INSTRUCTOR'S MANUAL

To Accompany

ENGINEERING MECHANICS - DYNAMICS

Volume 2

Fifth Edition, 2002

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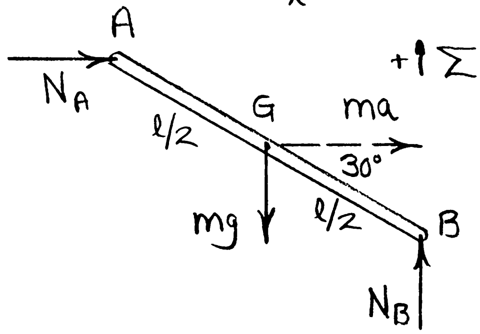
USE OF THE INSTRUCTOR'S MANUAL

The problem solution portion of this manual has been prepared for the instructor who wishes to occasionally refer to the authors' method of solution or who wishes to check the answer of his (her) solution with the result obtained by the authors. In the interest of space and the associated cost of educational materials, the solutions are very concise. Because the problem solution material is not intended for posting of solutions or classroom presentation, the authors request that it not be used for these purposes.

In the transparency master section there are approximately 65 solved problems selected to illustrate typical applications. These problems are different from and in addition to those in the textbook. Instructors who have adopted the textbook are granted permission to reproduce these masters for classroom use.

6/1

$\overset{+}{\rightarrow} \Sigma F_x = ma \quad : \quad N_A = ma \qquad (1)$

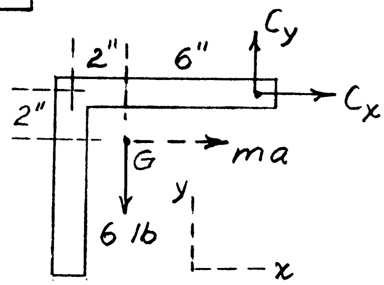


$+\uparrow \Sigma F_y = 0 \Rightarrow N_B = mg \quad (2)$
(not needed)

$$+\circlearrowright \Sigma M_B = mad: \quad N_A(\ell \sin 30^\circ) - mg\left(\frac{\ell}{2}\cos 30^\circ\right) = ma\left(\frac{\ell}{2}\sin 30^\circ\right) \quad (3)$$

Substitute (1) into (2) : $\underline{a = g\sqrt{3}}$

6/2

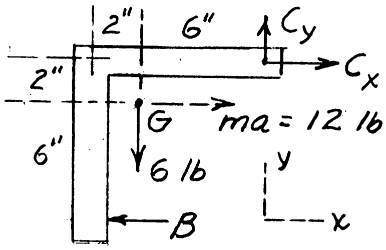


$\Sigma M_C = m a d$

$6(6) = \frac{6}{g} a (2)$

$\underline{a = 3g}$

6/3 | $ma = \frac{6}{32.2}(2)(32.2) = 12 \text{ lb}$



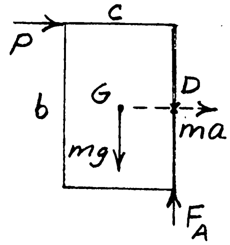
$\Sigma M_C = mad$

$6(6) - 8B = 12(2)$

$\underline{B = 1.5 \text{ lb}}$

6/4

(a)

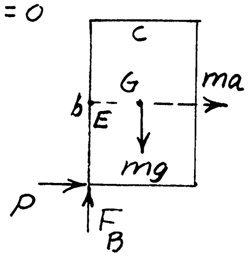


$\Sigma M_D = 0$

$P\frac{b}{2} - mg\frac{c}{2} = 0$

$P = mg\frac{c}{b}$

(b)

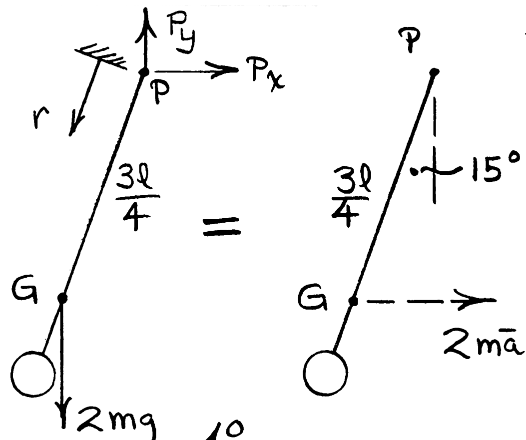


$\Sigma M_E = 0$

$mg\frac{c}{2} - P\frac{b}{2} = 0$

$P = mg\frac{c}{b}$

6/5



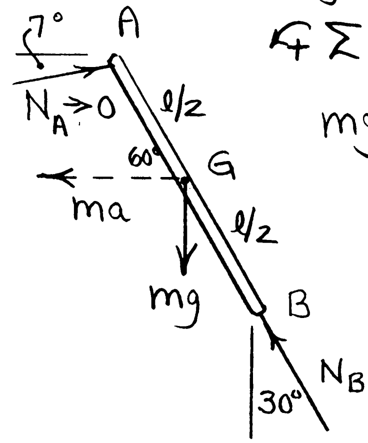
$2m\bar{a} = 2ma$

$$\bar{r} = \frac{\sum m_i r_i}{\sum m_i} = \frac{m(l/2) + m(l)}{m+m} = \frac{3}{4}l$$

$$\circlearrowleft + \quad \sum M_P = \cancelto{0}{\bar{I}\alpha} + m\bar{a}d \; : \; 2mg\left(\frac{3l}{4}\sin 15^\circ\right) = 2ma\left(\frac{3l}{4}\cos 15^\circ\right)$$

$$\Rightarrow a = g\tan 15^\circ = \underline{0.268g}$$

6/6



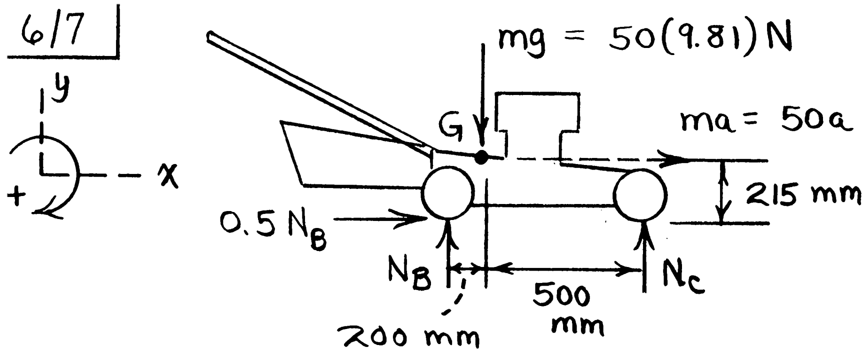
Tipping impends when $N_A \to 0$.

$\circlearrowleft + \Sigma M_B = mad$:

$$mg\,\frac{l}{2}\sin 30° = ma\,\frac{l}{2}\cos 30°$$

$$\underline{a = g\tan 30° = 5.66 \text{ m/s}^2}$$

6/7



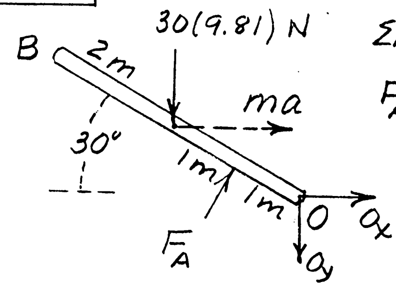
$\Sigma F_x = ma: \quad 0.5 N_B = 50a$

$\Sigma F_y = 0: \quad N_B + N_C - 50(9.81) = 0$

$\Sigma M_B = mad: \quad 50(9.81)(0.2) - N_C(0.7) = 50a(0.215)$

Simultaneous solution: $\begin{cases} N_B = 414 \text{ N} \\ N_C = 76.6 \text{ N} \\ a = 4.14 \text{ m/s}^2 \end{cases}$

6/8



$ma = 30(20) = 600\ N$

$\Sigma M_O = m a d_O$

$F_A(1) - 30(9.81)\,2\cos 30° = 600(2\sin 30°)$

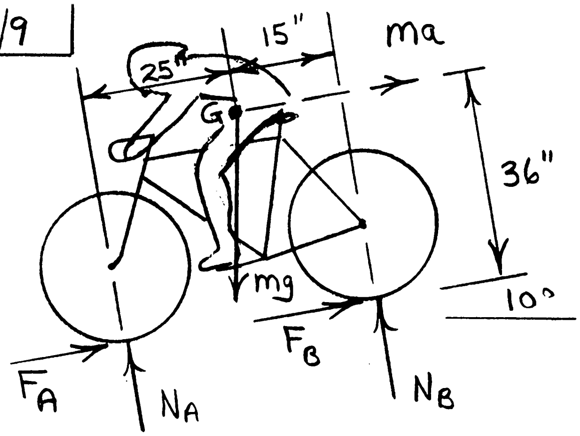
$F_A = 1110\ N$ or $\underline{F_A = 1.11\ kN}$

$\Sigma F_x = ma_x;\quad O_x + 1110\sin 30° = 600,\quad \underline{O_x = 45\ N}$

$\Sigma F_y = 0;\quad O_y + 30(9.81) - 1110\cos 30° = 0$

$\underline{O_y = 667\ N}$

6/9



Tipping at front wheel : $N_B, F_B \rightarrow 0$

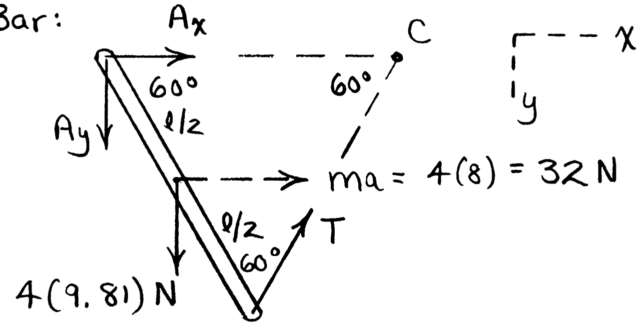
$+\circlearrowleft \Sigma M_A = mad$: $mg\,(25\cos 10° - 36\sin 10°)$

$= ma\,(36)$

Solve to obtain $a = \underline{0.510g}$ $(\underline{16.43 \text{ ft/sec}^2})$

6/10 As a whole: $\Sigma F_x = ma$: $80 = (6+4)a$, $a = 8\ m/s^2$

Bar:



$\curvearrowleft \Sigma M_C = m a d$: $A_y(2l\cos 60°) + 4(9.81)\frac{3l}{2}\cos 60°$

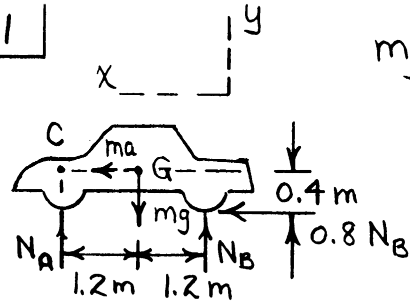
$= 32(\frac{l}{2}\sin 60°)$, $\underline{A_y = -15.57\ N}$

$\Sigma F_y = 0$: $-15.57 + 4(9.81) - T\sin 60° = 0$

$\underline{T = 27.3\ N}$

$\Sigma F_x = ma_x$: $A_x + 27.3\cos 60° = 32$, $\underline{A_x = 18.34\ N}$

6/11



$mg = 1650(9.81) = 16.19(10^3)$ N

$\curvearrowleft + \sum M_C = mad = 0: \quad N_B(2.4) - 0.8N_B(0.4) - 16.19(10^3)1.2 = 0$

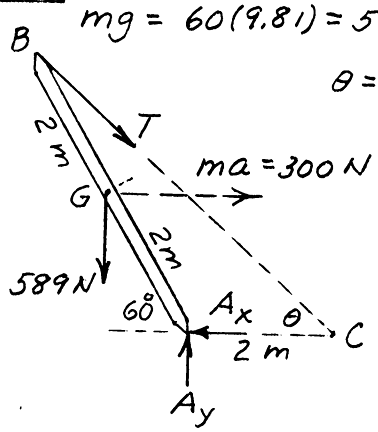
$N_B = 9.34(10^3)$ N or $\underline{N_B = 9.34 \text{ kN}}$

$\sum F_y = 0: \quad N_A + 9.34(10^3) - 16.19(10^3) = 0$

$N_A = 6.85(10^3)$ N or $\underline{N_A = 6.85 \text{ kN}}$

6/12 $ma = 60(5) = 300\ N$

$mg = 60(9.81) = 589\ N$



$$\theta = \tan^{-1}\frac{4\sin 60°}{4\cos 60° + 2} = 40.9°$$

$\Sigma M_A = ma d;\ (T\sin 40.9°)(2) - 589(1) = 300(2\sin 60°)$

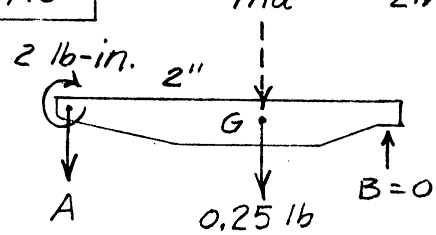
$T = 846\ N$

$\Sigma F_x = ma_x;\ 846\cos 40.9° - A_x = 300,\ A_x = 340\ N$

$\Sigma F_y = ma_y = 0;\ A_y - 589 - 846\sin 40.9° = 0,\ A_y = 1143\ N$

$A = \sqrt{(340)^2 + (1143)^2} = \underline{1192\ N}$

6/13



$$\Sigma M_A = mad$$

$$2 + 0.25(2) = \frac{0.25}{32.2(12)} a(2)$$

$$a = 1932 \text{ in./sec}^2$$

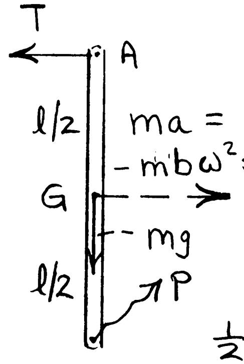
or $\underline{a = 161 \text{ ft/sec}^2}$

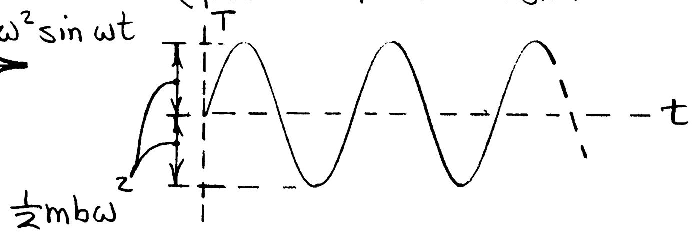
6/14

$$+\circlearrowright \sum M_P = mad: \quad -T\ell = -mb\omega^2 \sin\omega t\left(\frac{\ell}{2}\right)$$

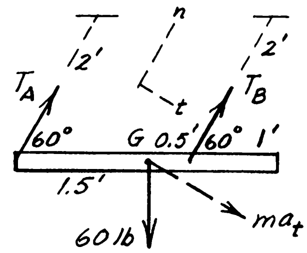
$$\Rightarrow T = \frac{1}{2} mb\omega^2 \sin\omega t$$

(positive T is tension in link)





6/15 | Curvilinear translation



$\Sigma F_t = ma_t$: $60 \cos 60° = \frac{60}{32.2} a_t$,

$a_t = 16.1$ ft/sec^2

$\alpha = a_t / r = 16.1/2 = \underline{8.05 \text{ rad/sec}^2}$

$\circlearrowleft + \Sigma M_G = 0$: $T_B \sin 60° \times 0.5 - T_A \sin 60° \times 1.5 = 0$, $T_A = \frac{1}{3} T_B$

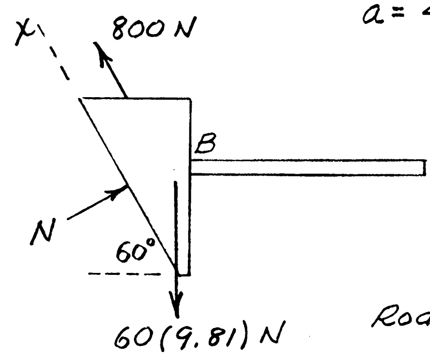
$\Sigma F_n = ma_n = 0$: $T_A + T_B - 60 \sin 60° = 0$, $T_A + T_B = 52.0$ lb

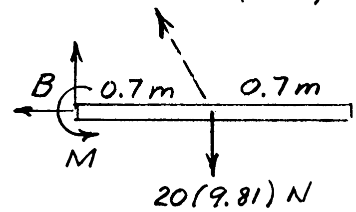
Combine & get $\underline{T_A = 12.99 \text{ lb}}$, $\underline{T_B = 39.0 \text{ lb}}$

6/16 $\Sigma F_x = ma_x$; $800 - 60(9.81)\sin 60^\circ = 60a$

$a = 4.84\ m/s^2$

$ma = 20(4.84) = 96.8\ N$



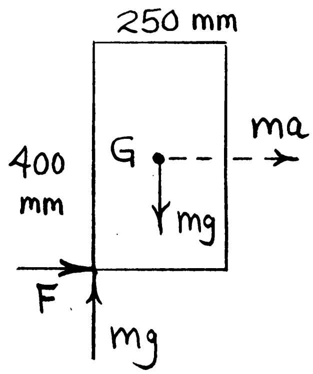


Rod: $\Sigma M_B = mad$

$M - 20(9.81)0.7 = 96.8(0.7\sin 60^\circ)$

$\underline{M = 196.0\ N\cdot m}$

6/17



$v = 1.2 + 0.9t^2$ m/s

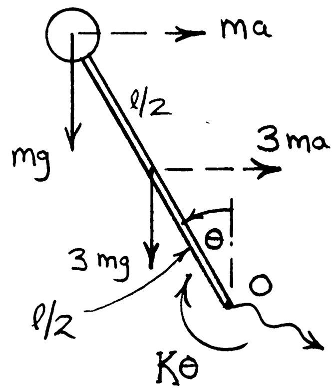
$a = \dot{v} = 1.8t$ m/s^2

$\circlearrowleft \Sigma M_A = ma d:$

$$mg\,\frac{250}{2} = m(1.8t)\left(\frac{400}{2}\right)$$

$$\underline{t = 3.41 \text{ s}}$$

6/18



$\overset{+}{\curvearrowright} \Sigma M_O = \Sigma mad: \; K\theta - 3mg\left(\frac{l}{2}\sin\theta\right)$

$- mg\,(l\sin\theta) = 3ma\left(\frac{l}{2}\cos\theta\right)$

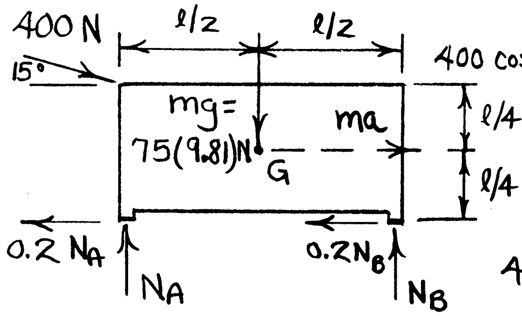
$+ \, ma\,(l\cos\theta)$

Simplify to

$K\theta - \frac{5}{2}mgl\sin\theta = \frac{5}{2}mal\cos\theta$

With $m = 0.5$ kg, $l = 0.6$ m, $a = 2g$, and $\theta = 20°$, K is found to be $\underline{K = 46.8 \ \frac{N\cdot m}{rad}}$

6/19 Static normals: $N_A = N_B = \dfrac{75(9.81)}{2} = 368$ N



$400 \cos 15° > 0.4[75(9.81) + 400 \sin 15°]$

so slippage occurs

$\overset{+}{\rightarrow} \Sigma F_x = ma$:

$400 \cos 15° - 0.2(N_A + N_B) = 75a$ (1)

$+\uparrow \Sigma F_y = 0$: $N_A + N_B - 400 \sin 15° - 75(9.81) = 0$ (2)

$\Sigma M_A = ma d$: $75(9.81)\left(\frac{l}{2}\right) - N_B(l) + 400 \cos 15°\left(\frac{l}{2}\right)$

$= 75a\,(l/4)$ (3)

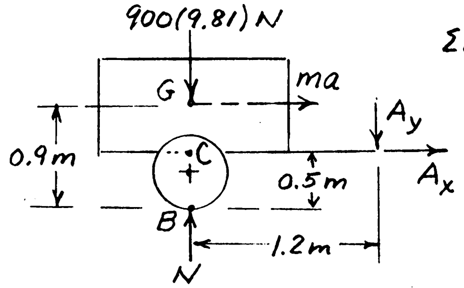
Solution of (1) - (3) : $N_A = 333$ N, $N_B = 506$ N

$(a = 2.91 \text{ m/s}^2)$

$n_A = \dfrac{333-368}{368}(100\%) = \underline{-9.52\%}$, $n_B = \dfrac{506-368}{368}(100\%) = \underline{37.7\%}$

6/20

$v^2 = 2as, \quad a = \frac{v^2}{2s} = \frac{(60/3.6)^2}{2(30)} = 4.63 \text{ m/s}^2$

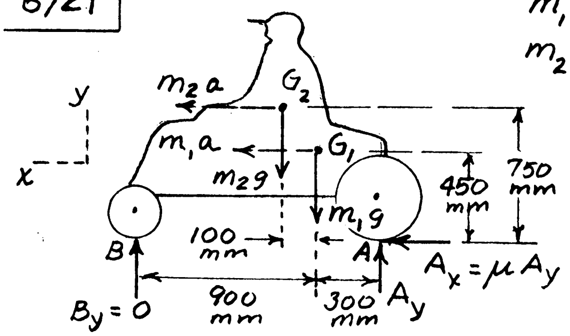


$\Sigma M_C = mad;$

$1.2 A_y = 900(4.63)(0.9 - 0.5)$

$\underline{A_y = 1389 \text{ N}}$

6/21



$m_1 g = 140(9.81) = 1373\ N$

$m_2 g = 90(9.81) = 883\ N$

$\Sigma F_y = 0$

$A_y - 1373 - 883 = 0$

$A_y = 2256\ N$

$\Sigma M_A = \Sigma mad;\quad 883(0.4) + 1373(0.3) = 90a(0.75) + 140a(0.45)$

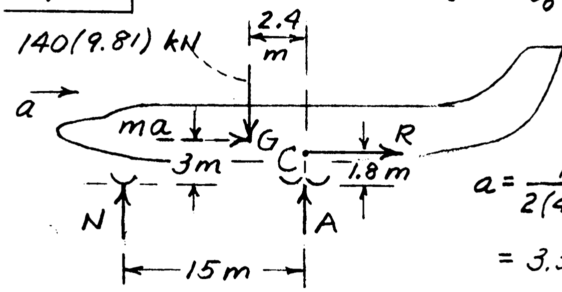
$765.2 = 130.5a$

$a = 5.86\ m/s^2$

$\Sigma F_x = \Sigma ma_x;\quad \mu(2256) = (140 + 90)5.86$

$\underline{\mu = 0.598}$

6/22



$$v^2 = v_0^2 + 2as$$

$$a = \frac{1}{2(425)}\left[(200)^2 - (60)^2\right]\frac{1}{(3.6)^2}$$

$$= 3.30 \text{ m/s}^2$$

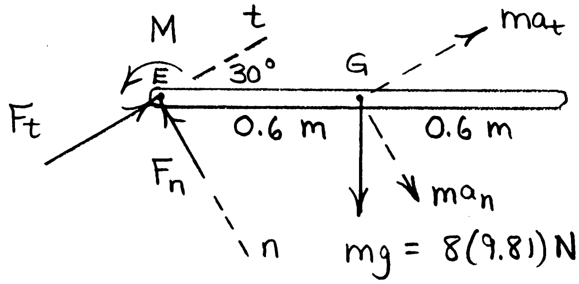
$\Sigma M_C = mad;$ $15N - 140(9.81)(2.4) = 140(3.30)(3-1.8)$

$\underline{N = 257 \text{ kN}}$

6/23 $a_E = a_A = a_C$

$(a_E)_n = r\omega^2 = 0.8(3)^2 = 7.2\ \text{m/s}^2$, $ma_n = 8(7.2) = 57.6\ \text{N}$

$(a_E)_t = r\alpha = 0.8(6) = 4.8\ \text{m/s}^2$, $ma_t = 8(4.8) = 38.4\ \text{N}$



$\curvearrowleft \Sigma M_E = \Sigma mad$: $M - 8(9.81)(0.6) = 38.4(0.6 \sin 30°) - 57.6(0.6 \cos 30°)$, $\underline{M = 28.7\ \text{N·m CCW}}$

$\Sigma F_t = ma_t$: $F_t - 8(9.81)\sin 30° = 38.4$, $F_t = 77.6\ \text{N}$

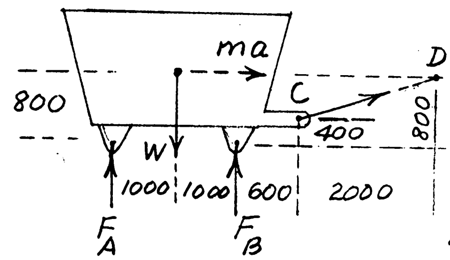
$\Sigma F_n = ma_n$: $-F_n + 8(9.81)\cos 30° = 57.6$, $F_n = 10.37\ \text{N}$

$F = \sqrt{F_t^2 + F_n^2} = \underline{78.3\ \text{N}}$

6/24 $ma = 2000(3) = 6000$ N or 6 kN

$W = mg = 2000(9.81) = 19\,620$ N or 19.62 kN

Dimensions in mm



$\Sigma M_C = mad_C$

$F_A(2.6) + F_B(0.6) - 19.62(1.6) = 6(0.8 - 0.4)$

$2.6F_A + 0.6F_B = 33.79$ kN·m

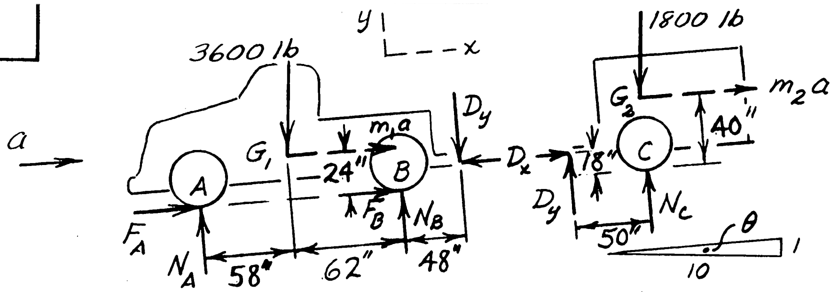
$\Sigma M_D = 0$; $F_A(4.6) + F_B(2.6) - 19.62(3.6) = 0$

$4.6F_A + 2.6F_B = 70.63$ kN·m

Solve simultaneously & get

$\underline{F_A = 11.37 \text{ kN}}$, $\underline{F_B = 7.05 \text{ kN}}$

6/25



For const. accel., $\theta = \tan^{-1}\frac{1}{10} = 5.71°$

$v^2 = v_0^2 + 2as$: $44^2 = 88^2 - 2a(360)$, $a = 8.07$ ft/sec² decel.

$m_1 a = \frac{3600}{32.2} \times 8.07 = 902$ lb, $m_2 a = \frac{1800}{32.2} \times 8.07 = 451$ lb

Trailer: $\Sigma F_x = ma_x$: $D_x - 1800 \sin 5.71° = 451$, $\underline{D_x = 630 \text{ lb}}$

$\circlearrowleft \Sigma M_C = mad$: $50 D_y + 630(18) - 1800 \sin 5.71°(40) = 451(40)$, $\underline{D_y = 277 \text{ lb}}$

$\Sigma F_y = 0$: $N_C - 1800 \cos 5.71° + 277 = 0$, $\underline{N_C = 1514 \text{ lb}}$

Truck:

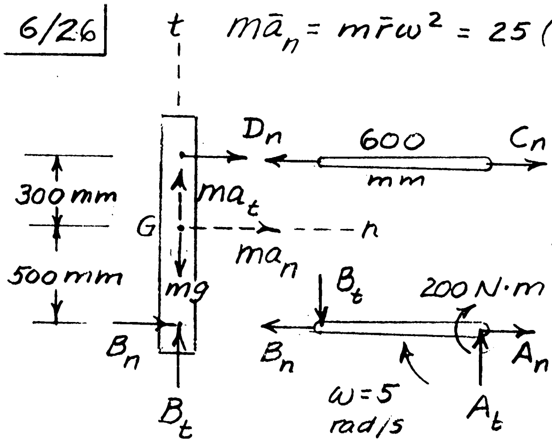
$\circlearrowleft \Sigma M_A = mad$: $3600 \cos 5.71° \times 58 - 3600 \sin 5.71° \times 24 - 120 N_B$

$$+ 277(168) - 630(18) = 902(24)$$

$$\underline{N_B = 1773 \text{ lb}}$$

6/26

$m\bar{a}_n = m\bar{r}\omega^2 = 25(0.600)(5^2) = 375\ N$



$\Sigma M_B = m\bar{a}d$

$D_n(0.800) = 375(0.500)$

$\underline{D = D_n = 234\ N}$

AB; $\Sigma M_A = 0, \quad 0.600 B_t - 200 = 0, \quad B_t = 333\ N$

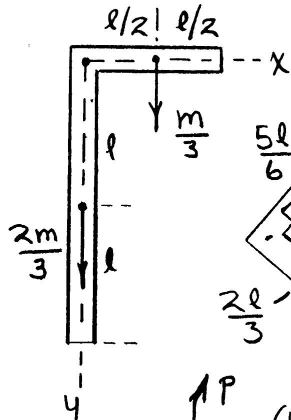
BGD; $\Sigma F_t = m\bar{a}_t = m\bar{r}\alpha; \quad 333 - 25(9.81) = 25(0.6\alpha)$

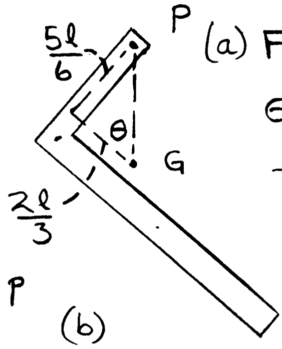
$\underline{\alpha = 5.87\ rad/s^2}$

6/27

$$\bar{X} = \frac{\Sigma \bar{x} m}{\Sigma m} = \frac{\frac{\ell}{2}\frac{m}{3}}{m} = \ell/6$$

$$\bar{Y} = \frac{\Sigma \bar{y} m}{\Sigma m} = \frac{\ell\left(\frac{2m}{3}\right)}{m} = \frac{2\ell}{3}$$

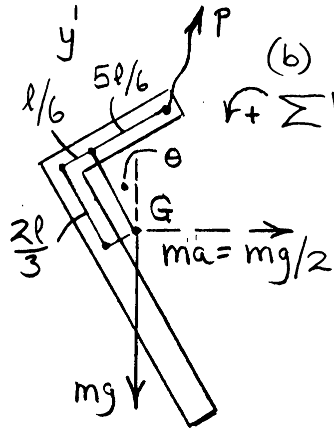




(a) For static equilibrium, G is vertically under P:

$\tan\theta = \dfrac{5\ell/6}{2\ell/3}$, $\underline{\theta = 51.3°}$

(b)



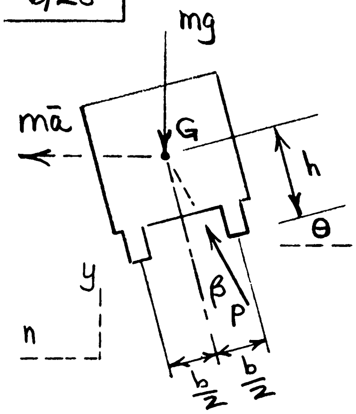
$$\curvearrowleft + \Sigma M_P = m\bar{a}d : \quad mg\left(\frac{5\ell}{6}\cos\theta - \frac{2\ell}{3}\sin\theta\right) = m\frac{g}{2}\left(\frac{5\ell}{6}\sin\theta + \frac{2\ell}{3}\cos\theta\right)$$

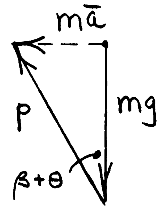
$$\Rightarrow \underline{\theta = 24.8°}$$

With $\theta = 0$: $\curvearrowleft + \Sigma M_P = m\bar{a}d$: $mg\left(\frac{5\ell}{6}\right) = ma\left(\frac{2\ell}{3}\right)$

$$\underline{a = \frac{5}{4}g}$$

6/28





(a) For no tendency to slip, $\beta = 0$.

From diagram,

$$\tan\theta = \frac{m\bar{a}}{mg} = \frac{v^2/r}{g}$$

$$\underline{\theta = \tan^{-1}\frac{v^2}{gr}}$$

(b) $$\tan(\beta+\theta) = \frac{m\bar{a}}{mg} = \frac{v^2/r}{g}$$

$$v^2 = gr\tan(\beta+\theta) = gr\,\frac{\tan\beta + \tan\theta}{1-\tan\beta\tan\theta}$$

Slips first if $\mu < \frac{b/2}{h}$ & $\mu = \tan\beta$

So $$\underline{v^2 = gr\,\frac{\mu + \tan\theta}{1-\mu\tan\theta}}$$

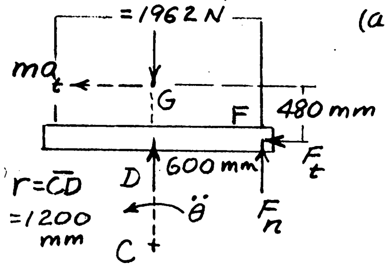
Tips first if $\mu > \frac{b/2}{h}$ & $\tan\beta = \frac{b}{2h}$:

$$\underline{v^2 = gr\,\frac{\frac{b}{2h} + \tan\theta}{1-\frac{b}{2h}\tan\theta}}$$

6/29

$mg = 200(9.81) = 1962\ N$

$\theta = \frac{\pi}{6}\left(1 - \cos\frac{\pi t}{2}\right)$, $\dot{\theta} = \frac{\pi^2}{12}\sin\frac{\pi t}{2}$, $\ddot{\theta} = \frac{\pi^3}{24}\cos\frac{\pi t}{2}$



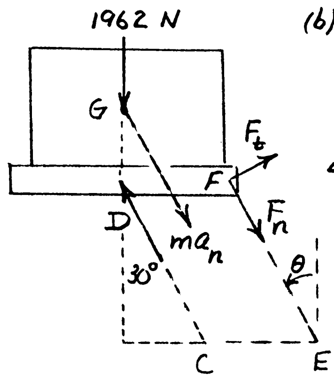
(a) For $\theta = 0$ & $t = 0$, $\ddot{\theta} = \frac{\pi^3}{24}$ rad/s^2

$ma_t = mr\ddot{\theta} = 200(1.2)(\pi^3/24)$

$= 310\ N$

$\Sigma M_F = ma_t d$; $(1962 - D)0.6 = 310(0.48)$

$\underline{D = 1714\ N}$ (compression)



(b) For $t = 1\,s$, $\theta = \frac{\pi}{6}$, $\dot{\theta} = \frac{\pi^2}{12}$ rad/s, $\ddot{\theta} = 0$

$ma_n = mr\dot{\theta}^2 = 200(1.2)(\pi^4/144)$

$= 162.4\ N$

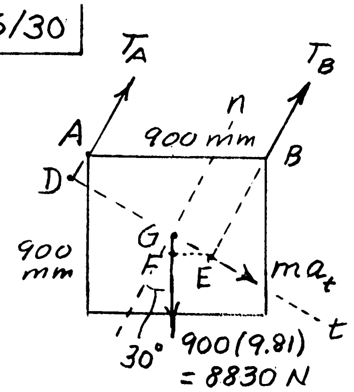
$\Sigma M_F = ma_n d$

$1962(0.6) - (0.6\cos 30°)D$

$= 162.4(0.6\cos 30° - 0.48\sin 30°)$

$\underline{D = 2178\ N}$ (compression)

6/30



Sol. I

$\Sigma F_n = ma_n = 0$; $T_A + T_B - 8830 \cos 30^\circ = 0$

$\Sigma F_t = ma_t$; $8830 \sin 30^\circ = 900\, a_t$

$\Sigma M_G = 0$

$T_A \cos 30^\circ (0.45) + T_A \sin 30^\circ (0.45)$

$- T_B \cos 30^\circ (0.45) + T_B \sin 30^\circ (0.45) = 0$

Solve simultaneously & get

$\underline{T_A = 1616\ N}$, $T_B = 6031\ N$, $a_t = 4.91\ \frac{m}{s^2}$

Sol. II $\overline{BE} = 0.45(\cos 30^\circ + \sin 30^\circ) = 0.615\ m$

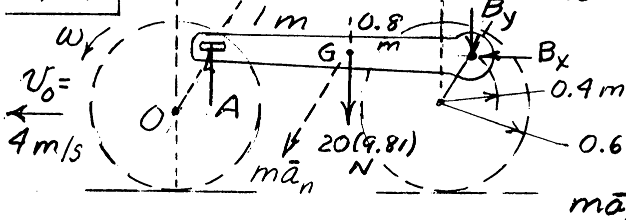
$\overline{FE} = 0.45 - 0.615 \sin 30^\circ = 0.1426\ m$

$\overline{DE} = 0.90 \cos 30^\circ = 0.779\ m$

$\Sigma M_E = 0$; $8830(0.1426) - 0.779\, T_A = 0$, $\underline{T_A = 1616\ N}$

Trade off between geometry & simultaneous sols.

6/31



$\omega = v/r = \dfrac{4}{0.6} = 6.67 \ \dfrac{rad}{s}$

$\alpha = 0 \ , \ \bar{a}_t = 0 \ , \ a_O = 0$

$\bar{a}_n = a_{A/O} = r\omega^2$

$= 0.4(6.67)^2 = 17.78 \ m/s^2$

$m\bar{a}_n = 20(17.78) = 356 \ N$

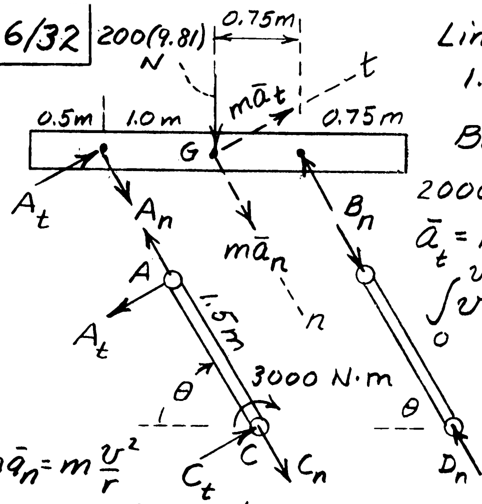
$\Sigma M_A = m\bar{a}d; \quad 1.8 B_y + 20(9.81)(1.0) = 356 \cos 30° (1.0)$

$B_y = 62.1 \ N$

$\Sigma F_x = m a_x \ ; \quad B_x = 356 \sin 30° = 177.8 \ N$

$B = \sqrt{B_x^2 + B_y^2} \ ; \quad B = \sqrt{(177.8)^2 + (62.1)^2} = \underline{188.3 \ N}$

▶ 6/32



Link CA; $\Sigma M_A \approx 0$,

$1.5A_t - 3000 = 0$, $A_t = 2000$ N

Bar; $\Sigma F_t = m\bar{a}_t$

$2000 - 200(9.81)\cos\theta = 200\,\bar{a}_t$

$\bar{a}_t = 10 - 9.81\cos\theta$

$$\int_0^v v\,dv = \int_0^{\pi/3} a_t\, r\,d\theta$$

$$\frac{v^2}{2} = 1.5\left(10\theta - 9.81\sin\theta\right)_0^{\pi/3}$$

$$v^2 = 3\left(\frac{10\pi}{3} - 9.81\frac{\sqrt{3}}{2}\right) = 5.93\ \frac{m^2}{s^2}$$

$m\bar{a}_n = m\dfrac{v^2}{r}$

$= 200(5.93)/1.5 = 791$ N, $\quad v = 2.43$ m/s

$m\bar{a}_t = 200(10 - 9.81\cos 60°) = 1019$ N

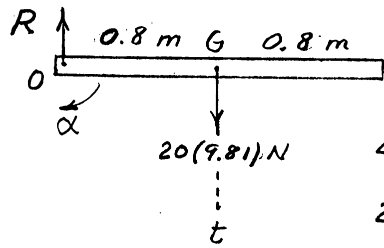
$\Sigma M_G = 0$; $2000(1.0)\cos 60° - A_n(1.0)\sin 60° - B_n(0.75)\sin 60° = 0$

$\Sigma F_n = m\bar{a}_n$; $200(9.81)\sin 60° + A_n - B_n = 791$

solve & get $A_n = 270$ N, $B_n = 1179$ N

$A = \sqrt{(270)^2 + (2000)^2} = 2018$ N or $\underline{A = 2.02\text{ kN}}$

6/33



$$\Sigma M_O = I_O \alpha$$

$$20(9.81)(0.8) = \frac{1}{3}(20)(1.6)^2 \alpha$$

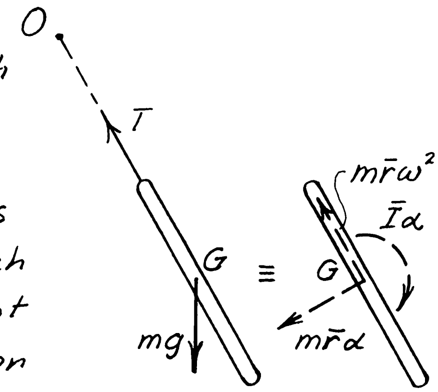
$$\alpha = 9.20 \text{ rad/s}^2$$

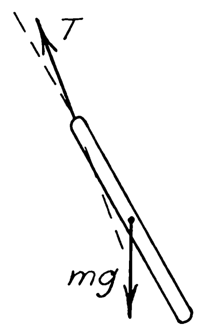
$$\Sigma F_t = m\bar{a}_t = m\bar{r}\alpha$$

$$20(9.81) - R = 20(0.8)(9.20)$$

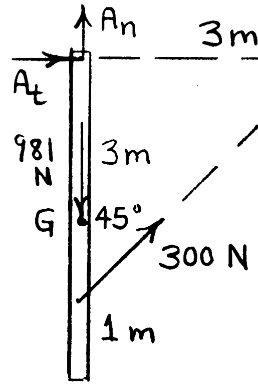
$$\underline{R = 49.0 \text{ N}}$$

6/34 Assume bar remains in line with the cord. To match the angular acceleration of the bar to that of the cord, the kinetic diagram requires a CW moment about G which the assumed position will not provide. Thus the assumption is in error. A correct free-body diagram would show





6/35



$$I_A = \frac{1}{3} m l^2 = \frac{1}{3}(100)(4)^2$$
$$= 533 \text{ kg}\cdot\text{m}^2$$

$\circlearrowleft + \Sigma M_A = I_A \alpha$:

$$300(3 \sin 45°) = 533\alpha$$
$$\underline{\alpha = 1.193 \text{ rad/s}^2}$$

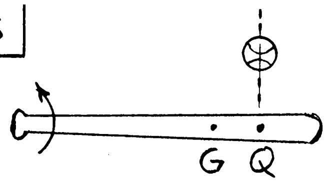
$\Sigma F_n = m\bar{a}_n$: $A_n + 300 \cos 45° - 981 = 0$, $A_n = 769$ N

$\Sigma F_t = m\bar{a}_t$: $A_t + 300 \sin 45° = 100(2)(1.193)$

$A_t = 26.5$ N

$$F_A = \sqrt{A_n^2 + A_t^2} = \underline{769 \text{ N}}$$

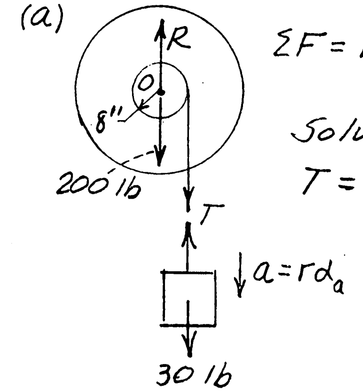
6/36



Considering the bat to be rotating around its handle, the force on the handle is least when contact occurs at the center of percussion Q

6/37

(a)



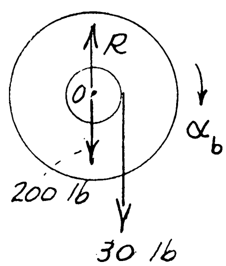
$\Sigma M_O = I_O \alpha; \quad T\frac{8}{12} = \frac{200}{32.2}\left(\frac{15}{12}\right)^2 \alpha_a$

$\Sigma F = ma; \quad 30 - T = \frac{30}{32.2}\left(\frac{8}{12}\alpha_a\right)$

Solve simultaneously & get

$T = 28.77 \text{ lb} \qquad \underline{\alpha_a = 1.976 \text{ rad/sec}^2}$

(b)



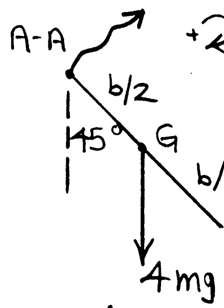
$\Sigma M_O = I_O \alpha; \quad 30\frac{8}{12} = \frac{200}{32.2}\left(\frac{15}{12}\right)^2 \alpha_b$

$\underline{\alpha_b = 2.06 \text{ rad/sec}^2}$

6/38 $I_{A-A} = 2\left(\frac{1}{3}mb^2\right) + mb^2 = \frac{5}{3}mb^2$ } m = mass of each side

$I_{B-B} = \frac{1}{12}mb^2 + 2\left[\frac{1}{12}mb^2 + m\left(\left(\frac{b}{2}\right)^2 + \left(\frac{b}{2}\right)^2\right)\right]$

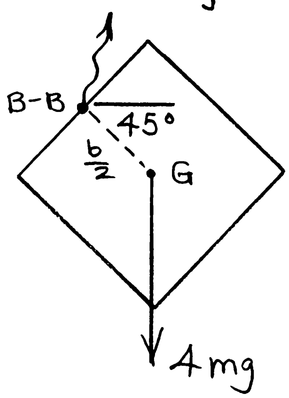
$+ \left[\frac{1}{12}mb^2 + mb^2\right] = \frac{7}{3}mb^2$



$+\circlearrowleft \Sigma M_{A-A} = I_{A-A}\alpha$:

$4mg\frac{b}{2}\frac{\sqrt{2}}{2} = \frac{5}{3}mb^2\alpha$

$\underline{\alpha = \frac{3\sqrt{2}}{5}\frac{g}{b}}$

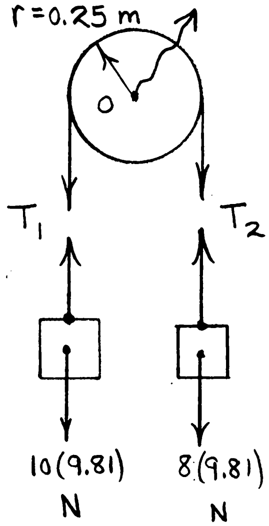


$+\circlearrowleft \Sigma M_{B-B} = I_{B-B}\alpha$:

$4mg\frac{b}{2}\frac{\sqrt{2}}{2} = \frac{7}{3}mb^2\alpha$

$\underline{\alpha = \frac{3\sqrt{2}}{7}\frac{g}{b}}$

6/39



Drum: $\circlearrowleft \Sigma M_O = I_O \alpha$:

$$(T_1 - T_2)(0.250) = \tfrac{1}{2} 15 (0.250)^2 \alpha \quad (1)$$

10-kg cylinder: $+\downarrow \Sigma F = ma$:

$$10(9.81) - T_1 = 10(0.250\alpha) \quad (2)$$

8-kg cylinder: $+\uparrow \Sigma F = ma$:

$$T_2 - 8(9.81) = 8(0.250\alpha) \quad (3)$$

Solution of Eqs. (1)–(3):

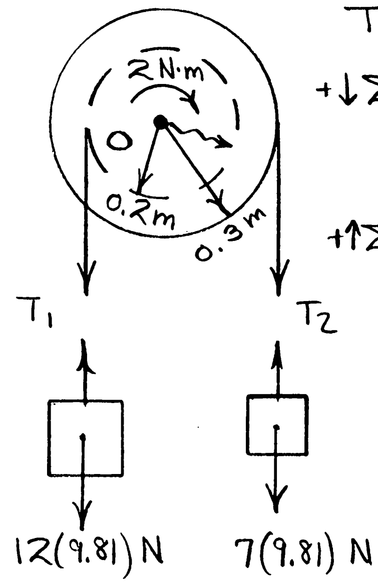
$\alpha = 3.08 \frac{\text{rad}}{\text{s}^2}$, $T_1 = 90.4$ N, $T_2 = 84.6$ N

$a_{10} = a = 0.250(3.08) = \underline{0.769 \text{ m/s}^2}$

Set I_O to zero & resolve: $\alpha = 4.36$ rad/s^2

$a_{10} = a = 0.250(4.36) = \underline{1.090 \text{ m/s}^2}$

6/40



$\circlearrowright + \sum M_O = I_O \alpha$ for drum:

$$T_1(0.2) - T_2(0.3) - 2 = 8(0.225)^2 \alpha \quad (1)$$

$+\downarrow \sum F = ma$ for 12-kg cylinder:

$$12(9.81) - T_1 = 12(0.2\alpha) \quad (2)$$

$+\uparrow \sum F = ma$ for 7-kg cylinder:

$$T_2 - 7(9.81) = 7(0.3\alpha) \quad (3)$$

Solution of Eqs. (1)-(3):

$$\begin{cases} T_1 = 116.2 \text{ N} \\ T_2 = 70.0 \text{ N} \\ \underline{\alpha = 0.622 \text{ rad/s}^2} \end{cases}$$

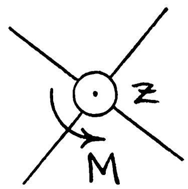
6/41 $\alpha = \dfrac{\omega}{\tau}$ (H = hub ; B = blades)

$$I_{zz} = \frac{1}{2} m_H r^2 + 4\left[\frac{1}{12} m_B l^2 + m_B \left(r + \frac{l}{2}\right)^2\right]$$

$$= \frac{1}{2}(\rho \pi r^2 d) r^2 + 4(\rho l d t)\left[\frac{1}{12} l^2 + r^2 + r l + \frac{l^2}{4}\right]$$

$$= \frac{1}{2}\rho \pi d r^4 + 4\rho l d t\left[\frac{1}{3} l^2 + r l + r^2\right]$$

$$= \rho d\left[\frac{1}{2}\pi r^4 + 4 l t\left(\frac{1}{3} l^2 + r l + r^2\right)\right]$$

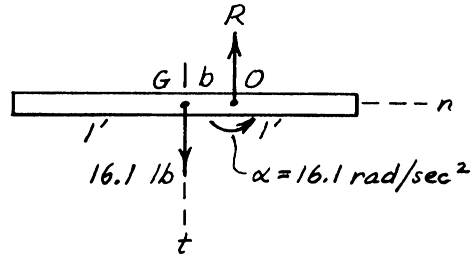


$\Sigma M_z = I_{zz}\alpha$:

$$\underline{M = \frac{\omega \rho d}{\tau}\left[\frac{1}{2}\pi r^4 + 4 l t\left(\frac{1}{3} l^2 + r l + r^2\right)\right]}$$

6/42

$$I_O = \frac{1}{12}mL^2 + mb^2$$
$$= \frac{16.1}{32.2}\left(\frac{2^2}{12} + b^2\right)$$
$$= \frac{1}{6} + \frac{b^2}{2} \text{ lb-ft-sec}^2$$



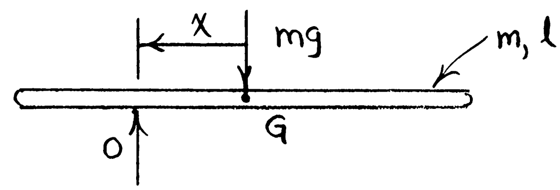
$$\Sigma M_O = I_O\alpha: \quad 16.1b = \left(\frac{1}{6} + \frac{b^2}{2}\right)16.1, \quad 3b^2 - 6b + 1 = 0$$

$$b = 1 \pm \sqrt{24}/6, \quad b = 0.1835 \text{ ft } (1.817 \text{ ft}),$$

$$\underline{b = 2.20 \text{ in.}}$$

$$\Sigma F_t = m\bar{r}\alpha: \quad 16.1 - R = \frac{16.1}{32.2}0.1835(16.1), \quad \underline{R = 14.62 \text{ lb}}$$

6/43



$$I_O = I_G + mx^2 = \frac{1}{12}ml^2 + mx^2 = m\left(\frac{l^2}{12} + x^2\right)$$

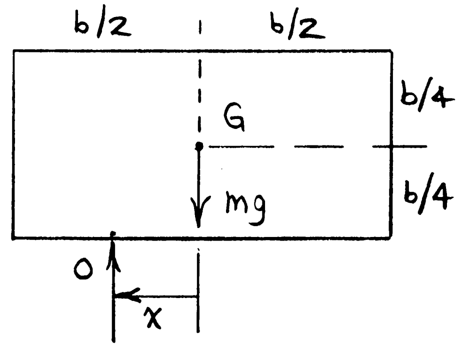
$$\circlearrowright \Sigma M_O = I_O\alpha : \quad mgx = m\left(\frac{l^2}{12} + x^2\right)\alpha$$

$$\alpha = \frac{gx}{\frac{1}{12}l^2 + x^2}$$

$$\frac{d\alpha}{dx} = \frac{\left(\frac{1}{12}l^2 + x^2\right)g - gx(2x)}{\left(\frac{1}{12}l^2 + x^2\right)^2} = 0 \Rightarrow \underline{x = \frac{l}{2\sqrt{3}}}$$

$$\alpha = \frac{g\frac{l}{\sqrt{12}}}{\frac{1}{12}l^2 + \frac{1}{12}l^2} = \underline{\sqrt{3}\,\frac{g}{l}}$$

6/44



$$I_G = \frac{1}{12}m\left[b^2 + \left(\frac{b}{2}\right)^2\right] = \frac{5}{48}mb^2$$

$$I_O = I_G + m\left[\left(\frac{b}{4}\right)^2 + x^2\right] = \frac{1}{6}mb^2 + mx^2$$

$$\circlearrowleft{+}\ \Sigma M_O = I_O\alpha: \quad mgx = \left(\frac{1}{6}mb^2 + mx^2\right)\alpha$$

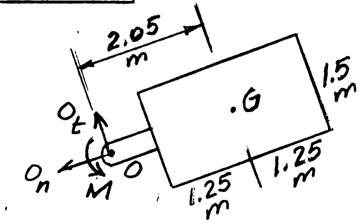
$$\alpha = \frac{gx}{\frac{1}{6}b^2 + x^2}$$

$$\frac{d\alpha}{dx} = \frac{\left(\frac{1}{6}b^2 + x^2\right)g - gx(2x)}{\left(\frac{1}{6}b^2 + x^2\right)} = 0 \Rightarrow \underline{x = \frac{b}{\sqrt{6}}}$$

$$\alpha = \frac{g\frac{b}{\sqrt{6}}}{\frac{1}{6}b^2 + \frac{1}{6}b^2} = \underline{\sqrt{\frac{3}{2}}\ \frac{g}{b}}$$

6/45

$m = 6000\,kg$; From Table D/4



$I_G = \frac{1}{12}(6000)\left[(1.5)^2 + (2.5)^2\right]$

$= 4250\ kg \cdot m^2$

$I_O = I_G + md^2$

$= 4250 + 6000(2.05)^2$

$= 29\,465\ kg \cdot m^2$

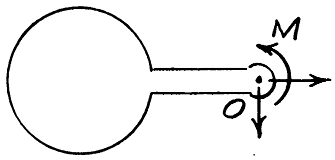
$M = 30\ N \cdot m$

$\Sigma M_O = I_O \alpha$; $30 = 29\,465\,\alpha$, $\alpha = 1.018(10^{-3})\ rad/s^2$

$\theta = \frac{1}{2}\alpha t^2$, $\frac{\pi}{4} = \frac{1}{2}\,1.018(10^{-3})\,t_1^2$, $t_1 = 39.28\,s$

Total time $t = 2t_1 = \underline{78.6\ s}$

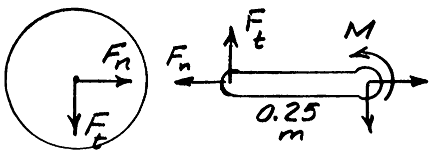
6/46



(a) Pin in place; $\Sigma M_O = I_O \alpha$

$30 = 43\left(\frac{1}{2}[0.2]^2 + [0.25]^2\right)\alpha$

$\underline{\alpha = 8.46 \text{ rad/s}^2}$



(b) Pin removed

Arm: $F_t \approx M/r = \frac{30}{0.25} = 120$ N

Rotor: $\Sigma F_t = m a_t$

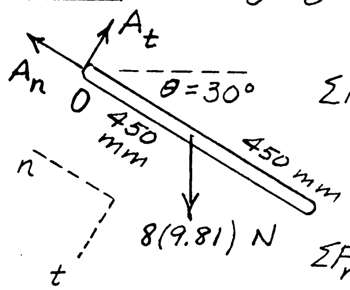
$120 = 43(0.25\alpha)$

$\underline{\alpha = 11.16 \text{ rad/s}^2}$

6/47

$\Sigma M_O = I_O\alpha$; $8(9.81)(0.450\cos 30^\circ) = \frac{1}{3}8(0.900)^2\alpha$

$\alpha = 14.16 \text{ rad/s}^2$



$\omega = \dot\theta = 2$ rad/s

$\Sigma F_t = m\bar r\alpha$; $8(9.81)\cos 30^\circ - A_t = 8(0.450)(14.16)$

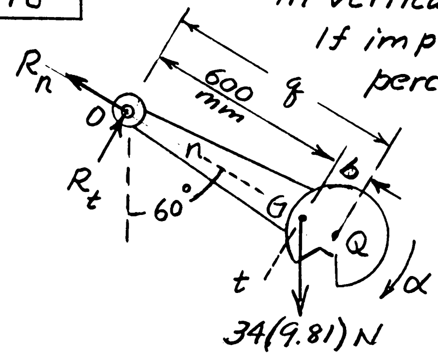
$A_t = 16.99$ N

$\Sigma F_n = m\bar r\omega^2$; $A_n - 8(9.81)\sin 30^\circ = 8(0.450)2^2$

$A_n = 53.64$ N

$A = \sqrt{\overline{16.99}^2 + \overline{53.64}^2} = \underline{56.3 \text{ N}}$

6/48



In vertical position $R_t = 0$

If impact occurs at center of percussion so

$$q = k_O^2/\bar{r},$$

$$0.600 + b = \frac{(0.620)^2}{0.600}$$

$$b = 0.0407 \text{ m}$$

or $\underline{b = 40.7 \text{ mm}}$

$\Sigma M_Q = 0;\ 34(9.81)(0.0407 \sin 60°) - (0.6407)R_t = 0$

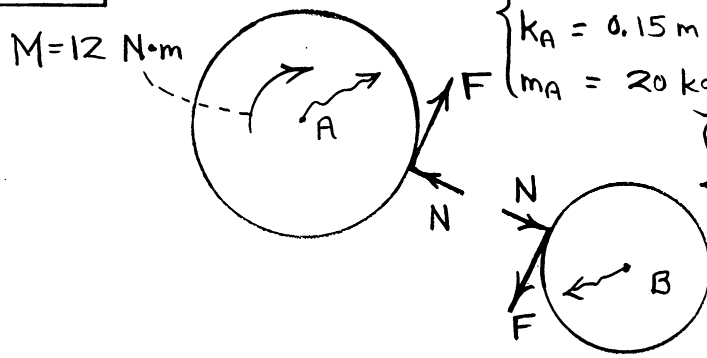
$R_t = 18.35$ N

$\Sigma F_n = m\bar{r}\omega^2 = 0;\ R_n - 34(9.81)\cos 60° = 0$

$R_n = 166.8$ N

$R = \sqrt{(166.8)^2 + (18.35)^2} = \underline{167.8 \text{ N}}$

6/49



$$\begin{cases} r_A = 0.24 \text{ m} \\ k_A = 0.15 \text{ m} \\ m_A = 20 \text{ kg} \end{cases} \qquad \begin{cases} r_B = 0.18 \text{ m} \\ k_B = 0.1 \text{ m} \\ m_B = 10 \text{ kg} \end{cases}$$

$$\overset{\curvearrowright}{+} \Sigma M_A = I_A \alpha_A: \quad 12 - F(0.24) = 20(0.15)^2 \alpha_A \quad (1)$$

$$\overset{\curvearrowleft}{+} \Sigma M_B = I_B \alpha_B: \quad F(0.18) = 10(0.1)^2 \alpha_B \quad (2)$$

Tangential accelerations match: $r_A \alpha_A = r_B \alpha_B$

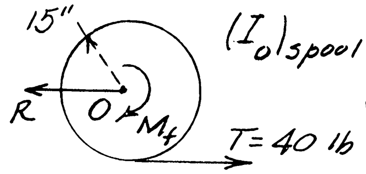
$$0.24 \alpha_A = 0.18 \alpha_B \quad (3)$$

Solution of Eqs. (1)–(3): $\begin{cases} F = 14.16 \text{ N} \\ \alpha_A = 19.12 \text{ rad/s}^2 \text{ (CW)} \\ \alpha_B = 25.5 \text{ rad/s}^2 \text{ (CCW)} \end{cases}$

6/50 The angular acceleration of the welded unit depends upon I_O and the moment (due to the weights) about O. Neither quantity depends upon the orientation θ of the slab.

6/51

$M_f = 20(15)/12 = 25$ lb-ft



$(I_O)_{spool} = mk^2 = \frac{62}{32.2}\left(\frac{12}{12}\right)^2 = 1.925$ ft-lb-sec^2

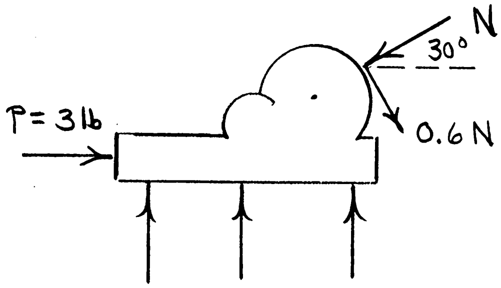
$(I_O)_{cable} = mr^2 = \frac{200(0.436)}{32.2}\left(\frac{15}{12}\right)^2$

$= 4.231$ ft-lb-sec^2

Total $I_O = 1.925 + 4.231 = 6.157$ ft-lb-sec^2

$\Sigma M_O = I_O\alpha$; $40(15/12) - 25 = 6.157\alpha$, $\underline{\alpha = 4.06 \text{ rad/sec}^2}$

6/52 Power unit C:

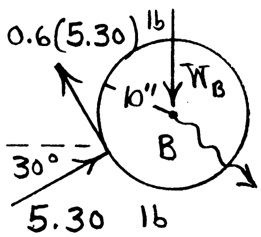


$\xrightarrow{+} \Sigma F = 0$:

$3 - N\cos 30° + 0.6 N \sin 30° = 0$

$N = 5.30$ lb

Wheel B: $\overset{+}{\circlearrowright} \Sigma M_B = I_B\alpha$: $0.6(5.30)\left(\frac{10}{12}\right) = \frac{50}{32.2}\left(\frac{8}{12}\right)^2 \alpha$



$\underline{\alpha = 3.84 \text{ rad/sec}^2}$

Steady-state speed:

$r_A\omega_A = r_B\omega_B$

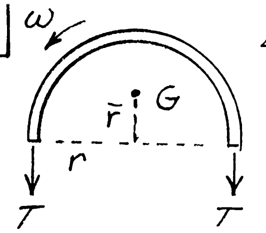
$\omega_B = \frac{r_A\omega_A}{r_B} = \frac{8\left[1600 \frac{2\pi}{60}\right]}{10}$

$= 134.0$ rad/sec

$\omega_B = \cancel{\omega_{B0}}^{\,0} + \alpha t$: $t = \frac{\omega_B}{\alpha} = \frac{134.0}{3.84} = \underline{34.9 \text{ sec}}$

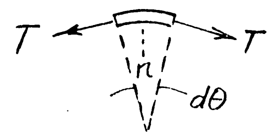
6/53

$\bar{r} = \frac{2r}{\pi}$



$\Sigma F_n = m\bar{r}\omega^2; \quad 2T = \rho\pi r \, \frac{2r}{\pi}\omega^2$

$\underline{T = \rho r^2 \omega^2}$

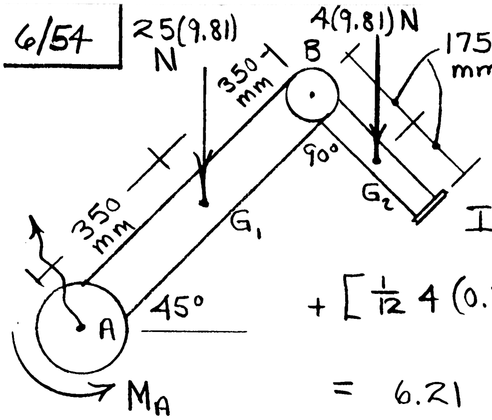


$\Sigma F_n = mr\omega^2; \quad 2T\sin\frac{d\theta}{2} = \rho r d\theta (r\omega^2)$

$2T\frac{d\theta}{2} = \rho r^2\omega^2 d\theta$

$\underline{T = \rho r^2 \omega^2}$

6/54

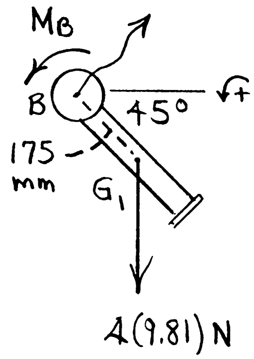


$$I_B = \frac{1}{3}(4)(0.350)^2 = 0.1633 \text{ kg}\cdot\text{m}^2$$

$$I_A = \frac{1}{3}(25)(0.7)^2 + \left[\frac{1}{12}4(0.350)^2 + 4(0.7^2 + 0.175^2)\right] = 6.21 \text{ kg}\cdot\text{m}^2$$

$$\circlearrowleft + \Sigma M_A = I_A\alpha: \quad M_A - 25(9.81)(0.350\cos 45^\circ) - 4(9.81)(0.700 + 0.175)\cos 45^\circ = 6.21(4)$$

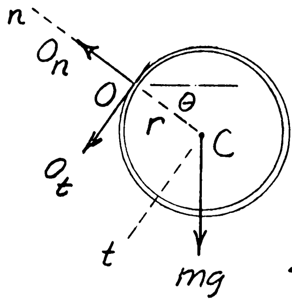
$$\underline{M_A = 109.8 \text{ N}\cdot\text{m}}$$



$$\circlearrowleft + \Sigma M_B = I_B\alpha: \quad M_B - 4(9.81)(0.175\cos 45^\circ) = 0.1633(4)$$

$$\underline{M_B = 5.51 \text{ N}\cdot\text{m}}$$

6/55



$\Sigma M_O = I_O\alpha$; $mgr\cos\theta = 2mr^2\alpha$

$\alpha = \dfrac{g}{2r}\cos\theta$

$\omega\, d\omega = \alpha\, d\theta$; $\displaystyle\int_0^\omega \omega\, d\omega = \frac{g}{2r}\int_0^\theta \cos\theta\, d\theta$

$\omega^2 = g/r \sin\theta$

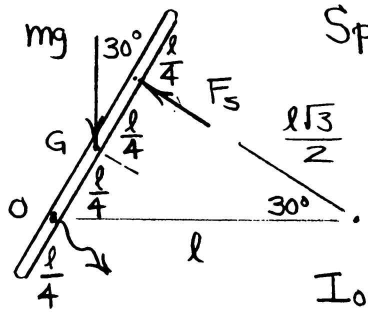
$\Sigma F_t = m\bar{r}\alpha$; $mg\cos\theta + O_t = mr\left(\dfrac{g}{2r}\cos\theta\right)$

$\underline{O_t = -\dfrac{mg}{2}\cos\theta}$

$\Sigma F_n = m\bar{r}\omega^2$; $O_n - mg\sin\theta = mr\left(\dfrac{g}{r}\sin\theta\right)$

$\underline{O_n = 2mg\sin\theta}$

6/56 Undeflected spring length is $\frac{\sqrt{5}}{2}l$



Spring length @ 30° is $\frac{l\sqrt{3}}{2}$

So $F_s = k\delta = kl\left(\frac{\sqrt{5}}{2} - \frac{\sqrt{3}}{2}\right)$

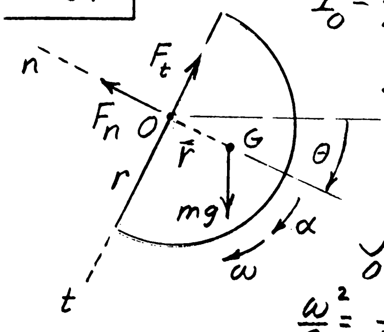
(spring in compression)

$$I_O = I_G + m\left(\frac{l}{4}\right)^2 = \frac{1}{12}ml^2 + \frac{1}{16}ml^2$$
$$= \frac{7}{48}ml^2$$

$$\curvearrowright \Sigma M_O = I_O\alpha: \quad mg\frac{l}{4}\sin 30° - kl\left(\frac{\sqrt{5}}{2} - \frac{\sqrt{3}}{2}\right)\frac{l}{2} = \frac{7}{48}ml^2\alpha$$

$$\underline{\alpha = \frac{6}{7}\frac{g}{l} - \frac{12}{7}\frac{k}{m}(\sqrt{5} - \sqrt{3})}$$

6/57



$$I_O = \frac{1}{2}mr^2\ ; \quad \bar{r} = \frac{4r}{3\pi}$$

$$\Sigma M_O = I_O\alpha\ ; \quad mg\bar{r}\cos\theta = I_O\alpha$$

$$\alpha = mg\cos\theta\,\frac{\bar{r}}{I_O} = \frac{8}{3\pi}\,\frac{g}{r}\cos\theta$$

$$\int_0^\omega \omega\,d\omega = \int_0^\theta \alpha\,d\theta$$

$$\frac{\omega^2}{2} = \frac{8}{3\pi}\,\frac{g}{r}\sin\theta\Big]_0^\theta, \quad \omega^2 = \frac{16}{3\pi}\,\frac{g}{r}\sin\theta$$

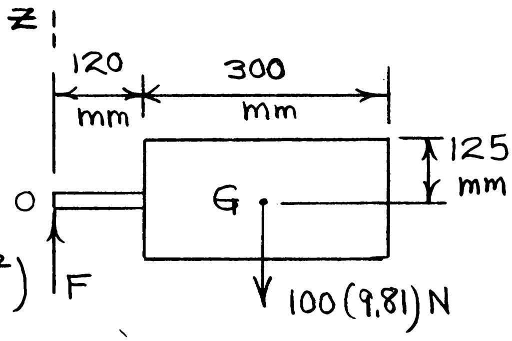
$$\Sigma F_n = m\bar{r}\omega^2; \quad F_n - mg\sin\theta = m\,\frac{4r}{3\pi}\,\frac{16}{3\pi}\,\frac{g}{r}\sin\theta$$

$$F_n = \left(\frac{64}{9\pi^2}+1\right)mg\sin\theta \;=\; \underline{1.721\,mg\sin\theta}$$

$$\Sigma F_t = m\bar{r}\alpha; \quad mg\cos\theta - F_t = m\,\frac{4r}{3\pi}\,\frac{8}{3\pi}\,\frac{g}{r}\cos\theta$$

$$F_t = \left(1-\frac{32}{9\pi^2}\right)mg\cos\theta \;=\; \underline{0.640\,mg\cos\theta}$$

6/58



From Table D/4,

$I_G = \frac{1}{4} m r^2 + \frac{1}{12} m l^2$

$= \frac{1}{4} 100 \left(0.125^2 + \frac{1}{3} 0.3^2\right)$

$= 1.141 \text{ kg}\cdot\text{m}^2$

$I_O = I_G + md^2 = 1.141 + 100(0.120 + 0.150)^2 = 8.43 \text{ kg}\cdot\text{m}^2$

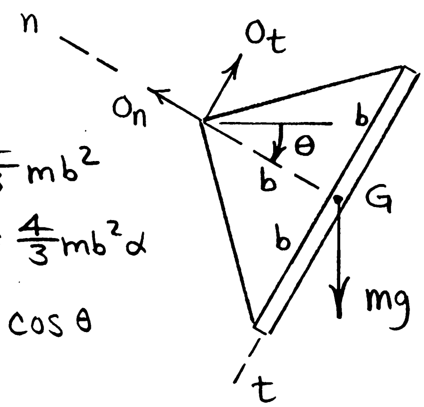
$\Sigma M_O = I_O \alpha$: $981(0.120 + 0.150) = 8.43\alpha$

$\underline{\alpha = 31.4 \text{ rad/s}^2}$

$\Sigma F_z = m\bar{a}_z$: $F - 981 = 100(-0.27)(31.4)$

$\underline{F = 132.7 \text{ N}}$

6/59



$$I_o = \bar{I} + mb^2$$

$$= \frac{1}{12}m(2b)^2 + mb^2 = \frac{4}{3}mb^2$$

$$\Sigma M_o = I_o\alpha : \quad mgb\cos\theta = \frac{4}{3}mb^2\alpha$$

$$\alpha = \frac{3}{4}\frac{g}{b}\cos\theta$$

$$\int \omega\, d\omega = \int \alpha\, d\theta :$$

$$\int_0^\omega \omega\, d\omega = \int_0^\theta \frac{3}{4}\frac{g}{b}\cos\theta\, d\theta, \quad \omega^2 = \frac{3g}{2b}\sin\theta$$

$$\Sigma F_n = m\bar{a}_n : \quad O_n - mg\sin\theta = mb\omega^2$$

$$O_n = mg\sin\theta + mb\,\frac{3g}{2b}\sin\theta = \frac{5}{2}mg\sin\theta$$

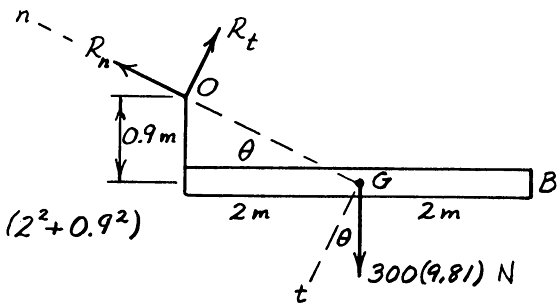
$$\Sigma F_t = m\bar{a}_t : \quad mg\cos\theta - O_t = mb\,\frac{3}{4}\frac{g}{b}\cos\theta$$

$$O_t = \frac{1}{4}mg\cos\theta$$

$$O = \sqrt{O_n^2 + O_t^2} = \underline{\frac{1}{4}mg\sqrt{\cos^2\theta + 100\sin^2\theta}}$$

6/60

$\theta = \tan^{-1}\dfrac{0.9}{2} = 24.2°$



$I_O = I_G + mr^2$

$= \frac{1}{12}(300)(4^2) + 300(2^2 + 0.9^2)$

$= 1843\ \text{kg}\cdot\text{m}^2$

$\curvearrowright \Sigma M_O = I_O\alpha$: $300(9.81)(2) = 1843\,\alpha$, $\alpha = 3.19\ \text{rad/s}^2$

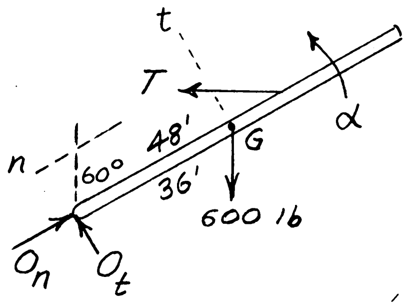
$\Sigma F_t = m\bar{r}\alpha$: $300(9.81)\cos 24.2° - R_t = 300(\sqrt{2^2+0.9^2})\,3.19$,

$R_t = 582\ \text{N}$

$\Sigma F_n = m\bar{r}\omega^2$: $R_n - 300(9.81)\sin 24.2° = 300(0)$, $R_n = 1208\ \text{N}$

$R = \sqrt{582^2 + 1208^2} = \underline{1341\ \text{N}}$

6/61 $M = \frac{T}{2} r$, $T = \frac{2(900)}{2} = 900$ lb



$\Sigma M_O = I_O \alpha$;

$900(48\cos 60°) - 600(36\sin 60°)$

$= \frac{1}{3}\frac{600}{32.2}\overline{72}^2\alpha$

$\underline{\alpha = 0.090 \text{ rad/sec}^2}$

$\Sigma F_t = m\bar{a}_t$; $O_t + 900\cos 60° - 600\sin 60° = \frac{600}{32.2}(36 \times 0.090)$

$O_t = 129.9$ lb

$\Sigma F_n = m\bar{a}_n = 0$; $900\sin 60° + 600\cos 60° - O_n = 0$

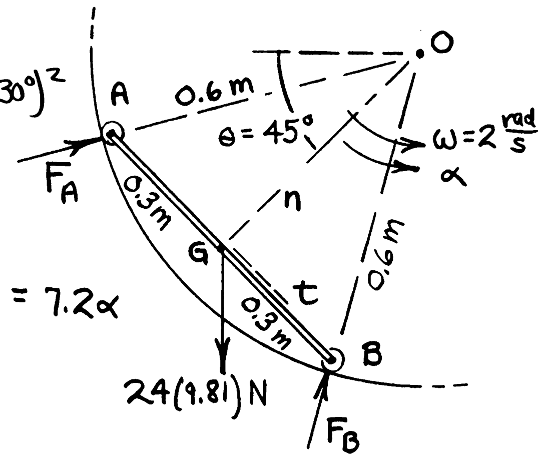
$O_n = 1079.4$ lb

$O = \sqrt{\overline{129.9}^2 + \overline{1079.4}^2} = \underline{1087 \text{ lb}}$

6/62 $\quad I_O = \bar{I} + md^2$

$= \frac{1}{12}(24)(0.6)^2 + 24(0.6\cos 30°)^2$

$= 7.2 \text{ kg}\cdot\text{m}^2$



$\circlearrowright \Sigma M_O = I_O\alpha$:

$24(9.81)\left(0.6\frac{\sqrt{3}}{2}\cos 45°\right) = 7.2\alpha$

$\alpha = 12.01 \text{ rad/s}^2$

$\Sigma F_n = m\bar{a}_n = m\bar{r}\omega^2$:

$(F_A + F_B)\cos 30° - 24(9.81)\cos 45° = 24(0.6\cos 30°)2^2$

$F_A + F_B = 250 \text{ N}$

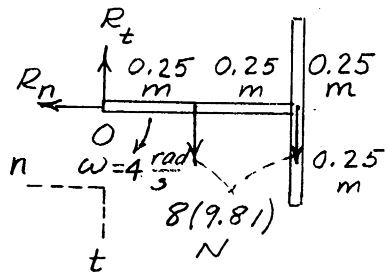
$\Sigma F_t = m\bar{a}_t = m\bar{r}\alpha$:

$(F_A - F_B)\cos 60° + 24(9.81)\cos 45° = 24(0.6\cos 30°)12.01$

$F_A - F_B = -33.3 \text{ N}$

Solve to obtain $\underline{F_A = 108.3 \text{ N}}$, $\underline{F_B = 141.6 \text{ N}}$

6/63 $I_O = \frac{1}{3}ml^2 + \left(\frac{1}{12}ml^2 + ml^2\right) = \frac{17}{12}(8)(0.5)^2$ $kg \cdot m^2$



$\Sigma M_O = I_O \alpha$

$8(9.81)(0.50 + 0.25) = \frac{17}{12}(8)(0.5)^2 \alpha$

$\alpha = 20.8 \ rad/s^2$

$\Sigma F_t = \Sigma m\bar{a}_t$

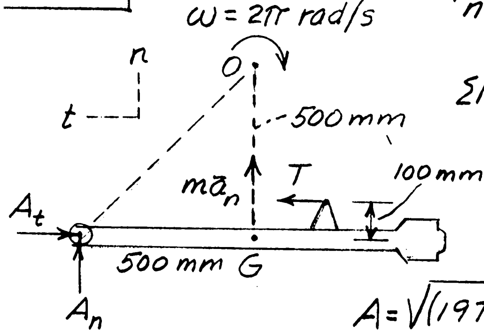
$2(8)(9.81) - R_t = 8(0.25)(20.8) + 8(0.50)(20.8)$

$R_t = 32.3 \ N$

$\Sigma F_n = \Sigma m\bar{a}_n$; $R_n = 8(0.25)4^2 + 8(0.50)4^2 = 96.0 N$

$R = \sqrt{32.3^2 + 96.0^2} = \underline{101.3 N}$

6/64



$\Sigma F_n = m\bar{r}\omega^2;\quad A_n = 10(0.500)(2\pi)^2$
$= 197.4\ N$

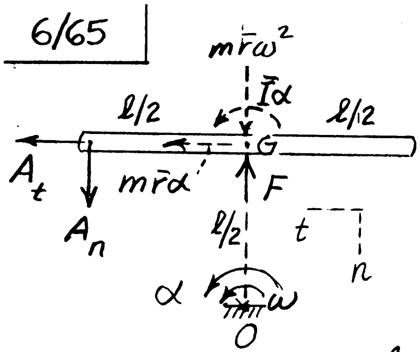
$\Sigma M_A = m\bar{a}d;\quad 0.100T = 197.4(0.500)$
$\underline{T = 987\ N}$

$\Sigma F_t = m\bar{r}\alpha = 0;\quad A_t = 987\ N$

$A = \sqrt{(197.4)^2 + (987)^2} = 1007\ N$

or $\underline{A = 1.007\ kN}$

6/65



$\bar{I} = \frac{1}{12} m l^2$

$\Sigma M_A = \bar{I}\alpha + \Sigma m\bar{a}d$

$F\frac{l}{2} = \frac{1}{12} m l^2 \alpha + 0 - m\frac{l}{2}\omega^2 \frac{l}{2}$

$F = m\frac{l}{2}\left(\frac{\alpha}{3} - \omega^2\right)$

$\Sigma F_t = m\bar{r}\alpha$; $A_t = m\frac{l}{2}\alpha$

$\Sigma F_n = m\bar{r}\omega^2$; $A_n - F = m\frac{l}{2}\omega^2$, $A_n = m\frac{l}{2}\left(\frac{\alpha}{3} - \omega^2\right) + m\frac{l}{2}\omega^2$

$= \underline{\frac{ml\alpha}{6}}$ independent of ω

(a) $\omega = 0$, $\underline{F = ml\alpha/6}$

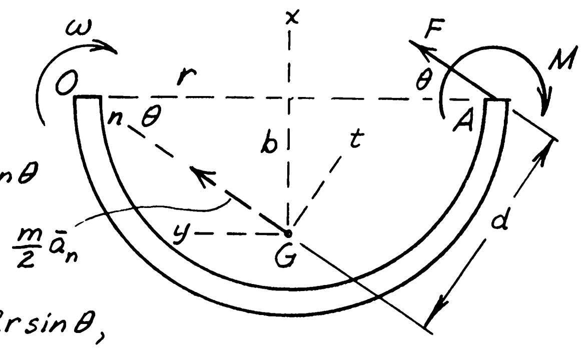
(b) $A = \sqrt{A_n^2 + A_t^2} = \frac{ml\alpha}{2}\sqrt{1^2 + (1/3)^2} = \underline{\frac{\sqrt{10}}{6} ml\alpha}$

(c) $F = 0$ when $\alpha/3 = \omega^2$ or $\underline{\omega = \sqrt{\alpha/3}}$

6/66 | $b = 2r/\pi$,

$\frac{m}{2}\bar{a}_n = \frac{m}{2}\overline{OG}\,\omega^2$

$\overline{OG} = r/\cos\theta$, $d = 2r\sin\theta$



$\Sigma M_A = \frac{m}{2}\bar{a}_n d$:

$$M = \frac{m}{2}\frac{r}{\cos\theta}\omega^2 \times 2r\sin\theta,$$

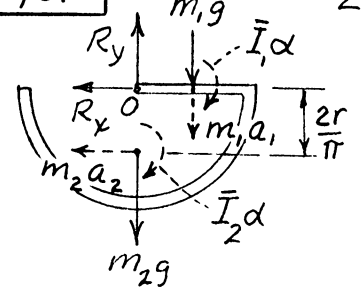
$$M = mr^2\omega^2\tan\theta$$

but $\tan\theta = b/r = \frac{2r}{\pi r} = \frac{2}{\pi}$ so $\underline{M = \frac{2}{\pi}mr^2\omega^2}$

$\Sigma F_y = \frac{m}{2}\bar{a}_y$: $V = F\cos\theta = \frac{m}{2}\overline{OG}\,\omega^2\cos\theta = \frac{m}{2}\frac{r}{\cos\theta}\omega^2\cos\theta$

$$\underline{V = \frac{m}{2}r\omega^2}$$

6/67



$r = 0.20$ m

$m_1 = 0.20(0.6) = 0.12$ kg

$m_2 = \pi(0.20)(0.6) = 0.377$ kg

$\Sigma M_O = \Sigma \bar{I}\alpha + \Sigma m\bar{a}d = \Sigma I_O\alpha$

$I_{1_O} = \frac{1}{3}m_1r^2,\ I_{2_O} = m_2r^2;$ Thus

$0.12(9.81)0.1 = \left(\frac{0.12}{3} + 0.377\right)(0.2)^2\alpha$

$\alpha = 7.06$ rad/s^2

$\Sigma F_x = \Sigma m\bar{a}_x;\ R_x = m_2a_2$

$= m_2\frac{2r}{\pi}\alpha$

$= 0.377\frac{2(0.2)}{\pi}7.06$

$= \underline{0.339\text{ N}}$

$\Sigma F_y = \Sigma m\bar{a}_y;\ (0.12+0.377)9.81 - R_y = 0.12(0.1)7.06$

$\underline{R_y = 4.79\text{ N}}$

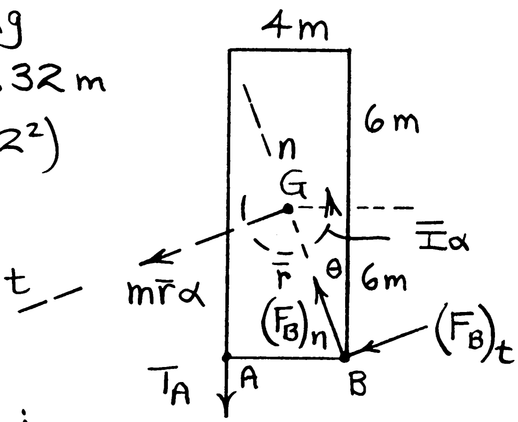
$\underline{6/68}$ $m = 3000\ kg$

$\bar{r} = \sqrt{6^2+2^2} = 6.32\ m$

$\bar{I} = \frac{1}{12}(3000)(4^2+12^2)$

$= 40\,000\ kg \cdot m^2$

$\theta = \tan^{-1}\frac{2}{6} = 18.43°$



$\circlearrowleft + \ \Sigma M_B = \bar{I}\alpha + m\bar{a}\bar{r}$:

$4T_A = 40\,000\alpha + 3000(6.32\alpha)(6.32)$

$\underline{\alpha = 0.05\ rad/s^2}$

$\Sigma F_n = m\bar{a}_n = 0 : (F_B)_n - 2000\cos 18.43° = 0$

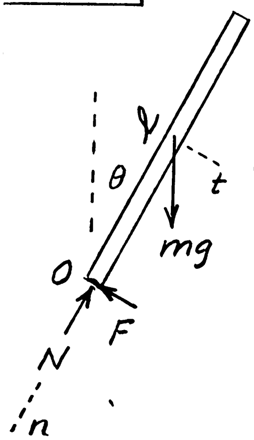
$(F_B)_n = 1897\ N$

$\Sigma F_t = m\bar{a}_t : \ 2000\sin 18.43° + (F_B)_t = 3000(6.32)(0.05)$

$(F_B)_t = 316\ N$

$F_B = \sqrt{1897^2 + 316^2} = \underline{1924\ N}$

6/69



$\Sigma M_O = I_O\alpha;\ mg\frac{l}{2}\sin\theta = \frac{1}{3}ml^2\alpha$

$$\alpha = \frac{3g}{2l}\sin\theta$$

$\int_0^\omega \omega\,d\omega = \int_0^\theta \alpha\,d\theta,\ \omega^2 = \frac{3g}{l}(-\cos\theta)\Big|_0^\theta$

$$\omega^2 = \frac{3g}{l}(1-\cos\theta)$$

$\Sigma F_n = m\bar{a}_n;\ mg\cos\theta - N = m\frac{l}{2}\omega^2$

$N = mg\cos\theta - m\frac{l}{2}\frac{3g}{l}(1-\cos\theta)$

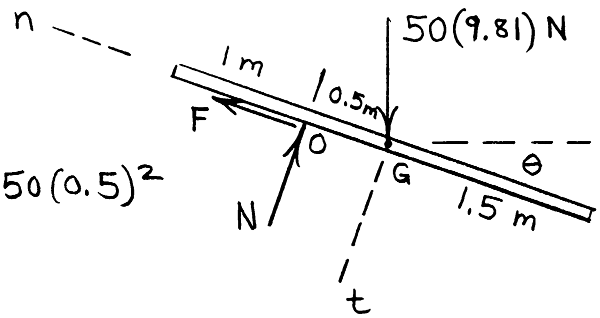
$= mg\left[\cos\theta - \frac{3}{2}(1-\cos\theta)\right] = \frac{mg}{2}(5\cos\theta - 3)$

$\Sigma F_t = m\bar{a}_t;\ mg\sin\theta - F = m\frac{l}{2}\frac{3g}{2l}\sin\theta,\ F = \frac{mg}{4}\sin\theta$

(a) Slips at $\theta = 30°$, $\mu_s = F/N = \dfrac{mg\sin 30°/4}{\frac{mg}{2}(5\cos 30° - 3)} = \underline{0.188}$

(b) No slip: $N = 0$ when $\cos\theta = 3/5$, $\underline{\theta = 53.1°}$

6/70



$$I_o = \bar{I} + mr^2$$
$$= \frac{1}{12} 50(3)^2 + 50(0.5)^2$$
$$= 50 \text{ kg}\cdot\text{m}^2$$

$$\Sigma M_o = I_o \alpha : \quad 50(9.81)(0.5 \cos\theta) = 50\alpha$$

$$\alpha = 4.905 \cos\theta = \omega \frac{d\omega}{d\theta}, \quad \int_0^\omega \omega\, d\omega = \int_0^\theta 4.905 \cos\theta\, d\theta$$

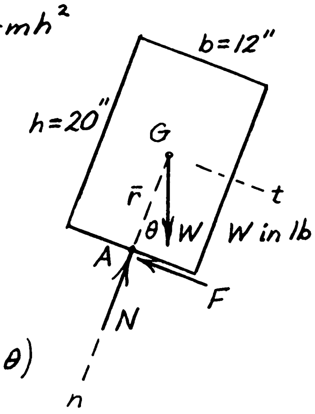
$$\omega^2 = 9.81 \sin\theta$$

$$\begin{cases} \Sigma F_t = m\bar{a}_t : & 50(9.81)\cos\theta - N = 50(0.5)(4.905\cos\theta) \\ \Sigma F_n = ma_n : & F - 50(9.81)\sin\theta = 50(0.5)(9.81\sin\theta) \end{cases}$$

Slipping occurs when $F = 0.30N$

2^{nd} eq: $\quad 0.3N = 75(9.81)\sin\theta$
1^{st} eq: $\quad N = \frac{75}{2}(9.81)\cos\theta$
$\Big\}$ Divide to obtain $\underline{\theta = 8.53°}$

▸6/71 Using Table D/4 $I_A = \frac{1}{12}mb^2 + \frac{1}{3}mh^2$



$$I_A = \frac{W}{3g}\left(\frac{[12/12]^2}{4} + \left[\frac{20}{12}\right]^2\right) = \frac{1}{3}\frac{W}{32.2} \times 3.03$$
$$= 0.0313\,W \text{ lb-ft-sec}^2$$

$\Sigma M_A = I_A\alpha$: $W\left(\frac{10}{12}\right)\sin\theta = 0.0313\,W\alpha$

$$\alpha = 26.6\sin\theta \text{ rad/sec}^2$$

$\Sigma F_t = m\bar{r}\alpha$: $W\sin\theta - F = \frac{W}{g}\frac{10}{12}\alpha$

$$F = W\left(\sin\theta - \frac{10}{12\times 32.2}\times 26.6\sin\theta\right)$$
$$= 0.312\,W\sin\theta$$

$\int_0^\omega \omega\, d\omega = \int_0^\theta \alpha\, d\theta$: $\frac{\omega^2}{2} = 26.6\int_0^\theta \sin\theta\, d\theta$, $\omega^2 = 53.2(1-\cos\theta)$

$\Sigma F_n = m\bar{r}\omega^2$: $W\cos\theta - N = \frac{W}{g}\frac{10}{12}53.2(1-\cos\theta)$

$$N = W(2.38\cos\theta - 1.376)$$

When $\theta = 30°$, $F = 0.312\,W\sin 30° = 0.1560\,W$

$$N = W(2.38\cos 30° - 1.376) = 0.682\,W$$

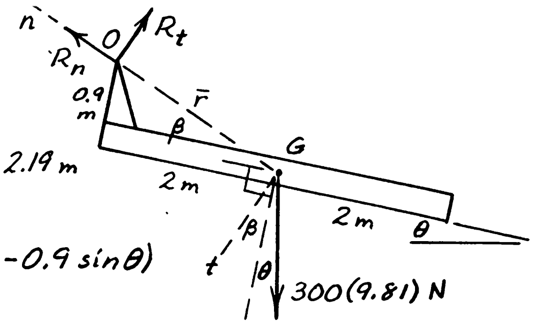
so $\mu_s = F/N = 0.1560\,W/0.682\,W$, $\underline{\mu_s = 0.229}$

(b) Block leaves corner when $N=0$

so $2.38\cos\theta - 1.376 = 0$

$$\theta = \cos^{-1}\frac{1.376}{2.38}, \quad \underline{\theta = 54.6°}$$

▶6/72



From solution to Prob. 6/60

$\beta = 24.2°$, $I_O = 1843\ \text{kg}\cdot\text{m}^2$, $\bar{r} = 2.19$ m

$$\circlearrowleft \Sigma M_O = I_O\alpha:\ 300(9.81)(2\cos\theta - 0.9\sin\theta) = 1843\alpha$$

$$\alpha = 1.597(2\cos\theta - 0.9\sin\theta)$$

$$\int_0^\omega \omega\, d\omega = \int_0^\theta \alpha\, d\theta:\ \frac{\omega^2}{2} = 1.597\int_0^\theta (2\cos\theta - 0.9\sin\theta)\,d\theta$$

$$\omega^2 = 3.19(2\sin\theta - 0.9[1-\cos\theta])$$

$$\frac{d(\omega^2)}{d\theta} = 3.19[2\cos\theta - 0.9\sin\theta] = 0 \text{ for max } \omega,\ \theta = \tan^{-1}\frac{2}{0.9} = 65.8°$$

$$\&\ \omega_{max}^2 = 3.19(2\sin 65.8° - 0.9[1-\cos 65.8°]) = 4.13\ (\text{rad/s})^2$$

$\alpha = 0$ for $\omega = \omega_{max}$ $\qquad$ $\underline{\omega_{max} = 2.03\ \text{rad/s}}$

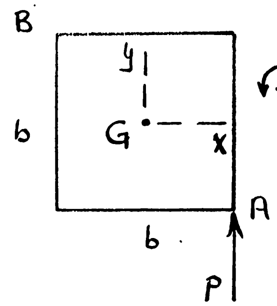
$$\Sigma F_n = m\bar{r}\omega^2:\ R_n - 300(9.81)\sin(\underbrace{24.2° + 65.8°}_{90°}) = 300(2.19)4.13$$

$$R_n = 5660\ \text{N}$$

$$\Sigma F_t = m\bar{r}\alpha:\ 300(9.81)\cos(\underbrace{24.2° + 65.8°}_{90°}) - R_t = 300(0),\ R_t = 0$$

$$\underline{R = R_n = 5.66\ \text{kN}}$$

6/73



$$\Sigma F_x = m\bar{a}_x : \quad \bar{a}_x = 0$$

$$\Sigma F_y = m\bar{a}_y : \quad P = m\bar{a}_y \,, \quad \bar{a}_y = \frac{P}{m}$$

$$\circlearrowleft + \; \Sigma M_G = \bar{I}\alpha : \quad P\frac{b}{2} = \frac{1}{6}mb^2\alpha$$

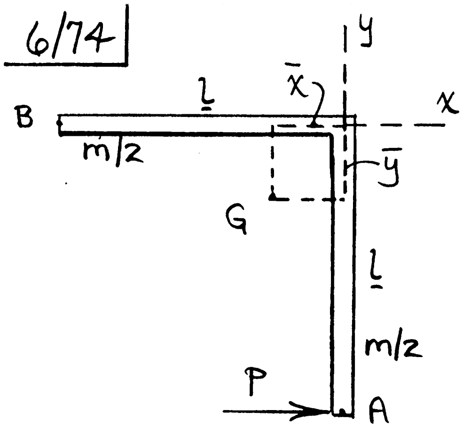
$$\alpha = \frac{3P}{mb}$$

$$\mathbf{a}_B = \mathbf{a}_G + \mathbf{a}_{B/G} = \bar{\mathbf{a}} + \boldsymbol{\alpha} \times \mathbf{r}_{B/G} + \cancelto{0}{\boldsymbol{\omega}} \times (\cancelto{0}{\boldsymbol{\omega}} \times \mathbf{r}_{B/G})$$

$$= \frac{P}{m}\mathbf{j} + \frac{3P}{mb}\mathbf{k} \times \left[-\frac{b}{2}\mathbf{i} + \frac{b}{2}\mathbf{j}\right]$$

$$= \underline{\frac{P}{2m}\left[-3\mathbf{i} - \mathbf{j}\right]}$$

6/74



$$\bar{x} = \frac{\Sigma \bar{x} m}{\Sigma m} = \frac{-\frac{m}{2}\left(\frac{l}{2}\right)}{m}$$
$$= -\,l/4$$

Similarly, $\bar{y} = -\frac{l}{4}$

$$I_G = 2\left[\frac{1}{12}\frac{m}{2}l^2 + \frac{m}{2}\left(\left(\frac{l}{4}\right)^2 + \left(\frac{l}{4}\right)^2\right)\right]$$
$$= \frac{5}{24} m l^2$$

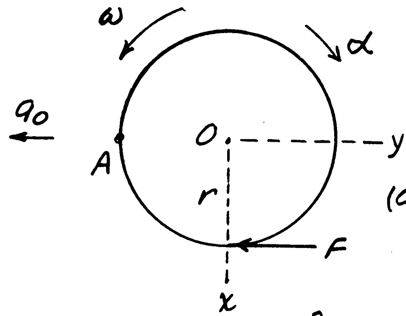
$$\Sigma F_x = m\bar{a}_x : \quad P = m\bar{a}_x \,, \quad a_x = \frac{P}{m}$$

$$\Sigma F_y = m\bar{a}_y : \quad \bar{a}_y = 0$$

$$\circlearrowleft + \Sigma M_G = \bar{I}\alpha : \quad P\left(\frac{3l}{4}\right) = \frac{5}{24} m l^2 \alpha, \quad \alpha = \frac{18}{5}\frac{P}{ml}$$

$$\underline{a}_A = \underline{a}_G + \underline{a}_{A/G} = \underline{a}_G + \underline{\alpha} \times \underline{r}_{A/G} - \cancelto{0}{\omega^2} \underline{r}_{A/G}$$
$$= \frac{P}{m}\underline{i} + \frac{18P}{5ml}\underline{k} \times \left[\frac{l}{4}\underline{i} - \frac{3l}{4}\underline{j}\right]$$
$$= \underline{\frac{P}{10m}\left[37\underline{i} + 9\underline{j}\right]}$$

6/75 $\Sigma M_O = I_O\alpha$; $Fr = mk^2\alpha$, $\alpha = \dfrac{Fr}{mk^2}$



$\Sigma F_y = ma_y$; $-F = m(-a_O)$

$a_O = F/m$

$\underline{a}_A = \underline{a}_O + \underline{a}_{A/O}$

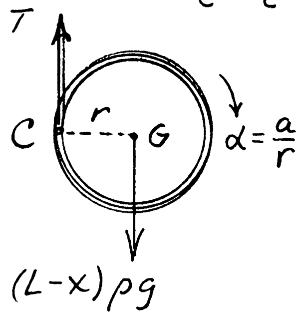
$(a_{A/O})_n = r\omega^2\underline{j}$, $(a_{A/O})_t = -r\alpha\,\underline{i} = -\dfrac{Fr^2}{mk^2}\underline{i}$

$\underline{a}_O = -\dfrac{F}{m}\underline{j}$

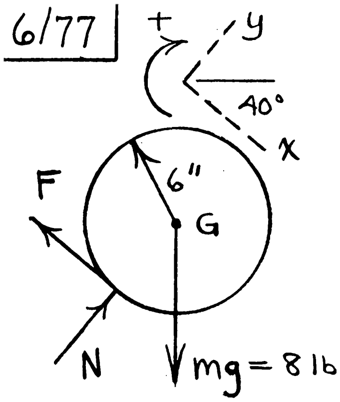
so $\underline{\underline{a}_A = -\dfrac{Fr^2}{mk^2}\underline{i} - \left(\dfrac{F}{m} - r\omega^2\right)\underline{j}}$

6/76 $\Sigma M_C = I_C \alpha$; $(L-x)\rho g r = 2(L-x)\rho r^2 \frac{a}{r}$

$\underline{a = g/2 \text{ constant}}$



6/77



$$\begin{cases} mg = 8 \text{ lb}, & \bar{I} = \frac{1}{2}mr^2 \\ \mu_s = 0.3, & \mu_k = 0.20 \\ \theta = 40° & \end{cases}$$

$\Sigma F_x = m\bar{a}_x$: $-F + 8\sin 40° = \frac{8}{32.2}a$ (1)

$\Sigma F_y = 0$: $N - 8\cos 40° = 0$ (2)

$\Sigma M_G = \bar{I}\alpha$: $F\left(\frac{6}{12}\right) = \frac{1}{2}\frac{8}{32.2}\left(\frac{6}{12}\right)^2\alpha$ (3)

Assume rolling with no slip : $a = \frac{6}{12}\alpha$ (4)

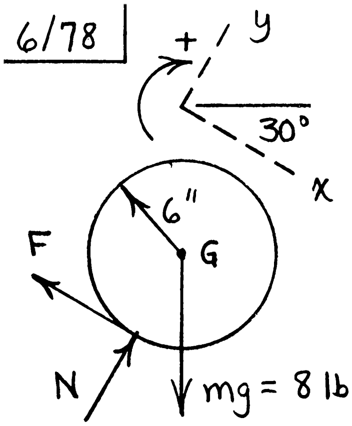
Solution of (1) - (4) : $\underline{F = 1.714 \text{ lb}}$ $\underline{a = 13.80 \frac{\text{ft}}{\text{sec}^2}}$

$N = 6.13$ lb $\alpha = 27.6 \frac{\text{rad}}{\text{sec}^2}$

$F_{max} = \mu_s N = 0.3\,(6.13) = 1.839 \text{ lb} > F$

Assumption valid.

6/78



$$\begin{cases} mg = 8 \text{ lb}, \quad \bar{I} = \frac{1}{2} m r^2 \\ \mu_s = 0.15, \quad \mu_k = 0.10 \\ \theta = 30° \end{cases}$$

$$\Sigma F_x = m\bar{a}_x : \quad -F + 8 \sin 30° = \frac{8}{32.2} a \qquad (1)$$

$$\Sigma F_y = 0 : \quad N - 8 \cos 30° = 0 \qquad (2)$$

$$\Sigma M_G = \bar{I}\alpha : \quad F\left(\frac{6}{12}\right) = \frac{1}{2} \frac{8}{32.2} \left(\frac{6}{12}\right)^2 \alpha \qquad (3)$$

Assume rolling with no slip: $a = \frac{6}{12}\alpha$ (4)

Solution of (1) - (4): $F = 1.333$ lb, $a = 10.73 \frac{ft}{sec^2}$

$N = 6.93$ lb, $\alpha = 21.5 \frac{rad}{sec^2}$

$F_{max} = \mu_s N = 0.15(6.93) = 1.039 \text{ lb} < F \Rightarrow$ Slips

$F = \mu_k N = 0.10(6.93) = \underline{0.693 \text{ lb}}$

From Eqs. (1) & (3): $\underline{\underline{a = 13.31 \text{ ft/sec}^2}}$, $\alpha = 11.15 \frac{rad}{sec^2}$

6/79

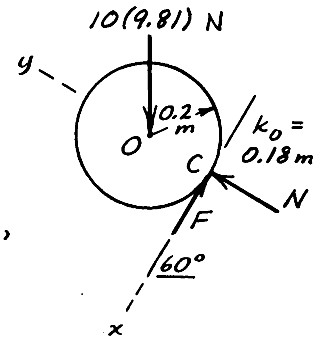
$\Sigma F_y = 0: \; N - 98.1 \cos 60° = 0$

$N = 49.0 \text{ N}$

$F = \mu_k N = 0.30(49.0) = 14.72 \text{ N}$

$\Sigma F_x = m\bar{a}_x: \; 10(9.81)\sin 60° - 14.72 = 10 a_o,$

$\underline{a_o = 7.02 \text{ m/s}^2}$



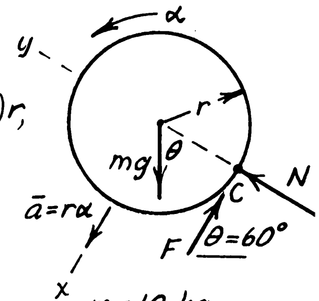
$\Sigma M_o = I_o \alpha: \; 14.72(0.2) = 10(0.18^2)\alpha,$

$\underline{\alpha = 9.08 \text{ rad/s}^2}$

6/80

$$\Sigma M_C = \bar{I}\alpha + m\bar{a}d: \; mgr\sin\theta = mk_o^2\alpha + m(r\alpha)r,$$

$$\alpha = \frac{gr\sin\theta}{k_o^2 + r^2} = \frac{9.81(0.2)\sin 60^\circ}{0.180^2 + 0.2^2} = 23.4 \text{ rad/s}^2$$



$m = 10$ kg

$mg = 10(9.81) = 98.1$ N

$r = 0.2$ m, $k_o = 0.180$ m

$$\Sigma F_x = m\bar{a}_x: \; mg\sin\theta - F = mr\alpha,$$

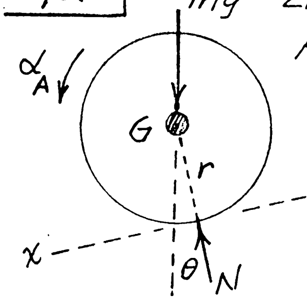
$$F = 98.1\sin 60^\circ - 10(0.2)(23.4) = 38.0 \text{ N}$$

$$\Sigma F_y = 0: \; N - mg\cos\theta = 0,$$

$$N = 98.1\cos 60^\circ = 49.0 \text{ N}$$

$$\mu_s = \frac{F}{N} = \underline{0.775 = (\mu_s)_{min}}$$

6/81

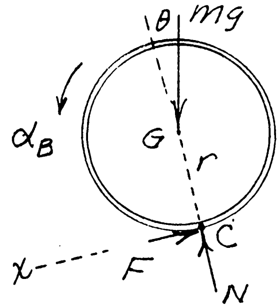


$\Sigma M_G = \bar{I}\alpha$, but $\bar{I} = 0$ so $\Sigma \bar{M} = 0$

Hence no friction force & $\underline{\mu_s = 0}$

$\Sigma F_x = ma_x$; $mg\sin\theta = mr\alpha_A$

$$\underline{\alpha_A = \frac{g}{r}\sin\theta}$$



$\Sigma M_C = I_C\alpha$; $mgr\sin\theta = 2mr^2\alpha_B$

$$\underline{\alpha_B = \frac{g}{2r}\sin\theta}$$

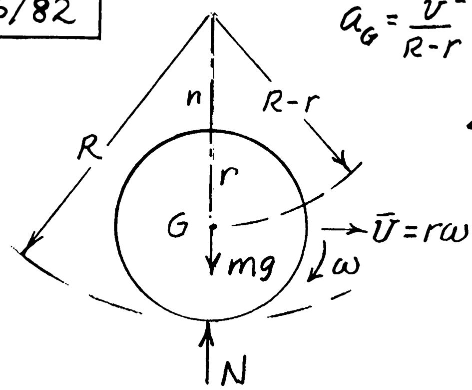
$\Sigma \bar{M} = \bar{I}\alpha$; $Fr = mr^2\frac{g}{2r}\sin\theta$

$$F = \frac{1}{2}mg\sin\theta$$

$\mu_s = \frac{F}{N} = \frac{1}{2}mg\sin\theta / mg\cos\theta$

$$\underline{\mu_s = \frac{1}{2}\tan\theta}$$

6/82



$$a_G = \frac{\bar{v}^2}{R-r} = \frac{r^2\omega^2}{R-r} = \bar{a}_n$$

$$\Sigma F_n = m\bar{a}_n$$

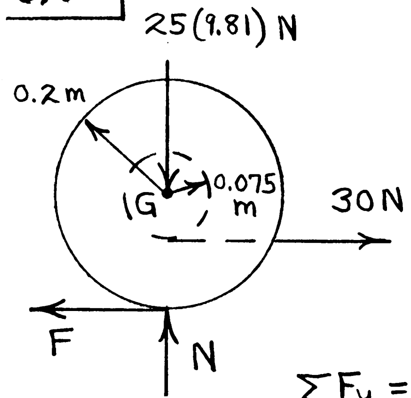
$$N - mg = m\frac{r^2\omega^2}{R-r}$$

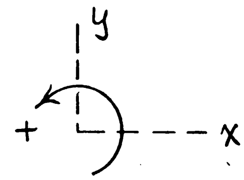
$$\underline{N = m\left(g + \frac{r^2\omega^2}{R-r}\right)}$$

6/83

$\bar{k} = 0.175$ m

$\mu_s = 0.1, \quad \mu_k = 0.08$





$\Sigma F_y = 0 \Rightarrow N = 25(9.81) = 245$ N

$\Sigma F_x = m\bar{a}_x : \quad 30 - F = 25a \qquad (1)$

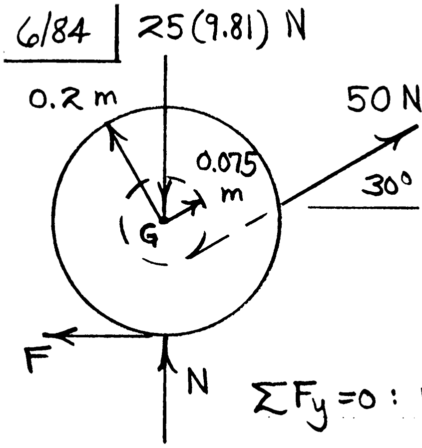
$\Sigma M_G = \bar{I}\alpha : \quad 30(0.075) - F(0.2) = 25(0.175)^2\alpha \qquad (2)$

Assume rolling with no slip : $\quad a = -r\alpha \qquad (3)$

Solution of Eqs. (1)-(3) : $\begin{cases} a = 0.425 \text{ m/s}^2 \\ \alpha = -2.12 \text{ rad/s}^2 \\ \underline{F = 19.38 \text{ N}} \end{cases}$

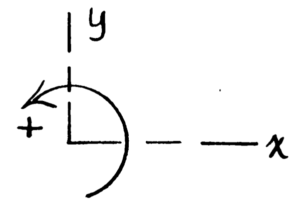
$F_{max} = \mu_s N = 0.1(245) = 24.5 \text{ N} > F$ (assumption OK)

6/84



$\bar{k} = 0.175$ m

$\mu_s = 0.1$, $\mu_k = 0.08$



$\Sigma F_y = 0: \; N - 25(9.81) + 50 \sin 30° = 0$

$N = 220$ N

$\Sigma F_x = m\bar{a}_x: \quad 50 \cos 30° - F = 25a \qquad (1)$

$\Sigma M_G = \bar{I}\alpha: \quad 50(0.075) - F(0.2) = 25(0.175)^2\alpha \qquad (2)$

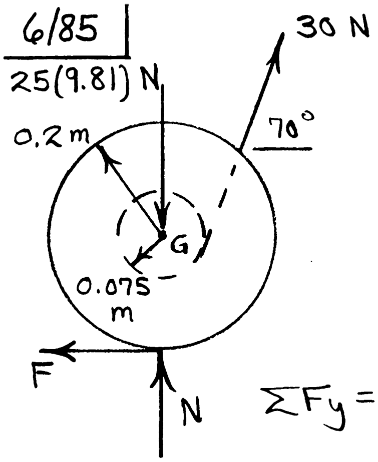
Assume rolling with no slip: $\quad a = -r\alpha \qquad (3)$

Solution of (1)-(3): $\begin{cases} a = 0.556 \text{ m/s}^2, \; \alpha = -2.78 \text{ rad/s}^2; \\ F = 29.4 \text{ N} \end{cases}$

$F_{max} = \mu_s N = 0.1(220) = 22.0 \text{ N} < F$: slips, $\underline{F = \mu_k N = 17.62 \text{ N}}$

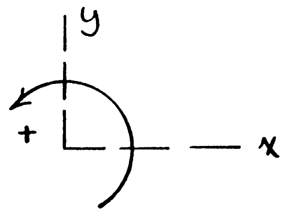
From Eqs. (1) & (2): $\underline{a = 1.027 \text{ m/s}^2, \quad \alpha = 0.295 \text{ rad/s}^2}$

6/85



$\bar{k} = 0.0175$ m

$\mu_s = 0.1$, $\mu_k = 0.08$



$\Sigma F_y = 0: \quad N - 25(9.81) + 30 \sin 70° = 0$

$N = 217$ N

$$\Sigma F_x = m\bar{a}_x: \quad 30 \cos 70° - F = 25a \qquad (1)$$

$$\Sigma M_G = \bar{I}\alpha: \quad 30(0.075) - F(0.2) = 25(0.175)^2\alpha \qquad (2)$$

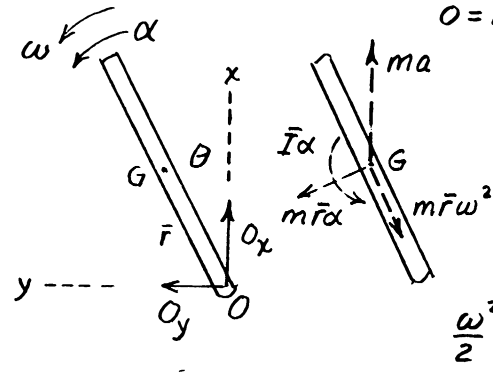
Assume rolling with no slip: $\quad a = -r\alpha \qquad (3)$

Solution of Eqs. (1)-(3): $\begin{cases} a = -0.0224 \text{ m/s}^2 \\ \alpha = 0.1121 \text{ rad/s}^2 \\ F = 10.82 \text{ N} \end{cases}$

$F_{max} = \mu_s N = 0.1(217) = 21.7 > F$ (assumption OK)

6/86

$$\Sigma M_O = \bar{I}\alpha + \Sigma m\bar{a}d\,; \quad \bar{I} = m(k_O^2 - \bar{r}^2)$$



$$0 = m(k_O^2 - \bar{r}^2)\alpha + m\bar{r}^2\alpha - ma\bar{r}\sin\theta$$

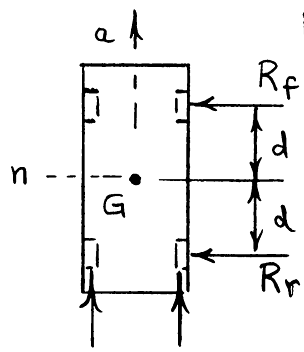
$$\alpha = \frac{a\bar{r}\sin\theta}{k_O^2}$$

$$\int_0^{\omega} \dot{\theta}\, d\dot{\theta} = \int_0^{\pi/2} \alpha\, d\theta$$

$$\frac{\omega^2}{2} = \frac{a\bar{r}}{k_O^2}\left(-\cos\theta\right)_0^{\pi/2} = \frac{a\bar{r}}{k_O^2}$$

$$\underline{\omega = \frac{1}{k_O}\sqrt{2a\bar{r}}}$$

6/87 R_f is lateral force at both front wheels, R_r is similar force at both rear wheels.



Car angular acceleration is

$$\alpha = \frac{a}{R} \quad \text{CCW}$$

(a) $v = 0$:

$$\Sigma F_n = ma_n : \quad R_f + R_r = 0 \qquad (1)$$

$$\Sigma M_G = \bar{I}\alpha : \quad (R_f - R_r)d = \bar{I}\frac{a}{R} \qquad (2)$$

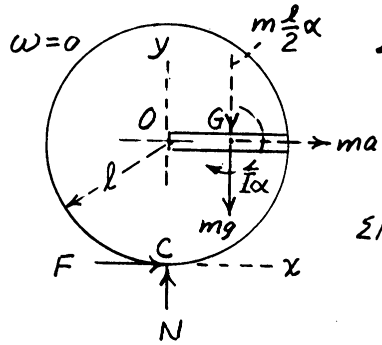
Solution: $\underline{R_f = \dfrac{\bar{I}a}{2dR}}$, $\underline{R_r = -\dfrac{\bar{I}a}{2dR}}$ (a couple)

(b) $v \neq 0$: Eq. (2) remains the same

$$\Sigma F_n = ma_n : \quad R_f + R_r = m\frac{v^2}{R} \qquad (1)$$

Solution: $\underline{R_f = \dfrac{mv^2 d + \bar{I}a}{2Rd}}$, $\underline{R_r = \dfrac{mv^2 d - \bar{I}a}{2Rd}}$

6/88



$$\bar{I}\alpha = \frac{1}{12} m l^2 \alpha = \frac{1}{12} m l^2 \frac{a}{l} = \frac{1}{12} m l a$$

$$\Sigma M_C = \bar{I}\alpha + \Sigma m \bar{a} d$$

$$mg\frac{l}{2} = \frac{1}{12} m l a + m a l + m \frac{l}{2}\frac{a}{l}\frac{l}{2}$$

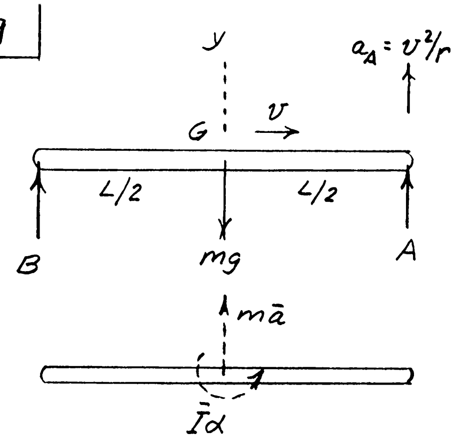
solve & get $a = \frac{3}{8} g$, $\alpha = \frac{a}{l} = \frac{3g}{8l}$

$$\Sigma F_x = m\bar{a}_x$$

$$F = m\frac{3}{8}g = \underline{\frac{3}{8}mg}$$

$\Sigma F_y = m\bar{a}_y$; $N - mg = -m\frac{l}{2}\frac{3g}{8l}$, $\underline{N = \frac{13}{16}mg}$

6/89



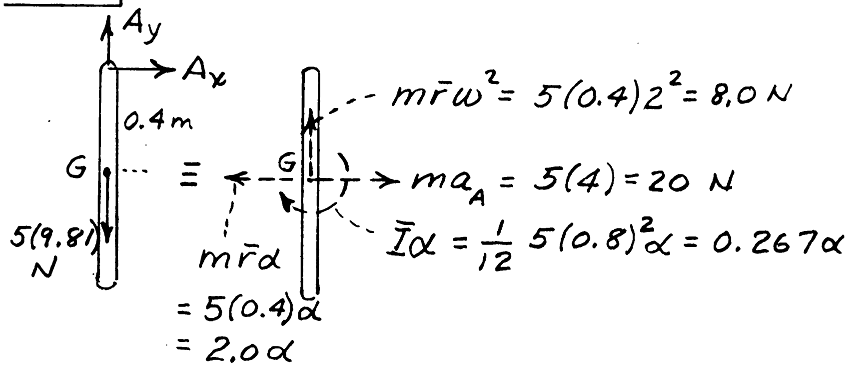
$$\bar{a} = a_B + a_{G/B}$$
$$= 0 + \frac{a_A}{2} j = \frac{v^2}{2r} j$$

$$\Sigma M_B = \bar{I}\alpha + m\bar{a}d; \quad AL - mg\frac{L}{2} = \frac{1}{12} m L^2 \alpha + m\frac{v^2}{2r}\frac{L}{2}$$

$$\alpha = a_A/L = \frac{v^2}{rL}$$

$$A = mg\left(\frac{1}{2} + \frac{1}{3}\frac{v^2}{gr}\right)$$

6/90



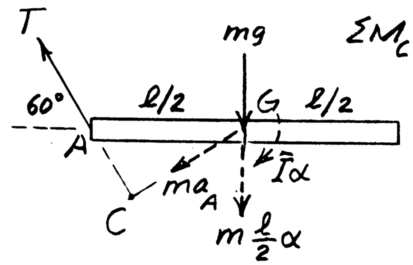
$\Sigma M_A = \bar{I}\alpha + \Sigma m\bar{a}d$; $\quad 0 = 0.267\alpha + 2.0\alpha(0.4) - 20(0.4)$

$\alpha = 7.50\ \text{rad/s}^2$

$\Sigma F_x = m\bar{a}_x$; $\quad A_x = 20 - 2.0(7.50) = \underline{5\ \text{N}}$

$\Sigma F_y = m\bar{a}_y$; $\quad A_y - 5(9.81) = 8, \quad A_y = \underline{57.1\ \text{N}}$

6/91 | 3 unknowns, T, a_A, α. Moments about C eliminates two of them.



$$\Sigma M_C = \bar{I}\alpha + m\bar{a}d$$

$$mg\frac{\ell}{2}\cos^2 30° = \frac{1}{12}m\ell^2\alpha + m\frac{\ell}{2}\alpha\left(\frac{\ell}{2}\cos^2 30°\right)$$

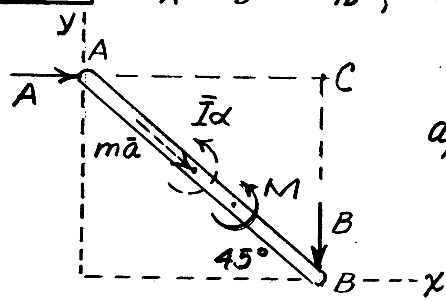
$$\alpha = \frac{18g}{13\ell}$$

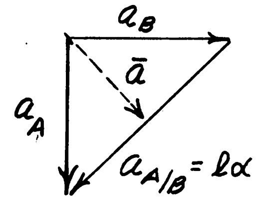
$\Sigma M_G = \bar{I}\alpha$; $T\frac{\ell}{2}\cos 30° = \frac{1}{12}m\ell^2\left(\frac{18g}{13\ell}\right)$, $\underline{T = \frac{2\sqrt{3}}{13}mg}$

independent of ℓ

6/92

$\underline{a}_A = \underline{a}_B + \underline{a}_{A/B}$, $a_{A/B} = (a_{A/B})_t = l\alpha$





$\bar{a} = l\alpha/2$

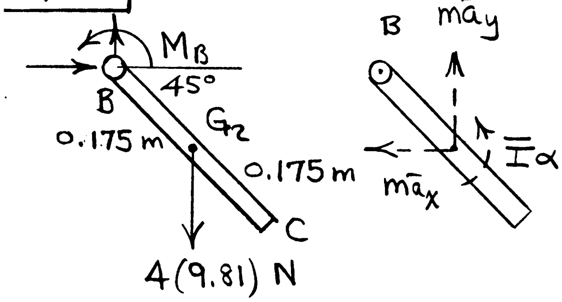
$\Sigma M_C = \bar{I}\alpha + m\bar{a}d$; $M = \frac{1}{12}ml^2\alpha + m\frac{l\alpha}{2}\frac{l}{2} = \frac{1}{3}ml^2\alpha$

$\alpha = \frac{3M}{ml^2}$

$\Sigma F_x = m\bar{a}_x$; $A = m\frac{l\alpha}{2}\frac{1}{\sqrt{2}} = \frac{ml}{2\sqrt{2}}\frac{3M}{ml^2}$, $\underline{A = \frac{3M}{2\sqrt{2}\,l}\,\mathbf{i}}$

$\Sigma F_y = m\bar{a}_y$; $-B = m\left(-\frac{l\alpha}{2}\frac{1}{\sqrt{2}}\right)$ $\underline{B = -\frac{3M}{2\sqrt{2}\,l}\,\mathbf{j}}$

6/93



$\underline{\omega}_{BC} = \underline{\omega}_{AB} = 2\underline{k}$ rad/s

$\underline{\alpha}_{BC} = \underline{\alpha}_{AB} = 4\underline{k}$ rad/s^2

$$\underline{a}_{G_2} = \underline{\alpha} \times \underline{r}_{AG_2} - \omega^2 \underline{r}_{AG_2} = 4\underline{k} \times [(0.7 + 0.175)\cos 45^\circ \underline{i} + (0.7 - 0.175)\sin 45^\circ \underline{j}] - 2^2[(0.7 + 0.175)\cos 45^\circ \underline{i} + (0.7 - 0.175)\sin 45^\circ \underline{j}] = -3.96\underline{i} + 0.990\underline{j} \text{ m/s}^2$$

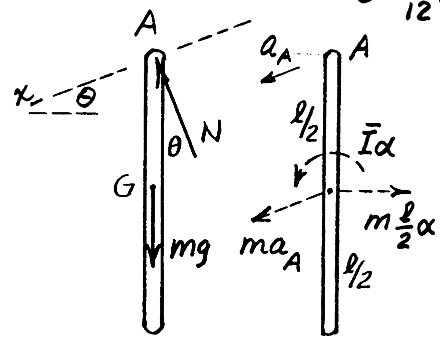
$$\Sigma M_B = \bar{I}\alpha + \Sigma m\bar{a}d: \quad M_B - 4(9.81)(0.175 \sin 45^\circ) = \tfrac{1}{12}(4)(0.35)^2(4) + 4(0.990)(0.175 \cos 45^\circ) - 4(3.96)(0.175 \sin 45^\circ),$$

$\underline{M_B = 3.55 \text{ N}\cdot\text{m}}$ (CCW)

6/94

$$\Sigma M_A = \bar{I}\alpha + \Sigma m\bar{a}d$$

$$0 = \frac{1}{12}ml^2\alpha + m\frac{l}{2}\alpha\frac{l}{2} - ma_A\frac{l}{2}\cos\theta$$

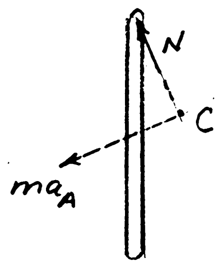


$$\Sigma F_x = m\bar{a}_x$$

$$mg\sin\theta = m\left(a_A - \frac{l}{2}\alpha\cos\theta\right)$$

Eliminate α & get

$$a_A = \frac{g\sin\theta}{1 - \frac{3}{4}\cos^2\theta}$$



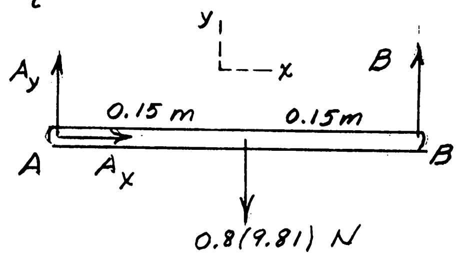
Alternative solution:

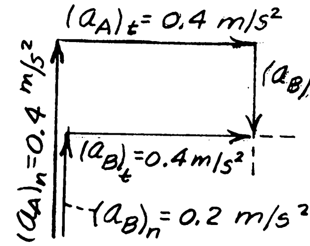
Pt. C may be used as a moment center thus eliminating reference to a_A & N giving one equation in α.

6/95 | $\omega_{AB} = 0$; $(\underline{a}_B)_n + (\underline{a}_B)_t = (\underline{a}_A)_n + (\underline{a}_A)_t + (\underline{a}_{B/A})_t$

$(a_A)_n = 0.1(2^2) = 0.4\ m/s^2$ $\quad (a_B)_n = \dfrac{v_B^2}{BC} = \dfrac{v_A^2}{BC} = \dfrac{[0.1(2)]^2}{0.2} = 0.2\ m/s^2$

$(a_A)_t = 0.1(4) = 0.4\ m/s^2$





Eq. 6/3 $\qquad \alpha_{AB} = \dfrac{(a_{B/A})_t}{\overline{AB}} = \dfrac{0.2}{0.3} = 0.667\ \dfrac{rad}{s^2}$ CW

$\Sigma \underline{M}_A = \underline{I}_A \underline{\alpha} + \bar{\underline{\rho}} \times m\underline{a}_A$

$(0.3B - 0.8[9.81]0.15)\underline{k} = \frac{1}{3}0.8(0.3)^2(-\frac{2}{3}\underline{k})$

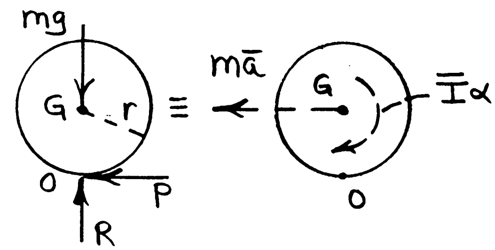
$+ (0.15\underline{i}) \times 0.8(0.4\underline{j} + 0.4\underline{i})$

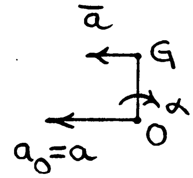
$0.3B = 1.177 - 0.016 + 0.048$

$\underline{B = BC = 4.03\ N}$ (tension)

6/96

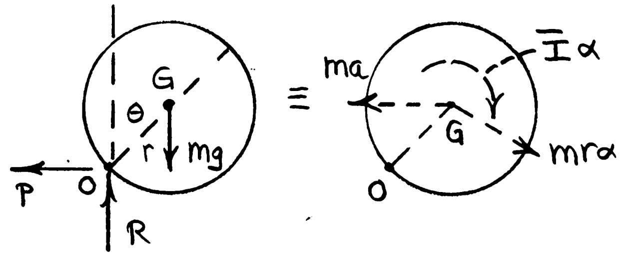
$\bar{I} = \frac{2}{5}mr^2$





$\Sigma M_O = \bar{I}\alpha - m\bar{a}r: \quad 0 = \frac{2}{5}mr^2\alpha - m\bar{a}r, \quad \bar{a} = \frac{2}{5}r\alpha$

$\underline{a}_G = \underline{a}_O + \underline{a}_{G/O}: \quad \bar{a} = a - r\alpha = \frac{2}{5}r\alpha \Rightarrow \underline{\alpha = \frac{5}{7}\frac{a}{r}}$



$a_O = a$

$\Sigma M_O = \bar{I}\alpha + \Sigma m\bar{a}d: \; mgr\sin\theta = \frac{2}{5}mr^2\alpha + mr^2\alpha - mar\cos\theta$

$\alpha = \frac{5}{7r}\left(g\sin\theta + a\cos\theta\right)$

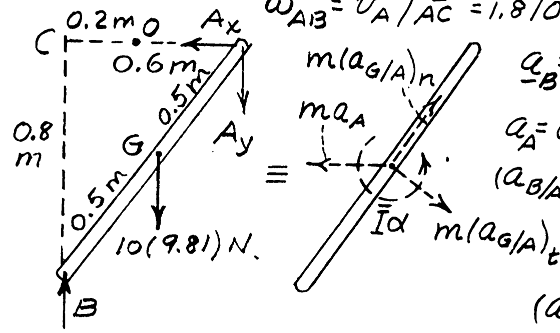
$\omega\,d\omega = \alpha\,d\theta: \quad \int_0^{\omega}\omega\,d\omega = \frac{5}{7r}\int_0^{\theta}(g\sin\theta + a\cos\theta)\,d\theta$

$\underline{\omega = \sqrt{\frac{10}{7r}}\sqrt{g(1-\cos\theta) + a\sin\theta}}$

6/97

$v_A = \overline{OA}\,\omega_0 = 0.4(4.5) = 1.8$ m/s

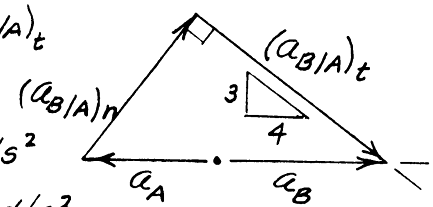
$\omega_{AB} = v_A / \overline{AC} = 1.8/0.6 = 3$ rad/s



$\underline{a}_B = \underline{a}_A + (\underline{a}_{B/A})_n + (\underline{a}_{B/A})_t$

$a_A = \overline{OA}\,\omega_0^2 = 0.4(4.5)^2 = 8.10\ \frac{m}{s^2}$

$(a_{B/A})_n = \overline{AB}\,\omega_{AB}^2 = 1.0(3)^2 = 9$ m/s^2



From diag., $(a_{B/A})_t = \frac{4}{3}9 = 12$ m/s^2

$\alpha = (a_{B/A})_t / \overline{AB} = 12/1.0 = 12$ rad/s^2

$\bar{I}\alpha = \frac{1}{12}10(1.0)^2(12) = 10$ N·m

$m(a_{G/A})_t = m\frac{(a_{B/A})_t}{2} = 10\,\frac{12}{2} = 60$ N

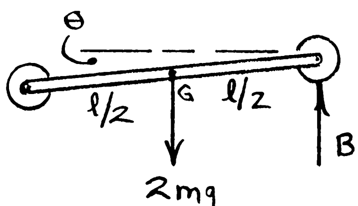
$ma_A = 10(8.10) = 81.0$ N

$\Sigma M_A = \bar{I}\alpha + \Sigma m\bar{a}d;\quad 10(9.81)(0.3) - 0.6B = 10 + 60(0.5) - 81.0(0.4)$

$0.6B = 21.8,\quad \underline{B = 36.4\ N}$

6/98 Assume that the angle θ present as B clears the surface is very small and that the speed of B is constant (while on surface).

Time t between A & B leaving surface : $t = \frac{l}{v}$.



$\bar{I} = 2m\left(\frac{l}{2}\right)^2 = ml^2/2$

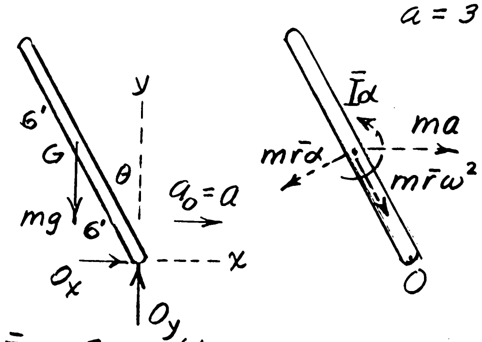
$I_B = \frac{ml^2}{2} + 2m\left(\frac{l}{2}\right)^2 = ml^2$

$\Sigma M_B = I_B\ddot{\theta} : 2mg\frac{l}{2} = ml^2\ddot{\theta}$, $\ddot{\theta} = \frac{g}{l}$ (CCW)

$\omega = \cancel{\omega_0}^{\,0} + \ddot{\theta}t = \frac{g}{l}\frac{l}{v} = \underline{\frac{g}{v}}$

6/99 $m\underline{\bar{a}} = m\underline{a}_0 + m\underline{a}_{G/0} = m\underline{a} + m\bar{r}\omega^2 + m\bar{r}\alpha$

$a = 3\ ft/sec^2,\ \bar{r} = 6\ ft$



$\Sigma M_0 = \bar{I}\alpha + \Sigma m\bar{a}d;$

$$mg(6\sin\theta) = \frac{1}{12}m(12^2)\alpha + m(6\alpha)(6) - m(3)6\cos\theta$$

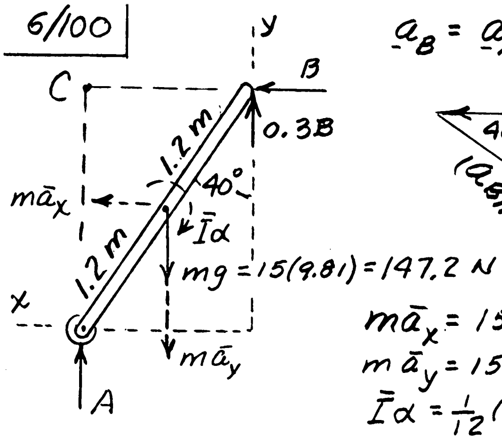
$$\alpha = \frac{1}{8}(g\sin\theta + 3\cos\theta)$$

$\int\omega\, d\omega = \int\alpha\, d\theta;\quad \int_0^{\omega}\omega\, d\omega = \frac{1}{8}\int_0^{\pi/2}(g\sin\theta + 3\cos\theta)d\theta$

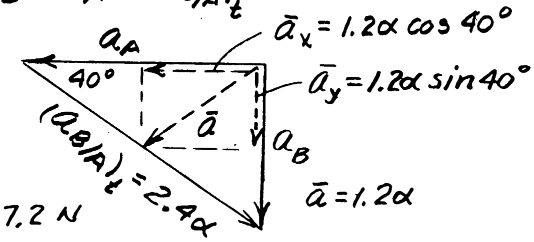
$$\omega^2 = \frac{1}{4}\Big[-32.2\cos\theta + 3\sin\theta\Big]_0^{\pi/2} = \frac{1}{4}[32.2 + 3] = \frac{35.2}{4}$$

$$\omega = \frac{1}{2}\sqrt{35.2} = \underline{2.97\ rad/sec}$$

6/100



$\underline{a}_B = \underline{a}_A + (\underline{a}_{B/A})_t$



$m\bar{a}_x = 15(1.2\alpha)\cos 40° = 13.79\alpha \quad N$

$m\bar{a}_y = 15(1.2\alpha)\sin 40° = 11.57\alpha \quad N$

$\bar{I}\alpha = \frac{1}{12}(15)(2.4)^2\alpha \quad = 7.2\alpha \quad N\cdot m$

$\Sigma M_C = \bar{I}\alpha + \Sigma m\bar{a}d$

$147.2(1.2 \sin 40°) - 0.3B(2.4 \sin 40°)$

$= 7.2\alpha + 13.79\alpha(1.2\cos 40°) + 11.57\alpha(1.2\sin 40°)$

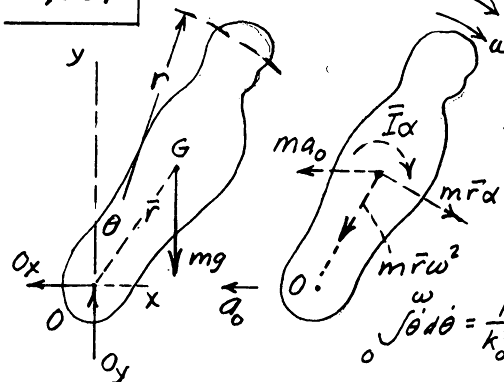
Simplify to $\alpha = 3.94 - 0.01607B$ ----(1)

$\Sigma F_x = m\bar{a}_x$; $B = 13.79\alpha$ ---------- (2)

Solve (1) & (2) & get $\alpha = 3.23$ rad/s²

$a_A = 2.4\alpha\cos 40° = 2.4(3.23)\cos 40° = \underline{5.93 \text{ m/s}^2}$

6/101



$\Sigma M_O = \bar I\alpha + \Sigma m\bar a d$

$mg\bar r\sin\theta = m(k_O^2 - \bar r^2)\alpha + m\bar r\alpha(\bar r) - ma_o\bar r\cos\theta$

$\alpha = \frac{1}{k_O^2}(g\bar r\sin\theta + a_o\bar r\cos\theta)$

$\dot\theta d\dot\theta = \ddot\theta d\theta$

$\int_0^\omega \dot\theta d\dot\theta = \frac{1}{k_O^2}\int (g\bar r\sin\theta + a_o\bar r\cos\theta)\,d\theta$

$\frac{\omega^2}{2} = \frac{1}{k_O^2}\left[g\bar r(1-\cos\theta) + a_o\bar r\sin\theta\right]$ where $a_o = a$

$v = r\omega = \frac{r\sqrt{2}}{k_O}\sqrt{g\bar r(1-\cos\theta) + a\bar r\sin\theta}$

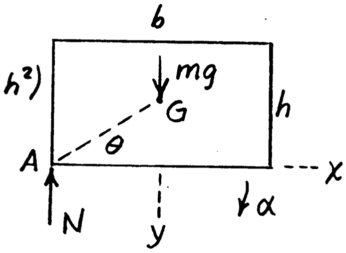
For $\bar r = 0.45$ m, $r = 0.8$ m, $k_O = 0.55$ m, $\theta = 45°$, $a = 10g$,

$v = \frac{0.80\sqrt{2}}{0.55}\sqrt{9.81(0.45)(1 - 1/\sqrt{2}) + 10(9.81)(0.45)\,1/\sqrt{2}} = \underline{11.73 \text{ m/s}}$

(Alternatively apply Eq. 6/3 with moment center at O)

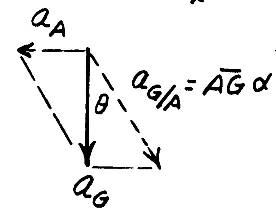
6/102

$\bar{I} = \frac{1}{12} m(b^2+h^2)$



$\underline{a}_G = \underline{a}_A + \underline{a}_{G/A}$

$\Sigma F_x = 0$ so $(a_G)_x = 0$



$\Sigma F_y = m\bar{a}_y$; $mg - N = m a_G$

$mg - N = m\, b\alpha/2$

$a_G = \overline{AG}\alpha \cos\theta$
$= b\alpha/2$

$\Sigma M_G = \bar{I}\alpha$; $N\frac{b}{2} = \frac{m}{12}(b^2+h^2)\alpha$

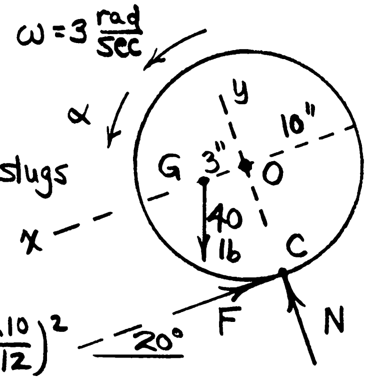
Eliminate N & get $\alpha = \dfrac{2bg}{b^2 + \frac{1}{3}(b^2+h^2)} = \dfrac{6bg}{4b^2+h^2}$

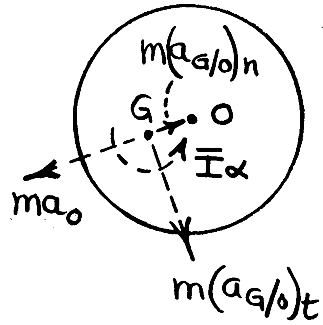
$a_A = \overline{AG}\alpha \sin\theta = \frac{h}{2}\alpha = \dfrac{3bhg}{4b^2+h^2}$ or $a_A = \dfrac{3g}{4\frac{b}{h} + \frac{h}{b}}$

6/103

$m = \dfrac{40}{32.2}$

$= 1.242$ slugs





$\bar{I} = mk^2$

$= 1.242\left(\dfrac{8.10}{12}\right)^2$

$= 0.566$ lb-ft-sec^2

$$\underline{a}_G = \underline{a}_O + (a_{G/O})_n + (a_{G/O})_t$$

$$a_O = r\alpha = \frac{10}{12}\alpha, \quad (a_{G/O})_n = \frac{3}{12}(3)^2 = 2.25 \text{ ft/sec}^2$$

$$(a_{G/O})_t = \frac{3}{12}\alpha = 0.25\alpha$$

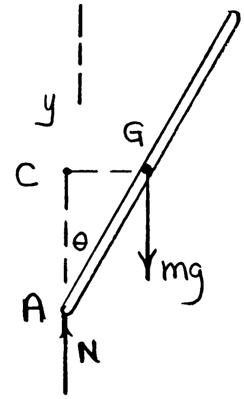
$$\begin{cases} ma_O = 1.242\left(\frac{10}{12}\alpha\right) = 1.035\alpha \\ m(a_{G/O})_n = 1.242(2.25) = 2.80 \text{ lb} \\ m(a_{G/O})_t = 1.242(0.25\alpha) = 0.311\alpha \end{cases}$$

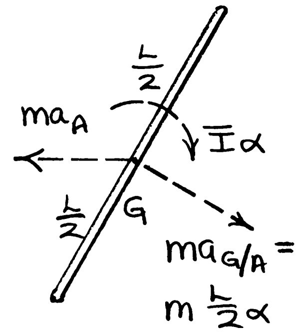
$$\begin{cases} \Sigma M_G = \bar{I}\alpha : \ \frac{10}{12}F + \frac{3}{12}N = 0.566\alpha \\ \Sigma F_x = m\bar{a}_x : \ 40\sin 20^\circ - F = 1.035\alpha - 2.80 \\ \Sigma F_y = m\bar{a}_y : \ N - 40\cos 20^\circ = -0.311\alpha \end{cases}$$

Simultaneous solution :

$$\begin{cases} F = 0.582 \text{ lb} \\ N = 32.8 \text{ lb} \\ \alpha = 15.35 \text{ rad/sec}^2 \text{ CCW} \end{cases}$$

6/104





$+\circlearrowleft$

$\Sigma M_C = \bar{I}\alpha + \Sigma m\bar{a}d$:

$$mg\frac{L}{2}\sin\theta = \frac{1}{12}mL^2\alpha + \frac{L}{2}\sin^2\theta\left(m\frac{L}{2}\alpha\right)$$

$$\underline{\alpha = \frac{2g}{L}\,\frac{\sin\theta}{\frac{1}{3}+\sin^2\theta}}$$

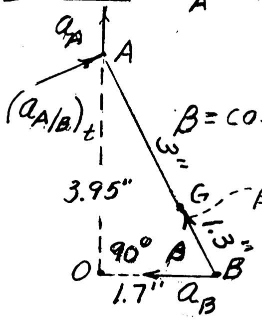
$$\Sigma F_y = m\bar{a}_y:\quad mg - N = m\frac{L}{2}\left(\frac{2g}{L}\,\frac{\sin\theta}{\frac{1}{3}+\sin^2\theta}\right)\sin\theta$$

$$\underline{N = \frac{mg}{1+3\sin^2\theta}}$$

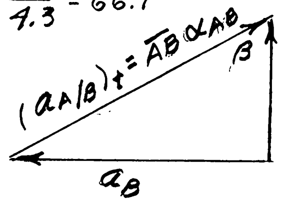
6/105

$\underline{a}_A = \underline{a}_B + (\underline{a}_{A/B})_t$; $(a_{A/B})_n = \overline{AB}\,\omega_{AB}^2 = 0$ since $v_A = v_B$

$a_B = r\omega^2 = \frac{1.7}{12}(100\pi)^2 = 13.98(10^3)$ ft/sec^2



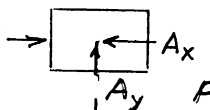
$\beta = \cos^{-1}\frac{1.7}{4.3} = 66.7°$



$(a_{A/B})_t = \frac{13.98(10^3)}{\sin 66.7°} = 15.22(10^3)\ \frac{ft}{sec^2}$

$\alpha_{AB} = \frac{15.22(10^3)}{4.3/12} = 42.5(10^3)$ rad/sec^2 CW

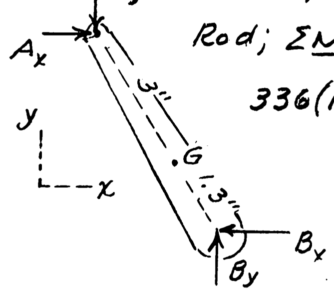
$a_A = 13.98(10^3)/\tan\beta = 6.02(10^3)$ ft/sec^2



Piston; $\Sigma F_y = ma_y$; $A_y = \frac{1.80}{32.2}\,6.02(10^3) = 336$ lb

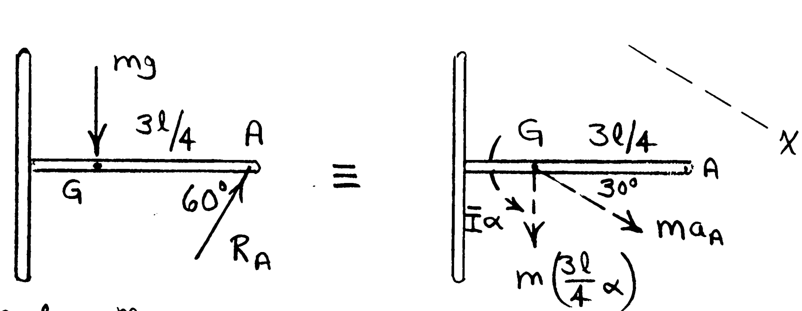
Rod; $\Sigma \underline{M}_B = I_B\underline{\alpha} + \bar{\underline{\rho}} \times m\underline{a}_B$ (Eq. 6/3)

$$336(1.7) - A_x(3.95) = \frac{1.20}{32.2(12)}\left[(1.12)^2 + (1.3)^2\right](-42.5)10^3$$
$$+\ 1.3\,\frac{1.20}{32.2(12)}(13.98)(10^3)12\sin\beta$$



$A_x = 85.6$ lb, $A = \sqrt{(85.6)^2 + (336)^2} = \underline{347\text{ lb}}$

6/106



$$\bar{r} = \frac{\frac{m}{2}\frac{l}{2} + \frac{m}{2}l}{m} = \frac{3l}{4}, \text{ as shown above}$$

$$\bar{I} = \left[\frac{1}{12}\frac{m}{2}l^2 + \frac{m}{2}\left(\frac{l}{4}\right)^2\right]2 = \frac{7}{48}ml^2$$

$$\circlearrowleft \Sigma M_A = \bar{I}\alpha + \Sigma m\bar{a}d:$$

$$mg\left(\frac{3l}{4}\right) = \frac{7}{48}ml^2\alpha + ma_A\left(\frac{3l}{4}\cdot\frac{1}{2}\right) + m\frac{3l}{4}\alpha\left(\frac{3l}{4}\right)$$

$$\Sigma F_x = m\bar{a}_x: \quad mg\sin 30° = m\left(a_A + \frac{3l}{4}\alpha\frac{1}{2}\right)$$

Solve simultaneously:
$$\begin{cases} \alpha = \frac{108}{109}\frac{g}{l} \\ a_A = \frac{14}{109}g \end{cases}$$

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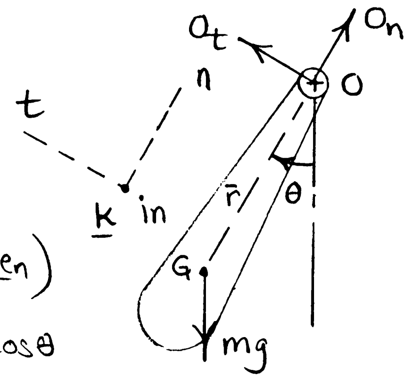
$\Sigma \underline{M}_O = \dot{\underline{H}}_O + \bar{\underline{\rho}} \times m\underline{a}_O$:



$-mg\bar{r}\sin\theta\,\underline{k} = mk_O^2\ddot{\theta}\,\underline{k}$

$+ \bar{r}\underline{e}_n \times ma(-\cos\theta\,\underline{e}_t + \sin\theta\,\underline{e}_n)$

$\Rightarrow -mg\bar{r}\sin\theta = mk_O^2\ddot{\theta} - ma\bar{r}\cos\theta$

$\ddot{\theta} = \dfrac{\bar{r}}{k_O^2}(a\cos\theta - g\sin\theta)$, $\dot{\theta}^2 = \dfrac{2\bar{r}}{k_O^2}\left[a\sin\theta - g(1-\cos\theta)\right]$

$\Sigma F_t = m\bar{a}_t$: $O_t - mg\sin\theta = m\dfrac{\bar{r}^2}{k_O^2}(a\cos\theta - g\sin\theta) - ma\cos\theta$

$$O_t = m\left[g\sin\theta - a\cos\theta\right]\left[1 - \frac{\bar{r}^2}{k_O^2}\right]$$

$\Sigma F_n = m\bar{a}_n$: $O_n - mg\cos\theta = m\dfrac{2\bar{r}^2}{k_O^2}\left[a\sin\theta - g(1-\cos\theta)\right]$

$+ ma\sin\theta$, $O_n = m\left[(g\cos\theta + a\sin\theta)\left(1 + \dfrac{2\bar{r}^2}{k_O^2}\right) - \dfrac{2\bar{r}^2}{k_O^2}g\right]$

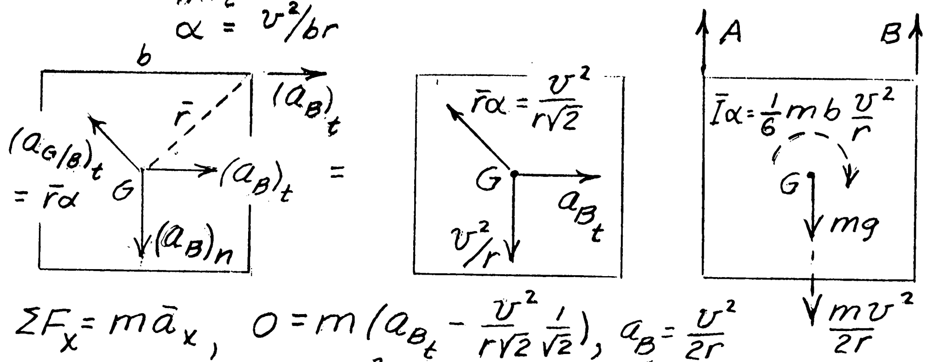
$\theta = \max$ when $\dot{\theta} = 0$: $a\sin\theta - g(1-\cos\theta) = 0$

Solve for θ when $a = \frac{g}{2}$: $\theta_{max} = 53.1°$

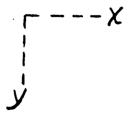
6/108 $v_{B/A} = 0$ so $\omega = 0$ $\qquad \underline{a}_G = (\underline{a}_B)_n + (\underline{a}_B)_t + (\underline{a}_{G/B})_t$

$(a_{B/A})_t = b\alpha = v^2/r$

$\alpha = v^2/br$



$\Sigma F_x = m\bar{a}_x, \quad 0 = m\left(a_{B_t} - \frac{v^2}{r\sqrt{2}}\frac{1}{\sqrt{2}}\right), \quad a_B = \frac{v^2}{2r}$

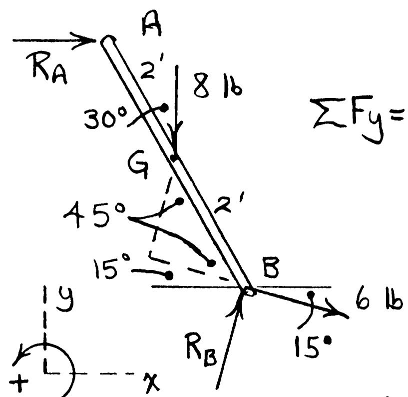


$a_G = \bar{a}_y = \frac{v^2}{r} - \frac{v^2}{r\sqrt{2}}\frac{1}{\sqrt{2}} = \frac{v^2}{2r}$

$\bar{I}\alpha = \frac{1}{6} mb^2 \frac{v^2}{br} = \frac{1}{6} mb\frac{v^2}{r}$

$\left.\begin{array}{l} \Sigma M_G = \bar{I}\alpha; \ (A - B)\frac{b}{2} = \frac{1}{6} mb\frac{v^2}{r} \\ \Sigma F_y = m\bar{a}_y; \ mg - (A + B) = \frac{mv^2}{2r} \end{array}\right\}$ solve & get $\qquad \underline{A = \frac{m}{2}\left(g - \frac{v^2}{6r}\right)}$ $\qquad \underline{B = \frac{m}{2}\left(g - \frac{5v^2}{6r}\right)}$

▶6/109



$\Sigma F_x = m\bar{a}_x$: $R_A + 6\cos 15° + R_B \sin 15° = \frac{8}{32.2}\bar{a}_x$ (1)

$\Sigma F_y = m\bar{a}_y$: $R_B \cos 15° - 6\sin 15° - 8 = \frac{8}{32.2}\bar{a}_y$ (2)

$\Sigma M_G = \bar{I}\alpha$:

$-R_A(2\cos 30°) + R_B(2\cos 45°) + 6(2\sin 45°) = \frac{1}{12}\frac{8}{32.2}(4)^2\alpha$ (3)

Kinematics :

$$\underline{a}_A = \underline{a}_G + \underline{a}_{A/G} = \bar{a}_x\underline{i} + \bar{a}_y\underline{j} + \underline{\alpha}\times\underline{r}_{A/G} - \omega^2\underline{r}_{A/G}$$

With $\underline{r}_{A/G} = 2[-\sin 30°\,\underline{i} + \cos 30°\,\underline{j}]$, we have

$$a_A\underline{j} = [\bar{a}_x - 2\cos 30°\alpha - 2^2\cdot(-2\sin 30°)]\,\underline{i} + [\bar{a}_y - 2\sin 30°\alpha - 2^2\cdot(2\cos 30°)]\,\underline{j}$$

$$\Rightarrow \begin{cases} 0 = \bar{a}_x - \sqrt{3}\alpha + 4 & (4) \\ a_A = \bar{a}_y - \alpha - 4\sqrt{3} & (5) \end{cases}$$

$$\underline{a}_B = \underline{a}_G + \underline{a}_{B/G} = \bar{a}_x \underline{i} + \bar{a}_y \underline{j} + \alpha \underline{k} \times \underline{r}_{B/G} - \omega^2 \underline{r}_{B/G}$$

With $\underline{r}_{B/G} = 2\left[\sin 30^\circ \underline{i} - \cos 30^\circ \underline{j}\right]$, we have

$$a_B\left[\cos 15^\circ \underline{i} - \sin 15^\circ \underline{j}\right] =$$

$$\left[\bar{a}_x + 2\cos 30^\circ \alpha - 2^2 \cdot (2\sin 30^\circ)\right] \underline{i}$$

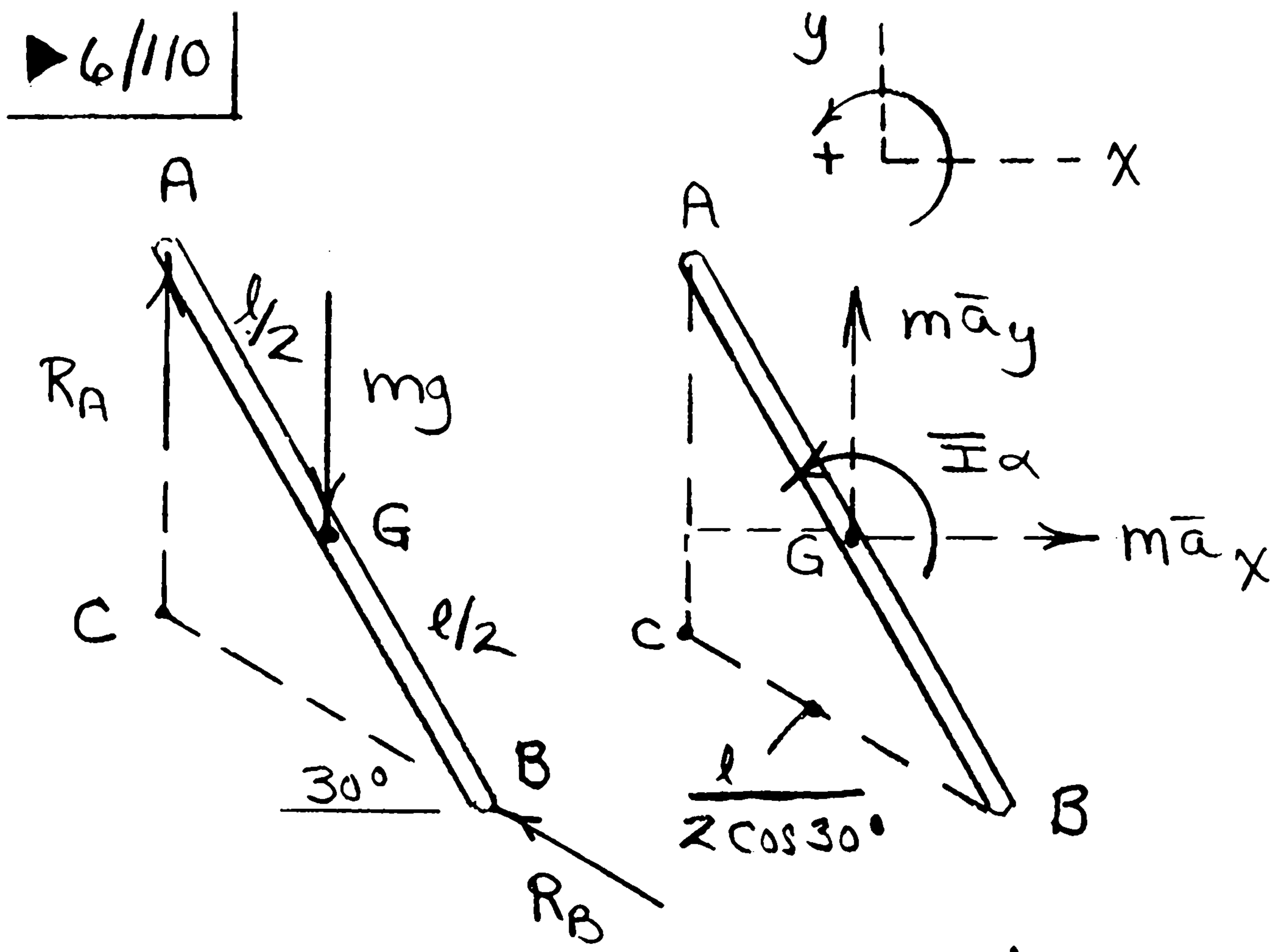
$$+ \left[\bar{a}_y + 2\sin 30^\circ \alpha - 2^2 \cdot (-2\cos 30^\circ)\right] \underline{j}$$

$$\Rightarrow \begin{cases} a_B \cos 15^\circ = \bar{a}_x + \sqrt{3}\,\alpha - 4 & (6) \\ -a_B \sin 15^\circ = \bar{a}_y + \alpha + 4\sqrt{3} & (7) \end{cases}$$

Solution of Eqs. (1)-(7) :

$$\begin{cases} \underline{R_A = 1.128 \text{ lb}} & \underline{\alpha = 18.18 \text{ rad/sec}^2} \\ \underline{R_B = -0.359 \text{ lb}} & \underline{a_A = -65.0 \text{ ft/sec}^2} \\ \bar{a}_x = 27.5 \text{ ft/sec}^2 & \underline{a_B = 56.9 \text{ ft/sec}^2} \\ \bar{a}_y = -39.8 \text{ ft/sec}^2 & \end{cases}$$

▶6/110



$\curvearrowleft \Sigma M_C = \bar{I}\alpha + \Sigma m\bar{a}d: \; -mg\frac{l}{2}\sin 30° = \frac{1}{12}ml^2\alpha$

$+ m\bar{a}_y\left(\frac{l}{2}\sin 30°\right) - m\bar{a}_x\left(\frac{l}{2\cos 30°} - \frac{l}{2}\cos 30°\right) \quad (1)$

Kinematics : $\underline{a}_A = \underline{a}_G + \underline{a}_{A/G}$

$a_A\underline{i} = \bar{a}_x\underline{i} + \bar{a}_y\underline{j} + \alpha\underline{k} \times \frac{l}{2}\left[-\sin 30°\underline{i} + \cos 30°\underline{j}\right]$

$\underline{j}: \; 0 = \bar{a}_y - \alpha\frac{l}{2}\sin 30° \quad (2)$

$\underline{a}_B = \underline{a}_G + \underline{a}_{B/G}$

$a_B(\sin 30°\underline{i} + \cos 30°\underline{j}) = \bar{a}_x\underline{i} + \bar{a}_y\underline{j} + \alpha\underline{k} \times \frac{l}{2}\left[\sin 30°\underline{i} - \cos 30°\underline{j}\right]$

$$\Rightarrow \begin{cases} a_B\sin 30° = \bar{a}_x + \alpha\frac{l}{2}\cos 30° & (3) \\ a_B\cos 30° = \bar{a}_y + \alpha\frac{l}{2}\sin 30° & (4) \end{cases}$$

Solution strategy : Eliminate a_B from (3) & (4) :

(3) : $a_B = 2\bar{a}_x + \alpha l\frac{\sqrt{3}}{2}$

(4) : $\sqrt{3}\,\bar{a}_x + \alpha l\frac{3}{4} = \bar{a}_y + \alpha\frac{l}{4}$

With (2) : $\sqrt{3}\,\bar{a}_x + \frac{3}{4}\alpha l = \alpha\frac{l}{4} + \alpha\frac{l}{4}$

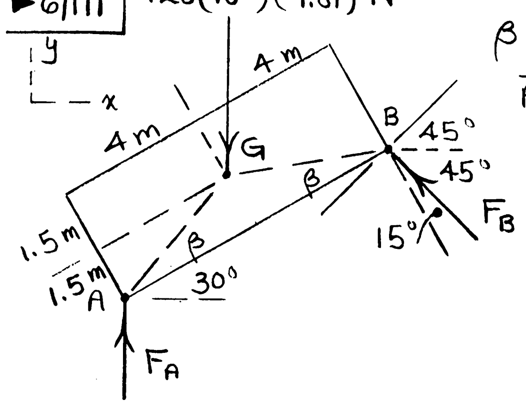
$\bar{a}_x = -\frac{1}{4\sqrt{3}}\alpha l \quad (5)$

(2) & (5) into (1) :

$-\frac{g}{4} = \frac{1}{12}l\alpha + \frac{1}{4}\alpha\left(\frac{l}{4}\right) - \alpha\left(\frac{1}{-4\sqrt{3}}\right)\left(\frac{l}{4\sqrt{3}}\right)$

$\alpha = -\frac{3}{2}\frac{g}{l}$ (CW)

▶6/111



$$\beta = \tan^{-1}\frac{1.5}{4} = 20.6°$$

$$\overline{AG} = \overline{BG} = (4^2 + 1.5^2)^{1/2}$$
$$= 4.27 \text{ m}$$

$$\bar{I} = \frac{120(10^3)}{12}\left[8^2 + 3^2\right]$$
$$= 73(10^4) \text{ kg}\cdot\text{m}^2$$

$$\Sigma F_x = m a_{Gx}: \quad -F_B \cos 45° = 120(10^3) a_{Gx} \qquad (1)$$

$$\Sigma F_y = m a_{Gy}: \quad F_A + F_B \sin 45° - 120(10^3)(9.81) = 120(10^3) a_{Gy} \qquad (2)$$

$$\circlearrowleft \Sigma M_G = \bar{I}\alpha: \quad -F_A\left[\overline{AG}\cos(30° + \beta)\right] + F_B \cos 15° (4)$$
$$- F_B \sin 15° (1.5) = 73(10^4)\alpha \qquad (3)$$

$$\underline{a}_A = \underline{a}_G + \underline{a}_{A/G}$$

$$a_A \underline{i} = a_{Gx}\underline{i} + a_{Gy}\underline{j} + \alpha \underline{k} \times \left[-\cos(30° + \beta)\underline{i} - \sin(30° + \beta)\underline{j}\right]\overline{AG}$$

$$\underline{i}: \quad a_A = a_{Gx} + \alpha \overline{AG} \sin(30° + \beta) \qquad (4)$$

$$\underline{j}: \quad 0 = a_{Gy} - \alpha \overline{AG} \cos(30° + \beta) \qquad (5)$$

$$\underline{a}_B = \underline{a}_G + \underline{a}_{B/G}$$

$$a_B\left[\cos 45° \underline{i} + \sin 45° \underline{j}\right] = a_{Gx}\underline{i} + a_{Gy}\underline{j}$$
$$+ \alpha \underline{k} \times \left[\cos(30° - \beta)\underline{i} + \sin(30° - \beta)\underline{j}\right]\overline{BG}$$

$$\underline{i}: \quad a_B \cos 45° = a_{Gx} - \alpha \overline{BG} \sin(30° - \beta) \qquad (6)$$

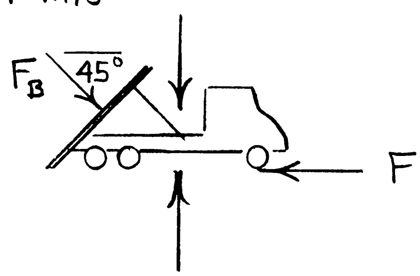
$$\underline{j}: \quad a_B \sin 45° = a_{Gy} + \alpha \overline{BG} \cos(30° - \beta) \qquad (7)$$

Now 7 eqs. in F_A, F_B, a_{Gx}, a_{Gy}, α, a_A, a_B

Solution :

$$\begin{cases} F_A = 716\ 000\ \text{N} & \alpha = -0.372\ \text{rad/s}^2 \\ F_B = 481\ 000\ \text{N} & a_A = -4.06\ \text{m/s}^2 \\ a_{Gx} = -2.83\ \text{m/s}^2 & a_B = -3.64\ \text{m/s}^2 \\ a_{Gy} = -1.009\ \text{m/s}^2 & \end{cases}$$

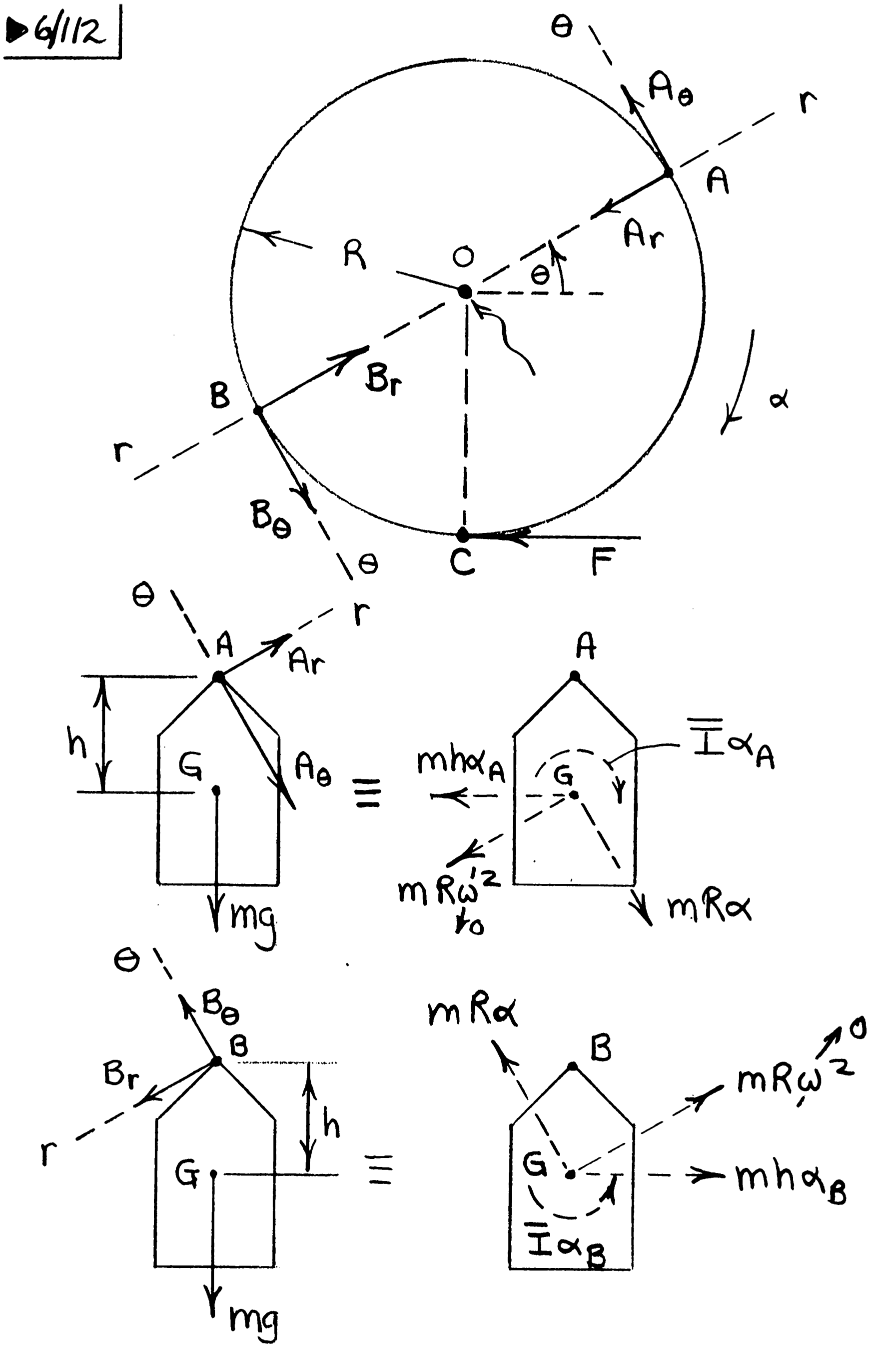
FBD of truck



$\Sigma F_x = 0: \quad F_B \cos 45° - F = 0$

$F = F_B \cos 45° = 481 \frac{\sqrt{2}}{2} = \underline{340\ \text{kN}}$

▶6/112



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• Gondola A — $\circlearrowleft + \ \sum M_A = \bar{I}\alpha_A + \sum m a_A d$:

$$0 = \bar{I}\alpha_A + mh^2\alpha_A - mR\alpha h \sin\theta$$

But $\bar{I} + mh^2 = I_A = mk^2$

So $I_A\alpha_A = mRh\alpha\sin\theta$ or $m\alpha_A = mRh\alpha\sin\theta / k^2$

$\sum F_\theta = m\bar{a}_\theta$: $A_\theta + mg\cos\theta = mR\alpha - mh\alpha_A\sin\theta$

$$A_\theta = m(R\alpha - g\cos\theta) - mR\alpha\left(\frac{h\sin\theta}{k}\right)^2 \qquad (1)$$

• Gondola B — $\circlearrowright + \ \sum M_B = \bar{I}\alpha_B + \sum m a_\theta d$:

$$0 = \bar{I}\alpha_B + mh^2\alpha_B - mR\alpha h\sin\theta$$

So $I_B\alpha_B = mRh\alpha\sin\theta$ or $m\alpha_B = mRh\alpha\sin\theta / k^2$

(where $I_B = I_A = mk^2$, as above)

$\sum F_\theta = m\bar{a}_\theta$: $B_\theta - mg\cos\theta = mR\alpha - mh\alpha_B\sin\theta$

$$B_\theta = m(R\alpha + g\cos\theta) - mR\alpha\left(\frac{h\sin\theta}{k}\right)^2 \qquad (2)$$

• Wheel — $\circlearrowleft + \ \sum M_0 = I_0\alpha$:

$$\left[F - \sum(A_\theta + B_\theta)\right]R = I_0\alpha \qquad (3)$$

Substitute (1) & (2) into (3)

$$FR - \sum_{1}^{n/2}\left[2mR\alpha - 2mR\alpha\left(\frac{h\sin\theta_n}{k}\right)^2\right]R = I_0\alpha$$

Simplify & solve for F :

$$F = \left\{mR\left[n - 2\frac{h^2}{k^2}\left(\sin^2\theta_1 + \sin^2\theta_2 + \cdots \sin^2\theta_{n/2}\right)\right] + \frac{I_0}{R}\right\}\alpha$$

($n = 0$ corresponds to $\theta = 0$; $n/2$ corresponds to $\theta < \pi$)

Note: The above expression for F simplifies to

$$F = \left\{mRn\left(1 - \frac{h^2}{2k^2}\right) + \frac{I_0}{R}\right\}\alpha$$

6/113

$$T_1 + U_{1-2} = T_2$$

$$T_1 = \frac{1}{2} 8(0.3)^2 + \frac{1}{2} 12(0.210)^2 \left(\frac{0.3}{0.2}\right)^2 = 0.955 \text{ J}$$

$$U_{1-2} = 8(9.81)(1.5) - 3\left(\frac{1.5}{0.2}\right) = 95.2 \text{ J}$$

$$T_2 = \frac{1}{2} 8 v^2 + \frac{1}{2} 12 (0.210)^2 \left(\frac{v}{0.2}\right)^2 = 10.62 v^2$$

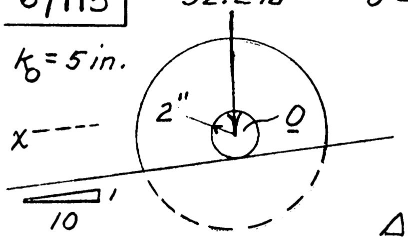
So $0.955 + 95.2 = 10.62 v^2$, $\underline{v = 3.01 \text{ m/s}}$

6/114 $U'_{1-2} = \Delta T + \Delta V_g$

$$0 = \frac{1}{2}m(4^2 - 0^2) - mg(5)(1 - \cos\theta)$$

$$\underline{\theta = 33.2^\circ}$$

6/115



$\theta = \tan^{-1} 1/10$, $\sin\theta = 0.0995$

$U = \Delta T$

$U = 32.2(0.0995)(10)$

$= 32.04$ ft-lb

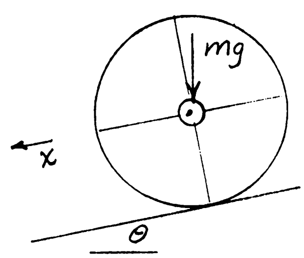
$\Delta T = \frac{1}{2} m v^2 + \frac{1}{2} I_O \omega^2 - 0$

$\Delta T = \frac{1}{2}\frac{32.2}{32.2} v^2 + \frac{1}{2}\frac{32.2}{32.2}\left(\frac{5}{12}\right)^2\left(\frac{v}{2/12}\right)^2 - 0 = \frac{29}{8} v^2$

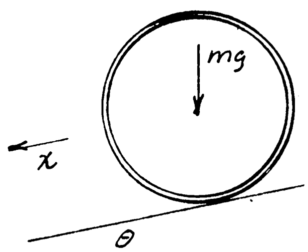
Thus $32.04 = \frac{29}{8} v^2$, $v^2 = 8.839$, $\underline{v = 2.97 \text{ ft/sec}}$

6/116

A



B



$U = \Delta T$

$U = mgx \sin\theta$

$\Delta T = \frac{1}{2} m \bar{v}^2 + \frac{1}{2} \bar{I} \omega^2$

Case A: $\Delta T = \frac{1}{2} m v^2 + 0$

case B: $\Delta T = \frac{1}{2} m v^2 + \frac{1}{2} m r^2 \left(\frac{v}{r}\right)^2$

$= m v^2$

Case A: $mgx \sin\theta = \frac{1}{2} m v^2$

$$\underline{v_A = \sqrt{2gx \sin\theta}}$$

Case B: $mgx \sin\theta = m v^2$

$$\underline{v_B = \sqrt{gx \sin\theta}}$$

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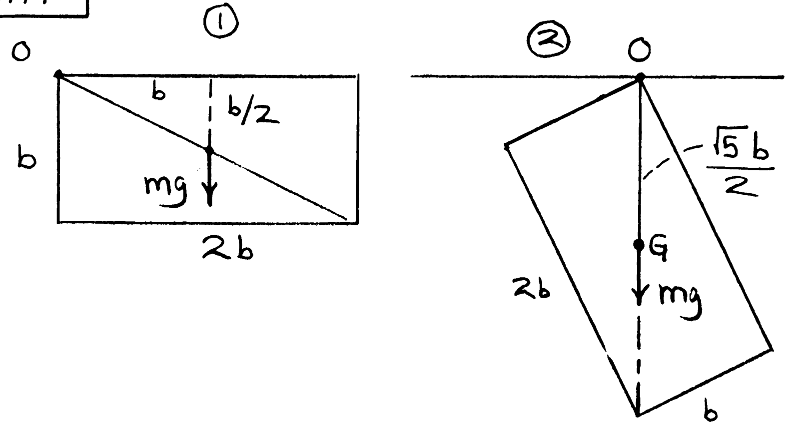


$$I_O = \bar{I} + md^2 = \frac{1}{12}m\left[b^2 + (2b)^2\right] + m\left[b^2 + \left(\frac{b}{2}\right)^2\right]$$
$$= \frac{5}{3}mb^2$$

$T_1 + U_{1\text{-}2} = T_2$:

$$0 + mgb\left[\frac{\sqrt{5}}{2} - \frac{1}{2}\right] = \frac{1}{2}\left[\frac{5}{3}mb^2\right]\omega^2$$

$$\omega^2 = \frac{3g}{5b}\left(\sqrt{5} - 1\right), \quad \underline{\omega = 0.861\sqrt{\frac{g}{b}}}$$

6/118

$U = \Delta T$: $U = M\theta = 5\left(\frac{6}{12}\right)6(2\pi) = 94.2$ ft-lb

$$\Delta T = \Sigma \frac{1}{2} I \omega^2 = \Sigma \frac{1}{2}\frac{W}{g}k^2\omega^2$$

$$= \frac{1}{2}\frac{3.94}{32.2}\left(\frac{2.85}{12}\right)^2\left(\frac{\omega}{4}\right)^2 + \frac{1}{2}\frac{1.22}{32.2}\left(\frac{2.14}{12}\right)^2\omega^2$$

$$= 0.216(10^{-3})\omega^2 + 0.602(10^{-3})\omega^2$$

$$= 0.818(10^{-3})\omega^2$$

$$\omega^2 = \frac{94.2}{0.818}(10^3) = 11.52(10^4)\ (\text{rad/sec})^2$$

$\omega = 339$ rad/sec, $N = \frac{339}{2\pi}60 = \underline{3240 \text{ rev/min}}$

$\underline{6/119}$ $\Delta V_g = -200(9.81)(1.25) = -2453$ J

Each spring stretches a distance of 1.25 m so that $\Delta V_e = 2\left[\frac{1}{2}k(1.25)^2 - 0\right] = 1.563\,k$ J

$\Delta T = \frac{1}{2}I_O\omega^2 = \frac{1}{2}\,200\left(\frac{1}{12}\overline{2.5}^2 + \overline{1.25}^2\right)\overline{1.5}^2 = 469$ J

$\Delta T + \Delta V_g + \Delta V_e = 0$; $469 - 2453 + 1.563k = 0$

$k = 1270$ N/m or $\underline{k = 1.270 \text{ kN/m}}$

6/120 Power $P = \frac{d(\text{Energy})}{dt} = \frac{\Delta E}{t}$

$$\Delta E = \frac{1}{2}\bar{I}(\omega_2^2 - \omega_1^2) = \frac{1}{2}(1200)(0.4)^2([5000]^2 - [3000]^2)\left(\frac{2\pi}{60}\right)^2$$

$$= 16.84(10^6) \text{ J}$$

$$P = \frac{16.84(10^6)}{2(60)} = 140.4(10^3) \text{ J/s or W}$$

so $\underline{P = 140.4 \text{ kW}}$ or $P = \frac{140.4(10^3)}{7.457(10^2)} = \underline{188 \text{ hp}}$

6/121 $U' = \Delta V_g + \Delta V_e + \Delta T = 0$

$\Delta V_g = -15(9.81)(0.3) = -44.1 \text{ J}$

$\Delta V_e = \frac{1}{2}(20)(10^3)h^2 = 10^4 h^2 \text{ J}$

$\Delta T = \frac{1}{2}\frac{1}{3}15(0.6)^2 4^2 = 14.40 \text{ J}$

So $0 = -44.1 + 10^4 h^2 + 14.40$

$\underline{h = 54.5 \text{ mm}}$

The distance x has no effect on the energy change & so does not affect h. The distance x does influence the bearing force during the spring compression, however.

6/122 $U'_{1-2} = \Delta T + \Delta V_g + \Delta V_e$

$U'_{1-2} = M\theta = \frac{\pi}{2}M = 1.571M$ in.-lb

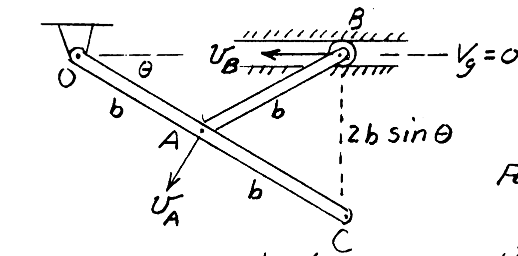
$\Delta T = \frac{1}{2}I_O\omega^2 - 0 = \frac{1}{2}\left(\frac{12}{32.2\times 12}\times 10^2\right)4^2 = 24.8$ in.-lb

$\Delta V_g = Wh = 12(-8) = -96$ in.-lb

$\Delta V_e = \frac{1}{2}k(x_2^2 - x_1^2) = \frac{1}{2}3\left([30-15\sqrt{2}]^2 - 0\right) = 115.8$ in.-lb

Thus $1.571M = 24.8 - 96 + 115.8$, $\underline{M = 24.8 \text{ lb-in.}}$

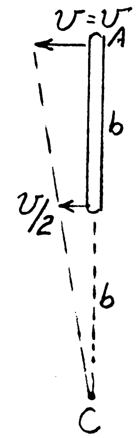
6/123 $\quad$ C = instant center for AB so



$$\frac{v_B}{2b\sin\theta} = \frac{v_A}{b}$$

$$v_B = 2v_A\sin\theta$$

For $\theta = \pi/2$, $v_B = 2v_A$



AB; $\Delta V_g = -\rho b g(b/2) = -\frac{1}{2}\rho g b^2$

$$\Delta T = \frac{1}{2}I_C\omega^2 = \frac{1}{2}\left(\frac{1}{12}\rho b^3 + \rho b\left[\frac{3b}{2}\right]^2\right)\left(\frac{v}{2b}\right)^2$$

$$= \frac{7}{24}\rho b v^2$$

OC; $\Delta V_g = -\rho(2b)g\,b = -2\rho g b^2$

$$\Delta T = \frac{1}{2}I_O\omega^2 = \frac{1}{2}\left(\frac{1}{3}2\rho b(2b)^2\right)\left(\frac{v/2}{b}\right)^2$$

$$= \frac{1}{3}\rho b v^2$$

$\Delta T + \Delta V_g = 0$; $\left(\frac{7}{24} + \frac{1}{3}\right)\rho b v^2 - \left(\frac{1}{2} + 2\right)\rho g b^2 = 0$

$v^2 = 4gb$, $\quad \underline{v = 2\sqrt{gb}}$

$\underline{6/124}$ $T_1 + U_{1-2} = T_2$

$T_1 = 0$

$$U_{1-2} = \int_1^2 M\,d\theta = \int_0^{5(2\pi)} 2(1-e^{-0.1\theta})\,d\theta$$

$$= \left(2\theta + 20e^{-0.1\theta}\right)\Big|_0^{5(2\pi)}$$

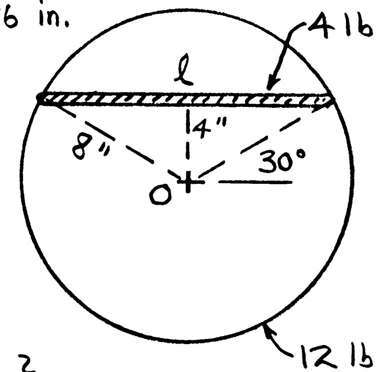
$$= 2(5)(2\pi) + 20e^{-0.1(5)(2\pi)} - 20$$

$$= 43.7\text{ J}$$

$$T_2 = \frac{1}{2}I\omega^2 = \frac{1}{2}(50)(0.4)^2\omega^2 = 4\omega^2$$

So $0 + 43.7 = 4\omega^2$, $\underline{\omega = 3.31 \text{ rad/s}}$

6/125 $l = 2(8)\cos 30° = 13.86$ in.



$$I_O = \frac{1}{2}\frac{12}{32.2}\left(\frac{8}{12}\right)^2 + \left[\frac{1}{12}\frac{4}{32.2}\left(\frac{13.86}{12}\right)^2 + \frac{4}{32.2}\left(\frac{4}{12}\right)^2\right]$$

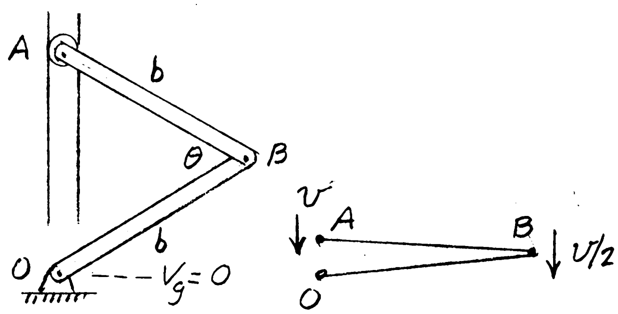
$$= 0.1104 \text{ lb-sec}^2\text{-ft}$$

$$T_1 + U_{1-2} = T_2$$

$$0 + 4\left(\frac{8}{12}\right) = \frac{1}{2}(0.1104)\omega^2$$

$$\underline{\omega = 6.95 \text{ rad/sec}}$$

6/126



$OB;\quad \Delta V_g = -mg\,\frac{b}{2}\sin\frac{\theta}{2}$

$\Delta T = \frac{1}{2}I_O\omega^2 - 0 = \frac{1}{2}\frac{1}{3}mb^2\left(\frac{v/2}{b}\right)^2 = \frac{1}{24}mv^2$

$AB;\quad \Delta V_g = -mg\,\frac{3b}{2}\sin\frac{\theta}{2}$

$\Delta T = \frac{1}{2}\bar{I}\omega^2 + \frac{1}{2}m\bar{v}^2 = \frac{1}{2}\frac{1}{12}mb^2\left(\frac{v}{2b}\right)^2 + \frac{1}{2}m\left(\frac{3v}{4}\right)^2$

$= \frac{7}{24}mv^2$

$U' = 0 = \Delta T + \Delta V_g;\quad 0 = \left(\frac{7}{24} + \frac{1}{24}\right)mv^2 - \left(\frac{3b}{2} + \frac{b}{2}\right)mg\sin\frac{\theta}{2} = 0$

$v^2 = 6gb\sin\frac{\theta}{2},\quad \underline{v = \sqrt{6gb\sin\theta/2}}$

6/127 ① : horizontal at release ; ② : vertical

$$T_1 + U_{1-2} = T_2 \qquad T_2 = \frac{1}{2}\left(\frac{1}{3} m l^2\right)\omega^2$$

$$U_{1-2} = -mgh + \int_0^{\pi/2} K\theta \, d\theta$$

$$= -6(9.81)(0.4) + \int_0^{\pi/2} 30\theta \, d\theta$$

$$= -23.5 + 15\theta^2 \Big|_0^{\pi/2} = -23.5 + 15\left(\frac{\pi}{2}\right)^2$$

$$= -23.5 + 37.0 = 13.47 \text{ N·m}$$

So $0 + 13.47 = \frac{1}{6}(6)(0.8)^2\omega^2$

$$\underline{\omega = 4.59 \text{ rad/s}}$$

6/128

$$T_1 + U_{1-2} = T_2$$

$$mg\left(\frac{l}{2}-x\right) = \frac{1}{2}\left[\frac{1}{12}m\,l^2 + m\left(\frac{l}{2}-x\right)^2\right]\omega^2$$

$$\omega^2 = \frac{g\left(\frac{l}{2}-x\right)}{\frac{l^2}{6} - \frac{lx}{2} + \frac{x^2}{2}}$$

Set $\frac{d\omega^2}{dx} = 0$ & obtain $\underline{x = 0.789\,l}$

or $\underline{x = 0.211\,l}$

$$\omega_{max} = \omega_{x=0.211} = \sqrt{\frac{g\left(\frac{l}{2} - 0.211\,l\right)}{\frac{l^2}{6} - \frac{0.211\,l^2}{2} + \frac{(0.211\,l)^2}{2}}}$$

$$= \underline{1.861\sqrt{\frac{g}{l}}}$$

(The solution $x = 0.789\,l$ would yield the same ω_{max}, only then the motion is CCW.)

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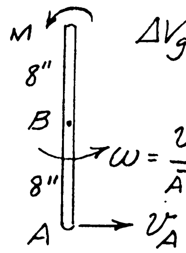
$U' = M\theta = \frac{72}{12}\frac{\pi}{4} = 4.712 \text{ ft-lb}$



$\Delta V_g = W\Delta h = 6\,\frac{8}{12}\left(1 - \frac{1}{\sqrt{2}}\right) = 1.172 \text{ ft-lb}$

$\omega = \frac{v_A}{\overline{AB}}$

$$\Delta T = \frac{1}{2} I_B \omega^2 = \frac{1}{2}\left(\frac{1}{12}\,\frac{6}{32.2}\left[\frac{16}{12}\right]^2\right)\left(\frac{10}{8/12}\right)^2$$

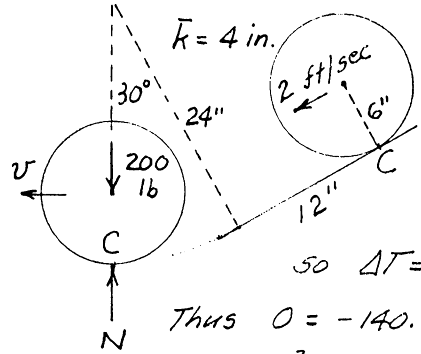
$$= 3.106 \text{ ft-lb}$$

ΔE = energy loss

$U' = \Delta T + \Delta V_g + \Delta E$, $\quad \Delta E = 4.712 - 3.105 - 1.172$

$$= \underline{0.435 \text{ ft-lb}}$$

6/130 $0 = \Delta V_g + \Delta T;\ \Delta V_g = -200\left[\frac{12}{12}\sin 30° + \frac{18}{12}(1-\cos 30°)\right]$



$= -140.2$ ft-lb

$\Delta T = \frac{1}{2} I_C (\omega_2^2 - \omega_1^2)$

$I_C = \frac{200}{32.2}\left(\left[\frac{4}{12}\right]^2 + \left[\frac{6}{12}\right]^2\right) = 2.24$ lb-ft-sec^2

$\omega_1 = \frac{2}{6/12} = 4$ rad/sec, $\omega_2 = \frac{v}{6/12} = 2v$

so $\Delta T = \frac{1}{2} 2.24 (4v^2 - 4^2) = 4.49v^2 - 17.94$ ft-lb

Thus $0 = -140.2 + 4.49v^2 - 17.94$

$v^2 = 35.30$, $v = 5.94$ ft/sec

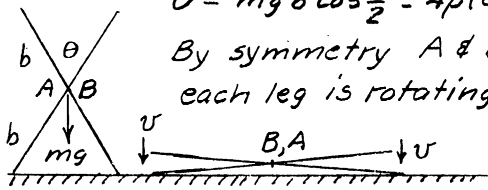
$\Sigma F_n = m\bar{a}_n;\ N - 200 = \frac{200}{32.2}\,\frac{35.30}{18/12}$, $\underline{N = 346 \text{ lb}}$

6/131 Let ρ = mass per unit length of bar

$$m = 4\rho c + 4\rho(2b) = 4\rho(c+2b)$$

$$U = mgb\cos\frac{\theta}{2} = 4\rho(c+2b)gb\cos\frac{\theta}{2}$$

By symmetry A & B move vertically, so each leg is rotating about its bottom end as the upper ends approach the floor.



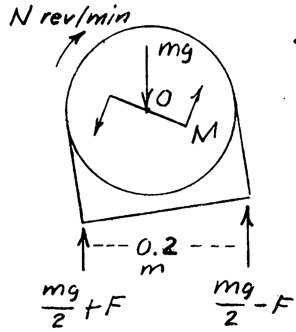
So $\Delta T_{\text{horiz. members}} = 2(\frac{1}{2})\rho c v^2 = \rho c v^2$

$$\Delta T_{\text{legs}} = 4(\tfrac{1}{2})I_{\text{end}}\,\omega^2 = 4(\tfrac{1}{2})\tfrac{1}{3}\rho(2b)(2b)^2\left(\frac{v}{2b}\right)^2 = \frac{4}{3}\rho b v^2$$

$U = \Delta T$; $4\rho(c+2b)gb\cos\frac{\theta}{2} = \rho c v^2 + \frac{4}{3}\rho b v^2$

$$v = \sqrt{12gb\,\frac{c+2b}{3c+4b}\cos\frac{\theta}{2}}$$

6/132

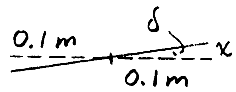


Power $P = M\omega = M\,\frac{2\pi N}{60}$, $M = \frac{4000(60)}{2\pi(1725)} = 22.14\ \text{N·m}$

$\Sigma M_O = 0;\ 2F(0.2) - 22.14 = 0,\ F = 55.4\ \text{N}$

$F = kx\ ,\ x = \frac{55.4}{15(10^3)} = 0.00369\ \text{m}$

or $x = 3.69\ \text{mm}$



$\delta = \tan^{-1}\frac{x}{0.1} = \tan^{-1}\frac{3.69}{100} = \underline{2.11°}$

<u>Motor shaft turns CW</u>

6/133 $U' = \Delta T + \Delta V_g + \Delta V_e$ for entire system

$U' = 0$

$\Delta T = \frac{1}{2} I_O \omega^2 = \frac{1}{2}\left[\frac{1}{3}\frac{10}{32.2}\left(\frac{20}{12}\right)^2\right](1.5)^2 = 0.323$ ft-lb

$\Delta V_g = -Wh = -10\left(\frac{10}{12}\right) = -8.33$ ft-lb

$\Delta V_e = 2\left(\frac{1}{2}kx^2\right) = k\left(\frac{2}{12}\right)^2 = 0.0278k$

Thus $0 = 0.323 - 8.33 + 0.0278k$

$k = 288$ lb/ft or $\underline{k = 24.0 \text{ lb/in.}}$

6/134 $\Delta V_e = \frac{1}{2}(1500)\left[(0.1+2\times0.05)^2 - \overline{0.1}^2\right] = 22.5 \text{ J}$

$\Delta V_g = -(150)(9.81)(0.05) = -73.58 \text{ J}$

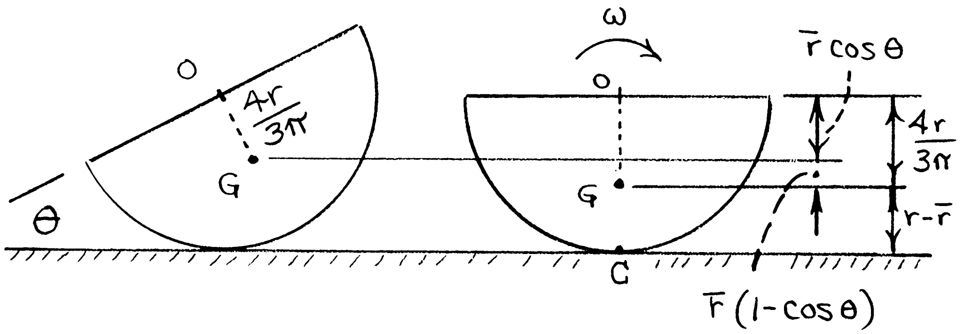
$\Delta T = \sum \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2 = \frac{1}{2}(150)v^2 + \frac{1}{2}(50)(0.3)^2\left(\frac{v}{0.4}\right)^2$

$= 75v^2 + 14.06v^2 = 89.06v^2$

$\Delta T + \Delta V_g + \Delta V_e = 0;\quad 89.06v^2 - 73.58 + 22.5 = 0$

$v^2 = 0.573, \quad \underline{v = 0.757 \text{ m/s}}$

6/135



$$U' = \Delta T + \Delta V_g = 0$$

$$I_C = \bar{I} + m(r-\bar{r})^2 = (I_O - m\bar{r}^2) + m(r-\bar{r})^2$$

$$= I_O + m(r^2 - 2r\bar{r}) = \frac{1}{2}mr^2 + mr^2\left(1 - \frac{8}{3\pi}\right)$$

$$= mr^2\left(\frac{3}{2} - \frac{8}{3\pi}\right)$$

$$\Delta T = \frac{1}{2}I_C\omega^2 - 0 = \frac{1}{2}mr^2\left(\frac{3}{2} - \frac{8}{3\pi}\right)\omega^2$$

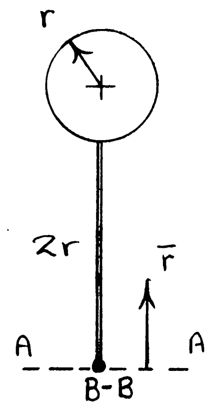
$$\Delta V_g = -mg\frac{4r}{3\pi}(1-\cos\theta)$$

So $0 = \frac{1}{2}mr^2\left(\frac{3}{2} - \frac{8}{3\pi}\right)\omega^2 - mg\frac{4r}{3\pi}(1-\cos\theta)$

$$\underline{\omega = 4\sqrt{\frac{g(1-\cos\theta)}{(9\pi-16)r}}}$$

6/136 Total mass $m = 2r\rho + 2\pi r\rho = 2r\rho(1+\pi)$

where ρ = mass per unit length.



$$\bar{r} = \frac{\Sigma \bar{r} m}{\Sigma m} = \frac{2r\rho(r) + 2\pi r\rho(3r)}{2r\rho + 2\pi r\rho}$$

$$= r\,\frac{1+3\pi}{1+\pi}$$

$$I_{B-B} = \frac{1}{3}(2r\rho)(2r)^2 + \left[2\pi r\rho r^2 + 2\pi r\rho(3r)^2\right]$$

$$= \frac{4+30\pi}{3(1+\pi)}\,mr^2$$

$$I_{A-A} = \frac{1}{3}(2r\rho)(2r)^2 + \left[\frac{1}{2}\,2\pi r\rho r^2 + 2\pi r\rho(3r)^2\right]$$

$$= \frac{8+57\pi}{6(1+\pi)}\,mr^2$$

$$T_1 + U_{1-2} = T_2$$

(a) $$0 + mgr\,\frac{1+3\pi}{1+\pi} = \frac{1}{2}\,\frac{8+57\pi}{6(1+\pi)}\,mr^2\omega^2$$

$$\underline{\omega = 2\sqrt{\frac{3+9\pi}{8+57\pi}\,\frac{g}{r}}}$$

(b) $$0 + mgr\,\frac{1+3\pi}{1+\pi} = \frac{1}{2}\,\frac{4+30\pi}{3(1+\pi)}\,mr^2\omega^2$$

$$\underline{\omega = \sqrt{\left(\frac{3+9\pi}{2+15\pi}\right)\frac{g}{r}}}$$

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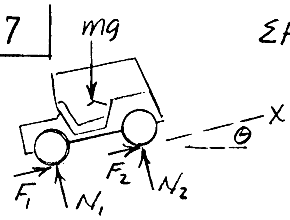
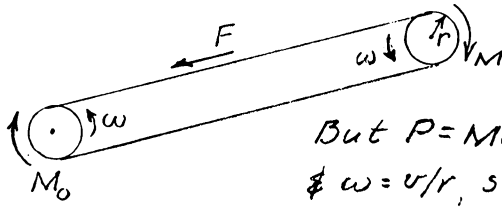


$\Sigma F_x = m a_x = 0; \quad F = F_1 + F_2 = mg\sin\theta$

For belt $dU = dT$, $dT = 0$

$$Fr\,d\theta - M\,d\theta - M_0\,d\theta = 0$$



$$M = Fr - M_0$$
$$= mgr\sin\theta - M_0$$

But $P = M\omega = mgr\omega\sin\theta - M_0\omega$

& $\omega = v/r$, so $P = mgv\sin\theta - M_0 v/r$

$$= (mg\sin\theta - M_0/r)\,v$$

Static friction forces do positive work on belt and negative work on cart, both of equal magnitude

$$\boxed{6/138}\quad \Delta T_{translational} = \frac{1}{2}mv^2 - 0 = \frac{1}{2}(10000)\left(\frac{72}{3.6}\right)^2 - 0$$

$$= 2(10^6)\ J$$

$$\Delta T_{rotation} = \frac{1}{2}I(\omega_2^2 - \omega_1^2)$$

$$= \frac{1}{2}(1500)(0.5)^2\left(\omega_2^2 - \left[\frac{4000\times 2\pi}{60}\right]^2\right)$$

$$= 187.5\,\omega_2^2 - 32.90\times 10^6\ J$$

$$\Delta E = 0.1(187.5\,\omega_2^2 - 32.90\times10^6) = 18.75\,\omega_2^2 - 3.29\times10^6\ J$$

$$\Delta V_g = mgh = 10000(9.81)(20) = 1.96\times10^6\ J$$

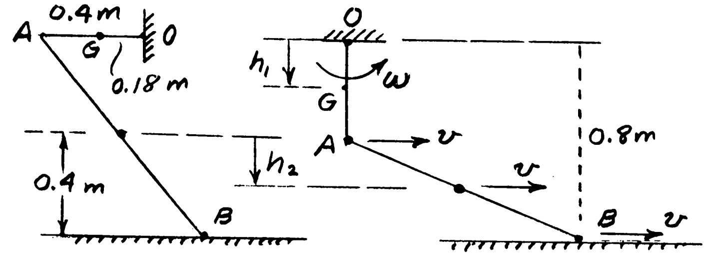
$$\Delta E = \Delta T + \Delta V_g\,;$$

$$18.75\,\omega_2^2 - 3.29\times10^6 = 2\times10^6 + 187.5\,\omega_2^2 - 32.90\times10^6 + 1.96\times10^6$$

$$168.75\,\omega_2^2 = 25.65\times10^6, \quad \omega_2^2 = 152\,000\ (rad/s)^2$$

$$\omega_2 = 390\ rad/s \quad or \quad N = \frac{390\times60}{2\pi} = \underline{3720\ rev/min}$$

6/139



$h_1 = 0.18$ m

$h_2 = 0.2$ m

OA; $\Delta V_g = -mgh_1 = -8(9.81)(0.18) = -14.13$ J

$\Delta T = \frac{1}{2} I_O \omega^2 = \frac{1}{2}(0.22)^2 8\left(\frac{v}{0.4}\right)^2 = 1.21 v^2$

AB; $\Delta V_g = -mgh_2 = -12(9.81)(0.2) = -23.54$ J

$\Delta T = \frac{1}{2} m v^2 = \frac{1}{2}(12) v^2 = 6 v^2$

For system, $U' = \Delta T + \Delta V_g$

$0 = 1.21 v^2 + 6 v^2 - 14.13 - 23.54$

$v^2 = 5.22$ m²/s²

$\underline{v = 2.29 \text{ m/s}}$

6/140 Each spring stretches 4 ft.

so $\Delta V_e = 2\left(\frac{1}{2}kx^2\right) = 2\left(\frac{1}{2}50[4]^2\right) = 800$ ft-lb

$\Delta V_g = -200\,(9-4) = -1000$ ft-lb

$U' = \Delta T + \Delta V_g + \Delta V_e$: $\quad 0 = \frac{1}{2}\frac{200}{32.2}v^2 - 1000 + 800$

$v^2 = 64.4$, $v = \underline{8.02 \text{ ft/sec}}$

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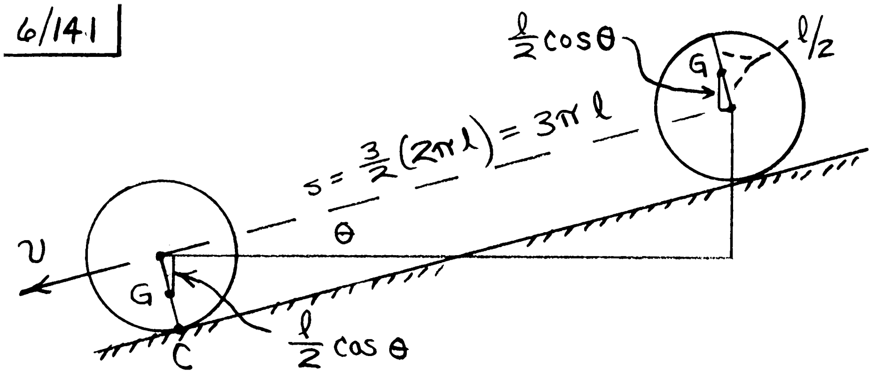

Mass center drops $h = 2\left(\frac{l}{2}\cos\theta\right) + (3\pi l)\sin\theta$

$$= l\left(\cos\theta + 3\pi\sin\theta\right)$$

$U' = \Delta T + \Delta V_g \quad : \quad U' = 0$

$T = \frac{1}{2} I_C \omega^2 = \frac{1}{2}\left(\frac{1}{3} m l^2\right)\left(\frac{v}{l}\right)^2 = \frac{1}{6} m v^2$

$\Delta V_g = -mgh = -mgl\left(\cos\theta + 3\pi\sin\theta\right)$

So $\quad 0 = \frac{1}{6} m v^2 - mgl\left(\cos\theta + 3\pi\sin\theta\right)$

$$\underline{v = \sqrt{6gl\left(\cos\theta + 3\pi\sin\theta\right)}}$$

6/142 Let x = distance moved by center O in m.

$\theta = \tan^{-1}\frac{1}{5} = 11.31°$, $\sin\theta = 0.1961$

$\Delta V_g = mg\Delta h = mgx\sin\theta = 10(9.81)x(0.1961) = 19.24x$

$\Delta V_e = \frac{1}{2}k(x_2^2 - x_1^2) = \frac{1}{2}(600)\left[\left(0.225 - \frac{275}{200}x\right)^2 - (0.225)^2\right]$

$567.2x^2 - 185.6x$

$\Delta T = \frac{1}{2}mv^2 + \frac{1}{2}\bar{I}\omega^2 = \frac{1}{2}(10)v^2 + \frac{1}{2}(10)(0.125)^2\left(\frac{v}{0.2}\right)^2$

$= 6.95v^2$. For system, $U' = \Delta T + \Delta V_g + \Delta V_e$:

$0 = 6.95v^2 + 19.24x + 567.2x^2 - 185.6x$

$v^2 = 23.93x - 81.57x^2$

Set $\frac{dv^2}{dx} = 0$ to get $x = 0.1467$ m for v_{max}

$v_{max}^2 = 23.93(0.1467) - 81.57(0.1467)^2$, $\underline{v_{max} = 1.325\ \frac{m}{s}}$

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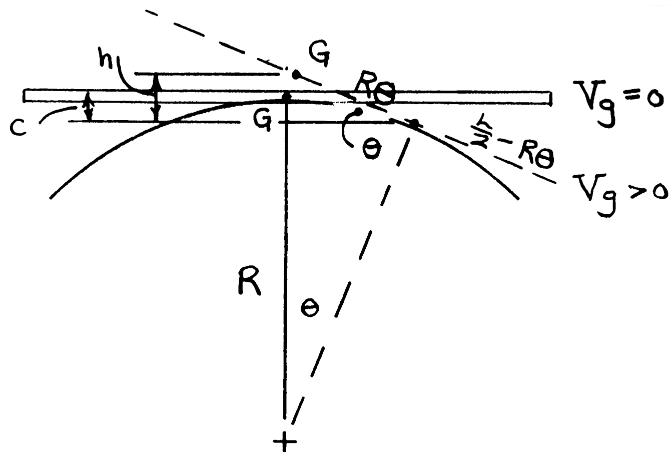

For arbitrary θ:

$$V_g = mg(h-c) = mg\left[R\theta\sin\theta - R(1-\cos\theta)\right]$$

$$= mgR(\theta\sin\theta + \cos\theta - 1)$$

$$T = \frac{1}{2}I_G\omega^2 = \frac{1}{24}mL^2\omega^2$$

$$\Delta T + \Delta V_g = 0: \quad (T-0) + (0-V_g) = 0, \; T = V_g$$

$$\frac{1}{24}mL^2\omega^2 = mgR(\theta\sin\theta + \cos\theta - 1)$$

$$\underline{\omega = \frac{2}{L}\sqrt{6gR(\theta\sin\theta + \cos\theta - 1)}}$$

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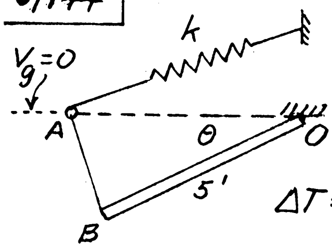

$\Delta V_g = 200(-2.5\sin\theta) = -500\sin\theta$ ft-lb

$\Delta V_e = \frac{1}{2}k(\overline{AB}^2 - 0) = \frac{15}{2}(2\times 5\sin\frac{\theta}{2})^2$

$= 750\sin^2\frac{\theta}{2}$ ft-lb

$\Delta T = \frac{1}{2}I_O\omega^2 = \frac{1}{2}\,\frac{1}{3}\,\frac{200}{32.2}5^2\omega^2 = 25.9\omega^2$ ft-lb

$\Delta T + \Delta V_g + \Delta V_e = 0$; $25.9\omega^2 - 500\sin\theta + 750\sin^2\frac{\theta}{2} = 0$

$\omega^2 = 19.32\sin\theta - 29.0\sin^2\frac{\theta}{2}$

$\frac{d\omega^2}{d\theta} = 19.32\cos\theta - 14.50\sin\theta = 0$ for max. ω^2

$\theta = \tan^{-1}\frac{19.32}{14.50} = \underline{53.1°}$, $\sin\theta = 0.800$, $\sin\frac{\theta}{2} = 0.447$

so $\omega^2 = 19.32(0.800) - 29.0(0.447)^2 = 9.66$

$\underline{\omega = 3.11 \text{ rad/sec}}$

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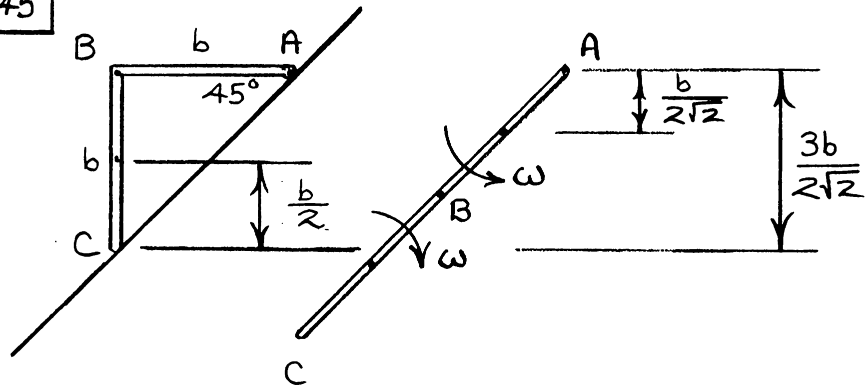


$AB: \quad \Delta V_g = -mg\frac{b}{2\sqrt{2}}$

$BC: \quad \Delta V_g = -mg\left(\frac{3b}{2\sqrt{2}} - \frac{b}{2}\right) = -mg\frac{b}{2}\left(\frac{3}{\sqrt{2}} - 1\right)$

$AB: \quad \Delta T = \frac{1}{2}I_A\omega^2 - 0 = \frac{1}{2}\frac{1}{3}mb^2\omega^2 = \frac{1}{6}mb^2\omega^2$

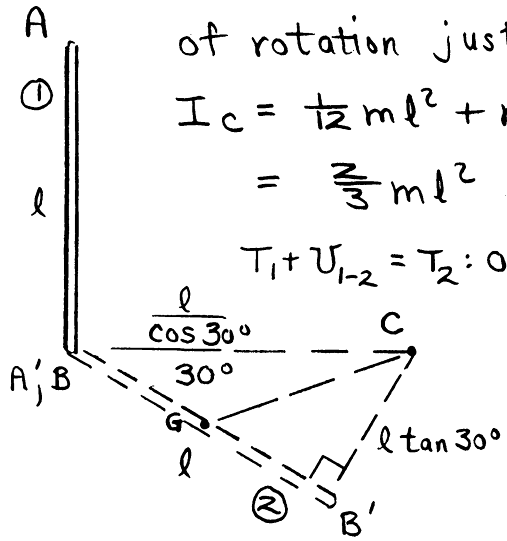
BC: Same as for AB

System: $\quad U' = \Delta T + \Delta V_g = 0$

$$0 = 2\left(\frac{1}{6}mb^2\omega^2\right) - mg\frac{b}{2}\left(\frac{1}{\sqrt{2}} + \frac{3}{\sqrt{2}} - 1\right)$$

$$\underline{\omega = \sqrt{\frac{3g}{2b}(2\sqrt{2}-1)}}$$

6/146 Point C is the instantaneous center of rotation just prior to impact.



$$I_C = \frac{1}{12} m l^2 + m\left[\left(\frac{l}{2}\right)^2 + (l \tan 30°)^2\right]$$

$$= \frac{2}{3} m l^2 \quad \left(\text{Note: } \tan 30° = \frac{\sqrt{3}}{3}\right)$$

$$T_1 + U_{1\text{-}2} = T_2 : \; 0 + mg\left[\frac{l}{2} + \frac{l}{2}\sin 30°\right] = \frac{1}{2}\left[\frac{2}{3} m l^2\right]\omega^2$$

$$\omega = \frac{3}{2}\sqrt{\frac{g}{l}}$$

$$v_A = \frac{l}{\cos 30°}\,\frac{3}{2}\sqrt{\frac{g}{l}} = \underline{\sqrt{3gl}}$$

6/147 $U' = M\theta$

$\Delta V_g = 2mg\left(\frac{b}{2} - \frac{b}{2}\cos\theta\right) = mgb(1-\cos\theta)$

Bar BO is rotating about O so

$\Delta T_{BO} = \frac{1}{2}I_O\omega^2 - 0 = \frac{1}{2}\frac{1}{3}mb^2\left(\frac{v_B}{b}\right)^2$

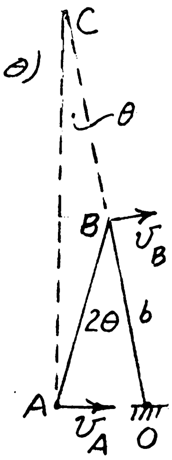
But in the limit as $\theta \to 0$, $v_B = \frac{1}{2}v_A$

so $\Delta T_{BO} = \frac{1}{6}m\frac{v_A^2}{4} = \frac{1}{24}mv_A^2$

Also AB is rotating about C so

$\Delta T_{AB} = \frac{1}{2}I_C\omega^2 = \frac{1}{2}\left[\frac{1}{12}mb^2 + m\left(\frac{3b}{2}\right)^2\right]\left(\frac{v_A}{2b}\right)^2$

$= \frac{7}{24}mv_A^2$



$U' = \Delta T + \Delta V_g$; $M\theta = \frac{7}{24}mv_A^2 + \frac{1}{24}mv_A^2 + mgb(1-\cos\theta)$

$$\underline{v_A = \sqrt{3}\sqrt{\frac{M\theta}{m} - gb(1-\cos\theta)}}$$

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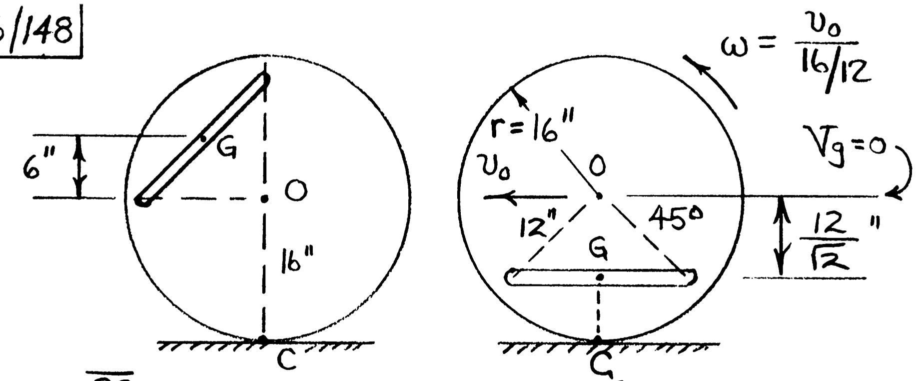

$v_G = \frac{\overline{CG}}{r} v_0$, $\quad \bar{I}_G = \frac{1}{12}\frac{20}{32.2}\left(\frac{12\sqrt{2}}{12}\right)^2 = 0.1035$ lb-ft-sec^2

$U_{1-2}' = 0 = \Delta T + \Delta V_g \qquad (1)$

$\Delta T_{disk} = \frac{1}{2}\frac{100}{32.2}v_0^2 + \frac{1}{2}\left[\frac{1}{2}\frac{100}{32.2}\left(\frac{16}{12}\right)^2\right]\left(\frac{v_0}{16/12}\right)^2 - 0$

$= 2.33\, v_0^2$

$\Delta T_{bar} = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2 = \frac{1}{2}\frac{20}{32.2}\left[\frac{16 - \frac{12}{\sqrt{2}}}{16}v_0\right]^2$

$+ \frac{1}{2}(0.1035)\left(\frac{v_0}{16/12}\right)^2 - 0 = 0.0976\, v_0^2$

$\Delta V_g = -mgh = -20\left(\frac{6 + 12/\sqrt{2}}{12}\right) = -24.1$ ft-lb

Eq. (1): $\quad 0 = (2.33 + 0.0976)v_0^2 - 24.1$

$\underline{v_0 = 3.15 \text{ ft/sec}}$

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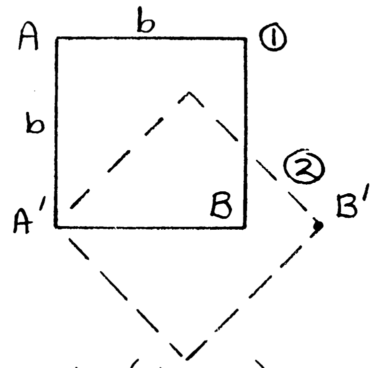
$$\bar{I} = 4\left[\frac{1}{12}\frac{m}{4}b^2 + \frac{m}{4}\left(\frac{b}{2}\right)^2\right]$$
$$= \frac{1}{3}mb^2$$

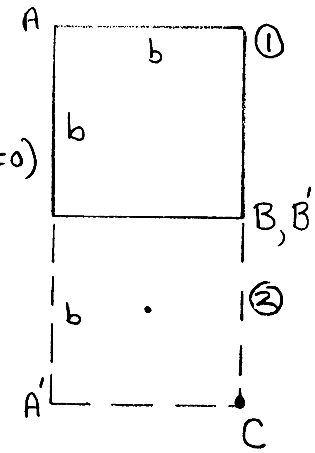
$$I_B = \frac{1}{3}mb^2 + m\left(\frac{b\sqrt{2}}{2}\right)^2$$
$$= \frac{5}{6}mb^2$$



(a) A has dropped distance b $(v_{B'} = 0)$

$$T_1 + U_{1-2} = T_2 : \quad 0 + \frac{mgb}{2} = \frac{1}{2}\left[\frac{5}{6}mb^2\right]\omega^2$$

$$\omega = \sqrt{\frac{6g}{5b}}, \quad v_A = b\omega\sqrt{2} = \underline{\sqrt{\frac{12}{5}gb}}$$



(b) A has dropped distance $2b$ $(v_C = 0)$

$$T_1 + U_{1-2} = T_2 :$$
$$0 + mgb = \frac{1}{2}\left[\frac{5}{6}mb^2\right]\omega^2$$
$$\omega = \sqrt{\frac{12g}{5b}}, \quad v_A = b\omega = \underline{\sqrt{\frac{12}{5}gb}}$$

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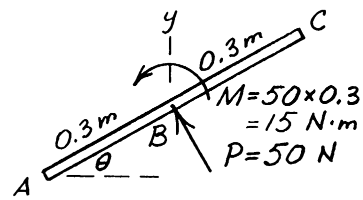
$U'_{1-2} = \Delta T + \Delta V_g$

Replace P by a force P at B
& a couple $M = 15$ N·m



$$U'_{1-2} = \int_0^{0.3\,m} 50 \cos\theta \, dy + 15\frac{\pi}{2}$$

where $\sin\theta = y/0.3$

so $\cos\theta = \frac{10}{3}\sqrt{0.09 - y^2}$

$$= \int_0^{0.3m} \frac{500}{3}\sqrt{0.09-y^2}\, dy + 7.5\pi$$

$$= 11.25\pi = 35.3 \text{ J}$$

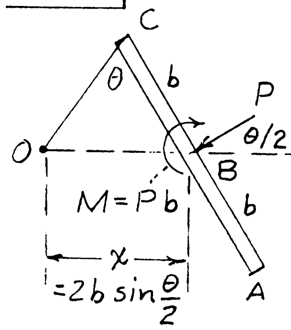
$$\Delta T = \frac{1}{2} I_B \omega^2 = \frac{1}{2}\left(\frac{1}{12} 4 \times 0.6^2\right)\left(\frac{v}{0.3}\right)^2 = \frac{2}{3}v^2 = 0.667v^2$$

$$\Delta V_g = mgh = 4 \times 9.81 \times 0.3 = 11.77 \text{ J}$$

Thus $35.3 = 0.667v^2 + 11.77$, $v^2 = 35.4\ (m/s)^2$,

$\underline{v = 5.95 \text{ m/s}}$

▶6/151 Replace P by force P at B and couple $M = Pb$



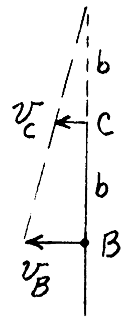
$$dU = M\left(-d\frac{\theta}{2}\right) + P\cos\frac{\theta}{2}(-dx)$$

$$= -\frac{M}{2}d\theta - P\cos\frac{\theta}{2}\left(b\cos\frac{\theta}{2}\right)d\theta$$

$$= \left(-\frac{M}{2} - Pb\cos^2\frac{\theta}{2}\right)d\theta$$

$$U = -\int_{\pi}^{0}\left(\frac{Pb}{2} + Pb\cos^2\frac{\theta}{2}\right)d\theta$$

$$U = \frac{Pb\pi}{2} - Pb\left[\frac{\theta}{2} + \frac{\sin\theta}{2}\right]_{\pi}^{0} = \frac{Pb\pi}{2} + \frac{Pb\pi}{2} = Pb\pi$$



When $\theta = 0$, $v_B = 2v_C$ & $\omega = v_B/2b$

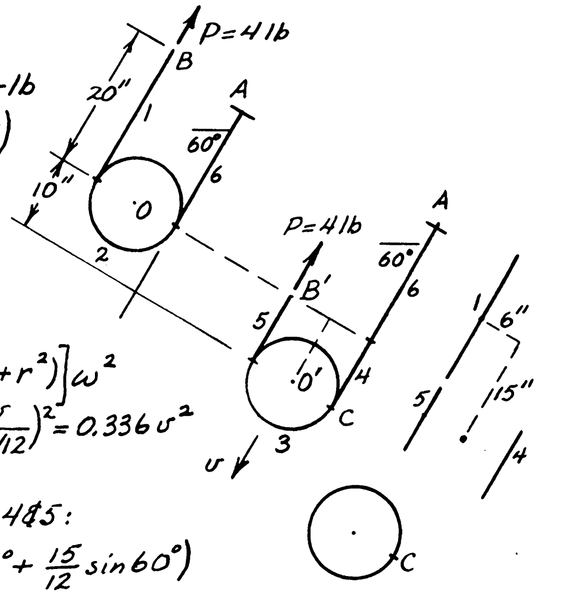
$$\Delta T = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2$$

$$= \frac{1}{2}m(2b\omega)^2 + \frac{1}{2}\left(\frac{1}{12}m[2b]^2\right)\omega^2$$

$$= \frac{13}{6}mb^2\omega^2$$

$U = \Delta T$; $Pb\pi = \frac{13}{6}mb^2\omega^2$, $\underline{\omega = \sqrt{\frac{6P\pi}{13mb}}}$

▸6/152 $U'_{1-2} = \Delta T + \Delta V_g$



$U'_{1-2} = -4(20/12) = -6.67$ ft-lb

$\Delta T_3 = \frac{1}{2} T_{entire\ hoop} = \frac{1}{2}\left(\frac{1}{2} I_C \omega^2\right)$

$= \frac{1}{2}\left(\frac{0.0390}{2}\right)\left(\frac{v}{6/12}\right)^2$

$= 0.0390\, v^2$

$\Delta T_5 = \frac{1}{2}\,\frac{0.8(10/12)}{32.2}(2v)^2$

$= 0.0414\, v^2$

$\Delta T_{wheel} = \frac{1}{2} I_C \omega^2 = \frac{1}{2}\left[m(k_O^2 + r^2)\right]\omega^2$

$= \frac{1}{2}\,\frac{15}{32.2}\left(\left[\frac{4}{12}\right]^2 + \left[\frac{6}{12}\right]^2\right)\left(\frac{v}{6/12}\right)^2 = 0.336\, v^2$

For ΔV_g compare 1 with 4 & 5:

$\Delta V_g = -0.8\left(\frac{20}{12}\right)\left(\frac{6}{12}\cos 60° + \frac{15}{12}\sin 60°\right)$

$= -1.777$ ft-lb

$(I_C)_{complete\ hoop} = 2mr^2$

$= 2\left(\frac{0.8 \times 2\pi \frac{6}{12}}{32.2}\right)\left(\frac{6}{12}\right)^2 = 0.0390$ lb-ft-sec^2

Compare 2 with 3:

$\Delta V_g = -0.8\left(\frac{6}{12}\pi\right)\frac{10}{12}\sin 60° = -0.907$ ft-lb

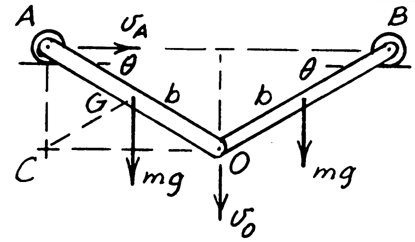
ΔV_g for wheel $= -15\,\frac{10}{12}\sin 60° = -10.83$ ft-lb

Thus $-6.67 = (0.0390 + 0.0414 + 0.336)v^2 - (1.777 + 0.907 + 10.83)$

$v^2 = 16.41$ (ft/sec)^2, $\underline{v = 4.05 \text{ ft/sec}}$

6/153 $dU = dT$

C = instantaneous center of zero velocity for AO



$$dU = 2mg\,d\left(\frac{b}{2}\sin\theta\right)$$
$$= mgb\cos\theta\,d\theta$$

$$dT = 2d\left(\frac{1}{2}I_c\omega^2\right) = 2I_c\,\omega\,d\omega = \frac{2}{3}mb^2\alpha\,d\theta$$

So $mgb\cos\theta\,d\theta = \frac{2}{3}mb^2\ddot{\theta}\,d\theta$ where $\alpha \equiv \frac{d^2\theta}{dt^2}$

$$\underline{\alpha = \ddot{\theta} = \frac{3g\cos\theta}{2b}}$$

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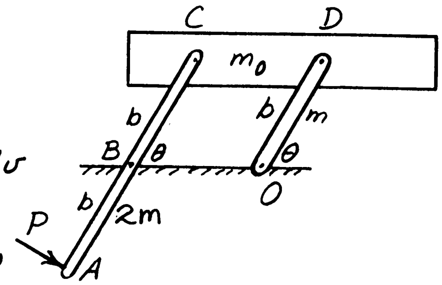


$$dU' = dT + dV_g$$

$$dU' = Pd(b\theta) = Pb\,d\theta$$

$$dT_{platform} = d(\tfrac{1}{2}m_0 v^2) = m_0 v\,dv$$
$$= m_0 a_t\,ds = m_0 a_t\, b\,d\theta$$

$$dT_{OD} = d(\tfrac{1}{2}I_O\omega^2) = \tfrac{1}{3}mb^2\omega\,d\omega$$
$$= \tfrac{1}{3}mb^2\alpha\,d\theta$$

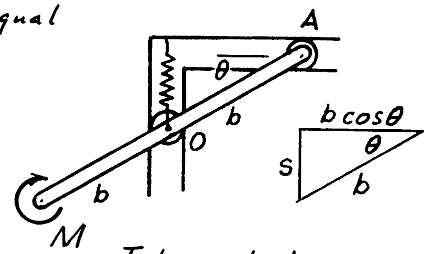
$$dT_{AC} = d(\tfrac{1}{2}I_B\omega^2) = 2dT_{OD} = \tfrac{2}{3}mb^2\alpha\,d\theta$$

$$dV_g = d(\Sigma mgh) = m_0 g\,d(b\sin\theta) + mg\,d(\tfrac{b}{2}\sin\theta)$$
$$= (m_0 + \tfrac{m}{2})gb\cos\theta\,d\theta$$

Thus $Pb\,d\theta = \frac{1}{3}mb^2\alpha\,d\theta + \frac{2}{3}mb^2\alpha\,d\theta + m_0 b^2\alpha\,d\theta$
$+ (m_0 + \frac{m}{2})gb\cos\theta\,d\theta$
$= (m+m_0)b^2\alpha\,d\theta + (m_0 + \frac{m}{2})gb\cos\theta\,d\theta$

$$\underline{\alpha = \ddot{\theta} = \frac{P - (\frac{m}{2}+m_0)g\cos\theta}{b(m+m_0)}}$$ if $P > (\frac{m}{2}+m_0)g\cos\theta$; otherwise $\alpha = 0$.

6/155 For the equilibrium position the work done by the spring is equal and opposite to the work done by the weight, so that



$dU' = dT + dV_e + dV_g$ becomes

$dU' = dT$

$dU' = M(-d\theta)$,

Take α clockwise so $\alpha = -\ddot{\theta}$

$dT = d(\frac{1}{2} m v_o^2 + \frac{1}{2} I_o \omega^2)$

$\quad = m a_o\, ds + I_o \ddot{\theta}\, d\theta$

But $s = b\sin\theta$, $\dot{s} = b\dot{\theta}\cos\theta$, $a_o = \ddot{s} = b\ddot{\theta}\cos\theta - b\dot{\theta}^2\sin\theta$

$\quad = b\ddot{\theta}\cos\theta$ since $\dot{\theta} = 0$

Thus $M(-d\theta) = mb\ddot{\theta}\cos\theta(b\cos\theta\, d\theta) + \frac{1}{12} m (2b)^2 \ddot{\theta}\, d\theta$

$\quad -M = mb^2(\cos^2\theta + \frac{1}{3})\ddot{\theta}$, $M = mb^2(\cos^2\theta + \frac{1}{3})\alpha$, α CW

$$\alpha = \frac{M}{mb^2(\cos^2\theta + \frac{1}{3})}$$

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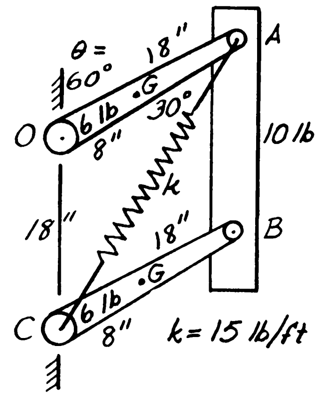
$$dU' = 0 = dT + dV_e + dV_g$$



$$dV_g = 2(6)\,d(8\cos\theta) + 10\,d(18\cos\theta)$$
$$= 276\,d(\cos\theta) = -276\sin\theta\,d\theta \text{ in.-lb}$$

$$dT_{bar} = d(\tfrac{1}{2}mv^2) = mv\,dv = ma_t\,ds$$
$$= \frac{10}{32.2\times12}(18\alpha)18\,d\theta = 8.39\,\alpha\,d\theta$$
where $a_t = r\alpha$, $ds = r\,d\theta$

$$dT_{links} = 2d(\tfrac{1}{2}I_o\omega^2) = 2I_o\omega\,d\omega = 2I_o\alpha\,d\theta$$
$$= 2\left(\frac{6}{32.2\times12}10^2\right)\alpha\,d\theta = 3.11\,\alpha\,d\theta$$

$\overline{CA} = 2(18)\cos30° = 31.2$ in.; stretch $x = 2(18)\cos\frac{\theta}{2} - 18$,

$$dx = 36\left(-\sin\frac{\theta}{2}\,\frac{d\theta}{2}\right)$$

$$dV_e = d(\tfrac{1}{2}kx^2) = kx\,dx = -\frac{15}{12}18\left(2\cos\frac{\theta}{2}-1\right)36\sin\frac{\theta}{2}\,\frac{d\theta}{2}$$
$$= -148.2\,d\theta \text{ in.-lb}$$

Thus $0 = (8.39 + 3.11)\alpha\,d\theta - 148.2\,d\theta - 276\sin60°\,d\theta$

$$\underline{\alpha = 33.7 \text{ rad/sec}^2}$$

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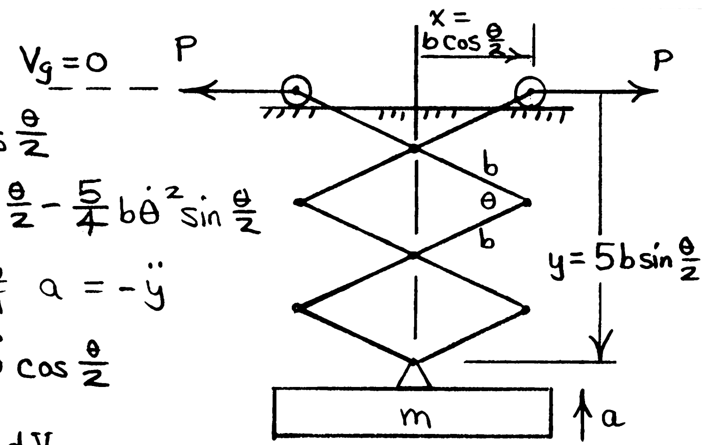


$\dot{y} = \frac{5}{2} b \dot{\theta} \cos\frac{\theta}{2}$

$\ddot{y} = \frac{5}{2} b \ddot{\theta} \cos\frac{\theta}{2} - \frac{5}{4} b \dot{\theta}^2 \sin\frac{\theta}{2}$

For $\dot{\theta} = 0$ & $a = -\ddot{y}$

$a = -\frac{5}{2} b \ddot{\theta} \cos\frac{\theta}{2}$

$dU' = dT + dV_g$

$dU' = +2P d\left(b\cos\frac{\theta}{2}\right) = -Pb\sin\frac{\theta}{2}\,d\theta$

$dT = d\left(\frac{1}{2}mv^2\right) = mv\,dv = ma\,(-dy)$

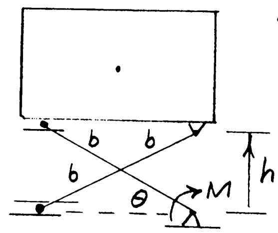
$\quad = -ma\left(\frac{5}{2}b\cos\frac{\theta}{2}\right)d\theta$

$dV_g = d(-mgy) = -mg\frac{5}{2}b\cos\frac{\theta}{2}\,d\theta$

Thus $-Pb\sin\frac{\theta}{2}\,d\theta = -\frac{5}{2}mab\cos\frac{\theta}{2}\,d\theta - \frac{5}{2}mgb\cos\frac{\theta}{2}\,d\theta$

$$\underline{a = \frac{2P}{5m}\tan\frac{\theta}{2} - g}$$

6/158 $dU' = dT + dV_g$; $dU' = M\,d\theta$



$dT = ma\,dh = ma\,d(2b\sin\theta)$

$= 2mba\cos\theta\,d\theta$

$dV_g = mg\,dh = 2mbg\cos\theta\,d\theta$

Thus $M\,d\theta = 2mb\cos\theta\,(a+g)\,d\theta$

$a+g = \dfrac{M}{2mb\cos\theta}$

But $2b\sin\theta = h$

so $\cos\theta = \sqrt{4b^2-h^2}/2b$

$= \sqrt{1-(h/2b)^2}$

so $\underline{a = \dfrac{M}{2mb\sqrt{1-(h/2b)^2}} - g}$

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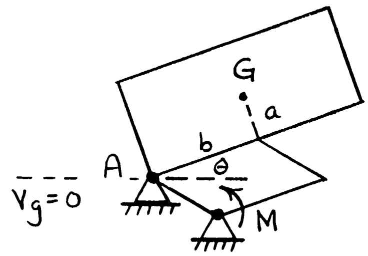

$dU' = dT + dV_g$

$dU' = M\,d\theta$

$dT = d\left(\frac{1}{2} I_A \omega^2\right)$

$= I_A \omega\, d\omega = I_A \alpha\, d\theta$

$dV_g = d(mg\,b \sin\theta + a\cos\theta) = mg\,(b\cos\theta - a\sin\theta)\,d\theta$

Thus $M\,d\theta = I_A \alpha\, d\theta + mg\,(b\cos\theta - a\sin\theta)\,d\theta$

$$\alpha = \frac{1}{I_A}\left[M - mg\,(b\cos\theta - a\sin\theta)\right]$$

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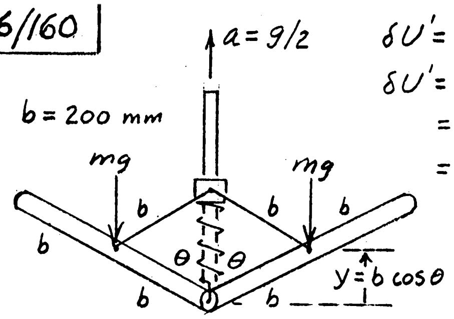


$$\delta U' = \delta T + \delta V_e$$

$$\delta U' = 2mg(-\delta y)$$

$$= 2(2 \times 9.81)(-\delta[b\cos\theta])$$

$$= -39.24(0.200)(-\sin\theta)\delta\theta$$

$$= 7.85 \sin\theta\,\delta\theta \quad J$$

$$\delta T = \Sigma m\underline{\bar{a}}\cdot\delta\underline{s} = 2(2)\frac{9.81}{2}\delta y = 19.62\,\delta(b\cos\theta)$$

$$= -19.62(0.200)\sin\theta\,\delta\theta$$

$$= -3.92 \sin\theta\,\delta\theta$$

$$\delta V_e = kx\,\delta x = 130(2b - 2b\cos\theta)\,\delta(2b - 2b\cos\theta)$$

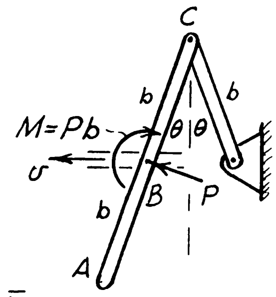
$$= 520\,b^2(1-\cos\theta)\sin\theta\,\delta\theta$$

Thus $7.85\sin\theta\,\delta\theta = -3.92\sin\theta\,\delta\theta + 520\,b^2(1-\cos\theta)\sin\theta\,\delta\theta$

$$\left[(7.85 + 3.92) - 520(0.200)^2(1-\cos\theta)\right]\sin\theta\,\delta\theta = 0$$

$$1-\cos\theta = \frac{11.77}{520(0.200)^2},\quad \cos\theta = 1 - 0.5660 = 0.4340,\quad \underline{\theta = 64.3°}$$

6/161 Replace P by force P at B and couple $M = Pb$



$dU = dT$

$dU = P\cos\theta \, d(2b\sin\theta) + Pb\,d\theta$

$= Pb(2\cos^2\theta + 1)\,d\theta$

$dT_{AC} = d\left(\frac{1}{2}2mv^2 + \frac{1}{2}\bar{I}\omega^2\right)$

$= 2mv\,dv + \bar{I}\omega\,d\omega = 2m\,a\,dx + \bar{I}\alpha\,d\theta$

where $x = 2b\sin\theta$, $v = 2b\dot{\theta}\cos\theta$, $a = 2b(\ddot{\theta}\cos\theta - \dot{\theta}^2\sin\theta)$

$= 2b\ddot{\theta}\cos\theta$ since $\dot{\theta} = 0$

So $dT_{AC} = 2m(2b\ddot{\theta}\cos\theta)\,d(2b\sin\theta) + \frac{1}{12}(2m)(2b)^2\ddot{\theta}\,d\theta$

$= 2mb^2(4\cos^2\theta + \frac{1}{3})\ddot{\theta}\,d\theta$

$dT_{OC} = d\left(\frac{1}{2}I_O\omega^2\right) = I_O\omega\,d\omega = I_O\alpha\,d\theta = \frac{1}{3}mb^2\ddot{\theta}\,d\theta$

So $dT = 2mb^2(4\cos^2\theta + \frac{1}{3})\ddot{\theta}\,d\theta + \frac{1}{3}mb^2\ddot{\theta}\,d\theta$

$= mb^2(8\cos^2\theta + 1)\ddot{\theta}\,d\theta$

$Pb(2\cos^2\theta + 1)\,d\theta = mb^2(8\cos^2\theta + 1)\ddot{\theta}\,d\theta$,

$$\ddot{\theta} = \alpha = \underline{\frac{P(2\cos^2\theta + 1)}{mb(8\cos^2\theta + 1)}}$$

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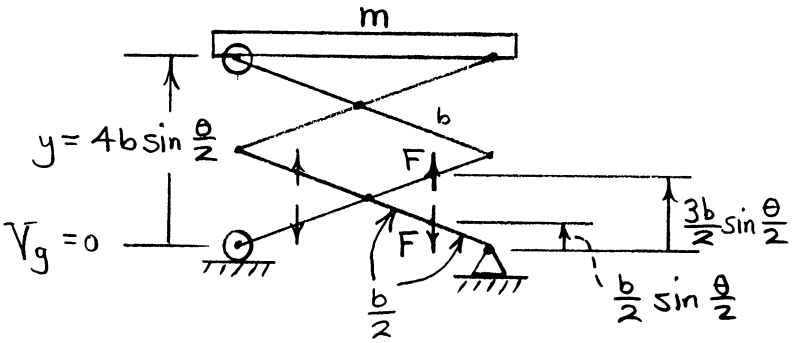

$dU' = dT + dV_g$

$dU' = 2F\,d\left(\frac{3b}{2}\sin\frac{\theta}{2}\right) - 2F\,d\left(\frac{b}{2}\sin\frac{\theta}{2}\right)$

$= 2F\,d\left(b\sin\frac{\theta}{2}\right) = Fb\cos\frac{\theta}{2}\,d\theta$

$dV_g = d\left(mg\,4b\sin\frac{\theta}{2}\right) = 2mgb\cos\frac{\theta}{2}\,d\theta$

$dT = d\left(\frac{1}{2}mv^2\right) = mv\,dv = ma\,dy$

$= ma\left(2b\cos\frac{\theta}{2}\,d\theta\right)$

Thus $Fb\cos\frac{\theta}{2}\,d\theta = 2mgb\cos\frac{\theta}{2}\,d\theta + 2mab\cos\frac{\theta}{2}\,d\theta$

$\underline{a = \frac{F}{2m} - g}$ } Both b and θ cancel so a is independent of both b and θ.

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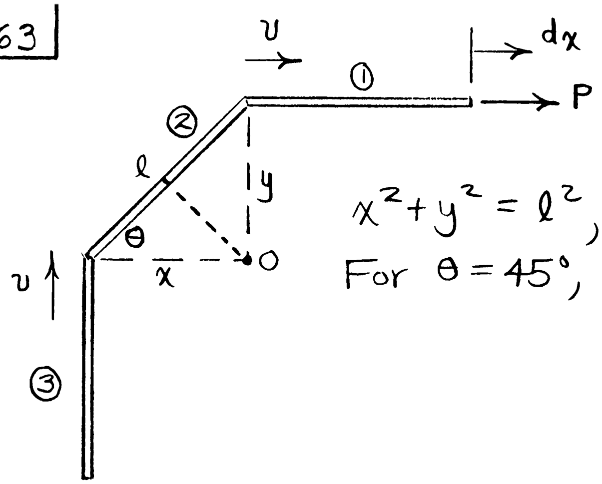

$x^2+y^2=l^2$, $x\,dx+y\,dy=0$

For $\theta=45°$, $x=y$, $dx=-dy$

$$dU' = dT + dV_g \quad ; \quad dU' = P\,dx$$

$$dT = d(T_1+T_2+T_3) = d\left(\tfrac{1}{2}mv^2+\tfrac{1}{2}I_O\omega^2+\tfrac{1}{2}mv^2\right)$$

$$= 2mv\,dv + I_O\omega\,d\omega = 2ma\,dx + I_O\alpha\,|d\theta|$$

$$= 2ma\,dx + \frac{1}{3}ml^2\,\frac{a}{l/\sqrt{2}}\,\frac{dx}{l/\sqrt{2}} = \frac{8}{3}ma\,dx$$

$$dV_g = d(V_{g_1}+V_{g_2}+V_{g_3}) = 0 + mg\,\frac{dx}{2} + mg\,dx$$

$$= \frac{3}{2}mg\,dx$$

Thus $P\,dx = \frac{8}{3}ma\,dx + \frac{3}{2}mg\,dx$

$$\underline{a = \frac{3}{8}\left(\frac{P}{m}-\frac{3g}{2}\right)}$$

6/164 | Radius to each weight is $r = 0.25 + 1.5 \sin\theta$ in.

$$\delta T = 2(m r \omega^2)(-\delta r) = 2 \frac{12}{16(32.2)} \frac{0.25 + 1.5 \sin\theta}{12} \omega^2 (-\delta r) \quad \text{ft-lb}$$

But $\delta r = 1.5 \cos\theta \, \delta\theta$ in.

$\not{c}$ $2(1.5) - 2(1.5)\cos\theta = 0.625 \sin\beta$, $\beta = 15°$

so $\cos\theta = \dfrac{3 - 0.625 \sin 15°}{3} = 0.9461$, $\theta = 18.90°$

$$\text{so } \delta T = \frac{0.25 + 1.5 \sin 18.90°}{8(32.2)} \omega^2 \left(-\frac{1.5 \cos 18.90°}{12}\right) \delta\theta$$

$$= -0.3378(10^{-3}) \omega^2 \delta\theta \quad \text{ft-lb}$$

$$\delta V_e = k x \delta x = 5(12) \frac{2(1.5)}{12}(1 - \cos\theta) \, \delta\left\{\frac{2(1.5)}{12}(1 - \cos\theta)\right\}$$

$$= 3.75(1 - \cos\theta)\sin\theta \, \delta\theta =$$

$$= 3.75(1 - \cos 18.90°)\sin 18.9° \delta\theta = 65.50(10^{-3}) \delta\theta$$

$$\delta U = \delta T + \delta V_e = 0; \; -0.3378(10^{-3})\omega^2 \delta\theta + 65.50(10^{-3}) \delta\theta = 0$$

$$\omega^2 = \frac{65.50}{0.3378} = 193.9 \quad (\text{rad/sec})^2$$

$$\omega = 13.92 \text{ rad/sec}, \quad N = \frac{13.92(60)}{2\pi} = \underline{133.0 \frac{\text{rev}}{\text{min}}}$$

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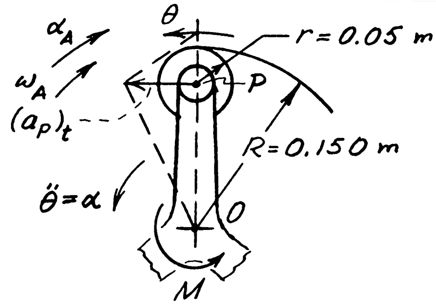

$ds_P = (R-r)\,d\theta$, $d\theta_A = ds_P / r$
$= \left(\frac{R}{r} - 1\right) d\theta$

$v_P = (R-r)\dot{\theta}$, $\omega_A = \frac{v_P}{r} = \left(\frac{R}{r} - 1\right)\dot{\theta}$

$(a_P)_t = (R-r)\alpha$, $\alpha_A = \frac{(a_P)_t}{r} = \left(\frac{R}{r} - 1\right)\alpha$

$$dU = dT_{spider} + dT_{gears}$$

$$dU = M\,d\theta$$

$$dT_{spider} = d\left(\tfrac{1}{2} I_O \omega^2\right) = I_O \omega\,d\omega = I_O \alpha\,d\theta$$

$$dT_{gears} = 3\left\{ d\left(\tfrac{1}{2} I_A \omega_A^2\right) + d\left(\tfrac{1}{2} m_A v_P^2\right)\right\} = 3\left\{ I_A \alpha_A\,d\theta_A + m_A (a_P)_t\,ds_P \right\}$$

$$= 3\left\{ I_A \left(\frac{R}{r} - 1\right)^2 \alpha\,d\theta + m_A (R-r)^2 \alpha\,d\theta \right\}$$

$$= 3(R-r)^2 \left(\frac{I_A}{r^2} + m_A\right)\alpha\,d\theta$$

So $M\,d\theta = \left[I_O + 3(R-r)^2\left(\frac{I_A}{r^2} + m_A\right)\right]\alpha\,d\theta$

$$5 = \left[1.2 \times 0.06^2 + 3(0.150 - 0.050)^2 \left(\frac{0.8 \times 0.030^2}{0.050^2} + 0.8\right)\right]\alpha$$

$$= \left[0.00432 + 0.03 \times 1.088\right]\alpha,$$

$$\underline{\alpha = 135.3 \text{ rad/s}^2}$$

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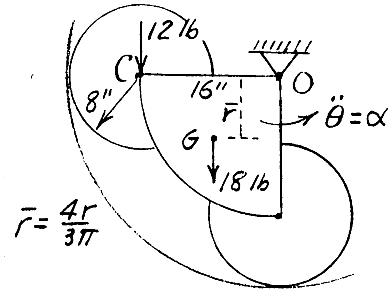
Each wheel: $dT = m_w \bar{a}_w ds_w + \bar{I}_w \alpha_w d\theta_w$



$$= \frac{12}{32.2}\,\frac{16}{12}\alpha\,\frac{16}{12}d\theta + \frac{1}{2}\,\frac{12}{32.2}\left(\frac{8}{12}\right)^2(2\alpha)(2d\theta)$$

$$= \frac{32}{32.2}\alpha\,d\theta$$

where $d\theta_w = 2d\theta$

$\alpha_w = 2\alpha$

Sector: $dT = I_o \alpha\,d\theta = \frac{1}{2}\,\frac{18}{32.2}\left(\frac{16}{12}\right)^2\alpha\,d\theta = \frac{16}{32.2}\alpha\,d\theta$

Combined $dT = 2\left(\frac{32}{32.2}\alpha\,d\theta\right) + \frac{16}{32.2}\alpha\,d\theta = \frac{80}{32.2}\alpha\,d\theta$

$dU = 12\,\frac{16}{12}d\theta + 18\,\frac{4\times 16}{3\pi}\frac{1}{12}d\theta = 16\left(1+\frac{2}{\pi}\right)d\theta = 26.19\,d\theta$

$dU = dT$; $26.19\,d\theta = \frac{80}{32.2}\alpha\,d\theta$, $\alpha = \frac{26.19(32.2)}{80} = \underline{10.54\ \frac{\text{rad}}{\text{sec}^2}}$

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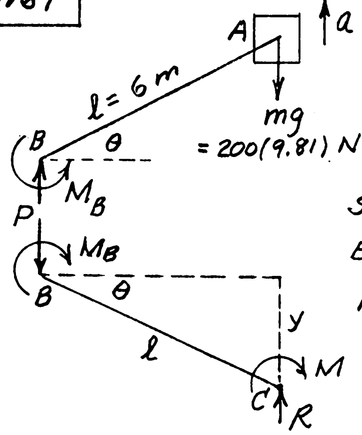


Upper arm ; $dU = dT$

$P\,dy - mg\,2\,dy + M_B\,d\theta = d(\frac{1}{2}mv^2)$

$y = l \sin\theta$, $dy = l\cos\theta\,d\theta$

$d(\frac{1}{2}ml^2) = ma\,d(2y)$

$= 2ma\,l\cos\theta\,d\theta$

So $(P - 2mg)l\cos\theta + M_B = 2mal\cos\theta$

But $P - mg = ma$ so

$M_B = mgl\cos\theta\,(\frac{a}{g} + 1)$

$= 200(9.81)(6)(0.866)(\frac{1.2}{9.81} + 1)$

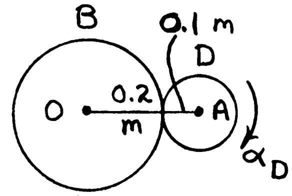
$= 11\,440$ N·m or $\underline{11.44 \text{ kN·m}}$

Lower arm; $\Sigma M = 0$; $M + M_B - Pl\cos\theta = 0$

$M = -mgl\cos\theta\,(\frac{a}{g}+1) + mg(\frac{a}{g}+1)l\cos\theta$, $\underline{M = 0}$

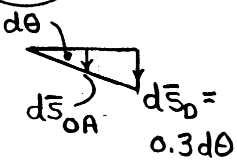
$M = 0$ can be obtained by inspection since m is directly above C. Also, problem can be solved directly by F-m-a equations.

$\boxed{6/168}$ $dU' = dT + dV_g$



$$dU' = \sum m_i a_i \cdot ds_i + \sum I_i \alpha_i \cdot d\theta_i + \sum m_i g\, dh_i$$

Let $\begin{cases} \alpha = \text{angular acceleration of } OA \\ d\theta = \text{angular displacement of } OA \end{cases}$



Arm OA : $\bar{a} = \frac{0.3}{2}\alpha$, $d\bar{s} = \frac{0.3}{2}d\theta$, $dh = -\frac{0.3}{2}d\theta$

$$\bar{I} = \frac{1}{12}(4)(0.3)^2 = 0.03 \text{ kg}\cdot\text{m}^2$$

$$dU'_{arm} = 4\left(\frac{0.3}{2}\alpha\right)\left(\frac{0.3}{2}d\theta\right) + 0.03\alpha\, d\theta - 4(9.81)\left(\frac{0.3}{2}d\theta\right)$$

$$= 0.12\alpha\, d\theta - 5.89\, d\theta$$

Gear D : $\bar{a} = a_A = 0.3\alpha$, $d\bar{s}_D = 0.3d\theta$, $dh = -0.3d\theta$

$$\alpha_D = 3\alpha, \quad d\theta_D = 3\,d\theta$$

$$\bar{I} = m\bar{k}^2 = 5(0.064)^2 = 0.0205 \text{ kg}\cdot\text{m}^2$$

$$dU'_D = 5(0.3\alpha)(0.3d\theta) + 0.0205(3\alpha)(3d\theta) - 5(9.81)(0.3d\theta) = 0.634\alpha\, d\theta - 14.72\, d\theta$$

For system : $dU' = dU'_{arm} + dU'_D = 0$

$$0.12\alpha\, d\theta - 5.89\, d\theta + 0.634\alpha\, d\theta - 14.72\, d\theta = 0$$

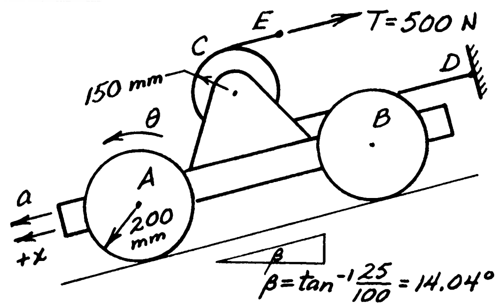
$$\underline{\alpha = 27.3 \text{ rad/s}^2}$$

6/169 $dU'_{1-2} = dT + dV_g$

Let x = displacement of vehicle down slope

$\bar{I}_A = \bar{I}_B = mk^2 = 140(0.150)^2$
$= 3.15 \text{ kg}\cdot\text{m}^2$

$\bar{I}_C = 40(0.100)^2 = 0.4 \text{ kg}\cdot\text{m}^2$



$a = r\alpha$

$dU'_{1-2} = -500\,(2\,dx) = -1000\,dx$

$(dT_{wheels})_{rotation\ only} = 2d\left(\frac{1}{2}\bar{I}_A\omega^2\right) = 2\bar{I}_A\omega\,d\omega = 2\bar{I}_A\alpha\,d\theta = 2\bar{I}_A\frac{a}{r_A}\frac{dx}{r_A}$
$= 2\times 3.15\,\frac{a\,dx}{0.2^2} = 157.5\,a\,dx$

$(dT_{drum})_{rotation\ only} = d\left(\frac{1}{2}\bar{I}_C\,\omega_C^2\right)$
$= \bar{I}_C\,\omega_C\,d\omega_C = \bar{I}_C\,\alpha_C\,d\theta_C = \bar{I}_C\frac{a}{r_C}\frac{dx}{r_C} = 0.4\frac{a\,dx}{0.150^2} = 17.78\,a\,dx$

$dT_{vehicle\ translation} = d\left(\frac{1}{2}mv^2\right) = mv\,dv = ma\,dx = 520\,a\,dx$

$dV_g = -mg\,dh = -520(9.81)\,dx\sin 14.04° = -1237\,dx$

Thus $-1000\,dx = (157.5 + 17.78 + 520)\,a\,dx - 1237\,dx$,

$\underline{a = 0.341 \text{ m/s}^2}$

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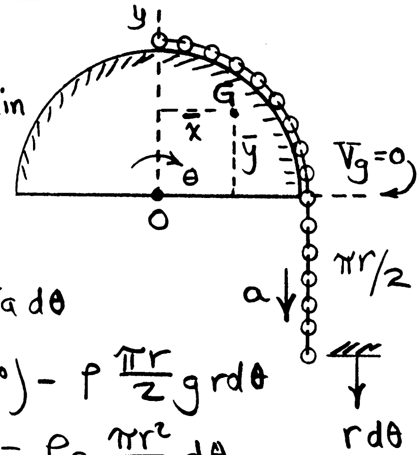


$dU' = dT + dV_g$ for entire chain

$dU' = 0$

$dT = d\left(\frac{1}{2}mv^2\right) = mv\,dv$

$\quad = ma \cdot r\,d\theta = \rho \pi r^2 a\,d\theta$

$dV_g = -\rho g \frac{\pi r}{2}\left(\bar{r}\,d\theta \cos 45°\right) - \rho \frac{\pi r}{2} g\,r\,d\theta$

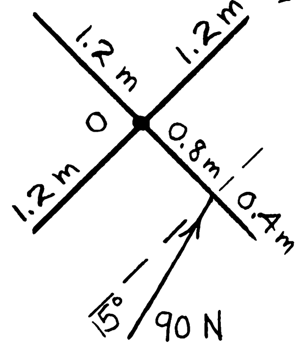
$\quad = -\rho g \frac{\pi r}{2} \frac{2r}{\pi} \sqrt{2}\,d\theta \frac{1}{\sqrt{2}} - \rho g \frac{\pi r^2}{2}\,d\theta$

$\quad = -\rho g \pi r^2 \left(\frac{1}{\pi} + \frac{1}{2}\right) d\theta$

So $0 = \rho \pi r^2 a\,d\theta - \rho g \pi r^2 \left(\frac{1}{\pi} + \frac{1}{2}\right) d\theta$

$$\underline{a = \left(\frac{1}{\pi} + \frac{1}{2}\right) g}$$

6/171 $I_O = 4\left(\frac{1}{3}ml^2\right) = 4\left(\frac{1}{3}60\,(1.2)^2\right) = 115.2 \text{ kg}\cdot\text{m}^2$



$\circlearrowleft + \Sigma M_O = I_O \alpha$:

$(90 \cos 15°)(0.8) = 115.2\,\alpha$

$\underline{\alpha = 0.604 \text{ rad/s}^2}$

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$$\int_{t_1}^{t_2} \Sigma M_G \, dt = \bar{I}(\omega_2 - \cancel{\omega_1}^{\,0}) = m\bar{k}^2 \omega$$

$$\int_0^3 10\left(1 - e^{-t}\right) dt = 75\,(0.5)^2\,\omega$$

$$10\left[t + e^{-t}\right]\Big|_0^3 = 75\,(0.5)^2\,\omega, \qquad \underline{\omega = 1.093 \text{ rad/s}}$$

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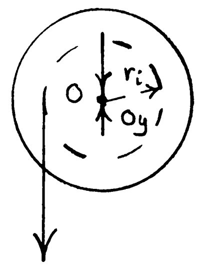
$m = 14\ \text{kg},\ \bar{k} = 0.225\ \text{m} \qquad t = 4\ \text{s}$

$r_i = 0.215\ \text{m},\ r_o = 0.325\ \text{m}$

(a)

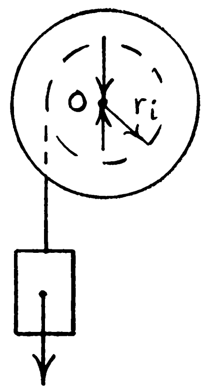


$$\circlearrowleft + \int_0^t \Sigma M_O\, dt = \bar{I}(\omega - \omega_0):$$

$$98.1(0.215)4 = 14(0.225)^2(\omega - 0)$$

$$\underline{\omega = 119.0\ \text{rad/s}}$$

(b)



$$\int_0^t \Sigma M_O\, dt = H_{O_2} - H_{O_1}$$

$$98.1(0.215)4 = 14(0.225)^2(\omega - 0) + 10(0.215\omega - 0)(0.215)$$

$$\underline{\omega = 72.0\ \text{rad/s}}$$

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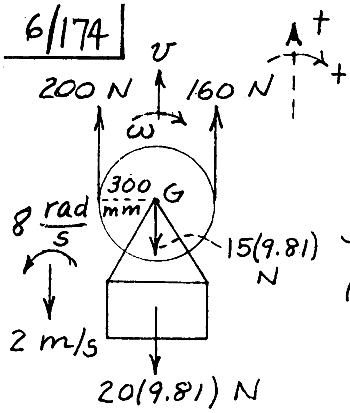

$$\int \Sigma F \, dt = \Delta G$$

$$(360 - 35 \times 9.81)5 = 35(v - [-2])$$

$$\underline{v = 0.379 \text{ m/s up}}$$

$$\int \Sigma M_G \, dt = \Delta H_G$$

$$(200-160)0.3(5) = 15(0.25)^2(\omega - [-8])$$

$$\underline{\omega = 56.0 \text{ rad/s cw}}$$

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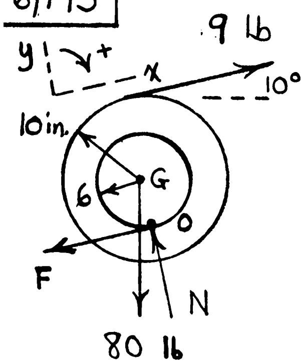

$$\int_{t_1}^{t_2} \Sigma M_O \, dt = H_{O_2} - H_{O_1}$$

$$\int_0^8 \left[\left(\frac{16}{12}\right)(9) - \left(\frac{6}{12}\right)(80 \sin 10^\circ)\right] dt$$

$$= \frac{80}{32.2}\left[\left(\frac{8}{12}\right)^2 + \left(\frac{6}{12}\right)^2\right] \omega$$

$\omega = 23.4$ rad/sec

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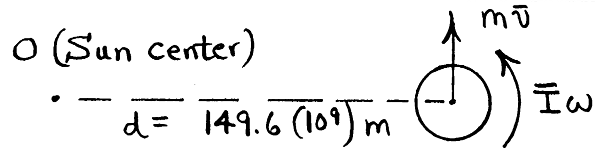

$$\bar{H} = \bar{I}\omega = \frac{2}{5}mr^2\left(\frac{2\pi}{\tau}\right)$$

$$= \frac{2}{5}(5.976\cdot10^{24})(6.371\cdot10^{6})^2 \frac{2\pi}{23.9344(3600)}$$

$$= 7.08(10^{33})\ \text{kg}\cdot\text{m}^2/\text{s}$$

$$\bar{v} = \sqrt{\frac{Gm_s}{d}} = \sqrt{\frac{6.673(10^{-11})(333{,}000)(5.976\cdot10^{24})}{149.6(10^9)}}$$

$$= 29\ 800\ \text{m/s}$$

$$m\bar{v}d = 5.976(10^{24})(29\ 800)(149.6\cdot10^9)$$

$$= 2.66(10^{40})\ \text{kg}\cdot\text{m}^2/\text{s}$$

$$H = \bar{I}\omega + m\bar{v}d = \underline{2.66(10^{40})\ \text{kg}\cdot\text{m}^2/\text{s}}$$

(The $\bar{I}\omega$ term is insignificant compared with the $m\bar{v}d$ term.)

6/177 | $H_{O_1} = H_{O_2}$: $mvb = (I_O + mb^2)\omega$

$$\frac{2}{16}\,\frac{1}{32.2}\,(1500)\,\frac{15}{12} = \left[\frac{1}{3}\,\frac{20}{32.2}\left(\frac{30}{12}\right)^2 + \frac{2}{16}\,\frac{1}{32.2}\left(\frac{15}{12}\right)^2\right]\omega$$

$\underline{\omega = 5.60 \text{ rad/sec}}$

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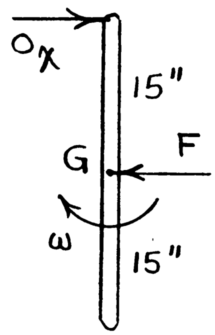

$$\int \Sigma M_G \, dt = \bar{H}_2 - \bar{H}_1 :$$

$$O_x \frac{15}{12}(0.001) = \frac{1}{12}\frac{20}{32.2}\left(\frac{30}{12}\right)^2(\omega - 0),$$

where $\omega = 5.60$ rad/sec from Prob. 6/177.

$$\Rightarrow \underline{O_x = 1449 \text{ lb}}$$

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$\overset{+}{\circlearrowleft}$ $H_{O_1} = H_{O_2}$ for system

$$m v h = (I_O + m h^2)\omega$$

$$\left(\frac{1/16}{32.2}\right)(1600)\left(\frac{43}{12}\right) = \left[\frac{55}{32.2}\left(\frac{37}{12}\right)^2 + \frac{1/16}{32.2}\left(\frac{43}{12}\right)^2\right]\omega$$

$$\underline{\omega = 0.684 \text{ rad/sec}}$$

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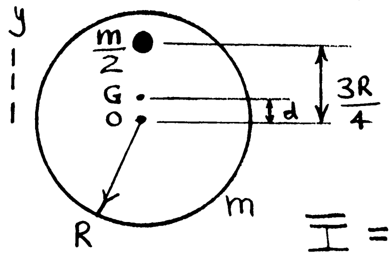

$$d = \frac{\Sigma \bar{y} m}{\Sigma m} = \frac{\left(\frac{m}{2}\right)\left(\frac{3R}{4}\right)}{m + m/2} = \frac{R}{4}$$

$$I_O = \frac{1}{2} m R^2 + \frac{m}{2}\left(\frac{3R}{4}\right)^2$$

$$= \frac{25}{32} m R^2$$

$$\bar{I} = I_O - m d^2 = \frac{25}{32} m R^2 - \frac{3}{2} m \left(\frac{R}{4}\right)^2$$

$$= \frac{11}{16} m R^2$$

$$H_G = \bar{I}\omega = \frac{11}{16} m R^2 \left(\frac{v_0}{R}\right) = \underline{\frac{11}{16} m R v_0}$$

$$H_O = \bar{I}\omega + m \bar{v} d = \frac{11}{16} m R v_0 + \frac{3}{2} m \left(\frac{5R}{4} \frac{v_0}{R}\right)\left(\frac{R}{4}\right)$$

$$= \underline{\frac{37}{32} m R v_0}$$

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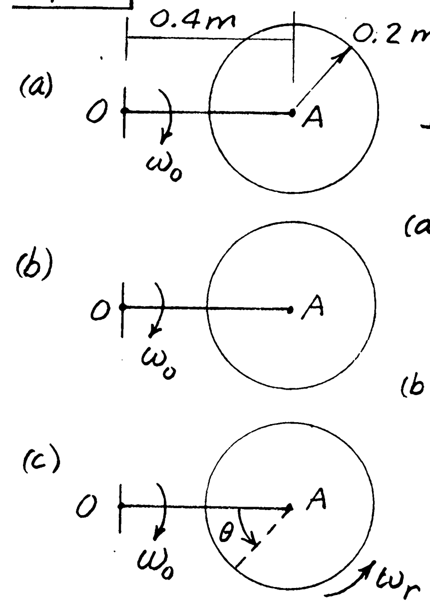


$\omega_0 = 4$ rad/s

ω = angular velocity of disk

$\bar{I} = I_A = \frac{1}{2} m r^2 = \frac{1}{2} 25 (0.2)^2$

$= \frac{1}{2}$ kg·m²

(a) $\omega = \omega_0$

$H_O = I_O \omega = \left(\frac{1}{2} + 25[0.4]^2\right) 4$

$= \underline{18 \text{ kg}\cdot\text{m}^2/\text{s}}$

(b) $\omega = 0$

$H_O = m\bar{v}d = 25(0.4)(4)(0.4)$

$= \underline{16 \text{ kg}\cdot\text{m}^2/\text{s}}$

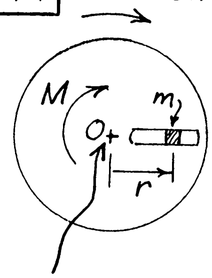
(c) $\omega = \omega_0 - \omega_r = 4 - 8 = -4$ rad/s

$H_O = \bar{I}\omega + m\bar{v}d$

$= \frac{1}{2}(-4) + 16$

$= \underline{14 \text{ kg}\cdot\text{m}^2/\text{s}}$

6/182 ω const.



$H_O = I_O\omega + mr^2\omega$

where I_O = moment of inertia of disk about O

$\Sigma M_O = \dot{H}_O$; $M = 0 + 2mr\dot{r}\omega$

$\underline{M = 2mr\dot{r}\omega}$

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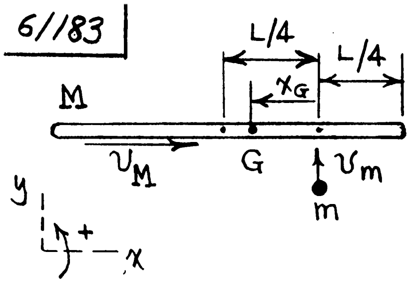


G is <u>system</u> mass center.

$$x_G = \frac{ML/4}{M+m}$$

x mom. : $M v_M = (M+m) v_x$, $\underline{v_x = \dfrac{M v_M}{M+m}}$

y mom : $m v_m = (M+m) v_y$, $\underline{v_y = \dfrac{m v_m}{M+m}}$

ang. mom_G: $m v_m \left(\dfrac{ML/4}{M+m}\right) = \left[\dfrac{1}{12} ML^2 + M\left(\dfrac{L}{4} - \dfrac{ML/4}{M+m}\right)^2 + m\left(\dfrac{ML/4}{M+m}\right)^2\right] \omega$

Solving, $\underline{\omega = \dfrac{12 v_m}{L}\left(\dfrac{m}{4M+7m}\right)}$ ↶

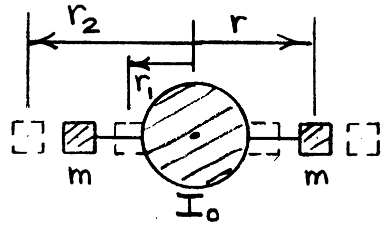
6/184 Approximate the diver's body as a uniform slender bar in the first case and as a sphere in the second case. Conservation of angular momentum $H_1 = H_2$:

$$\frac{1}{12}\cancel{m}l^2 N_1 = \frac{2}{5}\cancel{m}r^2 N_2$$

$$\frac{1}{12}(2)^2(0.3) = \frac{2}{5}\left(\frac{0.7}{2}\right)^2 N_2$$

$$\underline{N_2 = 2.04 \text{ rev/s}}$$

6/185



$H = I_0\omega_0 + 2mr^2\omega_0$

$\dot{H} = 4mr\dot{r}\omega_0$

$r = r_1 + \frac{\Delta r}{\Delta t}\, t$

$= 1.2 + \frac{4.5 - 1.2}{120}\, t$

$= 1.2 + 0.02750\, t$

$\dot{r} = 0.02750$ m/s

$M = \dot{H}$, $\quad 2T(1.1) = 4(10)(1.2 + 0.0275t)(0.0275)$

$\times (1.25)$

$\underline{T = 0.750 + 0.01719t \text{ N}}$

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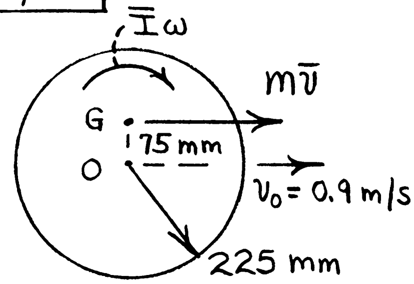
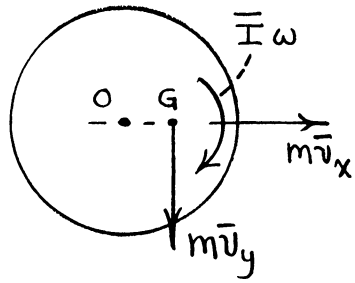


$$\bar{I} = I_0 - md^2$$
$$= 8\left[0.15^2 - 0.075^2\right]$$
$$= 0.1350 \text{ kg}\cdot\text{m}^2$$

(a) $H_0 = \bar{I}\omega + m\bar{v}d$

$$= 0.1350\frac{0.9}{0.225} + 8\left(\frac{300}{225}0.9\right)0.075$$
$$= \underline{1.260 \text{ kg}\cdot\text{m}^2/\text{s}}$$



(b) $H_0 = \bar{I}\omega + m\bar{v}_y d$

$$= 0.1350\,\frac{0.9}{0.225} + 8\,\frac{75}{225}\,0.9\,(0.075)$$
$$= \underline{0.720 \text{ kg}\cdot\text{m}^2/\text{s}}$$

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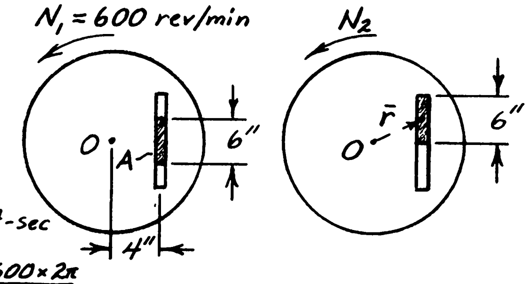

For system, $\Sigma M_O = 0$ so

$\Delta H_O = 0$, $H_1 = H_2$

$$(H_1)_{disk} = \frac{8}{32.2}\left(\frac{6}{12}\right)^2 \frac{600\times 2\pi}{60} \text{ lb-ft-sec}$$

$$(H_1)_{bar} = \left[\frac{1}{12}\frac{2}{32.2}\left(\frac{6}{12}\right)^2 + \frac{2}{32.2}\left(\frac{4}{12}\right)^2\right]\frac{600\times 2\pi}{60} \text{ lb-ft-sec}$$

$\bar{r}^2 = 4^2 + 3^2 = 5^2$ in.^{2}

$$(H_2)_{disk} = \frac{8}{32.2}\left(\frac{6}{12}\right)^2 \frac{2\pi N_2}{60} \text{ lb-ft-sec}, \ N_2 \text{ in rev/min}$$

$$(H_2)_{bar} = \left[\frac{1}{12}\frac{2}{32.2}\left(\frac{6}{12}\right)^2 + \frac{2}{32.2}\left(\frac{5}{12}\right)^2\right]\frac{2\pi N_2}{60} \text{ lb-ft-sec}$$

Factor out $\frac{1}{32.2}\times\frac{1}{12^2}\times\frac{2\pi}{60}$ & get

$$\left(8\times 6^2 + \frac{2}{12}\times 6^2 + 2\times 4^2\right)600 = \left(8\times 6^2 + \frac{2}{12}\times 6^2 + 2\times 5^2\right)N_2,$$

$$\underline{N_2 = 569 \text{ rev/min}}$$

Friction forces in the slot are internal so have no effect on ΣM_O. Hence the final value of N_2, as well as the loss of energy, is unaffected.

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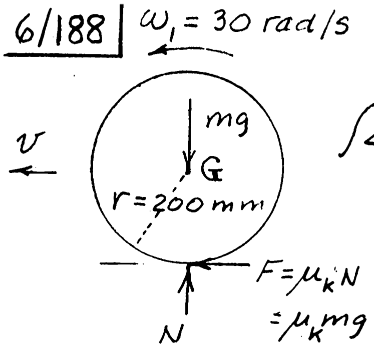


Slipping occurs until $v = r\omega$

$\int \Sigma F\,dt = \Delta G;\quad \mu_k m g t = m v - 0$

$\mu_k g t = v$

$\int \Sigma M_G\,dt = \Delta H_G;$ (+ counterclockwise)

$\mu_k m g r t = m r^2\left[-\frac{v}{r} - (-\omega_1)\right]$

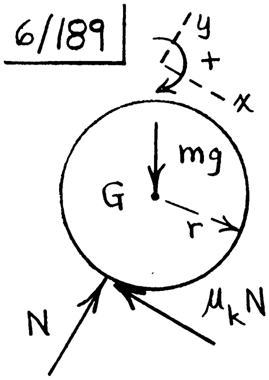
$\mu_k g t = -v + r\omega_1$

Eliminate v & get $2\mu_k g t = r\omega_1$

so $\mu_k = \frac{r\omega_1}{2gt} = \frac{0.2(30)}{2(9.81)(1.5)} = \underline{0.204}$

$v = \mu_k g t = \frac{r\omega_1}{2} = \frac{0.2(30)}{2} = \underline{3\ m/s}$

$\boxed{6/189}$



$$\int_0^t \Sigma F_y \, dt = m(v_y - v_{y_0}) = 0 \Rightarrow N = mg\cos\theta$$

$$\int_0^t \Sigma F_x \, dt = m(v_x - v_{x_0}) :$$

$$(-\mu_k mg\cos\theta + mg\sin\theta)t = m(v - v_0) \quad (1)$$

$$\int_0^t \Sigma M_G \, dt = \bar{I}(\omega - \omega_0) :$$

$$(\mu_k mg\cos\theta \, r)t = \frac{2}{5}mr^2\omega \quad (2)$$

We desire the time t when $v = r\omega$ (3)

Solution of Eqs. (1)–(3) :

$$\begin{cases} t = \dfrac{2v_0}{g(7\mu_k\cos\theta - 2\sin\theta)} \\[2ex] v = \dfrac{5v_0\mu_k}{7\mu_k - 2\tan\theta} \\[2ex] \omega = \dfrac{5v_0\mu_k/r}{7\mu_k - 2\tan\theta} \end{cases}$$

For slipping to cease,

$$7\mu_k\cos\theta > 2\sin\theta$$

or $\mu_k > \frac{2}{7}\tan\theta$

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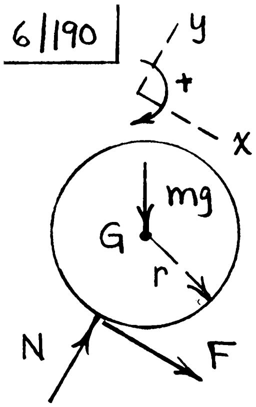

$$\int_0^t \Sigma F_y \, dt = m(v_y - v_{y_0}) = 0 \Rightarrow N = mg\cos\theta$$

$$\int_0^t \Sigma F_x \, dt = m(v_x - v_{x_0}):$$

$$(+\mu_k mg\cos\theta + mg\sin\theta)t = mv \quad (1)$$

$$\int_0^t \Sigma M_G \, dt = \bar{I}(\omega - \omega_0):$$

$$(-\mu_k mgr\cos\theta)t = \frac{2}{5}mr^2(\omega - \omega_0) \quad (2)$$

Slipping ceases when $v = r\omega$ (3)

Solution of Eqs. (1)-(3): $t = \dfrac{2r\omega_0}{g(2\sin\theta + 7\mu_k\cos\theta)}$

Note that the effect of the ramp is to decrease t.

$$v = \frac{2r\omega_0(\sin\theta + \mu_k\cos\theta)}{(2\sin\theta + 7\mu_k\cos\theta)}$$

$$\omega = \frac{2\omega_0(\sin\theta + \mu_k\cos\theta)}{(2\sin\theta + 7\mu_k\cos\theta)}$$

6/191 Conservation of angular momentum about the vertical spin axis of the platform:

$$H_1 = H_2$$

$$\left[10(0.3)^2\right]\left(250\,\frac{2\pi}{60}\right) = \left[I + \frac{1}{2}(10)(0.3)^2 + 10(0.6)^2\right] \times \left(30\,\frac{2\pi}{60}\right)$$

$$\underline{I = 3.45\ \mathrm{kg \cdot m^2}}$$

6/192 Conservation of angular momentum about the vertical spin axis of the platform:

$$H_1 = H_2$$

$$[10(0.3)^2][250] = [3.45 + 10(0.6)^2]N - 10(0.3)^2[250]$$

$$\underline{N = 63.8 \text{ rev/min}}$$

6/193 $\Delta H = 0$;

Initial: $H_{rods} = 2I\omega = 2(1.5)(0.060)^2 \frac{300 \times 2\pi}{60}$ N·m·s

$H_{base} = mk^2\omega = 4(0.040)^2 \frac{300 \times 2\pi}{60}$ N·m·s

Final: $H_{rods} = 2\left[\bar{I} + md^2\right]\omega = 2m\left[\frac{\ell^2}{12} + d^2\right]\frac{2\pi N}{60}$

$= 2(1.5)\left[\frac{0.3^2}{12} + (0.150 + 0.060)^2\right]\frac{2\pi N}{60}$

$= 0.1548\left(\frac{2\pi N}{60}\right)$ N·m·s

$H_{base} = 4(0.040)^2 \frac{2\pi N}{60} = 0.0064\left(\frac{2\pi N}{60}\right)$

Thus $\left[3(0.06)^2 + 4(0.04)^2\right]300 = [0.1548 + 0.0064]N$

$0.0172(300) = 0.1612\,N$, $\underline{N = 32.0 \text{ rev/min}}$

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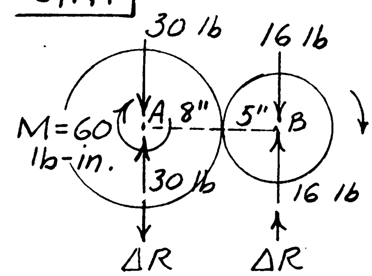

$$\int \Sigma M_A \, dt = \Delta H_{A_{system}}$$

$$\circlearrowright + \quad \left(\frac{60}{12} - \Delta R \frac{13}{12}\right) 4 = 12 - 4$$

$$\Delta R = 2.77 \text{ lb}$$

$$R_B = 16 + 2.77 = \underline{18.77 \text{ lb}}$$

$$R_A = 30 - 2.77 = \underline{27.23 \text{ lb}}$$

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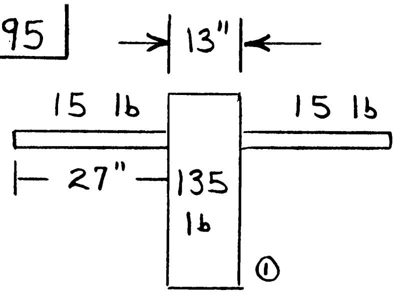
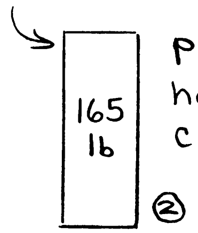


Assume arms become part of homogeneous cylinder



Conservation of angular momentum about a vertical axis : $H_1 = H_2$

$$\left\{\frac{1}{2}\frac{135}{32.2}\left(\frac{13}{2\cdot 12}\right)^2 + 2\left[\frac{1}{12}\frac{15}{32.2}\left(\frac{27}{12}\right)^2 + \frac{15}{32.2}\left(\frac{13+27}{2\cdot 12}\right)^2\right]\right\} \times 1 = \left\{\frac{1}{2}\frac{165}{32.2}\left(\frac{13}{2\cdot 12}\right)^2\right\} N$$

$\underline{N = 4.78 \text{ rev/sec}}$

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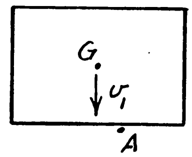
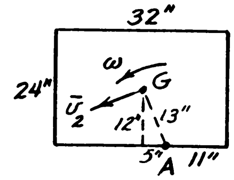
$v_1 = \sqrt{2gh} = \sqrt{2 \times 32.2 \times \frac{12}{12}}$

$= 8.02$ ft/sec



Before impact



After impact

Impulse of weight during impact is negligible, so

$\Delta H_A = 0$

$H_{A_1} = mvd = \frac{171}{32.2} \times 8.02 \times \frac{5}{12} = 17.76$ lb-ft-sec

$H_{A_2} = I_A \omega = \left[\frac{1}{12}\frac{171}{32.2}\left\{\left(\frac{32}{12}\right)^2 + \left(\frac{24}{12}\right)^2\right\} + \frac{171}{32.2}\left(\frac{13}{12}\right)^2\right]\omega = 11.15\omega$ lb-ft-sec

$17.76 = 11.15\,\omega$, $\underline{\omega = 1.593 \text{ rad/sec}}$

$$\left.\begin{aligned} T_1 &= \tfrac{1}{2} m v_1^2 = Wh = 171 \times \tfrac{12}{12} = 171 \text{ ft-lb} \\ T_2 &= \tfrac{1}{2} I_A \omega^2 = \tfrac{1}{2} \times 11.15 \times 1.593^2 = 14.14 \text{ ft-lb} \end{aligned}\right\} \; n = \frac{171 - 14.14}{171} \times 100\%,$$

$\underline{n = 91.7\%}$

6/197

$$H_{z_1} = H_{z_2} \quad \text{(for system)}$$

$$0 = (I - I_w)\omega_s + I_w(\omega_s + \omega_{w/s})$$

But for wheel alone: $\int \Sigma M_G \, dt = \Delta H_G$

$$Mt = I_w(\omega_s + \omega_{w/s})$$

So $\quad 0 = (I - I_w)\omega_s + Mt$

$$\underline{\omega_s = -Mt/(I - I_w)}$$

$$\omega_{w/s} = \frac{Mt}{I_w} - \omega_s = \underline{\frac{I}{I_w}\frac{Mt}{(I - I_w)}}$$

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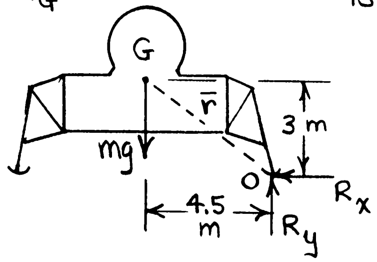
$k_G = 1.8$ m



Impulse of mg during impact interval is small and is neglected.

Before impact, $\bar{v} = \frac{8}{3.6} = 2.22 \frac{m}{s}$

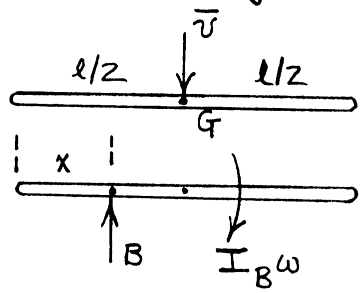
$\Delta H_O = 0$:

$$\cancel{m}\bar{v}d = \cancel{m}(\bar{k}^2 + \bar{r}^2)\omega$$

$$2.22(4.5) = ((1.8)^2 + (4.5)^2 + (3)^2)\omega$$

$$\underline{\omega = 0.308 \text{ rad/s}}$$

6/199 Velocity of bar at impact $= \sqrt{2gh} = \bar{v}$



Neglect small impulse of weight.

$\Delta H_B = 0$

$I_B \omega = m\bar{v}\left(\frac{l}{2} - x\right)$

$$I_B = \frac{1}{12} m l^2 + m\left(\frac{l}{2} - x\right)^2 = \frac{1}{3} m l^2 - m l x + m x^2$$

Thus $$\omega = \frac{\left(\frac{l}{2} - x\right)\sqrt{2gh}}{\left(\frac{1}{3} l^2 - l x + x^2\right)}$$

$$\omega_{x=0} = \frac{3}{2l}\sqrt{2gh}\ , \quad \omega_{x=l/2} = 0$$

$$\omega_{x=l} = -\frac{3}{2l}\sqrt{2gh}$$

6/200 $\Delta H_O = 0: \; I\omega_1 + 4mr_1^2\omega_1 - (I + 4mr_2^2)\omega_2 = 0$

$$6(175)^2(120) + 4(0.15)(100)^2(120) = \left[6(175)^2 + 4(0.15)(200)^2\right]\omega$$

$\underline{\omega = 109.6 \text{ rev/min}}$

$$|\Delta E| = T_1 - T_2 = \tfrac{1}{2}I\omega_1^2 + 4(\tfrac{1}{2})m(r_1\omega_1)^2 - \tfrac{1}{2}I\omega_2^2 - 4(\tfrac{1}{2})m(r_2\omega_2)^2$$

$$= \tfrac{1}{2}\left(120\tfrac{2\pi}{60}\right)^2\left[6(0.175)^2 + 4(0.15)(0.100)^2\right] - \tfrac{1}{2}\left(109.6\tfrac{2\pi}{60}\right)^2\left[6(0.175)^2 + 4(0.15)(0.200)^2\right]$$

$= \underline{1.298 \text{ J}}$

Neglecting diameter of ball amounts to neglecting $\bar{I}\omega$ of ball compared with the moment $mr^2\omega$ of its linear momentum.

$\underline{6/201}$ $\theta = 0$: $(I_{panel})_z = \frac{1}{12} m l^2 + m d^2$

$$= \frac{1}{12} \frac{16.1}{32.2} 6^2 + \frac{16.1}{32.2} 5^2 = 14 \text{ lb-ft-sec}^2$$

$$(I_{body})_z = \frac{322}{32.2} 1.5^2 = 22.5 \text{ lb-ft-sec}^2$$

$\theta = 90°$: $(I_{panel})_z = \frac{1}{12} m(a^2 + b^2) + md^2$

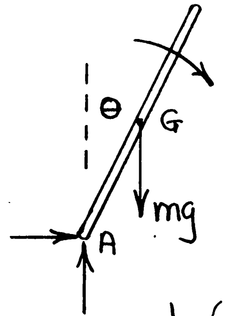
$$= \frac{1}{12} \frac{16.1}{32.2} (4^2 + 6^2) + \frac{16.1}{32.2} 5^2 = 14.67 \text{ lb-ft-sec}^2$$

$$H_{\theta=0} = (22.5 + 14 + 14)(1.0) = 50.5 \text{ lb-ft-sec}$$

$$H_{\theta=\frac{\pi}{2}} = (22.5 + 14.67 + 14.67)\omega = 51.8\omega \text{ lb-ft-sec}$$

$\Delta \overline{H}_z = 0$: $50.5 = 51.8\omega$, $\underline{\omega = 0.974 \text{ rad/sec}}$

6/202



Angular impulse of mg is negligible during impact.

$$\Delta H_A = 0: \quad mv\frac{L}{2}\sin\theta = \frac{1}{3}mL^2\omega$$

$$\omega = \frac{3v}{2L}\sin\theta$$

During rotation, $\Delta T + \Delta V_g = 0$

$$\frac{1}{2}\left(\frac{1}{3}ml^2\right)\left(\omega'^2 - \omega^2\right) - mg\frac{L}{2}\cos\theta = 0$$

With $v' = L\omega'$ and ω from above,

$$\underline{v' = \sqrt{\frac{9v^2}{4}\sin^2\theta + 3gL\cos\theta}}$$

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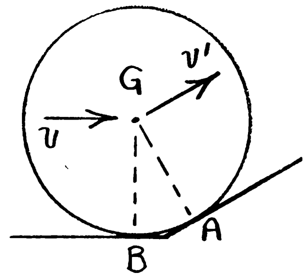
$$\Delta H_A = 0 : \bar{I}\omega + mvr\cos\theta = I_A \frac{v'}{r}$$



$$\frac{1}{2}mr^2\frac{v}{r} + mvr\cos\theta = \frac{3}{2}mr^2\frac{v'}{r}$$

$$\underline{v' = \frac{v}{3}(1 + 2\cos\theta)}$$

$$n = \frac{\frac{1}{2}I_B\omega^2 - \frac{1}{2}I_A\omega'^2}{\frac{1}{2}I_B\omega^2}$$

$$= 1 - \frac{\omega'^2}{\omega^2} = 1 - \frac{v'^2}{v^2}$$

So $n = 1 - \left(\frac{1 + 2\cos\theta}{3}\right)^2$

$$n_{10^\circ} = 1 - \left(\frac{1 + 2\cos 10^\circ}{3}\right)^2 = \underline{0.0202}$$

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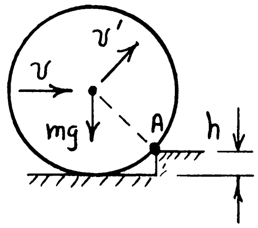
Angular impulse of mg is negligible.



Before impact: $H_A = \bar{I}\omega + mv(r-h)$

$$= mk^2 \frac{v}{r} + mv(r-h)$$

Just after impact:

$$H_A' = I_A \frac{v'}{r} = m(k^2+r^2)\frac{v'}{r}$$

$\Delta H_A = 0$: $mv\left(\frac{k^2}{r} + r - h\right) = m(k^2+r^2)\frac{v'}{r}$

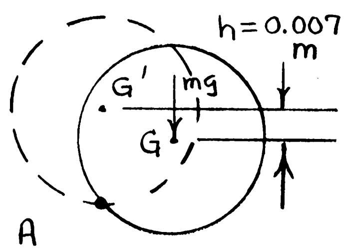
$$v' = v\left(1 - \frac{rh}{k^2+r^2}\right)$$

During roll on curb point, $\Delta T + \Delta V_g = 0$

$$\left[0 - \frac{1}{2}m(k^2+r^2)\frac{v'^2}{r^2}\right] + \left[mgh - 0\right] = 0$$

Solve for v: $\underline{v = \dfrac{r}{k^2+r^2-rh}\sqrt{2gh(k^2+r^2)}}$

6/205 Process II - roll about fixed point A



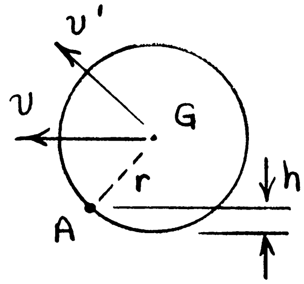
$$U_{1-2} = \Delta T$$

$$-mgh = \frac{1}{2}\left(\frac{3}{2}mr^2\right)\left(\omega'^2 - 0\right)$$

$$\omega' = \sqrt{\frac{4gh}{3r^2}} = \sqrt{\frac{4(9.81)(0.007)}{3(0.035)^2}}$$

$$= 8.65 \text{ rad/s}$$

Process I - impact at A



$$\Delta H_A = 0: \; mv(r-h) = I_A\omega'$$

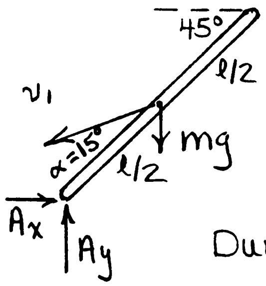
$$= \left(\frac{3}{2}mr^2\right)\omega'$$

With $v = 0.5\Omega$:

$$\frac{1}{2}\Omega(r-h) = \frac{3}{2}r^2\omega'$$

$$\Omega = \frac{3r^2\omega'}{r-h} = \frac{3(0.035)^2(8.65)}{0.035 - 0.007} = \underline{1.135 \frac{\text{rad}}{\text{s}}}$$

6/206 Neglect impulse of weight during impact so that $\Delta H_A = 0$



$$m v_1 \frac{l}{2} \sin 15° = \frac{1}{3} m l^2 \omega_2$$

$$\omega_2 = \frac{3 v_1}{2l} \sin 15°$$

During subsequent rotation about A,

$U = \Delta T$ or $-mg\frac{l}{2}(1 - \cos 45°) = 0 - \frac{1}{2} I_A \omega_2^2$

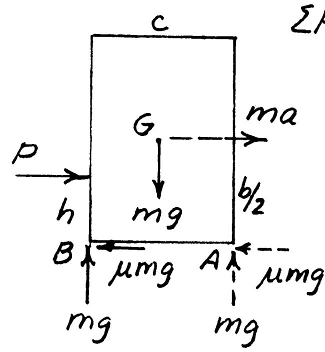
$$\omega_2 = \sqrt{\frac{3g}{l}\left(1 - \frac{\sqrt{2}}{2}\right)}$$

So $\sqrt{\frac{3g}{l}\left(1 - \frac{\sqrt{2}}{2}\right)} = \frac{3 v_1}{2l} \sin 15°$

$$\underline{v_1 = 2.41\sqrt{gl}}$$

6/207 P must be greater than μmg

$\Sigma F = ma$; $P - \mu mg = ma$



(a) Tipping about front edge A

$\Sigma M_A = m a d_A$

$$Ph - mg\frac{c}{2} = ma\frac{b}{2}$$

$$= (P - \mu mg)\frac{b}{2}$$

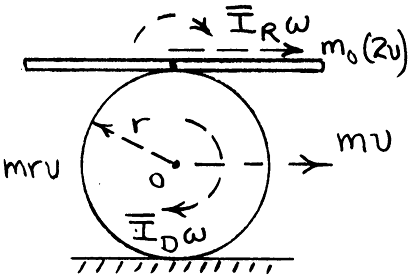
$$\underline{h_{max} = \frac{1}{2}\left[b + \frac{mg}{P}(c - \mu b)\right]}$$

(b) Tipping about rear edge B

$\Sigma M_B = m a d_B$; $Ph + mg\frac{c}{2} = ma\frac{b}{2} = (P - \mu mg)\frac{b}{2}$

$$\underline{h_{min} = \frac{1}{2}\left[b - \frac{mg}{P}(c + \mu b)\right]}$$

6/208



$\omega = v/r$

Disk: $\bar{I}_D \omega = \frac{1}{2} m r^2 \left(\frac{v}{r}\right) = \frac{1}{2} m r v$

Rod : $\bar{I}_R \omega = \frac{1}{12} m_0 l^2 \frac{v}{r}$

Combined : $H_O = \frac{1}{2} m r v + \frac{1}{12} m_0 l^2 \frac{v}{r} + m_0 (2v) r$

$$= vr\left[\frac{m}{2} + m_0\left(2 + \frac{l^2}{12r^2}\right)\right]$$

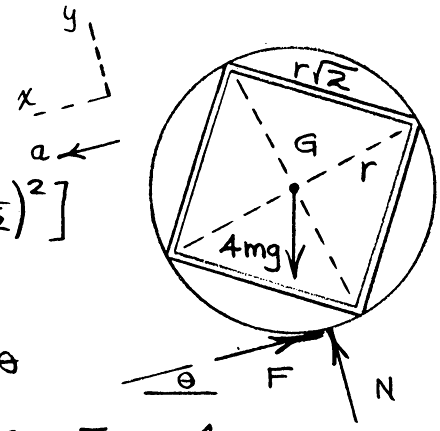
6/209 Max. power occurs when dV_g/dt is greatest, which occurs when $\bar{v}_y$ is max. at the start.

$\bar{v}_y = 1.500\,\omega = 1.500\,\frac{4\pi}{180} = 0.1047$ m/s

$P = mg\bar{v}_y = 1600(5)\,9.81(0.1047) = 8218$ W

or $\underline{P = 8.22 \text{ kW}}$

6/210



$$\bar{I} = 4\left[\frac{1}{12}m(2r^2) + m\left(\frac{r}{\sqrt{2}}\right)^2\right]$$
$$= \frac{8}{3}mr^2$$

$\Sigma F_y = 0: \quad N = 4mg\cos\theta$

$\Sigma F_x = ma_{G_x}: \quad 4mg\sin\theta - F = 4ma \qquad (1)$

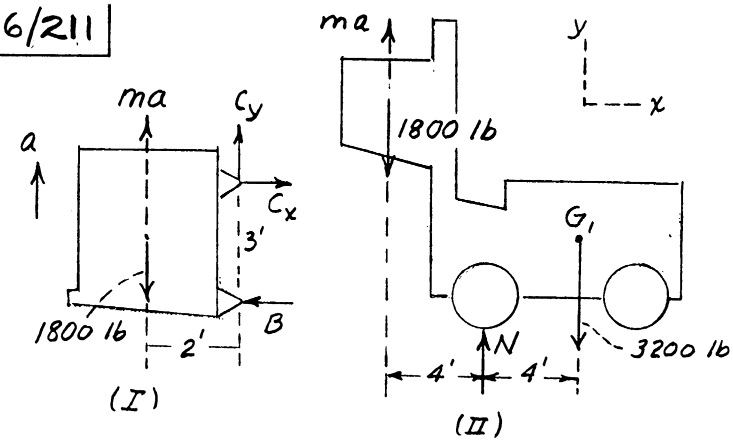
$\Sigma M_G = \bar{I}\alpha: \quad Fr = \frac{8}{3}mr^2\alpha \qquad (2)$

No slipping: $\quad a = r\alpha \qquad (3)$

Solution of (1)-(3): $\begin{cases} a = \frac{3}{5}g\sin\theta, \quad \alpha = \frac{3g}{5r}\sin\theta \\ F = \frac{8}{5}mg\sin\theta \end{cases}$

$$\mu_s = \frac{F}{N} = \frac{\frac{8}{5}mg\sin\theta}{4mg\cos\theta} = \underline{\frac{2}{5}\tan\theta}$$

6/211

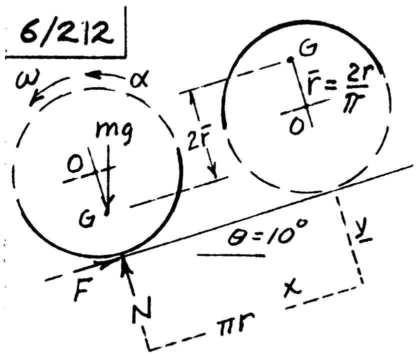


(II) $\Sigma M_N = m\bar{a}d$; $3200(4) - 1800(4) = \frac{1800}{32.2} a(4)$, $a = 25.04$ ft/sec^2

(I) $\Sigma M_C = m\bar{a}d$; $3B - 2(1800) = \frac{1800}{32.2}(25.04)(2)$

$\underline{B = 2130 \text{ lb}}$

6/212



$\bar{I} = I_O - m\bar{r}^2 = mr^2 - m(\frac{2r}{\pi})^2$

$= mr^2(1 - 4/\pi^2)$

$U = \Delta T$; $U = mg(2\bar{r}\cos\theta + \pi r\sin\theta)$

$= mgr(\frac{4}{\pi}\cos\theta + \pi\sin\theta)$

$\Delta T = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2$

$= \frac{1}{2}m[(r-\bar{r})\omega]^2 + \frac{1}{2}mr^2(1-\frac{4}{\pi^2})\omega^2$

$= mr^2\omega^2(1 - 2/\pi)$

Thus $mgr(\frac{4}{\pi}\cos\theta + \pi\sin\theta) = mr^2\omega^2(1 - 2/\pi)$

$$\omega^2 = \frac{(\frac{4}{\pi}\cos\theta + \pi\sin\theta)\frac{g}{r}}{(1-2/\pi)}, \quad \underline{\omega = \sqrt{\frac{g}{r}\frac{4\cos\theta + \pi^2\sin\theta}{\pi - 2}}}$$

$\Sigma F_y = m\bar{a}_y$; $N - mg\cos\theta = m\bar{r}\omega^2$

$$N = mg\cos\theta + m\frac{2r}{\pi}\,\frac{\frac{4}{\pi}\cos\theta + \pi\sin\theta}{1 - 2/\pi}\,\frac{g}{r}$$

$$N = mg\left[\frac{\pi^2 - 2\pi + 8}{\pi(\pi - 2)}\cos\theta + \frac{2\pi}{\pi - 2}\sin\theta\right]$$

For $\theta = 10°$, $N = mg[3.231\cos 10° + 5.504\sin 10°] = \underline{4.14\,mg}$

6/213 For the entire spacecraft,

$$\Sigma M_x = I_x \alpha : \quad 10^{-6} = 150{,}000\ \alpha$$

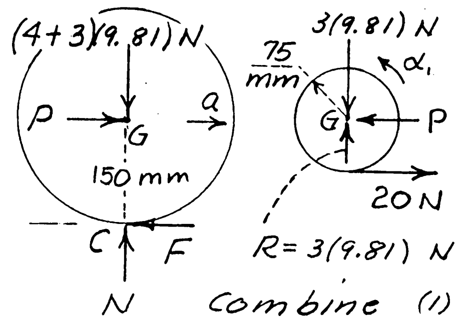
$$\alpha = 6.67 \times 10^{-12} \text{ rad/s}^2$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\frac{1}{3600}\left(\frac{\pi}{180}\right) = 0 + 0 + \frac{1}{2}\left(6.67 \times 10^{-12}\right) t^2$$

$$\underline{t = 1206 \text{ s}}$$

6/214



3 kg cyl.: $\Sigma F = ma$;

$20 - P = 3a$ - - - - - (1)

4 kg wheels:

Assume pure rolling

$\Sigma M_C = I_C \alpha$:

$P(0.150) = \frac{3}{2} 4 (0.150)^2 \frac{a}{0.150}$

$P = 6a$ - - - - - (2)

Combine (1) & (2) & get

$a = 20/9 = 2.22$ m/s^2

Check on assumption of no slipping

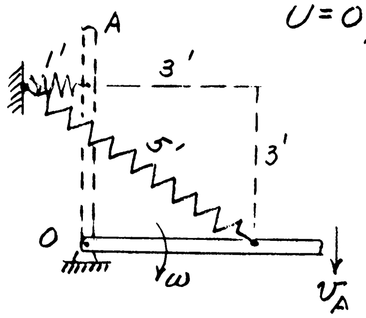
$\Sigma F = ma$; $P - F = ma$, $F = P - ma = 6(2.22) - 4(2.22)$

$= 4.44$ N

Req'd min. $\mu_s = \frac{F}{N} = \frac{4.44}{7(9.81)} = 0.06 < 0.2$

so assumption OK & $\underline{a = 2.22 \text{ m/s}^2}$

6/215 For system $U = \Delta T + \Delta V_g + \Delta V_e$



$U = 0$, $\Delta T = \frac{1}{2} I_o \omega^2 = \frac{1}{2}\left(\frac{1}{3} m l^2\right)\left(\frac{v_A}{l}\right)^2$

$= \frac{1}{6} m v_A^2 = \frac{1}{6} \frac{60}{32.2} v_A^2 = 0.3106\, v_A^2$

$\Delta V_e = \frac{1}{2} k x^2 - 0$

$= \frac{1}{2} 10 (5-1)^2 = 80$ ft-lb

$\Delta V_g = -60(2) = -120$ ft lb

Thus $0 = 0.3106\, v_A^2 - 120 + 80$

$v_A^2 = 128.8$, $\underline{v_A = 11.35 \text{ ft/sec}}$

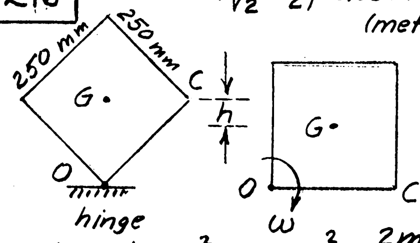
6/216 $h = 0.25\left(\frac{1}{\sqrt{2}} - \frac{1}{2}\right) = 0.05178$ m (meters); $\bar{I} = \frac{1}{6}m(0.25)^2$

$= 0.01042\,m$, (m = mass)

$I_O = \bar{I} + m(0.25/\sqrt{2})^2$

$= 0.04167\,m$

(a)

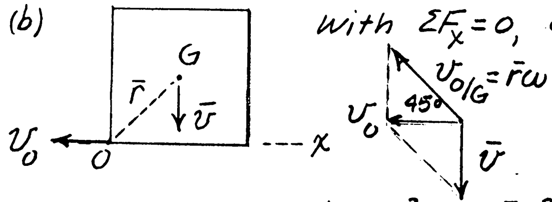


$\Delta V_g + \Delta T = 0$

$-mgh + \frac{1}{2}I_O\omega^2 = 0$, $\omega^2 = \frac{2mgh}{I_O} = \frac{2m(9.81)(0.05178)}{0.04167\,m} = 24.38\ (\text{rad/s})^2$

$\underline{\omega = 4.94\ \text{rad/s}}$

(b)



With $\Sigma F_x = 0$, $\bar{a}$ & hence $\bar{v}$ remain vertical

$\bar{v} = \frac{\bar{r}\omega}{\sqrt{2}} = \frac{0.25}{2}\omega = 0.125\omega$

$\Delta V_g + \Delta T = 0$; $\Delta T = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2 = \frac{1}{2}m(0.125\omega)^2 + \frac{1}{2}(0.01042m)\omega^2$

$= 13.02\,m\omega^2$

so $-mg(0.05178) + 0.01302\,m\omega^2 = 0$, $\omega^2 = \frac{9.81(0.05178)}{0.01302} = 39.01\ (\text{rad/s})^2$

$\underline{\omega = 6.25\ \text{rad/s}}$

6/217 $U = \Delta T + \Delta V_g + \Delta V_e$; $U = M\theta = 12\frac{\pi}{4} = 9.42$ J

$\Delta T = \frac{1}{2}(\frac{1}{12}6[0.50]^2)\omega^2 + \frac{1}{2}(\frac{1}{3}3[0.25]^2)\omega^2$

$= 0.0625\,\omega^2 + 0.0312\,\omega^2 = 0.0938\,\omega^2$ J

$\Delta V_g = 3(9.81)([0.25 + \frac{0.25}{2}] - [0.25 + \frac{0.25}{2}]/\sqrt{2}) = 3.23$ J

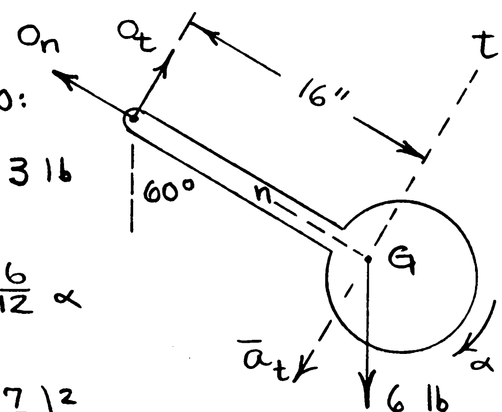
$\Delta V_e = \frac{1}{2}140(x_2^2 - x_1^2)$ where $x_2 = 0.15 + 0.50(1 - \frac{1}{\sqrt{2}}) = 0.296$ m

$= 70([0.296]^2 - [0.15]^2) = 4.58$ J

Thus $9.42 = 0.0938\,\omega^2 + 3.23 + 4.58$

$\omega^2 = 17.23$, $\underline{\omega = 4.15 \text{ rad/s}}$

6/218



$\Sigma F_n = m\bar{a}_n = m\bar{r}\omega^2 = 0$:

$O_n - 6\cos 60° = 0, \quad O_n = 3 \text{ lb}$

$\Sigma F_t = m\bar{a}_t = m\bar{r}\alpha$:

$6\sin 60° - O_t = \frac{6}{32.2}\,\frac{16}{12}\,\alpha$

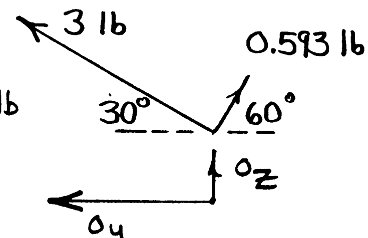
$\Sigma M_O = I_O\alpha$:

$(6\sin 60°)\frac{16}{12} = \frac{6}{32.2}\left(\frac{17}{12}\right)^2\alpha$

$\alpha = 18.53 \frac{\text{rad}}{\text{sec}^2}$; Then $O_t = 0.593$ lb

$O_y = 3\cos 30° - 0.593\cos 60° = 2.30$ lb

$O_z = 3\sin 30° + 0.593\sin 60° = 2.01$ lb

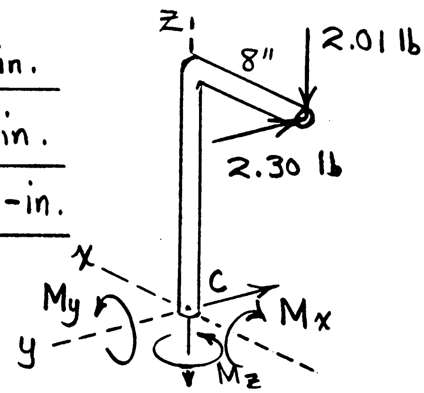


From equilibrium of CA:

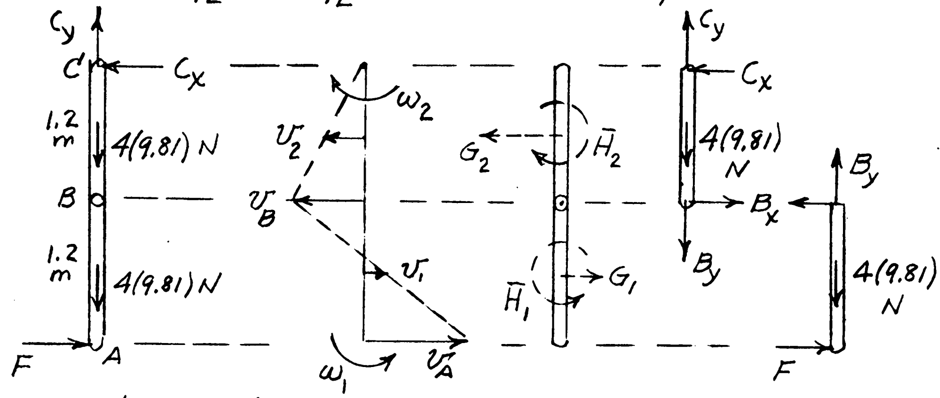
$M_x = -2.30(20) = \underline{-46.0 \text{ lb-in.}}$

$M_y = +2.01(8) = \underline{+16.11 \text{ lb-in.}}$

$M_z = -2.30(8) = \underline{-18.41 \text{ lb-in.}}$



▶ 6/219 $\bar{I} = \frac{1}{12} m l^2 = \frac{1}{12} 4(1.2)^2 = 0.48\ kg\cdot m^2$; $\int F dt = 14\ N\cdot s$



$\omega_2 = v_2/0.6$, $\omega_1 = (v_1 + 2v_2)/0.6$, $m = 4\ kg$ (a)

System: $\int \Sigma M_C\, dt = \Sigma \Delta H_C$; $14(2.4) = 4v_1(1.8) + 0.48\omega_1 - 4v_2(0.6) - 0.48\omega_2$

AB: $\int \Sigma M_C\, dt = \Delta H_C$; $14(2.4) - \int 1.2 B_x dt = 4v_1(1.8) + 0.48\omega_1$ (b)

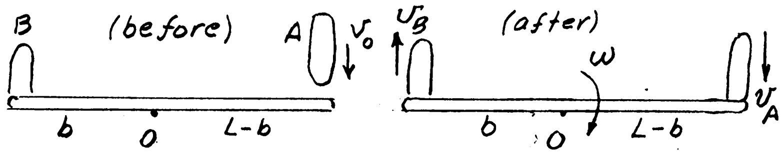
$\int \Sigma F_x\, dt = \Delta G_x$; $14 - \int B_x dt = 4v_1$ (c)

(b) & (c) & ω_1 give $2v_1 + v_2 = 10.5$; (a), ω_1, ω_2 give $5v_1 - v_2 = 21$

Combine & get $v_1 = 4.5\ m/s$, $v_2 = 1.5\ m/s$

& $\underline{\omega_2 = 2.50\ rad/s}$

6/220



Before: $H_O = m_A v_0 (L-b)$

After: $H_O = m_A v_A (L-b) + m_B v_B b$

$\Delta H_O = 0$ along with $\omega = v_B/b = v_A/(L-b)$ give

$$m_A v_0 (L-b) = m_A \frac{L-b}{b} v_B (L-b) + m_B v_B b$$

$$v_B = v_0 \frac{1}{\frac{L-b}{b} + n\frac{b}{L-b}} \quad \text{where } n = m_B/m_A$$

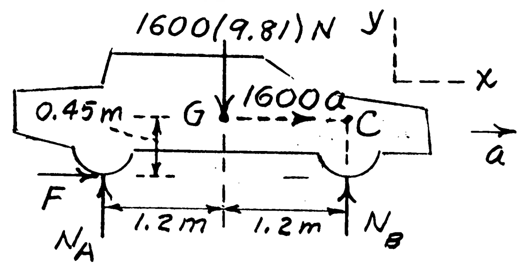
$$\frac{dv_B}{db} = v_0 \frac{-\left(\frac{L}{b^2} + n\frac{L-b-b(-1)}{(L-b)^2}\right)}{\left(\frac{L-b}{b} + n\frac{b}{L-b}\right)^2} = v_0 \frac{L\left(\frac{1}{b^2} - \frac{n}{(L-b)^2}\right)}{\left(\frac{L-b}{b} + n\frac{b}{L-b}\right)^2} = 0 \text{ for } v_{B_{max}}$$

So $\frac{1}{b^2} = \frac{n}{(L-b)^2}$, $b = \frac{L}{1 \pm \sqrt{n}}$ (+ sign gives positive v_B)

Thus $\underline{b = \frac{L}{1+\sqrt{n}}}$ which gives $v_B = \frac{v_0}{2\sqrt{n}}$

6/221

(a) Max. acceleration occurs when
$F = \mu N_A = 0.8 N_A$



$\circlearrowleft + \Sigma M_C = m a d = 0: \; 1600(9.81)(1.2) - 2.4 N_A + 0.8 N_A (0.45) = 0$

$N_A = 9233$ N, $F = 0.8(9233) = 7386$ N

$\Sigma F_x = m a_x: \; 7386 = 1600 a$, $\underline{a = 4.62 \text{ m/s}^2}$

(b) Each rear wheel:

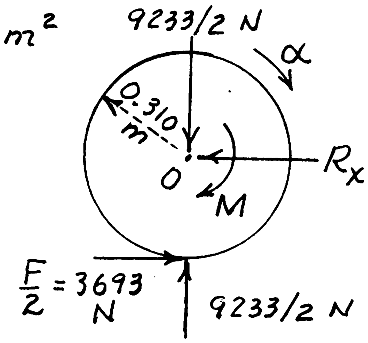
$I_O = m k^2 = 32(0.210)^2 = 1.411 \text{ kg}\cdot\text{m}^2$

$\alpha = \dfrac{a}{r} = \dfrac{4.62}{0.310} = 14.89 \text{ rad/s}^2$

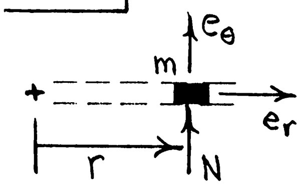
$\Sigma M_O = I_O \alpha$:

$M - 3693(0.310) = 1.411(14.90)$

$\underline{M = 1166 \text{ N}\cdot\text{m}}$



6/222



Conservation of angular momentum : $I_o \omega_o = (I_o + mr^2)\omega$

$$\dot{\theta} = \omega = \frac{I_o \omega_o}{I_o + mr^2}$$

$$\Sigma F_r = ma_r = m(\ddot{r} - r\dot{\theta}^2) : \quad 0 = m(\ddot{r} - r\dot{\theta}^2)$$

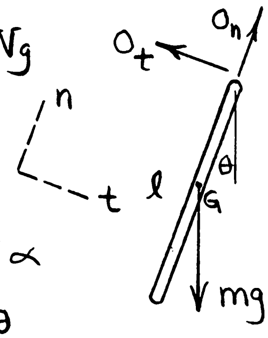
$$\ddot{r} = \dot{r}\frac{d\dot{r}}{dr} = r\left(\frac{I_o \omega_o}{I_o + mr^2}\right)^2$$

$$\int_0^{\dot{r}} \dot{r}\, d\dot{r} = I_o^2 \omega_o^2 \int_0^r \frac{r\,dr}{(I_o + mr^2)^2}$$

Integrating and solving for $\dot{r}$:

$$\dot{r} = \left(\frac{I_o \omega_o^2 r^2}{I_o + mr^2}\right)^{1/2} = \underline{\omega_o r \sqrt{\frac{I_o}{I_o + mr^2}}}$$

▶6/223 Entire bar: $U'_{1-2} = \Delta T + \Delta V_g$



$$0 = \frac{1}{2}\left(\frac{1}{3}m\ell^2\right)\omega^2 - mg\frac{\ell}{2}\cos\theta$$

$$\omega^2 = \frac{3g}{\ell}\cos\theta$$

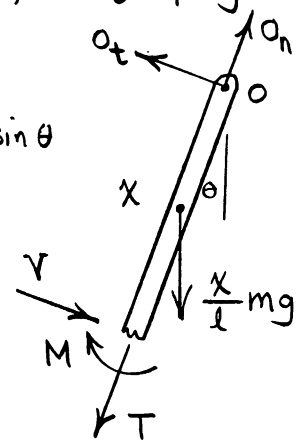
$$\Sigma M_O = I_O\alpha: \quad mg\frac{\ell}{2}\sin\theta = \frac{1}{3}m\ell^2\alpha$$

$$\alpha = \frac{3g}{2\ell}\sin\theta$$

$$\Sigma F_n = m\bar{a}_n: \quad O_n - mg\cos\theta = m\frac{\ell}{2}\omega^2, \quad O_n = \frac{5}{2}mg\cos\theta$$

$$\Sigma F_t = m\bar{a}_t: \quad mg\sin\theta - O_t = m\frac{\ell}{2}\alpha, \quad O_t = \frac{1}{4}mg\sin\theta$$

Upper section:



$$\Sigma F_t = m\bar{a}_t: \quad \frac{x}{\ell}mg\sin\theta + V - \frac{1}{4}mg\sin\theta = \frac{x}{\ell}m\left(\frac{x}{2}\alpha\right)$$

$$\underline{V = \frac{(\ell - x)(\ell - 3x)}{4\ell^2}mg\sin\theta}$$

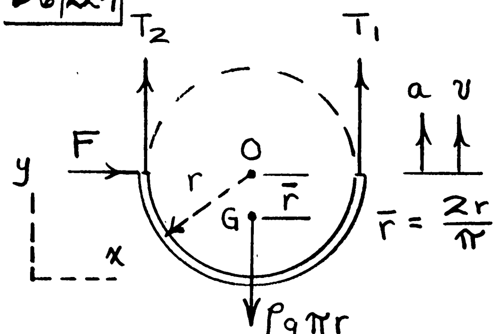
$$\Sigma F_n = m\bar{a}_n: \quad \frac{5}{2}mg\cos\theta - \frac{x}{\ell}mg\cos\theta - T = \frac{x}{\ell}m\left(\frac{x}{2}\omega^2\right)$$

$$\underline{T = \frac{5\ell^2 - 2\ell x - 3x^2}{2\ell^2}mg\cos\theta}$$

$$\Sigma M_O = I_O\alpha: \quad \frac{x}{\ell}mg\frac{x}{2}\sin\theta + Vx - M = \frac{1}{3}\frac{x}{\ell}mx^2(\alpha)$$

Substitute for V & α: $\underline{M = \frac{(\ell - x)^2 x}{4\ell^2}mg\sin\theta}$

▶6/224



$\bar{r} = \frac{2r}{\pi}$

$v_0 = \frac{v}{2}$, $\omega = \frac{v}{2r}$

$a_0 = \frac{a}{2}$, $\alpha = \frac{a}{2r}$

$\underline{a}_G = \underline{a}_0 + (\underline{a}_{G/0})_n + (\underline{a}_{G/0})_t$

$a_{Gy} = \frac{a}{2} + \bar{r}\omega^2 = \frac{a}{2} + \frac{v^2}{2\pi r}$

$a_{Gx} = \bar{r}\alpha = \frac{a}{\pi}$

$$I_G = I_0 - m\bar{r}^2 = \rho\pi r(r^2) - \rho\pi r\left(\frac{2r}{\pi}\right)^2 = \rho\pi r^3\left(1 - \frac{4}{\pi^2}\right)$$

$$\Sigma M_G = I_G\alpha: \quad (T_1 - T_2)r - F\frac{2r}{\pi} = \rho\pi r^3\left(1 - \frac{4}{\pi^2}\right)\frac{a}{2r}$$

$$T_1 - T_2 = F\frac{2}{\pi} + \frac{1}{2}\rho\pi r a\left(1 - \frac{4}{\pi^2}\right) \quad (1)$$

$$\Sigma F_x = ma_{Gx}: \quad F = \rho\pi r\frac{a}{\pi} = \rho r a \quad (2)$$

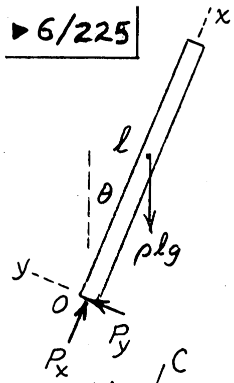
$$\Sigma F_y = ma_{Gy}: \quad T_1 + T_2 - \rho g\pi r = \rho\pi r\left(\frac{a}{2} + \frac{v^2}{2\pi r}\right) \quad (3)$$

Solve (1)-(3) for T_1 & T_2:

$$\begin{cases} T_1 = \frac{1}{2}\rho r a\pi + \frac{1}{2}\rho g r\pi + \frac{1}{4}\rho v^2 \\ T_2 = \frac{1}{2}\rho g r\pi + \frac{1}{4}\rho v^2 \end{cases}$$

When $r \to 0$, $\underline{T_1 = T_2 = \frac{1}{4}\rho v^2}$

▸ 6/225



For entire chimney (rod); ρ = mass/unit length

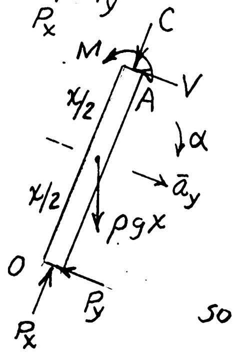
$\Sigma M_O = I_O\alpha$; $\rho l g \frac{l}{2}\sin\theta = \frac{1}{3}\rho l(l^2)\alpha$

$\alpha = \frac{3}{2}\frac{g}{l}\sin\theta$

$\Sigma F_y = m\bar{a}_y$; $P_y - \rho l g\sin\theta = \rho l\left(-\frac{l}{2}\frac{3}{2}\frac{g}{l}\sin\theta\right)$

$P_y = \frac{1}{4}\rho l g\sin\theta$

For section of length x,
M = bending moment, V = shear



$\Sigma M_A = \bar{I}\alpha + m\bar{a}d$

$\bar{a}d = \bar{a}_y\frac{x}{2} = \frac{x}{2}\alpha\frac{x}{2} = \frac{x^2}{4}\alpha$

$\bar{I}\alpha = \frac{1}{12}\rho x(x^2)\alpha = \frac{1}{12}\rho x^3\alpha$

$-M - \rho g x\frac{x}{2}\sin\theta + \frac{1}{4}\rho l g x\sin\theta = \frac{\rho x^3}{12}\alpha - \rho x\frac{x^2}{4}\alpha$

$= -\frac{1}{4}\rho x^3\frac{g}{l}\sin\theta$

So $M = \frac{\rho g\sin\theta}{4l}(x^3 - 2lx^2 + l^2x)$

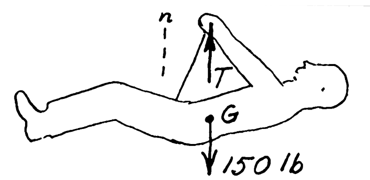
For max M, $\frac{dM}{dx} = \frac{\rho g\sin\theta}{4l}(3x^2 - 4lx + l^2) = 0$

$(3x - l)(x - l) = 0$, $\underline{x = \frac{l}{3}}$ (M_{max})

$x = l$ ($M = 0$)

$x = l/3$ locates center of percussion from upper end which is $q = k_O^2/\bar{r} = \frac{1}{3}l^2/\frac{l}{2} = \frac{2}{3}l$

▶6/226 Fixed-axis rotation



$\Sigma F_n = m\bar{a}_n: \; T - 150 = \dfrac{150}{32.2}\dfrac{13^2}{92/12},$

$T = 253 \text{ lb}$

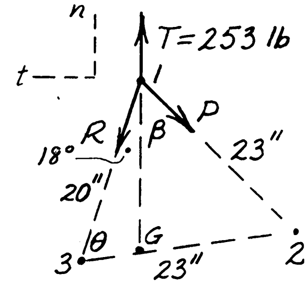
$\theta = \cos^{-1}(10/23) = 64.2°$

$\beta = \theta - 18° = 46.2°$

$\Sigma F_n \doteq 0: \; 253 - R\cos 18° - P\cos 46.2° = 0$

$\Sigma F_t \doteq 0: \; R\sin 18° - P\cos 46.2° = 0$

Solve & get $P = 86.7 \text{ lb}$, $R = 203 \text{ lb}$



$\gamma = \sin^{-1}\dfrac{13}{92} = 8.12°$

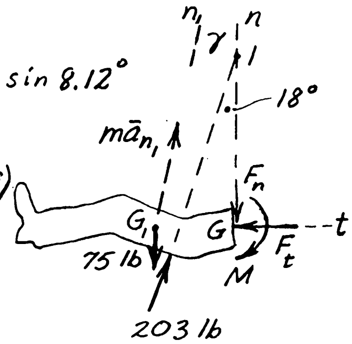
$\Sigma F_t = m\bar{a}_t: \; 203\sin 18° - F_t = \dfrac{75}{32.2}\dfrac{13^2}{92/12}\sin 8.12°$

$F_t = 55.4 \text{ lb}$

$\circlearrowleft + \Sigma M_O = I_O\alpha = 0: \; 203\sin 18°(92 - 18.18)$

$- 75(13) + 55.4(92) + M = 0$

$\underline{M = 504 \text{ lb-in.}}$



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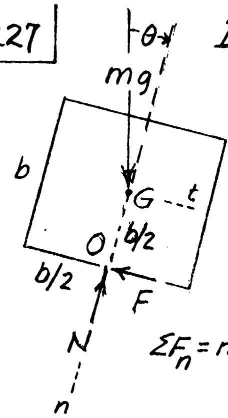


$I_O = \frac{1}{12}m(b^2+b^2) + m(b/2)^2 = \frac{5}{12}mb^2$

$\Sigma M_O = I_O\alpha;\ mg\frac{b}{2}\sin\theta = \frac{5}{12}mb^2\alpha$

$\alpha = \frac{6}{5}\frac{g}{b}\sin\theta$

$\int_0^\omega \omega\, d\omega = \int_0^\theta \alpha\, d\theta;\quad \omega^2 = \frac{12}{5}\frac{g}{b}\int_0^\theta \sin\theta\, d\theta$

$\omega^2 = \frac{12}{5}\frac{g}{b}(1-\cos\theta)$

$\Sigma F_n = m\bar{a}_n;\ mg\cos\theta - N = m\frac{b}{2}\left(\frac{12}{5}\frac{g}{b}\right)(1-\cos\theta)$

$N = \frac{mg}{5}(11\cos\theta - 6)$

$\Sigma F_t = m\bar{a}_t;\ mg\sin\theta - F = m\frac{b}{2}\left(\frac{6}{5}\frac{g}{b}\sin\theta\right)$

$F = \frac{2}{5}mg\sin\theta$

Compute & plot N/mg & F/mg

For $F = 0.8N$, $\frac{2}{5}\sin\theta = 0.8\left(\frac{1}{5}\right)(11\cos\theta - 6)$, $5\sin\theta = 22\cos\theta - 12$

Solve by Newton's method & get slip at $\underline{\theta = 45.1°}$

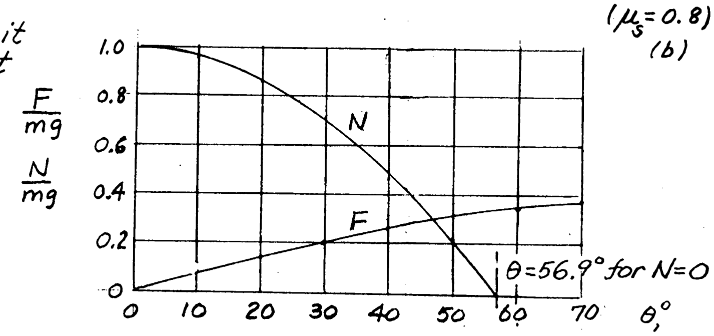
$(\mu_s = 0.8)$

(b)

(a) for no limit on F, contact ceases when $N = 0$ so

$\theta = \cos^{-1} 6/11$

$\underline{= 56.9°}$



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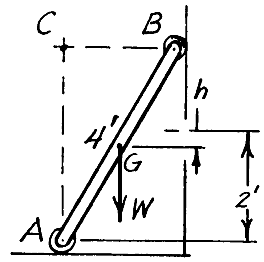
$U = \Delta T$: $T = \frac{1}{2} I_C \omega^2 = \frac{1}{2}\frac{1}{3}\frac{W}{g}4^2\omega^2$

$U = Wh = W(2 - 2\cos\theta)$

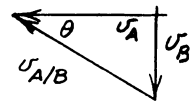
$= 2W(1-\cos\theta)$



Thus $2W(1-\cos\theta) = \frac{8W}{3g}\omega^2$, $\omega^2 = \frac{3g}{4}(1-\cos\theta)$

$\omega = \sqrt{(3 \times 32.2/4)(1-\cos\theta)} = 4.91\sqrt{1-\cos\theta}$ rad/sec

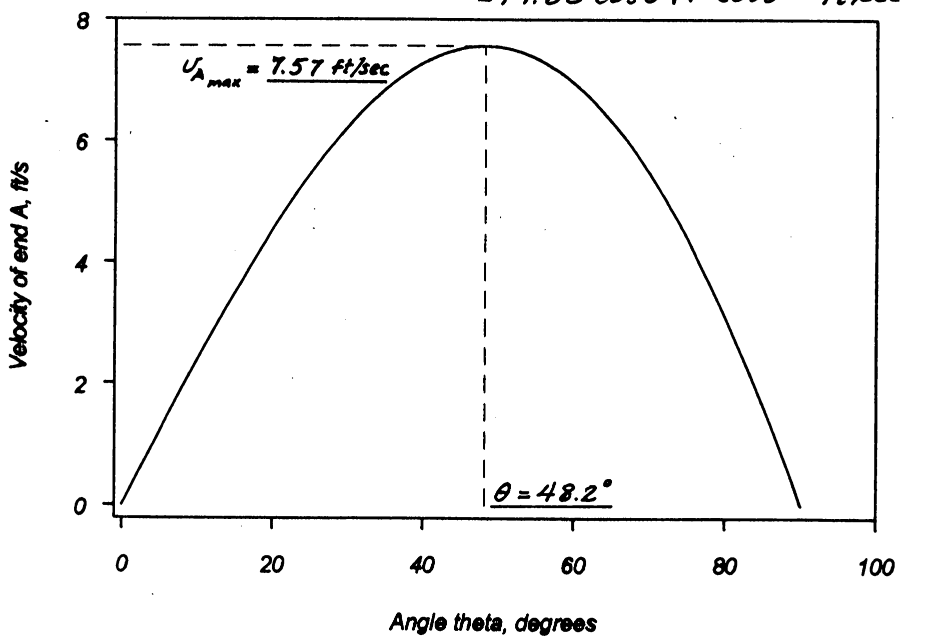
$\underline{v}_A = \underline{v}_B + \underline{v}_{A/B}$:



$v_A = v_{A/B}\cos\theta = L\omega\cos\theta$

$= 4(4.91)\sqrt{1-\cos\theta}\cos\theta$ ft/sec

$= 19.66\cos\theta\sqrt{1-\cos\theta}$ ft/sec



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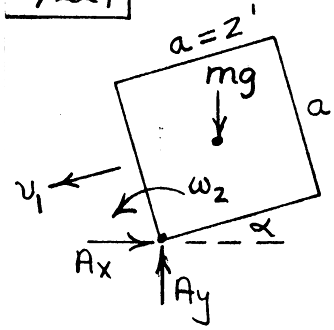


During impact; $\Delta H_A = 0$ if we neglect weight impulse.

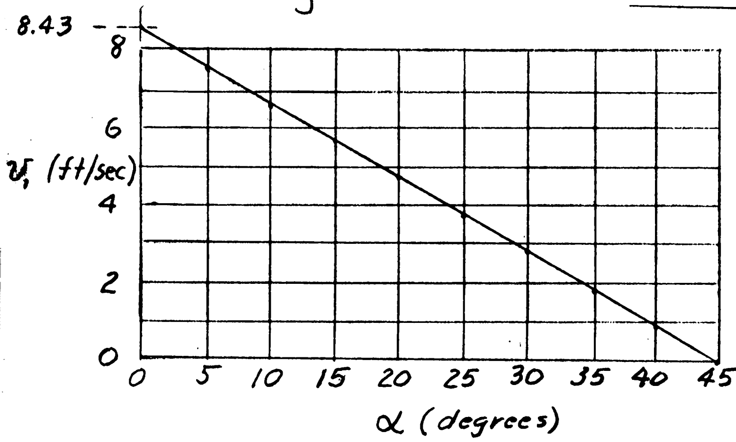
$$m v_1 \left(\frac{a}{2}\right) = \left[\frac{1}{6} m a^2 + m\left(a\frac{\sqrt{2}}{2}\right)^2\right]\omega_2$$

$$\omega_2 = \frac{3}{4}\frac{v_1}{a}$$

After impact, $U = \Delta T$

$$-mg\frac{a\sqrt{2}}{2}\left[1-\cos(45^\circ-\alpha)\right] = 0 - \frac{1}{2}\left(\frac{2}{3} m a^2\right)\left(\frac{3}{4}\frac{v_1}{a}\right)^2$$

$$v_1 = 2.746\sqrt{g[1-\cos(45^\circ-\alpha)]} = \underline{15.58\sqrt{1-\cos(45^\circ-\alpha)}}$$ ft/sec



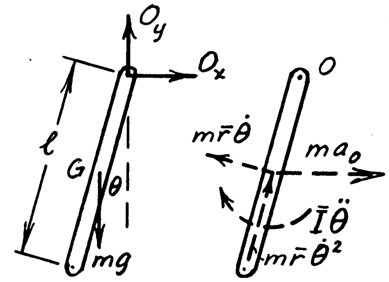
*6/230 From the solution of Prob. 6/18,

$$K\theta - \frac{5}{2} mgl \sin\theta - \frac{5}{2} mal \cos\theta = 0$$

With numbers: $75\theta - 7.36 \sin\theta - 14.72 \cos\theta = 0$

Numerical solution: $\underline{\theta = 12.17°}$

*6/231



$\Sigma M_O = \bar{I}\alpha + m\bar{a}d$:

$$-mg\frac{l}{2}\sin\theta = \frac{1}{12}ml^2\ddot{\theta} + m\frac{l}{2}\ddot{\theta}\left(\frac{l}{2}\right) - ma_o\frac{l}{2}\cos\theta$$

$$\ddot{\theta} = \frac{3}{l}\left(\frac{a_o}{2}\cos\theta - \frac{g}{2}\sin\theta\right)$$

$\int_0^{\dot{\theta}} \dot{\theta}\,d\dot{\theta} = \int_0^{\theta} \ddot{\theta}\,d\theta$:

$$\frac{\dot{\theta}^2}{2} = \frac{3}{l}\int_0^{\theta}\left(\frac{a_o}{2}\cos\theta - \frac{g}{2}\sin\theta\right)d\theta$$

$$\dot{\theta}^2 = \frac{6}{l}\left(\frac{a_o}{2}\sin\theta - \frac{g}{2}[1-\cos\theta]\right) = 1.5\left(\sin\theta - \frac{9.81}{2}[1-\cos\theta]\right)\frac{\text{rad}^2}{\text{s}^2}$$

$$\dot{\theta} = 0 \text{ when } R = \sin\theta - \frac{9.81}{2}(1-\cos\theta) = 0$$

Solve numerically & get

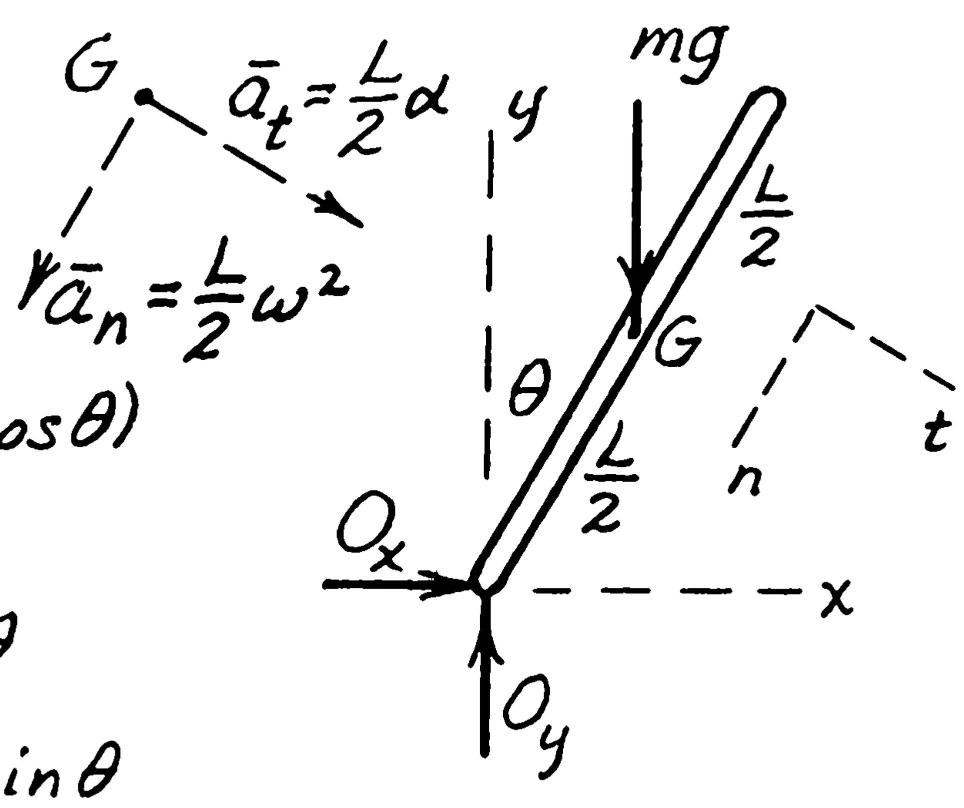
$$\theta_{max} = 23.0^\circ$$

$\dot{\theta}$ is max. when $\ddot{\theta} = 0$: $\frac{a_o}{2}\cos\theta - \frac{g}{2}\sin\theta = 0$ or $\theta = \tan^{-1}\frac{a_o}{g}$

$$\theta = 11.52^\circ,\ (\dot{\theta}^2)_{max} = 0.1513\ (\text{rad/s})^2,\ \dot{\theta}_{max} = 0.389\text{ rad/s}$$

*6/232 $\Sigma M_O = I_O\alpha$: $mg\frac{L}{2}\sin\theta = \frac{1}{3}mL^2\alpha$

$$\alpha = \frac{3}{2}\frac{g}{L}\sin\theta$$



$\omega\, d\omega = \alpha\, d\theta$:

$$\int_0^\omega \omega\, d\omega = \frac{3}{2}\frac{g}{L}\int_0^\theta \sin\theta\, d\theta, \quad \omega^2 = \frac{3g}{L}(1-\cos\theta)$$

$$\bar{a}_x = \bar{a}_t\cos\theta - \bar{a}_n\sin\theta = \frac{L}{2}\left(\frac{3g}{2L}\sin\theta\right)\cos\theta - \frac{L}{2}\left(\frac{3g}{L}[1-\cos\theta]\right)\sin\theta$$

$$= \frac{3g}{4}\sin\theta\,(3\cos\theta - 2)$$

$$\bar{a}_y = -\bar{a}_t\sin\theta - \bar{a}_n\cos\theta = -\frac{L}{2}\left(\frac{3g}{2L}\sin\theta\right)\sin\theta - \frac{L}{2}\left(\frac{3g}{L}[1-\cos\theta]\right)\cos\theta$$

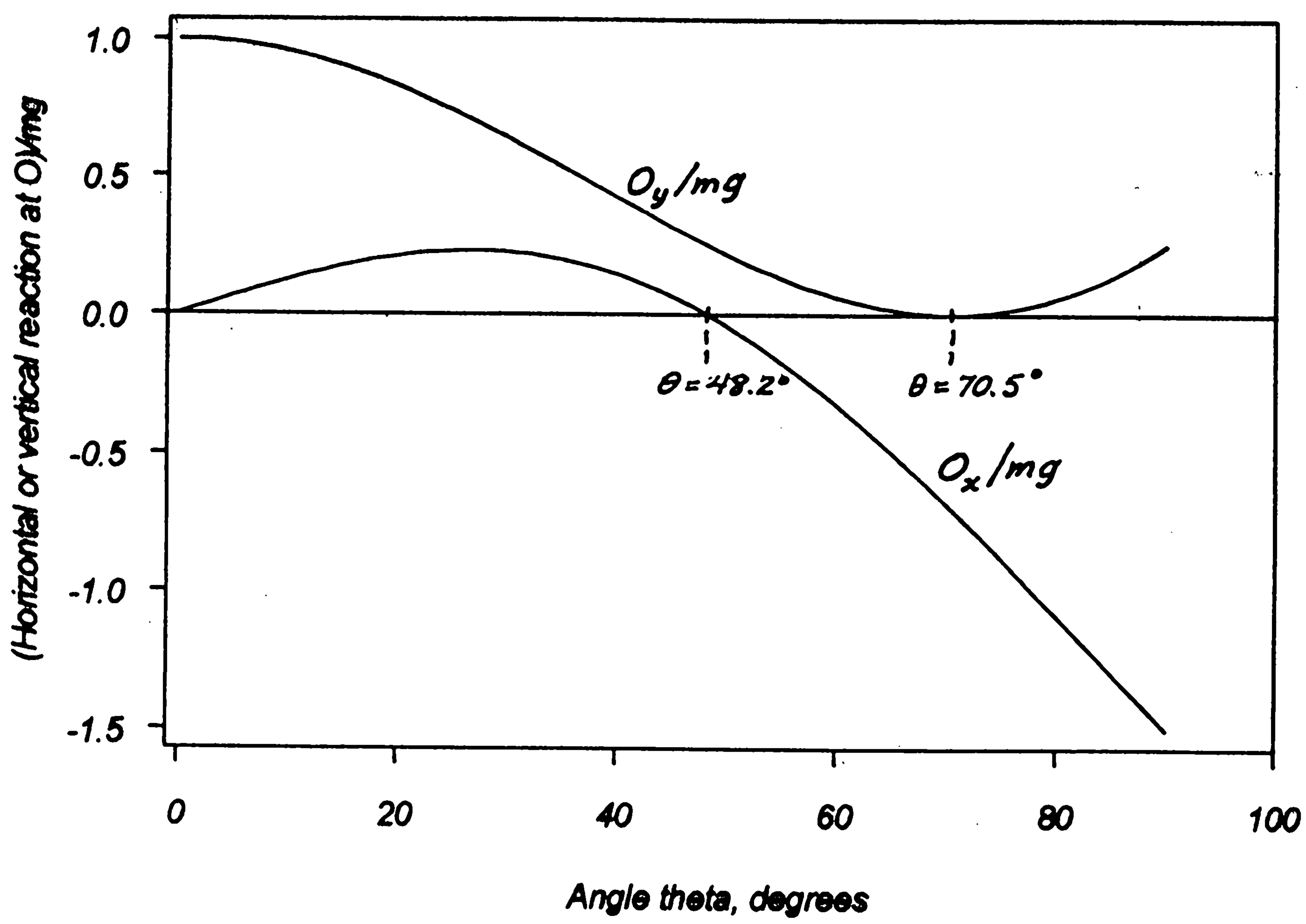
$$= -\frac{3g}{4}(1 + 2\cos\theta - 3\cos^2\theta)$$

$$= \frac{3g}{4}(3\cos\theta + 1)(\cos\theta - 1)$$

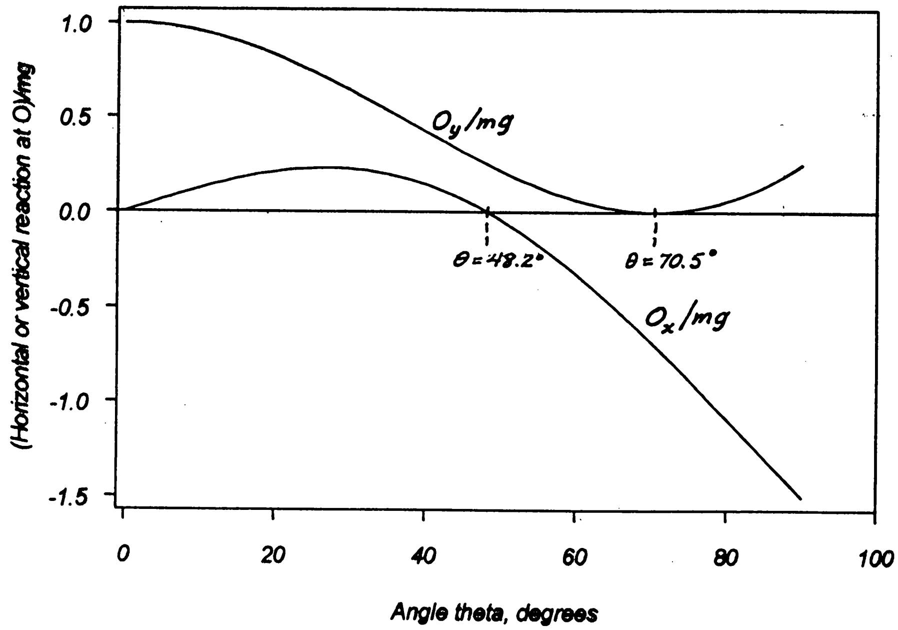
$\Sigma F_x = m\bar{a}_x$: $\underline{O_x = \frac{3mg}{4}\sin\theta\,(3\cos\theta - 2)}$

$\Sigma F_y = m\bar{a}_y$: $O_y - mg = \frac{3mg}{4}(3\cos\theta + 1)(\cos\theta - 1)$

$$O_y = \frac{mg}{4}(1 - 6\cos\theta + 9\cos^2\theta) = \underline{\frac{mg}{4}(3\cos\theta - 1)^2}$$



Increase in O_y from $\theta = 70.5°$ to $\theta = 90°$ reflects the fact that, in $-\bar{a}_y = \bar{a}_n\cos\theta + \bar{a}_t\sin\theta$, $\bar{a}_n\cos\theta$ is decreasing faster than $\bar{a}_t\sin\theta$ is increasing, showing the effect of the multipliers $\cos\theta$ and $\sin\theta$ for θ near $90°$.



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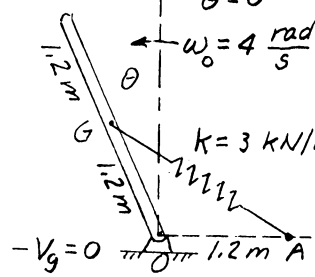
*6/233 $m = 30$ kg

$V_e = 0$ when $\theta = 0$



$-V_g = 0$

$\Delta V_g = -30(9.81)(1.2)(1-\cos\theta)$

$= -353(1-\cos\theta)$ J

$U = 0 = \Delta T + \Delta V_g + \Delta V_e$

$\overline{AG} = 1.2\sqrt{2(1+\sin\theta)}$

$\Delta V_e = \frac{1}{2}k\left[\overline{GA} - 1.2\sqrt{2}\right]^2$

$= \frac{3000}{2}\left[1.2\sqrt{2(1+\sin\theta)} - 1.2\sqrt{2}\right]^2$

$= 4320(2 + \sin\theta - 2\sqrt{1+\sin\theta})$ J

$\Delta T = \frac{1}{2}I_O(\omega^2 - \omega_0^2)$

$= \frac{1}{2}\left[\frac{1}{3}30(2.4)^2(\omega^2 - 16)\right]$

$= 28.8(\omega^2 - 16)$ J

Thus

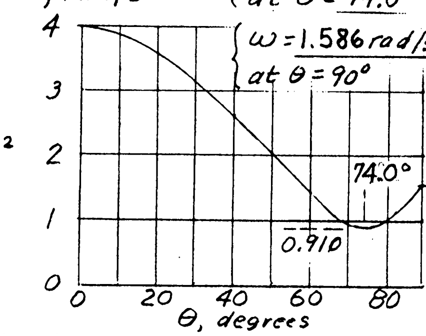
$0 = 28.8(\omega^2 - 16) - 353(1-\cos\theta)$
$+ 4320(2 + \sin\theta - 2\sqrt{1+\sin\theta})$

$\omega^2 = 300\sqrt{1+\sin\theta} - 150\sin\theta$
$-12.26\cos\theta - 271.7\ (rad/s)^2$

Set up computer program & solve for ω vs θ & get

$\left\{\begin{array}{l} \omega_{min} = \underline{0.910\ \frac{rad}{s}} \\ \text{at } \theta = \underline{74.0°} \end{array}\right.$

$\left\{\begin{array}{l} \omega = \underline{1.586\ rad/s} \\ \text{at } \theta = 90° \end{array}\right.$



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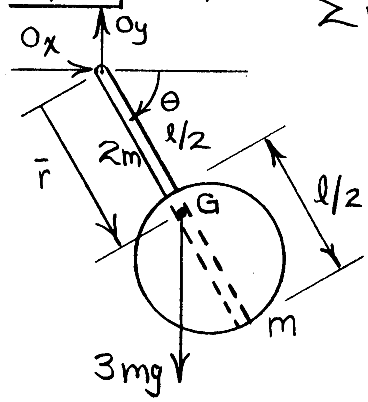
$$\bar{r} = \frac{\Sigma m\bar{r}}{\Sigma m} = \frac{2m(l/2) + m(3l/4)}{3m} = \frac{7}{12}l$$



$$I_O = \frac{1}{3}(2m)l^2 + \left[\frac{1}{2}m\left(\frac{l}{4}\right)^2 + m\left(\frac{3l}{4}\right)^2\right]$$

$$= \frac{121}{96}ml^2$$

$+\circlearrowright \Sigma M_O = I_O\alpha$:

$$3mg\left(\frac{7}{12}l\cos\theta\right) = \frac{121}{96}ml^2\alpha$$

$$\alpha = \frac{168}{121}\frac{g}{l}\cos\theta$$

$$\alpha = \omega\frac{d\omega}{d\theta} = \frac{168}{121}\frac{g}{l}\cos\theta$$

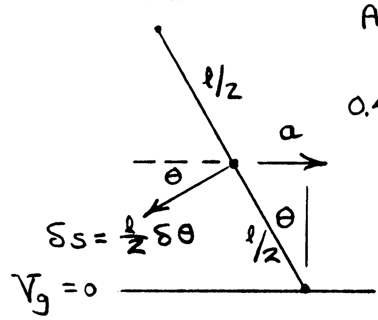
$$\int_{\omega_0}^{\omega}\omega\,d\omega = \frac{168}{121}\frac{g}{l}\int_0^{\theta}\cos\theta\,d\theta$$

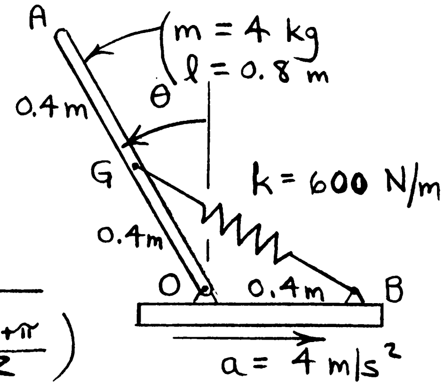
$$\Rightarrow \omega = \frac{d\theta}{dt} = \left[\omega_0^2 + \frac{336}{121}\frac{g}{l}\sin\theta\right]^{1/2}$$

$$\Rightarrow t = \int_0^{\theta}\frac{d\theta}{\left[\omega_0^2 + \frac{336}{121}\frac{g}{l}\sin\theta\right]^{1/2}}$$

Numerical solution with $\begin{cases}\omega_0 = 3 \text{ rad/s} \\ l = 0.8 \text{ m} \\ \theta = \pi/2\end{cases}$: $\underline{t = 0.302\text{ s}}$

*6/235





$\left(\text{Angle } GOB = \theta + \frac{\pi}{2} = \frac{2\theta+\pi}{2}\right)$

Consider a virtual displacement $\delta\theta$ from the steady-state position. Forces at O & B do no work.

$$\delta U' = 0 = \delta T + \delta V_g + \delta V_e$$

$$\delta T = \delta\left(\tfrac{1}{2} m \bar{v}^2\right) = m\bar{v}\,\delta\bar{v} = m\bar{a}\cdot\delta s$$

$$= ma\left(-\tfrac{l}{2}\delta\theta\right)\cos\theta = -ma\tfrac{l}{2}\cos\theta\,\delta\theta$$

$$= -4(4)(0.4)\cos\theta\,\delta\theta = -6.4\cos\theta\,\delta\theta$$

$$\delta V_g = \delta(mgh) = \delta\left(mg\tfrac{l}{2}\cos\theta\right) = -mg\tfrac{l}{2}\sin\theta\,\delta\theta$$

$$= -4(9.81)(0.4)\sin\theta\,\delta\theta = -15.70\sin\theta\,\delta\theta$$

$\overline{GB} = 2\left(\frac{l}{2}\right)\sin\frac{2\theta+\pi}{4}$, so stretch $x = l\sin\frac{2\theta+\pi}{4} - l\frac{\sqrt{2}}{2}$

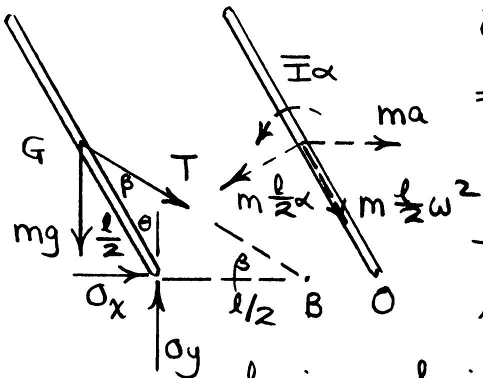
$$\delta V_e = \delta\left(\tfrac{1}{2}kx^2\right) = kx\,\delta x = kl\left(\sin\tfrac{2\theta+\pi}{4} - \tfrac{\sqrt{2}}{2}\right)\cdot\tfrac{1}{2}l\cos\tfrac{2\theta+\pi}{4}\,\delta\theta$$

$$= \frac{kl^2}{2}\left(\sin\tfrac{2\theta+\pi}{4} - \tfrac{\sqrt{2}}{2}\right)\cos\tfrac{2\theta+\pi}{4}\,\delta\theta = 192\left(\sin\tfrac{2\theta+\pi}{4} - \tfrac{\sqrt{2}}{2}\right)\cos\tfrac{2\theta+\pi}{4}\,\delta\theta$$

Thus $0 = -6.4\cos\theta\,\delta\theta - 15.70\sin\theta\,\delta\theta + 192\left(\sin\frac{2\theta+\pi}{4} - \frac{\sqrt{2}}{2}\right)\cos\frac{2\theta+\pi}{4}\,\delta\theta$

Numerical solution: $\underline{\theta = 15.62^\circ}$ ($\theta = 46.4^\circ$ is an unstable configuration)

*6/236 $2\beta + \theta + \pi/2 = \pi$, $\beta = \frac{\pi}{4} - \frac{\theta}{2}$



$$\overline{GB} = 2\left(\frac{l}{2}\right)\cos\beta$$

$$= l\left[\cos\frac{\pi}{4}\cos\frac{\theta}{2} + \sin\frac{\pi}{4}\sin\frac{\theta}{2}\right]$$

$$= \frac{l}{\sqrt{2}}\left(\cos\frac{\theta}{2} + \sin\frac{\theta}{2}\right)$$

$T = kx$, $x = \overline{GB} - \frac{l}{2}\sqrt{2}$

$$x = \frac{l}{\sqrt{2}}\left(\cos\frac{\theta}{2} + \sin\frac{\theta}{2} - 1\right)$$

$$\frac{l}{2}\sin\beta = \frac{l}{2}\sin\left(\frac{\pi}{4} - \frac{\theta}{2}\right) = \frac{l}{2\sqrt{2}}\left(\cos\frac{\theta}{2} - \sin\frac{\theta}{2}\right)$$

$$\Sigma M_O = \bar{I}\alpha + m\bar{a}d: \quad mg\frac{l}{2}\sin\theta - k\frac{l}{\sqrt{2}}\left(\cos\frac{\theta}{2} + \sin\frac{\theta}{2} - 1\right) \times \frac{l}{2\sqrt{2}}\left(\cos\frac{\theta}{2} - \sin\frac{\theta}{2}\right) = \frac{1}{12}ml^2\alpha + m\frac{l^2}{4}\alpha - ma\frac{l}{2}\cos\theta$$

$$\alpha = \frac{3}{2l}\left(a\cos\theta + g\sin\theta\right) - \frac{3k}{4m}\left(1 - 2\sin^2\frac{\theta}{2} - \cos\frac{\theta}{2} + \sin\frac{\theta}{2}\right)$$

$$\int_0^\omega \omega\, d\omega = \int_0^\theta \alpha\, d\theta :$$

$$\int_0^\theta (a\cos\theta + g\sin\theta)\, d\theta = \left[a\sin\theta - g\cos\theta\right]_0^\theta$$

$$= a\sin\theta + g(1 - \cos\theta)$$

$$2\int_0^\theta \sin^2\frac{\theta}{2}\, d\theta = \theta - \sin\theta$$

$$\int_0^\theta \cos\frac{\theta}{2}\, d\theta = 2\sin\frac{\theta}{2}$$

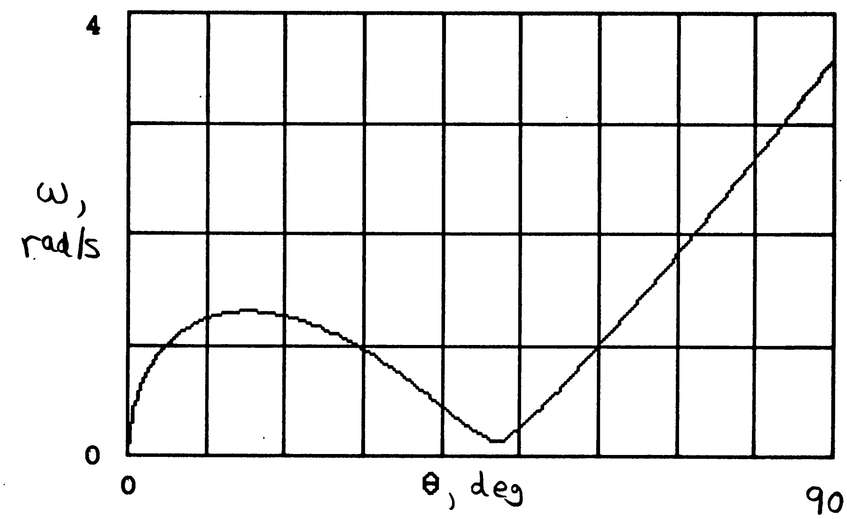
$$\int_0^\theta \sin\frac{\theta}{2}\, d\theta = 2\left(1-\cos\frac{\theta}{2}\right)$$

So $$\frac{\omega^2}{2} = \frac{3}{2l}\left[a\sin\theta + g(1-\cos\theta)\right] - \frac{3k}{4m}\left[\theta - (\theta-\sin\theta) - 2\sin\frac{\theta}{2} + 2\left(1-\cos\frac{\theta}{2}\right)\right]$$

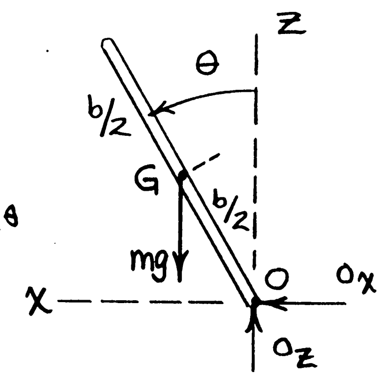
or $$\omega^2 = \frac{3}{l}\left[a\sin\theta + g(1-\cos\theta)\right] - \frac{3k}{2m}\left[\sin\theta - 2\left(\sin\frac{\theta}{2}+\cos\frac{\theta}{2}-1\right)\right]$$

For $l = 0.8$ m, $m = 4$ kg, $a = 0.4g = 3.92\ \text{m/s}^2$, $K = 600$ N/m :

$$\omega^2 = 36.8\left[0.4\sin\theta + 1 - \cos\theta\right] - 225\left[\sin\theta - 2\left(\sin\frac{\theta}{2}+\cos\frac{\theta}{2}-1\right)\right]$$



*6/237 $\circlearrowleft + \Sigma M_O = I_O \ddot{\theta}$:



$$mg\frac{b}{2}\sin\theta = \frac{1}{3}mb^2\ddot{\theta}$$

$$\ddot{\theta} = \frac{3}{2}\frac{g}{b}\sin\theta$$

$$\dot{\theta}\,d\dot{\theta} = \ddot{\theta}\,d\theta : \int_{\dot{\theta}_0}^{\dot{\theta}} \dot{\theta}\,d\dot{\theta} = \frac{3g}{2b}\int_{\theta_0}^{\theta} \sin\theta\,d\theta$$

$$\frac{\dot{\theta}^2}{2} - \frac{\dot{\theta}_0^2}{2} = \frac{3g}{2b}\left(\cos\theta_0 - \cos\theta\right)$$

$$\frac{d\theta}{dt} = \left[\dot{\theta}_0^2 + \frac{3g}{b}\left(\cos\theta_0 - \cos\theta\right)\right]^{1/2}$$

$$\int_0^t dt = \int_{\theta_0}^{\theta} \frac{d\theta}{\left[\dot{\theta}_0^2 + \frac{3g}{b}\left(\cos\theta_0 - \cos\theta\right)\right]^{1/2}}$$

With $\theta_0 = 10^\circ$ (0.1745 rad), $b = 60'$, $g = 32.2 \frac{\text{ft}}{\text{sec}^2}$ and $\dot{\theta}_0 = \frac{(v_A)_0}{b} = \frac{4.5}{60} = 0.0750$ rad/sec, a numerical solution yields $\underline{t = 2.85 \text{ sec}}$.

Energy considerations from $\theta_0 = 10^\circ$ to $\theta = 90^\circ$:

$$\Delta T + \Delta V_g = 0$$

$$\Delta T = \frac{1}{2}I_O\left[\frac{v_A}{b}\right]^2 - \frac{1}{2}I_O\left[\frac{(v_A)_0}{b}\right]^2 = \frac{1}{6}m\left[v_A^2 - (v_A)_0^2\right]$$

$$\Delta V_g = -mgh = -mg\frac{b}{2}\cos 10^\circ$$

So $\frac{1}{6}\cancel{m}\left[v_A^2 - 4.5^2\right] - \cancel{m}(32.2)\frac{60}{2}\cos 10^\circ = 0$

$$\underline{v_A = 75.7 \text{ ft/sec}}$$