# INSTRUCTOR'S MANUAL

To Accompany

# ENGINEERING MECHANICS - DYNAMICS

Volume 2

Fifth Edition, 2002

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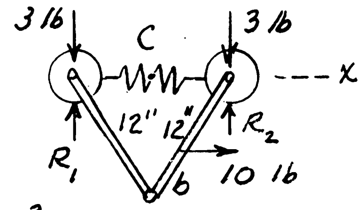
## USE OF THE INSTRUCTOR'S MANUAL

The problem solution portion of this manual has been prepared for the instructor who wishes to occasionally refer to the authors' method of solution or who wishes to check the answer of his (her) solution with the result obtained by the authors. In the interest of space and the associated cost of educational materials, the solutions are very concise. Because the problem solution material is not intended for posting of solutions or classroom presentation, the authors request that it not be used for these purposes.

In the transparency master section there are approximately 65 solved problems selected to illustrate typical applications. These problems are different from and in addition to those in the textbook. Instructors who have adopted the textbook are granted permission to reproduce these masters for classroom use.

4/1 | $C$ = mass center of system




$\Sigma F_x = m\bar{a}_x$ : $\bar{a}_x = a_c = \dfrac{10}{6/32.2}$

$$\underline{a_c = 53.7 \ ft/sec^2}$$

Dimension $b$ has no influence on $\Sigma F_x$ but it would influence $R_1$ & $R_2$.

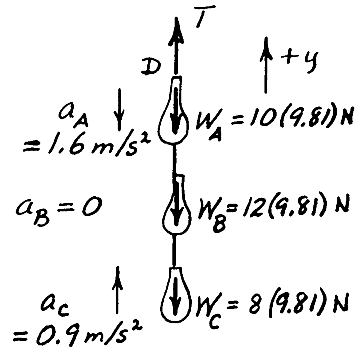
4/2

For system $\Sigma F_y = \Sigma m_i a_i$

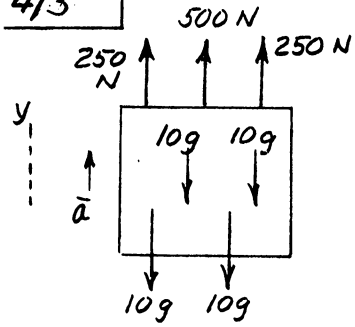
$T - 9.81(10+12+8)$

$= 10(-1.6) + 12(0) + 8(0.9)$

$T - 294 = -8.8$, $\underline{T = 286 \text{ N}}$



4/3




$$\Sigma F_y = m\bar{a}_y$$

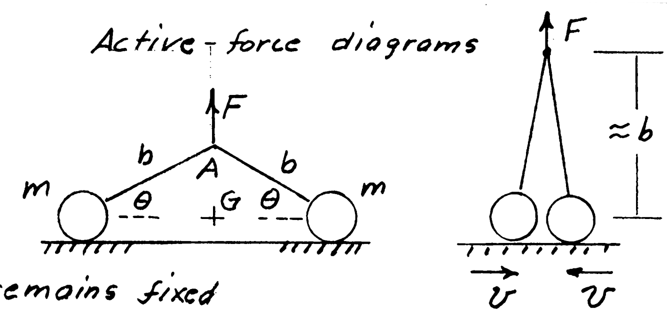
$$500 + 250 + 250 - 40(9.81) = 40\bar{a}$$

$$40\bar{a} = 1000 - 392$$

$$\underline{\bar{a} = 15.19 \text{ m/s}^2}$$

4/4

Active-force diagrams




Mass center remains fixed so long as $\underline{F < 2mg}$

For system $U = \Delta T$ : $F(b - b\sin\theta) = 2(\frac{1}{2}mv^2)$

$$v = \sqrt{\frac{Fb}{m}(1 - \sin\theta)}$$

**4/5**

$$\underline{F}_{av} = \frac{\Delta \underline{G}}{\Delta t} = \left[(3.7-3.4)\underline{i} + (-2.2+2.6)\underline{j} + (4.9-4.6)\underline{k}\right]/0.2$$

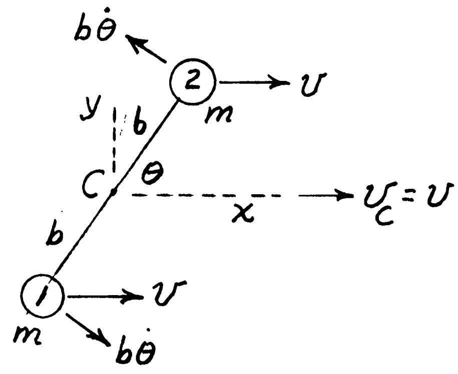
$$= 1.5\underline{i} + 2.0\underline{j} + 1.5\underline{k} \text{ N}$$

$$F = |\underline{F}_{av}| = \sqrt{1.5^2 + 2.0^2 + 1.5^2} = \underline{2.92 \text{ N}}$$

4/6

For sphere 1,

$$\underline{G}_1 = m\left[(v + b\dot{\theta}\sin\theta)\underline{i} - (b\dot{\theta}\cos\theta)\underline{j}\right]$$




For sphere 2

$$\underline{G}_2 = m\left[(v - b\dot{\theta}\sin\theta)\underline{i} + (b\dot{\theta}\cos\theta)\underline{j}\right]$$

$$\underline{G} = \underline{G}_1 + \underline{G}_2 = m\left[v + v\right]\underline{i} = \underline{2mv\,\underline{i}}$$

4/7

$$\underline{H}_O = \underline{H}_G + \bar{\underline{r}} \times \underline{G} \quad , \quad \underline{G} = 3(3\underline{i} + 4\underline{j}) \text{ kg}\cdot\text{m/s}$$

$$= 1.20\underline{k} + (0.4\underline{i} + 0.3\underline{j}) \times 3(3\underline{i} + 4\underline{j})$$

$$= 1.20\underline{k} + 3(1.6\underline{k} - 0.9\underline{k})$$

$$= 1.20\underline{k} + 3(0.7\underline{k}) = \underline{3.3\underline{k} \text{ kg}\cdot\text{m}^2/\text{s}}$$

4/8

$\Sigma M_O = \dot{H}_O$ where O-O is the axis of rotation

$$M = \frac{dH_O}{dt}, \quad \int_0^t M\,dt = \int_0^{H_O} dH_O = H_O$$

$$Mt = 4m(r\omega)r, \quad t = \frac{4mr^2\omega}{M}$$

4/9

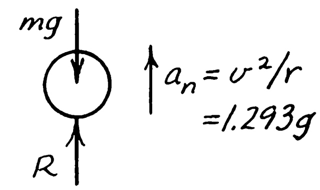
For the system of two spheres

$U' = 0 = \Delta V_g + \Delta T$

$0 = -mgr - mgr\left(1 - \frac{1}{\sqrt{2}}\right) + \frac{1}{2}2mv^2, \quad v^2 = gr\left(2 - \frac{1}{\sqrt{2}}\right)$

$\underline{v = 1.137\, gr}$

Sphere 1 just prior to reaching A




$\Sigma F_y = ma_y: \quad a_y = a_n = 1.293g$

$R - mg = m(1.293g)$

$\underline{R = 2.29\, mg}$

**4/10**  $\Sigma \underline{M}_O = \dot{\underline{H}}_O$ , $\underline{M}_{O_{av.}} = \dfrac{\Delta \underline{H}_O}{\Delta t}$

$$(\underline{M}_O)_{av} = \frac{1}{0.1}\left[(3.67-3.65)\underline{i} + (4.30-4.27)\underline{j} + (-5.30+5.36)\underline{k}\right]$$

$$= \frac{1}{0.1}(0.02\underline{i} + 0.03\underline{j} + 0.06\underline{k}) =$$

$$= (2\underline{i} + 3\underline{j} + 6\underline{k})10^{-1}\ N\cdot m$$

$$|\underline{M}_O|_{av} = \underline{0.7\ N\cdot m}$$

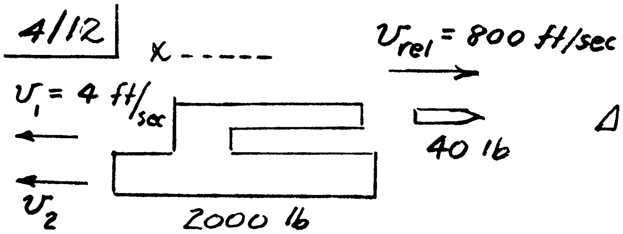
4/11 $\int_0^t M_z\,dt = H_{z_2} - H_{z_1}$ , $H_z = \sum m_i r_i (r_i \dot{\theta})$

$H_z = 2(3)(0.3)^2\dot{\theta} + 2(3)(0.5)^2\dot{\theta} = 2.04\dot{\theta}$

so $30t = 2.04(20-[-20]) = 81.6$

$\underline{t = 2.72\text{ s}}$

**4/12**




$\Delta G_x = 0, \quad G_1 = G_2$

$$\frac{1}{g}(2000 + 40)4 = \frac{1}{g}\left(2000\, v_2 - 40\left[800 - v_2\right]\right)$$

$$8160 = 2040\, v_2 - 32000$$

$$\underline{v_2 = 19.69 \text{ ft/sec}}$$

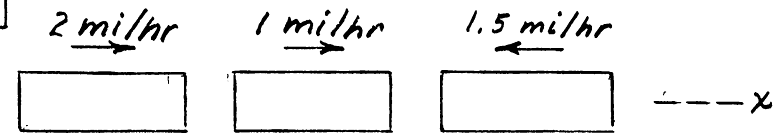
$\underline{4/13}$ For entire system $\Delta G_x = 0$, $x$ horiz.

$$(300 + 400 + 100)v - (300 \times 0.6 - 400 \times 0.3 + 100 \times 1.2 \cos 30°) = 0$$

$800 v = 163.9$, $\underline{v = 0.205 \text{ m/s}}$

Momentum is conserved regardless of sequence of events, so final velocity would be the same.

4/14




$W_A = 130{,}000$ lb $\quad W_B = 100{,}000$ lb $\quad W_C = 150{,}000$ lb

$\Sigma F_x = 0$ for system so $\Delta G_x = 0$

$$(130\times 2 + 100\times 1 - 150\times 1.5)\frac{44}{30}\frac{10^3}{32.2} - (130+100+150)v\frac{44}{30}\frac{10^3}{32.2} = 0$$

$$v = \frac{260+100-225}{130+100+150} = \underline{0.355 \text{ mi/hr}}$$

$$\% \text{ loss of energy} = \frac{T_i - T_f}{T_i}100 = 100\left(1 - \frac{T_f}{T_i}\right) = n$$

$$n = 100\left\{1 - \frac{\frac{1}{2g}(130+100+150)(0.355)^2}{\frac{1}{2g}(130\times 2^2 + 100\times 1^2 + 150\times 1.5^2)}\right\} = 100\left(1 - \frac{47.96}{957.5}\right)$$

$$\underline{n = 95.0\,\%}$$

4/15 Let $P_p$ = power to move 10 people

$P_b$ = " " " 3 boys

Velocity of people vertically up is $\frac{20}{40} = 0.5$ ft/sec

" " boys " down is $1 - 0.5 = 0.5$ ft/sec

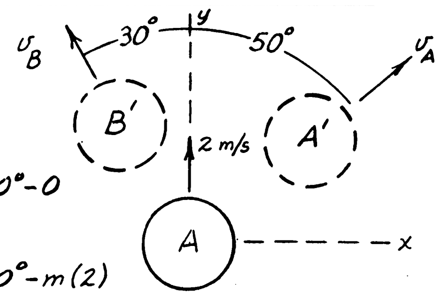
$P = dV_g/dt$, $P_p = \frac{10(150)(0.5)}{550} = 1.364$ hp

$P_b = \frac{3(120)(-0.5)}{550} = -0.327$ hp

$P_{fr} = 2.2$ hp

Thus $P = 1.364 - 0.327 + 2.2 = \underline{3.24 \text{ hp}}$

4/16




For the system as a whole

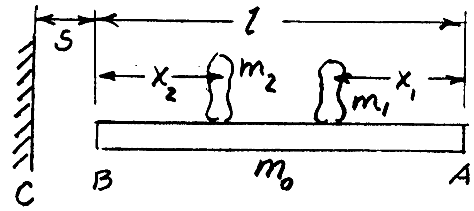
$\Sigma F_x = \Sigma F_y = 0$ so

$\Delta G_x = 0: \; -m v_B \sin 30° + m v_A \sin 50° - 0 = 0$

$\Delta G_y = 0: \; m v_B \cos 30° + m v_A \cos 50° - m(2) = 0$

Solve & get $\underline{v_A = 1.015 \text{ m/s}}$, $\underline{v_B = 1.556 \text{ m/s}}$

4/17 With respect to C, $\Sigma m_i x_i$ = constant



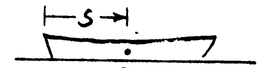

$m_1 l + m_2(0) + m_0 \frac{l}{2} =$
$m_1(s+l-x_1) + m_2(s+x_2)$
$+ m_0(s+\frac{l}{2})$

Simplify & get $s = \dfrac{m_1 x_1 - m_2 x_2}{m_0 + m_1 + m_2}$

But they meet when $x_2 + x_1 = l$ so

$$s = \frac{(m_1 + m_2)x_1 - m_2 l}{m_0 + m_1 + m_2}$$

**4/18** With neglect of hydraulic forces linear momentum is conserved & velocity $v_2 = v_1 = 1$ knot. Center of mass does not change position with respect to reference axes moving with constant speed of 1 knot.



G of skiff

Thus $(\Sigma m_i x_i)_1 = (\Sigma m_i x_i)_2$

$$\frac{1}{32.2}\left[120(2) + 180(8) + 160(16) + 300(s)\right]$$

$$= \frac{1}{32.2}\left[120(14+x) + 180(4+x) + 160(10+x) + 300(s+x)\right]$$

$$4240 = 4000 + 760x\,, \quad x = \frac{240}{760} = \underline{0.316 \text{ ft}}$$

Timing & sequence of changed positions does not affect final result because all forces are internal.

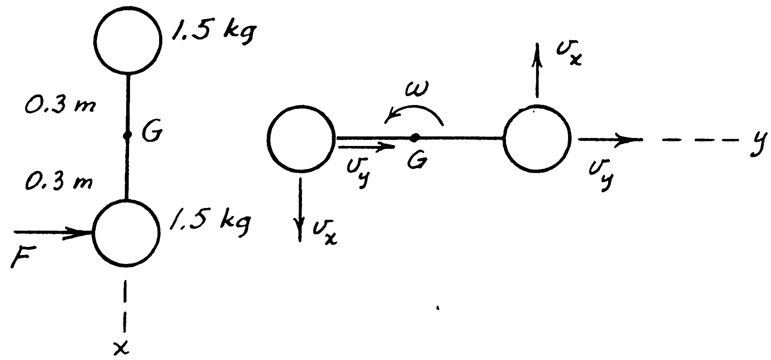
4/19 $\underline{H}_O = \underline{H}_G + \bar{\rho} \times m\bar{v}$

$\underline{H}_G = \Sigma \rho_i \times m_i \dot{\rho}_i = 2r \times m \times r\omega \underline{k} = 2mr^2\omega \underline{k}$

$\bar{\rho} \times 2m\bar{v} = (x\underline{i} + y\underline{j}) \times 2mv\underline{i} = -2mvy\underline{k}$

so $\underline{H}_O = 2mr^2\omega\underline{k} - 2mvy\underline{k}$, $\underline{H}_O = 2m(r^2\omega - vy)\underline{k}$

4/20




$$\int \Sigma F_x \, dt = 0 \quad \text{so} \quad \Delta G_x = 0$$

$$\int \Sigma F_y \, dt = \Delta G_y: \quad 10 = 2(1.5)v_y, \quad v_y = 3.33 \text{ m/s}$$

$$\int \Sigma M_G \, dt = \Delta H_G: \quad 10(0.3) = 2(1.5)v_x(0.3), \quad v_x = 3.33 \text{ m/s}$$

$$v = 3.33\sqrt{2} = \underline{4.71 \text{ m/s both spheres}}$$

4/21

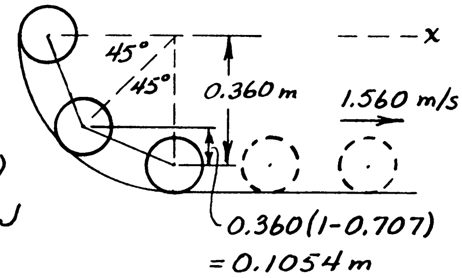
$U'_{1-2} = \Delta T + \Delta V_g$

$= 3(\frac{1}{2} \times 2.75 \times 1.560^2) - 0$

$- 2.75 \times 9.81(0.360 + 0.1054)$

$= 10.04 - 12.56 = -2.52$ J

so loss is $\underline{\Delta Q = 2.52 \text{ J}}$




$I_x = \int \Sigma F_x \, dt = \Delta G_x = G_2 - G_1, \; G_2 = 3mv = 3(2.75)(1.560)$

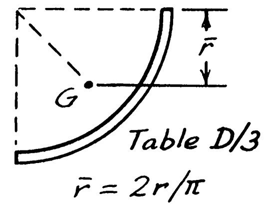
$= 12.87$ N·s, $G_1 = 0$

$\underline{I_x = 12.87 \text{ N·s}}$

4/22 $\Delta T = \Delta V_e = 0$ so $U' = \Delta V_g = -\Delta Q$

$$\Delta V_g = -mg\bar{r}, \quad |\Delta V_g| = \frac{\pi r \rho}{2} g \frac{2r}{\pi} = \underline{\rho g r^2}$$

$$= \Delta Q$$




Energy is lost in the generation of heat and sound upon impact of rope with fixed guide.

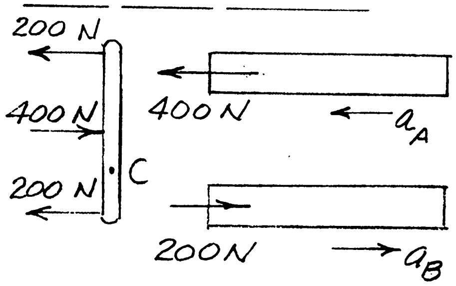
4/23 (a) $\Sigma F_x = m\bar{a}_x$; $F = 2m\bar{a}$, $\underline{\bar{a} = F/2m}$

(b) $H_G = 2m\left(\frac{L}{2}\right)^2\dot{\theta}$, $\dot{H}_G = mL^2\ddot{\theta}/2$

$\Sigma M_G = \dot{H}_G$; $Fb = mL^2\ddot{\theta}/2$, $\underline{\ddot{\theta} = \dfrac{2Fb}{mL^2}}$

4/24 For system of 2 bars & lever, mass center is in line with C & has the same acceleration as C.

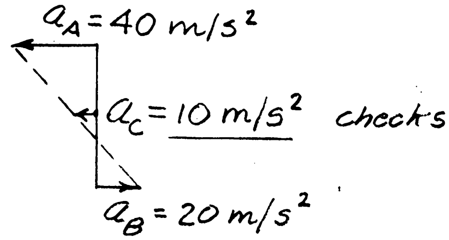
$\Sigma F = m a_C$ ; $200 = 2(10) a_C$ , $\underline{a_C = 10 \text{ m/s}^2}$



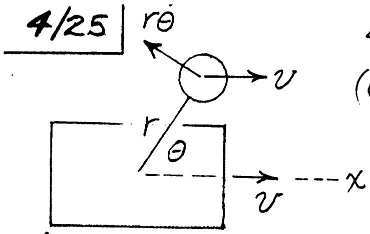

$\Sigma F = ma$

$400 = 10\, a_A$ , $a_A = 40 \frac{m}{s^2}$

$200 = 10\, a_B$ , $a_B = 20 \frac{m}{s^2}$



4/25




$\Sigma F_x = 0$ for system so $\Delta G_x = 0$

$(G_x)_{\theta=0} = (20+5)(0.6) = 15.0\ N\cdot s$

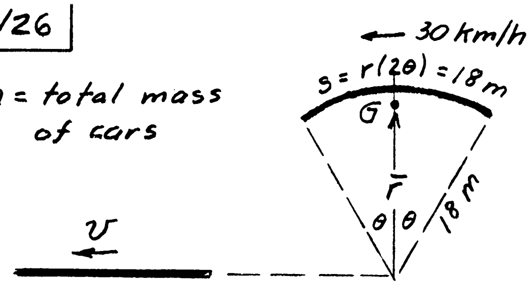
$(G_x)_{\theta=60^\circ} = (20+5)v - 5(1.6)\sin 60^\circ$
$= 25v - 6.93\ N\cdot s$

$r\dot\theta = 0.4(4) = 1.6\ m/s$

Thus $15.0 = 25v - 6.93$, $v = 21.9/25 = \underline{0.877\ m/s}$

4/26

m = total mass of cars




$\theta = \frac{s}{2r} = \frac{18}{2(18)} = \frac{1}{2}$ rad

$= \frac{1}{2}\,\frac{180}{\pi} = 28.65°$

$\bar{r} = \frac{r\sin\theta}{\theta} = \frac{18\sin 28.65°}{1/2}$

$= 17.26$ m

For system $\Delta T + \Delta V_g = 0$

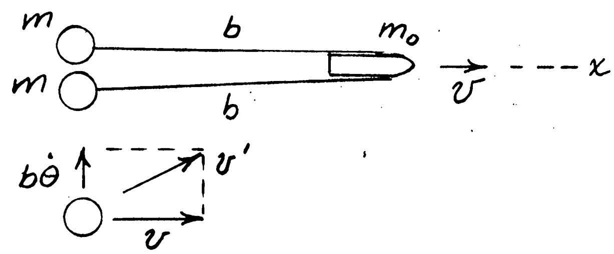
$\frac{1}{2}m\left(v^2 - \left[\frac{30}{3.6}\right]^2\right) - mg(17.26) = 0$

$v^2 = (30/3.6)^2 + 2(9.81)(17.26) = 69.4 + 338.6$

$= 408 \ (m/s)^2$

$v = 20.2$ m/s or $v = 20.2(3.6) = \underline{72.7 \text{ km/h}}$

4/27




For system $\Delta G_x = 0$: $(m_0 v + 2mv) - m_0 v_0 = 0$

$$v = \frac{m_0}{m_0 + 2m} v_0$$

$U = \Delta T$: $\quad 0 = \frac{1}{2} m_0 v^2 + 2\left[\frac{1}{2} m\,(v^2 + b^2\dot{\theta}^2)\right] - \frac{1}{2} m_0 v_0^2$

$$(m_0 + 2m)v^2 + 2mb^2\dot{\theta}^2 = m_0 v_0^2$$

Substitute $v$ & get $\dfrac{m_0^2 v_0^2}{m_0 + 2m} + 2mb^2\dot{\theta}^2 = m_0 v_0^2$

Solve for $\dot{\theta}$ & get $\quad \dot{\theta} = \dfrac{v_0}{b}\sqrt{\dfrac{m_0}{m_0 + 2m}}$

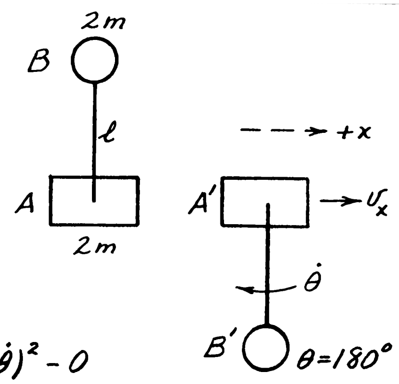
4/28

For entire system,

$$\int \Sigma F_x \, dt = \Delta G_x = G_{A'} + G_{B'} - 0$$

$$0 = 2m v_x + 2m(v_x - l\dot{\theta})$$

$$2v_x = l\dot{\theta} \qquad \text{----(1)}$$




For entire system,

$$U'_{1-2} = \Delta T + \Delta V_g$$

$$\Delta T = \frac{1}{2}(2m)v_x^2 + \frac{1}{2}(2m)(v_x - l\dot{\theta})^2 - 0$$

$$= m(2v_x^2 - 2l\dot{\theta}v_x + l^2\dot{\theta}^2)$$

$$\Delta V_g = -2mg(2l) = -4mgl$$

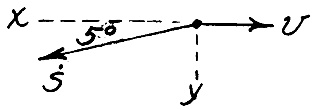
$$U'_{1-2} = 0 \text{ so } 0 = m(2v_x^2 - 2l\dot{\theta}v_x + l^2\dot{\theta}^2) - 4mgl$$

$$\text{or } 2v_x^2 - 2l\dot{\theta}v_x + l^2\dot{\theta}^2 = 4gl \qquad \text{---(2)}$$

Combine (1) & (2): $2v_x^2 - 4v_x^2 + 4v_x^2 = 4gl$, $v_x^2 = 2gl$, $\underline{v_x = \sqrt{2gl}}$

$$\dot{\theta} = 2v_x/l = 2\sqrt{2gl}/l = \underline{2\sqrt{\frac{2g}{l}}}$$

**4/29** System is conservative so $\Delta T + \Delta V_g = 0$




Flatcar; $\Delta T = \frac{1}{2} m v^2 - 0$

$$= \frac{1}{2} \frac{50{,}000}{32.2} v^2$$

$$\Delta V_g = 0$$

Vehicle; $\Delta T = \frac{1}{2} m v^2 - 0 = \frac{1}{2} \frac{15000}{32.2} \left[ (\dot{s} \cos 5° - v)^2 + (\dot{s} \sin 5°)^2 \right] - 0$

$$\Delta V_g = -W \Delta h = -15{,}000\,(40 \sin 5°)$$

Thus $776.4\, v^2 + 232.9 \left[ (\dot{s} \cos 5° - v)^2 + (\dot{s} \sin 5°)^2 \right] - 52290 = 0$ ----- (1)

Also for system, $\Sigma F_x = 0$ so $\Delta G_x = 0$

$$\frac{15{,}000}{32.2} (\dot{s} \cos 5° - v) - \frac{50{,}000}{32.2} v = 0$$

$\dot{s} \cos 5° - v = 3.33\, v$ & $\dot{s} \sin 5° = 4.33\, v \tan 5° = 0.379\, v$

Substitute into (1) & get

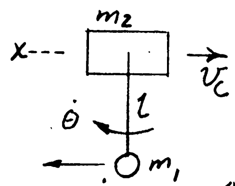
$$776.4\, v^2 + 232.9 \left[ (3.33\, v)^2 + (0.379\, v)^2 \right] - 52290 = 0$$

$$v^2 (776.4 + 2588 + 33.5) = 52290$$

$v^2 = 15.39\ (\text{ft/sec})^2$, $\underline{v = 3.92\ \text{ft/sec}}$

▶4/30 | $\Delta E = 0;\ \Delta V_g = -m_1 g l(1-\cos\theta)$

$$\Delta T = \tfrac{1}{2}m_1(l\dot\theta - v_c)^2 + \tfrac{1}{2}m_2 v_c^2$$




Thus

$$\tfrac{1}{2}m_1(l\dot\theta - v_c)^2 + \tfrac{1}{2}m_2 v_c^2 - m_1 g l(1-\cos\theta) = 0 \quad (1)$$

$v_b = l\dot\theta - v_c$ Also for system $\Sigma F_x = 0$ so $\Delta G_x = 0$

$$m_2 v_c - m_1(l\dot\theta - v_c) = 0 \quad (2)$$

Substitute (2) into (1) & get

$$\tfrac{1}{2}m_1\left(\frac{m_2 v_c}{m_1}\right)^2 + \tfrac{1}{2}m_2 v_c^2 = m_1 g l(1-\cos\theta)$$

$$v_c^2 = \frac{2gl(1-\cos\theta)}{(m_2/m_1)^2 + (m_2/m_1)}, \quad \underline{v_c = \sqrt{\frac{2gl(1-\cos\theta)}{(m_2/m_1)(1+m_2/m_1)}}}$$

$$v_{b/c} = l\dot\theta = v_c\left(1 + \frac{m_2}{m_1}\right), \quad v_{b/c} = \sqrt{\left(1+\frac{m_1}{m_2}\right)2gl(1-\cos\theta)}$$

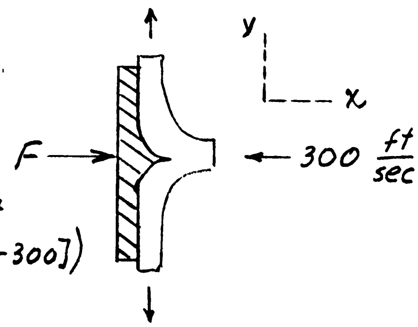
4/31 For max. speed, accel. = 0, so T = resistance = 225 lb.

$T = m'u$: $225 = \frac{3.5}{32.2} u$, $\underline{u = 2070 \text{ ft/sec}}$

4/32 $m' = \frac{\mu}{g} Q$

$= \frac{0.0753}{32.2}(6.50)$

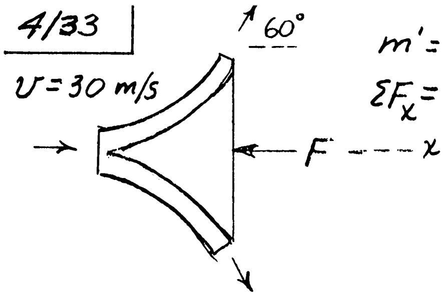
$= 0.0152 \text{ lb} \times \text{sec}/\text{ft}$




$\Sigma F_x = m' \Delta v_x : F = 0.0152(0 - [-300])$

$= \underline{4.56 \text{ lb}}$

4/33




$m' = \rho Q = 1000(0.05) = 50$ kg/s

$\Sigma F_x = m' \Delta v_x$; $-F = 50(30\cos 60^\circ - 30)$

$\underline{F = 750 \text{ N}}$

4/34 | Resistance $R$ equals net thrust $T$

where $T = m'(u - v)$

Nozzle velocity $u = Q/A = \dfrac{0.082}{\frac{\pi(0.050)^2}{4}} = 41.8\ m/s$

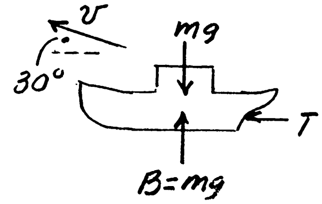
Density of salt water, Table D-1, $\rho = 1030\ kg/m^3$

$m' = \rho Q = 1030(0.082) = 84.5\ kg/s$

$v = 70\ \dfrac{1000}{3600} = 19.44\ m/s$

$R = T = 84.5(41.8 - 19.44) = \underline{1885\ N}$

4/35




$v = 40$ m/s , $m' = \rho Q = 1030(0.080)$

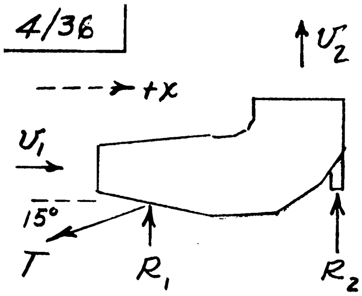
$= 82.4$ kg/s

$\Sigma F = m'\Delta v$ :

$T = 82.4\,(40 \cos 30° - 0) = 2850$ N

or $\underline{T = 2.85 \text{ kN}}$

4/36




$$\Sigma F_x = m' \Delta v_x$$

$$-T\cos 15^\circ = (43 + 0.8)(0 - 720)$$

$$T = 32\,600 \text{ N}$$

or $\underline{T = 32.6 \text{ kN}}$

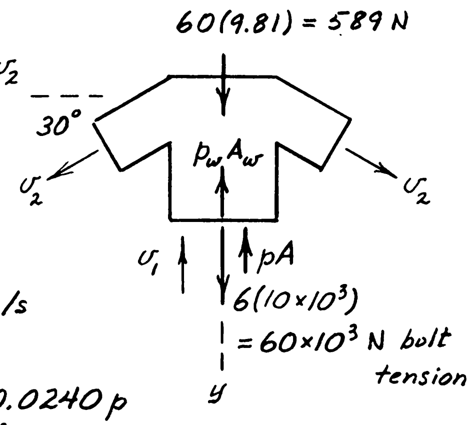
**4/37**

$Q = Av$: $\frac{30}{60} = \frac{2(\pi \times 0.1^2)}{4} v_2$

$v_2 = 31.8$ m/s

$v_1 = 2v_2 \frac{A_2}{A_1}$, $v_1 = 2(31.8)\left(\frac{0.1}{0.25}\right)^2$
$= 10.19$ m/s

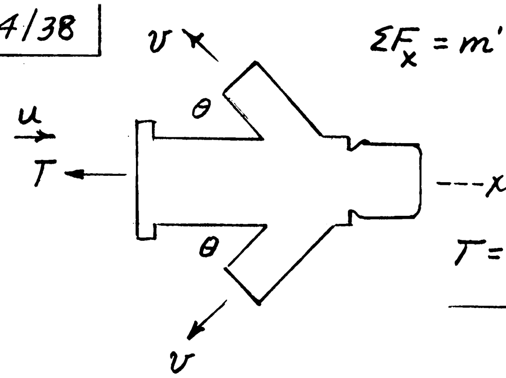
$m' = \rho Q = 1030 \times \frac{30}{60} = 515$ kg/s




$\Sigma F_y = m'\Delta v_y$: $60\times10^3 + 589 - 0.0240p$
$- 550(10^3)\frac{\pi \times 0.25^2}{4}$
$= 515(31.8 \sin 30° - [-10.19])$

$p = 840(10^3)$ Pa or $\underline{p = 840 \text{ kPa}}$

4/38

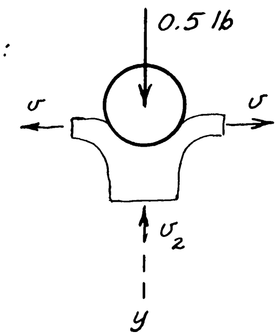



$$\Sigma F_x = m' \Delta v_x \ ,$$

$$-T = m'(-v\cos\theta - u)$$

$$T = \frac{\pi d^2}{4} \rho u \left(v\cos\theta + u\right)$$

**4/39** Ball & stream just under it:




$\Sigma F_y = m' \Delta v_y$:

$m'$ at ball $= m'$ at nozzle

$= \rho A v = \dfrac{62.4}{32.2} \dfrac{\pi (0.5)^2}{4} \left(\dfrac{1}{12}\right)^2 \cdot 35$

$= 0.925$ lb-sec/ft

so $0.5 = 0.925 (0 - [-v_2])$

$v_2 = 5.41$ ft/sec

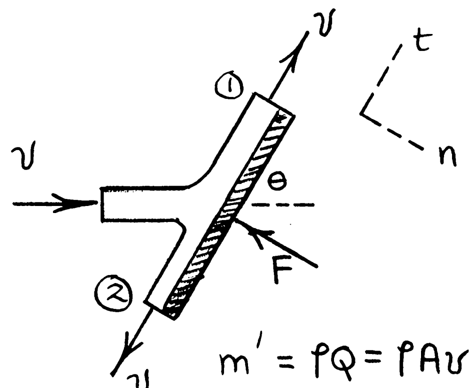
For water stream $\Delta V_g + \Delta T = 0$:

$mgh + \frac{1}{2} m (v_2^2 - v_1^2) = 0$,

$h = \dfrac{1}{2 \times 32.2} (35^2 - 5.41^2) = \underline{18.57 \text{ ft}}$

4/40

The system consists of the vane and the fluid shown. $Q$ is the volume rate of flow.




$m' = \rho Q = \rho A v$

For $Q_1$: $\begin{cases} \Delta v_n = 0 - v\sin\theta \\ \Delta v_t = v(1-\cos\theta) \end{cases}$

For $Q_2$: $\begin{cases} \Delta v_n = 0 - v\sin\theta \\ \Delta v_t = -v - v\cos\theta = -v(1+\cos\theta) \end{cases}$

For system, $\Sigma F_n = m'\Delta v_n$: $-F = \rho Q(0) - (\rho Q v\sin\theta)$ (1)

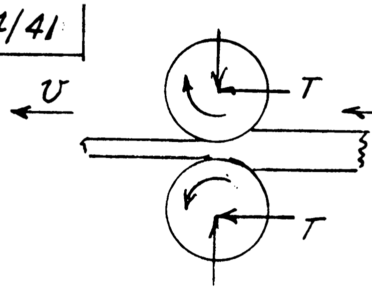
$\Sigma F_t = m'\Delta v_t$: $0 = \rho Q_1 v - \rho Q_1 v\cos\theta - \rho Q_2 v - \rho Q v\cos\theta$ (2)

(1): $F = \rho Q v\sin\theta$ or $\underline{F = \rho A v^2 \sin\theta}$

(2): $0 = Q_1(1-\cos\theta) - Q_2(1+\cos\theta)$

With $Q = Q_1 + Q_2$: $\begin{cases} Q_1 = \frac{Q}{2}(1+\cos\theta) \\ Q_2 = \frac{Q}{2}(1-\cos\theta) \end{cases}$

4/41




$v = \frac{25}{19}(0.4) = 0.526$ m/s

$\rho_{steel} = 7.83$ Mg/m$^3$

$m' = \rho A v$

$= 7.83(10^3)(25 \times 10^{-3})(1.2)(0.4)$

$= 94.0$ kg/s

$\Sigma F = m'\Delta v$: $2T = 94.0(0.526 - 0.4)$

$\underline{T = 5.93 \text{ N}}$

4/42 | $\Sigma F = \Sigma m' u$

With reversers in place,

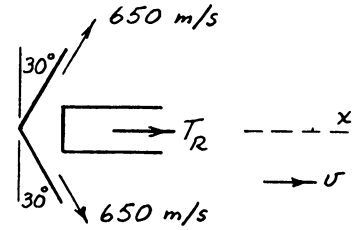
$T_R = m_g' u \sin 30° + m_a' v$

$T_R = (50+0.65)(650)\sin 30°$

$+ 50(55.6 - 0)$

$= 16460 + 2780$

$= 19\,240$ N




$v = 200/3.6 = 55.6$ m/s

Without reversers $T = m_g' u - m_a' v$

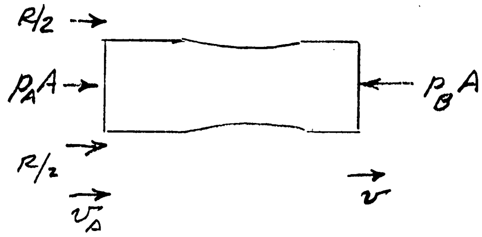
$T = (50+0.65)650 - 50(55.6)$

$= 32\,900 - 2780 = 30\,100$ N

so $n = \dfrac{19\,240}{30\,100} = \underline{0.638}$

**4/43** continuity requires $\rho_A A v_A = \rho_B A v$

so $v_A = v \rho_B / \rho_A$




$m' = \rho A v = \rho_B A v$ at $B$

$\Sigma F = m' \Delta v$; $\quad R + p_A A - p_B A = \rho_B A v (v - v_A)$

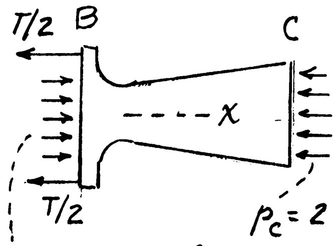
$$= \rho_B A v^2 (1 - \rho_B/\rho_A)$$

Also, $A = \pi d^2/4$, so

$$R = \rho_B \frac{\pi d^2}{4} v^2 \left(1 - \frac{\rho_B}{\rho_A}\right) + (p_B - p_A)\frac{\pi d^2}{4}$$

$$\underline{R = \frac{\pi d^2}{4}\left[\rho_B\left(1 - \frac{\rho_B}{\rho_A}\right) v^2 + (p_B - p_A)\right]}$$

**4/44** $A_C = \frac{\pi 4^2}{4(144)} = 0.0873\ ft^2$, $A_B = 4A_C = 0.349\ ft^2$




$p_C = 2\ lb/in.^2$

$p_B = 150\ lb/in.^2$

$\longrightarrow v_B = 50\ ft/sec$

$m' = \rho A v$

$m'_B = \frac{0.840}{32.2}(0.349)50 = 0.455\ \frac{lb\text{-}sec}{ft}$

$m'_C = \frac{0.0760}{32.2}(0.0873)v_C = 2.06(10^{-4})v_C$

$m'_B = m'_C$ so $v_C = \frac{0.455}{2.06(10^{-4})}$

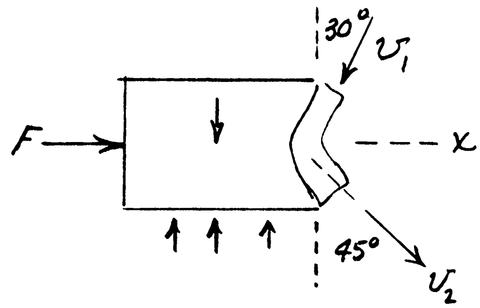
$= 2210\ ft/sec$

$\Sigma F_x = m'\Delta v_x$: $150(0.349)144 - 2(0.0873)(144) - T$

$= 0.455(2210 - 50)$

$\underline{T = 6530\ lb}$

4/45




$Q = Av$:

$\frac{1}{2}(231) = \frac{0.01^2\pi}{4} v_1$

$v_1 = 1.471(10^6)$ in./min

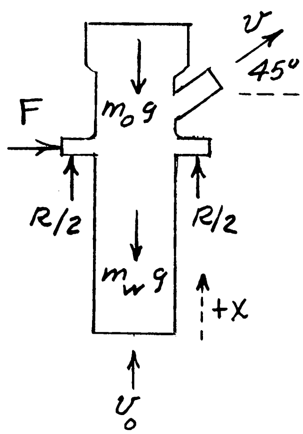
or $v_1 = 2042$ ft/sec & $v_2 = 0.60(2042) = 1225$ ft/sec

$\Sigma F_x = m'\Delta v_x$: $m' = \frac{231}{2}\frac{1}{60}\frac{1}{1728}\frac{68}{32.2} = 2.35(10^{-3})$ lb-sec/ft (slugs/sec)

$F = 2.35(10^{-3})(1225 \sin 45° - [-2042 \sin 30°])$

$F = 4.44$ lb

4/46




$\Sigma F_x = m' \Delta v_x$:

$R - m_o g - m_w g = \rho Q (v \cos 45° - v_o)$

$m_o = 310$ kg

Mass of water $m_w = \rho V$

$= 1000 \frac{\pi}{4} (0.2)^2 (6)$

$= 188.5$ kg

$Q = 0.125$ m$^3$/s

$A = \frac{\pi}{4}(0.1)^2 = 0.00785$ m$^2$

$A_o = \frac{\pi}{4}(0.25)^2 = 0.0491$ m$^2$

$v = Q/A = 0.125/0.00785 = 15.92$ m/s

$v_o = Q/A_o = 0.125/0.0491 = 2.55$ m/s

Thus $R - (310 + 188.5)\, 9.81 = 1000\,(0.125)(15.92 \cos 45° - 2.55)$

$= 1088$ N

$\underline{R = 5980 \text{ N}}$

4/47 $kx = m'\Delta v, \quad m' = \rho A v = 1000 \frac{\pi}{4}(0.030)^2 v$

$$= 0.7069 v$$

$$15000(0.150) = 0.7069 v (v - 0)$$

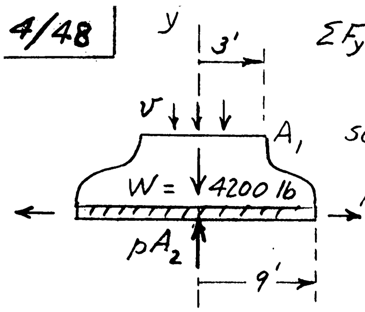
$$v^2 = 3183, \quad \underline{v = 56.4 \text{ m/s}}$$

$\Sigma M_A = m'vd; \; M = 15(150)(15 \sin 75° - 4.8 \cos 75°)$

$$= 2250(13.25) = 29\,800 \text{ N·m}$$

$$\text{or } \underline{M = 29.8 \text{ kN·m}}$$

**4/48**




$\Sigma F_y = m' \Delta v_y$ ; $m' = \rho A_1 v$

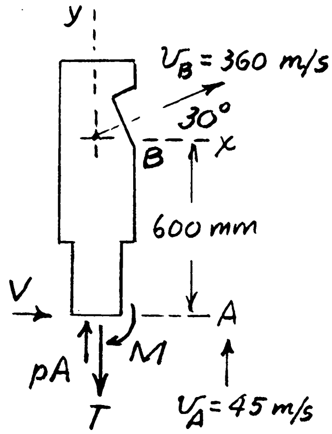
$\Delta v_y = 0 - (-v) = v$

so $pA_2 - W = \rho A_1 v^2$

$p(\pi \times 9^2)(144) - 4200 = \frac{0.076}{32.2} \pi \times 3^2 (150^2)$

$\underline{p = 0.1556 \text{ lb/in.}^2}$

4/49 | $m' = 6$ kg/s, $pA = 1400(10^3)(7500)(10^{-6}) = 10\,500$ N




$\Sigma F_x = m'\Delta v_x$: $V = 6(360\cos 30° - 0)$

$V = 1871$ N

or $\underline{V = 1.871 \text{ kN}}$

$\Sigma F_y = m'\Delta v_y$: $10\,500 - T = 6(360 \sin 30° - 45)$

$T = 9690$ N

or $\underline{T = 9.69 \text{ kN}}$

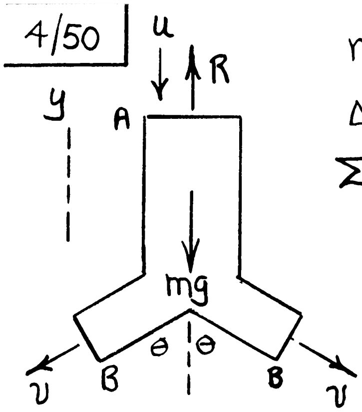
$\Sigma M_A = m'(v_2 d_2 - v_1 d_1)$

$M = 6(360\cos 30°[0.6] - 0)$

$= 1122$ N·m

or $\underline{M = 1.122 \text{ kN·m}}$

4/50




$m' = \rho A v_A = \rho \frac{\pi d^2}{4} u$

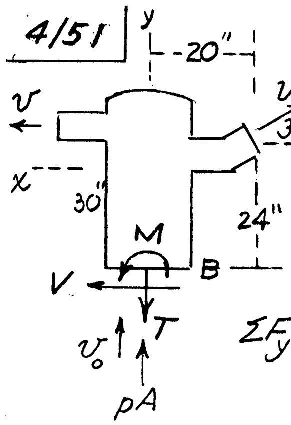
$\Delta v_y = -v\cos\theta - (-u) = u - v\cos\theta$

$\Sigma F_y = m' \Delta v_y$:

$R - mg = \rho \frac{\pi d^2}{4} u\,(u - v\cos\theta)$

$$R = mg + \rho \frac{\pi d^2}{4} u\,(u - v\cos\theta)$$

4/51




$v = \frac{\text{volume rate}}{\text{area}} = \frac{10/2}{0.040} = 125 \text{ ft/sec}$

For each outlet $m' = \frac{5 \times 62.4}{32.2}$

$= 9.69 \text{ lb-ft}^{-1}\text{-sec}$

$v_o = \frac{10}{0.75} = 13.33 \text{ ft/sec}$

$pA = 120(0.75)(144)$
$= 12{,}960 \text{ lb}$

$\Sigma F_y = m' \Delta v_y$; $-T + 12{,}960 = 9.69(0 - 13.33)$
$+ 9.69(+125 \sin 30° - 13.33)$
$= -129.2 + 476.5 - 12{,}960$

$\underline{T = 12{,}610 \text{ lb}}$

$\Sigma F_x = m' \Delta v_x$; $V = 9.69(125 - 0) + 9.69(-125 \cos 30° - 0)$
$= 1211 - 1049 = \underline{162.3 \text{ lb}}$

$\Sigma M_B = \Sigma m' v d$; $M = 9.69(125)\frac{30}{12} - 9.69(125 \cos 30°)\frac{24}{12}$
$+ 9.69(125 \sin 30°)\frac{20}{12}]$

$M = 3028 - 2098 + 1009 = \underline{1939 \text{ lb-ft}}$

4/52 For the truck and plow as a system:

$$\Sigma F_x = m'\Delta v_x: \quad P = \frac{60\,000}{60}\left[\frac{20}{3.6} - 0\right] = 5560 \text{ N}$$

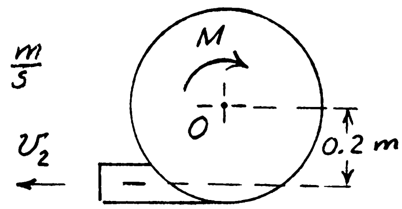
or $\underline{P = 5.56 \text{ kN}}$

$$\Sigma F_y = m'\Delta v_y: \quad R = \frac{60\,000}{60}\left[12\cos 45° - 0\right] = 8490 \text{ N}$$

or $\underline{R = 8.49 \text{ kN}}$

**4/53** $M = M_0 = m'(v_2 d_2 - 0)$

$v_2 = \frac{Q}{A} = \frac{16}{\pi(0.150)^2/4}\,\frac{1}{60} = 15.09\ \frac{m}{s}$




From Table D-1, air density is 1.206 kg/m$^3$

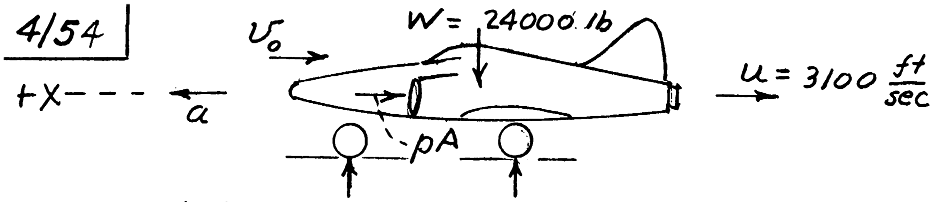
so $m' = \rho Q = 1.206(16)/60 = 0.322$ kg/s

$M_0 = 0.322(15.09 \times 0.2 - 0) = 0.971$ N·m

$P = 0.32 + M_0\omega/1000 = 0.32 + \frac{0.971(3450 \times 2\pi/60)}{1000}$

$P = 0.32 + 0.351 = \underline{0.671\ kW}$

**4/54**




$$m'_{air} = \frac{106}{32.2} \; \frac{lb/sec}{ft/sec^2} = 3.29 \; lb \times sec/ft$$

$$m'_{fuel} = 3.29/18 = 0.1829 \; lb \times sec/ft$$

air intake velocity $V_0 = \dfrac{m'_{air}}{\rho A} = \dfrac{3.29}{(0.0753/32.2)(1800/144)}$

$= 112.6$ ft/sec

$$m'_{exhaust} = 3.29 + 0.1829$$
$$= 3.47 \; lb \times sec/ft$$

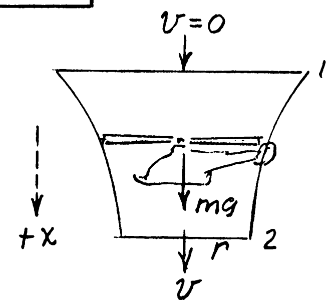
$$pA = -0.30(1800) = -540 \; lb$$

Net thrust $T = m'_{ex} u - m'_{air} V_0 - pA$

$$= 3.47(3100) - 3.29(112.6) - (-540)$$
$$= 10{,}940 \; lb$$

$\Sigma F_x = ma_x$: $10940 = \dfrac{24000}{32.2} a$, $\underline{a = 14.68 \; ft/sec^2}$

4/55




$mg$ = weight of helicopter
= force on system of air stream & helicopter

For system between sections 1 & 2

$\Sigma F_x = m' \Delta v_x$

$mg = \rho \pi r^2 v (v - 0)$

$v = \frac{1}{r}\sqrt{\frac{mg}{\pi \rho}}$

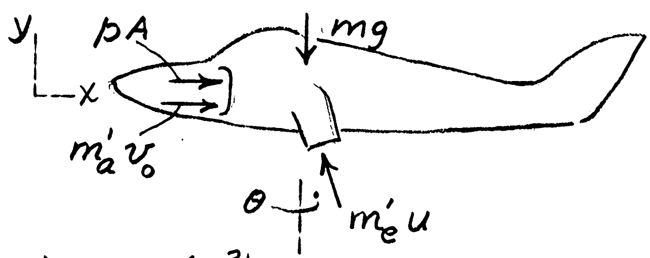
Power = rate of increase of kinetic energy

$$P = \frac{1}{2} m' (v_2^2 - v_1^2) = \frac{1}{2} m' v^2 = m' v \frac{v}{2} = mg \frac{v}{2}$$

$$P = \frac{mg}{2r}\sqrt{\frac{mg}{\pi \rho}}$$

4/56

Simulated FBD




$mg = 8600(9.81) = 84.4(10^3)\ N$

mass rate of air $= m_a' = 90$ kg/s

" " " fuel $= 90/18 = 5$ kg/s

" " " exhaust $= m_e' = 95$ kg/s

$pA = -2(10^3)(1.10) = -2200\ N$

$v_0 = m_a'/\rho A = \frac{90}{1.206(1.10)} = 67.8$ m/s, $m_a' v_0 = 90(67.8) = 6110\ N$

$m_e' u = 95(1020) = 96\,900\ N$

For vertical take off $\Sigma F_x = 0$: $6110 - 2200 - 96\,900 \sin\theta = 0$

$\sin\theta = 0.0403$, $\underline{\theta = 2.31^\circ}$

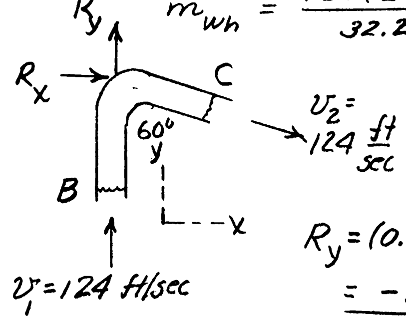
$\Sigma F_y = ma_y$: $96\,900 \cos 2.31^\circ - 84\,400 = 8600\, a_y$

$a_y = \underline{1.448\ m/s^2}$

**4/57**

$$m'_{air} = \frac{18(2000)}{32.2}\frac{1}{3600} = 0.3106 \text{ slugs/sec}$$

$$m'_{wh} = \frac{150(2000)}{32.2}\frac{1}{3600} = 2.588 \text{ slugs/sec}$$




$\Sigma F = m' \Delta v$

$R_x = (0.3106 + 2.588)(124 \sin 60° - 0)$

$= \underline{311 \text{ lb}}$

$R_y = (0.3106 + 2.588)(-124 \cos 60° - 124)$

$= \underline{-539 \text{ lb}}$

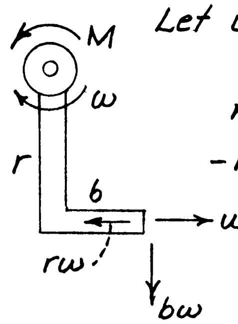
Forces acting on pipe bend & mass within it

1) tension $pA = 4.42 \frac{\pi(14)^2}{4} = 680$ lb due to vacuum
2) tension in pipe at B
3) " " " " C
4) weight of bend
5) balance of external support forces from crane
6) shear force and bending moment at C

4/58

For entire system $\Sigma M = m'(v_2 d_2 - v_1 d_1)$

Let $u$ = velocity of water relative to nozzle $= Q/4A$




components of absolute velocity of water at exit

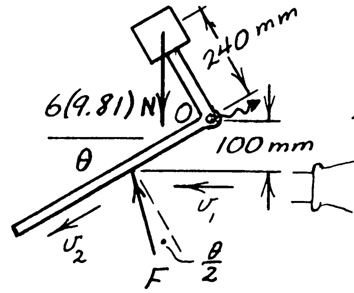
$m' = \rho Q$

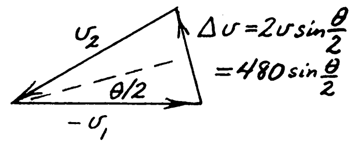
$-M = \rho Q\left(r^2\omega + b^2\omega - \frac{Q}{4A}r - 0\right)$

$\underline{M = \rho Q\left(\frac{Qr}{4A} - [r^2+b^2]\omega\right)}$

For $M = 0$, $\quad \omega = \omega_0 = \underline{\dfrac{Qr}{4A(r^2+b^2)}}$

4/59




$F = m'\Delta v$: $m' = \rho A v = 1.206\,\frac{\pi \times 0.040^2}{4}\,240 = 0.364$ kg/s

$$F = 0.364 \times 480 \sin\frac{\theta}{2} = 174.6\ \sin\frac{\theta}{2}\ \text{N}$$

For vane:

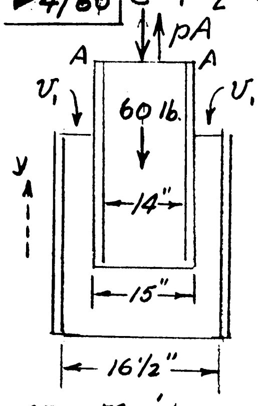
$$\Sigma M_O = 0:\ 174.6 \sin\frac{\theta}{2}\cos\frac{\theta}{2}\left(\frac{0.100}{\sin\theta}\right) - 6(9.81)(0.240\sin\theta) = 0$$

$$87.3 \times 0.100 = 6(9.81)(0.240\sin\theta)$$

$$\sin\theta = 0.618,\quad \underline{\theta = 38.2^\circ}$$

**▶4/60**

$\uparrow v_2 = 124$ ft/sec




$$\text{Air inlet area} = \frac{\pi}{4}\left([16.5]^2 - [15]^2\right) = 37.1 \text{ in.}^2$$

$$= 37.1/144 = 0.258 \text{ ft}^2$$

$$\text{Exit area} = \frac{\pi}{4}(14)^2 = 153.9 \text{ in.}^2$$

$$= 153.9/144 = 1.069 \text{ ft}^2$$

$$pA = -(-4.42)(153.9) = 680 \text{ lb}$$

$$m'_{air} = \frac{18(2000)}{32.2}\,\frac{1}{3600} = 0.3106 \text{ slugs/sec}$$

$$m'_{wh} = \frac{150(2000)}{32.2}\,\frac{1}{3600} = 2.588 \text{ slugs/sec}$$

$$v_1 = m'_{air}/\rho A = \frac{0.3106}{\left(\frac{0.075}{32.2}\right)(0.258)} = 517 \text{ ft/sec}$$

$$\Sigma F_y = m'\Delta v_y; \quad -C + 680 - 60 = 0.3106\left(124 - [-517]\right) + 2.588(124 - 0)$$

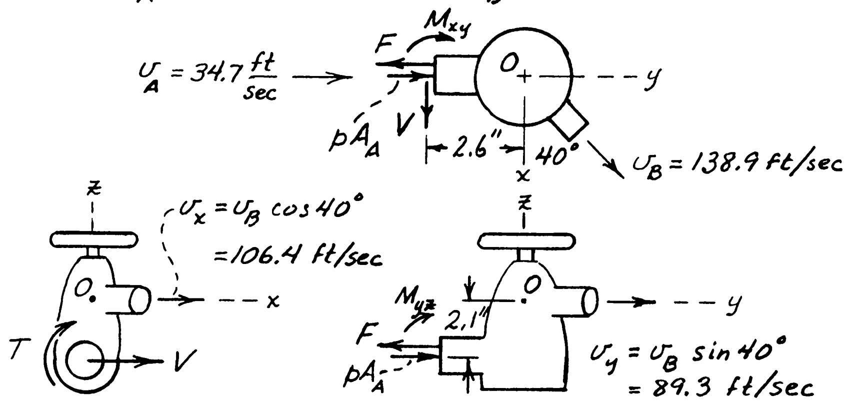
$$= 199.2 + 320.9$$

$$\underline{C = 100.3 \text{ lb}}$$

**▶4/61** Flow rate $Q = \frac{340 \times 231}{1728 \times 60} = 0.758 \frac{ft^3}{sec}$, $m' = \rho Q = \frac{62.4}{32.2} 0.758 = 1.468 \frac{lb\text{-}sec}{ft}$

Flow area $A_A = \frac{\pi 2^2}{4} / 144 = 0.0218 \; ft^2$, $A_B = \frac{\pi \times 1^2}{4} / 144 = 0.00545 \; ft^2$

Velocity $v_A = \frac{Q}{A_A} = \frac{0.758}{0.0218} = 34.7 \frac{ft}{sec}$, $v_B = \frac{Q}{A_B} = \frac{0.758}{0.00545} = 138.9 \frac{ft}{sec}$




$(x\text{-}y) \; \Sigma F_x = m' \Delta v_x: \; 150\left(\frac{\pi 2^2}{4}\right) - F = 1.468(89.3 - 34.7)$, $\underline{F = 391 \text{ lb}}$

$\Sigma F_y = m' \Delta v_y: \; V = 1.468\,(106.4 - 0)$, $\underline{V = 156.2 \text{ lb}}$

$\Sigma M_{A\text{-}A} = m' \Delta (vd): \; M_{xy} = 1.468\left(106.4 \times \frac{2.6}{12}\right) = 33.8 \text{ lb-ft}$

$(y\text{-}z) \; \Sigma M_{A\text{-}A} = m' \Delta (vd): \; M_{yz} = 1.468\left(89.3 \times \frac{2.1}{12}\right) = 22.9 \text{ lb-ft}$

$M = \sqrt{M_{xy}^2 + M_{yz}^2} = (33.8^2 + 22.9^2)^{1/2} = \underline{40.9 \text{ lb-ft}}$

$(x\text{-}z) \; \Sigma M_O = 0: \; T - Vd = 0$, $T = 156.2\left(\frac{2.1}{12}\right) = \underline{27.3 \text{ lb-ft}}$

**▸4/62** From Part (b) of Sample Problem $m' = \rho A(v-u)$

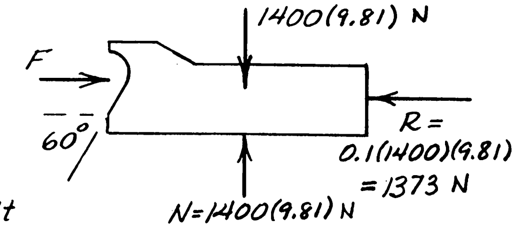
$$= (1000)\frac{\pi \times 0.140^2}{4}(150-u)$$

$$= 15.39(150-u) \text{ kg/s}$$

& $F = \rho A(v-u)^2(1-\cos 120°)$, $\theta = 90° + 30° = 120°$

$$= 15.39(150-u)^2(1-(-0.5))$$

$$= 23.1(150-u)^2$$




$\Sigma F = m\dot{u}$: $23.1(150-u)^2 - 1373 = 1400\,\dot{u}$

$$\int_0^u \frac{du}{0.01649(150-u)^2 - 0.981} = \int_0^3 dt$$

To integrate let $w = 150-u$, $\displaystyle\int_{u=0}^{u=u} \frac{dw}{0.981 - 0.01649w^2} = 3$

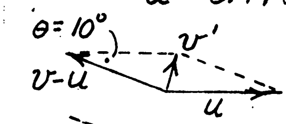
$$\frac{1}{2\sqrt{0.01649}\sqrt{0.981}} \ln\left|\frac{0.990 + 0.1284(150-u)}{0.990 - 0.1284(150-u)}\right|_0^u = 3$$

$$3.93 \ln\frac{1-0.00634u}{1-0.00703u} = 3, \quad \frac{1-0.00634u}{1-0.00703u} = 2.145$$

Solve for $u$ & get $\underline{u = 131.0 \text{ m/s}}$

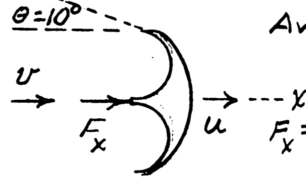
**▸4/63** $v = \sqrt{2gh} = \sqrt{2(9.81)(300)} = 76.7\ \text{m/s}$

$u = 0.47v = 36.1\ \text{m/s}$




$\Delta v_x = [u - (v-u)\cos\theta] - v = -(v-u)(1+\cos\theta)$

$= -(76.7 - 36.1)(1 + 0.985) = -80.7\ \text{m/s}$




Average tangential thrust per jet is

$\Sigma F_x' = m'\Delta v_x$

$F_x = 1000(76.7A)|-80.7| = 6.19(10^6)A$ N

where $A$ = jet area, $\text{m}^2$

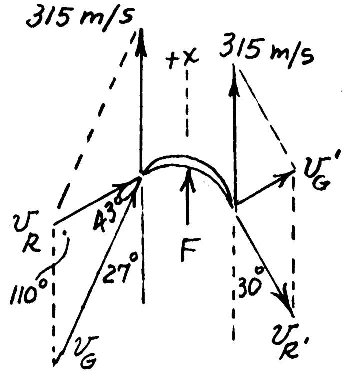
Theor. Power $P = 6F_x u = 6(6.19)(10^6)A(36.1) = 1.340A(10^9)$ W

$\dfrac{\text{Actual power}}{\text{Theor. power}} = \dfrac{22(10^6)}{1.340A(10^9)} = 0.90(0.85)$, $A = 0.0215\ \text{m}^2$

Thus $\pi d^2/4 = 0.0215$, $d = 0.1653$ m or $\underline{d = 165.3\ \text{mm}}$

$u = \dfrac{D}{2}\omega$, $D = \dfrac{2u}{\omega} = \dfrac{2(36.1)}{270(2\pi/60)} = \underline{2.55\ \text{m}}$

▶ 4/64 Entrance: $\frac{v_G}{315} = \frac{\sin 110°}{\sin 43°}$, $v_G = 434$ m/s




$\frac{v_R}{315} = \frac{\sin 27°}{\sin 43°}$, $v_R = 210$ m/s

Exit: $v_R' = v_R$ (negligible friction)

$v_{G_x}' = 315 - 210 \cos 30° = 133.4$ m/s

$|\Delta v_x| = 434 \cos 27° - 133.4 = \underline{253 \text{ m/s}}$

$\Sigma F_x = m' \Delta v_x$;

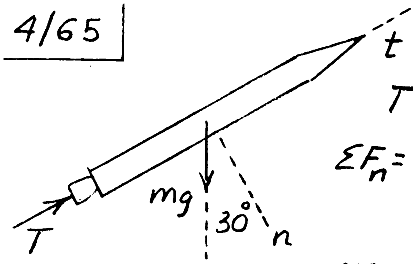
$m' = 15$ kg/s, $\Sigma F_{\text{all blades}} = 15(253) = 3800$ N

or $F = 3.8$ kN

Power $P = \Sigma F v$, $P = 3800\,(315) = 1.197(10^6)$ W

or $\underline{P = 1.197 \text{ MW}}$

4/65




$mg = 3(10^3)(9.60) = 28.8(10^3)$ N

$T = m'\Delta v = 130(600) = 78(10^3)$ N

$\Sigma F_n = ma_n$: $28.8(10^3)\cos 30° = 3(10^3)a_n$

$\underline{a_n = 8.31 \text{ m/s}^2}$

$\Sigma F_t = ma_t$: $78(10^3) - 28.8(10^3)\sin 30° = 3(10^3)a_t$

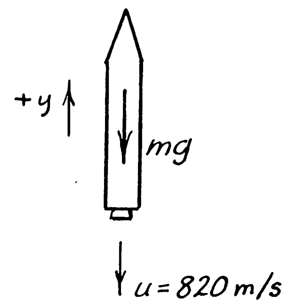
$\underline{a_t = 21.2 \text{ m/s}^2}$

4/66

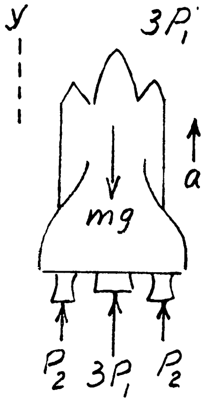
$$\Sigma F_y = ma + \dot{m}u: \; -9.81m = 6.80m - 220(820)$$

$$m = 10.86\,(10^3)\,\text{kg}$$

or $\underline{m = 10.86 \text{ Mg}}$



**4/67**




$mg = 2.04(10^6)(9.81) = 20.0(10^6)$ N

$3P_1 = 3(2.00)(10^6) = 6.00(10^6)$ N

$2P_2 = 2(11.80)(10^6) = 23.6(10^6)$ N

Specific impulse $I = \dfrac{u}{g} = 455$ s

So $u = 455(9.81) = 4460$ m/s

$\Sigma F_y = ma_y$: $(6.00)10^6 + (23.6)10^6 - 20.0(10^6)$

$= 2.04(10^6)\,a$

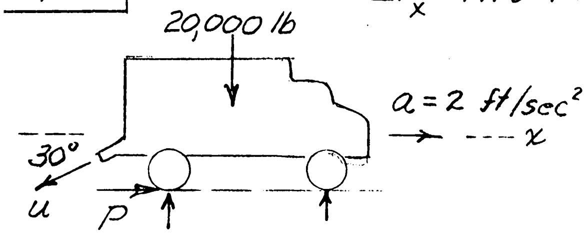
$\underline{a = 4.70 \text{ m/s}^2}$

$P_1 = m'u$, $2.00(10^6) = m'(4460)$

$\underline{m' = 448 \text{ kg/s}}$

4/68

$\Sigma F_x = m\dot{v} + \dot{m}u$




$u = 60$ ft/sec

$\dot{m} = -\dfrac{80}{32.2} = -2.48 \ \dfrac{\text{slugs}}{\text{sec}}$

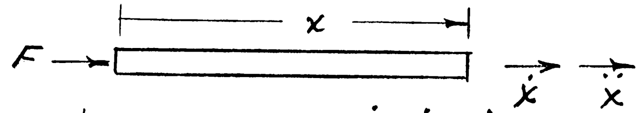
$m = \dfrac{20{,}000}{32.2} = 621$ slugs

(a) Water on; $P = 621(2) - 2.48(60)\cos 30^\circ$

$= 1242 - 129 = \underline{1113 \text{ lb}}$

(b) water off; $\dot{m} = 0$, $P = \underline{1242 \text{ lb}}$

4/69



$\Sigma F = m\dot{v} + \dot{m}u$ where $m = \rho x$, $v = \dot{x}$, $\dot{m} = \rho\dot{x}$, $u = \dot{x}$

Thus $F = \rho x\ddot{x} + \rho\dot{x}\dot{x}$ , $\underline{F = \rho(x\ddot{x} + \dot{x}^2)}$

4/70

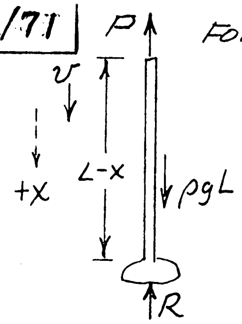

With added moisture particles initially at rest, relative velocity of attachment of mass is $u = v$

Thus with $\Sigma F = m\dot{v} + \dot{m}u$

we have $\Sigma F = m\dot{v} + \dot{m}v = \underline{\frac{d}{dt}(mv)}$

where $\Sigma F = mg - R$

**4/71**




For $\dot{x} = v = \text{const}$, $P$ = weight of descending links

$= \rho g (L - x)$

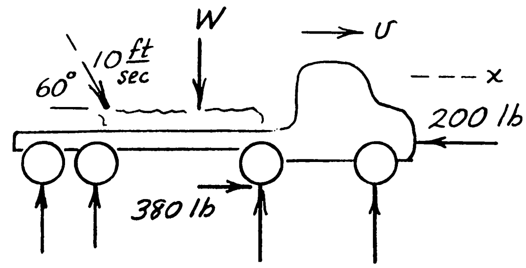
$\Sigma F_x = \dfrac{dG_x}{dt}$; $\rho g L - R - \rho g (L - x) = \dfrac{d}{dt}\left(\rho [L - x] v\right)$

$\rho g x - R = -\rho v \dot{x} = -\rho v^2$

so $\underline{R = \rho g x + \rho v^2}$

4/72

$\Sigma F_x = m\dot{v} + \dot{m}u$:




$\Sigma F_x = 380 - 200 = 180$ lb

$m = \dfrac{12000 + 4(220)}{32.2}$

$= 400$ lb-sec$^2$/ft

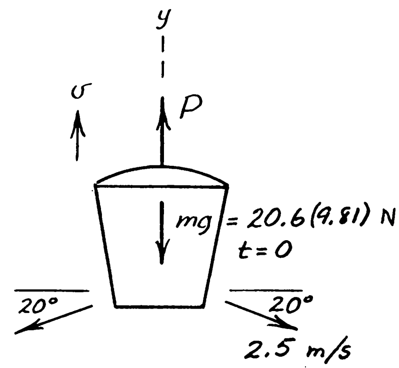
at $t = 4$ sec.

$\dot{m} = 220/32.2 = 6.83$ lb-sec/ft $\qquad$ $1.5$ mi/hr $= 2.20$ ft/sec

$u = 2.20 - 10\cos 60° = -2.80$ ft/sec

So $180 = 400\dot{v} + 6.83(-2.80)$, $a = \dot{v} = \underline{0.498\ \text{ft/sec}^2}$

4/73 $\Sigma F_y = m\dot{v} + \dot{m}u$




$\Sigma F_y = P - mg = P - 20.6(9.81)$

$= P - 202$ N

$m = 20.6$ kg

$\dot{v} = 0.5$ m/s$^2$

$\dot{m} = -\rho A v$

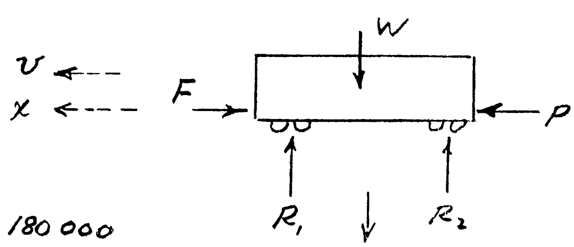
$= -2(1000)\dfrac{\pi \times 0.030^2}{4}(2.5)$

$= -3.53$ kg/s

$u = 2.5 \sin 20° - 0 = 0.855$ m/s

So $P - 202 = 20.6(0.5) - 3.53(0.855)$, $\underline{P = 209 \text{ N}}$

4/74




$$\Sigma F_x = m\dot{v}_x + \dot{m}u_x$$

$$F = 4\,\frac{54{,}600 + \frac{1}{2}\,180\,000}{2000}$$

$= 289$ lb frictional resistance

$u = v_y$

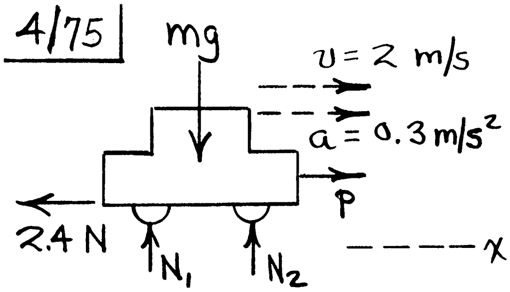
$u_x = 0$

$u_x =$ rel. velocity with respect to the car in $x$-dir. $= 0$

Thus $$P - 289 = \frac{54{,}600 + \frac{1}{2}\,180{,}000}{32.2}\,0.15 + 0$$

So $P = 674 + 289$, $\underline{P = 963 \text{ lb}}$

4/75




$m = 40 + 30(1.2)$
$= 76 \text{ kg}$

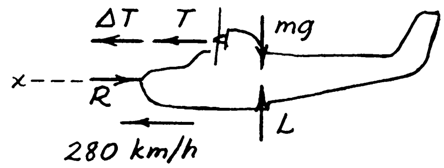
$\dot{m} = -\rho v = -1.2(2)$
$= -2.4 \text{ kg/s}$

$\Sigma F_x = m\dot{v} + \dot{m}u$ : $P - 2.4 = 76(0.3) - 2.4(2)$

$\underline{P = 20.4 \text{ N}}$

4/76

For constant initial speed propeller thrust $T$ = drag $R$.




Added power = $\Delta T \cdot v$,

$$\Delta T \times \frac{280 \times 1000}{3600} = 223.8\,(10^3) \text{ watts (joules/second)}$$

$$\Delta T = 2880 \text{ N}$$

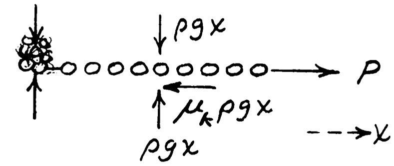
$\Sigma F_x = m\dot{v} + \dot{m}u$ where $\dot{m} = 4.5 \times 1000/12 = 375$ kg/s,

$$u = v = \frac{280 \times 1000}{3600} = 77.8 \text{ m/s}$$

So $2880 = 16.4(10^3)\,\dot{v} + 375(77.8)$, $\dot{v} = \underline{a = -1.603 \text{ m/s}^2}$

(deceleration)

4/77 Sol. I: entire chain




$\Sigma F_x = \dot{G}_x$; $P - \mu_k \rho g x = \frac{d}{dt}(\rho x \dot{x})$

$= \rho(\dot{x}^2 + x\ddot{x})$

$$a = \ddot{x} = \frac{P}{\rho x} - \mu_k g - \frac{\dot{x}^2}{x}$$

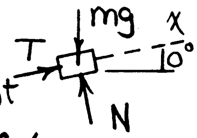
Sol. II: Eq. 4/20 for moving portion

$\Sigma F = m\dot{v} + \dot{m}u$

$P - \mu_k \rho g x = \rho x \ddot{x} + \rho \dot{x}\dot{x}$ where $u = \dot{x}$

$$a = \ddot{x} = \frac{P}{\rho x} - \mu_k g - \frac{\dot{x}^2}{x}$$

4/78 $\Sigma F_x = ma_x: \quad T - mg\sin\theta = ma_x$




$T = m'u = \frac{2}{32.2}(400) = 24.8$ lb constant

$m = m_0 - m't = \frac{1}{32.2}(125 - 2t)$ lb-sec$^2$/ft

Propulsion time $t = \frac{20}{2} = 10$ sec

So $m'u - (m_0 - m't)g\sin\theta = (m_0 - m't)\frac{dv}{dt}$

$$\int_0^t \left[\frac{m'u}{m_0 - m't} - g\sin\theta\right] dt = \int_0^v dv$$

$$\Rightarrow v = u\ln\left(\frac{m_0}{m_0 - m't}\right) - gt\sin\theta$$

When $t = 10$ sec, $v = 400\ln\left(\frac{125}{125-20}\right) - 32.2(10)\sin 10°$

$= \underline{13.83 \text{ ft/sec}}$

4/79 With $v = \text{const.}$, $\dot{v} = \text{accel.} = 0$

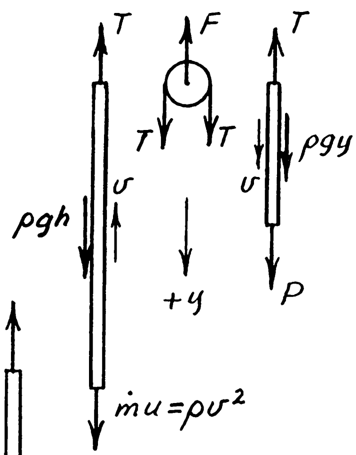
so $\Sigma F_y = 0$ for all bodies:

$$P + \rho g y - T = 0 \qquad \text{---(1)}$$

$$\rho v^2 + \rho g h - T = 0 \qquad \text{---(2)}$$

Eliminate $T$ & get

$$\underline{P = \rho v^2 + \rho g(h-y)}$$




Left-hand portion (constant mass, upper end moving up):

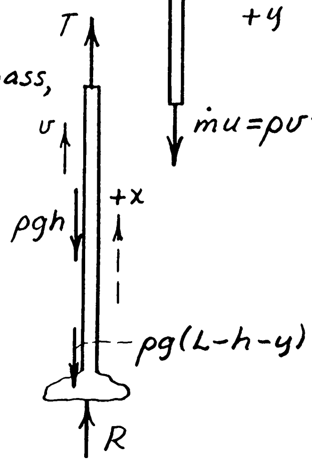
$$\Sigma F_x = \dot{G}_x : \ T + R - \rho g h - \rho g(L-h-y) = \frac{d}{dt}(\rho h v)$$

$$\rho v^2 + \rho g h + R - \rho g h - \rho g(L-h-y) = \frac{d}{dt}(\rho h v) = \rho v^2$$

where $\dot{h} = v$

$$\rho v^2 + R - \rho g(L-h-y) = \rho v^2$$

$R = \underline{\rho g(L-h-y)}$ = weight of pile of chain



4/80 Let $m_0$ = initial mass of car = $25(10^3)$ kg

$\dot{m} = 4(10^3)$ kg/s

The car acquires mass which has zero initial horizontal velocity, so for horizontal x-dir., $\Sigma F_x = \frac{d}{dt}(mv)$

$0 = \frac{d}{dt}(m_0 + \dot{m}t)v$, $(m_0 + \dot{m}t)a + \dot{m}v = 0$,

$$a = \frac{dv}{dt} = -\frac{\dot{m}v}{m_0 + \dot{m}t}$$

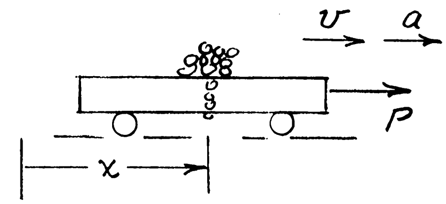
$$\int_{v_0}^{v} \frac{dv}{v} = -\int_0^t \frac{\dot{m}}{m_0 + \dot{m}t}\,dt \Rightarrow v = \frac{dx}{dt} = \frac{m_0 v_0}{m_0 + \dot{m}t}$$

Then $\displaystyle\int_0^x dx = \int_0^t \frac{m_0 v_0}{m_0 + \dot{m}t}\,dt \Rightarrow x = \frac{m_0 v_0}{\dot{m}} \ln\left(\frac{m_0 + \dot{m}t}{m_0}\right)$

With $t = \frac{32}{4} = 8$ s, $x = \dfrac{25(10^3)(1.2)}{4(10^3)} \ln\left(\dfrac{25 + 4(8)}{25}\right)$

$$\underline{x = 6.18 \text{ m}}$$

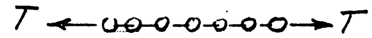
**4/81** $m = m_0 - \rho x$




T is not transmitted to cart so $\Sigma F = ma$

$$P = (m_0 - \rho x)a$$

$$\underline{a = \frac{P}{m_0 - \rho x}}$$




$v\,dv = a\,dx, \quad \int_{v_0}^{v} v\,dv = \int_0^x \frac{P\,dx}{m_0 - \rho x}$

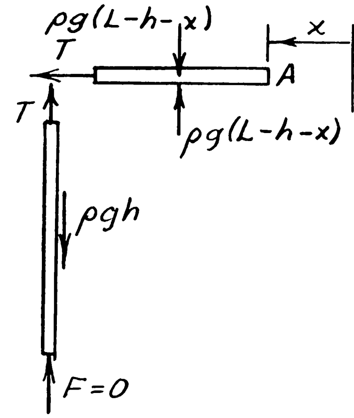
$$\frac{v^2}{2}\Big]_{v_0}^{v} = -\frac{P}{\rho}\ln(m_0 - \rho x)\Big]_0^x = \frac{P}{\rho}\ln\frac{m_0}{m_0 - \rho x}$$

Thus $\underline{v = \sqrt{v_0^2 + \frac{2P}{\rho}\ln\frac{m_0}{m_0 - \rho x}}}$

$T = m'\Delta v, \quad T = \rho v(v), \quad \underline{T = \rho v^2}$

There is no reaction (R in Fig. 4/6) between departing links & cart, so $\dot{m}u$ is zero & $\Sigma F = ma$

**4/82** Let $x$ be the displacement of the chain & $T$ be the tension in the chain at the corner.




Horiz. part $\Sigma F_x = ma_x$:

$$T = \rho(L-h-x)\ddot{x}$$

Vert. part $\Sigma F_v = ma_v$:

$$\rho g h - T = \rho h \ddot{x}$$

Eliminate $T$ & get

$$\ddot{x} = \frac{gh}{L-x}$$

$$\dot{x}\,d\dot{x} = \ddot{x}\,dx: \quad \int_0^{v_1^2} \frac{1}{2} d(\dot{x}^2) = \int_0^{L-h} \frac{gh}{L-x}\,dx$$

$$\left.\frac{\dot{x}^2}{2}\right|_{\dot{x}=0}^{v_1} = -gh \ln(L-x)\Big|_0^{L-h}, \quad \frac{v_1^2}{2} = gh \ln(L-x)\Big|_{L-h}^{0} = gh \ln\frac{L}{h}$$

(a) $\underline{v_1 = \sqrt{2gh \ln(L/h)}}$

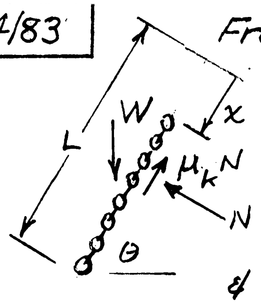
(b) Free fall of end A gives $v_2^2 = v_1^2 + 2gh = 2gh \ln\frac{L}{h} + 2gh$

$\underline{v_2 = \sqrt{2gh(1 + \ln[L/h])}}$

(c) $Q$ = loss of potential energy since $\Delta T = 0$

$Q = \rho g h \frac{h}{2} + \rho g (L-h) h, \quad \underline{Q = \rho g h (L - \frac{h}{2}) \text{ loss}}$

4/83



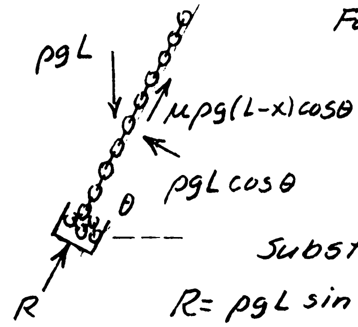

From FBD of links in motion with no $x$-force at bottom, $\Sigma F_x = ma_x$ gives

$W \sin\theta - \mu_k N = m\ddot{x}$ or

$\rho g(L-x)\sin\theta - \mu_k \rho g(L-x)\cos\theta = \rho(L-x)\ddot{x}$

$\ddot{x} = g\sin\theta - \mu_k g\cos\theta$ const.

& $\dot{x}^2 = 2\ddot{x}x$




For entire chain, $\Sigma F_x = \dot{G}_x$

$$\rho g L\sin\theta - \mu_k \rho g(L-x)\cos\theta - R = \frac{d}{dt}\left(\rho[L-x]\dot{x}\right) = \rho(L-x)\ddot{x} - \rho\dot{x}^2 = \rho\ddot{x}(L-3x)$$

Substitute $\ddot{x}$, solve for $R$ & get

$$R = \rho g L\sin\theta - \mu_k\rho g(L-x)\cos\theta - \rho g(\sin\theta - \mu_k\cos\theta)(L-3x)$$

$\underline{R = \rho g x(3\sin\theta - 2\mu_k\cos\theta)}$ $\quad \mu_k < \tan\theta$

4/84 For airplane plus moving portion of chains

$$\Sigma F = 0 = m\dot{v} + \dot{m}u = (m + 2\rho\tfrac{x}{2})\dot{v} + [2\tfrac{d}{dt}(\rho\tfrac{x}{2})]v$$

$$-(m+\rho x)\frac{dv}{dt} = \rho v\frac{dx}{dt} \qquad \frac{dv}{v} = -\frac{\rho\,dx}{m+\rho x}$$

$$\int_{v_0}^{v}\frac{dv}{v} = -\int_0^x \frac{\rho\,dx}{m+\rho x}\,; \quad \ln\frac{v}{v_0} = -\ln\frac{m+\rho x}{m}, \quad \frac{v}{v_0} = \frac{m}{m+\rho x}$$

$$\text{or } v = \frac{v_0}{1+\rho x/m} \quad \text{\& for } x = 2L, \quad \underline{v = \frac{v_0}{1+\frac{2\rho L}{m}}}$$

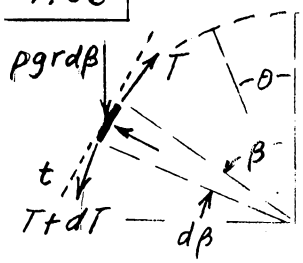
$$\text{Also, } v = \frac{dx}{dt} \text{ so } \int_0^x \left(1+\frac{\rho x}{m}\right)dx = \int_0^t v_0\,dt$$

$$x + \frac{\rho x^2}{2m} = v_0 t, \quad x^2 + \frac{2m}{\rho}x - \frac{2mv_0 t}{\rho}$$

$$x = -\frac{m}{\rho} \pm \frac{1}{2}\sqrt{\frac{4m^2}{\rho^2} + \frac{8mv_0 t}{\rho}}, \quad \underline{x = \frac{m}{\rho}\left[\sqrt{1+\frac{2v_0 t\rho}{m}} - 1\right]}$$

for + root

4/85




$\Sigma F_t = m a_t$;

$(T+dT)\cos\frac{d\beta}{2} - T\cos\frac{d\beta}{2} + \rho g r\, d\beta \sin\beta$

$= \rho r\, d\beta\, a_t$

Simplify & get

$dT = \rho r\,(a_t - g\sin\beta)\, d\beta$

$$\int_0^0 dT = \rho r \int_\theta^{\pi/2} (a_t - g\sin\beta)\, d\beta$$

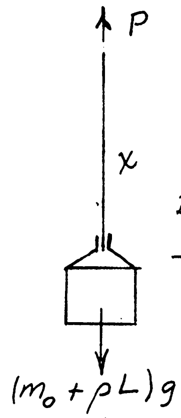
$$0 = \rho r\left[a_t\{\pi/2 - \theta\} + g\{\cos\pi/2 - \cos\theta\}\right]$$

$$a_t(\pi/2 - \theta) = g\cos\theta\,, \quad \underline{a_t = g\,\frac{\cos\theta}{\frac{\pi}{2} - \theta}}$$

Energy loss $Q = |\Delta V_g| = mg\Delta h = \rho g\,\frac{\pi r}{2}\,\bar{r}\cos 45°$

$= \rho g\,\frac{\pi r}{2}\,\frac{2r}{\pi} = \underline{\rho g r^2}$

▶ 4/86 $U = \Delta T + \Delta V_g$ ; $U = -Fx$



$\Delta T = \frac{1}{2}(m_0 + \rho[L-x])v^2$

$\Delta V_g = -[\{m_0 + \rho(L-x)\}x + \rho x \frac{x}{2}]g$

$-Fx = \frac{1}{2}(m_0 + \rho[L-x])v^2 - [(m_0 + \rho[L - \frac{x}{2}])gx$

Differentiate with time & get

$-Fv = (m_0 + \rho[L-x])va - \frac{1}{2}\rho v^3 - (m_0 + \rho[L - \frac{x}{2}])gv$
$+ \rho g v x/2$

or $[m_0 + \rho(L-x)]a = [m_0 + \rho(L-x)]g + \frac{\rho v^2}{2} - F$

so $\underline{a = g + \dfrac{\rho v^2/2 - F}{m_0 + \rho(L-x)}}$

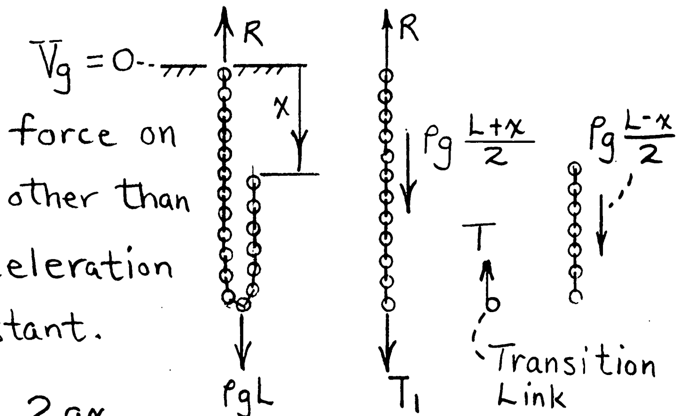
For entire system, $\Sigma F_x = \dot{G}_x$

$(m_0 + \rho L)g - P = \frac{d}{dt}[m_0 + \rho(L-x)]v = [m_0 + \rho(L-x)]a - \rho v^2$

Substitute $a$, simplify & get

$P = \rho g x + \frac{\rho v^2}{2} + F$

▶4/87




There is no force on moving part other than weight so acceleration $\ddot{x} = g =$ constant.

Thus, $v^2 = 2gx$

$T_1 = m'\Delta v$ : $T_1 = \left[\frac{d}{dt}\left(\rho \frac{L-x}{2}\right)\right]\left[0 - v\right]$

$= \frac{1}{2}\rho v^2 = \frac{1}{2}\rho(2gx) = \underline{\rho g x}$

Equilibrium of links at rest:

$\Sigma F_x = 0$ : $T_1 + \rho g \frac{L+x}{2} - R = 0$

$\Rightarrow \underline{R = \frac{1}{2}\rho g(L + 3x)}$

Loss $Q = |V_{g_1} - V_{g_2}| = |\rho g L(-\frac{L}{4}) - \rho g L(-\frac{L}{2})|$

$= \underline{\frac{1}{4}\rho g L^2}$

When $x \to L$, $v \to \infty$. The loss of potential energy equals the gain in kinetic energy, so the gain is concentrated in the last element and is lost during impact when the last element is abruptly brought to rest.

▸ 4/88

$\Delta V_g = \rho g \frac{L-x}{2}(-x) + \rho g \frac{x}{2}\left(-\frac{x}{2}\right)$

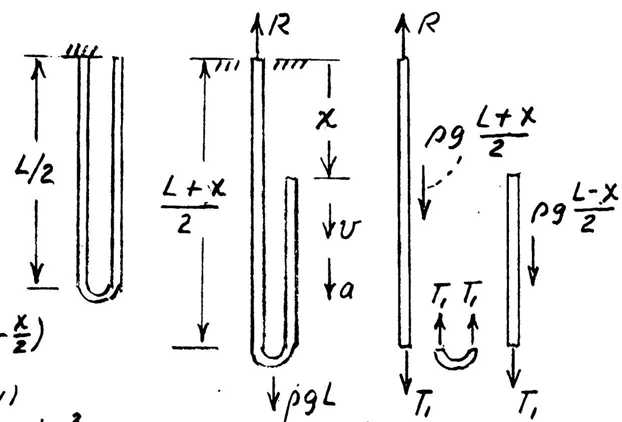
$= -\frac{1}{2}\rho g x \left(L - \frac{x}{2}\right)$

$\Delta T = \frac{1}{2}\rho \frac{L-x}{2} v^2$

$\Delta V_g + \Delta T = 0$

$\frac{1}{4}\rho (L-x) v^2 = \frac{1}{2}\rho g x \left(L - \frac{x}{2}\right)$

$v^2 = 2gx \frac{L - x/2}{L-x}$ ---(1)




$v\,dv = a\,dx$ so $a = \frac{1}{2}\frac{dv^2}{dx} = g \frac{(L-x)(L-x) - x(L - x/2)(-1)}{(L-x)^2}$

$$a = g\left(1 + \frac{x(L - x/2)}{(L-x)^2}\right) \quad \text{---(2)}$$

For entire rope

$\Sigma F_x = \dot{G}_x$: $\rho g L - R = \frac{d}{dt}\left(\rho \frac{L-x}{2} v\right) = \frac{\rho}{2}\left[(L-x)a - v^2\right]$

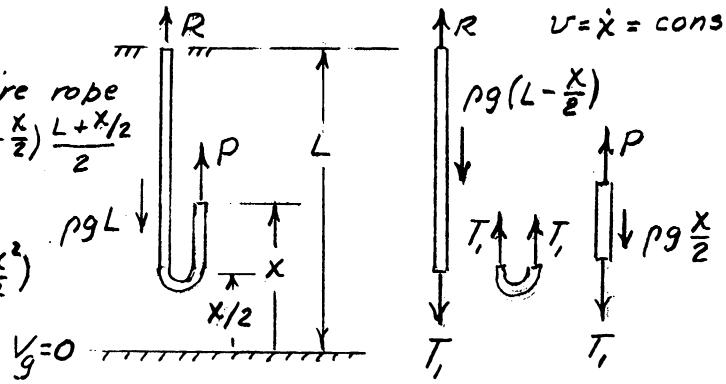
Sub. (1) & (2) & get $R = \frac{1}{2}\rho g\left[(L+x) + \frac{x(L - x/2)}{L-x}\right]$

Equil of fixed part gives $T_1 = \frac{1}{2}\rho g x (L - x/2)/(L-x)$

**▶ 4/89**

For entire rope

$$V_g = \rho g\left(L-\frac{x}{2}\right)\frac{L+x/2}{2} + \rho g\,\frac{x}{2}\,\frac{3x}{4} = \frac{\rho g}{2}\left(L^2+\frac{x^2}{2}\right)$$




$v = \dot{x} = \text{const.}$

Entire rope: $\Sigma F_x = \dot{G}_x$: $R + P - \rho g L = \frac{d}{dt}\left(\rho \frac{x}{2} v\right) = \frac{1}{2}\rho v^2$ ---(1)

Work-energy: $dU' = dT + dV_g$: $P\,dx = d\left(\frac{1}{2}\rho\frac{x}{2}v^2\right) + d\left\{\frac{\rho g}{2}\left(L^2+\frac{x^2}{2}\right)\right\}$

$$= \frac{1}{4}\rho v^2\,dx + \frac{1}{2}\rho g x\,dx$$

$$P = \frac{1}{4}\rho v^2 + \frac{1}{2}\rho g x \quad \text{---- (2)}$$

Sub. (2) into (1)

$$\underline{R = \frac{1}{4}\rho v^2 + \rho g\left(L - x/2\right)}$$

Equil. of part not moving $\Sigma F_y = 0$: $R - \rho g\left(L-\frac{x}{2}\right) - T_1 = 0$, $\underline{T_1 = \frac{1}{4}\rho v^2}$

▶ 4/90 For falling part $\Sigma F = m\dot{v} + \dot{m}u$

where $\Sigma F = \rho g x$, $m = \rho x$, $\dot{m} = \rho v$, $u = v = \dot{x}$

Thus $\rho g x = \rho x \dot{v} + \rho v \dot{x}$, $g x\, dt = x\, dv + v\, dx$

or $g x\, dt = d(xv)$; $g x^2 v\, dt = xv\, d(xv)$

so $g x^2 dx = \frac{1}{2} d[(xv)^2]$ & $g \int_0^x x^2 dx = \frac{1}{2} \int_0^{(xv)^2} d[(xv)^2]$

$\frac{g x^3}{3} = \frac{1}{2} (xv)^2$, $\underline{v = \sqrt{\frac{2gx}{3}}}$

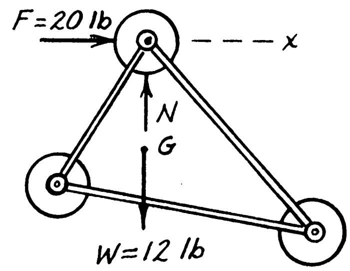
$a = \dot{v} = \sqrt{\frac{2g}{3}} \frac{1}{2} x^{-1/2} \dot{x} = \sqrt{\frac{2g}{3}} \frac{1}{2\sqrt{x}} \sqrt{\frac{2gx}{3}}$, $\underline{a = \frac{g}{3} \text{ constant}}$

$Q = -\Delta V_g - \Delta T = + \frac{\rho g L^2}{2} - \frac{\rho L}{2} v^2_{x=L} = \frac{\rho g L^2}{2} - \frac{\rho g L^2}{3} = \underline{\frac{\rho g L^2}{6}}$

4/91

$$\Sigma F_x = m\bar{a}_x : \; 20 = \frac{12}{32.2}\bar{a}_x$$

$$\bar{a}_x = \bar{a} = \underline{53.7 \text{ ft/sec}^2}$$



4/92 For the system, $\Sigma M_O = \dot{H}_O = 0$, so $H_O$ is conserved:

$$\frac{2}{16}(1000)\frac{10}{12} = \frac{2}{16}\left(\frac{10}{12}\right)^2\omega + 3\left(\frac{20}{12}\right)^2\omega$$

$$\underline{\omega = 12.37 \text{ rad/sec}}$$

A large horizontal force is exerted on the rod by the bearing so that $\Sigma F \neq 0$ in the horizontal direction. Thus $\dot{G}_x \neq 0$ and the linear momentum of the bullet-pendulum system is not conserved.

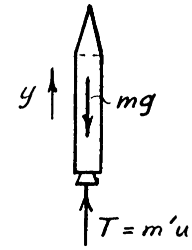
4/93

$\Sigma F_y = ma_y$: $T = m'u = 13(10^3)(2400) = 31.2(10^6)$ N

$mg = 2.7(10^6)(9.81) = 26.5(10^6)$ N

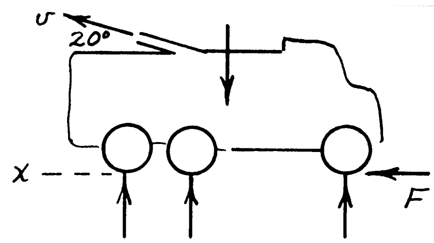
Thus $31.2(10^6) - 26.5(10^6) = 2.7(10^6)a$

$\underline{a = 1.746 \text{ m/s}^2}$



4/94 | $F = m'\Delta v$: $(30-20) = \frac{4.5}{60}(v-0)$, $\underline{v = 133.3 \text{ m/s}}$

4/95




$F = m'\Delta v_x$ : $\Delta v_x = v \cos 20°$

$Q = Av$: $\dfrac{1400 \times 231}{1728} \dfrac{1}{60} \dfrac{ft^3}{sec}$

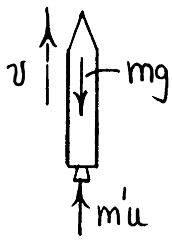
$= \dfrac{\pi \times 2^2/4}{144} v$, $v = 143.0$ ft/sec

$\Delta v_x = 143.0 \cos 20° - 0 = 134.4$ ft/sec

$m' = \rho Q = \dfrac{62.4}{32.2} \dfrac{1400 \times 231}{1728 \times 60} = 6.04$ lb-sec/ft

$F = 6.04(134.4) = \underline{812 \text{ lb}}$

4/96 $m = m_0 - m't$




$\Sigma F = ma: \quad m'u - (m_0 - m't)g = (m_0 - m't)a$

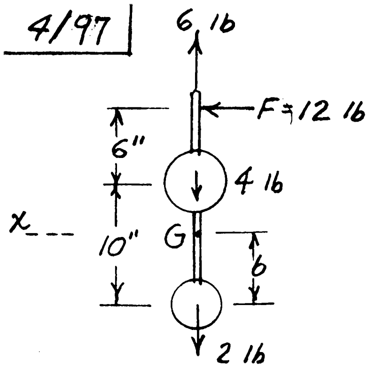
$$a = \frac{dv}{dt} = \frac{m'u}{m_0 - m't} - g$$

$$\int_0^v dv = \int_0^t \frac{m'u}{m_0 - m't}\,dt - \int_0^t g\,dt$$

$$v = -u \ln(m_0 - m't)\Big|_0^t - gt\Big|_0^t$$

$$v = u \ln\left(\frac{m_0}{m_0 - m't}\right) - gt$$

4/97




$\Sigma F_x = m\bar{a}_x; \quad 12 = \frac{4+2}{32.2}\bar{a}$

$\bar{a} = \underline{64.4 \text{ ft/sec}^2}$

$4(10-b) = 2b, \quad b = 6.67 \text{ in.}$

$H_G = \Sigma m r^2 \dot{\theta} = \frac{4(3.33)^2 + 2(6.67)^2}{32.2(12)^2}\dot{\theta}$

$= 0.0288\dot{\theta}$ lb-ft-sec

$\Sigma M_G = \dot{H}_G; \quad 12\,\frac{(6+3.33)}{12} = 0.0288\ddot{\theta}$

$\ddot{\theta} = \frac{9.33}{0.0288} = \underline{325 \text{ rad/sec}^2}$

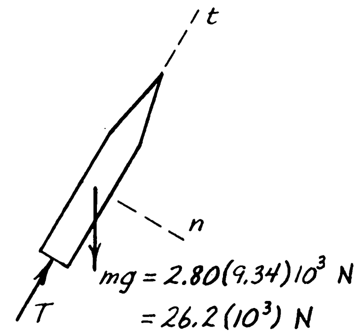
4/98

$T = m'u = 120(640) = 76.8(10^3)$ N

$\Sigma F_t = ma_t$ : $76.8(10^3) - 26.2(10^3)\cos 30°$

$= 2.80(10^3)\,a_t$

$\underline{a_t = 19.34 \text{ m/s}^2}$




$\Sigma F_n = ma_n$ : $26.2(10^3)\sin 30°$

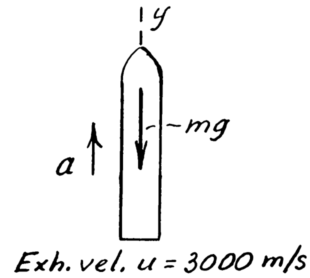
$= 2.80(10^3)\,a_n$

$\underline{a_n = 4.67 \text{ m/s}^2}$

4/99

$\dot{m} = -m' = -5.2\ kg/s$

$m = 200 + 1200 - 5.2t = 1400 - 5.2t\ kg$




$\Sigma F = m\dot{v} + \dot{m}u: \ -mg = ma - 5.2(3000)$

$(1400 - 5.2t)(a + 8.70) = 15\,600$

$a = \dfrac{15\,600}{1400 - 5.2t} - 8.70\ m/s^2$

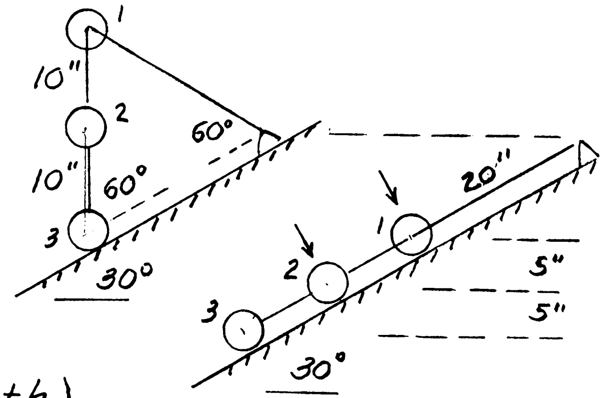
When $t = 60\ s$, $a = \dfrac{15\,600}{1400 - 5.2(60)} - 8.70 = 14.34 - 8.70$

$= \underline{5.64\ m/s^2}$

Max. accel. occurs when $5.2t = 1200$, $\underline{t = 231\ s}$

$a_{max} = \dfrac{15\,600}{1400 - 5.2(231)} - 8.70 = 78.0 - 8.70 = \underline{69.3\ m/s^2}$

4/100




Vertical drop of spheres is

$h_1 = 20''$

$h_2 = 15''$

$h_3 = 10''$

For system

$\Delta V_g = -mg(h_1 + h_2 + h_3)$

$\Delta T = \frac{1}{2}m(v_1^2 + v_2^2 + 0) = \frac{1}{2}m(v^2 + [\frac{v}{2}]^2) = \frac{5}{8}mv^2$

$U' = \Delta T + \Delta V : \quad 0 = \frac{5}{8}mv^2 - mg(h_1 + h_2 + h_3)$

$v^2 = \frac{8}{5}(32.2)\,\frac{20+15+10}{12} = 193.2 \ (ft/sec)^2$

$\underline{v = 13.90 \ ft/sec}$

Potential energy loss goes into impact energy loss

**4/101** $\Sigma F_x = m' \Delta v_x$

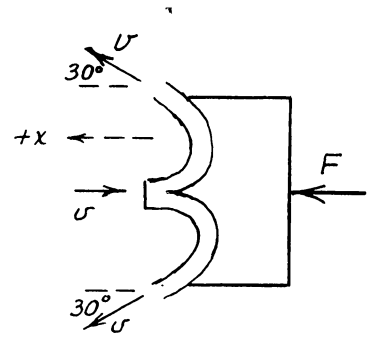
$$m' = \rho A v = \frac{62.4}{32.2}\left(\frac{\pi}{4}\frac{(3/4)^2}{144}\right)120$$

$$= 0.713 \text{ lb-sec/ft}$$

$$\Delta v_x = v \cos 30° - (-v) = v(1+\cos 30°)$$

$$= 120(1+0.866) = 224 \text{ ft/sec}$$

$$F = 0.713 \times 224 = \underline{159.8 \text{ lb}}$$



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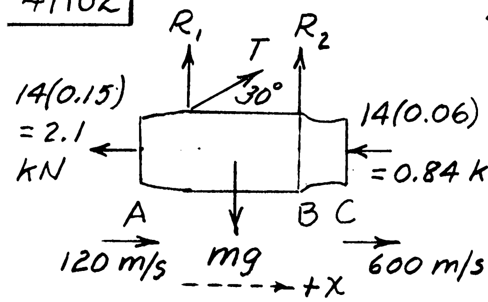
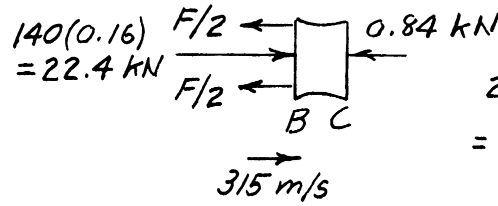



$$\Sigma F_x = m' \Delta v_x$$

$$-2.1 - 0.84 + T\cos 30° = [31.6(600) - 30(120)]10^{-3}$$

$$\underline{T = 21.1 \text{ kN}}$$



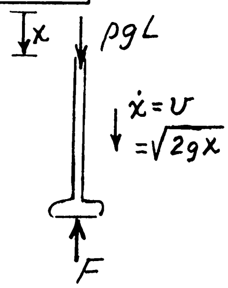

$$\Sigma F_x = m' \Delta v_x$$

$$22.4 - 0.84 - F = 31.6(600 - 315)10^{-3}$$

$$\underline{F = 12.55 \text{ kN}}$$

**4/103**

$$G_x = \rho(L-x)\sqrt{2gx} = \rho\sqrt{2g}\,(Lx^{1/2} - x^{3/2})$$




$\Sigma F_x = \dot{G}_x$ for entire system

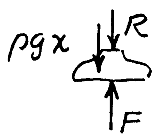
$$\rho g L - F = \rho\sqrt{2g}\left(\tfrac{1}{2}Lx^{-1/2} - \tfrac{3}{2}x^{1/2}\right)\dot{x}$$

$$= \rho\sqrt{2g}\left(\tfrac{1}{2}L\sqrt{2g} - \tfrac{3}{2}\sqrt{2g}\,x\right)$$

$$= \rho g L - 3\rho g x$$

So that $\underline{F = 3\rho g x}$

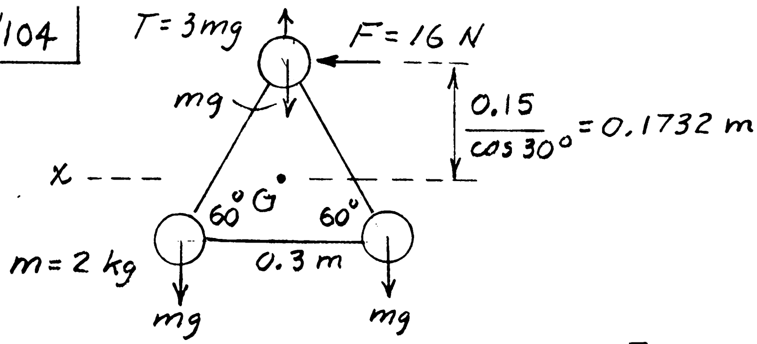
Alternatively




$$R = m'\Delta v = \rho\dot{x}(\dot{x}) = \rho\dot{x}^2 = \rho(2gx) = 2\rho g x$$

$\Sigma F_x = 0;\quad 2\rho g x + \rho g x - F = 0,\quad \underline{F = 3\rho g x}$

4/104




For system: $\Sigma F_x = m\bar{a}_x$: $16 = 3(2)\bar{a}$, $\underline{\bar{a} = 2.67 \frac{m}{s^2}}$

$\Sigma M_G = \dot{H}_G$: $16(0.1732) = \frac{d}{dt}\left(3 \times 2 \times \overline{0.1732}^2 \dot{\theta}\right)$

$\ddot{\theta} = \frac{16(0.1732)}{6(0.1732)^2} = \underline{15.40 \frac{rad}{s^2}}$

For top sphere

$a = \bar{a} + \bar{r}\ddot{\theta} = 2.67 + 0.1732(15.40) = \underline{5.33 m/s^2}$

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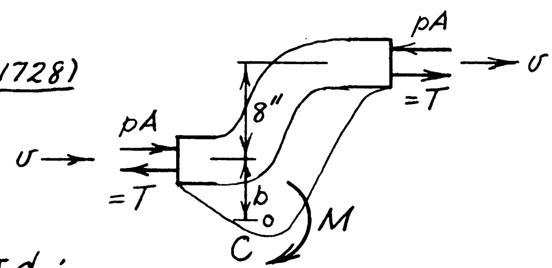
$Q = Av$

$$v = Q/A = \frac{5000 \times 231/(60 \times 1728)}{\frac{\pi \times 4^2}{4} \frac{1}{144}}$$

$= 127.7$ ft/sec




$\Sigma M_o = \dot{H}_o = m' v_2 d_2 - m' v_1 d_1$ :

$$m' = \frac{\mu}{g} Q = \frac{62.4}{32.2} \frac{5000 \times 231}{60 \times 1728} = 21.6 \text{ lb-sec/ft}$$

so $M = 21.6\,(127.7)(8/12) = \underline{1837 \text{ lb-ft}}$

4/106 $\Delta H_z = 0: \ \Sigma m v r = \Sigma m r^2 \omega = H_z$

so $4 m\omega(0.8^2 + 0.4^2) = 4 m\omega'(0.8^2 + 0.65^2)$

$\omega'/\omega = N'/N = (0.8^2 + 0.4^2)/(0.8^2 + 0.65^2) = 0.753$

Thus $N' = 0.753 \times 120 = \underline{90.4 \text{ rev/min}}$

$T = \Sigma \frac{1}{2} m v^2 = \Sigma \frac{1}{2} m (r\omega)^2$

$$|\Delta T| = 4 \times \frac{1}{2} \times 2 [0.8^2 + 0.4^2]\left[\frac{120 \times 2\pi}{60}\right]^2 - 4 \times \frac{1}{2} \times 2[0.8^2 + 0.65^2]\left[\frac{90.4 \times 2\pi}{60}\right]^2$$

$= 505 - 380 = 124.8 \text{ J}$

$\underline{|\Delta T| = 124.8 \text{ J loss}}$

$\Delta T$ due to impact energy loss upon impact of inner against outer spheres.

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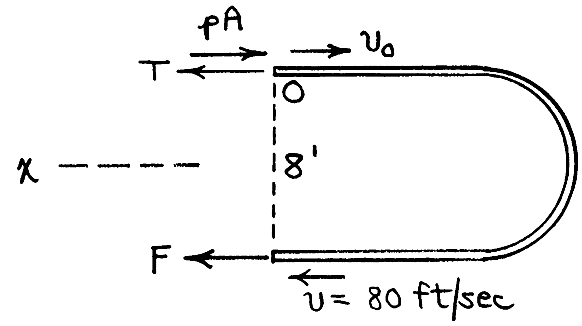



Velocity at O: $Q = Av_0$

$$\frac{780(231)}{1728(60)} = \frac{\pi(3/12)^2}{4} v_0, \quad v_0 = 35.4 \frac{\text{ft}}{\text{sec}}$$

Mass flow rate:

$$m' = \rho A v = \rho Q = \frac{62.4}{32.2} \frac{780(231)}{1728(60)} = 3.37 \frac{\text{lb-sec}}{\text{ft}}$$

$$\Sigma M_o = \dot{H}_o = m'vd - 0: \quad 8F = 3.37(80)(8)$$

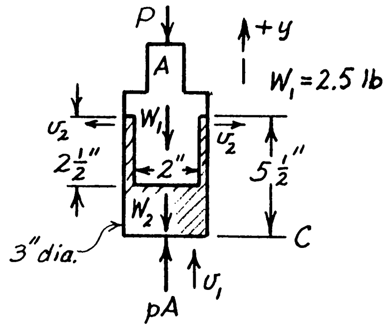
$$\underline{F = 269 \text{ lb}}$$

$$\Sigma F_x = \dot{G}_x = m'\Delta v_x:$$

$$T - 120\frac{\pi(3^2)}{4} + 269 = 3.37[80 - (-35.4)]$$

$$\underline{T = 967 \text{ lb}}$$

4/108 $\Sigma F_y = m' \Delta v_y$




Flow vol. per sec $Q = Av_1 = \frac{\pi \times 3^2}{4}\frac{1}{144}v_1$

$= 600\frac{231}{1728}\frac{1}{60}$, $v_1 = 27.2$ ft/sec

Weight of water

$$W_2 = \left[\frac{\pi \times 3^2}{4} \times 5.5 - \frac{\pi \times 2^2}{4} \times 2.5\right]\frac{62.4}{1728}$$

$= 1.120$ lb

$$pA = 12\frac{\pi \times 3^2}{4} = 84.8 \text{ lb}$$

$$m' = \rho Q = \frac{62.4}{32.2}\frac{600 \times 231}{1728}\frac{1}{60} = 2.59 \text{ lb-sec/ft}$$

So $P + 2.5 + 1.120 - 84.8 = 2.59\,(0 - [-27.2])$,

$\underline{P = 151.8 \text{ lb}}$

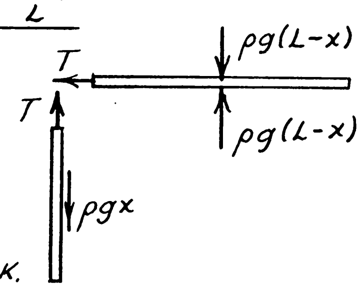
**4/109** System is conservative, so $\Delta V_g + \Delta T = 0$.

$$-\rho g x \frac{x}{2} + \frac{1}{2}\rho L \dot{x}^2 = 0, \quad \frac{g}{L}x^2 = \dot{x}^2, \quad \dot{x} = \sqrt{\frac{g}{L}}\,x$$

(a) accel $a = \ddot{x} = \sqrt{\frac{g}{L}}\,\dot{x} = \sqrt{\frac{g}{L}}\sqrt{\frac{g}{L}}\,x$, so $\underline{a = \frac{g}{L}x}$

(b) $\Sigma F = ma$: $T = \rho(L-x)\frac{g}{L}x$

$\underline{T = \rho g x\left(1 - \frac{x}{L}\right)}$

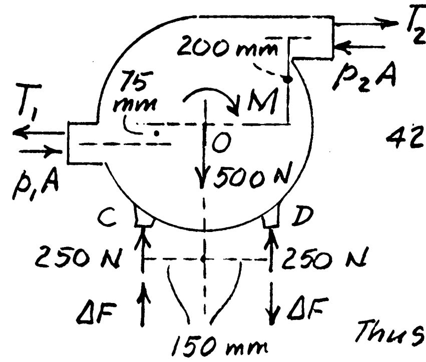



Check from vertical part

$\rho g x - T = \rho x \frac{g}{L} x$, $T = \rho g x\left(1 - \frac{x}{L}\right)$, OK.

(c) $v\,dv = a_x\,dx$: $\int_0^v v\,dv = \frac{g}{L}\int_0^L x\,dx$, $\frac{v^2}{2} = \frac{g}{L}\frac{L^2}{2}$, $\underline{v = \sqrt{gL}}$

**►4/110** $T_1 = p_1A$, $T_2 = p_2A$




Power $P = M\omega$

$$M = \frac{40(10^3)}{900(2\pi/60)} = 424 \text{ N·m}$$

$$\Sigma M_O = m'(v_2 d_2 - v_1 d_1)$$

$$424 + 0.3\Delta F = \frac{20}{60}(1000)\left[18(0.2) - (18)(-0.075)\right]$$

$$\Delta F = \frac{1650 - 424}{0.3} = 4090 \text{ N}$$

Thus $C = 250 + 4090 = \underline{4340 \text{ N}}$ (up)

$D = 4090 - 250 = \underline{3840 \text{ N}}$ (down)