INSTRUCTOR'S MANUAL

To Accompany

ENGINEERING MECHANICS - DYNAMICS

Volume 2

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USE OF THE INSTRUCTOR'S MANUAL

The problem solution portion of this manual has been prepared for the instructor who wishes to occasionally refer to the authors' method of solution or who wishes to check the answer of his (her) solution with the result obtained by the authors. In the interest of space and the associated cost of educational materials, the solutions are very concise. Because the problem solution material is not intended for posting of solutions or classroom presentation, the authors request that it not be used for these purposes.

In the transparency master section there are approximately 65 solved problems selected to illustrate typical applications. These problems are different from and in addition to those in the textbook. Instructors who have adopted the textbook are granted permission to reproduce these masters for classroom use.

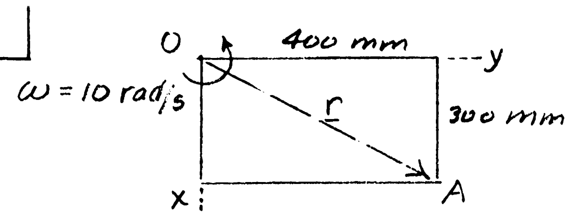
5/1

$$\alpha = \frac{\Delta\omega}{t} = \frac{800-200}{4/60} = 9000 \text{ rev/min}^2$$

$$\omega_2^2 = \omega_1^2 + 2\alpha\theta, \quad 800^2 = 200^2 + 2(9000)\theta \quad (\text{rev/min})^2$$

$$\theta = \frac{800^2 - 200^2}{2(9000)} = \underline{33.3 \text{ rev}} = N$$

5/2



$r = 500$ mm or 0.5 m

(a) Scalar: $v = r\omega = 0.5(10) = \underline{5 \text{ m/s}}$

$$a = a_n = r\omega^2 = 0.5(10^2) = \underline{50 \text{ m/s}^2}$$

(b) $\underline{r} = 0.3\underline{i} + 0.4\underline{j}$ m , $\underline{\omega} = 10\underline{k}$ rad/s

$$\underline{v} = \underline{\omega} \times \underline{r} = 10\underline{k} \times (0.3\underline{i} + 0.4\underline{j}) = 3\underline{j} + 4(-\underline{i})$$

$$v = \sqrt{3^2 + (-4)^2} = \underline{5 \text{ m/s}}$$

$$\underline{a} = \dot{\underline{\omega}} \times \underline{r} + \underline{\omega} \times \dot{\underline{r}} = 0 + \underline{\omega} \times \underline{v}$$

$$= \underline{0} + 10\underline{k} \times (3\underline{j} - 4\underline{i}) = -30\underline{i} - 40\underline{j} \text{ m/s}^2$$

$$|\underline{a}| = \sqrt{30^2 + 40^2} = \underline{50 \text{ m/s}^2}$$

5/3 | Let $\underline{k}$ be a unit vector out of paper.

(a) $\underline{v}_A = \underline{\omega} \times \underline{r}_{A/O} = 3\underline{k} \times (-0.4\underline{e}_n) = \underline{1.2\underline{e}_t \text{ m/s}}$

$\underline{a}_A = \underline{\alpha} \times \underline{r}_{A/O} - \omega^2 \underline{r}_{A/O} = -14\underline{k} \times (-0.4\underline{e}_n) - 3^2(-0.4\underline{e}_n)$

$= \underline{-5.6\underline{e}_t + 3.6\underline{e}_n \text{ m/s}^2}$

(b) $\underline{v}_B = \underline{\omega} \times \underline{r}_{B/O} = 3\underline{k} \times (-0.4\underline{e}_n + 0.1\underline{e}_t)$

$= \underline{1.2\underline{e}_t + 0.3\underline{e}_n \text{ m/s}}$

$\underline{a}_B = \underline{\alpha} \times \underline{r}_{B/O} - \omega^2 \underline{r}_{B/O}$

$= -14\underline{k} \times (-0.4\underline{e}_n + 0.1\underline{e}_t) - 3^2(-0.4\underline{e}_n + 0.1\underline{e}_t)$

$= \underline{-6.5\underline{e}_t + 2.2\underline{e}_n \text{ m/s}^2}$

5/4

$$\omega_{AB} = \omega_{cart} = \omega_{OG} = \frac{v_G}{\overline{OG}}, \quad \omega = \frac{14.20}{(56-8)/12} = \underline{3.55 \ \frac{rad}{sec}}$$

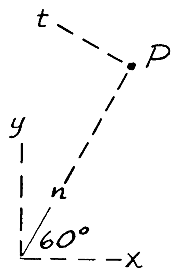
$$\omega_{av} = \frac{\Delta\theta}{\Delta t} = \frac{\pi/4}{0.638} = \underline{1.231 \ rad/sec}$$

5/5 | For $\theta = 90°$, $\underline{a} = -a_t \underline{i} - a_n \underset{\sim}{j}$ so $a_t = r\alpha = 1.8\ m/s^2$,

$$\alpha = \frac{1.8}{0.3} = \underline{6\ rad/s^2}$$

& $a_n = r\omega^2 = 4.8\ m/s^2$, $\omega = \sqrt{4.8/0.3} = \underline{4\ rad/s}$

5/6



$a_x = -3.02\ m/s^2$

$a_y = -1.624\ m/s^2$

$a_t = 3.02 \sin 60° - 1.624 \cos 60° = 1.803\ \frac{m}{s^2}$

$a_n = 3.02 \cos 60° + 1.624 \sin 60° = 2.92\ \frac{m}{s^2}$

$a_t = r\alpha: \quad \alpha = 1.803/0.3 = \underline{6.01\ rad/s^2}$

$a_n = r\omega^2: \quad \omega^2 = 2.92/0.3 = 9.72\ (rad/s)^2; \ \underline{\omega = 3.12\ rad/s}$

5/7

$\theta = 2t^3 - 3t^2 + 4$ rad.

$\dot{\theta} = 6t^2 - 6t$ rad/s

$\ddot{\theta} = 12t - 6$ rad/s^2

When $\ddot{\theta} = 42$ rad/s^2, $42 = 12t - 6$, $t = 4$ s

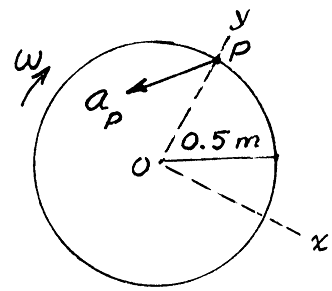
when $\ddot{\theta} = 66$ rad/s^2, $66 = 12t - 6$, $t = 6$ s

$\theta_{t=4s} = 2(4^3) - 3(4^2) + 4 = 84$ rad

$\theta_{t=6s} = 2(6^3) - 3(6^2) + 4 = 328$ rad

$\Delta\theta = 328 - 84 = \underline{244 \text{ rad}}$

5/8 For point P, $\underset{\sim}{a}_p = -3\underset{\sim}{i} - 4\underset{\sim}{j}$ m/s²



$a_n = r\omega^2$, $4 = 0.5\,\omega^2$, $\omega = \sqrt{8}$ $\frac{rad}{s}$

$\underset{\sim}{\omega} = -\sqrt{8}\,\underset{\sim}{k}$ rad/s

$a_t = r\alpha$, $3 = 0.5\alpha$, $\alpha = 6$ rad/s²

$\underset{\sim}{\alpha} = 6\underset{\sim}{k}$ rad/s²

5/9 $a_B = a_t = r\alpha$

$\omega = \omega_0 + \alpha t$: $300(2\pi)/60 = 0 + \alpha(2)$, $\alpha = 5\pi \text{ rad/s}^2$

Thus $5.5 = r(5\pi)$, $r = 0.350 \text{ m}$

$b = \sqrt{0.350^2 - 0.3^2} = 0.1806 \text{ m}$ or $\underline{b = 180.6 \text{ mm}}$

5/10

$$\underline{v}_P = \underline{\omega} \times \underline{r} = 2\underline{k} \times [0.5\underline{i} + 0.2\underline{j} + 0.050\underline{k}]$$
$$= \underline{-0.4\underline{i} + \underline{j} \quad \text{m/s}}$$

$$\underline{a}_P = \underline{\alpha} \times \underline{r} + \underline{\omega} \times (\underline{\omega} \times \underline{r})$$
$$= -3\underline{k} \times [0.5\underline{i} + 0.2\underline{j} + 0.050\underline{k}]$$
$$+ 2\underline{k} \times [2\underline{k} \times (0.5\underline{i} + 0.2\underline{j} + 0.050\underline{k})]$$
$$= \underline{-1.4\underline{i} - 2.3\underline{j} \quad \text{m/s}^2}$$

Note that $\underline{r}$ could have been taken as $0.5\underline{i} + 0.2\underline{j}$ m

The magnitudes of the above results are

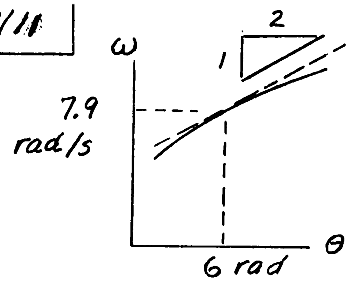
$v_P = 1.077$ m/s and $a_P = 2.69$ m/s^2.

These magnitudes check with

$$v_P = r_{xy}\,\omega = \sqrt{0.5^2 + 0.2^2}\,(2) = 1.077 \text{ m/s}^2 \checkmark$$

and $$a_P = \sqrt{a_t^2 + a_n^2} = \sqrt{(r_{xy}\alpha)^2 + (r_{xy}\omega^2)^2}$$
$$= \sqrt{0.5^2 + 0.2^2}\,\sqrt{3^2 + 2^4} = 2.69 \text{ m/s}^2 \checkmark$$

5/11



$$\alpha = \omega \frac{d\omega}{d\theta}$$

$$= 7.9\left(1/2\right) = \underline{3.95\ \text{rad/s}^2}$$

5/12

$$\theta = (-1 + 1.5t)e^{-0.5t}$$

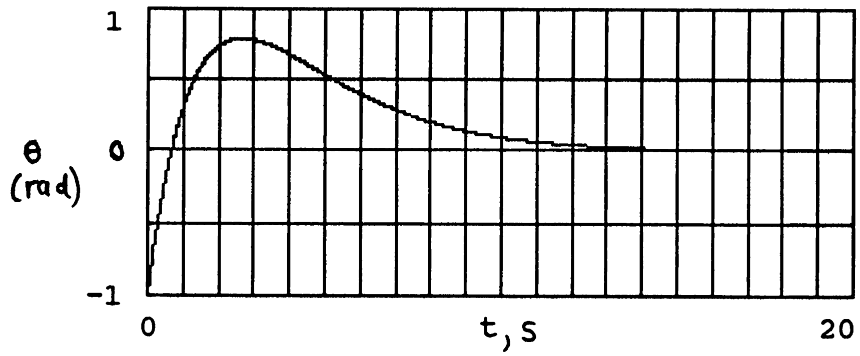
$$\omega = \frac{d\theta}{dt} = -0.5(-1+1.5t)e^{-0.5t} + 1.5e^{-0.5t}$$

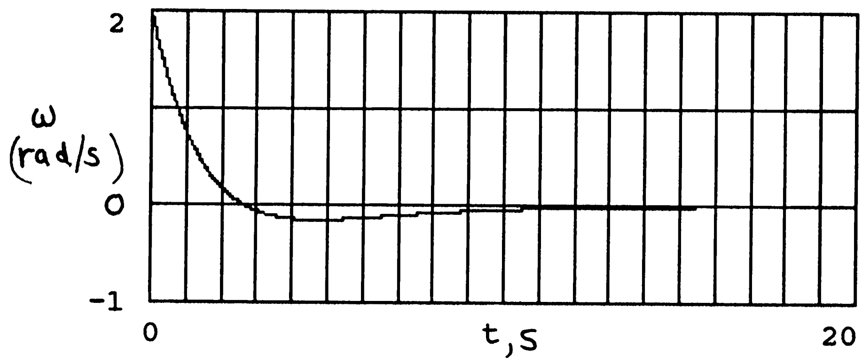
$$= \underline{(2 - 0.75t)e^{-0.5t}}$$

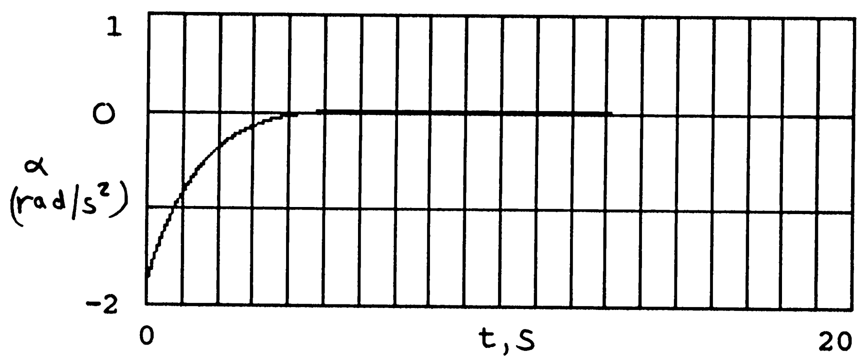
$$\alpha = \frac{d\omega}{dt} = -0.5(2-0.75t)e^{-0.5t} - 0.75e^{-0.5t}$$

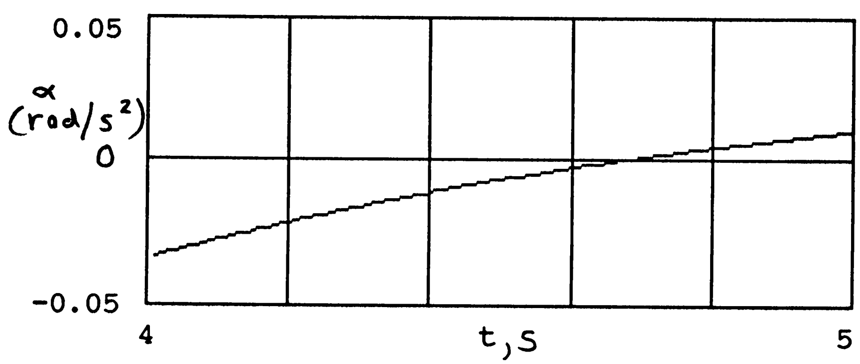
$$= \underline{(-1.75 + 0.375t)e^{-0.5t}}$$

$\alpha = 0$ when $-1.75 + 0.375t = 0$, $\underline{t = 4.67 \text{ s}}$


1
θ
(rad)
0
-1
0
t, s
20


2
ω
(rad/s)
0
-1
0
t, s
20


1
0
α
(rad/s²)
-2
0
t, s
20


0.05
α
(rad/s²)
0
-0.05
4
t, s
5

5/13 | $\underline{\omega}_{OA} = \underline{\omega}_{BC} = -6\underline{k}$ rad/s

$\underline{r}_A = 0.3\underline{i} + 0.28\underline{j}$ m

$\underline{v}_A = \underline{\omega} \times \underline{r}_A = -6\underline{k} \times (0.3\underline{i} + 0.28\underline{j}) = -1.8\underline{j} + 1.68\underline{i}$ m/s

$$\underline{\underline{v}_A = 1.68\underline{i} - 1.8\underline{j} \text{ m/s}}$$

$\underline{a}_A = \dot{\underline{\omega}} \times \underline{r}_A + \underline{\omega} \times \underline{v}_A = 0 + (-6\underline{k}) \times (1.68\underline{i} - 1.8\underline{j})$

$= -10.08\underline{j} - 10.8\underline{i}$

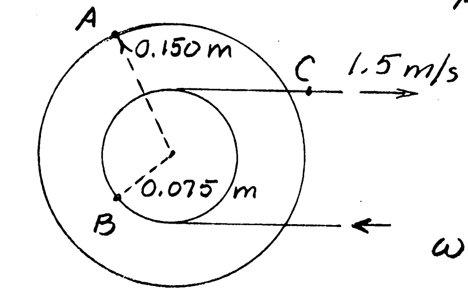
$$\underline{\underline{a}_A = -10.8\underline{i} - 10.08\underline{j} \text{ m/s}^2}$$

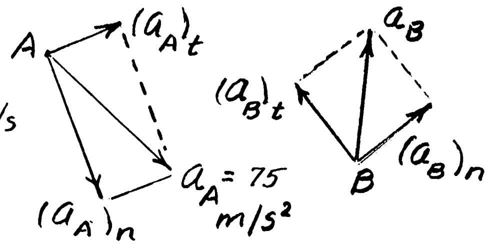
5/14 | At B, $v = \frac{50}{30} 44 = 73.3$ ft/sec, $r = 180 - \frac{18}{12} = 178.5$ ft

$\omega = v/r = 73.3/178.5 = \underline{0.411 \text{ rad/sec}}$

Between A & B $\omega_{av} = \frac{\Delta\theta}{\Delta t} = \frac{30}{180}\pi/1.52 = \underline{0.344 \text{ rad/sec}}$

5/15





$$\omega = v/r = \frac{1.5}{0.075} = 20 \text{ rad/s}$$

$$(a_A)_n = r\omega^2 = 0.15(20)^2 = 60 \text{ m/s}^2$$

$$(a_A)_t = \sqrt{(75)^2 - (60)^2} = 45 \text{ m/s}^2$$

$$\alpha = a_t/r = 45/0.15 = \underline{300 \text{ rad/s}^2}$$

$$(a_B)_n = 0.075(20)^2 = 30 \text{ m/s}^2$$

$$(a_B)_t = r\alpha = 0.075(300) = 22.5 \text{ m/s}^2$$

$$a_B = \sqrt{(30)^2 + (22.5)^2} = \underline{37.5 \text{ m/s}^2}$$

$$a_C = a_{B_t} = \underline{22.5 \text{ m/s}^2}$$

5/16

$a = a_n = \dfrac{v^2}{r}, \quad \left(\dfrac{v^2}{r}\right)_A = \dfrac{4}{3}\left(\dfrac{v^2}{4}\right)_B$

$r = \dfrac{4}{4/3} = \underline{3 \text{ in.}}$

5/17 $\underline{v}_A = \underline{\omega} \times \underline{r}_A$; $8\underline{j} = \omega \underline{k} \times 4\underline{i}$, $\omega = 2$ rad/sec

$\underline{\underline{\omega} = 2\underline{k} \text{ rad/sec}}$

$(\underline{a}_B)_t = \underline{\alpha} \times \underline{r}_B$; $6\underline{i} = \alpha \underline{k} \times 4\underline{j}$, $\alpha = -3/2$ rad/sec^2

$\underline{\underline{\alpha} = -\frac{3}{2}\underline{k} \text{ rad/sec}^2}$

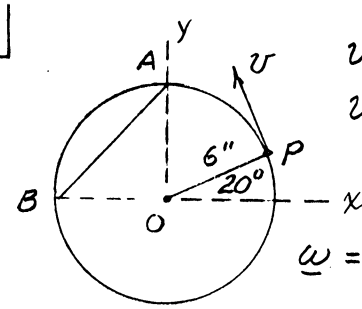
$\underline{r}_C = \frac{4}{\sqrt{2}}(\underline{i} - \underline{j})$ in.

$$\underline{a}_C = \underline{\alpha} \times \underline{r}_C + \underline{\omega} \times (\underline{\omega} \times \underline{r}_C)$$

$$= -\frac{3}{2}\underline{k} \times \frac{4}{\sqrt{2}}(\underline{i} - \underline{j}) + 2\underline{k} \times \left(2\underline{k} \times \frac{4}{\sqrt{2}}[\underline{i} - \underline{j}]\right)$$

$$= \frac{6}{\sqrt{2}}(-\underline{i} - \underline{j}) + \frac{16}{\sqrt{2}}(-\underline{i} + \underline{j}) = \underline{\sqrt{2}(-11\underline{i} + 5\underline{j})} \text{ in./sec}^2$$

5/18



$v_x = -4.2$ ft/sec

$v = 4.2/\sin 20° = 12.28$ ft/sec

$\omega = \frac{v}{r} = \frac{12.28}{6/12} = 24.6 \frac{\text{rad}}{\text{sec}}$

CCW

$\underline{\omega} = \underline{\omega}_{OP} = +24.6\underline{k}$ rad/sec

Element BC remains parallel to z-axis so has no angular velocity

5/19

$\alpha = \frac{d\omega}{dt} = 2 - kt = 2 - 0.2t$

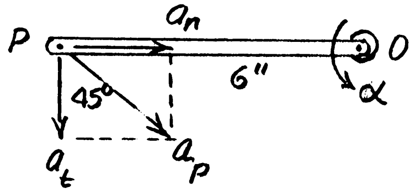
$\int_{\omega_0}^{\omega} d\omega = \int_0^t (2 - 0.2t)\,dt, \quad \omega = \omega_0 + 2t - 0.1t^2$

$\omega_0 = 200 \times 2\pi/60 = 20.9 \text{ rad/s}$

For $t = 5$ s, $\omega = 20.9 + 2(5) - 0.1(5^2) = 28.4 \text{ rad/s}$

$N = 28.4 \times 60/2\pi = \underline{272 \text{ rev/min}}$

5/20



$$\alpha = \frac{600(2\pi)}{60}\frac{1}{2} = 10\pi \text{ rad/sec}^2$$

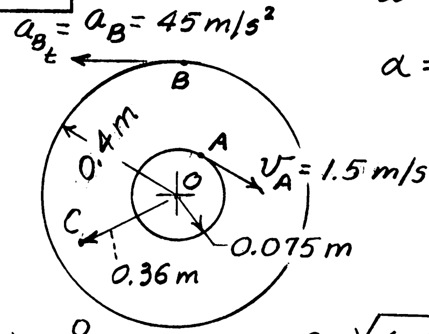
$$a_t = r\alpha = 6(10\pi) = 60\pi \text{ in./sec}^2$$

$$a_n = r\omega^2 = 60\pi \text{ in./sec}^2 \text{ for } 45°$$

so $\omega^2 = 60\pi/6 = 10\pi$, $\omega = 5.60$ rad/s

$\omega = \omega_0 + \alpha t$: $5.60 = 0 + 10\pi t$, $\underline{t = 0.1784 \text{ sec}}$

5/21

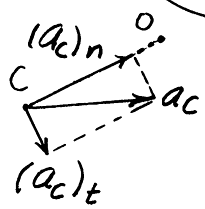


$\omega = v/r = \frac{1.5}{0.075} = 20\ rad/s$

$\alpha = a_t/r = \frac{45}{0.4} = 112.5\ rad/s^2$

$(a_C)_n = r\omega^2 = 0.36(20)^2$
$= 144\ m/s^2$

$(a_C)_t = r\alpha = 0.36(112.5)$
$= 40.5\ m/s^2$



$a_C = \sqrt{(144)^2 + (40.5)^2} = \underline{149.6\ m/s^2}$

5/22 $\alpha = 1.8 - k\theta$ rev/s^2 , θ in revolutions

$0.6 = 1.8 - k(20)$, $k = 0.06$ 1/s^2

so $\alpha = 1.8 - 0.06\theta$ rev/s^2 ; $\omega_0 = 300/60 = 5$ rev/s

$\omega\, d\omega = \alpha\, d\theta$: $\int_5^{\omega} \omega\, d\omega = \int_0^{20} (1.8 - 0.06\theta)\, d\theta$

$\omega^2 = 5^2 + 2\left[1.8\theta - 0.03\theta^2\right]_0^{20} = 25 + 48 = 73$ (rev/s)^2

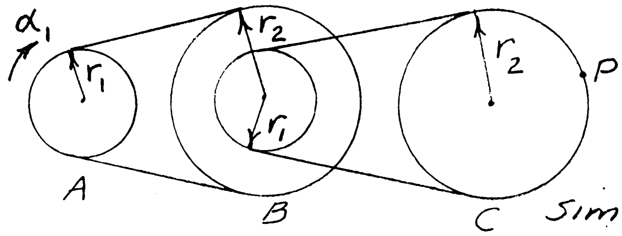
$\omega = \sqrt{73} = 8.54$ rev/s or $N = 8.54(60) = \underline{513 \text{ rev/min}}$

5/23 For gear A, $\Delta\omega = \int_2^6 \alpha_A \, dt$, $N_A = 2N_B$

$$(N_A - 600)\frac{2\pi}{60} = \frac{4+8}{2}(6-2), \quad N_A = 600 + 229 = 829 \text{ rev/min}$$

so at $t = 6$ s, $N_B = \frac{829}{2} = \underline{415 \text{ rev/min}}$

5/24



$r_2\omega_C = r_1\omega_B$

$r_2\omega_B = r_1\omega_A$

so that

$\omega_C = \left(\frac{r_1}{r_2}\right)^2\omega_A$

Similarly $\alpha_C = \left(\frac{r_1}{r_2}\right)^2\alpha_1$

$\omega_A = \alpha_1 t$ so that $\underline{\omega_C = \left(\frac{r_1}{r_2}\right)^2\alpha_1 t}$

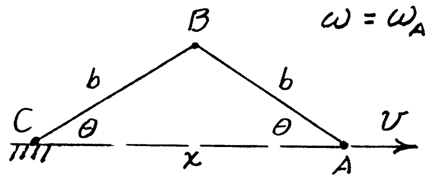
For P, $a_n = r_2\omega_C^2 = r_2\left[\left(\frac{r_1}{r_2}\right)^2\alpha_1 t\right]^2$

$a_t = r_2\alpha_C = r_2\left(\frac{r_1}{r_2}\right)^2\alpha_1$

$a_P = \sqrt{a_n^2 + a_t^2} = \underline{\frac{r_1^2}{r_2}\alpha_1\sqrt{1 + \left(\frac{r_1}{r_2}\right)^4\alpha_1^2 t^4}}$

5/25 $x = 2b\cos\theta$, $\dot{x} = -2b\dot{\theta}\sin\theta$, $v = \dot{x}$

$\omega = \omega_{AB} = \dot{\theta}$ so $\omega = \dfrac{-v}{2b\sin\theta}$ CW



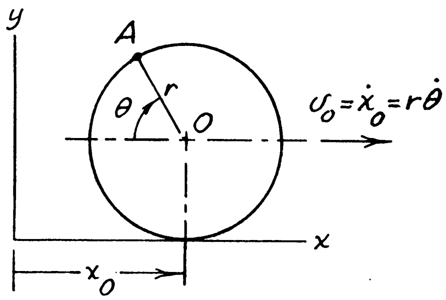
For $a = \ddot{x}$ const., $\dot{x}^2 = 2ax$

$v = \sqrt{2ax}$

so $\omega = \dfrac{\sqrt{2ax}}{2b\sqrt{1-\cos^2\theta}} = \dfrac{\sqrt{2ax}}{\sqrt{4b^2 - x^2}}$

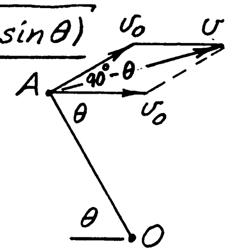
5/26 Coordinates of A are

$$x = x_0 - r\cos\theta$$
$$y = r + r\sin\theta$$



$$\dot{x} = \dot{x}_0 + r\dot{\theta}\sin\theta = v_0(1+\sin\theta)$$
$$\dot{y} = r\dot{\theta}\cos\theta = v_0\cos\theta$$

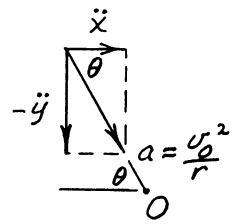
$$v = \sqrt{\dot{x}^2+\dot{y}^2} = v_0\sqrt{(1+\sin\theta)^2+\cos^2\theta} = \underline{v_0\sqrt{2(1+\sin\theta)}}$$



$$\ddot{x} = v_0\dot{\theta}\cos\theta = v_0\left(\frac{v_0}{r}\right)\cos\theta = \frac{v_0^2}{r}\cos\theta$$

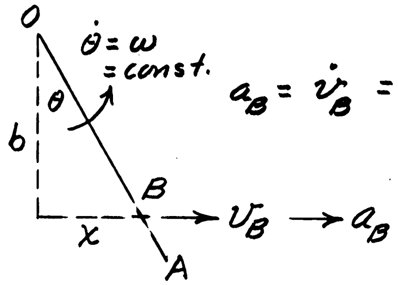
$$\ddot{y} = -v_0\dot{\theta}\sin\theta = -v_0\left(\frac{v_0}{r}\right)\sin\theta = -\frac{v_0^2}{r}\sin\theta$$

$$a = \sqrt{\ddot{x}^2+\ddot{y}^2} = \frac{v_0^2}{r}\sqrt{\cos^2\theta+\sin^2\theta} = \underline{\frac{v_0^2}{r}\text{ toward } O}$$



5/27

$x = b \tan\theta, \quad v_B = \dot{x} = b\dot{\theta}\sec^2\theta$



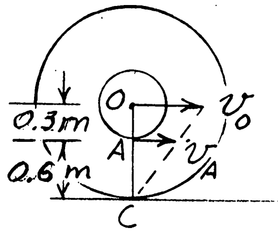
$$\underline{v_B = b\omega\sec^2\theta}$$

$$a_B = \dot{v}_B = 2b\omega\dot{\theta}\sec\theta\,(\sec\theta\tan\theta)$$

$$\underline{a_B = 2b\omega^2\sec^2\theta\tan\theta}$$

5/28

$$v_O = \overline{OC}\,\omega = \frac{\overline{OC}}{\overline{AC}} v_A = \frac{0.9}{0.6} 0.8 = \underline{1.2 \text{ m/s}}$$

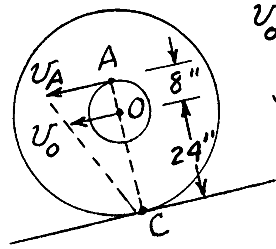


$$\omega = \frac{v_A}{\overline{AC}} = \frac{v_O}{\overline{OC}} = \frac{1.2}{0.9} = \underline{1.333 \text{ rad/s}} \text{ CW}$$

5/29

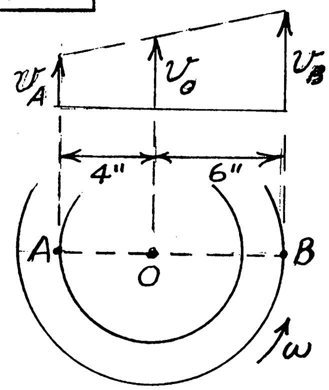
$v_A = r_1\omega_1 = 12(2) = 24$ in./sec

$v_O = \frac{\overline{OC}}{\overline{AC}} v_A = \frac{24}{32} 24 = 18$ in./sec



$s = v_O t, \quad t = \frac{100(12)}{18} = \underline{66.7 \text{ sec}}$

5/30



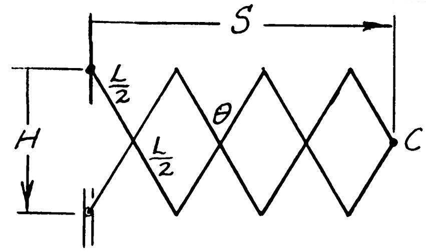
$v_A = 3$ ft/sec, $v_B = 4$ ft/sec

$$\omega = \frac{v_B - v_A}{\overline{AB}} = \frac{4-3}{10/12} = 1.2 \ \frac{\text{rad}}{\text{sec}}$$

$$v_O = v_A + \frac{\overline{AO}}{\overline{AB}}(v_B - v_A)$$

$$= 3 + \frac{4}{10}(4-3) = 3.4 \text{ ft/sec}$$

5/31

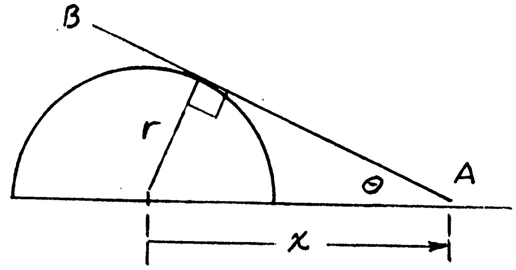


$$H = L\cos\frac{\theta}{2},\ S = \frac{7}{2}L\sin\frac{\theta}{2}$$

$$\dot{H} = -\frac{L}{2}\dot{\theta}\sin\frac{\theta}{2} = -u,\quad \dot{S} = v = \frac{7}{4}L\dot{\theta}\cos\frac{\theta}{2}$$

so $v = \frac{7}{4}L\cos\frac{\theta}{2}\left(\frac{2u}{L}\Big/\sin\frac{\theta}{2}\right)$, $\underline{v = \frac{7}{2}u\cot\frac{\theta}{2}}$

5/32



$r = x \sin\theta$

$0 = \dot{x}\sin\theta + x\dot{\theta}\cos\theta$

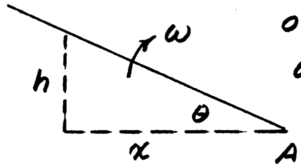
$\dot{x} = v$, $\omega = \dot{\theta}$ so

$\omega = -\frac{v}{x}\tan\theta$

$= -\frac{v}{x}\frac{r}{\sqrt{x^2 - r^2}}$

5/33

$v_A = r\omega_0 = -\dot{x}, \quad h = x\tan\theta$



$0 = \dot{x}\tan\theta + x\dot{\theta}\sec^2\theta$

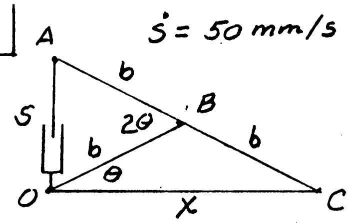
$\omega = \dot{\theta} = -\frac{\dot{x}}{x}\sin\theta\cos\theta$

$= -\frac{\dot{x}}{x}\frac{hx}{x^2+h^2}$

$\omega = \frac{rh\omega_0}{x^2+h^2}$

5/34

$\dot{s} = 50$ mm/s , $b = 150$ mm

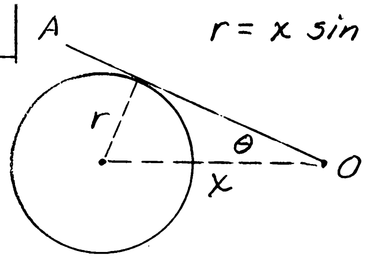


$s = 2b\sin\theta$, $\dot{s} = 2b\dot{\theta}\cos\theta$

$x = 2b\cos\theta$, $\dot{x} = -2b\dot{\theta}\sin\theta = -v_C$

$v_C = \dot{s}\,\dfrac{\sin\theta}{\cos\theta} = 50\tan 50° = \underline{59.6 \text{ mm/s}}$

5/35



$r = x \sin\theta$, $0 = \dot{x} \sin\theta + x\dot{\theta}\cos\theta$

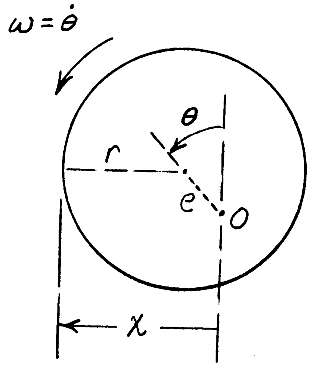
$\omega = \dot{\theta} = -\frac{\dot{x}}{x} \tan\theta$

But $v = -\dot{x}$

& $\tan\theta = \frac{r}{\sqrt{x^2 - r^2}}$

so $\omega = \frac{v}{x} \frac{r}{\sqrt{x^2 - r^2}} = \underline{\frac{v}{x\sqrt{(x/r)^2 - 1}}}$

5/36 Displacement of fork & rod is



$$x = r + e \sin\theta$$

$$v_x = \dot{x} = 0 + e\dot{\theta}\cos\theta$$

$$v_x = \underline{e\omega\cos\theta}$$

$$a_x = \dot{v}_x = -e\omega\dot{\theta}\sin\theta = \underline{-e\omega^2\sin\theta}$$

5/37

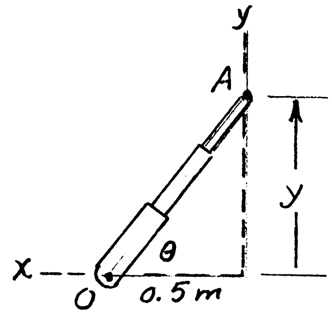
$y = 0.5 \tan\theta$

$\dot{y} = 0.5 \sec^2\theta\,\dot{\theta}$

$\ddot{y} = 0 = \sec\theta(\tan\theta\sec\theta)\dot{\theta}^2 + 0.5\sec^2\theta\,\ddot{\theta}$

$\dot{\theta} = 2\dot{y}/\sec^2\theta$

$\ddot{\theta} = -2\tan\theta\,\dot{\theta}^2$



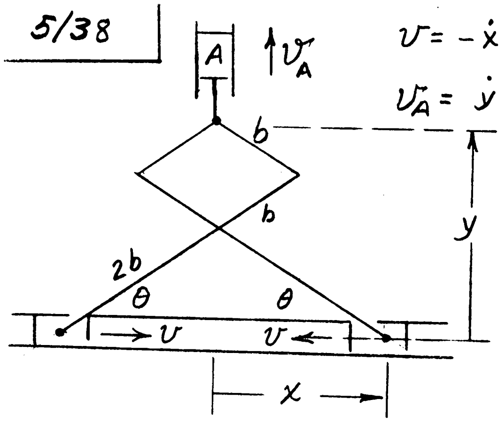
For $y = 0.6$ m, $\tan\theta = \frac{0.6}{0.5} = 1.2$, $\theta = 50.2°$

$\sec\theta = 1.562$

So for $\dot{y} = 0.2$ m/s, $\dot{\theta} = \frac{2(0.2)}{(1.562)^2} = \underline{0.1639 \text{ rad/s}}$

$\ddot{\theta} = -2(1.2)(0.1639)^2 = \underline{-0.0645 \text{ rad/s}^2}$

5/38



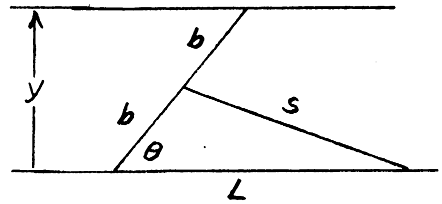
$v = -\dot{x} = -\frac{d}{dt}(2b\cos\theta) = 2b\sin\theta\,\dot{\theta}$

$v_A = \dot{y} = \frac{d}{dt}(4b\sin\theta) = 4b\cos\theta\,\dot{\theta}$

so $v = \dfrac{v_A}{2\cos\theta}\sin\theta$

$\underline{v = \frac{1}{2}v_A\tan\theta}$

5/39



$y = 2b \sin\theta$

$v = \dot{y} = 2b\dot{\theta}\cos\theta$

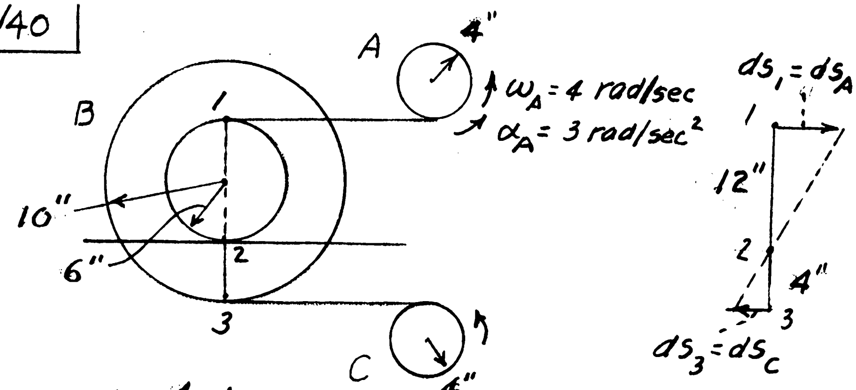
$s^2 = b^2 + L^2 - 2bL\cos\theta$

$2s\dot{s} = 0 + 0 + 2bL\dot{\theta}\sin\theta$

$\dot{\theta} = \dfrac{s\dot{s}}{bL\sin\theta}$

so $v = 2b\dfrac{s\dot{s}}{bL\sin\theta}\cos\theta = \underline{2\dfrac{\sqrt{b^2 + L^2 - 2bL\cos\theta}}{L\tan\theta}\dot{s}}$

5/40

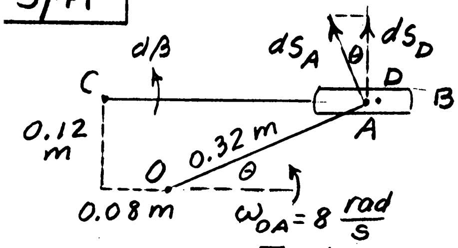


$(ds_3 = ds_C) = \frac{4}{12}(ds_1 = ds_A)$

so $ds_C = \frac{1}{3} ds_A$ Thus $\omega = \omega_C = \frac{1}{3} 4 = \underline{4/3 \text{ rad/sec CCW}}$

$\alpha = \alpha_C = \frac{1}{3} 3 = \underline{1 \text{ rad/sec}^2 \text{ CCW}}$

5/41



Let D be point on CB coincident with A

$\theta = \sin^{-1}\dfrac{0.12}{0.32} = 22.0°$

$ds_A = \overline{OA}\ d\theta$

$ds_D = \overline{CD}\ d\beta$

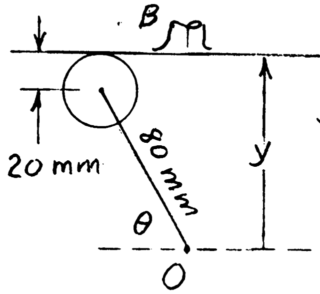
Thus $d\beta = \dfrac{\overline{OA}}{\overline{CD}}\ \dfrac{ds_D}{ds_A}\ d\theta$ & $\dot{\beta} = \dfrac{\overline{OA}}{\overline{CD}} \cos\theta\ \dot{\theta}$

so $\omega_{CB} = \dot{\beta} = \dfrac{0.32}{0.08 + 0.32 \cos 22°} \cos 22° \ (8) = \underline{6.30 \text{ rad/s}}$

5/42

$y = 20 + 80 \sin\theta, \quad \dot{y} = 80\dot{\theta}\cos\theta$

$\ddot{y} = 80\ddot{\theta}\cos\theta - 80\dot{\theta}^2\sin\theta$



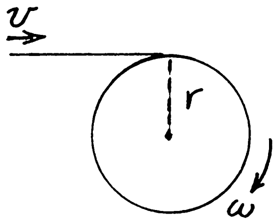
For $\theta = 60°$, $\dot{\theta} = 4\ \frac{rad}{s}$, $\ddot{\theta} = 8\ \frac{rad}{s^2}$,

$\ddot{y} = 80(8)(\frac{1}{2}) - 80(4)^2\frac{\sqrt{3}}{2}$

$= 320 - 1109 = -789\ mm/s^2$

Thus $a_B = \underline{789\ mm/s^2 \text{ down}}$

5/43

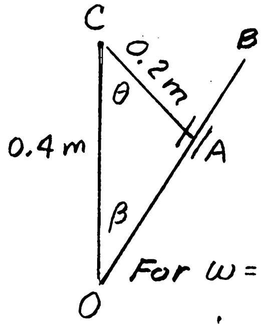


$v = r\omega$, $a = \dot{v} = \dot{r}\omega + r\dot{\omega}$, $\dot{\omega} = 0$

But $\dot{r} = \frac{t}{2\pi/\omega} = \frac{t\omega}{2\pi} = \frac{tv}{2\pi r}$

Thus $a = \frac{tv}{2\pi r}\omega = \underline{\frac{tv^2}{2\pi r^2}}$

5/44



$$\tan\beta = \frac{0.2\sin\theta}{0.4 - 0.2\cos\theta}, \quad \tan\beta\,(2-\cos\theta) = \sin\theta$$

$$\dot\beta \sec^2\beta\,(2-\cos\theta) + \tan\beta\,(\dot\theta\sin\theta) = \dot\theta\cos\theta$$

$$\dot\beta = \frac{\cos\theta - \sin\theta\tan\beta}{2-\cos\theta}\,\dot\theta\cos^2\beta$$

$$= \frac{2\cos\theta - 1}{(2-\cos\theta)^2}\,\dot\theta\cos^2\beta$$

For $\omega = -\dot\theta = 3\ \frac{\text{rad}}{\text{s}}$, $\theta = 45°$, $\beta = \tan^{-1}\frac{1/\sqrt2}{2 - 1/\sqrt2} = 28.7°$

$$\dot\beta = \frac{2/\sqrt2 - 1}{(2 - 1/\sqrt2)^2}(-3)\cos^2 28.7° = -0.572\ \text{rad/s}$$

So $\omega_{OB} = \underline{0.572\ \text{rad/s CCW}}$

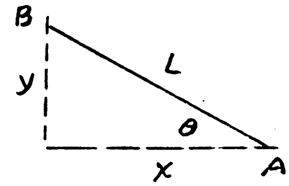
5/45 $x = L\cos\theta$

$\dot{x} = -v_0 = -L\dot{\theta}\sin\theta$

$\omega = \dot{\theta} = \dfrac{v_0}{L\sin\theta}$

where $L\sin\theta = y = \sqrt{L^2 - x^2}$

so $\underline{\omega = \dfrac{v_0}{\sqrt{L^2 - x^2}}}$



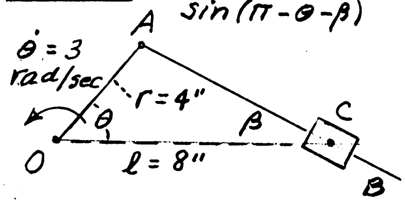
$\alpha = \ddot{\theta} = \dfrac{v_0}{L}\dfrac{d}{dt}\csc\theta = \dfrac{v_0}{L}(-\text{ctn}\,\theta\,\csc\theta)\dot{\theta}$

$= -\dfrac{v_0}{L}\dfrac{x}{y}\dfrac{L}{y}\dot{\theta} = \dfrac{-xv_0^2}{y^2\sqrt{L^2 - x^2}}$

$$\underline{= \dfrac{-xv_0^2}{(L^2 - x^2)^{3/2}}}$$

5/46

$$\frac{l}{\sin(\pi-\theta-\beta)} = \frac{l}{\sin(\theta+\beta)} = \frac{r}{\sin\beta}$$



$$r\sin(\theta+\beta) = l\sin\beta$$

$$r(\dot{\theta}+\dot{\beta})\cos(\theta+\beta) = l\dot{\beta}\cos\beta$$

$$\omega_{AB} = \omega = \dot{\beta} = \frac{r\dot{\theta}\cos(\theta+\beta)}{l\cos\beta - r\cos(\theta+\beta)}$$

where $\tan\beta = \dfrac{r\sin\theta}{l - r\cos\theta}$

For $\theta = 40°$, $\beta = \tan^{-1}\dfrac{4\sin 40°}{8-4\cos 40°} = \tan^{-1}0.5209 = 27.5°$

Thus $\omega = \dfrac{4(3)\cos 67.5°}{8\cos 27.5 - 4\cos 67.5} = \underline{0.825 \text{ rad/sec}}$

5/47 Belt velocity is the same for both pulleys

so $r_1\omega_1 = r_2\omega_2$

Thus $\dot{r}_1\omega_1 + r_1\dot{\omega}_1 = \dot{r}_2\omega_2 + r_2\dot{\omega}_2$

For $\dot{\omega}_1 = 0$ & $\alpha_2 = \dot{\omega}_2$, we have

$$\alpha_2 = \dot{\omega}_2 = \frac{\dot{r}_1\omega_1 - \dot{r}_2\omega_2}{r_2} = \underline{\frac{\dot{r}_1 r_2 - r_1\dot{r}_2}{r_2^2}\,\omega_1}$$

5/48 | Given $\dot{s} = 0.260$ m/s

$s = 2(0.2)\sin\frac{\theta}{2}$

$\dot{s} = 0.2\dot{\theta}\cos\frac{\theta}{2}$

For $\theta = 60°$

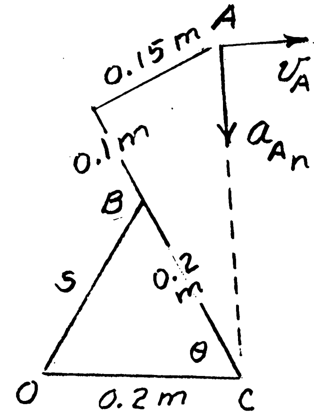
$\dot{s} = 0.260 = 0.2\dot{\theta}\cos\frac{60°}{2}$

$\dot{\theta} = \omega_{AC} = \frac{0.260}{0.2\cos 30°} = 1.501$ rad/s

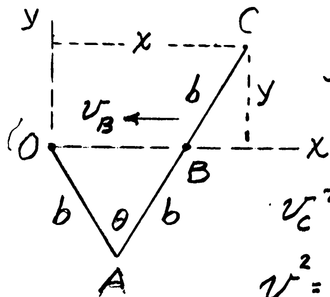
$\overline{AC} = \sqrt{0.3^2 + 0.15^2} = 0.335$ m

$a_{A_n} = \overline{AC}\,\omega_{AC}^2$

$= 0.335(1.501)^2 = \underline{0.756 \text{ m/s}^2}$



5/49 Coordinates of C are



$$x = 3b\sin\frac{\theta}{2}, \quad y = b\cos\frac{\theta}{2}$$

so $\dot{x} = \frac{3}{2}b\dot{\theta}\cos\frac{\theta}{2}, \quad \dot{y} = -\frac{b}{2}\dot{\theta}\sin\frac{\theta}{2}$

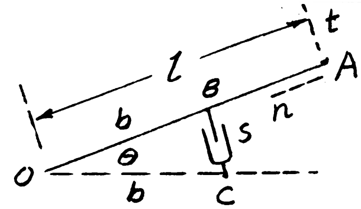
$$v_C^2 = \dot{x}^2 + \dot{y}^2 = \frac{b^2\dot{\theta}^2}{4}\left(9\cos^2\frac{\theta}{2} + \left[1-\cos^2\frac{\theta}{2}\right]\right)$$

$$v_C^2 = \frac{b^2\dot{\theta}^2}{4}\left(8\cos^2\frac{\theta}{2}+1\right) = 2b^2\dot{\theta}^2\cos^2\frac{\theta}{2} + \frac{1}{4}b^2\dot{\theta}^2$$

But $v_B = -\frac{d}{dt}\left(2b\sin\frac{\theta}{2}\right) = -b\dot{\theta}\cos\frac{\theta}{2}$

so $v_C^2 = 2v_B^2 + \frac{1}{4}\frac{v_B^2}{\cos^2\theta/2}, \quad \underline{v_C = \frac{v_B}{2}\sqrt{8+\sec^2\frac{\theta}{2}}}$

5/50 $a_{A_n} = l\dot{\theta}^2$, $a_{A_t} = l\ddot{\theta}$



$s = 2b \sin\frac{\theta}{2}$

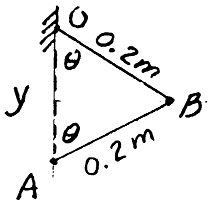
$\dot{s} = k = b\dot{\theta}\cos\frac{\theta}{2}$

$0 = b\ddot{\theta}\cos\frac{\theta}{2} - \frac{b}{2}\dot{\theta}^2 \sin\frac{\theta}{2}$

$\ddot{\theta} = \frac{1}{2}\dot{\theta}^2 \tan\frac{\theta}{2} = \frac{1}{2}\left(\frac{k}{b\cos\frac{\theta}{2}}\right)^2 \tan\frac{\theta}{2} = \frac{k^2}{2b^2} \frac{\tan\frac{\theta}{2}}{\cos^2\frac{\theta}{2}}$

$\underline{a}_A = l\dot{\theta}^2 \underline{e}_n + l\ddot{\theta}\underline{e}_t = \frac{k^2 l}{b^2\cos^2\frac{\theta}{2}}\left(\underline{e}_n + \frac{1}{2}\tan\frac{\theta}{2}\,\underline{e}_t\right)$

5/51



$y = 2(0.2)\cos\theta, \quad \dot{y} = -0.4\dot{\theta}\sin\theta$

$\ddot{y} = -0.4\ddot{\theta}\sin\theta - 0.4\dot{\theta}^2\cos\theta$

$= -0.4\ddot{\theta}\sin\theta - 0.4\left(\dfrac{\dot{y}}{-0.4\sin\theta}\right)^2\cos\theta$

For $y = 0.2\,m$, $\theta = 60^\circ$, $\sin\theta = \dfrac{\sqrt{3}}{2}$, $\cos\theta = \dfrac{1}{2}$

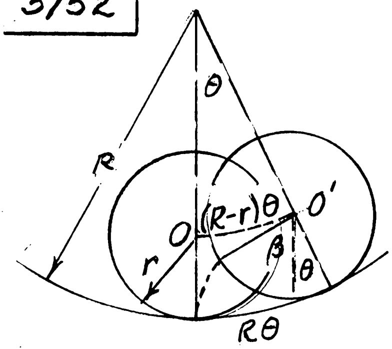
$\dot{y} = 0.4\,m/s, \quad \ddot{y} = -0.1\,m/s^2$

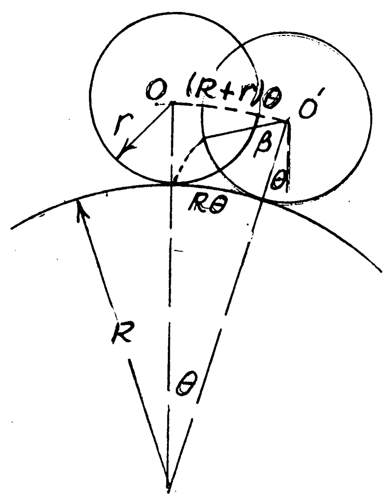
$$\dot{\theta} = -\frac{\dot{y}}{0.4\sin\theta} = -\frac{0.4}{0.4(\sqrt{3}/2)} = -1.155\ \frac{rad}{s}, \quad \underline{\omega = 1.155\ \frac{rad}{s}\ CCW}$$

$$\ddot{\theta} = \frac{-\ddot{y}}{0.4\sin\theta} - \frac{\dot{y}^2\cos\theta}{0.16\sin^3\theta} = \frac{+0.1}{0.4(\sqrt{3}/2)} - \frac{0.4^2(1/2)}{0.16(3\sqrt{3}/8)}$$

$$= 0.289 - 0.770 = -0.481\ rad/s^2, \quad \underline{\alpha = 0.481\ rad/s^2\ CCW}$$

5/52





$v_O = v = (R-r)\dot{\theta}$

$R\theta = r(\theta+\beta)$

so $\theta(R-r) = r\beta$

$\dot{\theta}(R-r) = r\dot{\beta}$

so $v = r\dot{\beta}$ & $\omega = \dot{\beta}$

$\underline{v = r\omega}$ so $\underline{a_t = r\alpha}$

(β = absolute angle)

$v_O = v = (R+r)\dot{\theta}$

$R\theta = r\beta$ so $\theta+\beta = (\frac{r+R}{r})\theta$

so $r(\dot{\theta}+\dot{\beta}) = (R+r)\dot{\theta}$

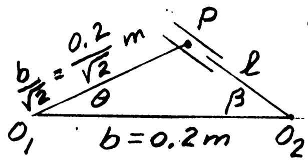
where $\omega = (\dot{\beta}+\dot{\theta})$

so $\underline{v = r\omega}$ so $\underline{a_t = r\alpha}$

($\beta+\theta$ = absolute angle)

▶5/53

$$\frac{b/\sqrt{2}}{\sin\beta} = \frac{b}{\sin(\pi-\theta-\beta)} = \frac{b}{\sin(\theta+\beta)}$$



so $\sqrt{2}\sin\beta = \sin(\theta+\beta)$ ----- (a)

$$\sqrt{2}\,\dot{\beta}\cos\beta = (\dot{\theta}+\dot{\beta})\cos(\theta+\beta)$$

$$\omega_2 = -\dot{\beta} = \dot{\theta}\,\frac{\cos(\theta+\beta)}{\cos(\theta+\beta)-\sqrt{2}\cos\beta} \quad \text{--- (b)}$$

From (a) $\sin\beta(\sqrt{2}-\cos\theta) = \sin\theta\cos\beta$, $\tan\beta = \dfrac{\sin\theta}{\sqrt{2}-\cos\theta}$

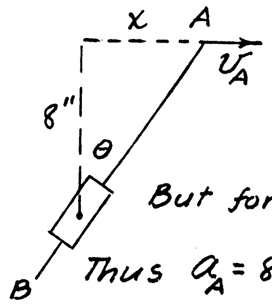
For $\theta = 20^\circ$, $\beta = \tan^{-1}\dfrac{0.3420}{\sqrt{2}-0.9397} = \tan^{-1}0.7208 = 35.8^\circ$

& for $\dot{\theta} = -2$ rad/s, Eq. (b) gives

$$\omega_2 = -2\,\frac{\cos(20^\circ+35.8^\circ)}{\cos(20^\circ+35.8^\circ)-\sqrt{2}\cos 35.8^\circ} = -2\,\frac{0.5623}{-0.5849}$$

$\underline{\omega_2 = 1.923 \text{ rad/s}}$

▶ 5/54 $x = 8\tan\theta$, $v_A = \dot{x} = 8\dot{\theta}\sec^2\theta$



$$a_A = \ddot{x} = 8\ddot{\theta}\sec^2\theta + 16\dot{\theta}^2\sec^2\theta\tan\theta$$

$$= 8\ddot{\theta}\sec^2\theta + 16\frac{v_A^2}{64\sec^4\theta}\sec^2\theta\tan\theta$$

$$= 8\ddot{\theta}\sec^2\theta + \frac{v_A^2}{4}\sin\theta\cos\theta$$

But for $\ddot{x} = \text{const.} = a_A$, $v_A^2 = 2a_A x$

Thus $a_A = 8\ddot{\theta}\sec^2\theta + \dfrac{a_A x}{2}\sin\theta\cos\theta$

Substitute $a_A = 4$ in./sec^2, $\sec\theta = 5/4$, $\cos\theta = 4/5$

$\sin\theta = 3/5$ for $x = 6$ in.

$4 = 8\ddot{\theta}(5/4)^2 + \dfrac{4(6)}{2}\dfrac{3}{5}\dfrac{4}{5}$, $\quad\ddot{\theta} = -0.1408$ rad/sec^2

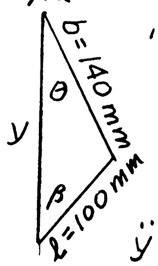
So $\underline{\alpha = 0.1408 \text{ rad/sec}^2 \text{ CCW}}$

▸5/55 $\theta = \theta_0 \sin 2\pi t$, $\dot\theta = 2\pi\theta_0 \cos 2\pi t$, $\ddot\theta = -4\pi^2\theta_0 \sin 2\pi t$

$\theta_0 = \pi/12$

When $\theta = 0$, $t = 1/2\,s$ & $\dot\theta = 2\pi\theta_0 = \pi^2/6$ rad/s, $\ddot\theta = 0$

" $\theta = \frac{\pi}{12}$, $t = 1/4\,s$ & $\dot\theta = 0$, $\ddot\theta = -4\pi^2\theta_0 = -\pi^3/3$ rad/s²



$\ell^2 = y^2 + b^2 - 2yb\cos\theta$, $0 = y\dot y + yb\dot\theta\sin\theta - \dot y b\cos\theta$

$0 = y\ddot y + \dot y^2 + \dot y b\dot\theta\sin\theta + yb\ddot\theta\sin\theta + yb\dot\theta^2\cos\theta$
$\qquad - \ddot y b\cos\theta + \dot y b\dot\theta\sin\theta$

$\ddot y(b\cos\theta - y) = \dot y^2 + 2\dot y b\dot\theta\sin\theta + yb\ddot\theta\sin\theta + yb\dot\theta^2\cos\theta$

(a) $\theta = 0$, $\ddot y(b - [b+\ell]) = 0 + 0 + 0 + (b+\ell)b(\pi^2/6)^2$

$\dot y = 0$

$\ddot\theta = 0$

$$\ddot y = \frac{\pi^4}{36}\frac{b(b+\ell)}{-\ell} = \frac{-\pi^4}{36}\frac{0.14(0.24)}{0.1} = -0.909\ \frac{m}{s^2}$$

$\underline{(up)}$

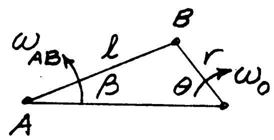
(b) $\theta = \pi/12$, $b/\sin\beta = \ell/\sin\frac{\pi}{12}$, $\beta = \sin^{-1}\left(\frac{0.14}{0.1}\sin\frac{\pi}{12}\right) = 21.24°$

$\dot y = 0$ $\quad y = b\cos\theta + \ell\cos\beta = 0.14\cos\frac{\pi}{12} + 0.1\cos 21.24° = 0.2284$ m

$\dot\theta = 0$ $\quad \ddot y(0.14\cos\frac{\pi}{12} - 0.2284) = 0 + 0 + 0.2284(0.14)\left(-\frac{\pi^3}{3}\right)\sin\frac{\pi}{12} + 0$

$\ddot y(-0.09320) = -0.08555$, $\underline{\ddot y = 0.918\ m/s^2}$ (down)

▶5/56 $l \sin\beta = r\sin\theta, \quad l\dot\beta\cos\beta = r\dot\theta\cos\theta$



so $\omega_{AB} = \dot\beta = \dfrac{r}{l}\dot\theta\dfrac{\cos\theta}{\cos\beta} = \dfrac{r}{l}\omega_0\dfrac{\cos\theta}{\sqrt{1-(\frac{r}{l}\sin\theta)^2}}$

$$l\ddot\beta\cos\beta - l\dot\beta^2\sin\beta = -r\dot\theta^2\sin\theta, \quad \ddot\theta = \dot\omega_0 = 0$$

$$\alpha_{AB} = \ddot\beta = \frac{l\dot\beta^2\sin\beta - r\dot\theta^2\sin\theta}{l\cos\beta} = \frac{r\omega_0^2}{l}\sin\theta\frac{\frac{r^2}{l^2}-1}{\left(1-\frac{r^2}{l^2}\sin^2\theta\right)^{3/2}}$$

5/57 $\underline{v}_A = \underline{v}_O + \underline{v}_{A/O}$ where $v_{A/O} = \overline{AO}\,\omega = \frac{10}{12}\,\omega \ \frac{ft}{sec}$

$v_O = 4$ ft/sec

(a) $v_A = 4 \quad v_O = 4$

$v_{A/O} = 8$ ft/sec $\qquad \omega = \frac{8}{10/12} = 9.6 \frac{rad}{sec}$, $N = 9.6 \frac{60}{2\pi} = 91.7 \frac{rev}{min}$ CCW

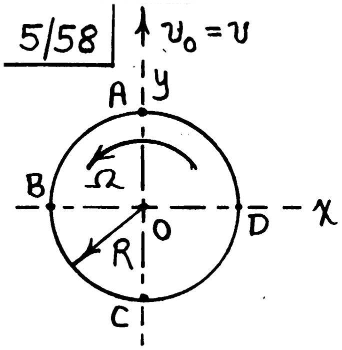
(b) $v_O = 4$

$v_{A/O} = 4$ ft/sec $\qquad v_A = 0$, $\omega = \frac{4}{10/12} = 4.8 \frac{rad}{sec}$, $N = 45.8 \frac{rev}{min}$ CCW

(c) $v_O = 4 \quad v_{A/O} = 4$

$v_A = 8$ ft/sec $\qquad \omega = \frac{4}{10/12} = 4.8 \frac{rad}{sec}$, $N = 45.8 \frac{rev}{min}$ CW

5/58



$\underline{v}_0 = 107\ 257\underline{j}$ km/h

$R\Omega = 6371(10^3)[7.292(10^{-5})]$

$= 465 \frac{m}{s}\left(3.6 \frac{km/h}{m/s}\right)$

$= 1672$ km/h

$\underline{v}_A = \underline{v}_0 + \underline{v}_{A/0} = \underline{-1672\underline{i} + 107\ 257\underline{j}\ \text{km/h}}$

$\underline{v}_B = \underline{v}_0 + \underline{v}_{B/0} = 107\ 257\underline{j} - 1672\underline{j} = \underline{105\ 585\underline{j}\ \text{km/h}}$

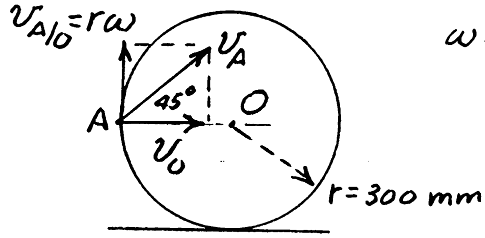
$\underline{v}_C = \underline{v}_0 + \underline{v}_{C/0} = \underline{1672\underline{i} + 107\ 257\underline{j}\ \text{km/h}}$

$\underline{v}_D = \underline{v}_0 + \underline{v}_{D/0} = (107\ 257 + 1672)\underline{j} = \underline{108\ 929\underline{j}\ \text{km/h}}$

$\underline{5/59}$ $v_{B/A} = \overline{AB}\,\omega$, $v_{C/D} = \overline{CD}\,\omega$

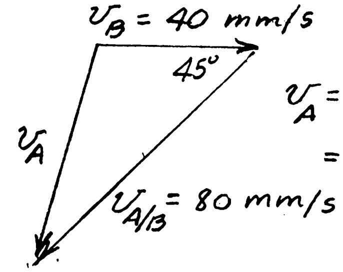
Therefore $v_{C/D} = \dfrac{\overline{CD}}{\overline{AB}} v_{B/A} = \dfrac{50}{80} 0.926 = \underline{0.579 \text{ m/s}}$

5/60 $|\underline{v}_O| = |v_{A/O}| = r\omega = 12 \cos 45^\circ = \underline{8.49 \text{ m/s}}$



$$\omega = \frac{8.49}{0.300} = \underline{28.3 \text{ rad/s}}$$

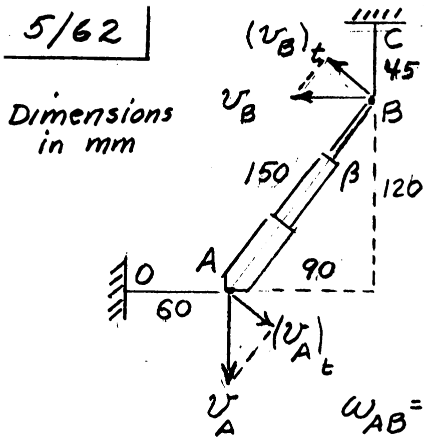
5/61 $\underset{\sim}{v}_A = \underset{\sim}{v}_B + \underset{\sim}{v}_{A/B}$, $v_{A/B} = \overline{AB}\,\omega_{AB} = 20(4) = 80 \dfrac{mm}{s}$



$$v_A = \sqrt{(80 \sin 45°)^2 + (80 \cos 45° - 40)^2}$$

$$= \sqrt{3470} = \underline{58.9 \text{ mm/s}}$$

5/62



$\sin\beta = \frac{90}{150} = \frac{3}{5}$, $\cos\beta = \frac{120}{150} = \frac{4}{5}$

$v_A = r\omega = 60(2) = 120$ mm/s

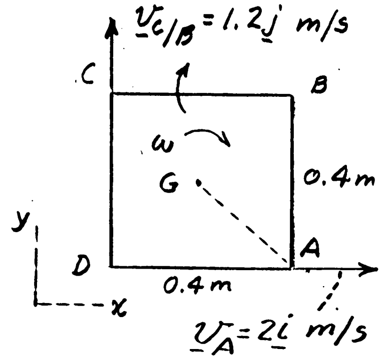
$(v_A)_t = v_A \sin\beta = 120\left(\frac{3}{5}\right) = 72\ \frac{mm}{s}$

$v_B = r\omega = 45(2) = 90$ mm/s

$(v_B)_t = v_B \cos\beta = 90\left(\frac{4}{5}\right) = 72$ mm/s

$$\omega_{AB} = \frac{(v_{A/B})_t}{\overline{AB}} = \frac{72 + 72}{150} = \underline{0.96\ \frac{rad}{s}\ \text{CCW}}$$

5/63



$v_{C/B} = \overline{CB}\,\omega$, $\omega = \frac{1.2}{0.4} = 3$ rad/s CW

$\underline{\omega} = -3\underline{k}$ rad/s

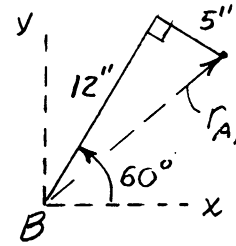
$\underline{v}_G = \underline{v}_A + \underline{\omega} \times \underline{r}_{AG}$

$= 2\underline{i} - 3\underline{k} \times (-0.2\underline{i} + 0.2\underline{j})$

$= 2\underline{i} + 0.6\underline{j} + 0.6\underline{i}$

$\underline{v}_G = 2.6\underline{i} + 0.6\underline{j}$ m/s

5/64



$$\underline{r}_{A/B} = (12\cos 60° + 5\sin 60°)\,\underline{i} + (12\sin 60° - 5\cos 60°)\,\underline{j}$$

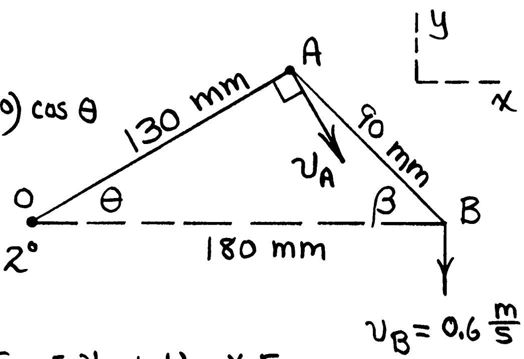
$$= 10.33\underline{i} + 7.89\underline{j} \text{ in.}$$

$$\omega_{AB} = -3\underline{k}\ \frac{\text{rad}}{\text{s}}$$

$$\underline{v}_{A/B} = \underline{\omega} \times \underline{r}_{A/B} = -3\underline{k} \times (10.33\underline{i} + 7.89\underline{j})$$

$$= \underline{23.7\underline{i} - 31.0\underline{j} \text{ in./sec}}$$

5/65



$90^2 = 180^2 + 130^2 - 2(180)(130)\cos\theta$

$\theta = 28.3°$

$\frac{130}{\sin\beta} = \frac{90}{\sin 28.3°}$, $\beta = 43.2°$

$\underline{v}_A = \underline{v}_B + \underline{v}_{A/B}$: $\underline{\omega}_{OA} \times \underline{r}_{OA} = \underline{v}_B + \underline{\omega}_{AB} \times \underline{r}_{BA}$

$\omega_{OA}\underline{k} \times 0.130(\cos 28.3°\underline{i} + \sin 28.3°\underline{j}) = -0.6\underline{j} +$

$\omega_{AB}\underline{k} \times 0.090(-\cos 43.2°\underline{i} + \sin 43.2°\underline{j})$

$\underline{i}$: $-0.0617\,\omega_{OA} = -0.0617\,\omega_{AB}$

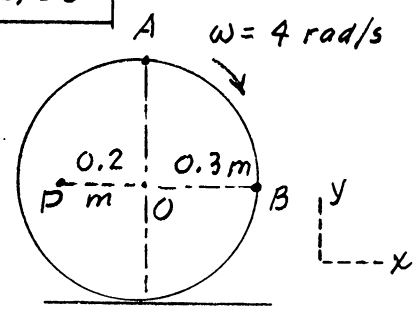
$\underline{j}$: $0.1144\,\omega_{OA} = -0.6 - 0.0656\,\omega_{AB}$

Solve simultaneously to obtain

$\omega_{AB} = \omega_{OA} = -3.33$ rad/s

So $\underline{\omega}_{OA} = -3.33\underline{k}$ rad/s

5/66



$\underline{v}_{A/B} = \underline{\omega} \times \underline{r}_{A/B}$

$\underline{\omega} = -4\underline{k}$ rad/s

$\underline{r}_{A/B} = -0.3\underline{i} + 0.3\underline{j}$ m

$\underline{v}_{A/B} = -4\underline{k} \times 0.3(-\underline{i} + \underline{j})$

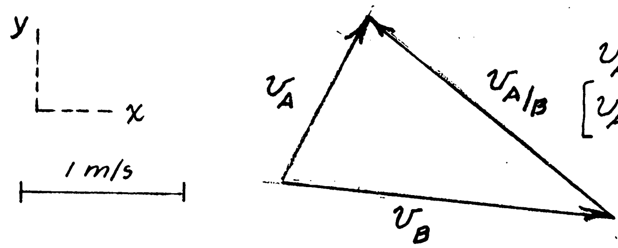
$= \underline{1.2(\underline{i} + \underline{j})\ \text{m/s}}$

$\underline{v}_P = \underline{v}_O + \underline{v}_{P/O} = r\omega\underline{i} + \overline{PO}\,\omega\underline{j} = \underline{4(0.3\underline{i} + 0.2\underline{j})}$ m/s

5/67 $\underline{v}_A = \underline{v}_B + \underline{v}_{A/B}$ where $\underline{v}_B = 2\underline{i} - 0.3\underline{j}$ m/s

$\underline{v}_{A/B} = \underline{\omega} \times \underline{r}_{A/B} = 3\underline{k} \times (0.4\underline{i} + 0.5\underline{j}) = 1.2\underline{j} - 1.5\underline{i}$ m/s

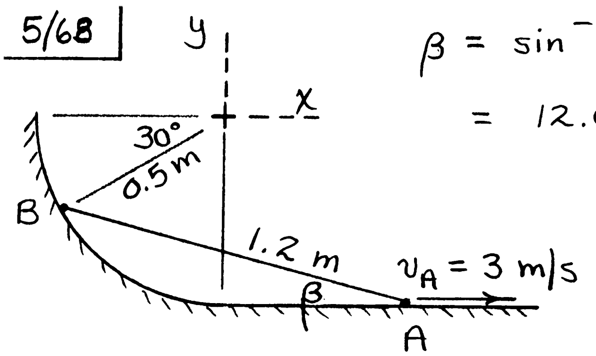
So $\underline{v}_A = 2\underline{i} - 0.3\underline{j} + 1.2\underline{j} - 1.5\underline{i} = \underline{0.5\underline{i} + 0.9\underline{j} \text{ m/s}}$



$v_A \approx 1$ m/s

$\left[v_{A_{\text{Exact}}} = \sqrt{0.5^2 + 0.9^2} = 1.030 \text{ m/s} \right]$

5/68



$$\beta = \sin^{-1}\frac{0.5 - 0.5\sin 30°}{1.2} = 12.02°$$

$$\underline{v}_B = \underline{v}_A + \underline{v}_{B/A} = \underline{v}_A + \underline{\omega} \times \underline{r}_{B/A}$$

$$v_B(\sin 30°\,\underline{i} - \cos 30°\,\underline{j}) = 3\underline{i} + \omega\underline{k} \times 1.2(-\cos\beta\,\underline{i} + \sin\beta\,\underline{j})$$

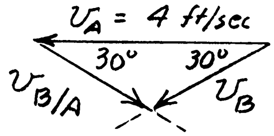
$$= 3\underline{i} + \omega\underline{k} \times 1.2(-\cos 12.02°\,\underline{i} + \sin 12.02°\,\underline{j})$$

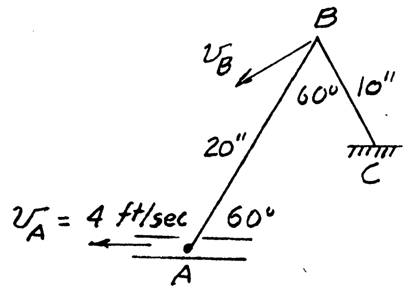
$$= 3\underline{i} - 1.174\omega\underline{j} - 0.250\omega\underline{i}$$

$$\left.\begin{array}{l} \underline{i}:\ \frac{1}{2}v_B = 3 - 0.250\omega \\ \underline{j}:\ -\frac{\sqrt{3}}{2}v_B = -1.174\omega \end{array}\right\} \quad \begin{array}{l} v_B = 4.38 \text{ m/s} \\ \omega = 3.23 \text{ rad/s} \end{array}$$

5/69

$\underline{v}_B = \underline{v}_A + \underline{v}_{B/A}$

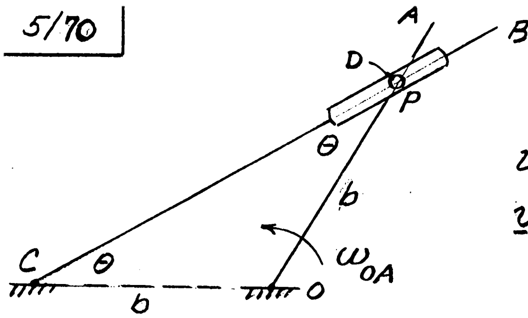




$v_B = \dfrac{2}{\cos 30^\circ} = 2.31$ ft/sec

$\omega_{BC} = v_B/\overline{BC} = \dfrac{2.31}{10/12} = \underline{2.77 \text{ rad/sec CCW}}$

5/70

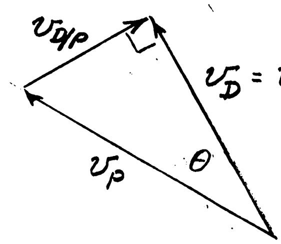


Let P = pin

D = point on CB coincident with P

$v_P = r\omega = b\omega_{OA}$

$\underline{v}_D = \underline{v}_P + \underline{v}_{D/P}$



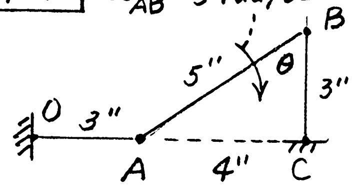
$v_D = v_P \cos\theta$

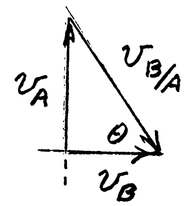
$v_D = \overline{CD}\,\omega_{CB} = 2b\cos\theta\ \omega_{CB}$

So $2b\cos\theta\ \omega_{CB} = b\omega_{OA}\cos\theta$

Thus $\underline{\omega_{CB} = \frac{1}{2}\omega_{OA}}$

5/71 $\omega_{AB} = 3$ rad/sec





$\underset{-}{v}_B = \underset{-}{v}_A + \underset{-}{v}_{B/A}$, $\omega_{BC} = \dfrac{v_B}{\overline{BC}}$

$v_{B/A} = \overline{AB}\,\omega_{AB}$

$= 5(3) = 15$ in./sec

$\theta = \cos^{-1}\dfrac{3}{5}$

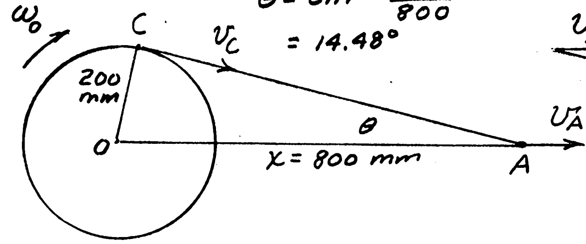
$v_B = v_{B/A}\cos\theta$

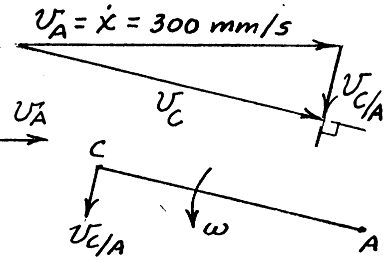
$= 15(3/5) = 9$ in./sec

$\omega_{BC} = 9/3 = \underline{3 \text{ rad/sec CW}}$

5/72 $\underline{v}_C = \underline{v}_A + \underline{v}_{C/A}$, $v_A = 300$ mm/s

$\theta = \sin^{-1}\dfrac{200}{800} = 14.48°$





$v_C = 300 \cos 14.48°$

$\quad = 300(0.9682) = 290$ mm/s

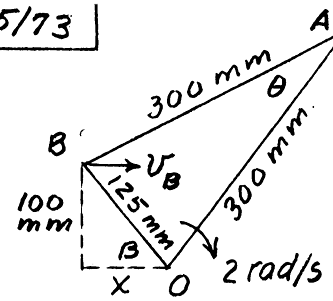
$v_{C/A} = 300 \sin 14.48° = 300/4 = 75$ mm/s

$\overline{CA} = 800 \cos 14.48° = 775$ mm

$\omega_{AB} = v_{C/A}/\overline{CA} = 75/775 = \underline{0.0968 \text{ rad/s CCW}}$

$\omega_0 = v_C/\overline{CO} = 290/200 = \underline{1.452 \text{ rad/s CW}}$

5/73

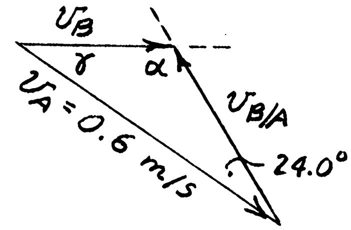


$125 = 2(300)\sin\theta/2$

$\frac{\theta}{2} = \sin^{-1}\frac{125}{600} = 12.02°$, $\theta = 24.0°$

$v_A = r\omega = 0.3(2) = 0.6$ m/s

$\underline{v}_B = \underline{v}_A + v_{B/A}$



$x = 75$ mm

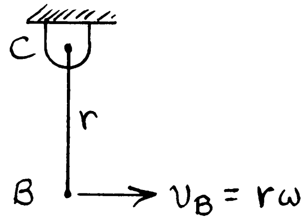
$\beta = \tan^{-1}\frac{4}{3} = 53.1°$

$\gamma = \beta - \frac{\theta}{2} = 53.1 - 12.02 = 41.1°$

$\alpha = 180 - 41.1 - 24.0 = 114.8°$

$$\frac{v_B}{\sin 24.0°} = \frac{0.6}{\sin 114.8},\quad \underline{v_B = 0.269 \text{ m/s}}$$

5/74 (a) $v_A = v_B = r\omega$ (right)



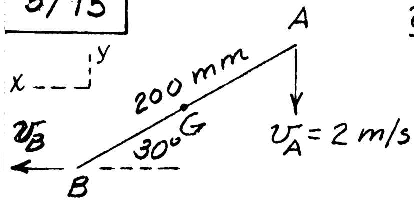
$$\omega_{BC} = \frac{v_B}{\overline{BC}} = \frac{r\omega}{r}$$

$$= \underline{\omega \quad CCW}$$

(b) $v_A = v_B = 2r\omega$ (right)

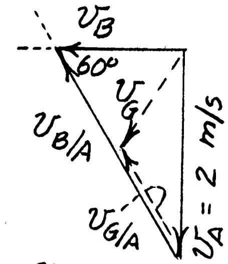
$$\omega_{BC} = \frac{v_B}{\overline{BC}} = \frac{2r\omega}{r} = \underline{2\omega \quad CCW}$$

5/75



$$\underline{v}_B = \underline{v}_A + \underline{v}_{B/A}$$

$$\omega = \frac{v_{B/A}}{\overline{BA}} = \frac{2/\cos 30°}{0.200} = \underline{11.55 \text{ rad/s CW}}$$



$$\underline{v}_G = \underline{v}_A + \underline{v}_{G/A}$$

$$v_{G/A} = \overline{GA}\,\omega = \frac{1}{2} v_{B/A}$$

From diagram $\quad v_G = 2/\sqrt{3} = \underline{1.155 \text{ m/s}}$

$\underline{v}_A = -2\underline{j}$ m/s , $\underline{v}_B = v_B\underline{i}$, $\underline{\omega}_{AB} = \omega_{AB}\underline{k}$

$$v_B\underline{i} = -2\underline{j} + \omega_{AB}\underline{k} \times (0.2\cos 30°\,\underline{i} - 0.2\sin 30°\,\underline{j})$$

$$= (-2 + 0.1732\,\omega_{AB})\underline{j} + 0.1\,\omega_{AB}\underline{i}$$

$$\omega_{AB} = \frac{2}{0.1732} = \underline{11.55 \text{ rad/s}} \text{ CW},$$

$$\underline{v}_G = -2\underline{j} + 11.55\underline{k} \times (0.1\cos 30°\,\underline{i} - 0.1\sin 30°\,\underline{j})$$

$$= (-2 + 1.00)\underline{i} + 0.577\underline{j} = -\underline{i} + 0.577\underline{j} \text{ m/s}$$

$$v_G = \sqrt{1^2 + 0.577^2} = \underline{1.155 \text{ m/s}}$$

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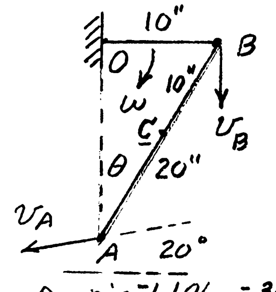
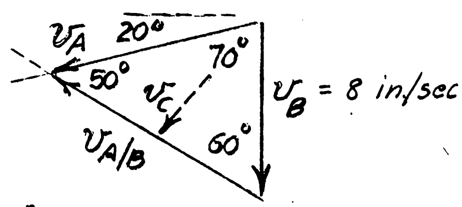
$\underline{v}_A = \underline{v}_B + \underline{v}_{A/B}$

$v_B = r\omega = 10(0.8) = 8$ in./sec





$\theta = \sin^{-1} 10/20 = 30°$

$$\frac{8}{\sin 50°} = \frac{v_A}{\sin 60°}$$

$v_A = \frac{\sin 60°}{\sin 50°} 8 = \underline{9.04 \text{ in./sec}}$

$\underline{v}_C = \underline{v}_B + \underline{v}_{C/B}$, $v_{C/B} = \frac{10}{20} v_{A/B}$ where $\frac{v_{A/B}}{\sin 70°} = \frac{8}{\sin 50°}$

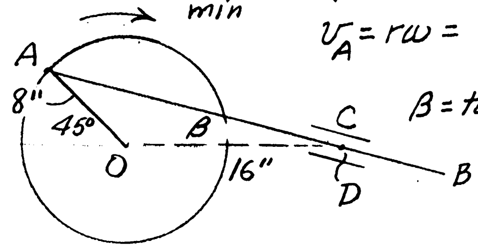
$v_{A/B} = 9.81$ in./sec

$v_C^2 = 8^2 + 4.91^2 - 2(8)(4.91)\cos 60°$
$= 48.8 \text{ (in./sec)}^2$

$\underline{v_C = 6.99 \text{ in./sec}}$

5/77

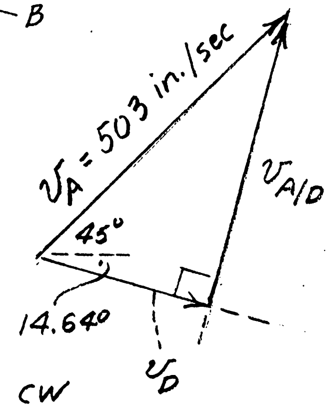
600 rev/min



$\underline{v}_A = \underline{v}_D + \underline{v}_{A/D}$

$v_A = r\omega = 8\,\frac{600(2\pi)}{60} = 503$ in./sec

$\beta = \tan^{-1}\frac{8\sin 45°}{16 + 8\cos 45°} = 14.64°$



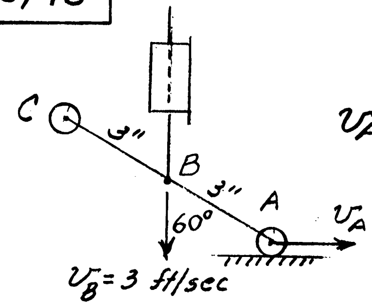
$v_{A/D} = 503\sin(45° + 14.64°)$
$= 434$ in./sec

$\omega_{AB} = \omega_{AD} = \frac{v_{A/D}}{\overline{AD}}$

$\overline{AD} = \frac{8\cos 45°}{\sin 14.64°} = 22.4$ in.

$\omega_{AB} = \frac{434}{22.4} = 19.38$ rad/sec CW

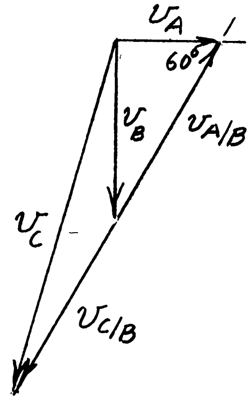
5/78



$\underline{v}_A = \underline{v}_B + \underline{v}_{A/B}$

$\underline{v}_C = \underline{v}_B + \underline{v}_{C/B}$

$v_{A/B} = \overline{AB}\omega$
$= \overline{CB}\omega$
$= v_{C/B}$



From geometry

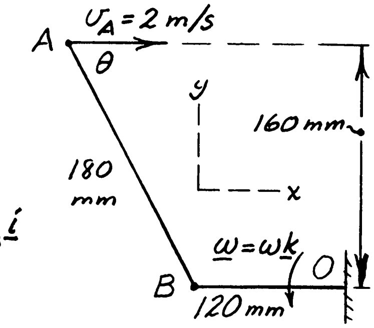
$v_{A/B} = 3/\sin 60° = 3.46$ ft/sec , $v_A = 3/\tan 60° = 1.732 \frac{ft}{sec}$

$v_{C/B} = 3.46$ ft/sec

$v_C = \sqrt{(3+3)^2 + (1.732)^2} = \sqrt{39} = \underline{6.24 \text{ ft/sec}}$

5/79

$$\underline{v}_B = \underline{v}_A + \underline{v}_{B/A}$$



$$\omega \underline{k} \times (-0.12\,\underline{i}) = 2\underline{i} + \omega_{AB}\underline{k} \times 0.18(0.458\,\underline{i} - 0.889\,\underline{j})$$

$$-0.12\,\omega \underline{j} = 2\underline{i} + 0.0825\,\omega_{AB}\underline{j} + 0.1600\,\omega_{AB}\underline{i}$$

$\underline{i}$-terms: $0 = 2 + 0.1600\,\omega_{AB}$

$$\omega_{AB} = -12.50 \text{ rad/s (CW)}$$

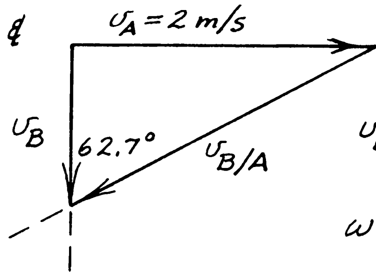
$\underline{j}$-terms: $-0.12\,\omega = 0.0825(-12.50)$

$$\underline{\omega = 8.59 \text{ rad/s CCW}}$$

$\sin\theta = \dfrac{160}{180} = 0.889$, $\theta = 62.7°$

$\cos\theta = 0.458$

(Alternatively & more simply)



$$v_B = \frac{2}{\tan 62.7°} = 1.031 \frac{\text{m}}{\text{s}}$$

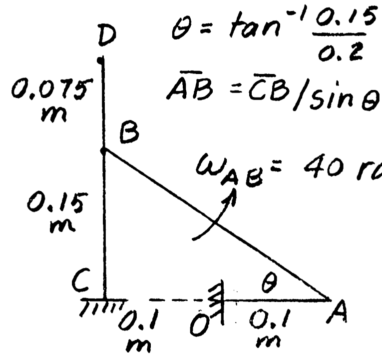
$$\omega = \frac{1.031}{0.12} = \underline{8.59 \frac{\text{rad}}{\text{s}} \text{ CCW}}$$

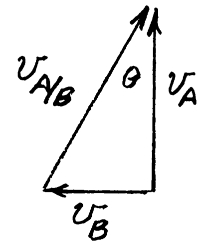
5/80

$\underline{v}_A = \underline{v}_B + \underline{v}_{A/B}$

$\theta = \tan^{-1}\frac{0.15}{0.2} = \tan^{-1}\frac{3}{4}$

$\overline{AB} = \overline{CB}/\sin\theta = \frac{0.15}{3/5} = 0.25\text{ m}$





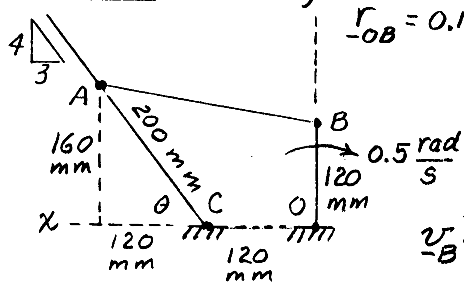
$v_{A/B} = \overline{AB}\,\omega_{AB}$

$= 0.25(40) = 10\text{ m/s}$

$v_B = v_{A/B}\sin\theta = 10\left(\frac{3}{5}\right) = 6\text{ m/s}$

$v_D = \frac{\overline{DC}}{\overline{BC}}v_B = \frac{0.225}{0.15}6 = \underline{9\text{ m/s}}$

5/81



$\underline{r}_{CA} = 0.12\underline{i} + 0.16\underline{j}$ m

$\underline{r}_{OB} = 0.12\underline{j}$ m, $\underline{r}_{BA} = 0.24\underline{i} + 0.04\underline{j}$ m

$$\underline{v}_A = \underline{\omega}_{AC} \times \underline{r}_{CA}$$
$$= \omega_{AC}\underline{k} \times (0.12\underline{i} + 0.16\underline{j})$$
$$= 0.12\omega_{AC}\underline{j} - 0.16\omega_{AC}\underline{i}$$

$$\underline{v}_B = \underline{\omega}_{OB} \times \underline{r}_{OB} = 0.5\underline{k} \times 0.12\underline{j}$$
$$= -0.06\underline{i} \text{ m/s}$$

$$\underline{v}_{A/B} = \underline{\omega}_{AB} \times \underline{r}_{BA} = \omega_{AB}\underline{k} \times (0.24\underline{i} + 0.04\underline{j})$$
$$= 0.24\omega_{AB}\underline{j} - 0.04\omega_{AB}\underline{i}$$

$\underline{v}_A = \underline{v}_B + \underline{v}_{A/B}$, so $0.12\omega_{AC}\underline{j} - 0.16\omega_{AC}\underline{i} = -0.06\underline{i}$
$+ 0.24\omega_{AB}\underline{j} - 0.04\omega_{AB}\underline{i}$

Equate coefficients & get

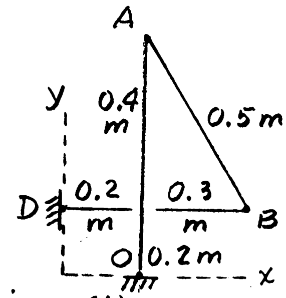
$$0.16\omega_{AC} - 0.04\omega_{AB} = 0.06$$
$$0.12\omega_{AC} - 0.24\omega_{AB} = 0$$

Solve & get

$\underline{\omega}_{AB} = 0.214\underline{k}$ rad/s, $\underline{\underline{\omega}_{CA} = 0.429\underline{k} \text{ rad/s}}$

5/82

$\underline{\omega}_{AB} = \underline{\omega}_P = 2\underline{k}$ rad/s



$\underline{v}_A = \underline{v}_B + \underline{\omega}_{AB} \times \underline{r}_{A/B}$

where $\underline{v}_A = \underline{\omega}_{AO} \times \underline{r}_{A/O}$

$\underline{v}_B = \underline{\omega}_{BD} \times \underline{r}_{B/D}$

so

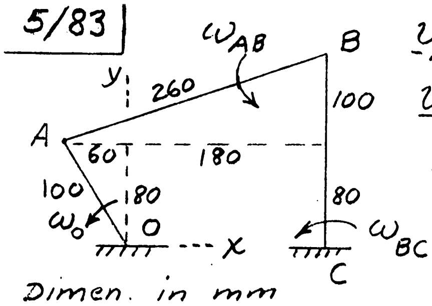
$\omega_{AO}\underline{k} \times 0.6\underline{j} = \omega_{BD}\underline{k} \times 0.5\underline{i} + 2\underline{k} \times (-0.3\underline{i} + 0.4\underline{j})$

$-0.6\,\omega_{AO}\underline{i} = 0.5\omega_{BD}\underline{j} - 0.6\underline{j} - 0.8\underline{i}$

Thus $-0.6\omega_{AO} = -0.8$, $\underline{\omega}_{AO} = \underline{1.333\underline{k}}$ rad/s

$0 = 0.5\omega_{BD} - 0.6$, $\underline{\omega}_{BD} = \underline{1.20\underline{k} \text{ rad/s}}$

5/83



Dimen. in mm

$\omega_{AO} = \omega_o = 10$ rad/s

$\underline{v}_A = \underline{v}_B + \underline{v}_{A/B}$

$\underline{v}_A = \underline{\omega}_{AO} \times \underline{r}_{AO}$

$= 10\underline{k} \times (-0.06\underline{i} + 0.08\underline{j})$

$= -0.6\underline{j} - 0.8\underline{i}$ m/s

$\underline{v}_B = \underline{\omega}_{BC} \times \underline{r}_{BC}$

$= \omega_{BC}\underline{k} \times 0.18\underline{j} = -0.18\omega_{BC}\underline{i}$ m/s

$\underline{v}_{A/B} = \underline{\omega}_{AB} \times \underline{r}_{A/B}$

$= \omega_{AB}\underline{k} \times (-0.24\underline{i} - 0.1\underline{j})$

$= -0.24\omega_{AB}\underline{j} + 0.1\omega_{AB}\underline{i}$ m/s

Thus,

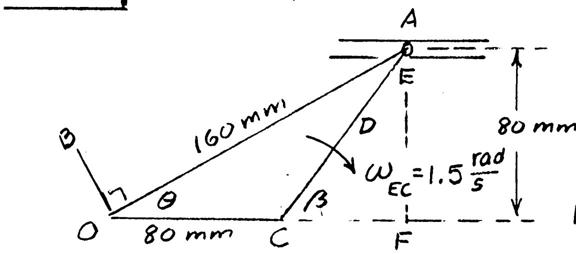
$-0.6\underline{j} - 0.8\underline{i} = -0.18\omega_{BC}\underline{i} - 0.24\omega_{AB}\underline{j} + 0.1\omega_{AB}\underline{i}$

Equate $\underline{j}$ terms & set $\omega_{AB} = \dfrac{0.6}{0.24} = 2.5$ rad/s

$\underline{\underline{\omega_{AB} = 2.5\underline{k} \text{ rad/s}}}$

$\underline{\underline{\omega_{BC} = 5.83\underline{k} \text{ rad/s}}}$

5/84 Let E be point on member D coincident with A



$\theta = \sin \frac{80}{160} = 30^\circ$

$\overline{CF} = 160 \cos 30^\circ - 80$
$= 58.6 \text{ mm}$

$\beta = \tan^{-1} \frac{80}{58.6} = 53.8^\circ$

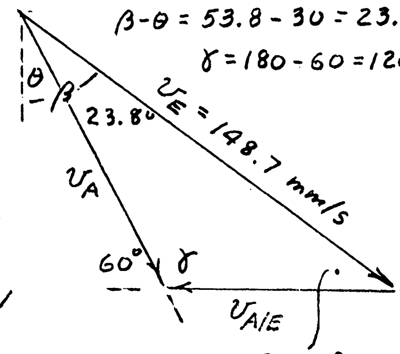
$\overline{EC} = \frac{80}{\sin 53.8^\circ} = 99.1 \text{ mm}$

$\underline{v}_A = \underline{v}_E + \underline{v}_{A/E}$

$v_E = 99.1(1.5) = 148.7 \frac{mm}{s}$

$\beta - \theta = 53.8 - 30 = 23.8^\circ$

$\gamma = 180 - 60 = 120^\circ$



$90 - \beta = 36.2^\circ$

$$\frac{v_A}{\sin 36.2^\circ} = \frac{148.7}{\sin 120^\circ}$$

$$v_A = 148.7 \frac{0.591}{0.866} = 101.4 \frac{mm}{s}$$

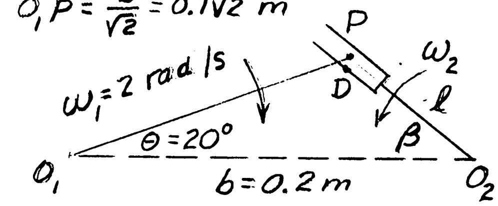
$$\omega_{AOB} = \frac{101.4}{160} = \underline{0.634 \text{ rad/s CW}}$$

Alternatively, draw vector triangle to scale & measure $v_A \approx 101$ mm/s. Etc.

5/85

$$l^2 = b^2 + \frac{b^2}{2} - 2b\left(\frac{b}{\sqrt{2}}\right)\cos 20^\circ = b^2\left(1 + \frac{1}{2} - 1.329\right)$$
$$= 0.00684,\quad l = 0.0827\ m$$

$\overline{O_1P} = \frac{b}{\sqrt{2}} = 0.1\sqrt{2}\ m$



$$\frac{0.1\sqrt{2}}{\sin\beta} = \frac{0.0827}{\sin 20^\circ},\quad \beta = 35.8^\circ$$

Let point D be point on wheel C coincident with point P

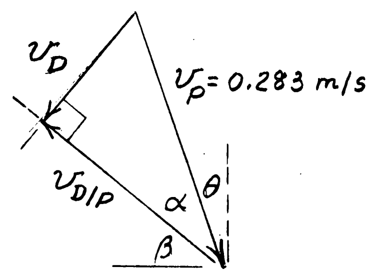
$\underline{v}_D = \underline{v}_P + \underline{v}_{D/P}$

$\underline{v}_P = 0.1\sqrt{2}(2) = 0.283$ m/s

$v_D = 0.283 \sin 34.2^\circ$
$= 0.1591$ m/s

$\omega_2 = v_D/l = 0.1591/0.0827$

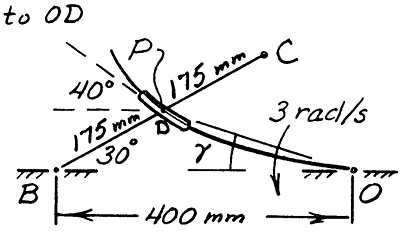
$\underline{\omega_2 = 1.923 \text{ rad/s}}$



$\alpha = 90 - \theta - \beta$
$= 90 - (20 + 35.8) = 34.2^\circ$

5/86 Let D be a point on OA coincident with P.

$\underline{v}_P = \underline{v}_D + \underline{v}_{P/D}$ where $\underline{v}_D$ is $\perp$ to OD



From triangle BOD,

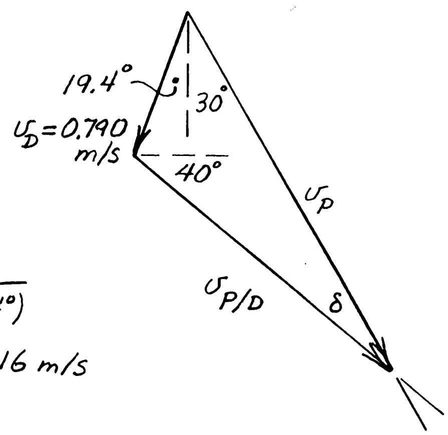
$175 \sin 30° = (400 - 175 \cos 30°) \tan \gamma$

$\gamma = 19.40°$

$\overline{DO} \cos 19.40° = 400 - 175 \cos 30°$

$\overline{DO} = 263$ mm

$v_D = \overline{DO}\, \omega_{DO} = 263(3) = 790$ mm/s $= 0.790$ m/s



$\delta + 30° + 90° + 40° = 180°$,

$\delta = 20°$

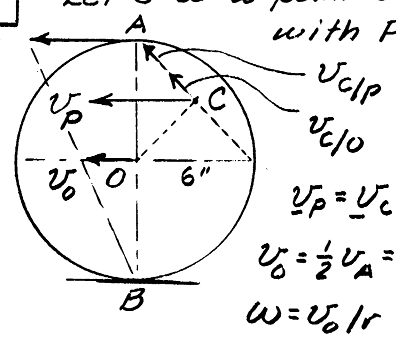
Law of sines: $\dfrac{v_P}{\sin(40° + 90° - 19.4°)} = \dfrac{0.790}{\sin 20°}$, $v_P = 2.16$ m/s

$v_C = 2 v_P = \underline{4.33 \text{ m/s}}$

5/87

$v_A = 0.4$ ft/sec

Let C be a point on wheel coincident with P

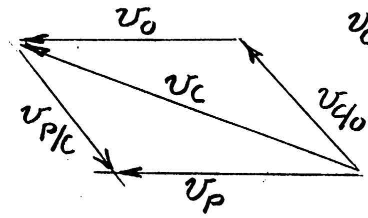


$\underline{v}_P = \underline{v}_C + \underline{v}_{P/C}$, $\underline{v}_C = \underline{v}_O + \underline{v}_{C/O}$

$v_O = \frac{1}{2} v_A = 0.2$ ft/sec

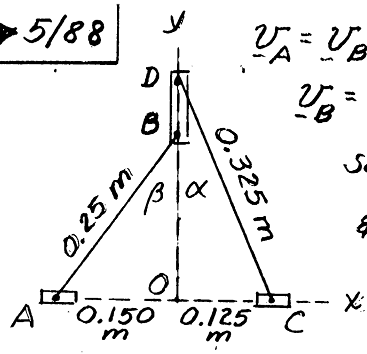
$\omega = v_O / r = 0.2/0.5 = 0.4$ rad/sec

$v_{C/O} = \overline{CO}\omega = \frac{1}{2\sqrt{2}}\, 0.4 = 0.1414$ ft/sec



From figure $v_P = v_O = \underline{0.2 \frac{ft}{sec}}$

▶5/88



$\underline{v}_A = \underline{v}_B + \underline{v}_{A/B}$, $\underline{v}_C = \underline{v}_D + \underline{v}_{C/D}$

$\underline{v}_B = \underline{v}_D$, $\underline{v}_C - \underline{v}_A = -0.2\underline{i}$ m/s

so $\underline{v}_A = (\underline{v}_C - \underline{v}_{C/D}) + \underline{v}_{A/B}$

& $\underline{v}_{C/D} - \underline{v}_{A/B} = \underline{v}_C - \underline{v}_A = -0.2\underline{i}$ m/s

$\overline{OD} = \sqrt{(0.325)^2 - (0.125)^2}$
$= 0.3 m$

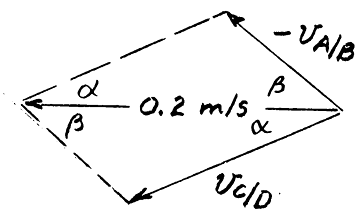
$\overline{OB} = \sqrt{(0.25)^2 - (0.15)^2}$
$= 0.2 m$

$\sin\alpha = 0.125/0.325 = 5/13$

$\cos\alpha = 0.3/0.325 = 12/13$

$\sin\beta = 0.150/0.250 = 3/5$

$\cos\beta = 0.2/0.25 = 4/5$



$v_{C/D}\cos\alpha + v_{A/B}\cos\beta = 0.2$

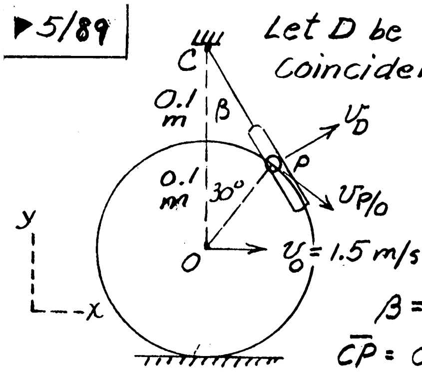
$v_{C/D}\sin\alpha - v_{A/B}\sin\beta = 0$

solve & get $v_{C/D} = \frac{39}{280} = 0.1393$ m/s

$\underline{v} = \underline{v}_D = \underline{v}_C - \underline{v}_{C/D}$; $v\underline{j} = v_C\underline{i} - \left[\frac{39}{280}\right](-\underline{i}\cos\alpha - \underline{j}\sin\alpha)$

so $v = \frac{39}{280}\frac{5}{13} = 3/56 = \underline{0.0536 \text{ m/s}}$

▶5/89



Let D be point on slotted arm coincident with P

$$\underline{v}_D = \underline{v}_P + \underline{v}_{D/P} \quad \text{- - - - (a)}$$

$$\underline{v}_P = \underline{v}_O + \underline{v}_{P/O}$$

$$v_{P/O} = \overline{PO}\,\omega_{PO} = \overline{PO}\,\frac{v_O}{\overline{PO}} = v_O$$

$$\beta = \tan^{-1}\frac{0.1\sin 30^\circ}{0.2 - 0.1\cos 30^\circ} = 23.8^\circ$$

$$\overline{CP} = 0.1\,\frac{\sin 30^\circ}{\sin 23.8^\circ} = 0.1239 \text{ m}$$

$$\underline{v}_D = v_D(\underline{i}\cos\beta + \underline{j}\sin\beta) = v_D(0.915\underline{i} + 0.403\underline{j})$$

$$\underline{v}_P = 1.5\underline{i} + (1.5\cos 30^\circ)\underline{i} - (1.5\sin 30^\circ)\underline{j} = 2.799\underline{i} - 0.75\underline{j} \text{ m/s}$$

$$\underline{v}_{D/P} = v_{D/P}(-\underline{i}\sin\beta + \underline{j}\cos\beta) = v_{D/P}(-0.403\underline{i} + 0.915\underline{j})$$

Substitute in Eq. (a) & separate $\underline{i}$ & $\underline{j}$ terms to get

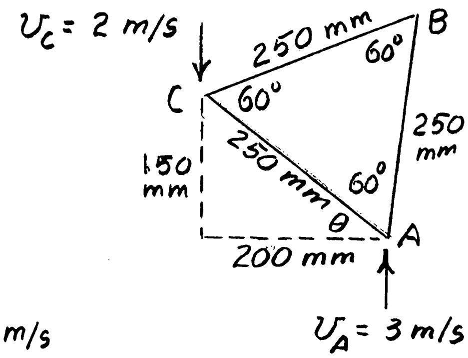
$$\left.\begin{aligned} 0.915v_D - 2.799 + 0.403v_{D/P} &= 0 \\ 0.403v_D + 0.75 - 0.915v_{D/P} &= 0 \end{aligned}\right\} \text{solve \& get}$$

$v_D = 2.26$ m/s, $v_{D/P} = 1.816$ $\frac{m}{s}$

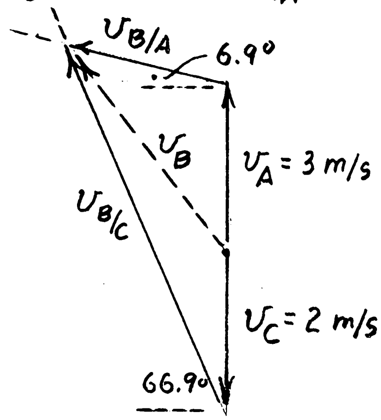
Thus $\omega = \omega_{CD} = \dfrac{2.26}{0.1239} = 18.22$ rad/s CCW

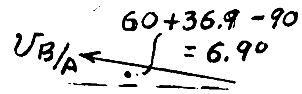
▶ 5/90

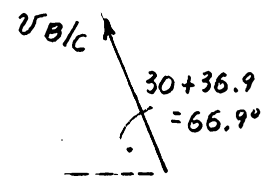
$\theta = \tan^{-1}\frac{150}{200} = 36.9^\circ$



$\underline{v}_C + \underline{v}_{B/C} = \underline{v}_B = \underline{v}_A + \underline{v}_{B/A}$







$v_{B/A} \cos 6.9^\circ = v_{B/C} \cos 66.9^\circ$

$-v_{B/A} \sin 6.9^\circ + v_{B/C} \sin 66.9^\circ = 5$

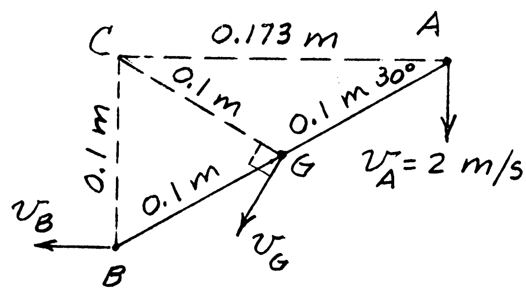
Solve simultaneously & get

$v_{B/C} = 5.73$ m/s, $v_{B/A} = 2.27$ m/s

From which $v_B^2 = 2.27^2 + 3^2 - 2(2.27)(3)\cos 96.9^\circ = 15.77$

$\underline{v_B = 3.97 \text{ m/s}}$

5/91



$$\omega = v_A / \overline{AC}$$

$$= 2/0.1732 = \underline{11.55 \text{ rad/s}}$$
$$\underline{\text{CW}}$$

$$v_G = \overline{CG}\,\omega$$
$$= 0.1(11.55)$$
$$= \underline{1.155 \text{ m/s}}$$

5/92

$\overline{CB} = \sqrt{180^2 - 160^2}$

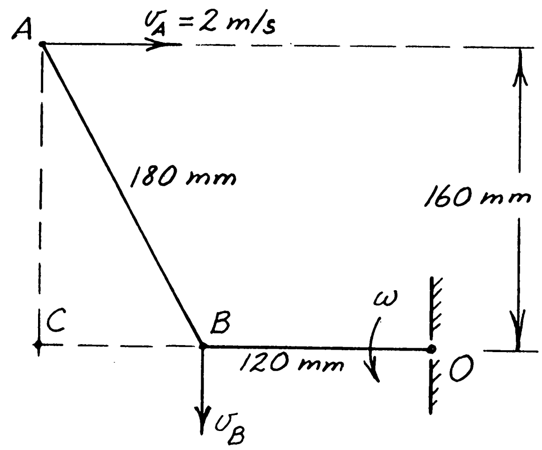
$= 82.5 \text{ mm}$

$\dfrac{v_B}{\overline{CB}} = \dfrac{v_A}{\overline{AC}}, \quad v_B = \dfrac{82.5}{160} 2$

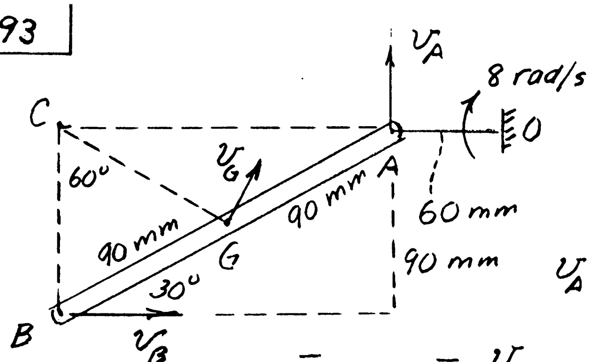
$= 1.031 \text{ m/s}$

$\omega_{OB} = \omega = \dfrac{1.031}{0.120}$

$= \underline{8.59 \text{ rad/s}}$



5/93



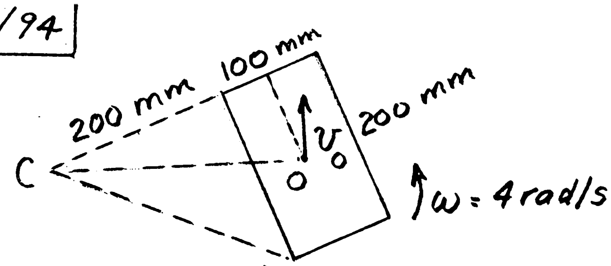
$v_A = \overline{OA}\,\omega_{OA}$

$= 60(8) = 480$ mm/s

$\overline{CG} = 90$ mm, $\quad v_G = \overline{CG}\,\omega_{CG} = \overline{CG}\,\dfrac{v_A}{\overline{AC}} = \dfrac{90}{180\cos 30^\circ}\,480$

$v_G = 277$ mm/s

5/94



$v_O = \overline{OC}\,\omega$, $\overline{OC} = \sqrt{100^2 + 250^2} = 269$ mm

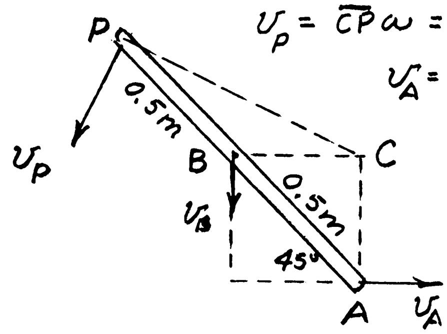
$v_O = 269(4) = 1077$ mm/s or $\underline{1.077 \text{ m/s}}$

5/95 $\overline{CP} = \sqrt{(1 \times \cos 45°)^2 + (0.5 \sin 45°)^2} = 0.791 \text{ m}$

$v_P = \overline{CP}\,\omega = 0.791\,(2) = \underline{1.581 \text{ m/s}}$

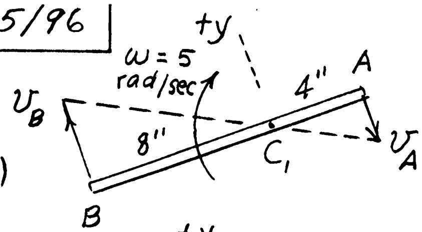
$v_A = \overline{CA}\,\omega = 0.5 \sin 45°\,(2)$

$\underline{v_A = 0.707 \text{ m/s}}$



5/96

(a)



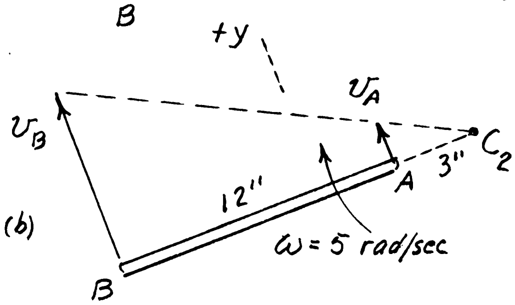
$v_A = \overline{C_1A}\,\omega = 4(5) = 20$ in./sec

$\underline{v}_A = -20\underline{j}$ in./sec

$v_B = \overline{C_1B}\,\omega = 8(5) = 40$ in./sec

$\underline{v}_B = 40\underline{j}$ in./sec

(b)



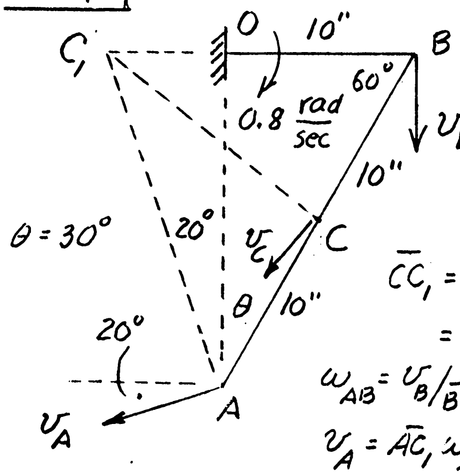
$v_A = \overline{C_2A}\,\omega = 3(5) = 15$ in./sec

$\underline{v}_A = 15\underline{j}$ in./sec

$v_B = \overline{C_2B}\,\omega = 15(5) = 75$ in./sec

$\underline{v}_B = 75\underline{j}$ in./sec

5/97



$v_B = r\omega = 10(0.8) = 8$ in./sec

$\overline{AO} = 20 \cos 30° = 17.32$ in.

$\overline{AC_1} = \overline{AO} / \cos 20°$

$= 17.32 / \cos 20° = 18.43$ in.

$\overline{C_1O} = 18.43 \sin 20° = 6.30$ in.

$\overline{C_1B} = 6.30 + 10 = 16.30$ in.

$\overline{CC_1} = \sqrt{10^2 + 16.30^2 - 2(10)16.30 \cos 60°}$

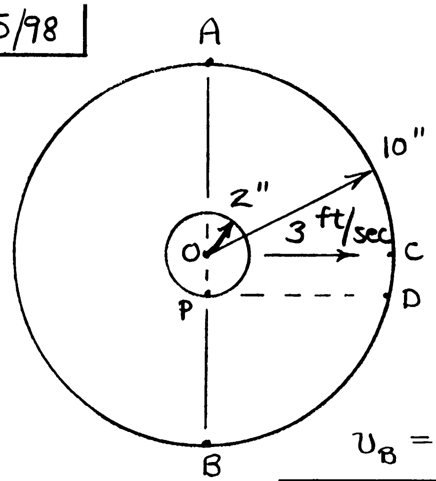
$= 14.24$ in.

$\omega_{AB} = v_B / \overline{BC_1} = \frac{8}{16.30} = 0.491$ rad/s

$v_A = \overline{AC_1}\, \omega_{AB} = 18.43(0.491) = \underline{9.04 \text{ in./sec}}$

$v_C = \overline{CC_1}\, \omega_{AB} = 14.24(0.491) = \underline{6.99 \text{ in./sec}}$

5/98



Point P is the instantaneous center

$v_O = \overline{OP}\,\omega, \quad \omega = \frac{3}{2/12}$

$\omega = 18$ rad/sec CW

$v_A = \overline{AP}\,\omega = \frac{12}{12}(18)$

$= \underline{18 \text{ ft/sec} \rightarrow}$

$v_B = \overline{BP}\,\omega = \frac{8}{12}(18) = \underline{12 \frac{\text{ft}}{\text{sec}} \leftarrow}$

$v_C = \overline{CP}\,\omega = \sqrt{\left(\frac{2}{12}\right)^2 + \left(\frac{10}{12}\right)^2}\,(18) = \underline{15.30 \text{ ft/sec}}$

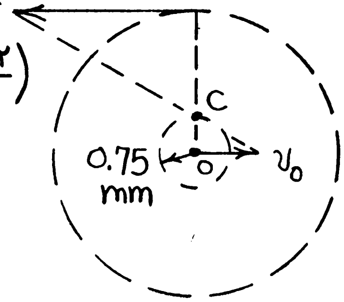
$\alpha = \tan^{-1}\frac{2}{12} = 9.46°$

$v_D = \overline{DP}\,\omega = \frac{10}{12}(18) = \underline{15 \text{ ft/sec} \downarrow}$

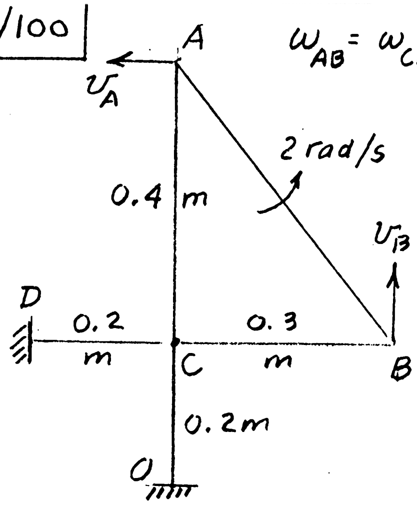
5/99 | C = instantaneous center

$v_o = \overline{OC}\,\omega = 0.75\,(10^{-3})\left(\frac{1800 \cdot 2\pi}{60}\right)$

$= \underline{0.1414 \text{ m/s}}$



5/100



$\omega_{AB} = \omega_{CB} = \omega_{CA} = 2$ rad/s

$v_B = \overline{CB}\,\omega_{CB} = 0.3(2) = 0.6$ m/s

$\omega_{BD} = v_B/\overline{BD} = \dfrac{0.6}{0.5} = 1.2$ rad/s

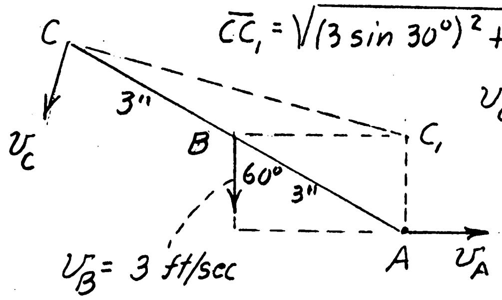
CCW

$v_A = \overline{CA}\,\omega_{CA} = 0.4(2) = 0.8$ m/s

$\omega_{AO} = v_A/\overline{AO} = \dfrac{0.8}{0.6} = 1.333\,\dfrac{rad}{s}$

CCW

5/101 $v_C = \overline{CC_1}\,\omega_{AC}$, $\omega_{AC} = \omega_{AB} = v_B/\overline{BC_1} = \dfrac{3}{\frac{3}{12}\sin 60^\circ} = 13.86$ rad/s



$\overline{CC_1} = \sqrt{(3\sin 30^\circ)^2 + (6\cos 30^\circ)^2} = 5.41$ in.

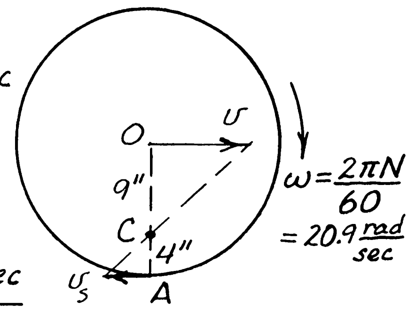
$v_C = \dfrac{5.41}{12}(13.86) = \underline{6.24 \dfrac{ft}{sec}}$

5/102

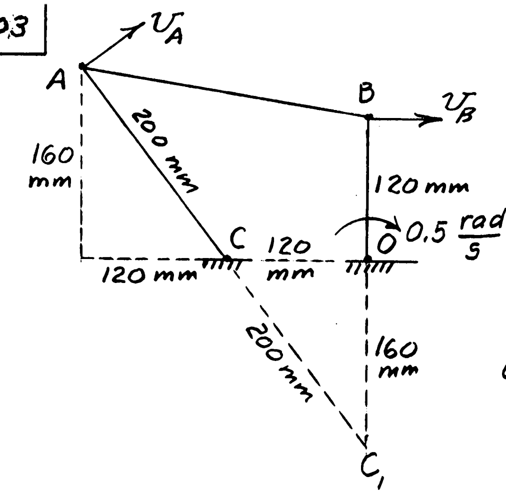
$\omega = v/\overline{OC}$, $v = \frac{9}{12}\cdot 20.9 = 15.71$ ft/sec

or $\underline{v = 10.71 \text{ mi/hr}}$

$v_s = \frac{4}{9}v = \frac{4}{9}(15.71)$, $\underline{v_s = 6.98 \text{ ft/sec}}$



5/103



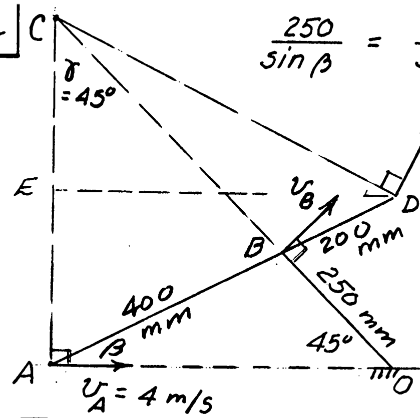
$$v_B = 0.120(0.5) = 0.060 \text{ m/s}$$

$$\omega_{CA} = v_A / \overline{CA}$$

$$v_A = \frac{\overline{AC_1}}{\overline{BC_1}} v_B = \frac{0.400}{0.280}(0.060) = 0.0857 \text{ m/s}$$

$$\omega_{CA} = \frac{0.0857}{0.200} = \underline{0.429 \ \frac{\text{rad}}{\text{s}}}$$

5/104



$$\frac{250}{\sin\beta} = \frac{400}{\sin 45^\circ}, \quad \beta = 26.2^\circ$$

$\overline{AO} = 400\cos 26.2^\circ + 250\cos 45^\circ = 535.6$ mm

$\overline{AC} = \overline{AO}\tan 45^\circ = 535.6$ mm

$\overline{ED} = 600\cos 26.2^\circ = 538.2$ mm

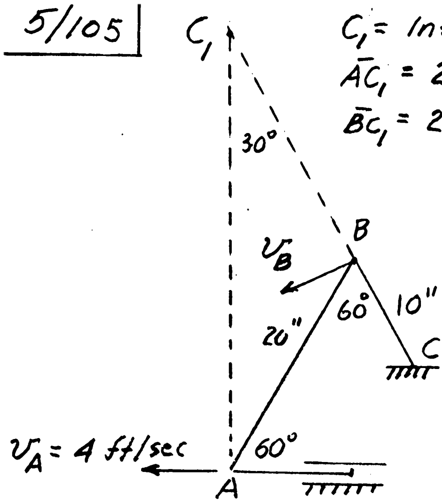
$\overline{CE} = 535.6 - 600\sin 26.2^\circ = 270.4$ mm

$$\overline{CD} = \sqrt{(270.4)^2 + (538.2)^2} = 602.4 \text{ mm}$$

$$v_D = v_A \frac{\overline{CD}}{\overline{CA}} = 4\,\frac{602.4}{535.6} = \underline{4.50 \text{ m/s}}$$

$$\omega_{ABD} = \omega = v_A/\overline{CA} = \frac{4000}{535.6} = \underline{7.47 \text{ rad/s}}$$

5/105



C_1 = Instant. center of AB

$\bar{AC_1} = 2(20 \cos 30°) = 34.6$ in.

$\bar{BC_1} = 20$ in.

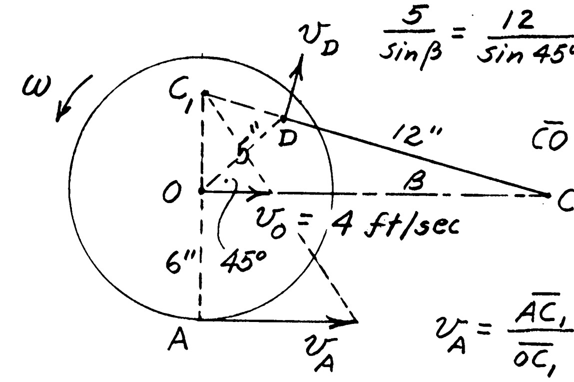
$$v_B = \bar{BC_1}\,\omega_{AB} = \bar{BC_1}\,\frac{v_A}{\bar{AC_1}}$$

$$= \frac{20}{34.6}(4) = 2.31 \text{ ft/sec}$$

$$\omega_{BC} = v_B/\bar{BC} = \frac{2.31}{10/12} = 2.77\ \frac{\text{rad}}{\text{sec}}$$

CCW

5/106 C_1 = instantaneous center



$$\frac{5}{\sin\beta} = \frac{12}{\sin 45^\circ}, \quad \beta = \sin^{-1}\left(\frac{5}{12}\frac{1}{\sqrt{2}}\right)$$

$$= 17.14^\circ$$

$$\overline{CO} = 5\cos 45^\circ + 12\cos 17.14^\circ$$

$$= 15.00 \text{ in.}$$

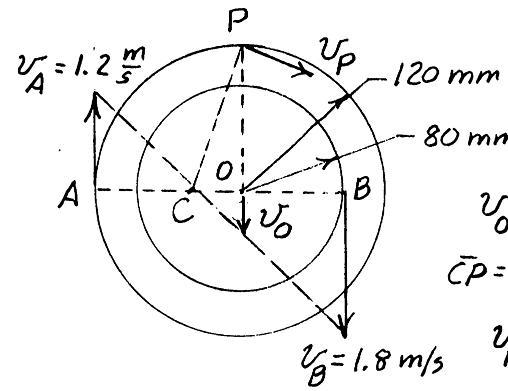
$$\overline{C_1O} = 15.00 \tan 17.14^\circ$$

$$= 4.63 \text{ in.}$$

$$v_A = \frac{\overline{AC_1}}{\overline{OC_1}} v_O = \frac{6 + 4.63}{4.63} 4$$

$$= \underline{9.19 \text{ ft/sec}}$$

5/107



$\overline{AC} / \overline{BC} = 1.2/1.8$ & $\overline{AC} + \overline{BC} = 200$ mm

so $\overline{AC} = 80$ mm

$\overline{OC} = 40$ mm

$\omega = \dfrac{v_A}{\overline{AC}} = \dfrac{1.2}{0.08} = \underline{15 \text{ rad/s}}$

$v_O = \overline{OC}\,\omega = 0.040(15) = 0.6 \frac{m}{s}$

$\overline{CP} = \sqrt{40^2 + 120^2} = 126.5$ mm

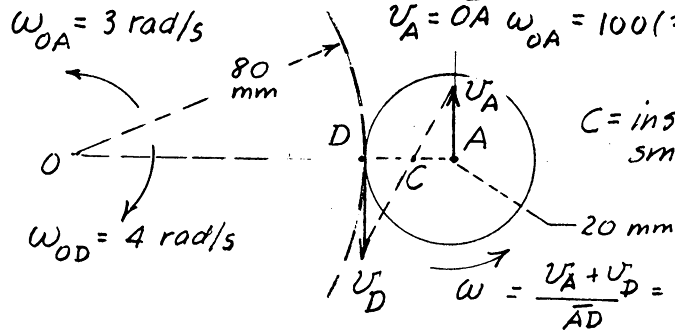
$v_P = \overline{CP}\,\omega = 0.1265(15)$

$= \underline{1.897 \text{ m/s}}$

5/108

$v_D = \overline{OD}\,\omega_{OD} = 80(4) = 320\ \text{mm/s}$

$v_A = \overline{OA}\,\omega_{OA} = 100(3) = 300\ \text{mm/s}$



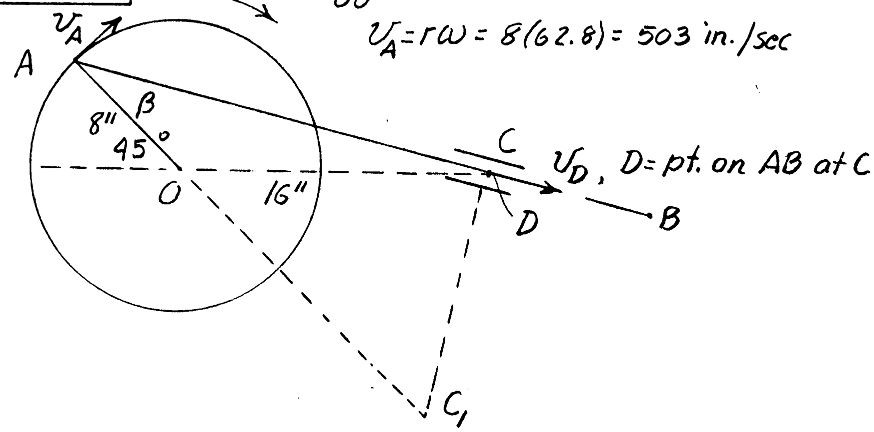
C = instant. center of small gear

$$\omega = \frac{v_A + v_D}{\overline{AD}} = \frac{320 + 300}{20}$$

$\underline{\omega = 31\ \text{rad/s CCW}}$

5/109 $\omega_{OA} = 600\frac{2\pi}{60} = 62.8$ rad/sec

$v_A = r\omega = 8(62.8) = 503$ in./sec



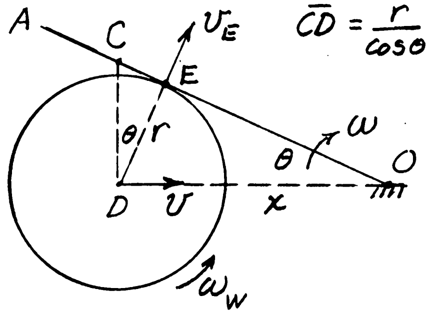
$\overline{AD} = \sqrt{8^2 + 16^2 - 2(8)(16)\cos 135°} = 22.4$ in.

$16/\sin\beta = 22.4/\sin 135°$, $\beta = 30.4°$

$\overline{AC_1} = 22.4/\cos 30.4° = 25.9$ in.

$\omega_{AB} = \omega_{AD} = \omega_{AC_1} = v_A/\overline{AC_1} = \frac{503}{25.9} = \underline{19.38 \text{ rad/sec CW}}$

5/110



$\overline{CD} = \frac{r}{\cos\theta}$, $\overline{CE} = r\tan\theta$

C = instantaneous center of zero velocity for wheel

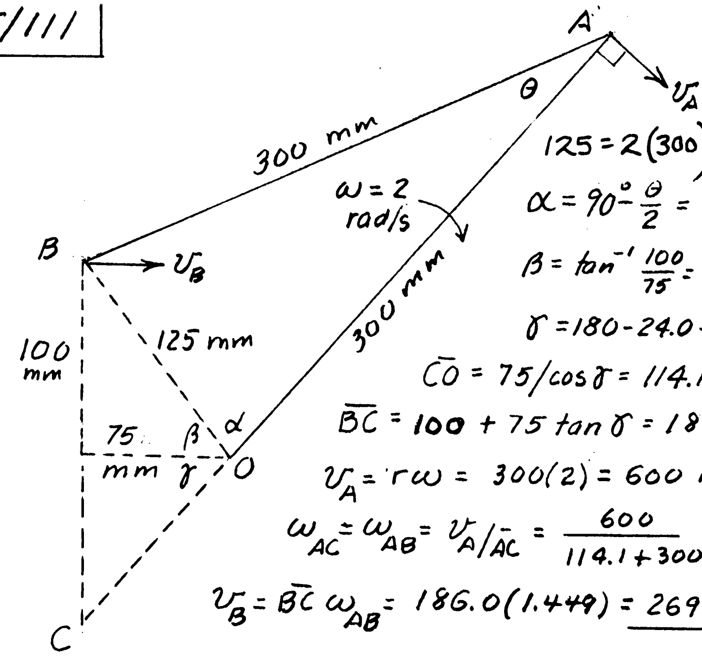
$$\omega_W = \frac{v_D}{\overline{CD}} = \frac{v}{r/\cos\theta} = \frac{v}{r}\frac{\sqrt{x^2-r^2}}{x}$$

$$\omega_W = \frac{v}{r}\sqrt{1-(r/x)^2}$$

$$\omega = \omega_{AO} = \frac{v_E}{\overline{EO}} = \frac{1}{\overline{EO}}\, v\, \frac{\overline{CE}}{\overline{CD}}$$

$$= \frac{1}{\sqrt{x^2-r^2}}\, v\, \frac{r\tan\theta}{r/\cos\theta} = \frac{v}{\sqrt{x^2-r^2}}\frac{r}{x}, \quad \omega = \frac{v}{x\sqrt{(x/r)^2-1}}$$

5/111



$125 = 2(300)\sin\frac{\theta}{2}, \quad \theta = 24.0°$

$\alpha = 90° - \frac{\theta}{2} = 78.0°$

$\beta = \tan^{-1}\frac{100}{75} = 53.1°$

$\gamma = 180 - 24.0 - 53.1 = 48.9°$

$\overline{CO} = 75/\cos\gamma = 114.1 \text{ mm}$

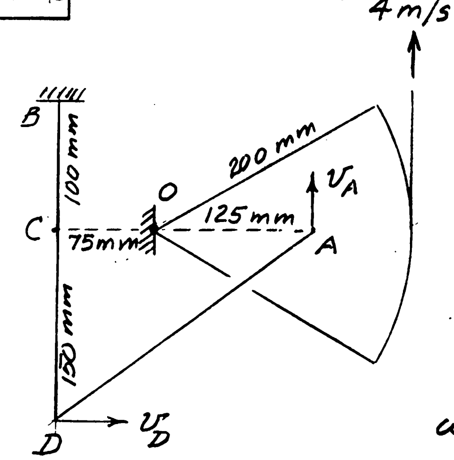
$\overline{BC} = 100 + 75\tan\gamma = 186.0 \text{ mm}$

$v_A = r\omega = 300(2) = 600 \text{ mm/s}$

$\omega_{AC} = \omega_{AB} = v_A/\overline{AC} = \frac{600}{114.1 + 300} = 1.449 \ \frac{\text{rad}}{\text{s}}$

$v_B = \overline{BC}\,\omega_{AB} = 186.0(1.449) = \underline{269 \text{ mm/s}}$

5/112



C = instantaneous center of zero velocity for AD.

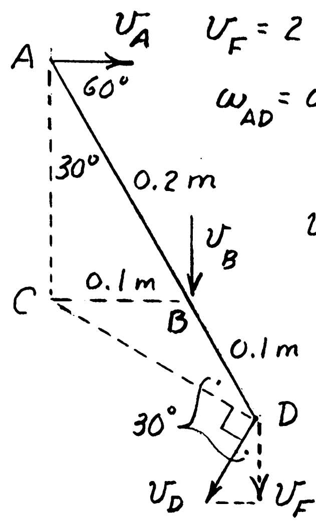
$$v_A = \frac{125}{200}(4) = 2.5 \text{ m/s}$$

$$\omega_{AD} = \frac{v_A}{\overline{AC}} = \frac{2.5}{0.200} = 12.5 \frac{\text{rad}}{\text{s}}$$

$$v_D = \overline{CD}\,\omega_{CD} = \overline{CD}\,\omega_{AD}$$
$$= 0.150\ (12.5)$$
$$= 1.875 \text{ m/s}$$

$$\omega_{BD} = \frac{v_D}{BD} = \frac{1.875}{0.25} = 7.5 \text{ rad/s}$$

5/113



C = instant. center for ABD

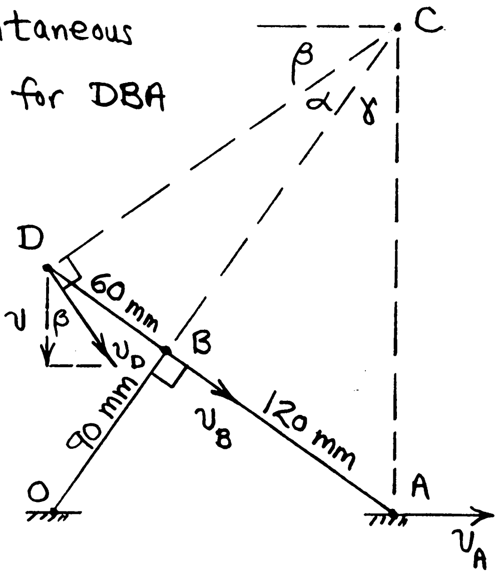
$v_F = 2$ m/s, so $v_D = \dfrac{2}{\cos 30°} = 2.31$ m/s

$\omega_{AD} = \omega_{CD} = \dfrac{v_D}{\overline{CD}} = \dfrac{2.31}{2(0.1)\cos 30°} = 13.33 \ \dfrac{\text{rad}}{\text{s}}$ $\underline{\text{CW}}$

$v_A = \overline{AC}\,\omega_{AC} = \overline{AC}\,\omega_{CD}$

$= 0.2 \cos 30° (13.33) = \underline{2.31 \text{ m/s}}$

5/114 C is the instantaneous center of zero velocity for DBA



From geometry,

$\overline{AC} = \frac{5}{3}(120) = 200 \text{ mm}$

$\overline{BC} = 160 \text{ mm}$

$\overline{DC} = \sqrt{60^2 + 160^2}$

$= 170.9 \text{ mm}$

$\gamma = \sin^{-1}\frac{120}{200} = 36.9°$

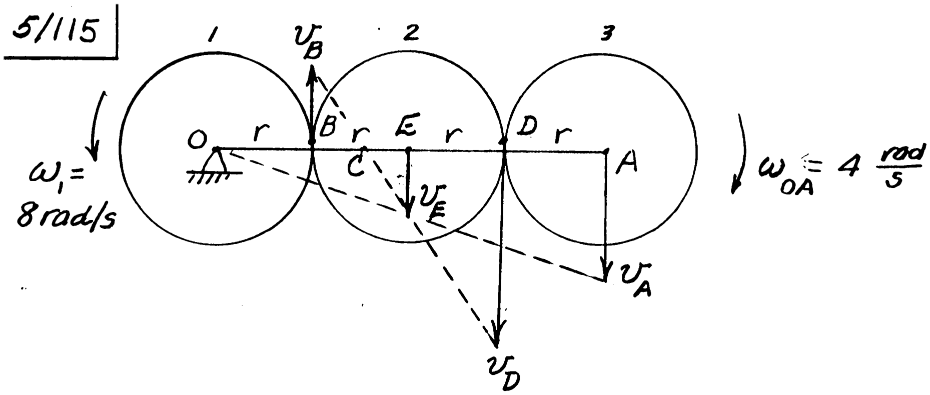
$\alpha = \tan^{-1}\frac{60}{160} = 20.6°$

$\beta = 90 - 36.9 - 20.6 = 32.6°$

$v_D = \frac{v}{\cos\beta} = \frac{0.2}{\cos 32.6°} = 0.237 \text{ m/s}$

$\frac{v_D}{\overline{DC}} = \frac{v_A}{\overline{AC}}$; $v_A = \frac{200}{170.9}(0.237) = \underline{0.278 \text{ m/s}}$

5/115



Let B be contact point common to gears 1 & 2

" D " " " " " gears 2 & 3

Point C is instantaneous center of zero velocity for gear 2.

By similar triangles, $v_D = 3(8r) = 24r$

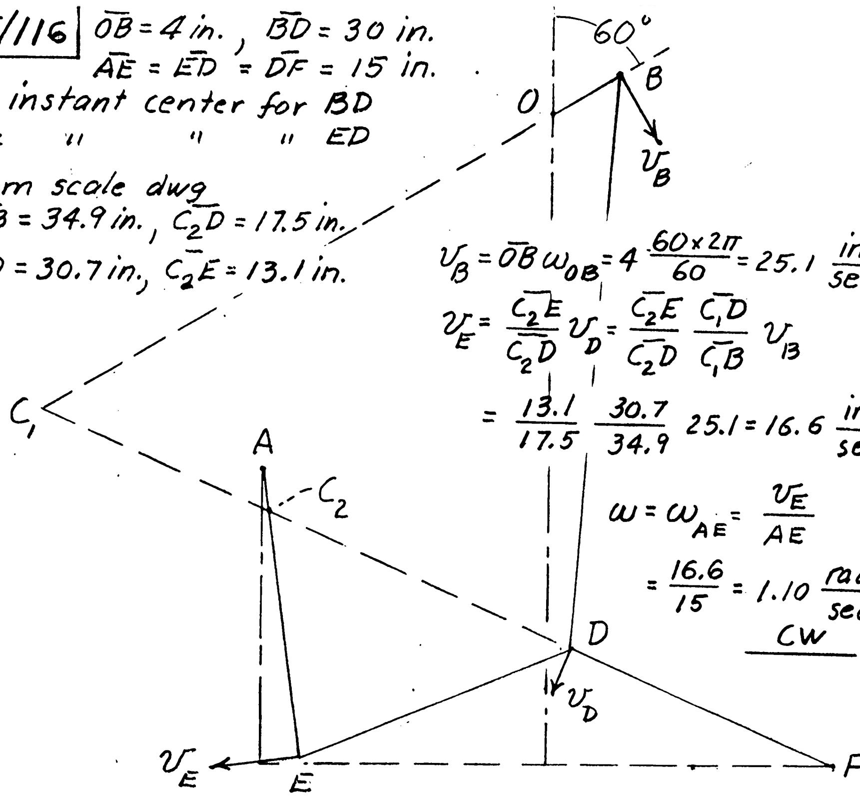
$v_B = r\omega_1 = 8r$ $\qquad v_A = \overline{OA}\,\omega_{OA} = 4r(4) = 16r$

$v_E = 2r\,\omega_{OA} = 8r$ $\qquad \omega_3 = \dfrac{v_{D/A}}{\overline{DA}} = \dfrac{24r - 16r}{r} = \underline{8 \text{ rad/s CCW}}$

▶ 5/116 $\overline{OB} = 4$ in., $\overline{BD} = 30$ in.
$\overline{AE} = \overline{ED} = \overline{DF} = 15$ in.

C_1 = instant center for BD
C_2 = " " " ED

From scale dwg
$\overline{C_1B} = 34.9$ in., $\overline{C_2D} = 17.5$ in.
$\overline{C_1D} = 30.7$ in., $\overline{C_2E} = 13.1$ in.



$$v_B = \overline{OB}\,\omega_{OB} = 4\,\frac{60 \times 2\pi}{60} = 25.1\ \frac{\text{in.}}{\text{sec}}$$

$$v_E = \frac{\overline{C_2E}}{\overline{C_2D}}\,v_D = \frac{\overline{C_2E}}{\overline{C_2D}}\,\frac{\overline{C_1D}}{\overline{C_1B}}\,v_B$$

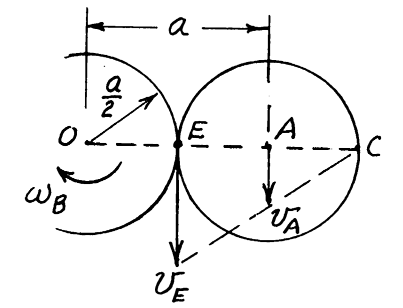
$$= \frac{13.1}{17.5}\,\frac{30.7}{34.9}\,25.1 = 16.6\ \frac{\text{in.}}{\text{sec}}$$

$$\omega = \omega_{AE} = \frac{v_E}{AE}$$

$$= \frac{16.6}{15} = 1.10\ \frac{\text{rad}}{\text{sec}}$$

CW

▶5/117

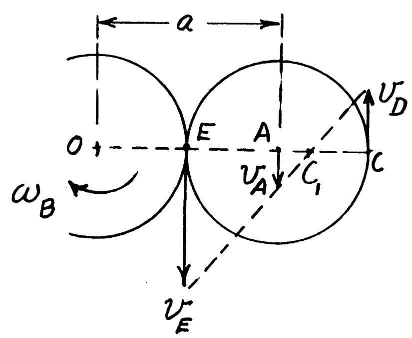


(a) $v_A = \omega_{OA}\, a$

$v_E = 2v_A = 2a\,\omega_{OA}$

$\omega_B = \dfrac{v_E}{a/2} = \dfrac{2a\,\omega_{OA}}{a/2} = 4(90)$

$= \underline{360 \text{ rev/min}}$



(b) $v_D = \overline{OC}\,\omega_D = \dfrac{3a}{2}\, 80 = 120a$

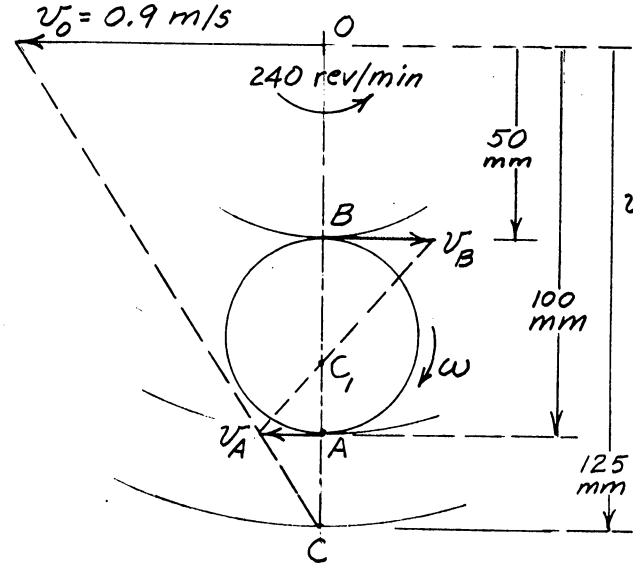
$v_A = \overline{OA}\,\omega_{OA} = 90a$

$\dfrac{v_A + v_D}{a} = \dfrac{v_E - v_A}{a}$, $v_E = v_D + 2v_A$

$= 300a$

$\omega_{OE} = \omega_B = \dfrac{v_E}{a/2} = \dfrac{300a}{a/2} = \underline{600 \dfrac{\text{rev}}{\text{min}}}$

▶5/118 $v_A = \dfrac{\overline{AC}}{\overline{OC}} v_o = \dfrac{25}{125} 0.9 = 0.18$ m/s ; $\underline{v}_B = \underline{v}_o + \underline{v}_{B/O}$



$v_{B/O} = \overline{BO}\,\omega_{BO}$

$= 0.5 \dfrac{240(2\pi)}{60}$

$= 1.257$ m/s

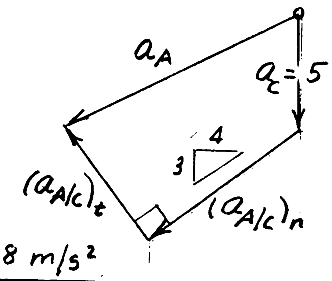
$v_o \longleftarrow \quad v_{B/O} \quad \dashrightarrow v_B$

$v_B = 1.257 - 0.9$

$= 0.357$ m/s

$\dfrac{v_B}{\overline{BC_1}} = \dfrac{v_A + v_B}{\overline{AB}} = \omega, \quad \omega = \dfrac{0.18 + 0.357}{0.05} = \underline{10.73 \text{ rad/s CW}}$

5/119 $\quad \underline{a}_A = \underline{a}_C + (\underline{a}_{A/C})_n + (\underline{a}_{A/C})_t$



<u>Scalar-geometric</u>

$(a_{A/C})_n = \overline{AC}\,\omega_{AC}^2 = 0.5(4^2) = 8 \text{ m/s}^2$

$(a_{A/C})_t = \overline{AC}\,\alpha_{AC} = 0.5(12) = 6 \text{ m/s}^2$

$a_A = \sqrt{(5 + \frac{3}{5}8 - \frac{4}{5}6)^2 + (\frac{4}{5}8 + \frac{3}{5}6)^2} = \underline{11.18 \text{ m/s}^2}$

<u>Vector-algebraic</u>

$(\underline{a}_{A/C})_n = \underline{\omega} \times (\underline{\omega} \times \underline{r}) = -4\underline{k} \times (-4\underline{k} \times [0.4\underline{i} + 0.3\underline{j}])$
$\qquad = -6.4\underline{i} - 4.8\underline{j} \text{ m/s}^2$

$(\underline{a}_{A/C})_t = \underline{\alpha} \times \underline{r} = 12\underline{k} \times (0.4\underline{i} + 0.3\underline{j}) = 4.8\underline{j} - 3.6\underline{i} \text{ m/s}^2$

$\underline{a}_C = -5\underline{j} \text{ m/s}^2$

Substitute, equate $\underline{i}$ & $\underline{j}$ coefficients, & get

$(a_A)_x = -10 \text{ m/s}^2, \; (a_A)_y = -5 \text{ m/s}^2$

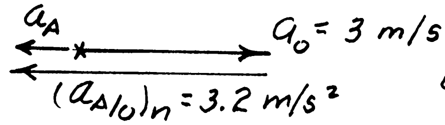
so $a_A = \sqrt{(-10)^2 + (-5)^2} = 11.18 \text{ m/s}^2$

5/120 $\underline{a}_A = \underline{a}_O + (\underline{a}_{A/O})_n + (\underline{a}_{A/O})_t$ not function of v_O or sense of ω

$(a_{A/O})_n = \overline{OA}\,\omega^2 = 0.8(2^2) = 3.2\ m/s^2$

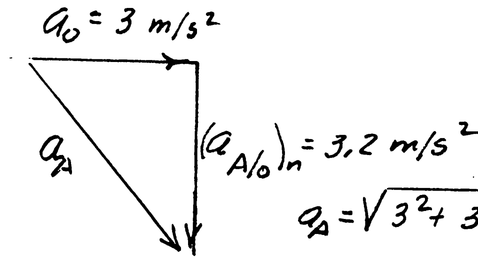
$(a_{A/O})_t = \overline{OA}\,\alpha = 0$

(a) $\theta = 0$



$\underline{a_A = 0.2\ m/s^2}$

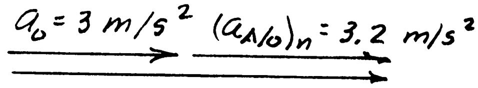
(b) $\theta = 90°$



$a_A = \sqrt{3^2 + 3.2^2}$

$= \underline{4.39\ m/s^2}$

(c) $\theta = 180°$



$a_A = 3 + 3.2 = \underline{6.2\ m/s^2}$

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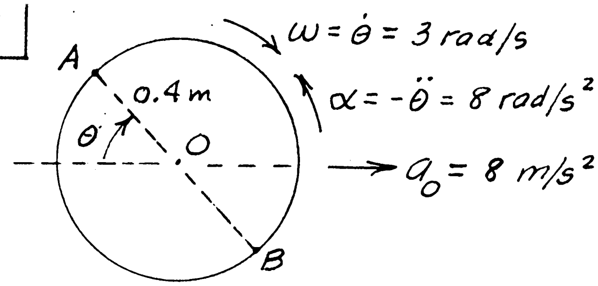


$\mathbf{a}_A = \mathbf{a}_O + (\mathbf{a}_{A/O})_n + (\mathbf{a}_{A/O})_t$

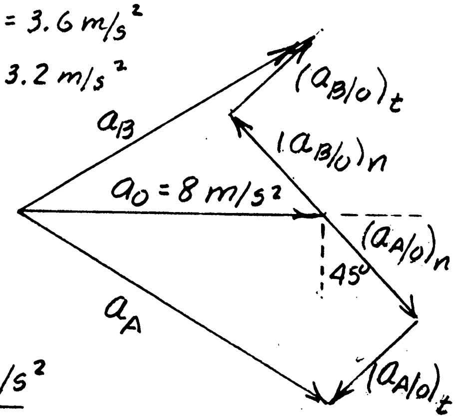
$(a_{A/O})_n = \overline{AO}\,\omega^2 = 0.4(3^2) = 3.6$ m/s^2

$(a_{A/O})_t = \overline{AO}\,\alpha = 0.4(8) = 3.2$ m/s^2

$\mathbf{a}_B = \mathbf{a}_O + (\mathbf{a}_{B/O})_n + (\mathbf{a}_{B/O})_t$

$(a_{B/O})_n = 3.6$ m/s^2

$(a_{A/O})_t = 3.2$ m/s^2



From polygons

$a_A = \underline{9.58 \text{ m/s}^2}$, $a_B = \underline{9.09 \text{ m/s}^2}$

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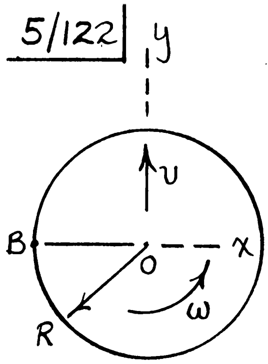

$$a_o = \frac{G m_s}{r^2}$$

$$= \frac{6.673(10^{-11})[5.976 \cdot 10^{24} \cdot 333\,000]}{[149.6(10^9)]^2}$$

$$= 0.00593 \text{ m/s}^2 \ (\leftarrow)$$

$$R\omega^2 = 6371(10^3)[7.292(10^{-5})]^2$$

$$= 0.0339 \text{ m/s}^2 \ (\rightarrow)$$

$$\underline{a}_B = \underline{a}_O + \underline{a}_{B/O} = -0.00593\,\underline{i} + 0.0339\,\underline{i}$$

$$= \underline{0.0279\,\underline{i} \text{ m/s}^2}$$

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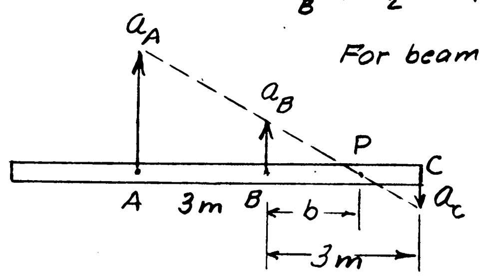
$a_A = r\alpha_1 = 0.5(0.5) = 0.25\ m/s^2$

$a_B = r\alpha_2 = 0.5(0.2) = 0.1\ m/s^2$



For beam $\alpha = \dfrac{a_{A/B}}{\overline{AB}} = \dfrac{0.25 - 0.1}{3}$

$= \underline{0.05\ rad/s^2\ CW}$

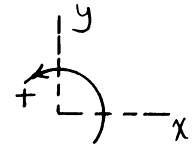
$\overset{+}{\downarrow}\ \underline{a}_C = \underline{a}_B + \underline{a}_{C/B} = -0.1 + 3(0.05) = \underline{0.05\ m/s^2\ \text{down}}$

$\overset{+}{\downarrow}\ \underline{a}_P = \underline{0} = \underline{a}_B + \underline{a}_{P/B} = -0.1 + b(0.05)$, $\underline{b = 2\ m}$

5/124 In the coordinates shown, the no-slip kinematic constraints are $v_O = -r\omega$, $a_O = -r\alpha$.

So $\omega = -\dfrac{v_O}{r} = -\dfrac{3}{0.4} = -7.5 \text{ rad/s}$

$\alpha = -\dfrac{a_O}{r} = -\dfrac{-5}{0.4} = 12.5 \text{ rad/s}^2$



$$\underline{v}_A = \underline{v}_O + \underline{v}_{A/O} = \underline{v}_O + \underline{\omega} \times \underline{r}_{A/O}$$

$$= 3\underline{i} + (-7.5\underline{k}) \times 0.4[-\cos 45^\circ \underline{i} + \sin 45^\circ \underline{j}]$$

$$= \underline{5.12\underline{i} + 2.12\underline{j} \text{ m/s}}$$

$$\underline{a}_B = \underline{a}_O + \underline{a}_{B/O} = \underline{a}_O + \underline{\alpha} \times \underline{r}_{B/O} - \omega^2 \underline{r}_{B/O}$$

$$= -5\underline{i} + 12.5\underline{k} \times 0.2\underline{i} - (-7.5)^2(0.2\underline{i})$$

$$= \underline{-16.25\underline{i} + 2.5\underline{j} \text{ m/s}^2}$$

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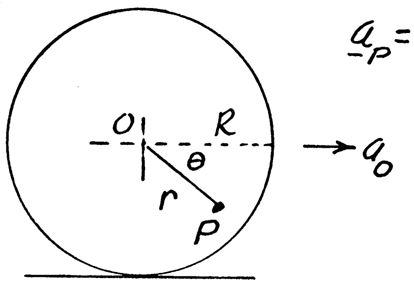
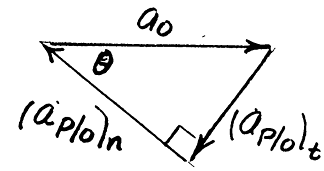


$$\underline{a}_P = \underline{0} = \underline{a}_O + (\underline{a}_{P/O})_n + (\underline{a}_{P/O})_t$$



$$\theta = \sin^{-1}\frac{(a_{P/O})_t}{a_O} = \sin^{-1}\frac{r\alpha}{R\alpha}, \quad \underline{\theta = \sin^{-1}\frac{r}{R}}$$

$$(a_{P/O})_n = r\omega^2 = r\left(\frac{v_O}{R}\right)^2 = \frac{r}{R^2}v_O^2 = a_O\cos\theta$$

$$\text{so} \quad v_O^2 = \frac{a_O R^2}{r}\frac{\sqrt{R^2-r^2}}{R} \quad \text{so} \quad \underline{v_O = \sqrt{\frac{R}{r}a_O}\sqrt[4]{R^2-r^2}}$$

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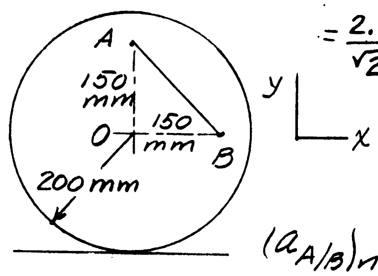

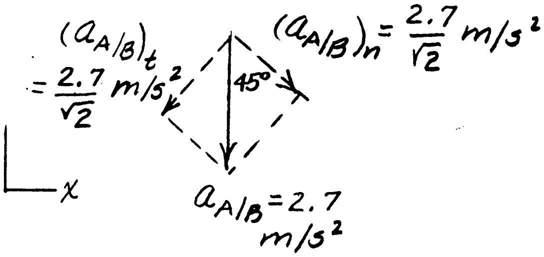

$(a_{A/B})_n = \overline{AB}\,\omega^2$; $\omega^2 = \dfrac{2.7}{\sqrt{2}}\dfrac{1}{0.15\sqrt{2}}$

$= 9 \text{ rad}^2/\text{s}^2$

$\omega = 3$ rad/s

$(a_{A/B})_t = \overline{AB}\,\alpha$

$\alpha = \dfrac{2.7}{\sqrt{2}}\dfrac{1}{0.15\sqrt{2}} = 9 \text{ rad/s}^2$ CCW

$v_O = r\omega = 0.2(3) = 0.6$ m/s or $\underline{\mathbf{v}_O = -0.6\mathbf{i}\ \text{m/s}}$

$a_O = r\alpha = 0.2(9) = 1.8 \text{ m/s}^2$ or $\underline{\mathbf{a}_O = -1.8\mathbf{i}\ \text{m/s}^2}$

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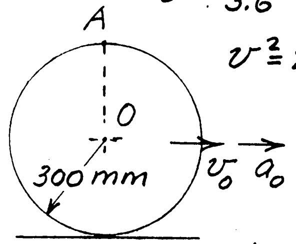
$v_O = \frac{10}{3.6} = 2.78$ m/s, $\omega = \frac{2.78}{0.300} = 9.26$ rad/s



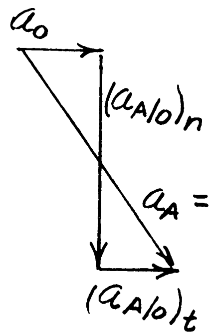
$v^2 = 2as$, $a_O = \frac{v^2}{2s} = \frac{(60/3.6)^2}{2(40)} = 3.47$ m/s²

$\alpha = a_O/r = 3.47/0.300 = 11.57 \frac{\text{rad}}{\text{s}^2}$

$\underline{a}_A = \underline{a}_O + (\underline{a}_{A/O})_n + (\underline{a}_{A/O})_t$

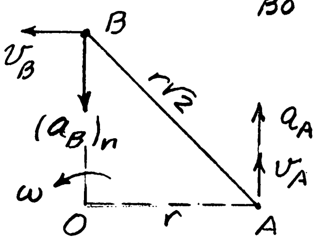
$(a_{A/O})_n = 0.300\,(9.26)^2 = 25.72$ m/s²

$(a_{A/O})_t = 0.300\,(11.57) = 3.47$ m/s²



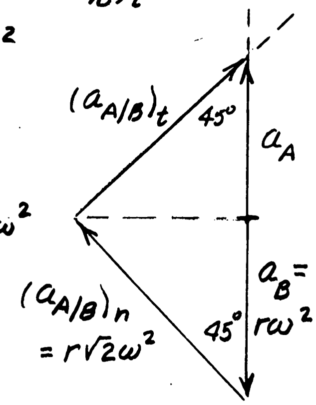
$a_A = \sqrt{(3.47 + 3.47)^2 + (25.72)^2} = \underline{26.6 \text{ m/s}^2}$

5/128 $\omega_{AB} = \frac{v_B}{BO} = \frac{r\omega}{r} = \omega$



$\underline{a}_A = \underline{a}_B + (\underline{a}_{A/B})_n + (\underline{a}_{A/B})_t$

$(a_{A/B})_n = r\sqrt{2}\,\omega^2$



From diagram $(a_{A/B})_t = r\sqrt{2}\alpha = r\sqrt{2}\,\omega^2$

$\underline{\alpha_{AB} = \omega^2}$

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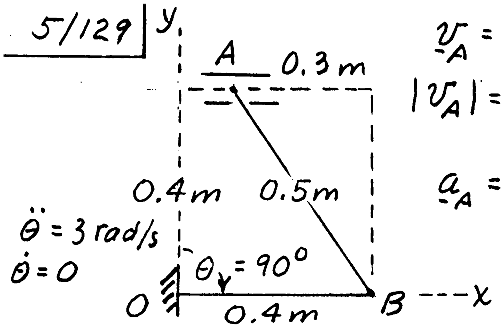

$\underline{v}_A = \underline{v}_B + \underline{\omega}_{AB} \times \underline{r}_{A/B}$

$|v_A| = |v_B| = |\omega_{AB}| = 0$ for $\dot{\theta} = 0$

$\underline{a}_A = \underline{a}_B + (\underline{a}_{A/B})_t\ ,\ (\underline{a}_{A/B})_n = \underline{0}$

$a_A \underline{i} = 0.4(3)(-\underline{j}) + \alpha_{AB}\underline{k} \times (-0.3\underline{i} + 0.4\underline{j})$

$= -1.2\underline{j} - 0.3\alpha_{AB}\underline{j} - 0.4\alpha_{AB}\underline{i}$

$a_A = -0.4\alpha_{AB}$ & $0 = -1.2 - 0.3\alpha_{AB}$

$\alpha_{AB} = -4 \text{ rad/s}^2$, $\underline{\alpha}_{AB} = \underline{-4\underline{k} \text{ rad/s}^2}$

$a_A = -0.4(-4) = +1.6 \text{ m/s}^2$, $\underline{a}_A = \underline{1.6\underline{i} \text{ m/s}^2}$

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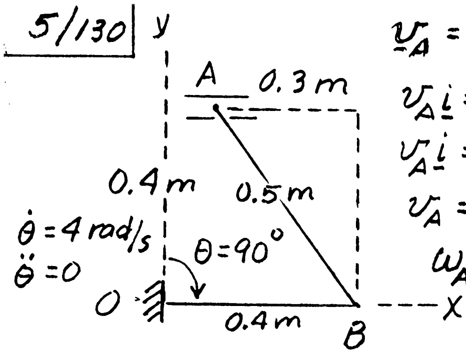

$\underline{v}_A = \underline{v}_B + \underline{\omega}_{AB} \times \underline{r}_{A/B}$

$v_A \underline{i} = 0.4(4)(-\underline{j}) + \omega_{AB}\underline{k} \times (-0.3\underline{i} + 0.4\underline{j})$

$v_A \underline{i} = -1.6\underline{j} - 0.3\omega_{AB}\underline{j} - 0.4\omega_{AB}\underline{i}$

$v_A = -0.4\omega_{AB}$, $0 = -1.6 - 0.3\omega_{AB}$

$\omega_{AB} = -5.33$ rad/s, $v_A = 2.13$ m/s

$\underline{a}_A = \underline{a}_B + (\underline{a}_{A/B})_n + (\underline{a}_{A/B})_t$ where $\underline{a}_B = \underline{a}_{B_n}$, $\underline{a}_{B_t} = 0$

$\underline{a}_A = a_A \underline{i}$, $\underline{a}_B = \overline{OB}\,\omega_{OB}^2(-\underline{i}) = 0.4(4^2)(-\underline{i}) = -6.4\underline{i}$ m/s^2

$(\underline{a}_{A/B})_n = \underline{\omega}_{AB} \times (\underline{\omega}_{AB} \times \underline{r}_{A/B}) = \omega_{AB}\underline{k} \times (\omega_{AB}\underline{k} \times [-0.3\underline{i} + 0.4\underline{j}])$

$= \omega_{AB}^2(0.3\underline{i} - 0.4\underline{j}) = (-5.33)^2(0.3\underline{i} - 0.4\underline{j})$

$(\underline{a}_{A/B})_t = \underline{\alpha}_{AB} \times \underline{r}_{A/B} = \alpha_{AB}\underline{k} \times (-0.3\underline{i} + 0.4\underline{j}) = \alpha_{AB}(-0.3\underline{j} - 0.4\underline{i})$

Substitute, equate $\underline{i}$ & $\underline{j}$ coefficients & get

$\underline{\alpha}_{AB} = \underline{-37.9\underline{k} \text{ rad/s}^2}$, $\underline{a}_A = \underline{17.30\,\underline{i} \text{ m/s}^2}$

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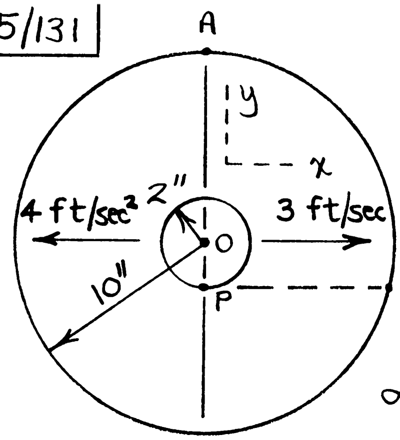

From the solution to Prob. 5/98 or from $v_0 = r\omega$, $\omega = \frac{v_0}{r} = \frac{3}{2/12} = 18$ rad/sec CW.

From $a_0 = r\alpha$,

$$\alpha = \frac{a_0}{r} = \frac{4}{2/12} = 24\ \frac{\text{rad}}{\text{sec}^2}\ \text{CCW}$$

$$\underline{a}_A = \underline{a}_0 + \underline{a}_{A/0} = \underline{a}_0 + \underline{\alpha} \times \underline{r}_{A/0} - \omega^2 \underline{r}_{A/0}$$

$$= -4\underline{i} + 24\underline{k} \times \frac{10}{12}\underline{j} - 18^2\left(\frac{10}{12}\underline{j}\right)$$

$$= \underline{-24\underline{i} - 270\underline{j}\ \ \text{ft/sec}^2}$$

$$\underline{a}_D = \underline{a}_0 + \underline{a}_{D/0} = \underline{a}_0 + \underline{\alpha} \times \underline{r}_{D/0} - \omega^2 \underline{r}_{D/0}$$

$$= -4\underline{i} + 24\underline{k} \times \left(\frac{10}{12}\cos\sin^{-1}\frac{2}{10}\underline{i} - \frac{2}{12}\underline{j}\right)$$

$$- 18^2\left(\frac{10}{12}\cos\sin^{-1}\frac{2}{10}\underline{i} - \frac{2}{12}\underline{j}\right)$$

$$= \underline{-265\underline{i} + 34.4\underline{j}\ \ \text{ft/sec}^2}$$

(Could use P as a base point for $\underline{a}_D$.)

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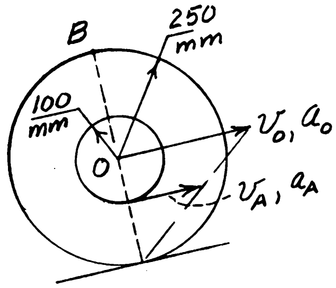
$v_0^2 = 2a_0 s$, $a_0 = \frac{(1.2)^2}{2(2.4)} = 0.30\ m/s^2$



$a_A = \frac{250-100}{250}(0.30) = \underline{0.18\ m/s^2}$

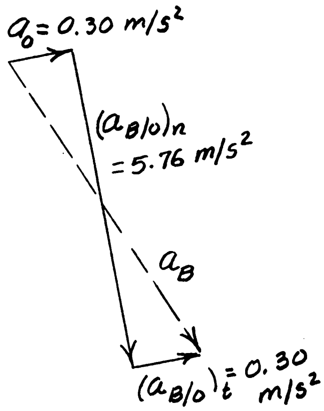
$\mathbf{a}_B = \mathbf{a}_0 + (\mathbf{a}_{B/0})_n + (\mathbf{a}_{B/0})_t$

$(a_{B/0})_n = \overline{BO}\,\omega^2 = 0.25\left(\frac{1.2}{0.25}\right)^2 = 5.76\ m/s^2$

$(a_{B/0})_t = \overline{BO}\,\alpha = 0.25\left(\frac{0.3}{0.25}\right) = 0.30\ m/s^2$

$a_B = \sqrt{(0.30+0.30)^2 + (5.76)^2}$

$= \underline{5.79\ m/s^2}$



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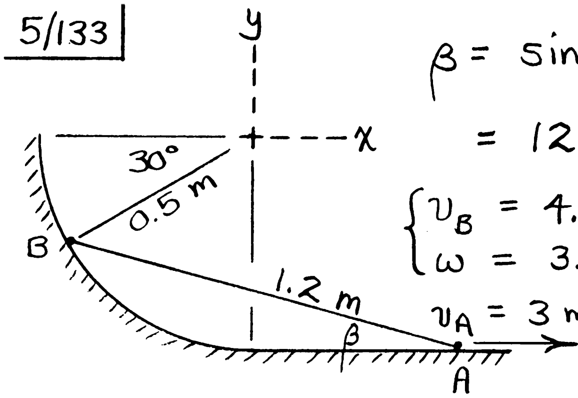

$$\beta = \sin^{-1}\frac{0.5 - 0.5\sin 30^\circ}{1.2}$$

$$= 12.02^\circ$$

$$\begin{cases} v_B = 4.38 \text{ m/s (Prob. 5/68)} \\ \omega = 3.23 \text{ rad/s} \end{cases}$$

$v_A = 3$ m/s = constant

$$\underline{a}_B = \underline{a}_A + \underline{a}_{B/A} = \underline{a}_A + \underline{\alpha} \times \underline{r}_{B/A} - \omega^2 \underline{r}_{B/A}$$

$$a_{Bt}(\sin 30^\circ \underline{i} - \cos 30^\circ \underline{j}) + \frac{4.38^2}{0.5}(\cos 30^\circ \underline{i} + \sin 30^\circ \underline{j})$$

$$= \underline{0} + \alpha \underline{k} \times 1.2(-\cos 12.02^\circ \underline{i} + \sin 12.02^\circ \underline{j})$$

$$- 3.23^2 (1.2)(-\cos 12.02^\circ \underline{i} + \sin 12.02^\circ \underline{j})$$

Carry out vector algebra & equate coefficients:

$$\underline{i}: \ \tfrac{1}{2} a_{Bt} + 33.3 = -0.250\alpha + 12.28$$

$$\underline{j}: \ -\tfrac{\sqrt{3}}{2} a_{Bt} + 19.21 = -1.174\alpha - 2.61$$

Solution: $\underline{a_{Bt} = -23.9 \text{ m/s}^2}$, $\underline{\alpha = -36.2 \text{ rad/s}^2}$

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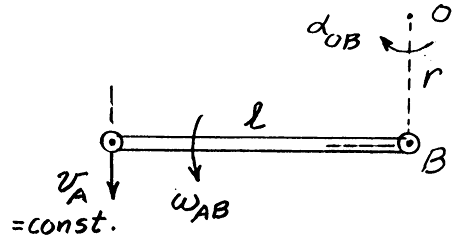

$\omega_{AB} = v_A/l$

$v_B = 0$ so $(a_B)_n = \dfrac{v_B^2}{r} = 0$

$\underline{a}_B = \underline{a}_A + (\underline{a}_{B/A})_n + (\underline{a}_{B/A})_t$

$\underline{a}_{B_t} = 0 + (\underline{a}_{B/A})_n + 0$

$(a_B)_t$

$(a_{B/A})_n = l\omega_{AB}^2 = v_A^2/l$

Thus $\alpha_{OB} = \dfrac{(a_B)_t}{r} = \underline{\dfrac{v_A^2}{rl}}$

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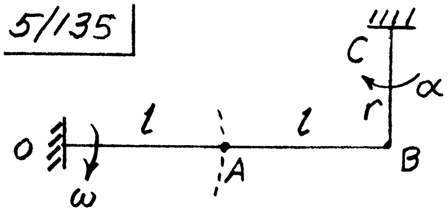


$v_B = 0$ so $(a_B)_n = 0$

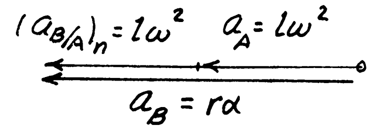
$a_B = (a_B)_t = r\alpha$

$\omega_{BA} = \dfrac{v_A}{l} = \omega$

$\underline{a}_B = \underline{a}_A + (\underline{a}_{B/A})_n + (\underline{a}_{B/A})_t$

$a_A = l\omega^2$

$(a_{B/A})_n = \overline{BA}\,\omega_{BA}^2 = l\omega^2$

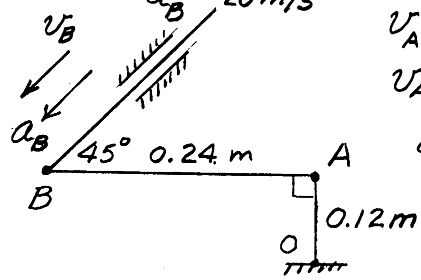


$(a_{B/A})_t = 0$

Thus $r\alpha = 2l\omega^2$, $\underline{\alpha = 2l\omega^2/r \text{ CW}}$

5/136 $v_B = 4$ m/s

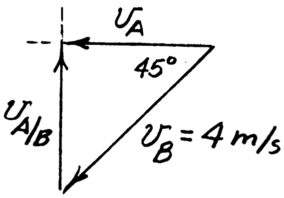
$a_B = 20$ m/s²



$\underline{v}_A = \underline{v}_B + \underline{v}_{A/B}$

$v_A = \frac{4}{\sqrt{2}}$ m/s

$v_{A/B} = \frac{4}{\sqrt{2}}$ m/s



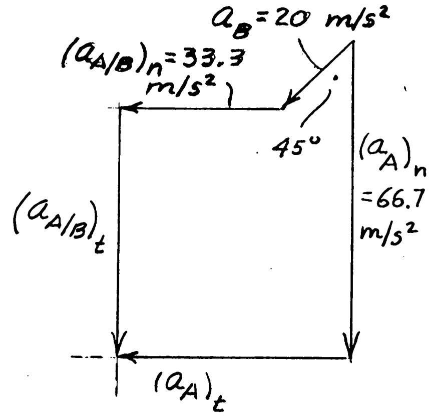
$\omega_{AB} = \dfrac{4/\sqrt{2}}{0.24} = 11.79$ rad/s CCW

$\omega_{OA} = \dfrac{4/\sqrt{2}}{0.12} = 23.57$ rad/s CCW

$\underline{a}_A = \underline{a}_B + (\underline{a}_{A/B})_n + (\underline{a}_{A/B})_t$

$(a_{A/B})_n = 0.24(11.79)^2 = 33.3$ m/s²

$(a_A)_n = 0.12(23.57)^2 = 66.7$ m/s²



From diagram

$(a_A)_t = 33.3 + 20/\sqrt{2} = 47.5$ m/s²

$\alpha_{OA} = \dfrac{(a_A)_t}{\overline{OA}} = \dfrac{47.5}{0.12} = 396$ rad/s² CCW

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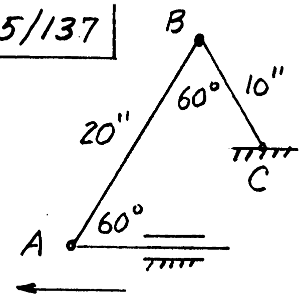


$v_A = 4$ ft/sec

$a_A = 3$ ft/sec^2

From Prob. 5/69, $v_B = 2.31$ ft/sec

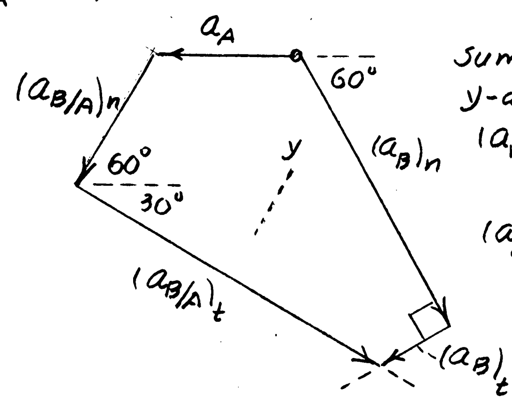
$v_{B/A} = 2.31$ ft/sec, $\omega_{BC} = 2.77$ rad/sec CCW

$$\underline{a}_B = \underline{a}_A + (\underline{a}_{B/A})_n + (\underline{a}_{B/A})_t$$

$$a_A = 3 \text{ ft/sec}^2$$

$$(a_{B/A})_n = v_{B/A}^{\,2}/\overline{BA} = 2.31^2/\tfrac{20}{12} = 3.2 \ \frac{\text{ft}}{\text{sec}^2}$$

$$(a_B)_n = v_B^2/\overline{BC} = 2.31^2/\tfrac{10}{12} = 6.4 \text{ ft/sec}^2$$



Summing component in y-dir. gives

$$(a_B)_t \cos 30° + 6.4 \sin 30°$$

$$= 3.2 + 3 \sin 30°$$

$$(a_B)_t = 1.732 \text{ ft/sec}^2$$

$$\alpha_{BC} = \frac{(a_B)_t}{\overline{BC}} = \frac{1.732}{10/12}$$

$$= 2.08 \ \frac{\text{rad}}{\text{sec}^2} \text{ CCW}$$

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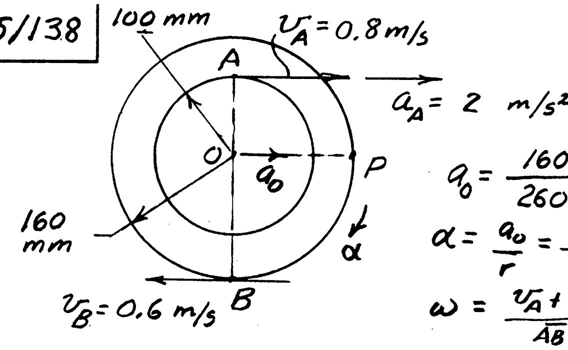


$$a_0 = \frac{160}{260}(2) = 1.231 \text{ m/s}$$

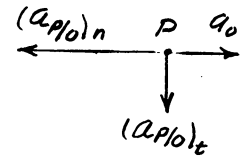
$$\alpha = \frac{a_0}{r} = \frac{1.231}{0.160} = 7.69 \text{ rad/s}^2$$

$$\omega = \frac{v_A + v_B}{\overline{AB}} = \frac{0.8+0.6}{0.260} = 5.38 \frac{\text{rad}}{\text{s}}$$

$$\underline{a}_P = \underline{a}_0 + (\underline{a}_{P/O})_n + (\underline{a}_{P/O})_t$$

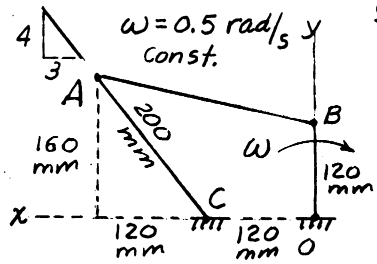
$$(a_{P/O})_n = \overline{PO}\,\omega^2 = 0.160(5.38)^2 = 4.64 \text{ m/s}^2$$

$$(a_{P/O})_t = \overline{PO}\,\alpha = 0.16(7.69) = 1.231 \text{ m/s}^2$$



$$a_P = \sqrt{(4.64 - 1.231)^2 + (1.231)^2} = \underline{3.62 \text{ m/s}^2}$$

5/139 From Prob. 5/81, $\underline{\omega}_{AB} = 0.214\underline{k}$, $\underline{\omega}_{CA} = 0.429\underline{k}$ $\frac{rad}{s}$



$\underline{a}_A = \underline{a}_B + (\underline{a}_{A/B})_n + (\underline{a}_{A/B})_t$ ---- (a)

$\underline{a}_A = \underline{\omega}_{CA} \times (\underline{\omega}_{CA} \times \underline{r}_{CA}) + \underline{\alpha}_{CA} \times \underline{r}_{CA}$

$= (0.429)^2(\underline{k} \times [\underline{k} \times \{0.12\underline{i} + 0.16\underline{j}\}])$

$+ \alpha_{CA}\underline{k} \times (0.12\underline{i} + 0.16\underline{j})$

$= 0.1837(-0.12\underline{i} - 0.16\underline{j})$

$+ 0.12\alpha_{CA}\underline{j} - 0.16\alpha_{CA}\underline{i}$

$\underline{a}_B = \underline{\omega} \times (\underline{\omega} \times \underline{r}_{OB}) = (0.5)^2(0.12)(-\underline{j}) = -0.03\underline{j}$ m/s²

$(\underline{a}_{A/B})_n = \underline{\omega}_{AB} \times (\underline{\omega}_{AB} \times \underline{r}_{BA}) = (0.214)^2(\underline{k} \times [\underline{k} \times \{0.24\underline{i} + 0.04\underline{j}\}])$

$= 0.0459(-0.24\underline{i} - 0.04\underline{j})$ m/s²

$(\underline{a}_{A/B})_t = \alpha_{AB}\underline{k} \times \underline{r}_{BA} = \alpha_{AB}\underline{k} \times (0.24\underline{i} + 0.04\underline{j}) = \alpha_{AB}(0.24\underline{j} - 0.04\underline{i})$

Substitute terms into Eq. (a) & equate separately $\underline{i}$ & $\underline{j}$ coefficients & get $\alpha_{AB} - 4\alpha_{CA} = 0.2755$

$2\alpha_{AB} - \alpha_{CA} = 0.02041$

Solve & get $\alpha_{CA} = -0.0758$ rad/s², $\alpha_{AB} = -0.0277$ rad/s²

$\underline{\alpha}_{CA} = \underline{-0.0758\,\underline{k}\ \text{rad/s}^2}$

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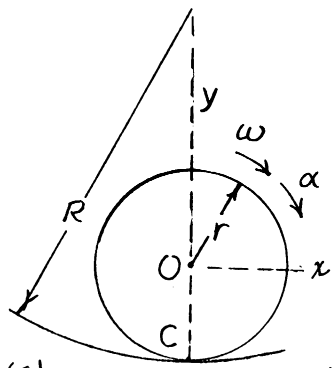
$$\underline{a}_C = \underline{a}_O + (\underline{a}_{C/O})_n + (\underline{a}_{C/O})_t$$



(a)

$(\underline{a}_O)_n = \dfrac{v_O^2}{R-r}\underline{j} = \dfrac{r^2\omega^2}{R-r}\underline{j}$

$(\underline{a}_O)_t = r\alpha\underline{i}$

$(\underline{a}_{C/O})_n = r\omega^2\underline{j}$

$(\underline{a}_{C/O})_t = -r\alpha\underline{i}$

Add & get $\underline{a}_C = \dfrac{r^2\omega^2}{R-r}\underline{j} + r\omega^2\underline{j} = \dfrac{r\omega^2}{1-r/R}\underline{j}$

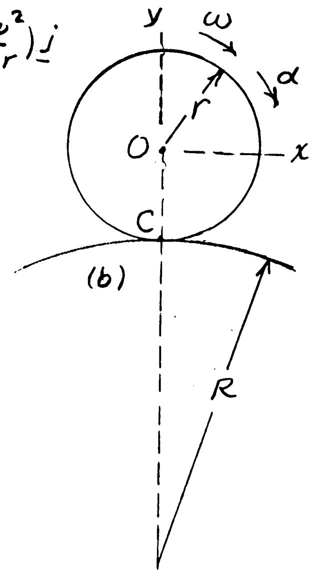
(b) $(\underline{a}_O)_n = \dfrac{v_O^2}{R+r}(-\underline{j})$; $(\underline{a}_O)_t = r\alpha\underline{i}$

$(\underline{a}_{C/O})_n = r\omega^2\underline{j}$; $(\underline{a}_{C/O})_t = -r\alpha\underline{i}$

Add & get

$\underline{a}_C = \left(r\omega^2 - \dfrac{r^2\omega^2}{R+r}\right)\underline{j}$

$\underline{a}_C = \dfrac{r\omega^2}{1+\frac{r}{R}}\underline{j}$



(b)

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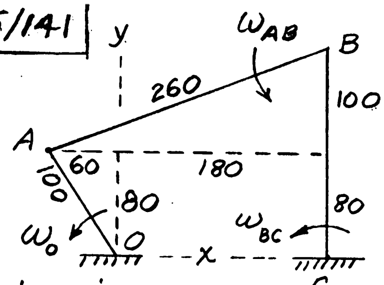


Dim. in mm

$\underline{a}_B = \underline{a}_A + \underline{a}_{B/A}$

$\underline{a}_B = \underline{\omega}_{BC} \times (\underline{\omega}_{BC} \times \underline{r}_{B/C}) + \underline{\alpha}_{BC} \times \underline{r}_{B/C}$

$= 5.83\underline{k} \times (5.83\underline{k} \times 0.18\underline{j})$

$+ \alpha_{BC}\underline{k} \times 0.18\underline{j} \quad m/s^2$

$= -6.125\underline{j} - 0.18\alpha_{BC}\underline{i} \quad m/s^2$

$\underline{a}_A = \underline{\omega}_0 \times (\underline{\omega}_0 \times \underline{r}_{A/O}) = 10\underline{k} \times (10\underline{k} \times [-0.06\underline{i} + 0.08\underline{j}])$

$= 6\underline{i} - 8\underline{j} \ m/s^2 \qquad (\underline{\alpha}_{OA} = 0)$

$(a_{B/A})_n = \underline{\omega}_{AB} \times (\underline{\omega}_{AB} \times \underline{r}_{B/A}) = 2.5\underline{k} \times (2.5\underline{k} \times [0.24\underline{i} + 0.1\underline{j}])$

$= -1.5\underline{i} - 0.625\underline{j} \ m/s^2$

$(a_{B/A})_t = \alpha_{AB}\underline{k} \times (0.24\underline{i} + 0.1\underline{j}) = -0.1\alpha_{AB}\underline{i} + 0.24\alpha_{AB}\underline{j}$

Substitute in accel. equation & equate coefficients & get

$$\left.\begin{aligned} -0.18\alpha_{BC} &= 6 - 1.5 - 0.1\alpha_{AB} \\ -6.125 &= -8 - 0.625 + 0.24\alpha_{AB} \end{aligned}\right\} \quad \text{Sol. is} \quad \underline{\alpha}_{AB} = \underline{10.42\underline{k} \ rad/s^2}$$

$(\underline{\alpha}_{BC} = -19.21\underline{k} \ rad/s^2)$

$\underline{5/142}$ Using C as the instant. center for AB gives

$v_B = 0.150(40) = 6 \text{ m/s}, \quad \omega_{BC} = \frac{6}{0.15} = 40 \text{ rad/s}$

$v_A = 0.2(40) = 8 \text{ m/s}, \quad \omega_{AO} = \frac{8}{0.1} = 80 \text{ rad/s}$

$(\underline{a}_A)_n + (\underline{a}_A)_t = (\underline{a}_B)_n + (\underline{a}_B)_t + (\underline{a}_{A/B})_n + (\underline{a}_{A/B})_t$

$(\underline{a}_A)_n = 0.1(80)^2(-\underline{i}) = -640\underline{i} \text{ m/s}^2$

$(\underline{a}_B)_n = 0.150(40)^2(-\underline{j}) = -240\underline{j} \text{ m/s}^2$

$(\underline{a}_{A/B})_n = \underline{\omega}_{AB} \times (\underline{\omega}_{AB} \times \underline{r}_{A/B}) = 40\underline{k} \times (40\underline{k} \times [0.2\underline{i} - 0.15\underline{j}])$

$= -320\underline{i} + 240\underline{j} \text{ m/s}^2$

$(\underline{a}_{A/B})_t = \underline{\alpha}_{AB} \times \underline{r}_{A/B} = 0$ for ω_{AB} const.

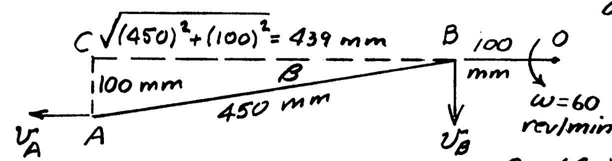
Substitute & equate $\underline{i}$ & $\underline{j}$ coefficients & get

$(\underline{a}_A)_t = -240\underline{j} + 240\underline{j} = 0$ so $\underline{\alpha_{OA} = 0}$

$(\underline{a}_B)_t = -320\underline{i} \text{ m/s}^2, \ (\underline{a}_D)_n = \frac{225}{150}(240)(-\underline{j}) = -360\underline{j} \text{ m/s}^2$

$(\underline{a}_D)_t = \frac{225}{150}(320)(-\underline{i}) = -480\underline{i} \text{ m/s}^2, \ \underline{a}_D = \underline{-120(4\underline{i} + 3\underline{j}) \text{ m/s}^2}$

5/143 C = instantaneous center of zero velocity for AB

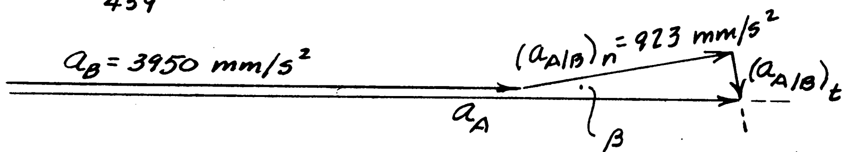


$$\omega_{AB} = \frac{v_B}{\overline{CB}} = \frac{\overline{OB}\,\omega}{\overline{CB}} = \frac{100}{439}\,\frac{60(2\pi)}{60} = 1.432\ \frac{\text{rad}}{\text{s}}$$

$$a_B = (a_B)_n = 100(2\pi)^2 = 3950\ \text{mm/s}^2$$

$$\mathbf{a}_A = \mathbf{a}_B + (\mathbf{a}_{A/B})_n + (\mathbf{a}_{A/B})_t\,;\qquad (a_{A/B})_n = 450(1.432)^2 = 923\ \text{mm/s}^2$$

$$\beta = \tan^{-1}\frac{100}{439} = 12.82°$$



$$(a_{A/B})_t = 923 \tan 12.82 = 923\,\frac{100}{439} = 210\ \text{mm/s}^2$$

$$\alpha_{AB} = \frac{(a_{A/B})_t}{\overline{AB}} = \frac{210}{450} = \underline{0.467\ \text{rad/s}^2\ \text{CCW}}$$

$$a_A = 3950 + 923\cos 12.82° + 210 \sin 12.82° = 4890\ \text{mm/s}^2$$

$$\text{or}\quad \underline{a_A = 4.89\ \text{m/s}^2}$$

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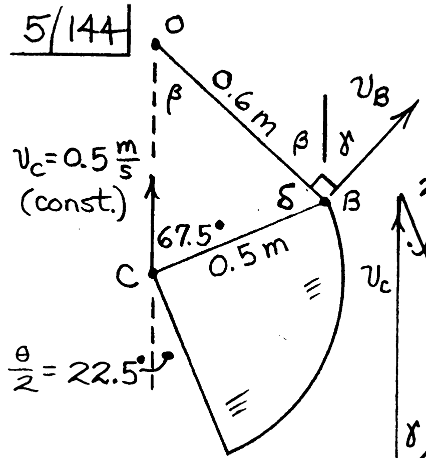
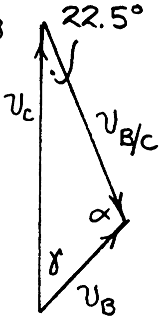


$$\frac{0.5}{\sin\beta} = \frac{0.6}{\sin 67.5°},\quad \beta = 50.3°$$

Then $\delta = 180° - 50.3° - 67.5°$
$= 62.2°$



$\gamma = 90 - \beta = 39.7°$

$\alpha = 180 - \gamma - 22.5° = 117.8°$

$$\frac{v_{B/C}}{\sin 39.7°} = \frac{v_C}{\sin 117.8°}$$

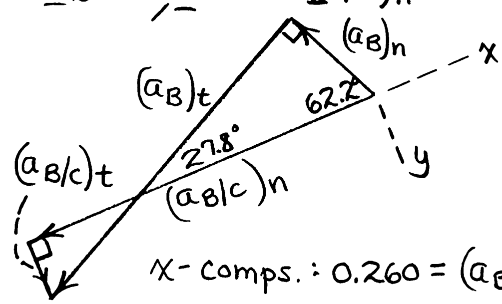
$v_{B/C} = 0.361$ m/s

$$\omega_{BC} = \frac{v_{B/C}}{\overline{BC}} = 0.722\ \frac{\text{rad}}{\text{s}}$$

$$\frac{v_B}{\sin 22.5°} = \frac{v_C}{\sin 117.8°},\quad v_B = 0.216 \text{ m/s}$$

$$(a_B)_n = v_B^2/r = 0.216^2/0.6 = 0.0780 \text{ m/s}^2$$

$$\underline{a}_B = \underline{a}_C^{\nearrow 0} + (\underline{a}_{B/C})_n + (\underline{a}_{B/C})_t$$



$(a_{B/C})_n = 0.5(0.722)^2$
$= 0.260$ m/s^2

x-comps.: $0.260 = (a_B)_t \cos 27.8° + 0.078 \cos 62.2°$

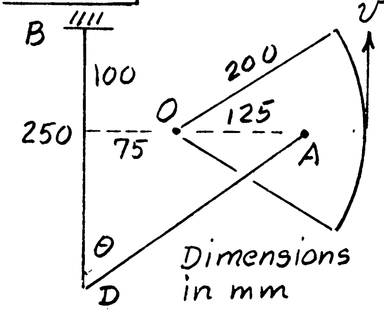
$(a_B)_t = 0.253$ m/s^2

y-comps: $0.253 \sin 27.8° = 0.078 \sin 62.2° + (a_{B/C})_t$

$(a_{B/C})_t = 0.0493$ m/s^2

$$\alpha_{CB} = \frac{(a_{B/C})_t}{\overline{CB}} = \frac{0.0493}{0.5} = \underline{0.0986 \text{ rad/s}^2 \text{ CW}}$$

5/145 $v = 4\ m/s$ const.



From Prob. 5/112

$v_A = 2.5\ \frac{m}{s}$, $v_D = 1.875\ \frac{m}{s}$

$\omega_{AD} = 12.5\ rad/s$

$\theta = \tan^{-1}\frac{200}{150} = 53.1°$

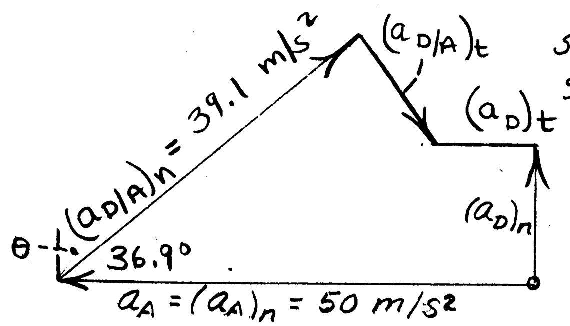
$\overline{AD} = 250\ mm$

$\underline{a}_D = \underline{a}_A + \underline{a}_{D/A}$

$(a_D)_n = v_D^2/\overline{BD} = \frac{1.875^2}{0.250} = 14.06\ m/s^2$

$a_A = (a_A)_n = v_A^2/\overline{OA} = \frac{2.5^2}{0.125} = 50\ m/s^2$

$(a_{D/A})_n = \overline{AD}\,\omega_{AD}^2 = 0.250(12.5)^2 = 39.1\ m/s^2$



Solution of polygon gives $(a_{D/A})_t = 11.72\ m/s^2$

$(a_D)_t = 11.72\ m/s^2$

$\alpha_{BD} = (a_D)_t/\overline{BD}$

$= \frac{11.72}{0.25} = 46.9\ \frac{rad}{s^2}$ CW

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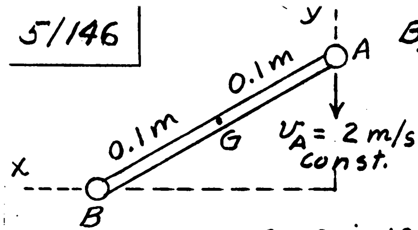


By inspection using instant center,

$\underline{\omega}_{AB} = \frac{2}{0.2}\,\frac{2}{\sqrt{3}}\,\underline{k} = \frac{20}{\sqrt{3}}\,\underline{k}$ rad/s

$\underline{a}_B = \underline{a}_A + (\underline{a}_{B/A})_n + (\underline{a}_{B/A})_t\,,\ \underline{a}_A = \underline{0}$

Vector algebra: $\underline{a}_B = a_B\underline{i}\,,\ (\underline{a}_{B/A})_n = \underline{\omega}_{AB}\times(\underline{\omega}_{AB}\times\underline{r}_{AB})$

$$= \left(\frac{20}{\sqrt{3}}\right)^2\underline{k}\times\left[\underline{k}\times 0.2\left(\frac{\sqrt{3}}{2}\underline{i}-\frac{1}{2}\underline{j}\right)\right]$$

$$= \frac{40}{3}(-\sqrt{3}\,\underline{i}+\underline{j})\ \text{m/s}^2$$

$$(\underline{a}_{B/A})_t = \underline{\alpha}_{AB}\times\underline{r}_{AB} = \alpha_{AB}\underline{k}\times 0.2\left(\frac{\sqrt{3}}{2}\underline{i}-\frac{1}{2}\underline{j}\right)$$

$$= \frac{\alpha_{AB}}{10}(\sqrt{3}\,\underline{j}+\underline{i})$$

Thus $a_B\underline{i} = \underline{0} + \frac{40}{3}(-\sqrt{3}\,\underline{i}+\underline{j}) + \frac{\alpha_{AB}}{10}(\sqrt{3}\,\underline{j}+\underline{i})$

so $a_B = -\frac{40}{\sqrt{3}} + \frac{\alpha_{AB}}{10}$ & $0 = \frac{40}{3} + \frac{\alpha_{AB}}{10}\sqrt{3}$

giving $\alpha_{AB} = -\frac{400}{3\sqrt{3}}$ rad/s² & $a_B = -\frac{160}{3\sqrt{3}}$ m/s²

$\underline{a}_G = \underline{a}_A + \underline{a}_{G/A} = 0 + \frac{1}{2}(\underline{a}_{B/A})_n + \frac{1}{2}(\underline{a}_{B/A})_t = \frac{20}{3}(-\sqrt{3}\,\underline{i}+\underline{j}) - \frac{20}{3\sqrt{3}}(\sqrt{3}\,\underline{j}+\underline{i})$

$$= -\frac{80}{3\sqrt{3}}\underline{i} = \underline{-15.40\,\underline{i}\ \text{m/s}^2}$$

Vector geometry:

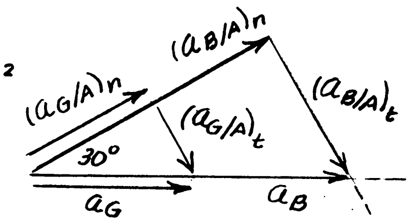
$(a_{B/A})_n = 0.2(20/\sqrt{3})^2 = 80/3$ m/s²

$(a_{B/A})_t = 0.2\,\alpha_{AB}$

$(a_{G/A})_n = \frac{1}{2}(a_{B/A})_n = 40/3$ m/s²

$(a_{G/A})_t = \frac{1}{2}(a_{B/A})_t = 0.1\,\alpha_{AB}$

$a_G = \frac{40}{3}\frac{2}{\sqrt{3}} = \underline{15.40\ \text{m/s}^2}$



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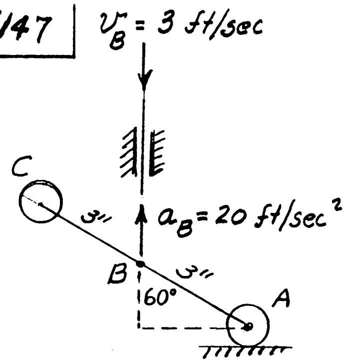


From solution to Prob. 5/78

$v_{A/B} = 3.46$ ft/sec

$\omega_{AB} = 3.46/\frac{3}{12} = 13.86$ rad/sec

$\underline{a}_A = \underline{a}_B + (\underline{a}_{A/B})_n + (\underline{a}_{A/B})_t$

$(a_{A/B})_n = \frac{3}{12}(13.86)^2 = 48$ ft/sec²

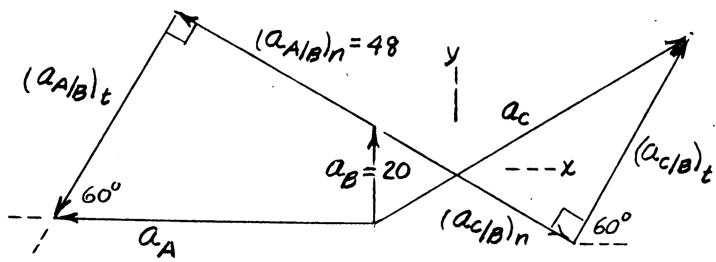
From diagram $(a_{A/B})_t = \dfrac{20+24}{\sin 60°}$

$= 50.8 \ \frac{ft}{sec^2}$

$= (a_{C/B})_t$

Also $(a_{C/B})_n$

$= (a_{A/B})_n = 48 \ \frac{ft}{sec^2}$



Components of a_C are

$(a_C)_x = 48\cos 30° + 50.8\cos 60° = 66.97 \ \frac{ft}{sec^2}$

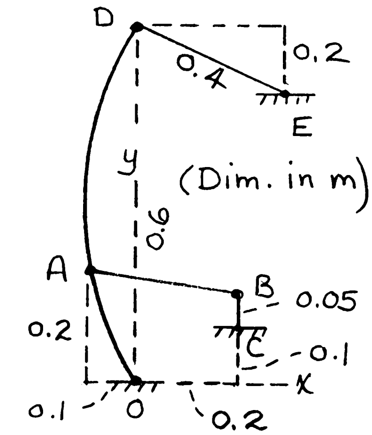
$(a_C)_y = 20 - 48\sin 30° + 50.8\sin 60° = 40 \ \frac{ft}{sec^2}$

$a_C = \sqrt{(66.97)^2 + (40)^2}$

$= \underline{78.0 \text{ ft/sec}^2}$

▶5/148

$\underline{\omega}_{CB} = -\pi \underline{k}$ rad/s , $\underline{r}_{OA} = -0.1\underline{i} + 0.2\underline{j}$ m



$\underline{r}_{CB} = 0.05\underline{j}$ m, $\underline{r}_{BA} = -0.3\underline{i} + 0.05\underline{j}$ m

$\underline{r}_{OD} = 0.6\underline{j}$ m , $\underline{v}_B = 0.05\pi\,\underline{i}$ m/s

$\underline{v}_{A/B} = \omega_{AB}\underline{k} \times (-0.3\underline{i} + 0.05\underline{j})$

$= -0.3\,\omega_{AB}\,\underline{j} - 0.05\,\omega_{AB}\,\underline{i}$

$\underline{v}_A = \omega_{OA}\underline{k} \times (-0.1\underline{i} + 0.2\underline{j})$

$= -0.1\,\omega_{OA}\,\underline{j} - 0.2\,\omega_{OA}\,\underline{i}$

From $\underline{v}_A = \underline{v}_B + \underline{v}_{A/B}$:

$-0.1\,\omega_{OA}\,\underline{j} - 0.2\,\omega_{OA}\,\underline{i} = 0.05\pi\,\underline{i} - 0.3\,\omega_{AB}\,\underline{j} - 0.05\,\omega_{AB}\,\underline{i}$

Equate like coefficients: $\begin{cases} \underline{\omega}_{AB} = -0.286\underline{k} \text{ rad/s} \\ \underline{\omega}_{OA} = -0.857\underline{k} \text{ rad/s} \end{cases}$

Now, $\underline{a}_A = \underline{a}_B + (\underline{a}_{A/B})_n + (\underline{a}_{A/B})_t$ $*$

$\underline{a}_A = -\omega_{OA}^2 \underline{r}_{OA} + \underline{\alpha}_{OA} \times \underline{r}_{OA}$

$= 0.734(0.1\underline{i} - 0.2\underline{j}) + \alpha_{OA}(-0.1\underline{j} - 0.2\underline{i})$

$(\underline{a}_{A/B})_n = -\omega_{AB}^2 \underline{r}_{BA} = 0.0816(0.3\underline{i} - 0.05\underline{j})$ m/s^2

$(\underline{a}_{A/B})_t = \underline{\alpha}_{AB} \times \underline{r}_{BA} = \alpha_{AB}(-0.3\underline{j} - 0.05\underline{i})$ m/s^2

Substitute into $*$, equate like coefficients, & obtain

$\alpha_{OA} = -0.0519$ rad/s^2, $\alpha_{AB} = -1.186$ rad/s^2

$\underline{a}_E = (\underline{a}_D)_n + (\underline{a}_D)_t + (\underline{a}_{E/D})_n + (\underline{a}_{E/D})_t$, $(\underline{a}_{E/D})_n = \underline{0}$ since $\omega_{DE} = 0$

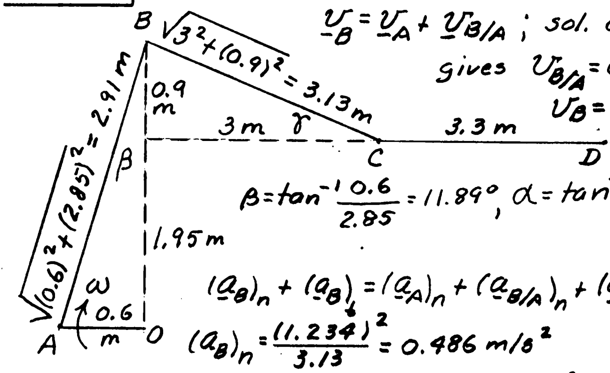
$a_E\,\underline{i} = -0.6(0.857)^2\underline{j} + 0.6(0.0519)\underline{i} + \alpha_{ED}\underline{k} \times (\sqrt{0.12}\,\underline{i} - 0.2\underline{j})$

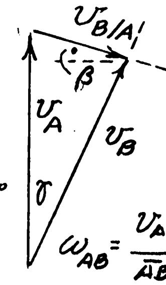
Solve to obtain $\alpha_{ED} = 1.272$ rad/s^2, $\underline{a_E = 0.285 \text{ m/s}^2}$

▶5/149 $\omega = \frac{2\pi}{3} = 2.09$ rad/s, $v_A = 0.6(2.09) = 1.257$ m/s

$\underline{v}_B = \underline{v}_A + \underline{v}_{B/A}$; sol. of polygon gives $v_{B/A} = 0.362$ m/s

$v_B = 1.234$ m/s





$\beta = \tan^{-1}\frac{0.6}{2.85} = 11.89°$, $\alpha = \tan^{-1}\frac{0.9}{3} = 16.70°$

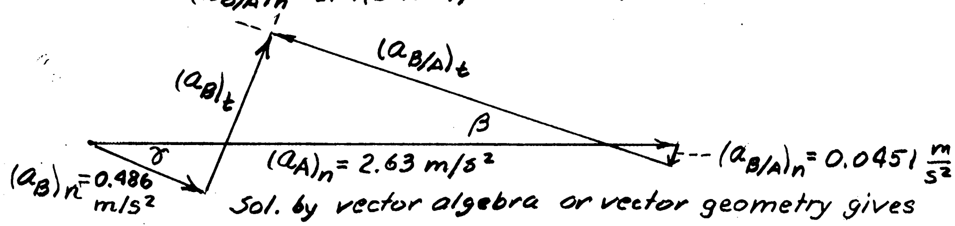
$\omega_{AB} = \frac{v_{A/B}}{\overline{AB}} = \frac{0.362}{2.91} = 0.1244$ rad/s

$(\underline{a}_B)_n + (\underline{a}_B)_t = (\underline{a}_A)_n + (\underline{a}_{B/A})_n + (\underline{a}_{B/A})_t$

$(a_B)_n = \frac{(1.234)^2}{3.13} = 0.486 \text{ m/s}^2$

$(a_A)_n = 0.6(2.09)^2 = 2.63 \text{ m/s}^2$

$(a_{B/A})_n = 2.91(0.1244)^2 = 0.0451 \text{ m/s}^2$

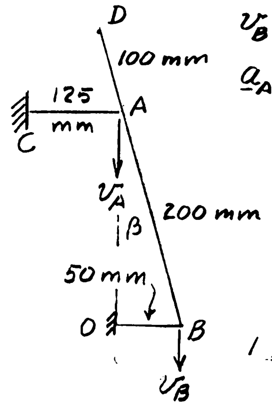


Sol. by vector algebra or vector geometry gives

$(a_B)_t = 0.539 \text{ m/s}^2$, $a_D = \overline{CD}\alpha_{CD} = \overline{CD}\alpha_{BC} = 3.3\frac{0.539}{3.13} = \underline{0.568 \text{ m/s}^2}$

▶5/150 $v_B = r\omega = 50\,\frac{(120)2\pi}{60} = 200\pi = 628$ mm/s

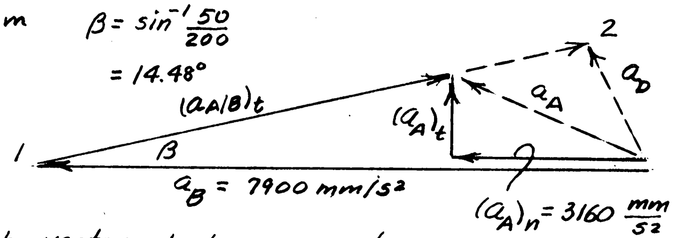
v_B parallel to v_A so $v_A = v_B$ & $\omega_{AB} = 0$



$\underline{a}_A = \underline{a}_B + \underline{a}_{A/B}$; $a_B = (a_B)_n = \frac{v_B^2}{r} = \frac{(200\pi)^2}{50} = 7900$ mm/s²

$(a_A)_n = \frac{(200\pi)^2}{125} = 3160$ mm/s²

$\beta = \sin^{-1}\frac{50}{200} = 14.48°$



From solution by vector algebra or vector geometry, $(a_{A/B})_t = 4890$ mm/s²

$\underline{a}_D = \underline{a}_B + \underline{a}_{D/B}$; $a_{D/B} = (a_{D/B})_t = \frac{\overline{BD}}{\overline{BA}}(a_{A/B})_t = \frac{300}{200}(4890) = 7340\ \frac{\text{mm}}{\text{s}^2}$

(= 1-2)

$a_D = \sqrt{(7340\sin 14.48°)^2 + (7900 - 7340\cos 14.48°)^2}$

$= \underline{1997\ \text{mm/s}^2}$

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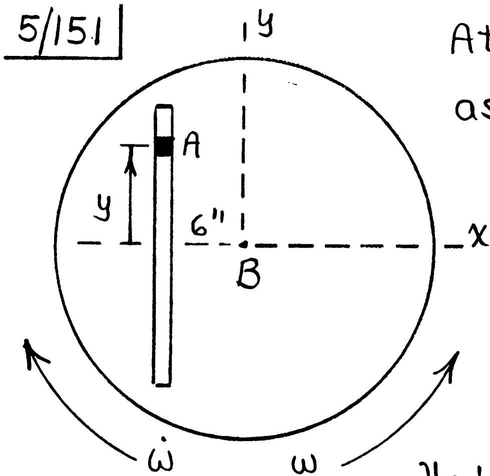

Attach Bxy to disk as shown. In Eqs. 5/12 & 5/14:

$\underline{v}_B = \underline{a}_B = \underline{0}$

$\underline{\omega} = 5\underline{k}\ \frac{\text{rad}}{\text{sec}}$, $\underline{\dot{\omega}} = -3\underline{k}\ \frac{\text{rad}}{\text{sec}^2}$

$\underline{r} = -0.5\underline{i} + 0.667\underline{j}$ ft

$\underline{v}_{rel} = -2\underline{j}$ ft/sec

$\underline{a}_{rel} = 2.5\underline{j}$ ft/sec^2

Eq. 5/12, $\underline{v}_A = \underline{v}_B + \underline{\omega} \times \underline{r} + \underline{v}_{rel}$, yields

$$\underline{v}_A = -3.33\underline{i} - 4.5\underline{j} \quad \text{ft/sec}$$

Eq. 5/14, $\underline{a}_A = \underline{a}_B + \underline{\alpha} \times \underline{r} + \underline{\omega} \times (\underline{\omega} \times \underline{r}) + 2\underline{\omega} \times \underline{v}_{rel} + \underline{a}_{rel}$,

yields $\quad \underline{a}_A = 34.5\underline{i} - 12.67\underline{j}$ ft/sec^2

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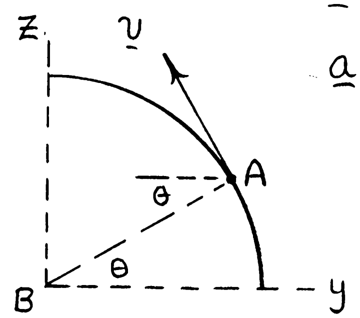


$$\underline{v} = v(-\sin\theta\,\underline{j} + \cos\theta\,\underline{k})$$

$$\underline{a}_{cor} = 2\underline{\omega} \times \underline{v}$$

$$= 2\Omega\underline{k} \times v(-\sin\theta\,\underline{j} + \cos\theta\,\underline{k})$$

$$= \underline{2\Omega v \sin\theta\,\underline{i}} \quad \text{(west)}$$

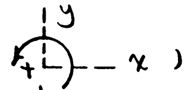
For $v = 500$ km/h,

(a) Equator, $\theta = 0$: $\underline{a_{cor} = 0}$

(b) North pole, $\theta = 90°$: $a_{cor} = 2(7.292 \cdot 10^{-5})\dfrac{500}{3.6}$

$$= \underline{0.0203 \text{ m/s}^2}$$

The track provides the necessary westward acceleration so that the velocity vector is properly rotated and reduced in magnitude.

5/153 For the coordinates  , the no-slip constraints are $v_0 = -r\omega$ & $a_0 = -r\alpha$. So

$$\omega = -\frac{v_0}{r} = -\frac{-3}{0.30} = 10 \text{ rad/s}$$

$$\alpha = -\frac{a_0}{r} = -\frac{5}{0.30} = -16.67 \text{ rad/s}^2$$

Use the frame Oxy as disk-fixed.

(5/12): $\underline{v}_A = \underline{v}_0 + \underline{\omega} \times \underline{r} + \underline{v}_{rel}$

(5/14): $\underline{a}_A = \underline{a}_0 + \underline{\alpha} \times \underline{r} + \underline{\omega} \times (\underline{\omega} \times \underline{r}) + 2\underline{\omega} \times \underline{v}_{rel} + \underline{a}_{rel}$

Ingredients: $\begin{cases} \underline{v}_0 = -3\underline{i} \text{ m/s} \\ \underline{a}_0 = 5\underline{i} \text{ m/s}^2 \\ \underline{\omega} = 10\underline{k} \text{ rad/s} \\ \underline{\alpha} = -16.67\underline{k} \text{ rad/s}^2 \end{cases}$

$\underline{r} = 0.24\underline{j}$ m

$\underline{v}_{rel} = 2\underline{i}$ m/s

$\underline{a}_{rel} = -7\underline{i} - \frac{2^2}{0.24}\underline{j}$

$= -7\underline{i} - 16.67\underline{j}$ m/s^2

Substitute into (5/12) & (5/14) & simplify:

$\underline{v}_A = -3.4\underline{i}$ m/s

$\underline{a}_A = 2\underline{i} - 0.667\underline{j}$ m/s^2

5/154 Let the Bxy axes shown in the figure be attached to the disk of the merry-go-round.

Eq. 5/12 : $\underline{v}_A = \underline{v}_B + \underline{\omega} \times \underline{r} + \underline{v}_{rel}$

$$\underline{0} = \underline{0} + \Omega \underline{k} \times d\underline{i} + \underline{v}_{rel}$$

$$\underline{\underline{v}_{rel} = -\Omega d \underline{j}}$$

This result does not depend on the location of P.

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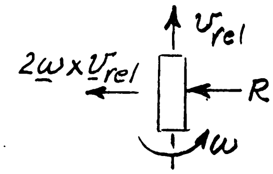
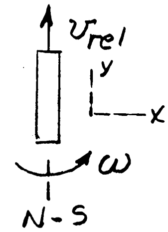
(a) North pole



Only horizontal component of acceleration is $|2\underline{\omega} \times \underline{v}_{rel}| = 2(0.7292)(10^{-4})(15) = 0.00219 \text{ m/s}^2$

$\Sigma F = ma;$ $R = 50\,000\,(0.00219) = \underline{109.4 \text{ N}}$

(b) Equator



$\omega_{xy} = 0$ where x-y is horizontal plane

so $2\underline{\omega} \times \underline{v}_{rel} = 0$ & there is no other horizontal acceleration so $\underline{R = 0}$

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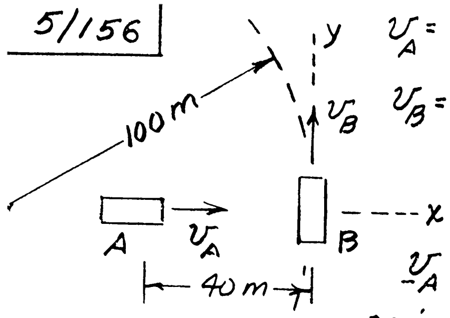


$v_A = \frac{72 \times 1000}{3600} = 20 \text{ m/s}$

$v_B = \frac{54 \times 1000}{3600} = 15 \text{ m/s}$

$\omega = \omega_B = \frac{15}{100} = 0.15 \text{ rad/s}$

$\underline{v}_A = \underline{v}_B + \underline{\omega} \times \underline{r} + \underline{v}_{rel}$

$20\underline{i} = 15\underline{j} + 0.15\underline{k} \times (-40\underline{i}) + \underline{v}_{rel}$

$\underline{v}_{rel} = 20\underline{i} - 9\underline{j}$ m/s

$(v_{rel})_{\text{rotating axes}}$ differs from $(v_{rel})_{\text{translating axes}}$ by $\underline{\omega} \times \underline{r}$

5/157 | From Prob. 5/156 $\underline{v}_{rel} = 20\underline{i} - 9\underline{j}$ m/s

$\underline{\omega} = 0.15\underline{k}$ rad/s

$$\underline{a}_A = \underline{a}_B + \underline{\omega}\times(\underline{\omega}\times\underline{r}) + \dot{\underline{\omega}}\times\underline{r} + 2\underline{\omega}\times\underline{v}_{rel} + \underline{a}_{rel}$$

$$\underline{a}_A = \underline{0}, \quad \underline{a}_B = \frac{v_B^2}{R}(-\underline{i}) = -\frac{15^2}{100}\underline{i} = -2.25\underline{i} \text{ m/s}^2$$

$$\underline{\omega}\times(\underline{\omega}\times\underline{r}) = 0.15\underline{k}\times(0.15\underline{k}\times[-40\underline{i}] = 0.90\underline{i} \text{ m/s}^2$$

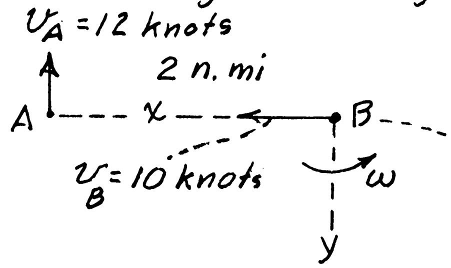
$$\dot{\underline{\omega}}\times\underline{r} = \underline{0}$$

$$2\underline{\omega}\times\underline{v}_{rel} = 2(0.15\underline{k})\times(20\underline{i} - 9\underline{j}) = 2.7\underline{i} + 6\underline{j} \text{ m/s}^2$$

Thus $\underline{0} = -2.25\underline{i} + 0.90\underline{i} + \underline{0} + 2.7\underline{i} + 6\underline{j} + \underline{a}_{rel}$

$$\underline{a}_{rel} = \underline{-1.35\underline{i} - 6\underline{j} \text{ m/s}^2}$$

5/158 | x-y axes attached to B and have the angular velocity $\omega = \frac{10}{180}\pi = 0.1745$ rad/min



$\underline{v}_A = \underline{v}_B + \underline{\omega} \times \underline{r} + \underline{v}_{rel}$

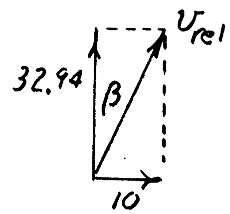
$\underline{\omega} \times \underline{r} = 0.1745\underline{k} \times 2\underline{i}\ (60)$
$= 20.94\underline{j}$ knots

Thus $-12\underline{j} = 10\underline{i} + 20.94\underline{j} + \underline{v}_{rel}$

$\underline{v}_{rel} = -10\underline{i} - 32.94\underline{j}$ knots

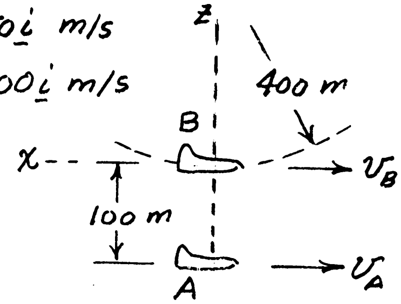
$|\underline{v}_{rel}| = \sqrt{10^2 + 32.94^2} = \underline{34.4 \text{ knots}}$

$\beta = \tan^{-1}\frac{10}{32.94} = \underline{16.89°}$



5/159 $\underline{v}_B = \frac{540(1000)}{3600}(-\underline{i}) = -150\underline{i}$ m/s

$\underline{v}_A = \frac{360(1000)}{3600}(-\underline{i}) = -100\underline{i}$ m/s



$\underline{v}_A = \underline{v}_B + \underline{\omega}\times\underline{r} + \underline{v}_{rel}$

$\underline{\omega} = \frac{v_B}{\rho}\underline{j} = \frac{150}{400}\underline{j} = \frac{3}{8}\underline{j}$ rad/s

$\underline{r} = -100\underline{k}$ m

so $-100\underline{i} = -150\underline{i} + \frac{3}{8}\underline{j}\times(-100\underline{k}) + \underline{v}_{rel}$, $\underline{v}_{rel} = 87.5\underline{i}$ m/s

$\underline{a}_A = \underline{a}_B + \dot{\underline{\omega}}\times\underline{r} + \underline{\omega}\times(\underline{\omega}\times\underline{r}) + 2\underline{\omega}\times\underline{v}_{rel} + \underline{a}_{rel}$

where $\underline{a}_A = \underline{0}$, $\dot{\underline{\omega}} = \underline{0}$, $\underline{a}_B = \frac{v_B^2}{\rho}\underline{k} = \frac{150^2}{400}\underline{k} = 56.2\underline{k}$ m/s²

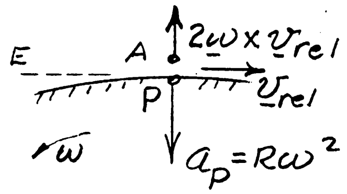
$\underline{\omega}\times(\underline{\omega}\times\underline{r}) = \frac{3}{8}\underline{j}\times(-37.5\underline{i}) = 14.06\underline{k}$ m/s²

$2\underline{\omega}\times\underline{v}_{rel} = 2\left(\frac{3}{8}\underline{j}\right)\times 87.5\underline{i} = -65.6\underline{k}$ m/s²

Thus $\underline{0} = 56.2\underline{k} + \underline{0} + 14.06\underline{k} - 65.6\underline{k} + \underline{a}_{rel}$

$\underline{a}_{rel} = -4.69\underline{k}$ m/s²

5/160 Let P be a point on the road coincident with A. $\underline{a}_A = \underline{a}_P + 2\underline{\omega} \times \underline{v}_{rel} + \underline{a}_{rel}$



For zero vertical acceleration,

$|2\underline{\omega} \times \underline{v}_{rel}| = R\omega^2$

$v_{rel} = \frac{1}{2} R\omega$

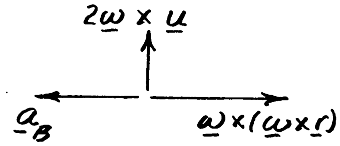
For $R = 6378$ km, $\omega = 0.7292(10^{-4})$ rad/s,

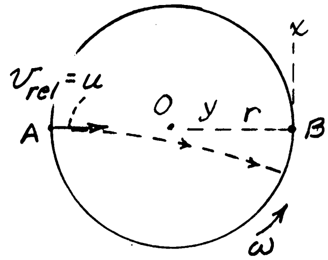
$v_{rel} = \frac{1}{2}(6378)(10^3)(0.7292 \times 10^{-4}) = 233$ m/s

or $v_{rel} = 233(3.6) = \underline{837 \text{ km/h}}$

5/161 (accel. of ball)$_{xy}$ $= \underline{a} = \underline{a}_B + \underline{\omega}\times(\underline{\omega}\times\underline{r}) + 2\underline{\omega}\times\underline{v}_{rel} + \underline{a}_{rel}$

Once ball leaves A, its horiz. accel. is zero, $\underline{a} = \underline{0}$





$\underline{a}_B = r\omega^2\underline{j}$

$\underline{\omega}\times(\underline{\omega}\times``\underline{r}") = 2r\omega^2(-\underline{j})$

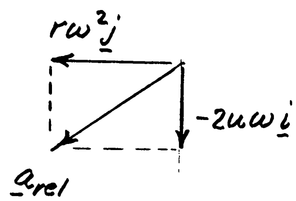
where $``r" = \overrightarrow{BA} = 2r$

$2\underline{\omega}\times\underline{v}_{rel} = 2\omega\underline{k}\times u(-\underline{j}) = 2u\omega\underline{i}$

$\underline{a}_{rel} = -r\omega^2\underline{j} + 2r\omega^2\underline{j} - 2u\omega\underline{i}$

$= r\omega^2\underline{j} - 2u\omega\underline{i}$

$a_{rel} = \omega\sqrt{r^2\omega^2 + 4u^2}$



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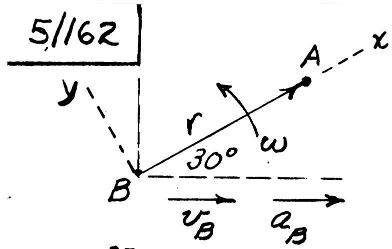

$\underline{r} = (20+b)\underline{i} = 25\underline{i}$ ft

$\underline{v}_{rel} = \dot{r}\underline{i} = 2\underline{i}$ ft/sec

$\underline{a}_{rel} = \ddot{r}\underline{i} = -1\underline{i}$ ft/sec^2

$\underline{\omega} = \frac{10}{180}\pi\underline{k} \doteq 0.1745\underline{k}$ rad/sec

$\dot{\underline{\omega}} = \underline{0}$

$v_B = \frac{35}{30}44 = 51.3$ ft/sec , $\underline{v}_B = 51.3(\underline{i}\cos 30° - \underline{j}\sin 30°)$

$a_B = -10$ ft/sec^2 $\quad \underline{a}_B = -10(\underline{i}\cos 30° - \underline{j}\sin 30°)$

Eq. 5/14, $\underline{a}_A = \underline{a}_B + \dot{\underline{\omega}}\times\underline{r} + \underline{\omega}\times(\underline{\omega}\times\underline{r}) + 2\underline{\omega}\times\underline{v}_{rel} + \underline{a}_{rel}$

so $\underline{a}_A = -10(0.866\underline{i} - 0.5\underline{j}) + \underline{0} + (0.1745)^2\underline{k}\times(\underline{k}\times 25\underline{i})$

$\quad + 2(0.1745\underline{k})\times 2\underline{i} - 1\underline{i}$

(b) $\underline{a}_A = -10.42\underline{i} + 5.70\underline{j}$ ft/sec^2 with respect to ground

(a) $\underline{a}_A - \underline{a}_B = -10.42\underline{i} + 5.70\underline{j} - (-10)(0.866\underline{i} - 0.5\underline{j})$

$= -1.76\underline{i} + 0.70\underline{j}$ ft/sec^2 with respect to truck

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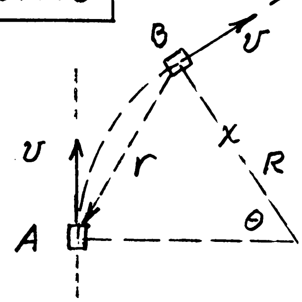


Attach x-y axes to B

$$\underline{v}_A = \underline{v}_B + \underline{\omega}\times\underline{r} + \underline{v}_{rel}$$

$$\underline{v}_A = -v\sin\theta\,\underline{i} + v\cos\theta\,\underline{j}$$

$$\underline{v}_B = v\underline{j}\ ,\quad \underline{\omega} = -\frac{v}{R}\underline{k}$$

$$|\underline{r}| = 2R\sin\frac{\theta}{2}$$

$$\underline{r} = 2R\sin\frac{\theta}{2}\left(\sin\frac{\theta}{2}\underline{i} - \cos\frac{\theta}{2}\underline{j}\right)$$

$$\underline{\omega}\times\underline{r} = -\frac{v}{R}\underline{k}\times 2R\sin\frac{\theta}{2}\left(\sin\frac{\theta}{2}\underline{i} - \cos\frac{\theta}{2}\underline{j}\right)$$

$$= -2v\sin^2\frac{\theta}{2}\underline{j} - 2v\sin\frac{\theta}{2}\cos\frac{\theta}{2}\underline{i}$$

$$= -v(1-\cos\theta)\underline{j} - v\sin\theta\,\underline{i}$$

Thus $-v\sin\theta\,\underline{i} + v\cos\theta\,\underline{j} = v\underline{j} - v(1-\cos\theta)\underline{j} - v\sin\theta\,\underline{j} + \underline{v}_{rel}$

so $\underline{\underline{v}_{rel} = \underline{0}}$

5/164 From Prob. 5/163 $\underline{v}_{rel} = \underline{0}$, $\underline{\omega} = -\frac{v}{R}\underline{k}$

$$\underline{\omega} \times \underline{r} = -v\sin\theta\,\underline{i} - v(1-\cos\theta)\underline{j}$$

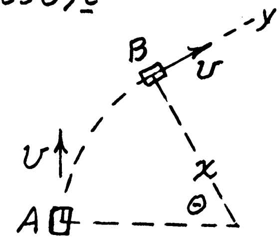
$$\underline{\omega} \times (\underline{\omega} \times \underline{r}) = -\frac{v}{R}\underline{k} \times \left[-v\sin\theta\,\underline{i} - v(1-\cos\theta)\underline{j}\right]$$

$$= \frac{v^2}{R}\sin\theta\,\underline{j} - \frac{v^2}{R}(1-\cos\theta)\underline{i}$$

$$\dot{\underline{\omega}} \times \underline{r} = \underline{0}$$

$$2\underline{\omega} \times \underline{v}_{rel} = \underline{0}$$

$$\underline{a}_B = \frac{v^2}{R}\underline{i}$$

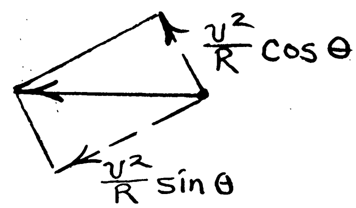


$$\underline{a}_A = \underline{a}_B + \underline{\omega} \times (\underline{\omega} \times \underline{r}) + \dot{\underline{\omega}} \times \underline{r} + 2\underline{\omega} \times \underline{v}_{rel} + \underline{a}_{rel}$$

so $\underline{0} = \frac{v^2}{R}\underline{i} + \frac{v^2}{R}\sin\theta\,\underline{j} - \frac{v^2}{R}(1-\cos\theta)\underline{i} + \underline{0} + \underline{0} + \underline{a}_{rel}$

$$\underline{\underline{a}_{rel} = -\frac{v^2}{R}(\cos\theta\,\underline{i} + \sin\theta\,\underline{j})}$$

$$a_{rel} = \frac{v^2}{R}$$



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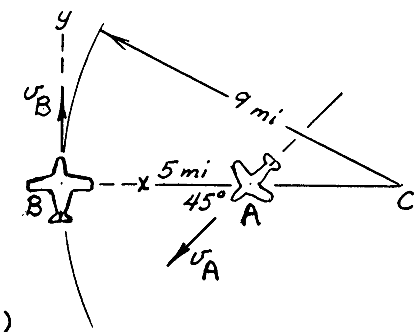
$v_B = 480\frac{44}{30} = 704$ ft/sec

$v_A = 360\frac{44}{30} = 528$ ft/sec



$\underline{v}_A = \underline{v}_B + \underline{\omega} \times \underline{r} + \underline{v}_{rel}$

Angular vel. of axes $= \underline{\omega} = \frac{v_B}{\rho}(-\underline{k})$

$= \frac{-704}{9 \times 5280}\underline{k} = -0.01481\underline{k}$ rad/sec

v_{rel} = vel. of A rel. to B

$\underline{r} = 5(5280)\underline{i} = 26{,}400\underline{i}$ ft

Thus $528(-0.707\underline{i} - 0.707\underline{j}) = 704\underline{j} - 0.01481\underline{k} \times 26{,}400\underline{i} + \underline{v}_{rel}$

$\underline{v}_{rel} = -373\underline{i} - 686\underline{j}$ ft/sec with $v_{rel} = 781$ ft/sec

or 533 mi/hr

5/166 | Refer to solution for Prob. 5/165 .

$\underline{a}_A = \underline{a}_B + \dot{\underline{\omega}} \times \underline{r} + \underline{\omega} \times (\underline{\omega} \times \underline{r}) + 2\underline{\omega} \times \underline{v}_{rel} + \underline{a}_{rel}$

$\underline{a}_A = \underline{0}, \ \underline{a}_B = \frac{v_B^2}{\rho}\underline{i} = \frac{704^2}{9 \times 5280}\underline{i} = 10.43\,\underline{i}$ ft/sec²

$\dot{\underline{\omega}} \times \underline{r} = \underline{0}$

$\underline{\omega} \times (\underline{\omega} \times \underline{r}) = -0.01481\,\underline{k} \times (-0.01481\,\underline{k} \times 26{,}400\,\underline{i}) = -5.79\,\underline{i}$ ft/sec²

$2\underline{\omega} \times \underline{v}_{rel} = 2(-0.1481\,\underline{k}) \times (-373\,\underline{i} - 686\,\underline{j}) = 11.05\,\underline{j} - 23.0\,\underline{i}$ ft/sec²

$\underline{a}_{rel} = \underline{0} - 10.43\,\underline{i} - \underline{0} + 5.79\,\underline{i} - 11.05\,\underline{j} + 20.3\,\underline{i} = \underline{15.69\,\underline{i} - 11.05\,\underline{j}}$ ft/sec²

where $\underline{a_{rel} = 19.19}$ ft/sec²

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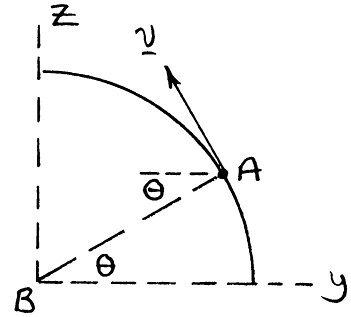


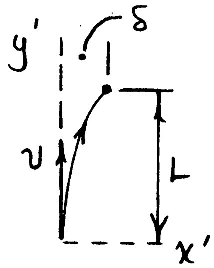
$\underline{v} = v(-\sin\theta\,\underline{j} + \cos\theta\,\underline{k})$

$\underline{a}_{cor} = 2\underline{\omega} \times \underline{v}$

$= 2\Omega\underline{k} \times v(-\sin\theta\,\underline{j} + \cos\theta\,\underline{k})$

$= 2\Omega v \sin\theta\,\underline{i}$ (west)

With no westward force mechanism available, the ball will drift to the east (relative to the ground) with an acceleration of magnitude a_{cor}.



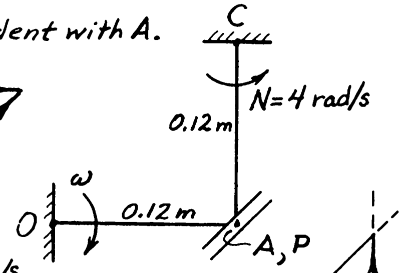
$a_{x'} = 2\Omega v \sin\theta$

$\delta = \frac{1}{2} a_{x'} t^2 = \frac{1}{2}(2\Omega v \sin\theta)\left(\frac{L}{v}\right)^2$

$= \underline{\frac{\Omega L^2}{v} \sin\theta}$ (assumes $\delta \ll L$)

With $\Omega = 7.292(10^{-5})$ rad/sec, $v = 15$ ft/sec, $L = 60$ ft, & $\theta = 40°$: $\delta = 0.01125$ ft
$\underline{(0.1350 \text{ in.})}$

5/168 | Let P = point on ODE coincident with A.



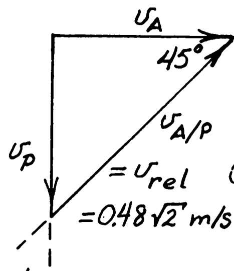
$\underline{v}_A = \underline{v}_P + \underline{v}_{A/P}$

$v_A = 0.12(4) = 0.48$ m/s

$v_P = 0.48$ m/s,

$\omega_{OP} = \omega = \dfrac{0.48}{0.12}$

$= \underline{4 \text{ rad/s CW}}$



$\underline{a}_A = \underline{a}_P + 2\underline{\omega}\times\underline{v}_{rel} + \underline{a}_{rel}$

$a_A = (a_A)_n = 0.12(4^2) = 1.92 \text{ m/s}^2 \uparrow$

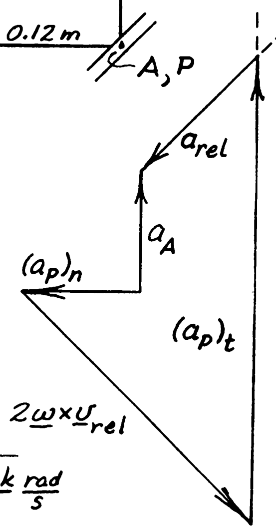
$|2\underline{\omega}\times\underline{v}_{rel}| = 2(4)(0.48\sqrt{2}) = 5.43 \text{ m/s}^2 \searrow$

$(a_P)_n = 0.12(4^2) = 1.92 \text{ m/s}^2 \leftarrow$

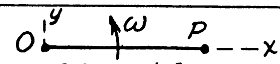
From diagram, $a_{rel} = 2.72$ m/s^2,

$(a_P)_t = 7.68$ m/s^2

$\alpha_{ODE} = \alpha = 7.68/0.12 = \underline{64.0 \text{ rad/s}^2 \text{ CCW}}$



Alternatively, with



$\underline{\omega} = -4\underline{k}\ \frac{\text{rad}}{\text{s}}$

$\underline{a}_A = (\underline{a}_A)_n = 0.12(4^2)\underline{j} = 1.92\underline{j}$ m/s^2

$(\underline{a}_P)_n = 0.12(4^2)(-\underline{i}) = -1.92\underline{i}$ m/s^2

$(\underline{a}_P)_t = \alpha\underline{k} \times 0.12\underline{i} = 0.12\alpha\underline{j}$

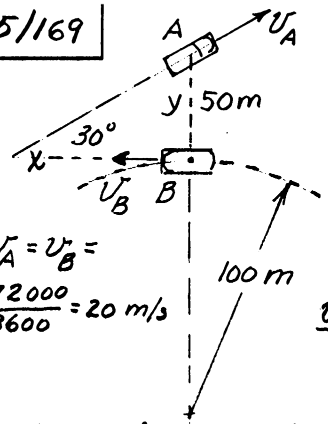
$2\underline{\omega}\times\underline{v}_{rel} = 2(-4\underline{k}) \times 0.48(\underline{i}+\underline{j}) = 3.84(\underline{i}-\underline{j})$ m/s^2

$\underline{a}_{rel} = a_{rel}\dfrac{1}{\sqrt{2}}(\underline{i}+\underline{j})$

Substitute: $\underline{i}$-terms give $a_{rel}/\sqrt{2} = -1.92$, $a_{rel} = -2.72$ m/s^2 so $a_{rel} = \swarrow$

$\underline{j}$-terms: $0.12\alpha = 1.92 + 3.84 + 1.92$, $\underline{\alpha = 64 \text{ rad/s}^2 \text{ CCW}}$

5/169



$v_A = v_B = \frac{72000}{3600} = 20$ m/s

$\underline{v}_A = \underline{v}_B + \underline{\omega} \times \underline{r} + \underline{v}_{rel}$

$\underline{v}_A = 20(-0.866\underline{i} + 0.5\underline{j})$ m/s

$\underline{v}_B = 20\underline{i}$ m/s

$\underline{\omega} = \frac{20}{100}(-\underline{k}) = -0.2\underline{k}$ rad/s

$\underline{\omega} \times \underline{r} = -0.2\underline{k} \times 50\underline{j} = 10\underline{i}$ m/s

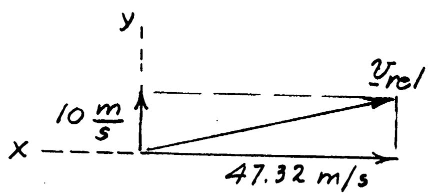
$\underline{v}_{rel} = 20(-0.866\underline{i} + 0.5\underline{j}) - 20\underline{i} - 10\underline{i}$

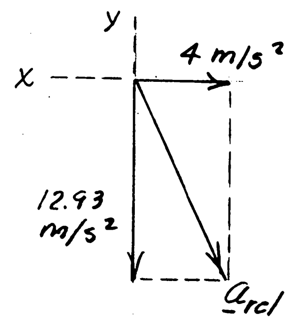
$= -47.3\underline{i} + 10\underline{j}$ m/s

$\underline{a}_A = \underline{a}_B + \dot{\underline{\omega}} \times \underline{r} + \underline{\omega} \times (\underline{\omega} \times \underline{r}) + 2\underline{\omega} \times \underline{v}_{rel} + \underline{a}_{rel}$

$\underline{0} = \frac{(20)^2}{100}(-\underline{j}) + \underline{0} - 0.2\underline{k} \times 10\underline{i} - 2(0.2\underline{k}) \times (-47.32\underline{i} + 10\underline{j}) + \underline{a}_{rel}$

$\underline{a}_{rel} = -4\underline{i} - 12.93\underline{j}$ m/s²





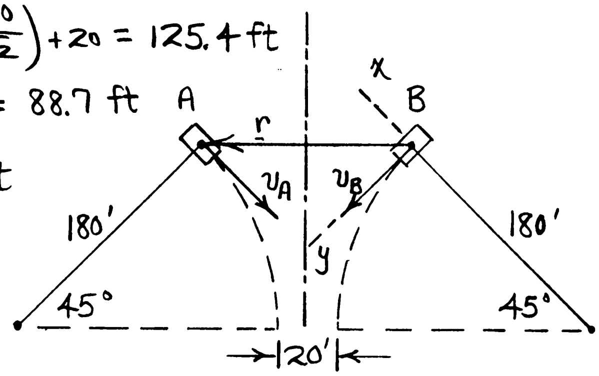
5/170 $r = 2\left(180 - \frac{180}{\sqrt{2}}\right) + 20 = 125.4 \text{ ft}$

$r_x = r_y = 125.4\frac{\sqrt{2}}{2} = 88.7 \text{ ft}$

$\underline{r} = 88.7(\underline{i} + \underline{j}) \text{ ft}$

$\underline{v}_A = -44\underline{i} \text{ ft/sec}$

$\underline{v}_B = 44\underline{j} \text{ ft/sec}$



$\underline{\omega} = \frac{44}{180}\underline{k} = 0.244\underline{k} \text{ rad/s}$

(5/12): $\underline{v}_A = \underline{v}_B + \underline{\omega} \times \underline{r} + \underline{v}_{rel}$

Substitute & obtain $\underline{v}_{rel} = -22.3\underline{i} - 65.7\underline{j} \text{ ft/sec}$

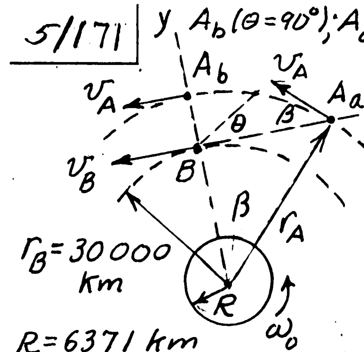
(5/14): $\underline{a}_A = \underline{a}_B + \underline{\alpha} \times \underline{r} + \underline{\omega} \times (\underline{\omega} \times \underline{r}) + 2\underline{\omega} \times \underline{v}_{rel} + \underline{a}_{rel}$

$\underline{a}_A = \frac{44^2}{180}\underline{j} = 10.76\underline{j} \text{ ft/sec}^2$; $\underline{a}_B = -10.76\underline{i} \text{ ft/sec}^2$

$\dot{\underline{\omega}} = \underline{0}$

Substitute & obtain $\underline{a}_{rel} = -16.06\underline{i} + 27.0\underline{j} \text{ ft/sec}^2$

5/171



For circular orbit $v = R\sqrt{g/r}$

For geosyn. orbit $v = r_A \omega_0$

so $r_A \omega_0 = R\sqrt{g/r_A}$, $r_A = \left(\frac{R^2 g}{\omega_0^2}\right)^{1/3}$

$r_A = \left\{\frac{(6371)^2(9.825/10^3)}{[0.7292(10^{-4})]^2}\right\}^{1/3} = 42\,171$ km

$\beta = \cos^{-1}\frac{30000}{42171} = 44.65°$

$\overline{BA} = \sqrt{(42171)^2 - (30000)^2} = 29638$ km

$R = 6371$ km
$g = 9.825$ m/s^2
$\omega_0 = 0.7292(10^{-4})$ rad/s

$v_A = 6371\sqrt{\frac{9.825/10^3}{42171}}(3600) = 11070$ km/h

$\omega = \omega_{xy} = v_B/r_B$
$= \frac{13125}{30000} = 0.4375\ \frac{\text{rad}}{\text{h}}$

$v_B = 6371\sqrt{\frac{9.825/10^3}{30000}}(3600) = 13125$ km/h

$\underline{v}_A = \underline{v}_B + \underline{\omega}\times\underline{r} + \underline{v}_{rel}$, $\underline{v}_{rel} = \underline{v}_A - \underline{v}_B - \underline{\omega}\times\underline{r}$

(a) $\theta = 0$; $\underline{r} = 29638\underline{i}$ km

$\underline{v}_{rel} = 11070(-\underline{i}\cos\beta + \underline{j}\sin\beta) - (-13125\underline{i}) - 0.4375\underline{k}\times 29638\underline{i}$

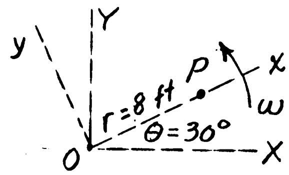
$= 5250\underline{i} - 5190\underline{j}$ km/h

(b) $\theta = 90°$, $\underline{r} = (42171 - 30000)\underline{j} = 12171\underline{j}$ km

$\underline{v}_{rel} = -11070\underline{i} - (-13125\underline{i}) - 0.4375\underline{k}\times 12171\underline{j}$

$= 7380\underline{i}$ km/h

5/172 $\underline{r} = 8\underline{i}$ ft, $\underline{v}_{rel} = \dot{r}\underline{i} = 2\underline{i}$ ft/sec, $\underline{a}_{rel} = \ddot{r}\underline{i} = -1\underline{i}$ ft/sec^2



$\underline{\omega} = \omega\underline{k} = \frac{2\pi}{10}\underline{k} = 0.628\underline{k}$ rad/sec, $\dot{\underline{\omega}} = \underline{0}$

$\underline{v}_B = \underline{v}_0 = \underline{0}$, $\underline{a}_B = \underline{a}_0 = \underline{0}$

$\underline{v}_A = \underline{v}_B + \underline{\omega}\times\underline{r} + \underline{v}_{rel}$

$\quad = \underline{0} + 0.628\underline{k}\times 8\underline{i} + 2\underline{i}$

$\quad = 2\underline{i} + 5.03\underline{j}$ ft/sec

$\left.\begin{array}{l}\underline{i} = \underline{I}\cos\theta + \underline{J}\sin\theta \\ \underline{j} = -\underline{I}\sin\theta + \underline{J}\cos\theta\end{array}\right\}$

so $\underline{v}_A = 2[0.866\underline{I} + 0.5\underline{J}] + 5.03[-0.5\underline{I} + 0.866\underline{J}]$

$\quad = \underline{-0.781\underline{I} + 5.35\underline{J}}$ ft/sec (a)

$\underline{a}_A = \underline{a}_B + \dot{\underline{\omega}}\times\underline{r} + \underline{\omega}\times(\underline{\omega}\times\underline{r}) + 2\underline{\omega}\times\underline{v}_{rel} + \underline{a}_{rel}$

$\quad = \underline{0} + \underline{0} + (0.628)^2\underline{k}\times(\underline{k}\times 8\underline{i}) + 2(0.628\underline{k})\times 2\underline{i} - 1\underline{i}$

$\quad = -3.16\underline{i} + 2.51\underline{j} - \underline{i}$

$\underline{a}_A = \underline{-4.16\underline{i} + 2.51\underline{j}}$ ft/sec^2 (b)

5/173 Let Oxy be attached to OA. Eq. (5/12):

$\underline{v}_B = \overset{\nearrow 0}{\underline{v}_0} + \underline{\omega} \times \underline{r} + \underline{v}_{rel}$

$\underline{\omega}_{BC} \times \underline{r}_{B/C} = \underline{\omega}_{OA} \times \underline{r}_{B/O} + v_{rel}\,\underline{i}$

$\omega_{BC}\underline{k} \times 0.2(\cos 60^\circ \underline{i} + \sin 60^\circ \underline{j})$

$= -3\underline{k} \times 0.2\underline{i} + v_{rel}\,\underline{i}$

$0.10\,\omega_{BC}\,\underline{j} - 0.2\frac{\sqrt{3}}{2}\omega_{BC}\underline{i} = -0.6\underline{j} + v_{rel}\,\underline{i}$

$2\beta + \theta = \pi$

$2\beta + 60^\circ = 180^\circ$

$\beta = 60^\circ$

$\left.\begin{aligned} \underline{i}&: -0.1732\,\omega_{BC} = v_{rel} \\ \underline{j}&: \ \ 0.10\,\omega_{BC} = -0.6 \end{aligned}\right\}$ $\quad \omega_{BC} = -6$ rad/s, $\quad v_{rel} = +1.039$ m/s

Now Eq. (5/14):

$\underline{a}_B = \underline{a}_0 + \underline{\alpha} \times \underline{r} + \underline{\omega} \times (\underline{\omega} \times \underline{r}) + 2\underline{\omega} \times \underline{v}_{rel} + \underline{a}_{rel}$

$\underline{a}_B = \underline{\omega}_{BC} \times (\underline{\omega}_{BC} \times \underline{r}_{B/C}) + \underline{\alpha}_{BC} \times \underline{r}_{B/C}$

$= 6^2 \cdot 0.2(-\cos 60^\circ \underline{i} - \sin 60^\circ \underline{j}) + \alpha_{BC}\underline{k} \times 0.2(+\cos 60^\circ \underline{i} + \sin 60^\circ \underline{j})$

$= -3.6\underline{i} - 6.24\underline{j} + 0.10\alpha_{BC}\,\underline{j} - 0.2\frac{\sqrt{3}}{2}\alpha_{BC}\,\underline{i}$

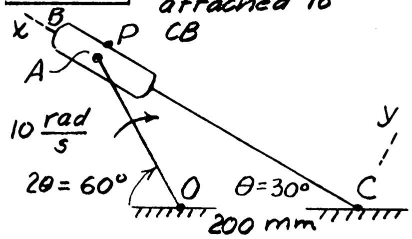
$\underline{a}_0 = \underline{0}$, $\underline{r} = \underline{r}_{B/O} = 0.2\underline{i}$, $\underline{\omega} = \underline{\omega}_{OA} = -3\underline{k}\ \frac{\text{rad}}{\text{s}}$, $\underline{\alpha} = \underline{\alpha}_{OA} = \underline{0}$

$\underline{a}_{rel} = a_{rel}\,\underline{i}$ → Substitute into (5/14) & solve to obtain $\underline{\alpha_{BC} = 0}$, $a_{rel} = -1.8$ m/s^2

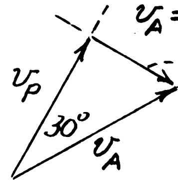
Absolute motion approach: $2\beta + \theta = \pi$,

$2\overset{\nearrow 0}{\ddot{\beta}} + \ddot{\theta} = 0 \Rightarrow \underline{\ddot{\theta} = \alpha_{BC} = 0\ !!}$

5/174 x-y axes are attached to CB



$\overline{OA} = 200$ mm



$v_A = 200(10) = 2000$ mm/s

$v_{A/P} = v_{rel} = 2000\left(\frac{1}{2}\right) = 1000$ mm/s

$v_P = 2000\,\frac{\sqrt{3}}{2} = 1732$ mm/s

$\omega_{xy} = \omega = \dfrac{v_P}{\overline{PC}} = \dfrac{1732}{2(200)\sqrt{3}/2} = 5$ rad/s CW

$\underline{a}_A = \underline{a}_C + \underline{\dot\omega}\times\underline{r} + \underline{\omega}\times(\underline{\omega}\times\underline{r}) + 2\underline{\omega}\times\underline{v}_{rel} + \underline{a}_{rel}$

$\dot\omega = \ddot\theta = 0$ since $\dfrac{d^2(2\theta)}{dt^2} = 0$; thus $(\underline{a}_P)_t = \underline{\dot\omega}\times\underline{r} = 0$

$\underline{\omega}\times(\underline{\omega}\times\underline{r}) = 5^2\underline{k}\times(\underline{k}\times 200\sqrt{3}\,\underline{i}) = -8660\,\underline{i}$ mm/s²

$2\underline{\omega}\times\underline{v}_{rel} = 2(5\underline{k})\times 1000(-\underline{i}) = -10\,000\,\underline{j}$ mm/s²

$\underline{a}_{rel} = \ddot x\,\underline{i}$; $\underline{a}_A = 200(10)^2(-0.866\,\underline{i} - 0.5\,\underline{j})$ mm/s²

Thus $20\,000(-0.866\,\underline{i} - 0.5\,\underline{j}) = \underline{0} + \underline{0} - 8660\,\underline{i} - 10000\,\underline{j} + \ddot x\,\underline{i}$

$\ddot x = -8660$ mm/s², $\underline{a}_{rel} = -8660\,\underline{i}$ mm/s²

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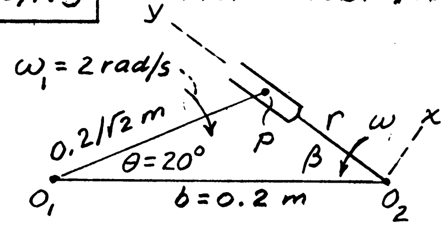
From Prob. 5/53 $\omega_2 = \omega = 1.923$ rad/s $\quad (= -\dot\beta)$



$\beta = 35.8°$

$$r = \frac{0.2}{\sqrt2}\,\frac{\sin 20°}{\sin 35.8°} = 0.0827 \text{ m}$$

$$\frac{r}{\sin\theta} = \frac{0.2/\sqrt2}{\sin\beta}$$

$$r\sin\beta = 0.2/\sqrt2\,\sin\theta$$

$$\dot r\sin\beta + r\dot\beta\cos\beta = 0.2/\sqrt2\,\dot\theta\cos\theta$$

$$v_{rel} = \dot r = \frac{1}{\sin 35.8°}\left(0.2/\sqrt2\,[-2]\cos 20° - 0.0827(-1.923)\cos 35.7°\right)$$

$$= -0.2339 \text{ m/s}$$

$$\underline{a}_P = \dot{\underline{\omega}}\times\underline{r} + \underline{\omega}\times(\underline{\omega}\times\underline{r}) + 2\underline{\omega}\times\underline{v}_{rel} + \underline{a}_{rel}$$

$$\underline{a}_P = \frac{0.2}{\sqrt2}(2)^2\left[-\underline{i}\sin(\theta+\beta) + \underline{j}\cos(\theta+\beta)\right] = \frac{0.8}{\sqrt2}(-0.8269\underline{i} + 0.5623\underline{j})\ \frac{m}{s^2}$$

$$\dot{\underline{\omega}}\times\underline{r} = \dot\omega\underline{k}\times r\underline{j} = -0.0827\dot\omega\underline{i}$$

$$\underline{\omega}\times(\underline{\omega}\times\underline{r}) = 0.0827(1.923)^2(-\underline{j}) = -0.3058\underline{j}\ \text{m/s}^2$$

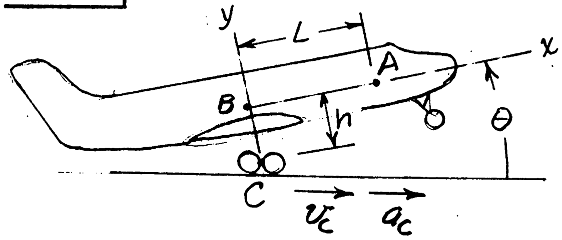
$$2\underline{\omega}\times\underline{v}_{rel} = 2(1.923\underline{k})\times(-0.2339\underline{j}) = 0.8994\,\underline{i}\ \text{m/s}^2$$

$$\underline{a}_{rel} = \ddot r\underline{j}$$

Substitute, separate terms & get

$$\left.\begin{aligned}\frac{0.8}{\sqrt2}(-0.8269) &= -0.0827\dot\omega + 0.8994\\ \frac{0.8}{\sqrt2}(0.5623) &= -0.3058 + \ddot r\end{aligned}\right\}\quad \begin{aligned}&\dot\omega = \alpha_2 = \underline{16.53 \text{ rad/s}^2 \text{ CCW}}\\ &\ddot r = 0.624 \text{ m/s}^2\end{aligned}$$

▶5/176



$\underline{\omega} = \omega \underline{k} = \dot{\theta}\underline{k}$, $\underline{\alpha} = \dot{\omega}\underline{k}$

$\underline{r} = L\underline{i}$, $\underline{v}_{rel} = \dot{L}\underline{i}$

$\underline{a}_{rel} = \ddot{L}\underline{i}$

$\underline{v}_C = v_C(\underline{i}\cos\theta - \underline{j}\sin\theta)$

$\underline{a}_C = a_C(\underline{i}\cos\theta - \underline{j}\sin\theta)$

$$\underline{v}_B = \underline{v}_C + \underline{\omega} \times \underline{r}_{CB} = \underline{v}_C + \omega\underline{k} \times h\underline{j} = (v_C\cos\theta - \omega h)\underline{i} - (v_C\sin\theta)\underline{j}$$

$$\underline{a}_B = \underline{a}_C + \dot{\underline{\omega}} \times \underline{r}_{CB} + \omega \times (\underline{\omega} \times \underline{r}_{CB}) = \underline{a}_C + \alpha\underline{k} \times h\underline{j} + \omega\underline{k} \times (\omega\underline{k} \times h\underline{j})$$
$$= (a_C\cos\theta - \alpha h)\underline{i} - (a_C\sin\theta + h\omega^2)\underline{j}$$

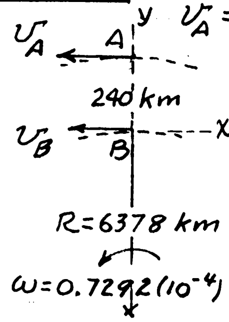
$$\underline{v}_A = \underline{v}_B + \underline{\omega} \times \underline{r} + \underline{v}_{rel} = \underline{v}_B + \omega\underline{k} \times L\underline{i} + \dot{L}\underline{i}$$

$$\underline{v}_A = \underline{(v_C\cos\theta - \omega h + \dot{L})\underline{i} + (\omega L - v_C\sin\theta)\underline{j}}$$

$$\underline{a}_A = \underline{a}_B + \dot{\underline{\omega}} \times \underline{r} + \underline{\omega} \times (\underline{\omega} \times \underline{r}) + 2\underline{\omega} \times \underline{v}_{rel} + \underline{a}_{rel}$$
$$= \underline{a}_B + \dot{\omega}\underline{k} \times L\underline{i} + \omega\underline{k} \times (\omega\underline{k} \times L\underline{i}) + 2\omega\underline{k} \times \dot{L}\underline{i} + \ddot{L}\underline{i}$$

$$\underline{a}_A = \underline{(a_C\cos\theta - \alpha h - L\omega^2 + \ddot{L})\underline{i} + (-a_C\sin\theta - h\omega^2 + L\alpha + 2\omega\dot{L})\underline{j}}$$

▶ 5/177 For circular orbit ; At equator $g = 9.814\ m/s^2$



$$v_A = R\sqrt{g/(R+h)} = 6378\sqrt{\frac{9.814/1000}{6378+240}}(3600) = 27\,960\ \frac{km}{h}$$

$$a_A = \frac{v_A^2}{R+h} = g\left(\frac{R}{R+h}\right)^2 = 9.814\left(\frac{6378}{6378+240}\right)^2 = 9.115\ m/s^2$$

$$v_B = R\omega = 6378(0.7292)(10^{-4})(3600) = 1674\ km/h$$

$$\underline{v}_A = \underline{v}_B + \underline{\omega}\times\underline{r} + \underline{v}_{rel}$$

$$\underline{\omega}\times\underline{r} = 0.7292(10^{-4})(3600)(240)(-\underline{i})$$
$$= -63.00\underline{i}\ km/h$$

$$\underline{v}_{rel} = -27\,960\underline{i} - (-1674\underline{i}) - (-63.00\underline{i})$$
$$= -26\,220\underline{i}\ km/h$$

$$\underline{a}_A = \underline{a}_B + \dot{\underline{\omega}}\times\underline{r} + \underline{\omega}\times(\underline{\omega}\times\underline{r}) + \overline{2\underline{\omega}\times\underline{v}_{rel} + \underline{a}_{rel}}$$

$$\underline{a}_B = -R\omega^2\underline{j} = -6378(10^3)(0.7292)^2(10^{-4})^2\underline{j} = -0.03391\underline{j}\ m/s^2$$

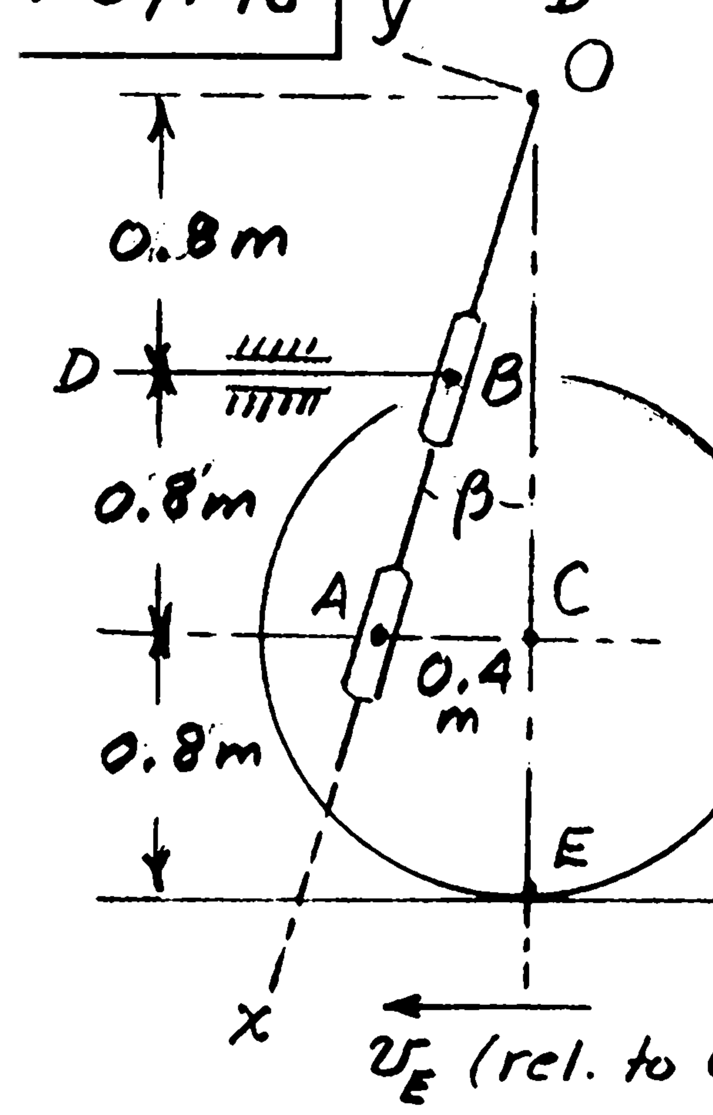
$$\dot{\underline{\omega}} = \underline{0};\ \underline{\omega}\times(\underline{\omega}\times\underline{r}) = (0.7292)(10^{-4})\underline{k}\times(-63.00\underline{i})\frac{1000}{3600} = -0.001276\underline{j}\ m/s^2$$

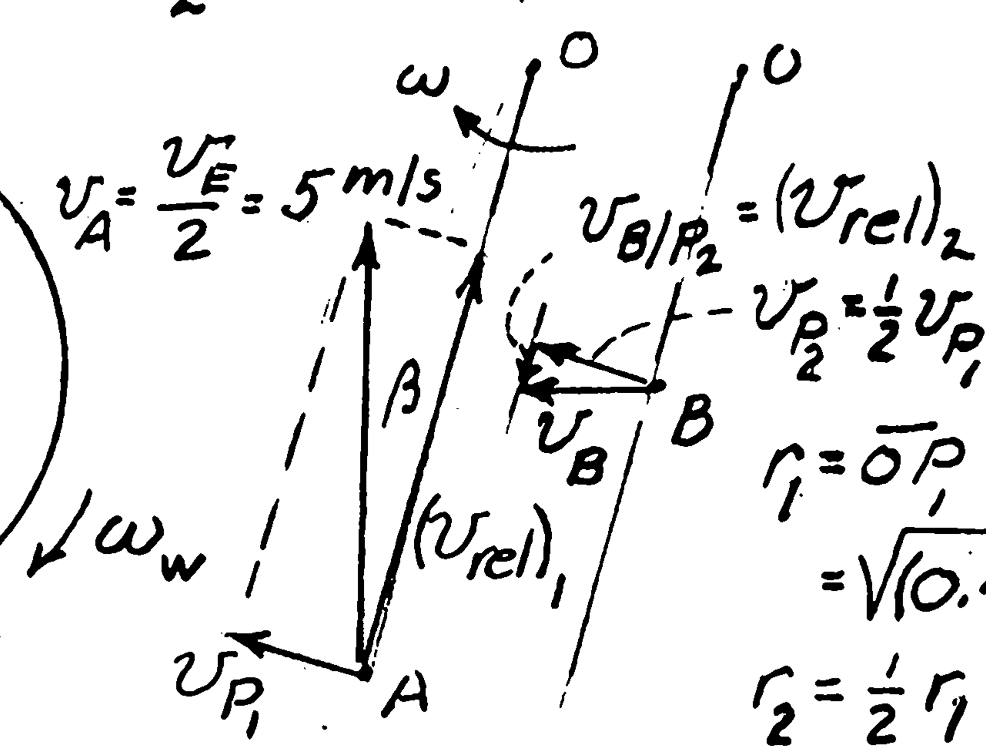
$$2\underline{\omega}\times\underline{v}_{rel} = 2(0.7292)(10^{-4})\underline{k}\times(-26\,220\underline{i})\frac{1000}{3600} = -1.0623\underline{j}\ m/s^2$$

So
$$-9.115\underline{j} = -0.03391\underline{j} - 0.001276\underline{j} - 1.0623\underline{j} + \underline{a}_{rel}$$

$$\underline{a}_{rel} = \underline{-8.018\underline{j}\ m/s^2}$$

▸5/178 $a_D = a_B$ = accel. measured relative to constant velocity frame F. Therefore point O can be considered to be fixed so v_E = 36 km/h or 10 m/s





P_1, P_2 = points on arm coincident with A & B, respectively.

$r_1 = \overline{OP_1} = \overline{OA}$
$= \sqrt{(0.4)^2 + (1.6)^2} = 1.649$ m

$r_2 = \frac{1}{2} r_1 = 0.825$ m

$\beta = \tan^{-1} \frac{0.4}{1.6} = 14.04°$

$\underline{\omega} = (v_{P_1}/r_1)\underline{k} = [(5 \sin 14.04°)/1.649]\underline{k}$
$= 0.735\,\underline{k}$ rad/s

$(\underline{v}_{rel})_1 = 5 \cos 14.04°(-\underline{i}) = -4.85\underline{i}$ m/s

$(\underline{v}_{rel})_2 = v_{P_2} \tan\beta\,(\underline{i}) = \frac{1}{2}(5)\sin\beta \tan\beta(\underline{i}) = 0.1516\underline{i}$ m/s

$\underline{a}_A = [5^2/0.4](-\underline{i}\sin\beta - \underline{j}\cos\beta) = -15.16\underline{i} - 60.63\underline{j}$ m/s²

$\dot{\underline{\omega}} \times \underline{r}_1 = \dot{\omega}\underline{k} \times r_1\underline{i} = 1.649\dot{\omega}\underline{j}$, $\dot{\underline{\omega}} \times \underline{r}_2 = 0.825\dot{\omega}\underline{j}$

$\underline{\omega} \times (\underline{\omega} \times \underline{r}_1) = (0.735)^2 \underline{k} \times (\underline{k} \times 1.649\underline{i}) = -0.892\underline{i}$ m/s²

$\underline{\omega} \times (\underline{\omega} \times \underline{r}_2) = -0.446\underline{i}$ m/s²

$2\underline{\omega} \times (\underline{v}_{rel})_1 = 2(0.735)\underline{k} \times (-4.85\underline{i}) = -7.13\underline{j}$ m/s²

$2\underline{\omega} \times (\underline{v}_{rel})_2 = 2(0.735)\underline{k} \times (0.1516\underline{i}) = 0.2229\underline{j}$ m/s²

$(\underline{a}_{rel})_1 = \ddot{r}_1\underline{i}$, $(\underline{a}_{rel})_2 = \ddot{r}_2\underline{i}$

$\underline{a}_O = \underline{0}$

$\underline{a}_A = \underline{a}_O + \dot{\underline{\omega}} \times \underline{r}_1 + \underline{\omega} \times (\underline{\omega} \times \underline{r}_1) + 2\underline{\omega} \times (\underline{v}_{rel})_1 + (\underline{a}_{rel})_1$

Substitute above terms, separate terms & get

$\ddot{r}_1 = -14.27$ m/s², $\dot{\omega} = -32.4$ rad/s²

$\underline{a}_B = \underline{a}_O + \dot{\underline{\omega}} \times \underline{r}_2 + \underline{\omega} \times (\underline{\omega} \times \underline{r}_2) + 2\underline{\omega} \times (\underline{v}_{rel})_2 + (\underline{a}_{rel})_2$

where $\underline{a}_B = a_B(\underline{i}\sin\beta + \underline{j}\cos\beta)$

$\dot{\underline{\omega}} \times \underline{r}_2 = -32.4\,\underline{k} \times 0.825\underline{i} = -26.75\underline{j}$ m/s²

Substitute & separate terms & get

$\ddot{r}_2 = -6.19$ m/s²

$a_B = \underline{-27.3 \text{ m/s}^2}$ (to the right)

$\boxed{5/179}$ $\alpha = \frac{d\omega}{dt} = -K - k\omega^2$

$$\int_{\omega_0}^{0} -\frac{d\omega}{K - k\omega^2} = \int_0^t dt, \quad \frac{1}{\sqrt{Kk}} \tan^{-1}\left(\omega\sqrt{\frac{k}{K}}\right)\Bigg]_0^{\omega_0} = t$$

$$\underline{t = \frac{1}{\sqrt{Kk}} \tan^{-1}\left(\omega_0\sqrt{\frac{k}{K}}\right)}$$

5/180 For v constant $a_t = 0$ & $a = a_n = v^2/r$

$\left(\frac{v^2}{r}\right)_A = \frac{2}{3}\left(\frac{v^2}{3}\right)_B$, $\underline{r = 4.5 \text{ in.}}$

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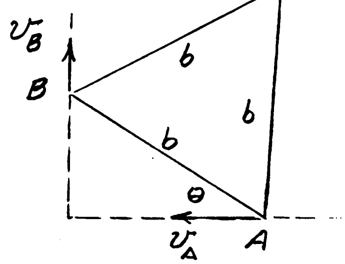


$x_C = \frac{b}{2}\cos\theta + \frac{b\sqrt{3}}{2}\sin\theta$

$\dot{x}_C = -\frac{b}{2}\dot{\theta}\sin\theta + \frac{b\sqrt{3}}{2}\dot{\theta}\cos\theta$

$\dot{x}_C = 0$ when $\frac{b}{2}\dot{\theta}\sin\theta = \frac{b\sqrt{3}}{2}\dot{\theta}\cos\theta$

so $\underline{\theta = \tan^{-1}\sqrt{3} = 60°}$

$\boxed{5/182}$ $\underline{v}_A = \underline{v}_C + \underline{v}_{A/C}$

where $\underline{v}_C = 120\left(\frac{44}{30}\right)\underline{i} = 176\underline{i}$ ft/sec

$v_{A/C} = \overline{AC}\,\Omega = 13\left[800 \cdot \frac{2\pi}{60}\right] = 1089$ ft/sec

So $\underline{v}_A = 176\underline{i} + 1089\left[+\cos 10°\,\underline{i} - \sin 10°\,\underline{k}\right]$

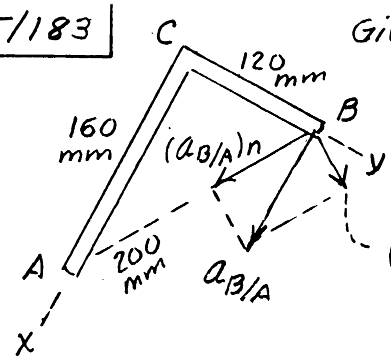
$= \underline{1249\underline{i} - 189.1\underline{k} \text{ ft/sec}}$

$\underline{v}_B = \underline{v}_C + \underline{v}_{B/C}$

$\underline{v}_{B/C} = 1089\underline{j}$ ft/sec

So $\underline{\underline{v}_B = 176\underline{i} + 1089\underline{j} \text{ ft/sec}}$

5/183



Given: $a_C = 0$, $\underline{a}_{B/A} = 20 \ m/s^2$

$(a_{B/A})_n = 20(4/5) = 16 \ m/s^2$

$= \overline{AB}\omega^2 = 0.2\omega^2$

so $\omega^2 = 80 \ (rad/s)^2$

$(a_{B/A})_t = 20(3/5) = 12 \ m/s^2$

$= \overline{AB}\alpha = 0.2\alpha$

so $\alpha = 60 \ rad/s^2$

$\underline{a}_A = \underline{a}_C + (\underline{a}_{A/C})_n + (\underline{a}_{A/C})_t$

$a_C = 0$; $(a_{A/C})_n = \overline{AC}\omega^2 = 0.16(80)$, $(\underline{a}_{A/C})_n = -12.8\underline{i} \ m/s^2$

$(a_{A/C})_t = \overline{AC}\alpha = 0.16(60)$, $(\underline{a}_{A/C})_t = -9.6\underline{j} \ m/s^2$

$\underline{a}_A = -12.8\underline{i} - 9.6\underline{j} \ m/s^2$

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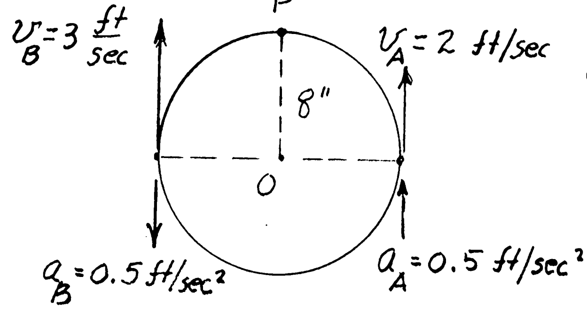


$$\omega = \omega_{PO} = \frac{3-2}{16/12} = \frac{3}{4} \ \frac{rad}{sec}$$

$$\alpha = \alpha_{PO} = \frac{0.5}{8/12} = \frac{3}{4} \ \frac{rad}{sec^2}$$

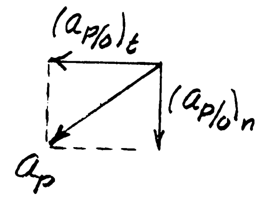
ccw

$$\underline{a}_P = \underline{a}_O + (\underline{a}_{P/O})_n + (\underline{a}_{P/O})_t$$

$$a_O = 0, \ (a_{P/O})_n = \frac{8}{12}\left(\frac{3}{4}\right)^2 = \frac{3}{8} \ ft/sec^2$$

$$(a_{P/O})_t = \frac{8}{12}\,\frac{3}{4} = \frac{1}{2} \ ft/sec^2$$

$$a_P = \sqrt{(3/8)^2 + (1/2)^2} = \underline{5/8 \ ft/sec^2}$$



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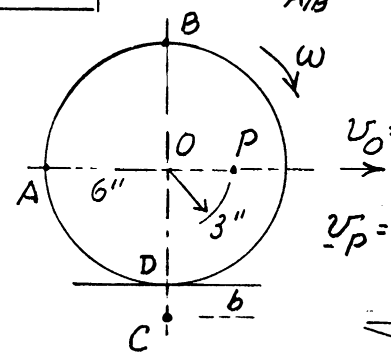
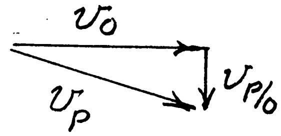
$v_{A/B} = \overline{AB}\,\omega$, $3\sqrt{2} = \frac{1}{2}\sqrt{2}\,\omega$, $\omega = 6$ rad/s



$v_O / \overline{OC} = \omega$, $\frac{4(12)}{6+b} = 6$

$v_O = 4$ ft/sec $\quad$ $\underline{b = 2 \text{ in}}$

$\underline{v}_P = \underline{v}_O + \underline{v}_{P/O}$, $v_{P/O} = \overline{PO}\,\omega = \frac{3}{12}\,6 = \frac{3}{2}$ ft/sec



$v_P = \sqrt{4^2 + (3/2)^2} = \underline{4.27 \text{ ft/sec}}$

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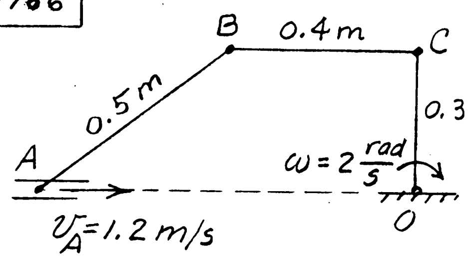


$\underline{v}_B = \underline{v}_A + \underline{v}_{B/A}$

$\underline{v}_B = \underline{v}_C + \underline{v}_{B/C}$

Thus

$\underline{v}_A + \underline{v}_{B/A} = \underline{v}_C + \underline{v}_{B/C}$

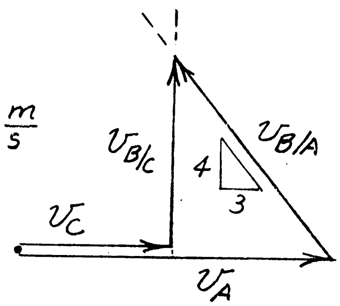
$\underline{v}_C = r\omega = 0.3(2) = 0.6$ m/s

From vector polygon

$v_{B/C} = \frac{4}{3}(v_A - v_C) = \frac{4}{3}\,0.6 = 0.8\ \frac{m}{s}$

$\omega_{BC} = v_{B/C} / \overline{BC} = 0.8/0.4$

$= \underline{2\ \text{rad/s CW}}$



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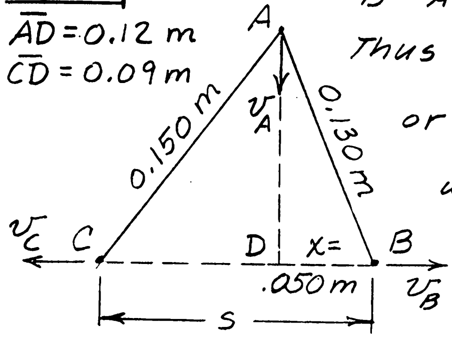
$\overline{AD} = 0.12$ m

$\overline{CD} = 0.09$ m

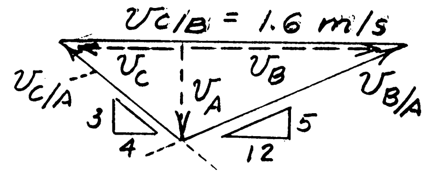


$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$, $\mathbf{v}_C = \mathbf{v}_A + \mathbf{v}_{C/A}$

Thus $\mathbf{v}_B = \mathbf{v}_C - \mathbf{v}_{C/A} + \mathbf{v}_{B/A}$

or $\mathbf{v}_{C/A} = \mathbf{v}_{C/B} + \mathbf{v}_{B/A}$

where $\mathbf{v}_{C/B} = \mathbf{v}_C - \mathbf{v}_B$



$v_{C/B} = \dot{s} = 1.6$ m/s

From diagram $\quad \frac{5}{12} v_B = \frac{3}{4} v_C \; ; \; v_B + v_C = 1.6$

$\frac{5}{12} v_B = \frac{3}{4}(1.6 - v_B)$, $\underline{v_B = 1.029 \text{ m/s}}$

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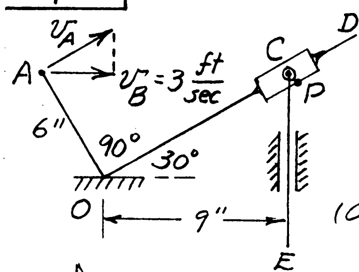
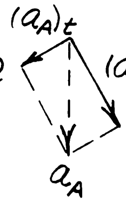




$v_A = v_B/\cos 30° = 3/0.866$
$= 3.46$ ft/sec

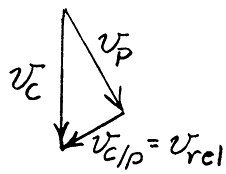
$(a_A)_{horiz.} = 0$ since $(v_A)_{horiz}$ is constant

Thus a_A is vertical

$(a_A)_n = v_A^2/\overline{AO} = (2\sqrt{3})^2/\frac{6}{12} = 24$ ft/sec²

$(a_A)_t = 24/\sqrt{3} = 13.86$ ft/sec²

$\alpha_{OD} = \alpha_{OA} = \frac{(a_A)_t}{\overline{OA}} = \frac{13.86}{6/12} = 27.7$ rad/sec² ccw



$v_P = \frac{\overline{OP}}{\overline{OA}} v_A = \frac{9(2/\sqrt{3})}{6}(3.46) = 6$ ft/sec

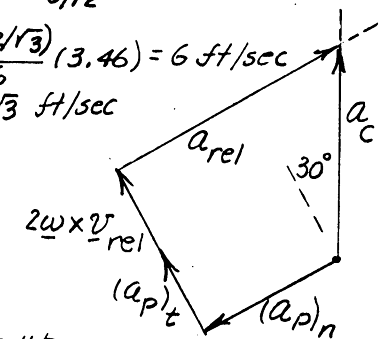
$v_{rel} = 6/\sqrt{3}$ ft/sec

$\underline{a}_C = \underline{a}_P + 2\underline{\omega} \times \underline{v}_{rel} + \underline{a}_{rel}$

$(a_P)_n = v_P^2/\overline{OP} = 6^2/\frac{9}{12}\frac{2}{\sqrt{3}} = 24\sqrt{3}$ ft/sec²

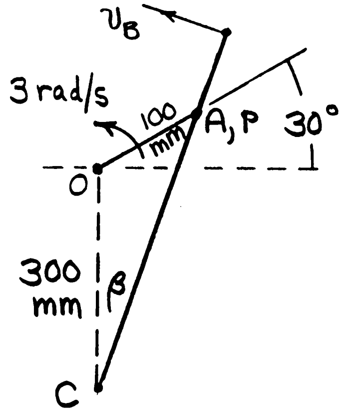
$(a_P)_t = \overline{OP}\alpha_{OD} = \frac{9}{12}\frac{2}{\sqrt{3}}(27.7) = 24$ ft/sec²

$|2\underline{\omega} \times \underline{v}_{rel}| = 2\frac{2\sqrt{3}}{6/12}\frac{6}{\sqrt{3}} = 48$ ft/sec²



From diag. $a_C = (24+48)\frac{2}{\sqrt{3}} = \underline{83.1 \frac{ft}{sec^2} \text{ up}}$

5/189 Let P be a point on BC coincident with A.



$CA^2 = 300^2 + 100^2 - 2(300)(100)\cos 120°$

$= 361$ mm

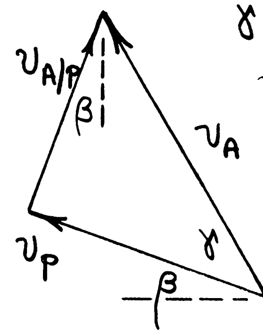
$\dfrac{100}{\sin\beta} = \dfrac{361}{\sin 120°}$, $\beta = 13.90°$

$\gamma = 60 - \beta = 46.1°$

$v_P = v_A \cos\gamma$

$= 300 \cos 46.1°$

$= 208$ mm/s



$\underline{v}_A = \underline{v}_P + \underline{v}_{A/P}$

$v_A = 100(3)$

$= 300$ mm/s

$v_B = \overline{CB}\,\dfrac{v_P}{\overline{CA}} = 500\,\dfrac{208}{361} = \underline{288 \text{ mm/s}}$

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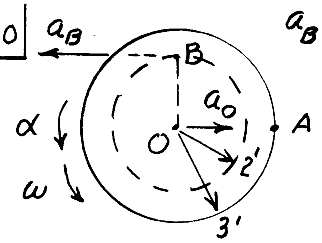
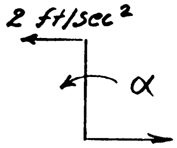




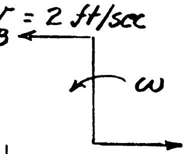
$a_B = 2$ ft/sec²

$a_O = 3$ ft/sec²

$(t = 0)$

$$\alpha = \frac{2+3}{2} = 2.5\ \frac{\text{rad}}{\text{sec}^2}$$

$$\omega = 0$$



$v_B = 2$ ft/sec

$v_O = at = 3\ \frac{\text{ft}}{\text{sec}}$

$(t = 1\ \text{sec})$

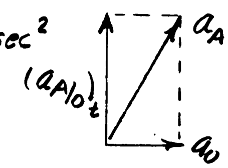
$$\alpha = 2.5\ \frac{\text{rad}}{\text{sec}^2}$$

$$\omega = \frac{5}{2} = 2.5\ \frac{\text{rad}}{\text{sec}}$$

$$\underline{a}_A = \underline{a}_O + \underline{a}_{A/O}$$

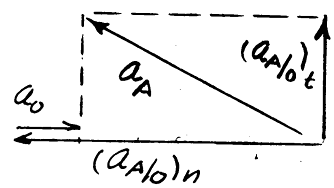
(a) $t = 0$; $a_{A/O} = (a_{A/O})_t = 3(2.5) = 7.5$ ft/sec²

$$a_A = \sqrt{3^2 + 7.5^2} = \underline{8.08\ \frac{\text{ft}}{\text{sec}^2}}$$



(b) $t = 1$ sec, $(a_{A/O})_n = 3(2.5)^2$
$= 18.75$ ft/sec²

$$\underline{a}_A = \sqrt{(18.75-3)^2 + 7.5^2}$$
$$= \underline{17.44\ \text{ft/sec}^2}$$



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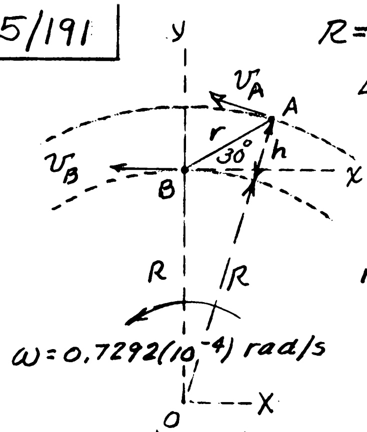


$R = 6378$ km, $h = 200$ km

Law of cosines gives

$(R+h)^2 = R^2 + r^2 - 2Rr\cos 120°$

$(6378+200)^2 = (6378)^2 + r^2 - 2(6378)(-0.5)r$

$r^2 + 6378r - 2\,591\,200 = 0$

$r = -\frac{6378}{2} \pm \frac{1}{2}\sqrt{(6378)^2 + 4(2\,591\,200)}$

$= 383.2$ km (or -6761 km)

Rel. velocity from nonrotating system X-Y at O is $\underline{v}_A - \underline{v}_B$. But $\underline{v}_A = \underline{v}_B + \underline{\omega}\times\underline{r} + \underline{v}_{rel}$

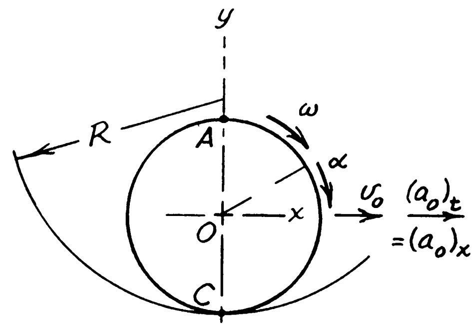
so $\underline{\omega}\times\underline{r} = (\underline{v}_A - \underline{v}_B) - (\underline{v}_{rel})$

$\underline{\omega}\times\underline{r} = 0.7292(10^{-4})(3600)\underline{k} \times 383.2(0.866\underline{i} + 0.5\underline{j})$

so $\Delta\underline{v}_{rel} = \underline{-50.3\underline{i} + 87.1\underline{j}}$ km/h

5/192

$v_O = r\omega,\ (a_O)_t = r\alpha$



$\underline{a}_C = \underline{a}_O + (\underline{a}_{C/O})_n + (\underline{a}_{C/O})_t$

$\underline{a}_O = r\alpha\underline{i} + \dfrac{(r\omega)^2}{R-r}\underline{j}$

$(\underline{a}_{C/O})_n = r\omega^2\underline{j},\ (\underline{a}_{C/O})_t = -r\alpha\underline{i}$

$\underline{a}_C = r\alpha\underline{i} + \dfrac{(r\omega)^2}{R-r}\underline{j} + r\omega^2\underline{j} - r\alpha\underline{i},\ \underline{\underline{a}_C = \dfrac{r\omega^2}{1-r/R}\underline{i}}$

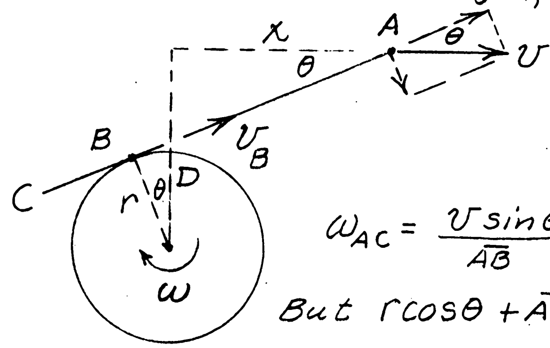
$\underline{a}_A = \underline{a}_O + (\underline{a}_{A/O})_n + (\underline{a}_{A/O})_t$

$(\underline{a}_{A/O})_n = -r\omega^2\underline{j},\ (\underline{a}_{A/O})_t = r\alpha\underline{i}$

$\underline{a}_A = r\alpha\underline{i} + \dfrac{(r\omega)^2}{R-r}\underline{j} - r\omega^2\underline{j} + r\alpha\underline{i},\ \underline{\underline{a}_A = 2r\alpha\underline{i} + r\omega^2\dfrac{2r/R-1}{1-r/R}\underline{j}}$

5/193

For constant length, $v_B = v\cos\theta$



$\omega = v_B/r$

$$\omega = \frac{v\cos\theta}{r}$$

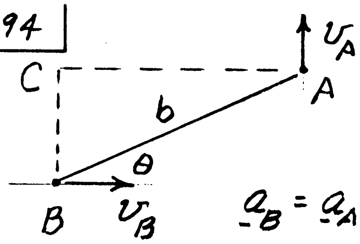
$$\omega_{AC} = \frac{v\sin\theta}{\overline{AB}}$$

But $r\cos\theta + \overline{AB}\sin\theta = D$

so $\overline{AB} = \frac{1}{\sin\theta}(D - r\cos\theta)$

Thus $$\omega_{AC} = \frac{v\sin^2\theta}{D - r\cos\theta}$$

5/194

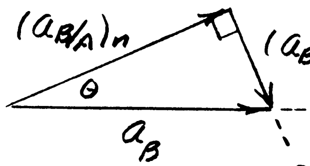


C = instant center

$$\omega_{AB} = \frac{v_A}{\overline{AC}} = \frac{v_A}{b\cos\theta}$$

$$\underline{a}_B = \underline{a}_A + (\underline{a}_{B/A})_n + (\underline{a}_{B/A})_t$$

$$a_A = 0, \quad (a_{B/A})_n = \overline{AB}\,\omega_{AB}^2 = \frac{v_A^2}{b\cos^2\theta}$$



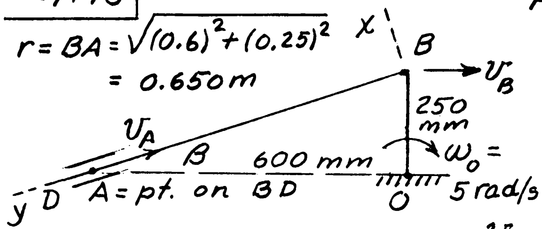
$$(a_{B/A})_t = \frac{v_A^2}{b\cos^2\theta}\tan\theta = \frac{v_A^2}{b}\,\frac{\sin\theta}{\cos^3\theta}$$

$$\alpha_{AB} = \frac{(a_{B/A})_t}{\overline{AB}} = \underline{\left(\frac{v_A}{b}\right)^2 \frac{\sin\theta}{\cos^3\theta} \text{ CCW}}$$

▶5/195 x-y axes attached to BD

$\beta = \tan^{-1}\dfrac{250}{600} = 22.6°$

$r = BA = \sqrt{(0.6)^2 + (0.25)^2} = 0.650\text{ m}$

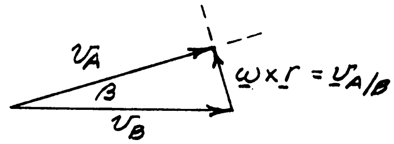


$v_B = 0.25(5) = 1.25\text{ m/s}$

$\underline{v}_A = \underline{v}_B + \underline{\omega}\times\underline{r} + \underline{v}_{rel}$

$\underline{v}_{rel} = \underline{0} \quad (\dot{r} = 0)$

$v_{A/B} = v_B \sin\beta = 1.25(0.3846) = 0.481\text{ m/s}$



$\omega = \dfrac{v_{A/B}}{\overline{AB}} = \dfrac{0.481}{0.650} = 0.740\text{ rad/s}$

$\underline{a}_A = \underline{a}_B + \dot{\underline{\omega}}\times\underline{r} + \underline{\omega}\times(\underline{\omega}\times\underline{r}) + 2\underline{\omega}\times\underline{v}_{rel} + \underline{a}_{rel},$

$a_{rel} = 0; \quad \underline{a}_B = 0.25(5^2)(-\underline{i}\cos\beta + \underline{j}\sin\beta) = -5.77\underline{i} + 2.40\underline{j}\ \frac{\text{m}}{\text{s}^2}$

$\underline{\omega}\times(\underline{\omega}\times r) = 0.650(0.740)^2(-\underline{j}) = -0.356\underline{j}\text{ m/s}^2$

$\dot{\underline{\omega}}\times\underline{r} = \dot{\omega}\underline{k}\times r\underline{j} = -0.650\dot{\omega}\underline{i}$

$2\underline{\omega}\times\underline{v}_{rel} = 0$

$\underline{a}_A = (a_A)_y\underline{j} + (a_A)_x\underline{i}$ where $\underline{a}_{A_x} = 2\underline{\omega}\times\underline{v}_A = 2\omega\underline{k}\times v_A\underline{j}$
$= 2(0.740)(1.25\cos\beta)(-\underline{i})$
$= -1.708\underline{i}\text{ m/s}^2$

Substitute & get

$-1.708 = -5.769 - 0.650\dot{\omega}, \quad \dot{\omega} = -6.25\text{ rad/s}^2$

$(a_A)_y = 2.404 - 0.356 = 2.05\text{ m/s}^2$

so $\underline{\alpha = 6.25\text{ rad/s}^2\text{ CW}}$

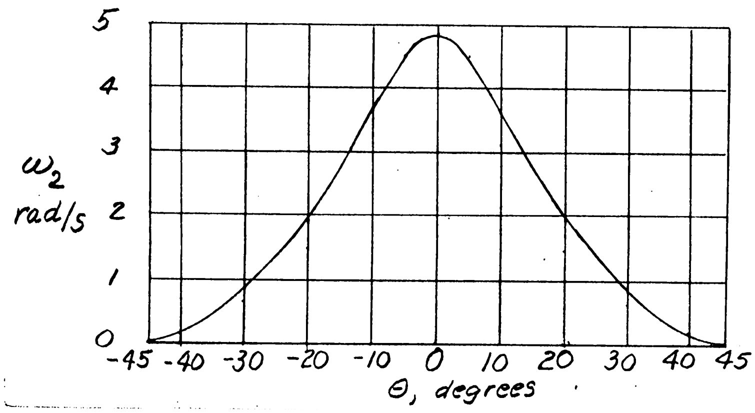
*5/196 From Prob. 5/53 we have

$$\omega_2 = \dot{\theta}\,\frac{\cos(\theta+\beta)}{\cos(\theta+\beta) - \sqrt{2}\cos\beta} \quad \text{where } \beta = \angle\, O, O_2 P$$

Also $\tan\beta = \dfrac{\sin\theta}{\sqrt{2} - \cos\theta}$

For $\dot{\theta} = -2$ rad/s, $\quad \omega_2 = 2\,\dfrac{\cos(\theta+\beta)}{\sqrt{2}\cos\beta - \cos(\theta+\beta)}$

Set up program to compute β & ω_2 & plot results.



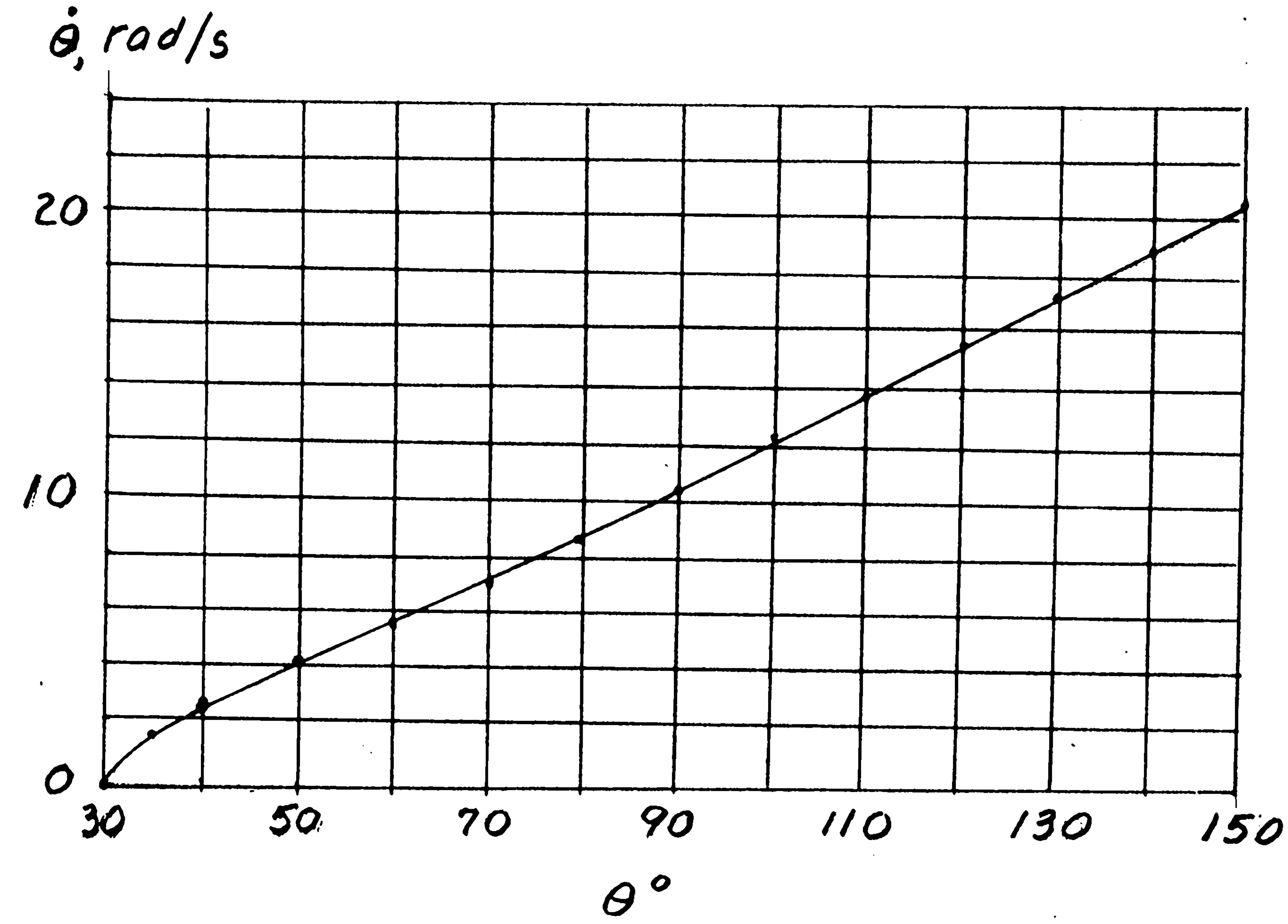
*5/197 $\ddot{\theta} = 100(1-\cos\theta)$ rad/s²

$\dot{\theta}\,d\dot{\theta} = \ddot{\theta}\,d\theta$ so $\int_0^{\dot{\theta}} \dot{\theta}\,d\dot{\theta} = 100\int_{\pi/6}^{\theta}(1-\cos\theta)\,d\theta$

$\dot{\theta}^2 = 200(\theta - \sin\theta)\Big|_{\pi/6}^{\theta} = 200(\theta - \sin\theta - 0.0236)$

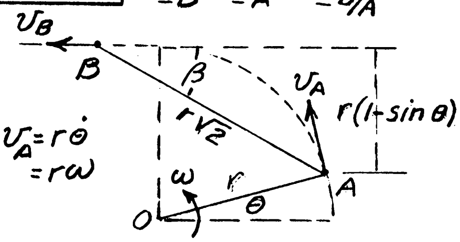
$\dot{\theta} = \frac{d\theta}{dt} = 10\sqrt{2}\sqrt{\theta - \sin\theta - 0.0236}$ rad/s

$\int_0^t dt = \int_{\pi/2}^{5\pi/6} \frac{d\theta}{10\sqrt{2}\sqrt{\theta - \sin\theta - 0.0236}}$ Numerical integration gives $t = 0.0701$ s

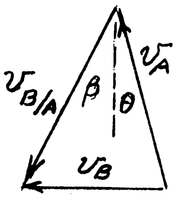


*5/198

$\underline{v}_B = \underline{v}_A + \underline{v}_{B/A}$



$v_A = r\dot{\theta}$
$= r\omega$



$\sin\beta = \dfrac{1-\sin\theta}{\sqrt{2}}$

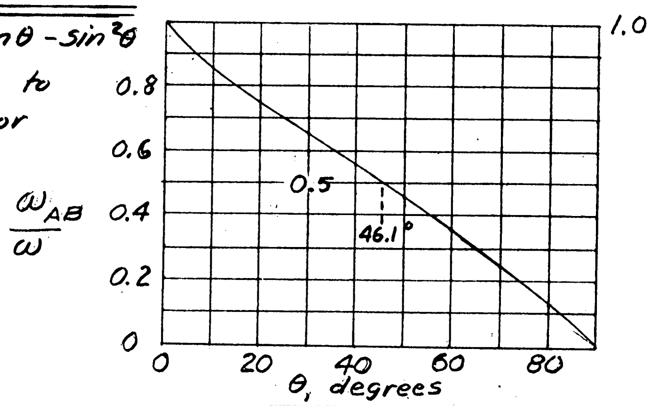
$\cos\beta = \sqrt{1-\sin^2\beta}$

$\cos\beta = \dfrac{1}{\sqrt{2}}\sqrt{1+2\sin\theta-\sin^2\theta}$

$v_{B/A}\cos\beta = v_A\cos\theta$ so $\omega_{AB} = \dfrac{v_{B/A}}{r\sqrt{2}} = \dfrac{r\omega}{r}\dfrac{\cos\theta}{\sqrt{1+2\sin\theta-\sin^2\theta}}$

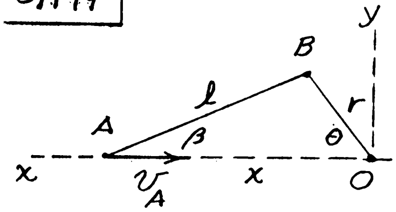
$\omega_{AB}/\omega = \dfrac{\cos\theta}{\sqrt{1+2\sin\theta-\sin^2\theta}}$

Set up program to solve & plot for $0 < \theta < 90°$.



$\omega_{AB} = 0.5\omega$
when $\theta = 46.1°$

*5/199



$x = l\cos\beta + r\cos\theta$

$r\sin\theta = l\sin\beta$

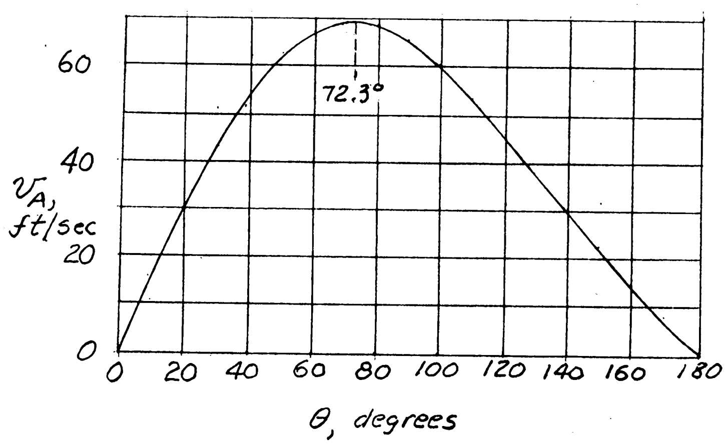
$v_A = -\dot{x} = l\dot{\beta}\sin\beta + r\dot{\theta}\sin\theta$

$r\dot{\theta}\cos\theta = l\dot{\beta}\cos\beta$

$$\dot{\beta} = \frac{r}{l}\frac{\dot{\theta}\cos\theta}{\sqrt{1-\sin^2\beta}} = \frac{\omega\cos\theta}{\sqrt{(l/r)^2 - \sin^2\theta}}$$

$$v_A = l\left[\frac{\omega\cos\theta}{\sqrt{(l/r)^2-\sin^2\theta}}\right]\frac{r}{l}\sin\theta + r\omega\sin\theta = r\omega\sin\theta\left(1 + \frac{\cos\theta}{\sqrt{(l/r)^2-\sin^2\theta}}\right)$$

From Sample Problem 5/15 substitute
$l = 14/12$ ft, $r = 5/12$ ft, $\omega = 1500(2\pi)/60 = 157.1$ rad/sec & get
$v_A = 65.45\sin\theta\left(1 + \dfrac{\cos\theta}{\sqrt{7.84 - \sin^2\theta}}\right)$, Set up computer program & solve for $0 < \theta < 180°$



$(v_A)_{max} = 69.6$ ft/sec at $\theta = 72.3°$

By symmetry $(v_A)_\theta = -(v_A)_{-\theta}$

***5/200** From the results of Prob. 5/199, we may write

$$a_A = \dot{v}_A = \frac{d}{dt}\left\{ r\omega\sin\theta + r\omega\frac{\sin\theta\cos\theta}{\sqrt{(l/r)^2 - \sin^2\theta}} \right\}$$

$$= r\omega\left\{ \dot{\theta}\cos\theta + \frac{\sqrt{(l/r)^2 - \sin^2\theta}\,(\dot{\theta}\cos 2\theta) - \frac{1}{2}\sin 2\theta \dfrac{(-\frac{1}{2}\sin 2\theta)\dot{\theta}}{\sqrt{(l/r)^2 - \sin^2\theta}}}{(l/r)^2 - \sin^2\theta} \right\}$$

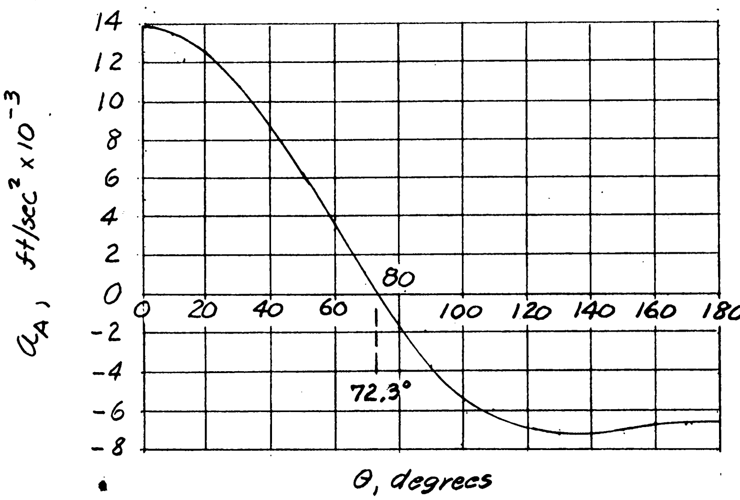
which reduces to

$$a_A = r\omega^2\left[\cos\theta + \frac{r}{l}\,\frac{1 - 2\sin^2\theta + \frac{r^2}{l^2}\sin^4\theta}{\left(1 - \frac{r^2}{l^2}\sin^2\theta\right)^{3/2}}\right]$$

From Sample Problem 5/15 substitute
$l = 14/12$ ft, $r = 5/12$ ft, $\omega = 1500(2\pi)/60 = 157.1$ rad/sec & get

$$a_A = 1.028(10^4)\left[\cos\theta + 0.357\,\frac{1 - 2\sin^2\theta + 0.1276\sin^4\theta}{(1 - 0.1276\sin^2\theta)^{3/2}}\right]$$

Set up computer program
& solve for $0 < \theta < 180°$



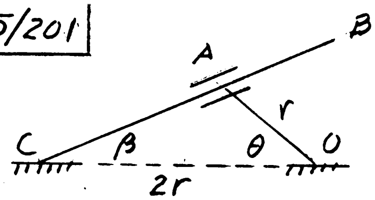
$(a_A)_{max} = 13.95(10^3)$ ft/sec^2 at $\theta = 0$

$a_A = -6.61(10^3)$ ft/sec^2 at $\theta = 180°$

$a_A = 0$ at $\theta = 72.3°$

$(a_A)_{+\theta} = +(a_A)_{-\theta}$

***5/201**



$r \sin\theta = (2r - r\cos\theta)\tan\beta$

$\tan\beta\,(2-\cos\theta) = \sin\theta$

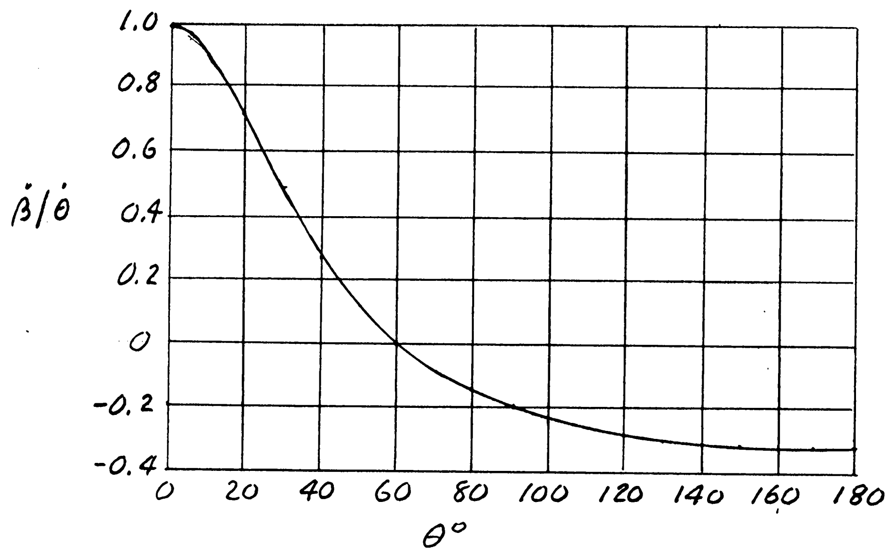
$\cos^2\beta = \dfrac{(2-\cos\theta)^2}{(2-\cos\theta)^2+\sin^2\theta} = \dfrac{(2-\cos\theta)^2}{5-4\cos\theta}$

$\dot\beta \sec^2\beta\,(2-\cos\theta) + \tan\beta\,(\dot\theta\sin\theta) = \dot\theta\cos\theta$

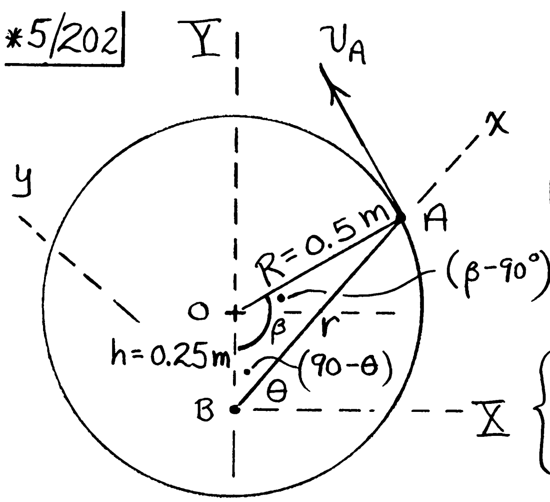
$\dot\beta = \dot\theta\left(\cos\theta - \dfrac{\sin^2\theta}{2-\cos\theta}\right)\dfrac{\cos^2\beta}{2-\cos\beta} = \dfrac{2\cos\theta-1}{5-4\cos\theta}\dot\theta$

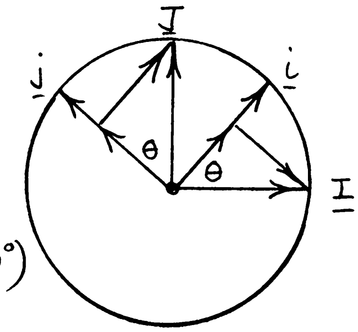
$\dot\beta/\dot\theta = \dfrac{2\cos\theta-1}{5-4\cos\theta}$

When $OA \perp$ to BC, $\beta = 30^\circ$, and v_A is along CB so $\dot\beta = 0$



*5/202





$$\begin{cases} \underline{I} = \underline{i}\cos\theta - \underline{j}\sin\theta \\ \underline{J} = \underline{i}\sin\theta + \underline{j}\cos\theta \end{cases}$$

$$r^2 = h^2 + R^2 - 2hR\cos\beta$$

$$\frac{\sin(90-\theta)}{R} = \frac{\cos\theta}{R} = \frac{\sin\beta}{r}, \quad \theta = \cos^{-1}\left[\frac{R}{r}\sin\beta\right]$$

$$\underline{v}_A = R\dot{\beta}\left[-\sin(\beta - 90°)\underline{I} + \cos(\beta - 90°)\underline{J}\right]$$

$$\underline{a}_A = R\dot{\beta}^2\left[-\cos(\beta - 90°)\underline{I} - \sin(\beta - 90°)\underline{J}\right]$$

Substitute the above transformation equations into the expressions for $\underline{v}_A$ & $\underline{a}_A$ and simplify to obtain (with c = cos, s = sin)

$$\underline{v}_A = R\dot{\beta}\left\{\left[-c\theta\, s(\beta-90^\circ)+s\theta\, c(\beta-90^\circ)\right]\underline{i} + \left[s\theta\, s(\beta-90^\circ)+c\theta\, c(\beta-90^\circ)\right]\underline{j}\right\}$$

$$\underline{a}_A = R\dot{\beta}^2\left\{\left[-c\theta\, c(\beta-90^\circ)-s\theta\, s(\beta-90^\circ)\right]\underline{i} + \left[s\theta\, c(\beta-90^\circ)-c\theta\, s(\beta-90^\circ)\right]\underline{j}\right\}$$

Eqs. 5/12 & 5/14, Bxy attached to BD:

$$\begin{cases} \underline{v}_A = \cancelto{0}{\underline{v}_B} + \underline{\omega}\times\underline{r} + \underline{v}_{rel} \\ \underline{a}_A = \cancelto{0}{\underline{a}_B} + \underline{\alpha}\times\underline{r} + \underline{\omega}\times(\underline{\omega}\times\underline{r}) + 2\underline{\omega}\times\underline{v}_{rel} + \underline{a}_{rel} \end{cases}$$

$$\begin{cases} \text{Note: } \underline{r} = r\underline{i}, \quad \underline{v}_{rel} = v_{rel}\underline{i} \\ \underline{a}_{rel} = a_{rel}\underline{i}, \quad \underline{\omega} = \omega\underline{k}, \quad \underline{\alpha} = \alpha\underline{k} \end{cases}$$

$$\Rightarrow v_{rel} = v_{A_x}, \quad \omega = \frac{v_{A_y}}{r}, \quad a_{rel} = a_{A_x} + r\omega^2,$$

and $\alpha = \frac{1}{r}\left(a_{A_y} - 2\omega v_{rel}\right)$

Program above equations & plot: