\underline{j}) = 1.084\underline{k} \text{ lb-ft-sec}$$

$$\underline{H}_O = 0.0867\underline{i} + 0.421\underline{j} + 0.1967\underline{k} + 1.084\underline{k} = \underline{0.0867\underline{i} + 0.421\underline{j} + 1.281\underline{k}}$$

lb-ft-sec

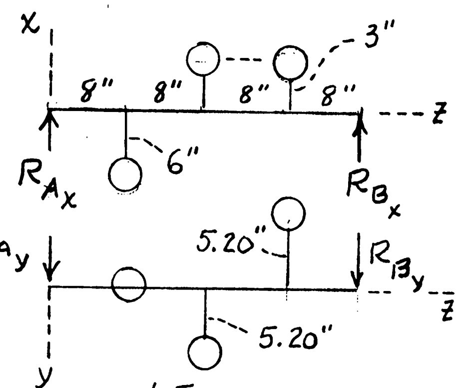
$$T = \frac{1}{2}\bar{\underline{v}} \cdot \underline{G} + \frac{1}{2}\underline{\omega} \cdot \underline{H}_G \quad (G = C \text{ here})$$

$$= \frac{1}{2} 5.24\underline{j} \cdot \frac{8}{32.2}(5.24\underline{j}) + \frac{1}{2}(12.57\underline{i} + 29.5\underline{j} + 17.03\underline{k}) \cdot (0.0867\underline{i} + 0.421\underline{j} + 0.1967\underline{k})$$

$$= \underline{11.85 \text{ ft-lb}}$$

7/143

$$\omega_z = \frac{1200(2\pi)}{60}$$
$$= 40\pi \text{ rad/sec}$$



Eq. 7/23

$$\Sigma M_x = I_{yz}\omega_z^2$$
$$\Sigma M_y = -I_{xz}\omega_z^2$$

Where

$$I_{yz} = m(5.20 \times 16 - 5.20 \times 24)$$
$$= -161.4(10^{-3}) \text{ in.-lb-sec}^2$$

$$m = \frac{1.5}{32.2}\frac{1}{12} = 3.88(10^{-3}) \text{ lb-sec}^2/\text{in.}$$

$$I_{xz} = m(-6 \times 8 + 3 \times 16 + 3 \times 24)$$
$$= 280(10^{-3}) \text{ in.-lb-sec}^2$$

$\Sigma M_x = -32R_{B_y} = -0.1614(40\pi)^2$, $R_{B_y} = 79.6$ lb

$\Sigma M_y = +32R_{B_x} = -0.280(40\pi)^2$, $R_{B_x} = -137.9$ lb

Because mass center has no acceleration

$R_{A_y} = -R_{B_y}$, $R_{A_x} = R_{B_x}$

$$|R_A| = |R_B| = \sqrt{79.6^2 + 137.9^2} = \underline{159.3 \text{ lb}}$$

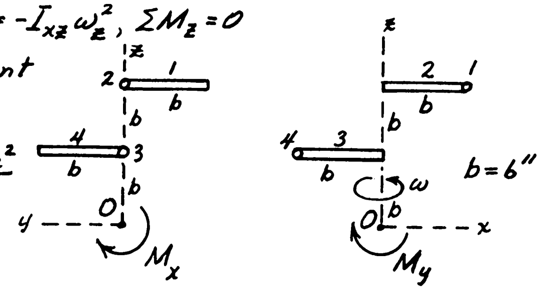
7/144 With $\omega_x = \omega_y = 0$, $\omega_z = \frac{1200 \times 2\pi}{60} = 125.7$ rad/sec,

$\dot{\omega}_x = \dot{\omega}_y = \dot{\omega}_z = 0$, Eqs. 7/23 about O become

$$\Sigma M_x = I_{yz}\omega_z^2, \quad \Sigma M_y = -I_{xz}\omega_z^2, \quad \Sigma M_z = 0$$

Let m = mass of each segment of length b

$$= \frac{1.4}{32.2} = 0.0435 \ \frac{\text{lb-sec}^2}{\text{ft}}$$



Static forces produce no moment so are not shown.

$$\begin{array}{ccccc} & ① & ② & ③ & ④ \end{array}$$

$$I_{xz} = m(b)(2b) + m\left(\frac{b}{2}\right)(2b) + m\left(-\frac{b}{2}\right)(b) + m(-b)(b) = \frac{3}{2}mb^2$$

$$M_y = -\frac{3}{2}mb^2\omega_z^2 = -\frac{3}{2}(0.0435)\left(\frac{6}{12}\right)^2(125.7)^2 = -257 \text{ lb-ft}$$

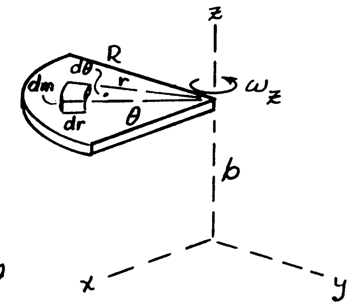
$$\begin{array}{ccccc} & ① & ② & ③ & ④ \end{array}$$

$$I_{yz} = m\left(-\frac{b}{2}\right)(2b) + m(0) + m(0) + m\left(\frac{b}{2}\right)(b) = -\frac{1}{2}mb^2$$

$$M_x = -\frac{1}{2}mb^2\omega_z^2 = -\frac{1}{2}(0.0435)\left(\frac{6}{12}\right)^2(125.7)^2 = -85.8 \text{ lb-ft}$$

$$M = \sqrt{M_x^2 + M_y^2} = \sqrt{85.8^2 + 257^2} = \underline{271 \text{ lb-ft}}$$

7/145 Let m = mass of each plate



mass per unit area $= m/(\pi R^2/4)$

$= 4m/\pi R^2$

$dm = \frac{4m}{\pi r^2} r\,dr\,d\theta$

$I_{xz} = \int xz\,dm = \frac{4m}{\pi R^2}\int_0^{\frac{\pi}{2}}\int_0^R (r\cos\theta)\,b\,r\,dr\,d\theta$

$= \frac{4mbR}{3\pi}$

$I_{yz} = \int yz\,dm = \frac{4m}{\pi R^2}\int_0^{\frac{\pi}{2}}\int_0^R (-r\sin\theta)\,b\,r\,dr\,d\theta = -\frac{4mbr}{3\pi}$

Top plate $I_{xz} = -I_{yz} = \frac{4(2)(0.150)(0.150)}{3\pi} = 0.01910\ kg\cdot m^2$

Lower plate $I_{xz} = -\frac{4mbR}{3\pi}$, $I_{yz} = \frac{4mbR}{3\pi}$ where $b = 0.075\,m$ ($\frac{1}{2}$ of 0.150)

$I_{xz} = -I_{yz} = -0.01910/2 = -0.00955\ kg\cdot m^2$

From Eq. 7/23 with $\omega_x = \omega_y = 0$, $\omega_z = \frac{2\pi(300)}{60} = 10\pi$ rad/s, $\dot{\omega}_z = 0$

$\Sigma M_x = I_{yz}\omega_z^2 = (-0.01910 + 0.00955)(10\pi)^2 = -9.42\ N\cdot m$

$\Sigma M_y = -I_{xz}\omega_z^2 = -(0.01910 - 0.00955)(10\pi)^2 = -9.42\ N\cdot m$

$M = \sqrt{9.42^2 + 9.42^2} = \underline{13.33\ N\cdot m}$

7/146 With $\omega_x = \omega_y = \omega_z = 0$ & $\dot{\omega}_z = 200\ \text{rad/s}^2$, Eq. 7/23 gives

$$\Sigma M_x = -I_{xz}\dot{\omega}_z\ , \quad \Sigma M_y = -I_{yz}\dot{\omega}_z$$

From solution to Prob. 7/145,

$$I_{xz} = 0.01910 - 0.00955 = 0.00955\ \text{kg}\cdot\text{m}^2$$

$$I_{yz} = -0.01910 + 0.00955 = -0.00955\ \text{kg}\cdot\text{m}^2$$

So $\Sigma M_x = -0.00955(200) = -1.910\ \text{N}\cdot\text{m}$

$\Sigma M_y = 0.00955(200) = 1.910\ \text{N}\cdot\text{m}$

$M = \sqrt{1.910^2 + 1.910^2} = \underline{2.70\ \text{N}\cdot\text{m}}$