INSTRUCTOR'S MANUAL

To Accompany

ENGINEERING MECHANICS - DYNAMICS

Volume 2

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USE OF THE INSTRUCTOR'S MANUAL

The problem solution portion of this manual has been prepared for the instructor who wishes to occasionally refer to the authors' method of solution or who wishes to check the answer of his (her) solution with the result obtained by the authors. In the interest of space and the associated cost of educational materials, the solutions are very concise. Because the problem solution material is not intended for posting of solutions or classroom presentation, the authors request that it not be used for these purposes.

In the transparency master section there are approximately 65 solved problems selected to illustrate typical applications. These problems are different from and in addition to those in the textbook. Instructors who have adopted the textbook are granted permission to reproduce these masters for classroom use.

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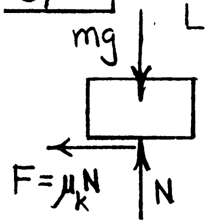


$$\begin{cases} \sum F_y = 0 : N - mg = 0, \quad N = mg \\ \sum F_x = ma_{Gx} : -\mu_k mg = ma_{Gx} \end{cases}$$

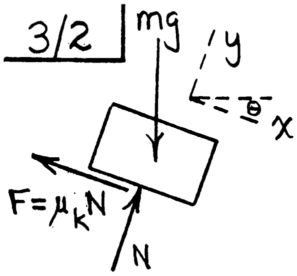
$$a_{Gx} = -\mu_k g = -(0.4)(9.81) = -3.92 \text{ m/s}$$

Kinematics :

$$v^2 - v_0^2 = 2a(x - x_0)$$
$$0 - 7^2 = 2(-3.92)(x - 0)$$
$$\underline{x = 6.24 \text{ m}}$$

$$v = v_0 + at$$
$$0 = 7 - 3.92t, \quad \underline{t = 1.784 \text{ s}}$$

3/2



$\Sigma F_y = 0: \quad N - mg\cos\theta = 0, \quad N = mg\cos\theta$

$\Sigma F_x = ma_x: \quad mg\sin\theta - \mu_k mg\cos\theta = ma_{Gx}$

$a_{Gx} = g(\sin\theta - \mu_k\cos\theta)$

(a) $\theta = 15°$: $\quad a_{Gx} = 9.81(\sin 15° - 0.4\cos 15°) = -1.251 \text{ m/s}^2$

$v^2 - v_0^2 = 2a(x - x_0)$

$0 - 7^2 = 2(-1.251)(x - 0)$, $\quad \underline{x = 19.58 \text{ m}}$

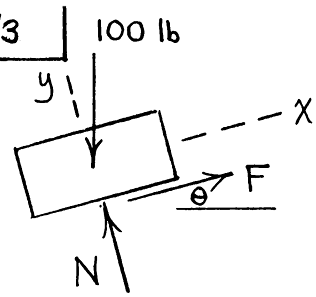
$v = v_0 + at$

$0 = 7 - 1.251t$, $\quad \underline{t = 5.59 \text{ s}}$

(b) $\theta = 30°$: $\quad a_{Gx} = 9.81(\sin 30° - 0.4\cos 30°) = 1.51 \text{ m/s}^2$

<u>Crate does not come to rest.</u>

3/3



$$\theta_{max} = \tan^{-1}\mu_s = \tan^{-1}(0.30) = 16.70°$$

$\therefore$ (a) $\theta = 15°$: No motion $\underline{a = 0}$

(b) $\Sigma F_y = 0$: $N = 100 \cos 20° = 94.0$ lb

$$F = \mu_k N = 0.25(94.0) = 23.5 \text{ lb}$$

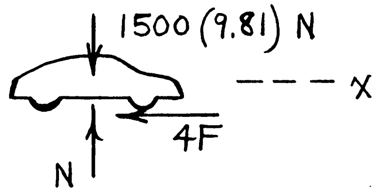
$$\Sigma F_x = ma_x : -100 \sin 20° + 23.5 = \frac{100}{32.2} a$$

$$\underline{a = -3.45 \text{ ft/sec}^2}$$

(Block accelerates down plane)

3/4 $v_2^2 - v_1^2 = 2a(x_2 - x_1)$

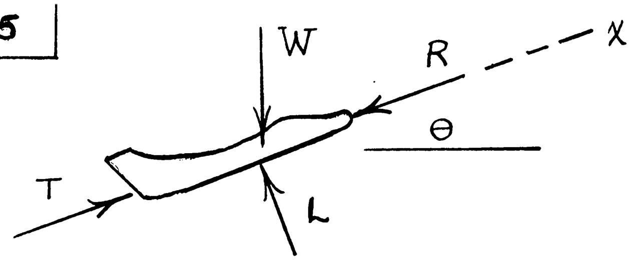
$0^2 - \left(\frac{100}{3.6}\right)^2 = 2a_x(50)$, $a_x = -7.72 \text{ m/s}^2$



$\Sigma F_x = ma_x: \quad -4F = 1500(-7.72)$

$\underline{F = 2890 \text{ N}}$

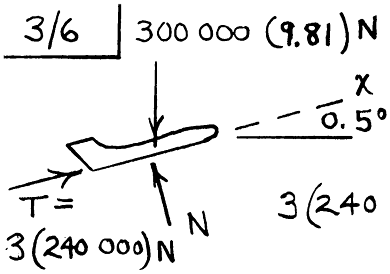
3/5



$$\Sigma F_x = ma_x: \quad T - R - W\sin\theta = \frac{W}{g}a$$

$$n = \frac{T-R}{W} = \underline{\sin\theta + \frac{a}{g}}$$

3/6

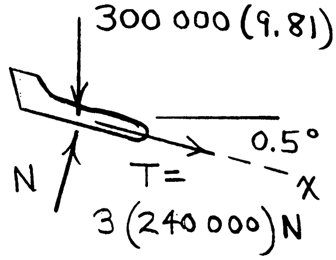


$\Sigma F_x = ma_x$:

$3(240\,000) - 300\,000(9.81)\sin\frac{1}{2}° = 300\,000\, a_x$

$a_x = 2.31$ m/s^2

$v^2 = 2a_x s$: $(220/3.6)^2 = 2(2.31)s$, $\underline{s_u = 807 \text{ m}}$



$\Sigma F_x = ma_x$:

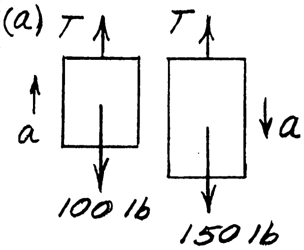
$3(240\,000) + 300\,000(9.81)\sin\frac{1}{2}° = 300\,000\, a_x$

$a_x = 2.49$ m/s^2

$v^2 = 2a_x s$: $(220/3.6)^2 = 2(2.49)s$, $\underline{s_d = 751 \text{ m}}$

3/7 $\Sigma F = ma;$ $T - 100 = \frac{100}{32.2}a$

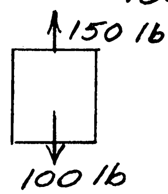
(a)



$$150 - T = \frac{150}{32.2}a$$

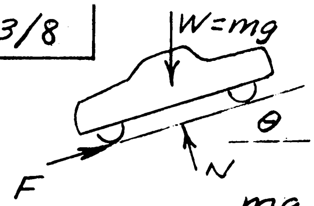
$$50 = \frac{250}{32.2}a\,, \quad a = \frac{32.2}{5} = 6.44\,\frac{ft}{sec^2}$$

(b)



$$150 - 100 = \frac{100}{32.2}a, \quad a = \frac{32.2}{2} = 16.10\,\frac{ft}{sec^2}$$

3/8



For $\theta = \theta_1$, accel. = 0

$\Sigma F = 0$; $F = W \sin\theta_1$

For $\theta = \theta_2$, $\Sigma F = ma$ so

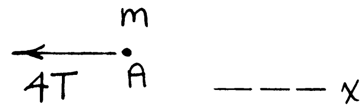
$$mg \sin\theta_1 - mg \sin\theta_2 = ma$$

$$\underline{a = g(\sin\theta_1 - \sin\theta_2)}$$

3/19

$\overset{+}{\leftarrow} \Sigma F = ma: \; 4(40{,}000) = \dfrac{750{,}000}{32.2}\, a$

$a = 6.87 \text{ ft/sec}^2$



$\underline{a}_{A/B} = \underline{a}_A - \underline{a}_B = -6.87\underline{i} - \underline{0} = \underline{-6.87\underline{i} \text{ ft/sec}^2}$

$v_A = (v_A)_0 + at = 0 + 6.87(10) = 68.7 \text{ ft/sec}$

$v_B = 15\left(\dfrac{88}{60}\right) = 22 \text{ ft/sec}$

$\underline{v}_{A/B} = \underline{v}_A - \underline{v}_B = -68.7\underline{i} - 22(\cos 30°\underline{i} + \sin 30°\underline{j})$

$= \underline{-87.7\underline{i} - 11\underline{j} \text{ ft/sec}}$

3/10

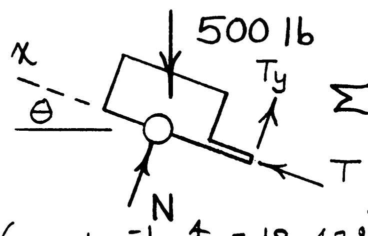
$$v^2 - v_0^2 = 2a(s - s_0)$$

$$0^2 - \left(5\,\frac{5280}{3600}\right)^2 = 2a(-4)$$

$$a = 6.72 \text{ ft/sec}^2$$

FBD of cart (treat as a particle):

(Static)



$$\Sigma F_x = 0: \quad T_s - 500 \sin 18.43° = 0$$

$$T_s = 158.1 \text{ lb}$$

$$\left(\theta = \tan^{-1}\frac{4}{12} = 18.43°\right)$$

$$\Sigma F_x = ma_x: \quad T - 500 \sin 18.43° = \frac{500}{32.2}(6.72)$$

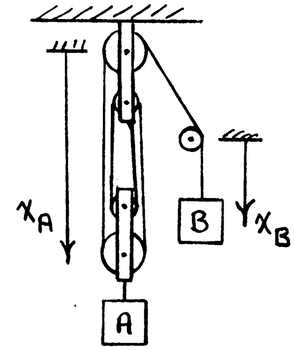
$$T = 262 \text{ lb}$$

Percent increase $n = \dfrac{262 - 158.1}{158.1}(100\%)$

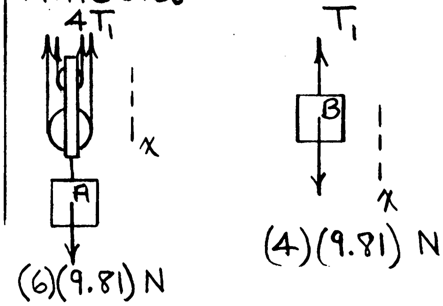
$$= \underline{66.0\%}$$

3/11 Kinematics: $4x_A + x_B = L_{rope} + \text{constant}$

$$\therefore \quad 4a_A + a_B = 0 \qquad (1)$$



Kinetics:



$A: \Sigma F_x = ma_x: \quad 6(9.81) - 4T_1 = 6a_A \qquad (2)$

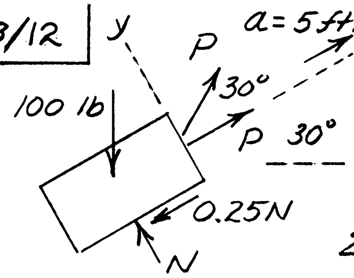
$B: \Sigma F_x = ma_x: \quad 4(9.81) - T_1 = 4a_B \qquad (3)$

Solution of Eqs. (1) – (3):
$$\begin{cases} a_A = -1.401 \text{ m/s}^2 \\ a_B = 5.61 \text{ m/s}^2 \\ \underline{T_1 = 16.82 \text{ N}} \end{cases}$$

Tension in cable above A

$T_2 = 4T_1 = \underline{67.3 \text{ N}}$

3/12



$\Sigma F_x = m a_x$;

$P(1 + \cos 30°) - 0.25N$
$- 100 \sin 30° = \frac{100}{32.2}(5)$

$\Sigma F_y = 0$; $N + P \sin 30° - 100 \cos 30° = 0$

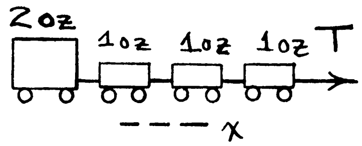
$$\left.\begin{aligned} 1.866P - 0.25N &= 65.53 \\ 0.5P + N &= 86.6 \end{aligned}\right\}$$

solve simultaneously & get $N = 64.7$ lb

$\underline{P = 43.8 \text{ lb}}$

3/13 | Coupler 1 will fail first, because it must accelerate more mass than any other coupler.

Rear part of train:

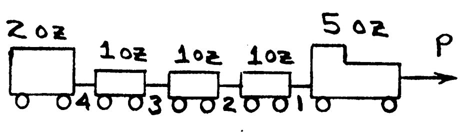


$$\Sigma F_x = ma_x$$

$$T = 0.2 = \left(\frac{5/16}{32.2}\right) a$$

$$a = 20.6 \text{ ft/sec}^2$$

Whole train:

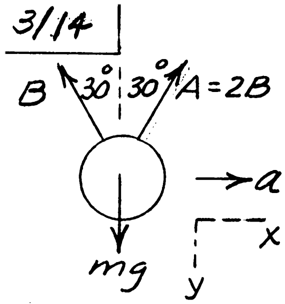


$$\Sigma F_x = ma_x$$

$$P = \left(\frac{10/16}{32.2}\right)(20.6)$$

$$\underline{P = 0.4 \text{ lb}}$$

3/14



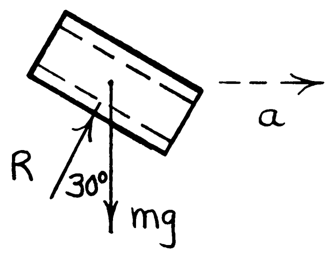
$\Sigma F_x = ma_x$; $2B \sin 30° - B \sin 30° = ma$

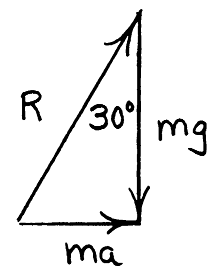
$\Sigma F_y = 0$; $2B \cos 30° + B \cos 30° - mg = 0$

Eliminate B & set $\underline{a = g/3\sqrt{3}}$

3/15

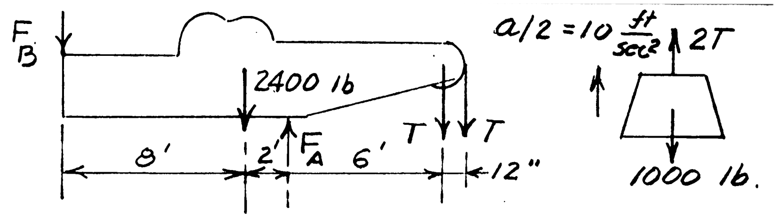
$\Sigma \underline{F} = m\underline{a}$:





$$ma = mg\tan 30° \Rightarrow a = g\tan 30° = \underline{5.66 \text{ m/s}^2}$$

3/16

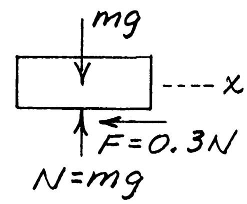


Load; $\Sigma F = ma$; $2T - 1000 = \frac{1000}{32.2}(10)$, $T = 655$ lb

Beam; $\Sigma M_B = 0$; $2400(8) - 10F_A + 655(16 + 17) = 0$

$F_A = \underline{4080 \text{ lb}}$

3/17 Let m = mass of crate



$\Sigma F_x = ma_x$; $-0.3mg = ma_x$

$a_x = -0.3g = -0.3(9.81) = -2.94 \text{ m/s}^2$

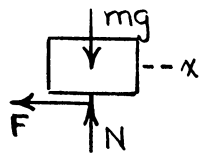
$\int_v^0 v\,dv = \int_0^s a_x\,dx$; $-\frac{v^2}{2} = a_x s$

$s = \frac{-(70/3.6)^2/2}{-2.94} = \underline{64.3 \text{ m}}$

3/18 Truck :

$$\begin{cases} v^2 - v_0^2 = 2a_T(x - x_0) \\ 0^2 - (19.44)^2 = 2a_T(50-0) \\ a_T = -3.78 \text{ m/s}^2 \end{cases}$$

Crate :



$$\Sigma F_x = ma_x : \; -F = m(-3.78)$$

$$F = 3.78\,m$$

$$F_{max} = \mu_s N = 0.3(m\, 9.81) = 2.94\, m$$

$F > F_{max}$, crate slips , $F = \mu_k N$

$$\therefore \; \Sigma F_x = ma_x : \; -0.25\, mg = m a_c , \; a_c = -2.45 \text{ m/s}^2$$

$$a_{c/T} = a_c - a_T = -2.45 - (-3.78) = 1.328 \text{ m/s}^2$$

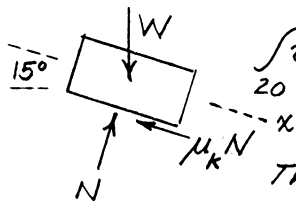
$$v_{c/T}^2 - v_{c/T_0}^2 = 2a_{c/T}(x_{c/T} - x_{c/T_0})$$

$$v_{c/T}^2 - 0^2 = 2(1.328)(3-0) , \quad \underline{v_{c/T} = 2.82 \text{ m/s}}$$

(Truck stopping time = 5.14 s, crate impacts at 2.13 s)

3/19

$\Sigma F_x = ma_x$; $\quad W\sin 15^\circ - \mu_k(W\cos 15^\circ) = \frac{W}{g}a_x$



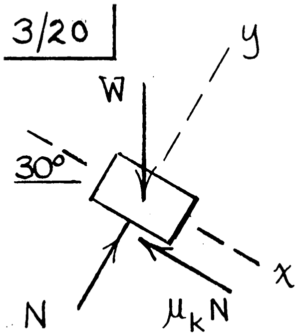
$$\int_{20}^{10} v\,dv = \int_{0}^{30} a_x\,dx, \quad a_x = \frac{100-400}{2(30)} = -5\ \frac{ft}{sec^2}$$

constant

Thus $\quad g(\sin 15^\circ - \mu_k\cos 15^\circ) = a_x = -5$

& $\mu_k = \tan 15^\circ + \frac{5}{32.2(0.9659)} = \underline{0.429}$

3/20



$\Sigma F_y = 0: \; N = W\cos 30°$

$\Sigma F_x = ma_x:$

$W\sin 30° - \mu_k W\cos 30° = \frac{W}{g} a_x$

$a_x = 32.2\left(\frac{1}{2} - \mu_k \frac{\sqrt{3}}{2}\right)$

$v_2^2 - v_1^2 = 2a(x_2 - x_1): \; 3^2 - 1.2^2 = 2(32.2)\left(\frac{1}{2} - \mu_k\frac{\sqrt{3}}{2}\right)6$

$\underline{\mu_k = 0.555}$

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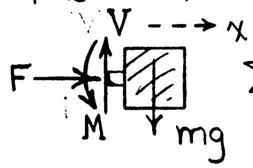
$$x = X \sin \omega t$$

$$\dot{x} = X \omega \cos \omega t$$

$$\ddot{x} = -X \omega^2 \sin \omega t \,, \quad \ddot{x}_{max} = X \omega^2$$

FBD of circuit board:



$$\Sigma F_x = m a_x : \quad F = m\left(-X \omega^2 \sin \omega t\right)$$

$$\underline{F_{max} = m X \omega^2}$$

3/22

$F = ma: \; 2.5 = 70(10^3)\,a, \quad a = 35.7(10^{-6})\ \frac{m}{s^2}$

$\Delta v = \int a\,dt = at, \quad t = \dfrac{(65-40)\,10^3}{35.7(10^{-6})(3600)^2\,24}$

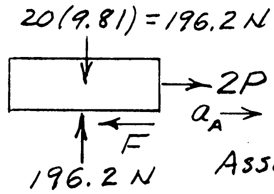
$= 2251 \text{ days or } \underline{6.16 \text{ years}}$

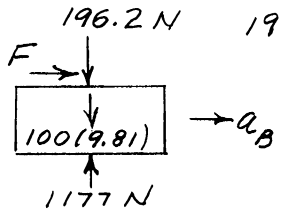
$s = \int v\,dt = v_{AV}\,t = \dfrac{65+40}{2}(10^3)(2251)(24)$

$= \underline{2.84(10^9)\text{ km}}$

3/23

$\mu = 0.5$





(a) $2P = 120$ N

$F_{max} = 0.5(196.2)$
$= 98.1 \text{ N} < 2P$

Assume slipping occurs & $F = 98.1$ N

A; $\Sigma F = ma$; $120 - 98.1 = 20\,a_A$

$\underline{a_A = 1.095 \text{ m/s}^2}$

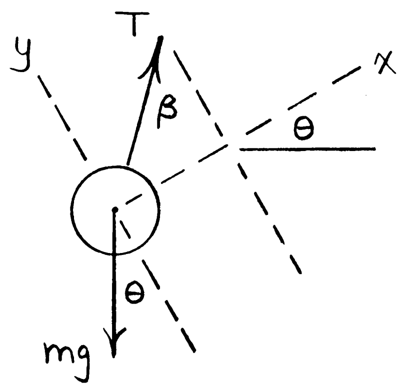
B; $\Sigma F = ma$; $98.1 = 100\,a_B$

$a_A > a_B$ so assumption OK. $\underline{a_B = 0.981 \text{ m/s}^2}$

(b) $2P = 80 \text{ N} < F_{max}$, so no slipping occurs & for block & cart combined,

$\Sigma F = ma$; $80 = 120a$, $\underline{a_A = a_B = a = 0.667 \text{ m/s}^2}$

3/24



$\Sigma F_x = ma_x$: $T \sin\beta - mg \sin\theta = ma$

$\Sigma F_y = 0$: $T \cos\beta - mg \cos\theta = 0$

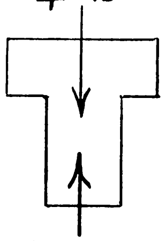
Eliminate T : $\underline{\beta = \tan^{-1} \dfrac{a + g \sin\theta}{g \cos\theta}}$

3/25

$+\uparrow \Sigma F = ma: \quad \frac{1}{4} + \frac{1}{4}k - \frac{1}{4} = \frac{1/4}{g}(5g)$

$\underline{k = 5 \text{ lb/in.}}$

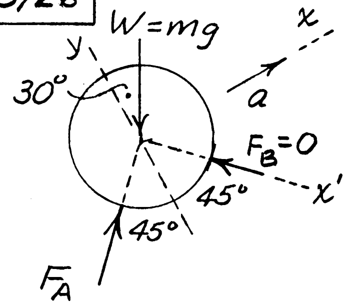
$\frac{1}{4}$ lb



$\frac{1}{4} + kx =$

$\frac{1}{4} + k\left(\frac{1}{4}\right)$

3/26



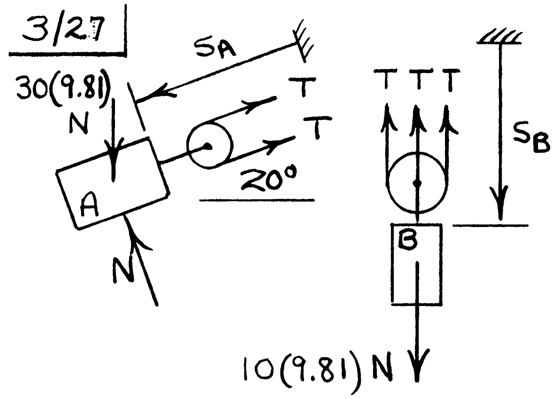
$\Sigma F_{x'} = m a_{x'}$;

$mg\cos(45° + 30°) = ma\cos 45°$

$a = g\,\dfrac{\cos 75°}{\cos 45°} = 9.81\,\dfrac{0.2588}{0.7071}$

$= \underline{0.366g}$

3/27



Kinematic constraint: $L = 2s_A + 3s_B$

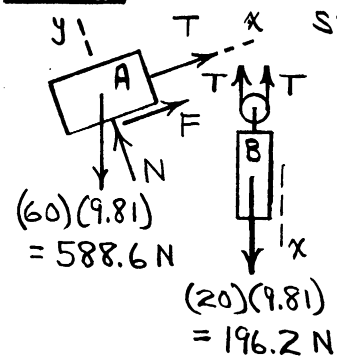
$$\Rightarrow \quad 0 = 2a_A + 3a_B \quad (1)$$

$\overset{+}{\swarrow}$ $\Sigma F = m_A a_A$: $30(9.81)\sin 20° - 2T = 30a_A$ (2)

$+\downarrow$ $\Sigma F = m_B a_B$: $10(9.81) - 3T = 10a_B$ (3)

Solution of Eqs. (1)-(3): $\begin{cases} \underline{a_A = 1.024 \text{ m/s}^2} \\ \underline{a_B = -0.682 \text{ m/s}^2} \\ T = 35.0 \text{ N} \end{cases}$

3/28



Check for motion by assuming static equilibrium.

B : $2T = 196.2$, $T = 98.1$ N

A : $\Sigma F_x = 0$: $98.1 - 588.6 \sin 30° + F = 0$, $F = 196.2$ N

$F_{max} = \mu_s N = (0.25)(588.6)\cos 30° = 127.4$ N

$F > F_{max} \Rightarrow$ motion ($\swarrow$)

From kinematics, $a_A = 2a_B = 2a$

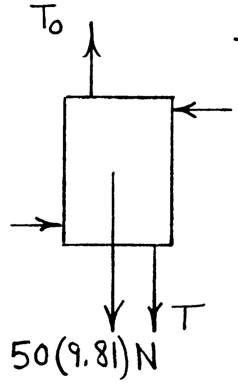
A : $\Sigma F_x = ma_x$: $T + 0.2(588.6 \cos 30°) - 588.6 \sin 30° = 60(2a)$

B : $\Sigma F_x = ma_x$: $-2T + 196.2 = 20a$

Solution : $\underline{a = -0.725 \text{ m/s}^2}$, $\underline{T = 105.4 \text{ N}}$

3/29

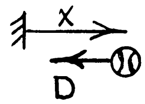
$$T_0 = Te^{\mu\beta} = Te^{0.2(\pi+\pi)} = 3.51T$$



$$+\downarrow \Sigma F = ma: \quad 50(9.81) + T - 3.51T = 50(1.2)$$

$$\underline{T = 171.3 \text{ N}}$$

3/30



(Neglect weight for now)

$$\Sigma F_x = ma_x: \; -D = -C_D \tfrac{1}{2}\rho v^2 S = m v \frac{dv}{dx}$$

$$\int_0^x \left(-C_D \tfrac{1}{2}\rho S\right) dx = m\int_{v_0}^{v} \frac{dv}{v}$$

$$\Rightarrow \quad v = v_0 e^{\left(-\frac{1}{2}C_D \rho S x/m\right)}$$

$$= v_0 e^{\left(-\frac{1}{2}(0.3)\left(\frac{0.07530}{32.2}\right)(\pi)\left(\frac{9.125/2\pi}{12}\right)^2 x \Big/ \frac{5.125}{16\cdot 32.2}\right)}$$

$$\underline{v = v_0 e^{-1.623(10^{-3})x}}$$

For $v_0 = 90$ mi/hr and $x = 60$ ft: $\underline{v = 81.7 \text{ mi/hr}}$

Comment on y-motion. Assume $v = 90$ mi/hr = constant. Time t to plate is

$$t = \frac{60}{90\left(5280/3600\right)} = 0.455 \text{ sec}$$

$v_y = v_{y_0} - gt = -32.2(0.455) = -14.64$ ft/sec, which would not appreciably change $v = \sqrt{v_x^2 + v_y^2}$.

3/31 $D = kv^2$: $120(10^3) = k\left(300/3.6\right)^2$

$k = 17.28\ \dfrac{\mathrm{N \cdot s^2}}{\mathrm{m^2}}$, $D = 17.28\, v^2$

$\Sigma F = ma$: $-17.28\, v^2 = 5000\, a$

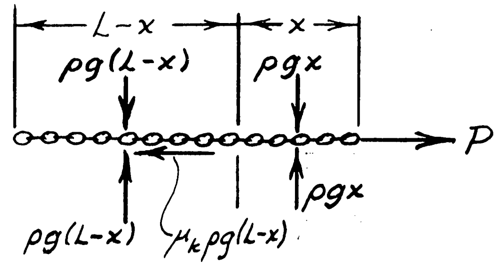
$v\,dv = a\,dx$: $-\dfrac{5000}{17.28}\displaystyle\int_{v_1}^{v_2} \frac{v\,dv}{v^2} = \int_0^x dx$

$v_1 = \left(300/3.6\right)$ m/s, $v_2 = v_1/2 = \left(150/3.6\right)$ m/s

So $-\dfrac{5000}{17.28} \ln v \Big|_{300/3.6}^{150/3.6} = x$, $\underline{x = 201\text{ m}}$

3/32

$\Sigma F_x = ma_x: \; P - \mu_k \rho g(L-x) = \rho L \ddot{x}$

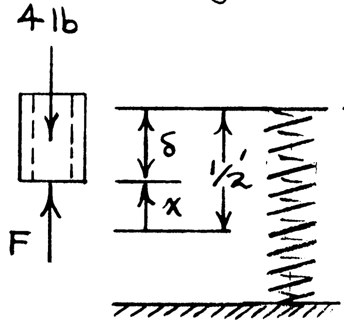


$$\int v\,dv = \int \ddot{x}\,dx: \; \int_0^v v\,dv = \int_0^L \left(\frac{P}{\rho L} - \mu_k g + \mu_k g \frac{x}{L}\right) dx$$

$$\frac{v^2}{2} = \left(\frac{P}{\rho L} - \mu_k g\right)L + \frac{\mu_k g}{2L}L^2; \quad \underline{v = \sqrt{\frac{2P}{\rho} - \mu_k g L}}$$

3/33 Spring force $F = k\delta = k\left(\frac{1}{2} - x\right)$

$$= 120\left(\frac{1}{2} - x\right)$$



$\uparrow \Sigma F_x = ma_x: \; -4 + 120\left(\frac{1}{2} - x\right) = \frac{4}{32.2} a_x$

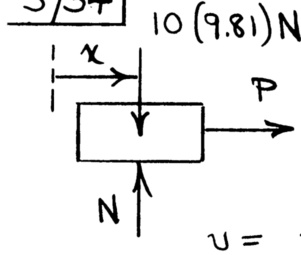
$$\underline{a_x = 32.2\,(14 - 30x)}$$

$$32.2\,(14 - 30x) = v\,\frac{dv}{dx}$$

$$\int_0^{1/2} 32.2\,(14 - 30x)\,dx = \int_0^v v\,dv$$

$$\underline{v = 14.47 \text{ ft/sec}}$$

3/34



$\Sigma F_x = ma_x$: $P = 10a_x$

$\frac{P}{10} = \frac{dv}{dt}$, $v = \int_0^t \frac{P}{10}\,dt$

For $P_1 = 10t$:

$v = t^2/2$, $s = t^3/6$

At $t = 5$ s, $\underline{v = 12.5 \text{ m/s}}$, $\underline{s = 20.8 \text{ m}}$

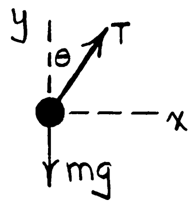
For $P_2 = kt^2$: $50 = k(5)^2$, $k = 2 \text{ N/s}^2$

So $P_2 = 2t^2$

$v = \int_0^t \frac{2t^2}{10}\,dt = \frac{t^3}{15}$, $s = t^4/60$

At $t = 5$s, $\underline{v = 8.33 \text{ m/s}}$, $\underline{s = 10.42 \text{ m}}$

3/35 Mass m :



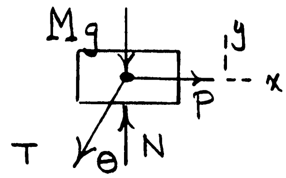
$\sum F_y = 0: \; T\cos\theta - mg = 0$

$T = mg/\cos\theta$

$\sum F_x = ma_x: \; T\sin\theta = ma$

$\left(\frac{\cancel{m}g}{\cos\theta}\right)\sin\theta = \cancel{m}a, \quad \underline{\theta = \tan^{-1}\left(\frac{a}{g}\right)}$

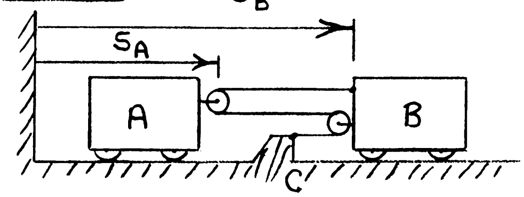
Cart M :



$\sum F_x = ma_x: \; P - T\sin\theta = Ma$

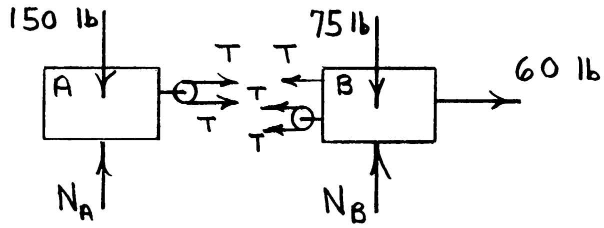
$P = ma + Ma = \underline{(m+M)a}$

3/36



$$L = 2(s_B - s_A) + (s_B - s_C) + \text{constants}$$

$$\Rightarrow \quad 0 = 3a_B - 2a_A \qquad (1)$$

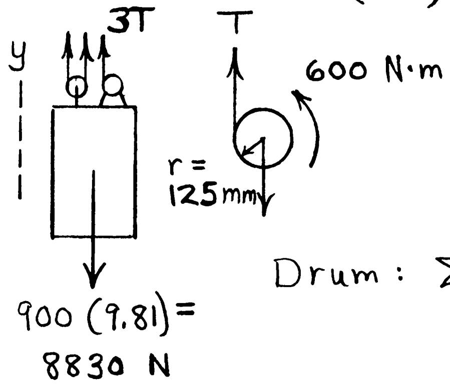


$\xrightarrow{+} \Sigma F = ma$: (A) $\quad 2T = \dfrac{150}{32.2} a_A \qquad (2)$

(B) $\quad 60 - 3T = \dfrac{75}{32.2} a_B \qquad (3)$

Solve Eqs. (1) − (3) : $\begin{cases} a_A = 7.03 \text{ ft/sec}^2 \\ a_B = 4.68 \text{ ft/sec}^2 \\ T = 16.36 \text{ lb} \end{cases}$

3/37 Case (a) has the higher acceleration because of three (> 2) supporting cables.



Drum : $\Sigma M = 0 : T(0.125) - 600 = 0$

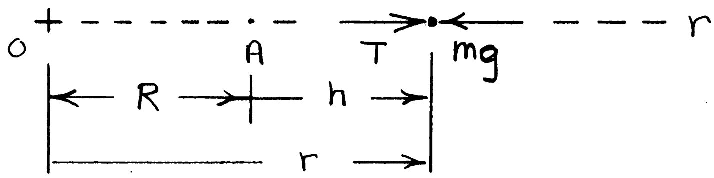
$T = 4800$ N

Elevator : $\Sigma F_y = ma_y : 3(4800) - 8830 = 900a$

$a = 6.19$ m/s^2

$v = v_0 + at = 0 + 6.19(1.2) = \underline{7.43 \text{ m/s}}$

3/38



$$g = g_0 \frac{R^2}{r^2} \quad \text{(all pertaining to moon)}$$

$$\Sigma F_r = ma_r: \quad T - mg_0 \frac{R^2}{r^2} = mv\frac{dv}{dr}$$

$$\int_R^{2R} \left(\frac{T}{m} - g_0 \frac{R^2}{r^2}\right) dr = \int_0^v v\,dv$$

$$\Rightarrow \quad v = \sqrt{\frac{2TR}{m} - g_0 R} = \sqrt{R\left(\frac{2T}{m} - g_0\right)}$$

Numbers :

$$v = \sqrt{\frac{3476(1000)}{2}\left(\frac{2(2500)}{1200} - 1.62\right)}$$

$$= \underline{2100 \text{ m/s}}$$

3/39

$R = kv$

$\Sigma F_y = ma_y;\quad mg - kv = ma$

$$a = g - \frac{k}{m}v$$

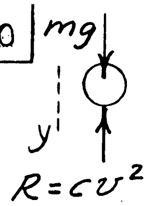
$$v\,dv = a\,dy, \quad \int_0^v \frac{v\,dv}{g - \frac{k}{m}v} = \int_0^h dy$$

$$\frac{m^2}{k^2}\left[\left(g - \frac{k}{m}v\right) - g\ln\left(g - \frac{k}{m}v\right)\right]_0^v = h$$

$$h = \frac{m^2}{k^2}\left[-\frac{k}{m}v - g\ln\left(1 - \frac{kv}{mg}\right)\right]$$

$$\underline{h = \frac{m^2}{k^2}\,g\ln\left(\frac{1}{1 - \frac{kv}{mg}}\right) - \frac{mv}{k}}$$

3/40



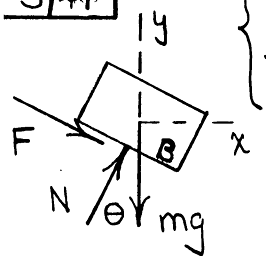
$\Sigma F_y = ma_y; \quad mg - cv^2 = ma$

$$a = g - \frac{c}{m}v^2$$

$v\,dv = a\,dy, \quad \int_0^v \frac{v\,dv}{g - \frac{c}{m}v^2} = \int_0^h dy$

$-\frac{m}{2c}\ln\left(g - \frac{c}{m}v^2\right)\Big]_0^v = h, \quad h = \frac{m}{2c}\ln\left(\frac{mg}{mg - cv^2}\right)$

3/41



$$\begin{cases} \Sigma F_x = ma_x : & N\sin\theta + F\cos\theta = ma \\ \Sigma F_y = 0 : & N\cos\theta - F\sin\theta - mg = 0 \end{cases}$$

Solving, $\begin{cases} F = m(a\cos\theta - g\sin\theta) \\ N = m(a\sin\theta + g\cos\theta) \end{cases}$

For slip impending, $F = \mu_s N$

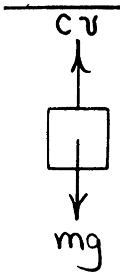
or $\cancel{m}(a\cos\theta - g\sin\theta) = \mu_s \cancel{m}(a\sin\theta + g\cos\theta)$

Solve for θ to obtain $\theta = \tan^{-1}\left(\frac{a - \mu_s g}{\mu_s a + g}\right)$

For large a $(a >> g)$, $\underline{\theta = \tan^{-1}\left(\frac{1}{\mu_s}\right)}$

$$\therefore \underline{\tan^{-1}\left(\frac{1}{\mu_s}\right) \le \theta \le \frac{\pi}{2}}$$

3/42



$\Sigma F_y = ma_y: \quad mg - cv = ma_y$

$\downarrow y$ (a) For $a_y = 0$, $\quad mg - cv = 0$

$$v_s = \frac{mg}{c} = \frac{100(9.81)}{3000} = \underline{0.327 \text{ m/s}}$$

(b) $mg - cv = m\frac{dv}{dt}$

$$\int_0^t dt = \int_0^v \frac{dv}{g - \frac{c}{m}v} = -\frac{m}{c}\int_0^v \frac{-\frac{c}{m}\,dv}{g - \frac{c}{m}v}$$

$$t = -\frac{m}{c}\ln\left(g - \frac{c}{m}v\right)\Big|_0^v = -\frac{m}{c}\ln\left(\frac{g - \frac{c}{m}v}{g}\right)$$

$$\Rightarrow v = \frac{mg}{c}\left[1 - e^{-ct/m}\right] = 0.327\left[1 - e^{-30t}\right]$$

$$\left[1 - e^{-30t}\right] = 0.9 \Rightarrow \underline{t = 0.0768 \text{ s}}$$

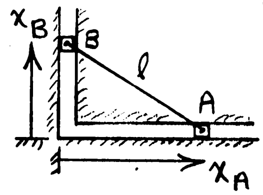
$$v = \frac{dy}{dt} = 0.327\left[1 - e^{-30t}\right]$$

$$\int_0^y dy = \int_0^t 0.327\left[1 - e^{-30t}\right]dt$$

$y = 0.327\left[t + \frac{1}{30}\left(e^{-30t} - 1\right)\right]$

y when $t = 0.0768$ s is $\underline{y = 0.01529 \text{ m}}$

3/43.



$$x_A^2 + x_B^2 = l^2$$

$$\cancel{2}x_A\dot{x}_A + \cancel{2}x_B\dot{x}_B = 0$$

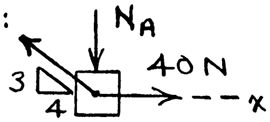
$$x_A\ddot{x}_A + \dot{x}_A^2 + x_B\ddot{x}_B + \dot{x}_B^2 = 0$$

So $\dot{x}_B = -\dfrac{x_A\dot{x}_A}{x_B} = \dfrac{-(0.4)(0.9)}{0.3} = -1.2$ m/s

$$\ddot{x}_B = \frac{-\dot{x}_B^2 - \dot{x}_A^2 - x_A\ddot{x}_A}{x_B} = \frac{-1.2^2 - 0.9^2 - 0.4\ddot{x}_A}{0.3}$$

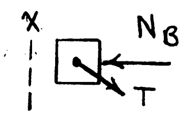
$$= -7.5 - \tfrac{4}{3}\ddot{x}_A \quad \text{or} \quad a_B = -7.5 - \tfrac{4}{3}a_A \quad (1)$$

A:



$\Sigma F_x = ma_x$: $40 - \frac{4}{5}T = 2a_A$ (2)

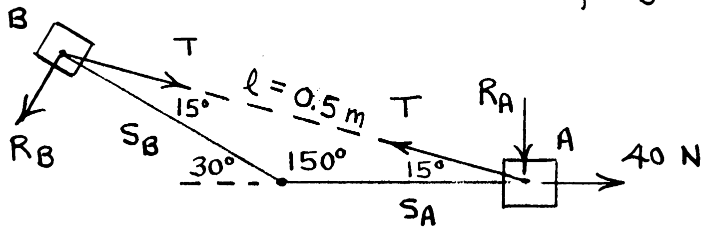
B:



$\Sigma F_x = ma_x$: $-\frac{3}{5}T = 3a_B$ (3)

Solution of Eqs. (1)-(3):
$$\begin{cases} a_A = 1.364 \text{ m/s}^2 \\ a_B = -9.32 \text{ m/s}^2 \\ T = 46.6 \text{ N} \end{cases}$$

3/44 $\sin 150°/l = \sin 15°/s_B$, $s_B = s_A = 0.259$ m



Law of cosines: $l^2 = s_A^2 + s_B^2 - 2s_A s_B \cos 150°$

$$2l\dot{l} = 0 = 2s_A v_A + 2s_B v_B - 2\left(-\frac{\sqrt{3}}{2}\right)(s_A v_B + s_B v_A)$$

$$s_A v_A + s_B v_B + \frac{\sqrt{3}}{2}(s_A v_B + v_A s_B) = 0^*$$

With $s_A = s_B = 0.259$ m, $v_A = 0.4$ m/s : $v_B = -0.4$ m/s

Differentiate $*$: $v_A^2 + s_A a_A + v_B^2 + s_B a_B + \frac{\sqrt{3}}{2}(s_A a_B + v_A v_B + a_A s_B + v_A v_B) = 0$

Numbers: $0.483\,a_A + 0.483\,a_B + 0.0429 = 0$ (1)

Kinetics:

$\overset{+}{\nwarrow}\ \Sigma F = ma_B : -T\cos 15° = 3a_B$ (2)

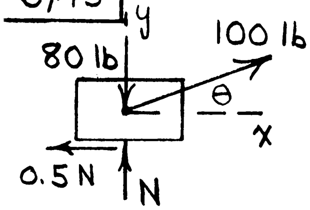
$\overset{+}{\rightarrow}\ \Sigma F = ma_A : 40 - T\cos 15° = 2a_A$ (3)

Solution of Eqs. (1)-(3): $T = 25.0$ N

$a_A = 7.95$ m/s^2

$a_B = -8.04$ m/s^2

3/45



$\Sigma F_x = ma_x$: $100 \cos\theta - 0.5N = \frac{80}{32.2}(26)$

$\Sigma F_y = 0$: $N - 80 + 100 \sin\theta = 0$

Eliminate N to obtain $2\cos\theta + \sin\theta = 2.0919$

Use $\sin\theta = \sqrt{1-\cos^2\theta}$ to obtain

$$5\cos^2\theta - 8.3677 \cos\theta + 3.3762 = 0$$

Solve the quadratic to get $\theta = \underline{5.88°}, \underline{47.2°}$

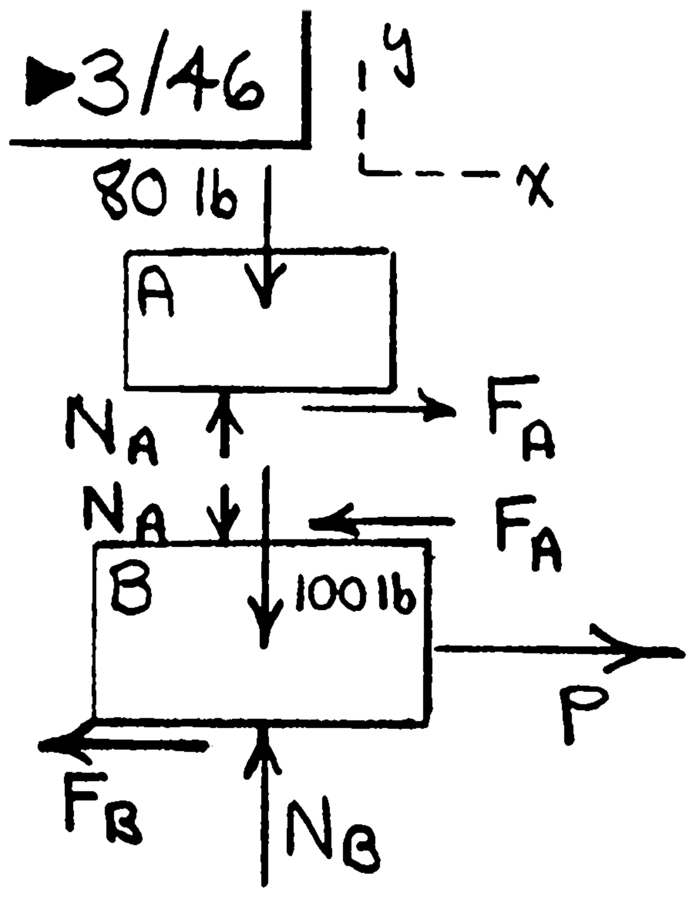
Check that block moves in both cases:

$\theta = 5.88°$: $\begin{cases} N = 80 - 100 \sin 5.88° = 69.8 \text{ lb} \\ F_{max} = 0.6(69.8) = 41.9 \text{ lb} < 100 \cos 5.88° = 99.5 \text{ lb} ✓ \end{cases}$

$\theta = 47.2°$: $N = 6.57$ lb, $F_{max} = 3.94$ lb

$100 \cos 47.2° = 67.9$ lb ✓

▶3/46



$N_B = 180$ lb

$F_{B\,max} = \mu_s N = (0.15)(180) = 27$ lb

$\therefore$ No motion for $0 \le P \le 27$ lb

$N_A = 80$ lb, $F_{A\,max} = (.2)(80) = 16$ lb

$\Sigma F_x = ma_x$ for A: $16 = \frac{80}{32.2} a_A$

$a_A = 6.44$ ft/sec² – max. for no slipping between A and B.

Corresponding $P \rightarrow \Sigma F_x = ma_x$ for system:

$P - 0.1(180) = \frac{180}{32.2}(6.44)$, $P = 54$ lb.

$27 \le P \le 54$ lb:

$\Sigma F = ma_x$: $P - 0.1(180) = \frac{180}{32.2} a$

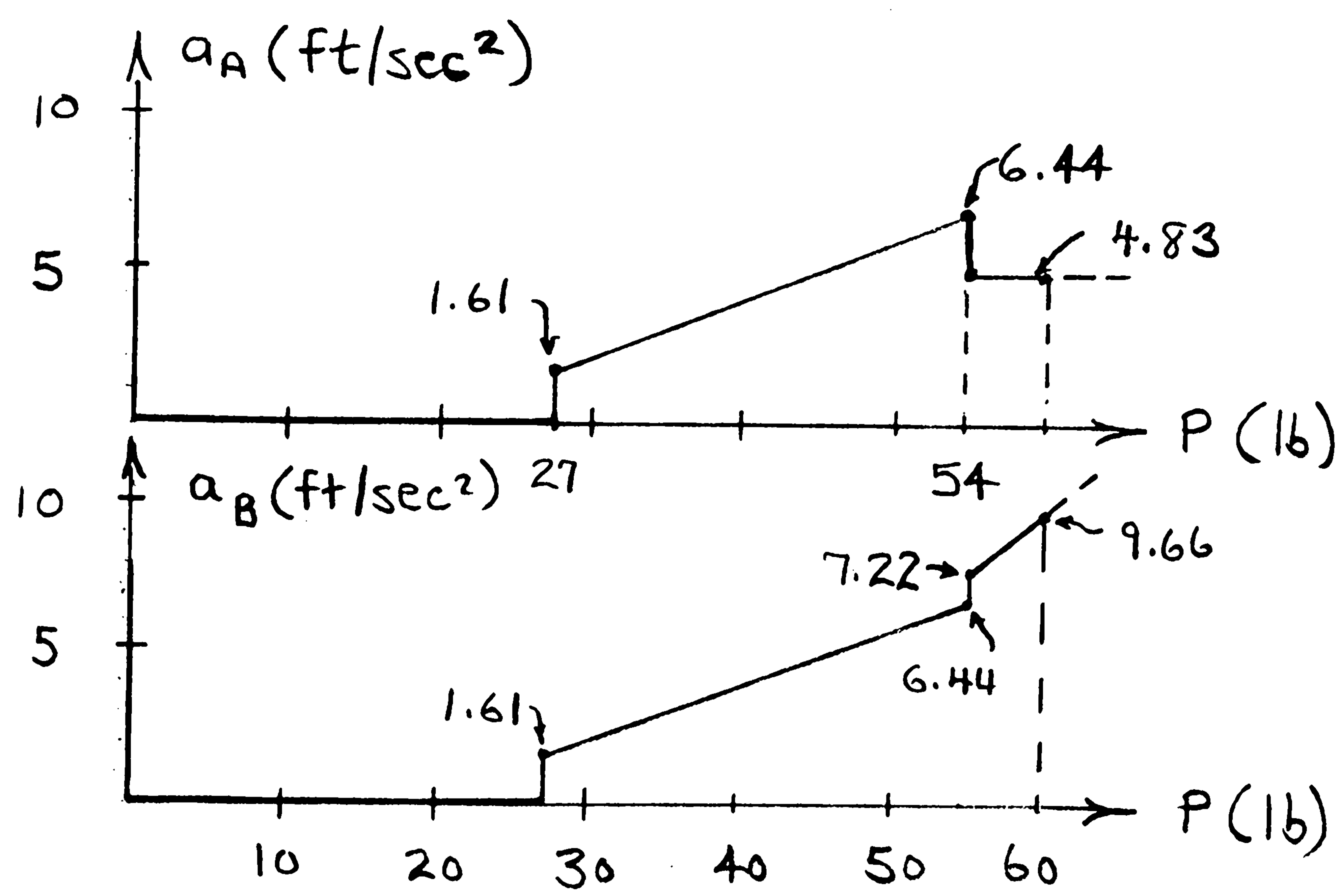
$a_A = a_B = a = 0.1789\,P - 3.22$

$P \ge 54$ lb:

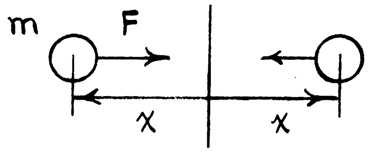
A: $0.15(80) = \frac{80}{32.2} a_A$, $a_A = 4.83$ ft/sec²

B: $P - (0.1)(180) - (0.15)(80) = \frac{100}{32.2} a_B$

$a_B = 0.322P - 9.66$



3/47



$$F = \frac{Gm^2}{x^2}$$

$$m = \rho V = 7210\left(\tfrac{4}{3}\pi\, 0.05^3\right) = 3.775 \text{ kg}$$

$$\Sigma F_x = ma_x: \quad -\frac{Gm^2}{(2x)^2} = mv\frac{dv}{dx}$$

$$-\frac{Gm}{4}\int_{x_0=0.5}^{x} \frac{dx}{x^2} = \int_{v_0=0}^{v} v\,dv$$

$$v = \sqrt{Gm}\sqrt{\frac{1}{2x}-1} = \sqrt{(6.673\times10^{-11})(3.775)}\sqrt{\frac{1}{2(0.05)}-1}$$

$$= \underline{4.76\times10^{-5} \text{ m/s}}$$

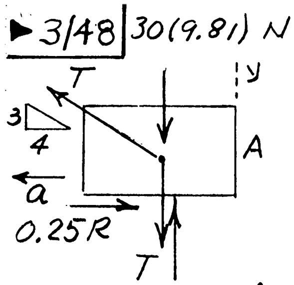
Now, $\dfrac{dx}{dt} = -\sqrt{Gm}\sqrt{\dfrac{\frac{1}{2}-x}{x}}$

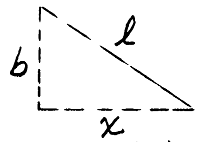
$$\int_{x_0=0.5}^{x=0.05} \frac{\sqrt{x}\,dx}{\sqrt{\frac{1}{2}-x}} = -\sqrt{Gm}\int_0^t dt$$

$$\left[-\sqrt{x}\sqrt{\tfrac{1}{2}-x} + \tfrac{1}{2}\sin^{-1}\sqrt{2x}\right]_{x_0=0.5}^{x=0.05} = -\sqrt{Gm}\; t$$

Solving, $\underline{t = 48{,}800 \text{ s}}$ or $\underline{t = 13 \text{ hr } 33 \text{ min}}$

▶ 3/48





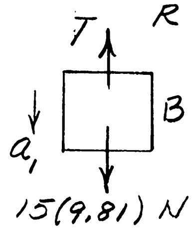
$a = -\ddot{x} \qquad a_1 = -\ddot{l}$

$l^2 = b^2 + x^2$

$l\dot{l} = 0 + x\dot{x}$

$\cancel{\dot{l}^2}^{\,0} + l\ddot{l} = \cancel{\dot{x}^2}^{\,0} + x\ddot{x}, \quad \ddot{l} = \frac{x}{l}\ddot{x}$

or $a_1 = \frac{4}{5}a$



A; $\Sigma F_y = 0$; $R + \frac{3}{5}T - 30(9.81) - T = 0$

$R = 0.4T + 294.3$

$\Sigma F_x = ma_x$; $0.8T - 0.25(0.4T + 294.3) = 30a$

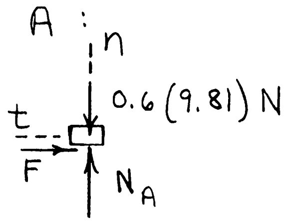
B; $\Sigma F_y = ma_y$; $15(9.81) - T = 15\left(\frac{4}{5}a\right)$

solve simultaneously to get $\underline{T = 138.0\text{ N}}$

$\underline{a = 0.766\text{ m/s}^2}$

3/49

A :

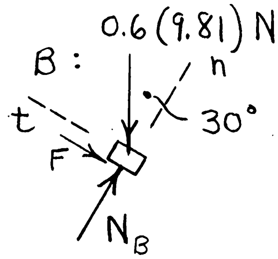


$\Sigma F_n = m\dfrac{v^2}{\rho}$:

$$N_A - 0.6(9.81) = 0.6\,\frac{5^2}{3}$$

$$\underline{N_A = 10.89 \text{ N}}$$

B :



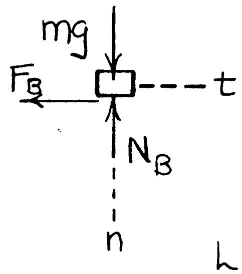
$\Sigma F_n = m\dfrac{v^2}{\rho}$:

$$N_B - 0.6(9.81)\cos 30° = 0.6\,\frac{4^2}{3}$$

$$\underline{N_B = 8.30 \text{ N}}$$

Note: Friction is along the t-axis and does not affect the above calculations.

3/50

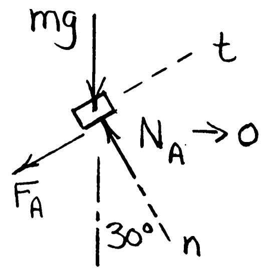


$$\Sigma F_n = ma_n = m\frac{v^2}{\rho}:$$

$$2(9.81) - N = 2\,\frac{3.5^2}{2.4}$$

$$\underline{N_B = 9.41\ \text{N}}$$

Loss of contact at A: $N_A \to 0$

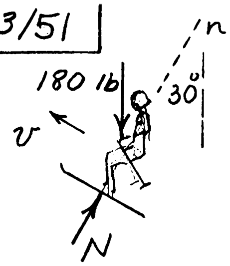


$$\Sigma F_n = ma_n = m\frac{v^2}{\rho}:$$

$$\cancel{m}g\cos 30^\circ = \cancel{m}\frac{v^2}{2.4}$$

$$\underline{v = 4.52\ \text{m/s}}$$

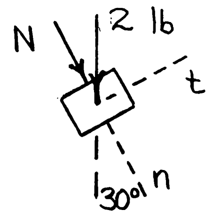
3/51



$$\Sigma F_n = ma_n;\quad N - 180\cos 30° = \frac{180}{32.2}\,\frac{(80)^2}{150}$$

$$N = 180(0.866 + 1.33) = \underline{394 \text{ lb}}$$

3/52



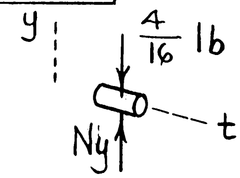
$$\sum F_n = m\frac{v^2}{\rho} : \; N + 2\cos 30° = \frac{2}{32.2}\,\frac{10^2}{2}$$

$$\underline{N = 1.374 \text{ lb}}$$

$$\sum F_t = ma_t : \; -2\sin 30° = \frac{2}{32.2}\,\dot{v}$$

$$\underline{\dot{v} = -16.10 \text{ ft/sec}^2}$$

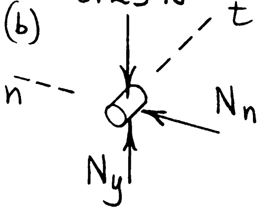
3/53 (a)



$\Sigma F_y = 0: \quad N_y - \frac{4}{16} = 0$

$N_y = \underline{R = 0.25 \text{ lb}}$

(b)



$N_y = 0.25$ lb, as in (a)

$\Sigma F_n = m\frac{v^2}{\rho}: \; N_n = \frac{0.25}{32.2} \; \frac{3^2}{8/12}$

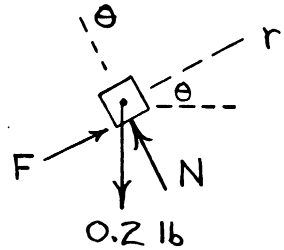
$N_n = 0.1048$ lb

$R = \sqrt{N_y^2 + N_n^2} = \underline{0.271 \text{ lb}}$

3/54 $\Sigma F_\theta = ma_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$: $N - 0.2\cos 30^\circ$

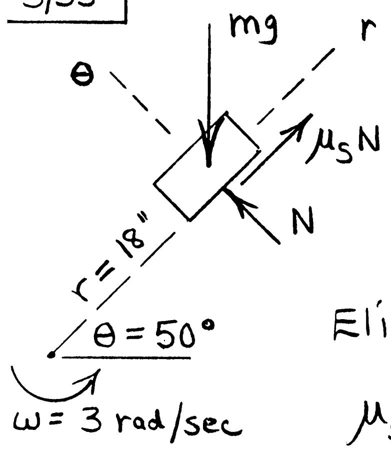
Slider:

$$= \frac{0.2}{32.2}\left(r\cancelto{0}{\ddot{\theta}} + 2(-4)(3)\right)$$



$\underline{N = 0.024 \text{ lb}}$

3/55



$\Sigma F_\theta = ma_\theta : N - mg\cos\theta = 0$

$N = mg\cos\theta$

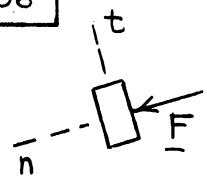
$\Sigma F_r = ma_r : \mu_s N - mg\sin\theta = m(0 - r\omega^2)$

Eliminate N:

$$\mu_s = \tan\theta - \frac{r\omega^2}{g\cos\theta}$$

Numbers: $\mu_s = \tan 50° - \dfrac{(18/12)(3)^2}{32.2\cos 50°} = \underline{0.540}$

3/56



$$a_n = \frac{v^2}{\rho} = \frac{\left[(35)\left(\frac{5280}{3600}\right)\right]^2}{100}$$

$$= 26.4 \frac{\text{ft}}{\text{sec}^2}\left(\frac{1 g}{32.2 \text{ ft/sec}^2}\right)$$

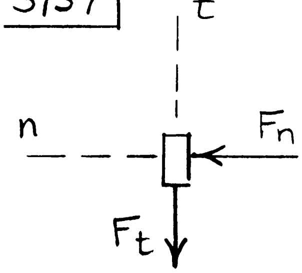
$$= \underline{0.818 g}$$

$\Sigma F_n = ma_n$: $\quad F = \frac{3000}{32.2}(26.4)$

$$= \underline{2455 \text{ lb}}$$

(An average of 614 lb per tire!)

3/57



$\Sigma F_n = ma_n: \; F_n = \frac{3000}{32.2} \frac{\left(25 \cdot \frac{5280}{3600}\right)^2}{100}$

$F_n = 1253 \text{ lb}$

$\sqrt{F_n^{\,2} + F_t^{\,2}} = F_{tot}$

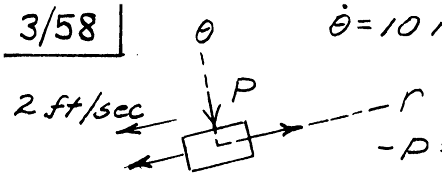
$1253^2 + F_t^{\,2} = 2400^2$

$F_t = 2047 \text{ lb}$

$\Sigma F_t = ma_t: \; -2047 = \frac{3000}{32.2} a_t$

$\underline{a_t = -22.0 \text{ ft/sec}^2}$

3/58



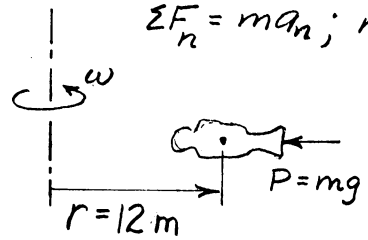
$\dot{\theta} = 10$ rad/sec, $\dot{r} = -2$ ft/sec

$$\Sigma F_{\theta} = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$$

$$-P = \frac{3.22}{32.2}(0 + 2[-2]10)$$

$\underline{P = 4 \text{ lb} \quad (\text{side } A)}$

3/59 | g = surface gravitational acceleration on earth

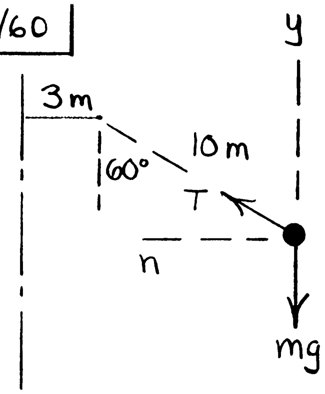


$\Sigma F_n = ma_n$; $mg = mr\omega^2$, $\omega = \sqrt{g/r}$

$\omega = \sqrt{\frac{9.81}{12}} = 0.904$ rad/s

so $N = 0.904\left(\frac{60}{2\pi}\right) = \underline{8.63 \text{ rev/min}}$

3/60



$\Sigma F_n = ma_n$:

$T \sin 60° = m[3 + 10 \sin 60°]\omega^2$

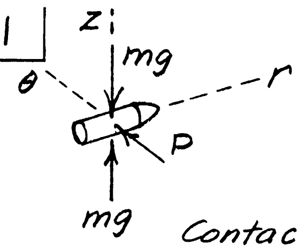
$\Sigma F_y = 0$: $T \cos 60° - mg = 0$

$\Rightarrow \tan 60° = \dfrac{3 + 10 \sin 60°}{9.81} \omega^2$

$\omega = 1.207$ rad/s

$N = 1.207\left(\dfrac{60}{2\pi}\right) = \underline{11.53 \text{ rev/min}}$

3/61



$$\Sigma F_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$$

$$P = 0.06\,(0 + 2[600][0.5])$$

$$\underline{= 36 \text{ N}}$$

Contact is against <u>right-hand</u> side of barrel.

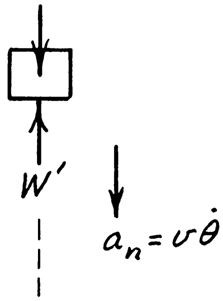
3/62

Make $a_n = g$ $\qquad a_n = \frac{v^2}{\rho} = v\dot{\theta}$, $\dot{\theta} = \frac{a_n}{v} = \frac{g}{v}$

$$\dot{\theta} = \frac{9.81}{\frac{600(1000)}{3600}} = 5.89(10^{-2}) \text{ rad/s} \quad \text{or}$$

$$\underline{\dot{\theta} = 3.37 \text{ deg/s}}$$

3/63



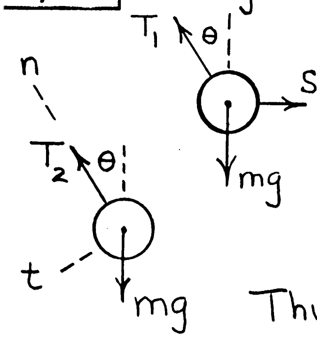
$W = mg: \quad m = 20/9.81 = 2.04 \text{ kg}$

$\Sigma F_n = ma_n: \quad 20 - W' = 2.04\left(\frac{800\times1000}{3600}\,\frac{(1)\pi}{180}\right)$

$= 2.04(3.88) = 7.91 \text{ N}$

$W' = 20 - 7.91 = \underline{12.09 \text{ N}}$

3/64



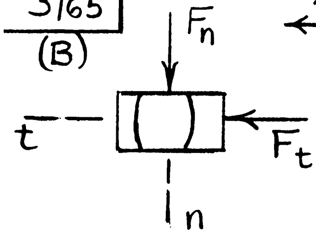
Before: $\sum F_y = 0: \; T_1 \cos\theta = mg$

After: $\sum F_n = ma_n = m\frac{v^2}{r} = 0$

$T_2 = mg\cos\theta$

Thus $k = T_2/T_1 = \dfrac{mg\cos\theta}{mg/\cos\theta} = \underline{\cos^2\theta}$

3/65
(B)



$$\overset{+}{\leftarrow} \quad v_B^2 = v_A^2 + 2a_t s$$

$$(+88)^2 = (-66)^2 + 2a_t(-300)$$

$$a_t = +5.65 \text{ ft/sec}^2$$

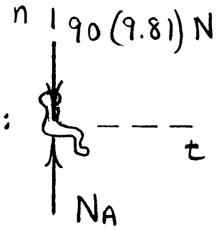
$$a_n = v^2/\rho = (66)^2/600 = 7.26 \text{ ft/sec}^2$$

$$\begin{cases} \Sigma F_t = ma_t: & F_t = \dfrac{3220}{32.2}(5.65) = 565 \text{ lb} \\ \Sigma F_n = ma_n: & F_n = \dfrac{3220}{32.2}(7.26) = 726 \text{ lb} \end{cases}$$

$$F = \sqrt{565^2 + 726^2} = \underline{920 \text{ lb}}$$

3/66

A:

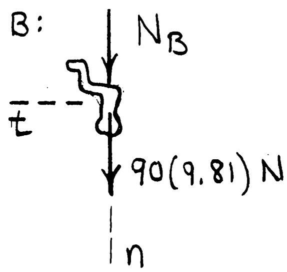


$\Sigma F_n = ma_n$:

$$N_A - 90(9.81) = 90\,\frac{(600/3.6)^2}{1000}$$

$$\underline{N_A = 3380\ \text{N}}$$

B:



$\Sigma F_n = ma_n$:

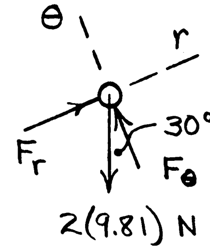
$$N_B + 90(9.81) = 90\,\frac{(600/3.6)^2}{1000}$$

$$\underline{N_B = 1617\ \text{N}}$$

(Note static normal $mg = 90(9.81) = 883$ N)

3/67

F_r and F_θ are the r- and θ-components of the total friction force F.



$\Sigma F_r = ma_r = m(\ddot{r} - r\dot{\theta}^2)$:

$F_r - 19.62 \sin 30° = 2[0 - 1(-0.873)^2]$

$F_r = 8.29$ N

$\Sigma F_\theta = ma_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$

$F_\theta - 19.62 \cos 30° = 2[(1)(3.49) + 2(-0.5)(-0.873)]$

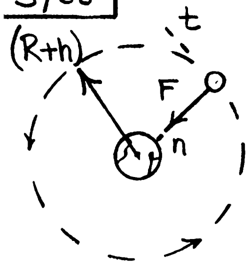
$F_\theta = 25.7$ N

$F = \sqrt{F_r^2 + F_\theta^2} = 27.0$ N

$P = \frac{F/2}{\mu_s} = \frac{27.0/2}{0.5} = \underline{27.0 \text{ N}}$

(Static gripping force $= \underline{19.62 \text{ N}}$)

3/68



$\Sigma F_n = ma_n : F = \frac{Gm_e m}{(R+h)^2} = m \frac{v^2}{(R+h)}$

But $v = \frac{s}{t} = \frac{2\pi (R+h)}{(23.944)(3600)}$

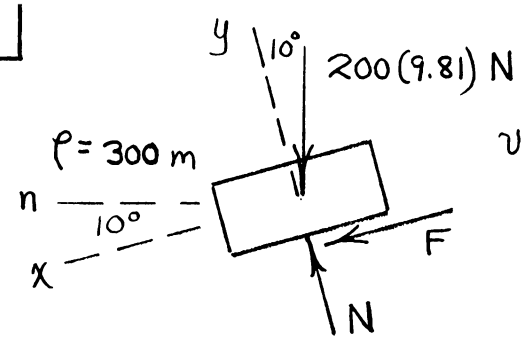
Combining the two equations :

$$v = \frac{2\pi (R+h)}{(23.944)(3600)} = \sqrt{\frac{Gm_e}{(R+h)}}$$

Solve for h to obtain $\underline{h = 3.580 \times 10^7 \text{ m}}$

(35,800 km)

3/69



$v = \frac{100}{3.6} = 27.8 \text{ m/s}$

$\Sigma F_x = ma_x$: $F + 200(9.81)\sin 10° = 200\frac{27.8^2}{300}\cos 10°$

$\underline{F = 165.9 \text{ N}}$

Check: $\Sigma F_y = ma_y$:

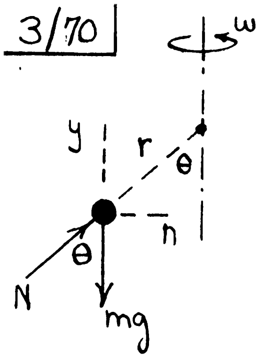
$N - 200(9.81)\cos 10° = 200\frac{27.8^2}{300}\sin 10°$

$N = 2020 \text{ N}$

$F_{max} = \mu_s N = 0.70(2020) = 1415 \text{ N} > F$

Crate does not slip.

3/70



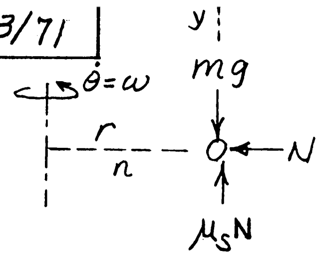
$$\Sigma F_y = 0: \quad N\cos\theta - mg = 0$$
$$N = mg/\cos\theta$$

$$\Sigma F_n = ma_n: \quad N\sin\theta = m(r\sin\theta)\omega^2$$
$$\left(\frac{\cancel{m}g}{\cos\theta}\right)\cancel{\sin}\theta = \cancel{m}r\cancel{\sin}\theta\,\omega^2$$
$$\underline{\omega = \sqrt{\frac{g}{r\cos\theta}}}$$

Note that $\cos\theta = \dfrac{g}{r\omega^2} \leq 1$

$\therefore \quad \omega^2 \geq \dfrac{g}{r}$ is a restriction.

3/71

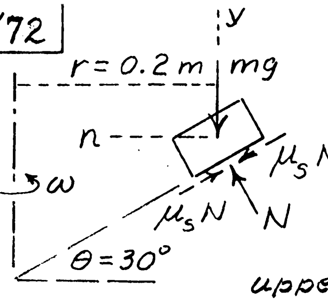


$\Sigma F_n = ma_n;\ N = mr\omega^2$

$\Sigma F_y = 0;\ \mu_s(mr\omega^2) = mg$

$\omega^2 = \dfrac{g}{\mu_s r}, \quad \underline{\omega = \sqrt{\dfrac{g}{\mu_s r}}}$

3/72



$\mu_s N$ is down for ω_{max}
" " up " ω_{min}

$\Sigma F_y = 0;\ N\cos\theta \mp \mu_s N\sin\theta = mg$

$\Sigma F_n = ma_n;\ N\sin\theta \pm \mu_s N\cos\theta = mr\omega^2$

upper sign for ω_{max}
lower sign for ω_{min}

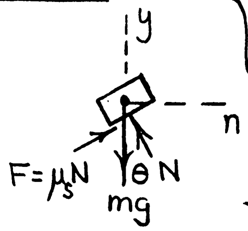
Combine & get $\dfrac{\sin\theta \pm \mu_s\cos\theta}{\cos\theta \mp \mu_s\sin\theta} = \dfrac{r\omega^2}{g}$

$$\omega = \sqrt{\frac{g}{r}}\sqrt{\frac{\sin\theta \pm \mu_s\cos\theta}{\cos\theta \mp \mu_s\sin\theta}} = \sqrt{\frac{9.81}{0.2}}\sqrt{\frac{0.5 \pm 0.3(0.866)}{0.866 \mp 0.3(0.5)}}$$

Upper sign $\omega_{max} = \underline{7.21\ \text{rad/s}}$

Lower sign $\omega_{min} = \underline{3.41\ \text{rad/s}}$

3/73

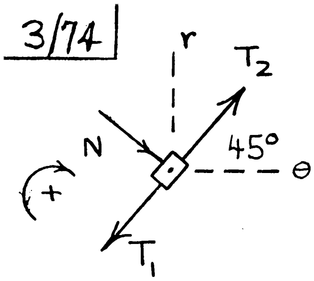


$$\begin{cases} \Sigma F_y = 0: & N\cos\theta - mg + \mu_s N \sin\theta = 0 \\ \Sigma F_n = ma_n: & -N\sin\theta + \mu_s N\cos\theta = m r \omega^2 \end{cases}$$

Solving for ω:

$$\omega = \sqrt{\frac{g}{r}\,\frac{(\mu_s\cos\theta - \sin\theta)}{(\cos\theta + \mu_s\sin\theta)}} = \underline{2.73 \text{ rad/s}}$$

3/74



$\omega = \omega_0 + \alpha t = +0.5t$

$a_r = \ddot{r} - r\dot{\theta}^2 = -0.1(+0.5t)^2$

$= -0.025t^2$ (m/s²)

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = (0.1)(+0.5)$

$= 0.05$ m/s²

Component of total acceleration in direction of wire #2: $0.05 \cos 45° - 0.025t^2 \cos 45°$

Setting this to zero yields $t = 1.414$ s as the time when tension switches from wire #2 to wire #1.

$0 < t < 1.414$ s :

$\Sigma F_r = m(\ddot{r} - r\dot{\theta}^2)$: $-N\cos 45° + T_2 \cos 45°$

$= 2(-0.1)(+0.5t)^2$

$\Sigma F_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$: $+N\cos 45° + T_2 \cos 45°$

$= 2(0.1)(+0.5)$

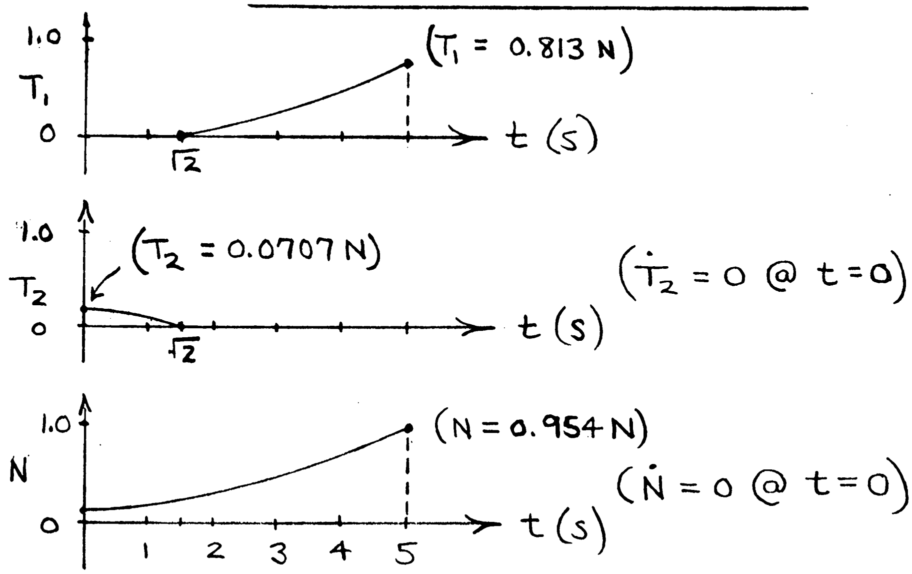
Solving, $\begin{cases} T_2 = 0.0707 - 0.0354t^2 \\ N = 0.0707 + 0.0354t^2 \end{cases}$

$1.414 \le t \le 5$ s

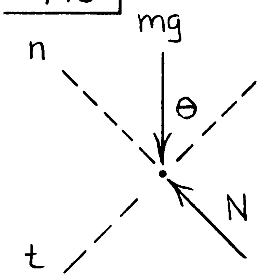
$$\sum F_r = m(\ddot{r} - r\dot{\theta}^2): -N\cos 45^\circ + T_1 \cos 45^\circ = 2(-0.1)(+0.5t)^2$$

$$\sum F_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta}): N\cos 45^\circ - T_1 \cos 45^\circ = 2(0.1)(-0.5)$$

Solving, $\begin{cases} T_1 = -0.0707 + 0.0354t^2 \\ N = 0.0707 + 0.0354t^2 \end{cases}$



3/75



Treat the child as a particle.

$$\begin{cases} \Sigma F_t = ma_t : \ mg\cos\theta = ma_t & (1) \\ \Sigma F_n = ma_n : \ N - mg\sin\theta = m\dfrac{v^2}{R} & (2) \end{cases}$$

From (1) : $g\cos\theta = v\dfrac{dv}{ds} = v\dfrac{dv}{R\,d\theta}$

$$\int_{\theta_0 = 20^\circ}^{\theta} Rg\cos\theta\, d\theta = \int_{v_0 = 0}^{v} v\, dv$$

$$v = \left[2Rg\left(\sin\theta - \sin 20^\circ\right)\right]^{1/2}$$

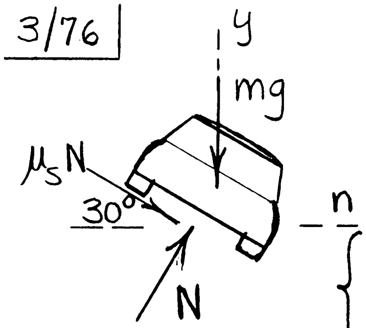
(2) : $N = m\left(g\sin\theta + \dfrac{v^2}{R}\right)$

Numbers ($R = 2.5$ m , $g = 9.81$ m/s^2)

$\theta = 30^\circ$: $\begin{cases} \underline{a_t = 8.50 \text{ m/s}^2} \\ \underline{v = 2.78 \text{ m/s}} \\ \underline{N = 280 \text{ N}} \end{cases}$

$\theta = 90^\circ$: $\begin{cases} \underline{a_t = 0} \\ \underline{v = 5.68 \text{ m/s}} \\ \underline{N = 795 \text{ N}} \end{cases}$

3/76



For no slipping tendency, set $F_{max} = \mu_s N$ to zero in FBD.

$$\begin{cases} \Sigma F_y = 0: & N\cos 30° - mg = 0 \\ \Sigma F_n = m\frac{v^2}{\rho}: & N \sin 30° = m\frac{v^2}{1200} \end{cases}$$

Solve: $N = 1.155mg$, $v = 149.4$ ft/sec

or $\underline{v = 101.8 \text{ mi/hr}}$

$\underline{v_{min} = 0}$, as $\theta_{max} = \tan^{-1}\mu_s = \tan^{-1}(0.9)$
$= 42.0° > 30°$

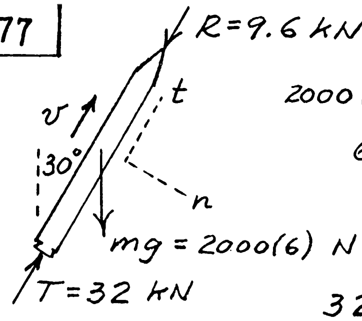
For v_{max}, use F_{max} as shown in FBD.

$$\begin{cases} \Sigma F_y = 0: & N\cos 30° - mg - \mu_s N \sin 30° = 0 \\ \Sigma F_n = m\frac{v^2}{\rho}: & \mu_s N\cos 30° + N\sin 30° = m\frac{v_{max}^2}{\rho} \end{cases}$$

With $\mu_s = 0.9$: $N = 2.40mg$

$v_{max} = 345$ ft/sec ($\underline{235 \text{ mi/hr}}$)

3/77



$\Sigma F_n = m a_n;$

$2000(6)\sin 30° = 2000\,\dfrac{(3000)^2}{\rho}$

$6(\tfrac{1}{2}) = 9(10^6)/\rho\,,\quad \underline{\rho = 3000\text{ km}}$

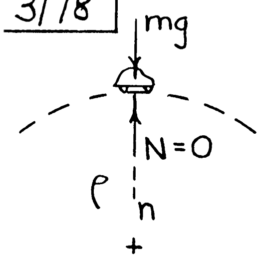
$\Sigma F_t = m\dot{v}$

$32(10^3) - 2000(6)\cos 30° - 9600$

$= 2000\dot{v}$

$\underline{\dot{v} = 6.00\text{ m/s}^2}$

3/78



$\Sigma F_n = ma_n: \quad mg = m\, v^2/\rho\,, \quad v = \sqrt{g\rho}$

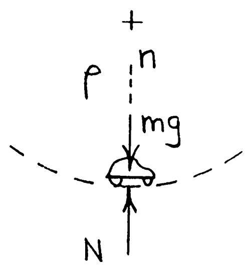
At top of hump, $\frac{dy}{dx} = 0$ and radius of curvature is

$\rho = \left(\frac{d^2y}{dx^2}\right)^{-1}$ where $y = b \sin \frac{2\pi x}{L}$

$$\frac{dy}{dx} = \frac{2\pi b}{L} \cos \frac{2\pi x}{L}\,, \quad \frac{d^2y}{dx^2} = -\frac{4\pi^2 b}{L^2} \sin \frac{2\pi x}{L}$$

At top of hump, $x = L/4$, $\sin \frac{2\pi x}{L} = 1$,

and $\left|\frac{d^2y}{dx^2}\right| = \frac{1}{\rho} = \frac{4\pi^2 b}{L^2}$, thus $\underline{v = \frac{L}{2\pi}\sqrt{g/b}}$



At bottom of dip:

$\Sigma F_n = ma_n: \quad N - mg = m\,\frac{g\rho}{\rho}$

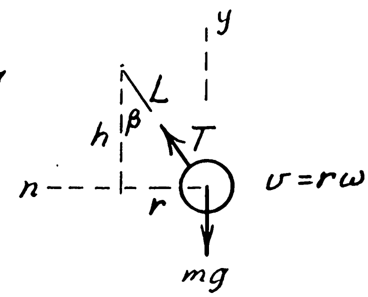
$\underline{N = 2mg}$

3/79

$\Sigma F_y = 0$: $T\cos\beta - mg = 0$, $T\cos\beta = mg$

$\Sigma F_n = ma_n$: $T\sin\beta = mv^2/r$

Divide & get $\tan\beta = \frac{v^2}{gr} = \frac{r\omega^2}{g}$



But $r = L\sin\beta$ so $\tan\beta = L\omega^2 \sin\beta / g$

or $L\cos\beta = g/\omega^2$

And $h = L\cos\beta$ so $\underline{h = g/\omega^2}$ (depends only on ω & g)

Then $\underline{T} = \frac{mg}{\cos\beta} = \frac{mg}{h/L} = \frac{mgL}{g/\omega^2} \underline{= mL\omega^2}$

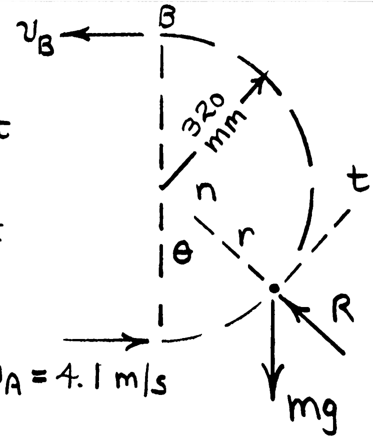
3/80

$\Sigma F_t = ma_t: \; -mg\sin\theta = ma_t$

$a_t = -g\sin\theta$

$\Sigma F_n = ma_n: \; R - mg\cos\theta = m\dfrac{v^2}{r}$

$R = m\left(g\cos\theta + \dfrac{v^2}{r}\right)$



$v\,dv = a_t(r\,d\theta): \; \int_{4.1}^{v} v\,dv = \int_0^{\theta} (-9.81\sin\theta)(0.320)\,d\theta$

$v^2 = 10.53 + 6.28\cos\theta$

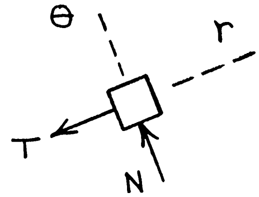
Thus $R = 0.065\left(9.81\cos\theta + \dfrac{10.53 + 6.28\cos\theta}{0.320}\right)$

$= \underline{2.14 + 1.913\cos\theta \text{ N}}$

For $\theta = 180°$: $v_B^2 = 10.53 + 6.28(-1)$

$\underline{v_B = 2.06 \text{ m/s}}$

3/81



$\Sigma F_r = ma_r = m(\ddot{r} - r\dot{\theta}^2)$:

$-T = \frac{3}{32.2}\left(0 - \frac{9}{12}6^2\right)$

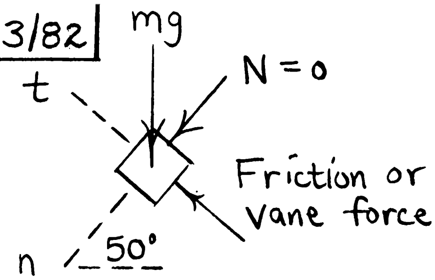
$\underline{T = 2.52 \text{ lb}}$

$\Sigma F_\theta = ma_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$:

$N = \frac{3}{32.2}\left[\frac{9}{12}(-2) + 2\left(-\frac{2}{12}\right)(6)\right]$

$\underline{N = -0.326 \text{ lb}}$ (Contact on side B)

3/82

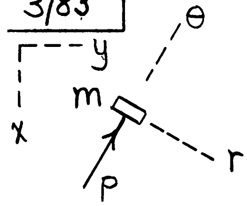


$$\Sigma F_n = ma_n: \quad mg\sin 50° = mr\Omega^2$$

$$\Omega = \sqrt{\frac{g\sin 50°}{r}} = \sqrt{\frac{9.81\sin 50°}{0.330}} = \underline{4.77 \text{ rad/s}}$$

$$(45.6 \text{ rev/min})$$

3/83



(Note : mg $\&$ static normal $\perp$ to paper)

$\Sigma F_r = ma_r: \quad 0 = m(\ddot{r} - r\Omega^2) \quad (1)$

$\Sigma F_\theta = ma_\theta: \quad P = m(r\underset{0}{\ddot{\theta}} + 2\dot{r}\Omega) \quad (2)$

(1): $\ddot{r} = \dot{r}\dfrac{d\dot{r}}{dr} = r\Omega^2$

$$\int_{\dot{r}_0}^{\dot{r}} \dot{r}\, d\dot{r} = \int_{r_0}^{r} \Omega^2 r\, dr \Rightarrow \dot{r}^2 = \dot{r}_0^2 + \Omega^2(r^2 - r_0^2)$$

Numbers (at end of tube) : $\dot{r} = \left[60^2 + 7^2\left(3^2 - \left(\frac{6}{12}\right)^2\right)\right]^{1/2} = 63.5 \dfrac{\text{ft}}{\text{sec}}$

(2): $P = m(2\dot{r}\Omega) = \dfrac{5/16}{32.2}(2)(63.5)(7)$

$= \underline{8.62 \text{ lb}}$

3/84 The distance traveled from A to C is

$(s_C - s_A) = 100 + 250\left(30 \frac{\pi}{180}\right) = 231$ ft

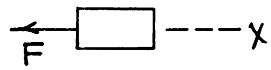
Uniform tangential acceleration : $v_C^2 = v_A^2 + 2a_t(s_C - s_A)$

$0^2 = \left[60 \frac{5280}{3600}\right]^2 + 2a_t(231)$, $a_t = -16.77$ ft/sec^2

Speed at B: $v_B^2 = v_A^2 + 2a_t(s_B - s_A)$

$v_B^2 = \left[60 \frac{5280}{3600}\right]^2 + 2(-16.77)(100)$, $v_B = 66.3$ ft/sec

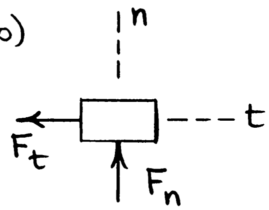
(a)



$\Sigma F_x = ma_x: \; -F = \frac{3000}{32.2}(-16.77)$

$\underline{F = 1562 \text{ lb}}$

(b)



$\Sigma F_t = ma_t: \; -F_t = \frac{3000}{32.2}(-16.77)$

$F_t = 1562$ lb

$\Sigma F_n = m\frac{v^2}{\rho}: \; F_n = \frac{3000}{32.2} \frac{66.3^2}{250}$

$F_n = 1636$ lb

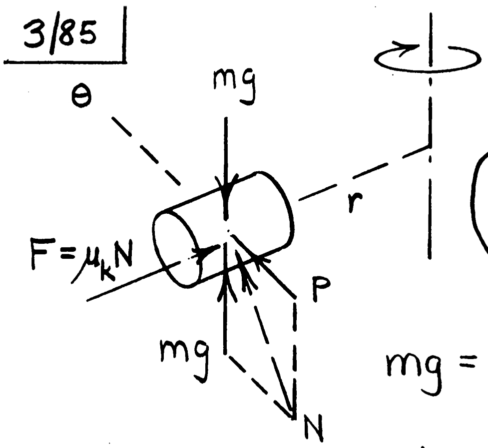
$F = \sqrt{F_t^2 + F_n^2} = \underline{2260 \text{ lb}}$

(c) v and therefore F_n go to zero;

$\underline{F = F_t = 1562 \text{ lb}}$

(In all FBDs, there is a weight into the paper and a static normal force out of the paper.)

3/85



(Note: Spring forces not shown on FBD.)

$mg = 0.8(9.81) = 7.85$ N

$\sum F_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$

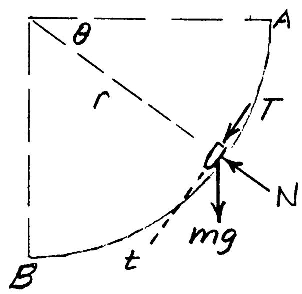
$P = 0.8(0 + 2(0.8)6) = 7.68$ N

$N = \sqrt{7.68^2 + 7.85^2} = 10.98$ N

$F = \mu_k N = 0.4\,(10.98) = \underline{4.39 \text{ N}}$

3/86 $\Sigma F_t = ma_t$; $T + mg\cos\theta = m\,a_t$

$a_t = \frac{T}{m} + g\cos\theta$



$v\,dv = a_t(r\,d\theta)$

$$\int_0^v v\,dv = \int_0^{\pi/2}\left(\frac{T}{m} + g\cos\theta\right) r\,d\theta$$

$$\frac{v^2}{2} = \frac{Tr}{m}\frac{\pi}{2} + gr\sin\theta\Big]_0^{\pi/2}$$

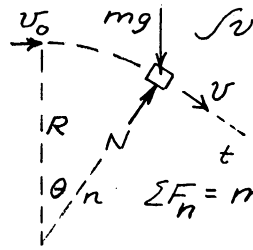
$$= \frac{Tr\pi}{2m} + gr$$

$v^2 = r\left(\frac{\pi T}{m} + 2g\right)$ $\quad \underline{v = \sqrt{r\left(\frac{\pi T}{m} + 2g\right)}}$

At B, $\Sigma F_n = ma_n$; $N - mg = m\frac{v^2}{r}$

$N = mg + T\pi + 2mg$, $\underline{N = T\pi + 3mg}$

3/87 $\Sigma F_t = ma_t$; $mg\sin\theta = ma_t$, $a_t = g\sin\theta$



$\int v\,dv = \int a_t\,ds$; $\int_{v_0}^{v} v\,dv = \int_0^{\theta} g\sin\theta\,(R\,d\theta)$

$v^2 = v_0^2 + 2gR(1-\cos\theta)$

$\Sigma F_n = ma_n$; $mg\cos\theta - N = m\dfrac{v^2}{R}$

$N = mg\cos\theta - \dfrac{m}{R}v_0^2 - 2mg(1-\cos\theta)$

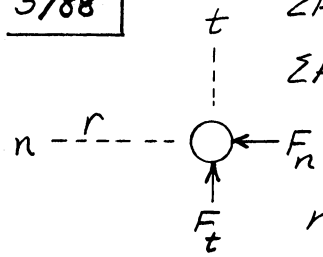
$= mg\left(3\cos\theta - 2 - \dfrac{v_0^2}{gR}\right)$

When $N = 0$, $\theta = \beta$ so $3\cos\beta = 2 + \dfrac{v_0^2}{gR}$

$$\beta = \cos^{-1}\left(\frac{2}{3} + \frac{v_0^2}{3gR}\right)$$

For $v_0 = 0$, $\beta = \cos^{-1}\left(\frac{2}{3}\right) = \underline{48.2^\circ}$

3/88



$\Sigma F_t = ma_t;\ F_t = mr\alpha$

$\Sigma F_n = ma_n;\ F_n = mr\omega^2$

$F_{max} = \mu_s N = \mu_s mg$ so

$$mr\sqrt{\alpha^2+\omega^4} = \mu_s mg$$

& $\omega^2 = \frac{1}{r}\sqrt{\mu_s^2 g^2 - r^2\alpha^2}$ But for constant α,

$\omega^2 = 2\alpha\theta = 2\alpha(2\pi N)$

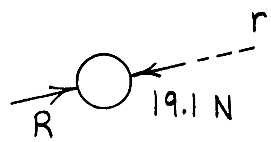
Thus no. of rev. $N = \dfrac{\omega^2}{4\pi\alpha} = \dfrac{1}{4\pi}\sqrt{\left(\dfrac{\mu_s g}{r\alpha}\right)^2 - 1}$

3/89

$$\begin{cases} r = 100 + 10 \sin 6(12)t & \text{mm} \\ \dot{r} = 720 \cos 72t & \text{mm/s} \\ \ddot{r} = -51.84 \sin 72t & \text{m/s}^2 \end{cases}$$

For r a maximum, $\cos 72t = 0$

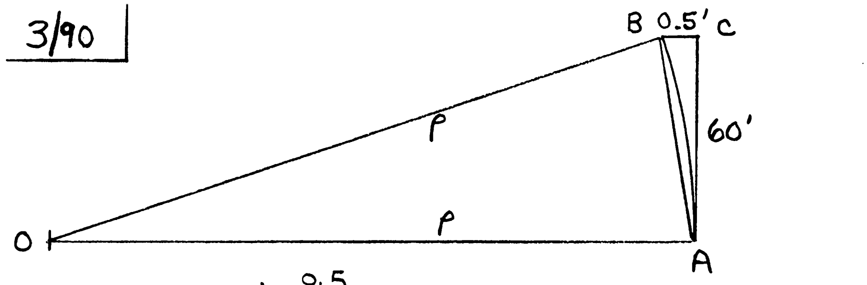
$$a_r = \ddot{r} - r\omega^2 = -51.84 - 0.110(12)^2 = -66.7 \ \frac{\text{m}}{\text{s}^2}$$



$$\Sigma F_r = ma_r : R - 19.1 = 0.1(-67.7)$$

$$\underline{R = 12.33 \text{ N}}$$

3/90



$\measuredangle BAC = \tan^{-1}\frac{0.5}{60} = 0.477°$

$\measuredangle OBA = \measuredangle OAB = (90 - 0.477) = 89.5°$

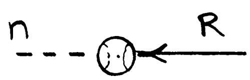
$\measuredangle BOA = 180 - 2(89.5) = 0.955° = 2\measuredangle BAC$

$\overline{AB} = \sqrt{60^2 + 0.5^2}' = 60.002'$

$\frac{\sin 0.955°}{60.002'} = \frac{\sin 89.5°}{\rho}$, $\quad \underline{\rho = 3600 \text{ ft}}$

FBD: (horizontal forces)

$\Sigma F_n = ma_n: \ R = \frac{5.125/16}{32.2}\ \frac{120^2}{3600}$



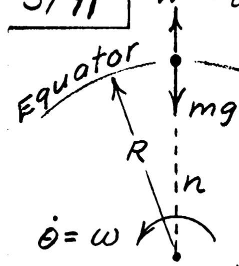
$\underline{R = 0.0398 \text{ lb}}$

(Note: R = 0.637 oz represents 12.4% of the weight of the baseball)

3/91

W' = apparent weight (force to support m)

mg = true gravitational attraction where g = absolute acceleration due to gravity



$\Sigma F_n = ma_n$; $mg - W' = mR\omega^2$

$W' = m(g - R\omega^2)$

$$k = \frac{W'}{mg} = \underline{1 - \frac{R\omega^2}{g}}$$

For values given $W' = mg\left(1 - \frac{R\omega^2}{g}\right)$

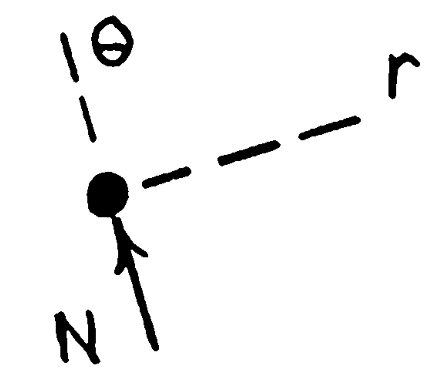
$$= 100\left(1 - \frac{6378(10^3)(0.729)^2(10^{-8})}{9.815}\right) = 100(1 - 0.00345)$$

$$= \underline{99.655 \text{ N}}$$

3/92

$$\Sigma F_r = ma_r : \quad 0 = m(\ddot{r} - r\dot{\theta}^2)$$

Particle:



$$\ddot{r} = r\dot{\theta}^2 = r\omega_0^2$$

$$\dot{r}\frac{d\dot{r}}{dr} = r\omega_0^2$$

$$\int_0^{\dot{r}} \dot{r}\,d\dot{r} = \omega_0^2 \int_{r_0}^{r} r\,dr$$

$$\Rightarrow \quad \underline{\dot{r} = \omega_0\sqrt{r^2 - r_0^2} = v_r}$$

$$\frac{dr}{dt} = \omega_0\sqrt{r^2 - r_0^2}$$

$$\int_{r_0}^{r} \frac{dr}{\sqrt{r^2 - r_0^2}} = \omega_0 \int_0^t dt$$

$$\ln\left[r + \sqrt{r^2 - r_0^2}\right]\Big|_{r_0}^{r} = \omega_0 t \Rightarrow \underline{r = \frac{r_0}{2}\left[e^{-\omega_0 t} + e^{\omega_0 t}\right]}$$

$$v_\theta = r\dot{\theta} = r\omega_0 = \underline{\frac{r_0\omega_0}{2}\left[e^{-\omega_0 t} + e^{\omega_0 t}\right]}$$

As a function of t, $\underline{v_r = \frac{r_0\omega_0}{2}\left(e^{\omega_0 t} - e^{-\omega_0 t}\right)}$

In terms of hyperbolic functions,

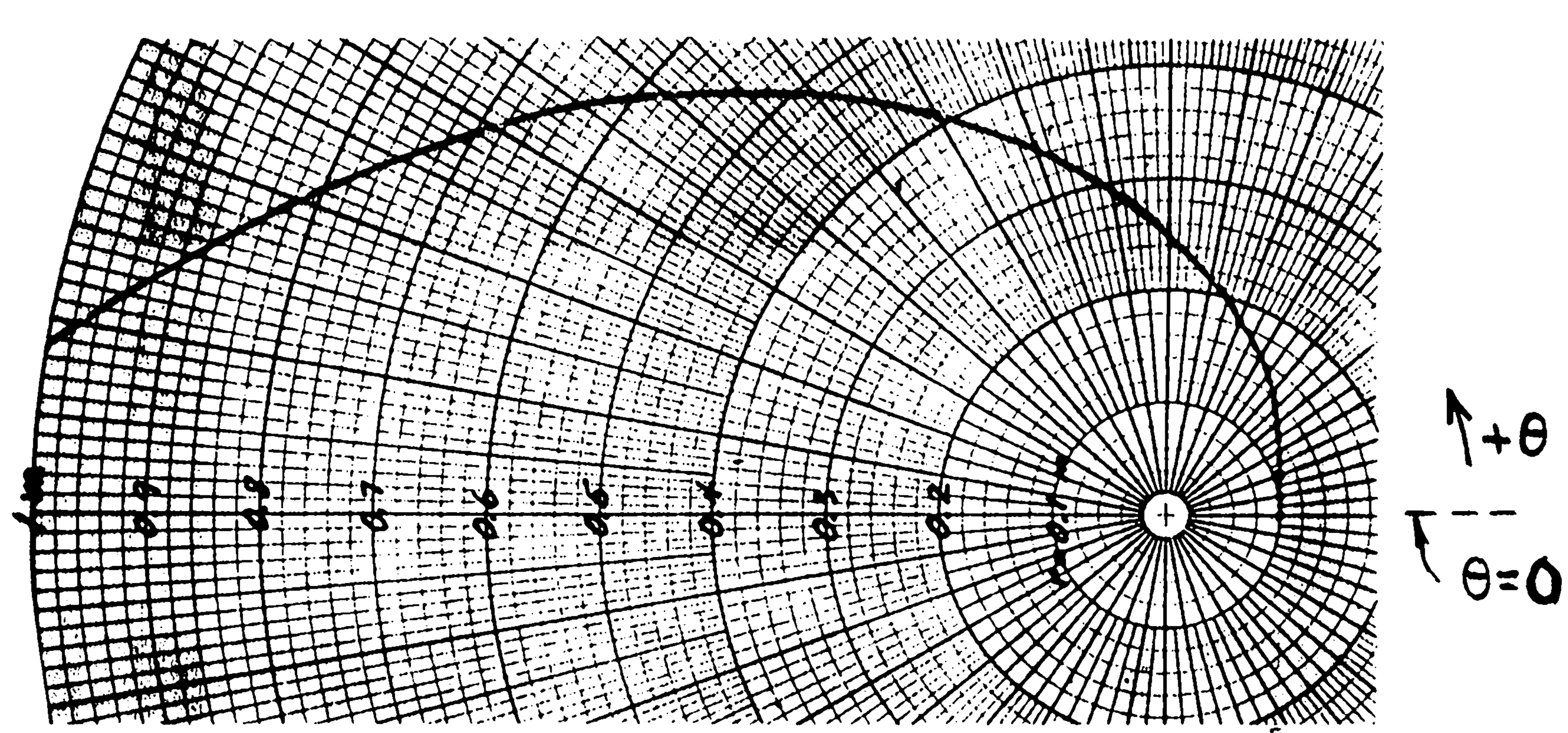
$$v_r = r_0\omega_0 \sinh \omega_0 t$$

$$r = r_0 \cosh \omega_0 t$$

$$\underline{v_\theta = r_0\omega_0 \cosh \omega_0 t}$$

With numbers,

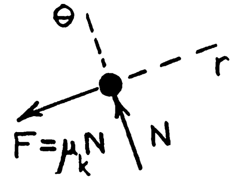
$$\begin{cases} v_r = 0.1 \sinh t \\ r = 0.1 \cosh t \\ v_\theta = 0.1 \cosh t \end{cases}$$



($r = 1.0$ m @ $\theta = 171.5°$)

3/93

Particle : $\begin{cases} \Sigma F_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta}) : N = m(2\dot{r}\omega_o) \\ \Sigma F_r = m(\ddot{r} - r\dot{\theta}^2) : -\mu_k N = m(\ddot{r} - r\omega_o^2) \end{cases}$



Eliminating N :

$$-\mu_k(2\cancel{m} r \omega_o) = \cancel{m}(\ddot{r} - r\omega_o^2)$$

$$\ddot{r} + 2\mu_k\omega_o \dot{r} - \omega_o^2 r = 0$$

Assume a solution of form $r = Ce^{st}$ and obtain $s_{1,2} = \omega_o\left[-\mu_k \pm \sqrt{\mu_k^2+1}\right]$.

$$\therefore \quad r = C_1 e^{s_1 t} + C_2 e^{s_2 t}$$

Initial conditions : $\begin{cases} r(0) = r_o \\ \dot{r}(0) = 0 \end{cases}$

So $\begin{cases} r_o = C_1 + C_2 \\ 0 = C_1 s_1 + C_2 s_2 \end{cases} \Rightarrow \begin{matrix} C_1 = -s_2 r_o/(s_1 - s_2) \\ C_2 = s_1 r_o/(s_1 - s_2) \end{matrix}$

Final Solution :

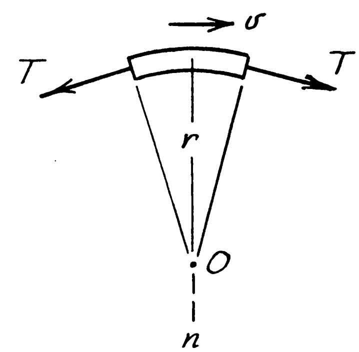
$$r(t) = \frac{r_o}{2\sqrt{\mu_k^2+1}}\left[\left(\mu_k + \sqrt{\mu_k^2+1}\right)e^{\omega_o\left(-\mu_k + \sqrt{\mu_k^2+1}\right)t} + \left(-\mu_k + \sqrt{\mu_k^2+1}\right)e^{\omega_o\left(-\mu_k - \sqrt{\mu_k^2+1}\right)t}\right]$$

3/94

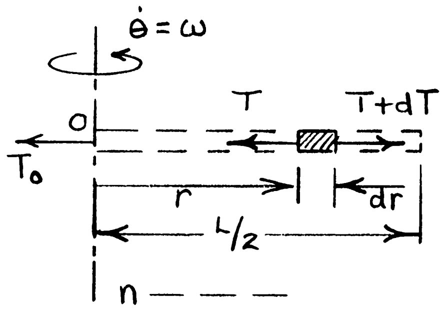
$$\Sigma F_n = ma_n : 2T \sin\frac{d\theta}{2} = \rho r\, d\theta \times \frac{v^2}{r}$$

$$T\,d\theta = \rho v^2 d\theta, \quad T = \rho v^2$$

so $\quad \sigma_t = T/A = \underline{\rho v^2/A}$



3/95



The mass per unit length is m/L

$$\Sigma F_n = ma_n : \quad T - (T + dT) = \frac{m}{L}\, dr\, (r\omega^2)$$

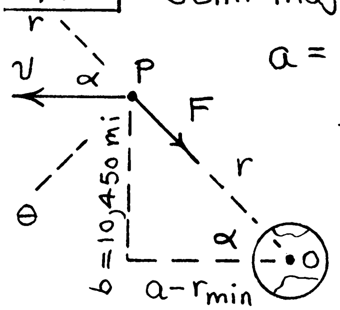
$$\int_{T_0}^{T} -dT = \int_0^r \frac{m\omega^2}{L}\, r\, dr$$

$$\Rightarrow \quad T = T_0 - \frac{mr^2\omega^2}{2L}$$

When $r = L/2$, $T = 0$: $\quad 0 = T_0 - \dfrac{m(L/2)^2\omega^2}{2L}$

$$T_0 = \frac{mL\omega^2}{8} \Rightarrow \underline{\sigma = \frac{T}{A} = \frac{mL\omega^2}{2A}\left(\frac{1}{4} - \frac{r^2}{L^2}\right)}$$

3/96 Semi-major axis of ellipse is



$$a = \frac{r_{max} + r_{min}}{2} = \frac{26,259 + 4159}{2}$$

$$= 15,209 \text{ mi}$$

$$\alpha = \tan^{-1} \frac{b}{a - r_{min}}$$

$$= \tan^{-1} \frac{10,450}{15,209 - 4159} = 43.4°$$

$$\dot{r} = v_r = v \cos\alpha = 13,244 \cos 43.4° = \underline{9620 \text{ ft/sec}}$$

$$r\dot{\theta} = v_\theta = v \sin\alpha, \quad \dot{\theta} = \frac{v \sin\alpha}{r}, \quad \text{where}$$

$$r = \sqrt{b^2 + (a - r_{min})^2} = \sqrt{10,450^2 + (15,209 - 4159)^2}$$

$$= 15,208 \text{ mi}; \text{ so } \dot{\theta} = \frac{13,244 \sin 43.4°}{15,208(5280)} = \underline{1.133(10^{-4}) \frac{\text{rad}}{\text{sec}}}$$

$$\Sigma F_r = ma_r: \; -\cancel{m}\frac{gR^2}{r^2} = \cancel{m}(\ddot{r} - r\dot{\theta}^2)$$

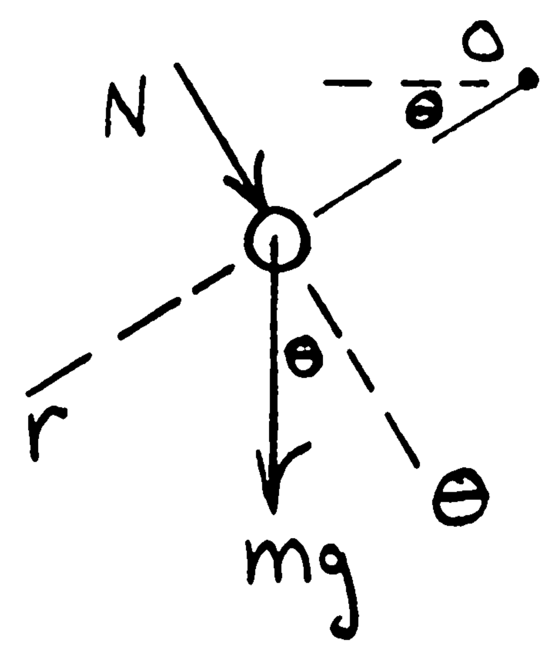
$$\ddot{r} = r\dot{\theta}^2 - \frac{gR^2}{r^2} = 15,208(5280)\left[(1.133)(10^{-4})\right]^2 - \frac{32.23(3959)^2}{15,208^2}$$

$$= \underline{-1.153 \text{ ft/sec}^2}$$

$$\Sigma F_\theta = ma_\theta: \; 0 = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$$

$$\ddot{\theta} = -\frac{2\dot{r}\dot{\theta}}{r} = -\frac{2(9620)(1.133)(10^{-4})}{15,208(5280)} = \underline{-2.72(10^{-8}) \frac{\text{rad}}{\text{sec}^2}}$$

▸3/97



$$\Sigma F_r = ma_r = m(\ddot{r} - r\dot{\theta}^2):$$

$$\cancel{m}g \sin\theta = \cancel{m}(\ddot{r} - r\omega_0^2)$$

$$\ddot{r} - \omega_0^2 r = g \sin\omega_0 t$$

Assume $r_h = Ce^{st}$ and substitute into equation to obtain $s_1 = -\omega_0$, $s_2 = \omega_0$. Also, assume a particular solution of form $r_p = D\sin\omega_0 t$, substitute, and obtain $D = -g/2\omega_0^2$.

So $\quad r = r_h + r_p = C_1 e^{-\omega_0 t} + C_2 e^{\omega_0 t} - \frac{g}{2\omega_0^2}\sin\omega_0 t$

Initial conditions :

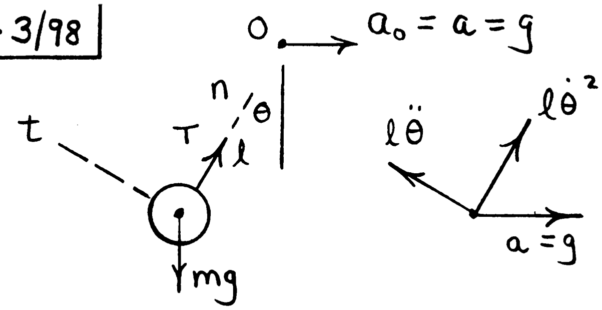
$$\begin{cases} r(0) = C_1 + C_2 = 0 \\ \dot{r}(0) = -\omega_0 C_1 + \omega_0 C_2 - \frac{g}{2\omega_0} = 0 \end{cases}$$

Solve for C_1 and C_2 to obtain

$$r = -\frac{g}{4\omega_0^2}e^{-\theta} + \frac{g}{4\omega_0^2}e^{\theta} - \frac{g}{2\omega_0^2}\sin\theta$$

or $\quad \underline{r = \frac{g}{2\omega_0^2}\left[\sinh\theta - \sin\theta\right]}$

▸ 3/98



$$\underline{a}_m = \underline{a}_o + \underline{a}_{m/o}$$

$$\Sigma F_t = ma_t: \quad -mg\sin\theta = m(\ell\ddot{\theta} - g\cos\theta)$$

$$\ddot{\theta} = \frac{g}{\ell}(\cos\theta - \sin\theta) = \dot{\theta}\frac{d\dot{\theta}}{d\theta}$$

$$\int_0^\theta \frac{g}{\ell}(\cos\theta - \sin\theta)\,d\theta = \int_0^{\dot{\theta}} \dot{\theta}\,d\dot{\theta}$$

$$\frac{g}{\ell}\left[\sin\theta + \cos\theta - 1\right] = \dot{\theta}^2/2 \qquad *$$

$\theta = \theta_{max}$ when $\dot{\theta} = 0$: $\sin\theta + \cos\theta = 1$

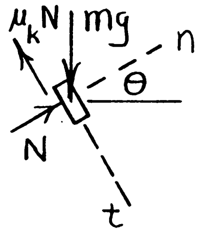
$$\Rightarrow \theta_{min} = 0, \quad \underline{\theta_{max} = \frac{\pi}{2}}$$

$$\Sigma F_n = ma_n: \quad T - mg\cos\theta = m(\ell\dot{\theta}^2 + g\sin\theta)$$

$$T = m(\ell\dot{\theta}^2 + g\sin\theta + g\cos\theta)$$

With $*$: $\underline{T = mg(3\sin\theta + 3\cos\theta - 2)}$

▶ 3/99



$$\begin{cases} \Sigma F_n = ma_n: & N - mg\sin\theta = m\dfrac{v^2}{r} \quad (1) \\ \Sigma F_t = ma_t: & mg\cos\theta - \mu_k N = ma_t \quad (2) \end{cases}$$

Solve (1) for N & sub. into (2):

$$g\cos\theta - \mu_k g\sin\theta - \mu_k\frac{v^2}{r} = a_t$$

But $a_t = r\ddot{\theta} = r\dot{\theta}\dfrac{d\dot{\theta}}{d\theta} = r\,\dfrac{1}{2}\dfrac{d}{d\theta}(\dot{\theta}^2) = \dfrac{1}{2r}\dfrac{d}{d\theta}(r^2\dot{\theta}^2)$

Let $u = (r\dot{\theta})^2 = v^2$ and the EOM becomes

$$\frac{du}{d\theta} + 2\mu_k u = 2gr(\cos\theta - \mu_k\sin\theta)$$

For homogeneous solution, assume $u_H = Ce^{\lambda\theta}$, substitute in to find $u_H = Ce^{-2\mu_k\theta}$

For particular solution, assume $u_p = A\cos\theta + B\sin\theta$, substitute in to find $u_p = \dfrac{2gr}{[1+4\mu_k^2]}\left[+3\mu_k\cos\theta + (1-2\mu_k^2)\sin\theta\right]$

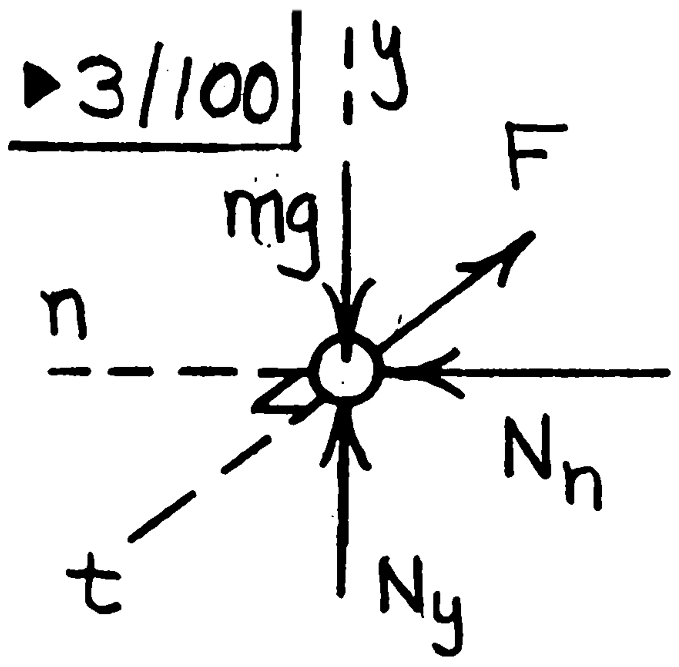
Assemble $u = u_H + u_p$ and apply initial condition $u = v^2 = 0$ @ $\theta = 0$ to obtain

$$u = \frac{-6\mu_k gr}{1+4\mu_k^2}e^{-2\mu_k\theta} + \frac{2gr}{1+4\mu_k^2}\left[3\mu_k\cos\theta + (1-2\mu_k^2)\sin\theta\right]$$

For $\mu_k = 0.2$, $g = 9.81\ m/s^2$, $r = 3\ m$, & $\theta = \frac{\pi}{2}$:

$v^2 = 30.4\ m^2/s^2$, $\underline{v = 5.52\ m/s}$

▶3/100



$$\Sigma F_y = 0 : \quad N_y = mg$$

$$\Sigma F_n = ma_n : \quad N_n = m\frac{v^2}{r}$$

$$F = \mu_k N_{tot} = \mu_k \sqrt{(mg)^2 + \left(\frac{mv^2}{r}\right)^2}$$

$$= \frac{\mu_k m}{r}\sqrt{r^2 g^2 + v^4}$$

$$\Sigma F_t = ma_t : \quad -\frac{\mu_k \cancel{m}}{r}\sqrt{r^2g^2+v^4} = \cancel{m} v \frac{dv}{ds}$$

$$-\frac{\mu_k}{r}\int_0^s ds = \int_{v_0}^0 \frac{v\,dv}{\sqrt{v^4 + r^2g^2}} = \int_{v_0^2}^0 \frac{\frac{1}{2}dx}{\sqrt{x^2+r^2g^2}}$$

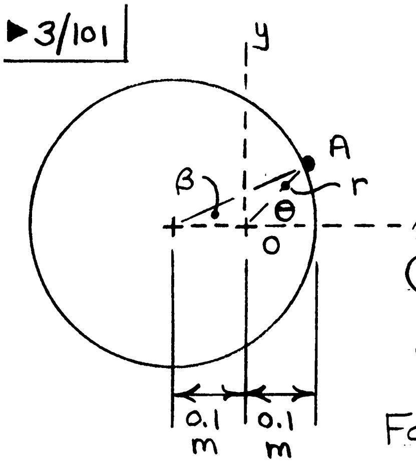
where $x = v^2$, $dx = 2v\,dv$

Integrating,

$$-\frac{\mu_k}{r}s = \frac{1}{2}\ln\left[x + \sqrt{x^2+r^2g^2}\right]_{v_0^2}^{0}$$

$$\text{or} \quad s = \frac{r}{2\mu_k}\ln\left[\frac{v_0^2 + \sqrt{v_0^4 + r^2g^2}}{rg}\right]$$

▶ 3/101



$(x-x_0)^2 + (y-y_0)^2 = R_0^2$

$(x+0.1)^2 + y^2 = 0.2^2$

Switch to polar form:

$(r\cos\theta + 0.1)^2 + (r\sin\theta)^2 = 0.2^2$

or * $r^2 + 0.2r\cos\theta - 0.03 = 0$

For $\theta = 45°$, quadratic formula yields $r = 0.1164$ m

Differentiate * with respect to time:

$2r\dot{r} + 0.2\dot{r}\cos\theta - 0.2r\dot{\theta}\sin\theta = 0 \qquad (1)$

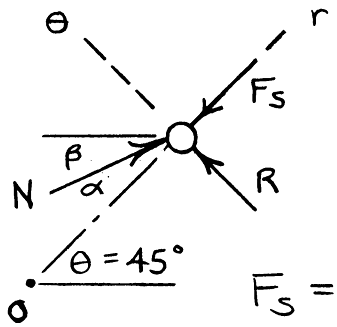
Again: $2\dot{r}^2 + 2r\ddot{r} + 0.2\ddot{r}\cos\theta - 0.2\dot{r}\dot{\theta}\sin\theta$

$\qquad - 0.2\dot{r}\dot{\theta}\sin\theta - 0.2r\overset{0}{\ddot{\theta}}\sin\theta - 0.2r\dot{\theta}^2\cos\theta = 0 \quad (2)$

With $r = 0.1164$ m, $\theta = 45°$, & $\dot{\theta} = 15$ rad/s,

Eqs. (1) & (2) yield $\dot{r} = 0.660$ m/s

$\ddot{r} = 15.05$ m/s^2



$$\frac{\sin\beta}{0.1164} = \frac{\sin 135°}{0.2}$$

$$\beta = 24.3°$$

$$\alpha = 45° - \beta = 20.7°$$

$$F_s = k\delta = 5000\,(0.1164 - 0.1)$$

$$= 81.9 \text{ N}$$

$$\Sigma F_r = ma_r : \quad N\cos\alpha - F_s = m(\ddot{r} - r\dot{\theta}^2)$$

$$N\cos 20.7° - 81.9 = 0.5(15.05 - 0.1164(15^2))$$

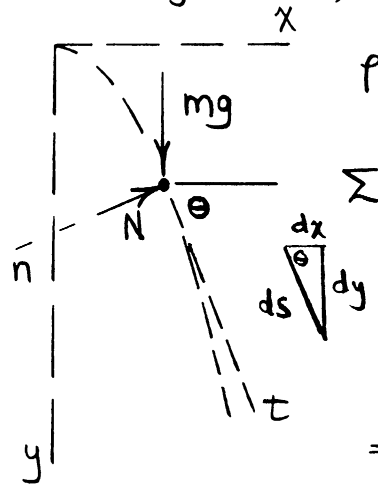
$$\underline{N = 81.6 \text{ N}}$$

$$\Sigma F_\theta = ma_\theta : \quad R - N\sin\alpha = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$$

$$R - 81.6\sin 20.7 = 0.5\,(0 + 2(0.660)(15))$$

$$\underline{R = 38.7 \text{ N}}$$

▶ **3/102** $y = kx^2, \quad \tan\theta = \frac{dy}{dx} = 2kx, \quad \frac{d^2y}{dx^2} = 2k$



$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{d^2y/dx^2} = \frac{\left[1+4k^2x^2\right]^{3/2}}{2k}$$

$$\Sigma F_t = ma_t: \; mg\sin\theta = mv\frac{dv}{ds}$$

$$\int_0^s g\sin\theta\, ds = \int_0^v v\,dv$$

$$g\int_0^y dy = \int_0^v v\,dv$$

$$\Rightarrow v^2 = 2gy$$

$$\Sigma F_n = ma_n: \; mg\cos\theta - N = m\frac{v^2}{\rho}$$

$$N = m\left[g\cos\theta - \frac{v^2}{\rho}\right] = m\left[g\cos\theta - \frac{2g(kx^2)}{\frac{(1+4k^2x^2)^{3/2}}{2k}}\right]$$

$$\underline{N = \frac{mg}{(1+4k^2x^2)^{3/2}}} \qquad \left(\begin{array}{c}\text{Always} > 0, \text{ unless} \\ x \to \infty\end{array}\right)$$

(Note: $\sqrt{1+4k^2x^2}$ θ 1 $2kx$; $\cos\theta = \frac{1}{\sqrt{1+4k^2x^2}}$)

3/103 (a) spring : $U_{1-2} = \frac{1}{2}k(x_1^2 - x_2^2)$

$U_{1-2} = \frac{1}{2}(4000)[0.1^2 - 0.2^2] = \underline{-60\text{ J}}$

(b) weight : $U_{1-2} = mg(y_1 - y_2)$

$U_{1-2} = 7(9.81)[0.1 \sin 20°] = \underline{2.35\text{ J}}$

3/104 $T_A + U_{A-B} = T_B$: $\frac{1}{2}\cancel{m}v_A^2 - \cancel{m}gh = \frac{1}{2}\cancel{m}v_B^2$

$v_B^2 = v_A^2 - 2gh = 5^2 - 2(9.81)(0.8)$

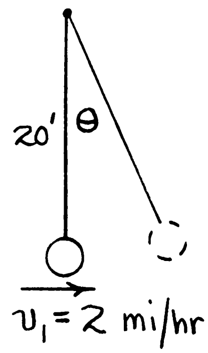
$\underline{v_B = 3.05 \text{ m/s}}$

In the absence of friction, only the altitude change, and not the shape of the path followed, is a factor in the work calculation.

3/105 $U = \Delta T;\quad 64.4(20) + U_f = \frac{1}{2}\,\frac{64.4}{32.2}(25^2 - 3^2)$

$$U_f = 616 - 1288 = \underline{-672\ \text{ft-lb}}$$

3/106

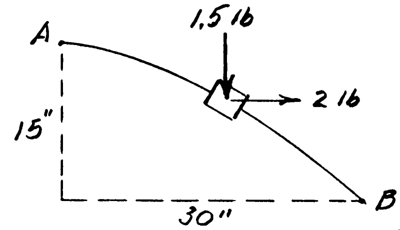


$$T_1 + U_{1-2} = T_2: \quad \frac{1}{2}\cancel{m}v_1^2 - \cancel{m}gh = 0$$

$$\frac{1}{2}\left[2\,\frac{5280}{3600}\right]^2 - 32.2\left[20\,(1-\cos\theta)\right] = 0$$

$$\underline{\theta = 6.63^\circ}$$

3/107



$U_{1\text{-}2} = \Delta T;\quad 2(30/12) + 1.5(15/12) = \frac{1}{2}\frac{1.5}{32.2}(v^2 - 0)$

$v^2 = 295.2,\quad \underline{v = 17.18 \text{ ft/sec}}$

3/108 For collar, $U_{1-2} = \Delta T = 0$

$$U_{1-2} = 50\left(\frac{50-30}{12}\right) - 30\,\frac{40}{12}\sin 30° - \frac{1}{2}k\left(\frac{6}{12}\right)^2 = 0$$

$$\underline{k = 267 \text{ lb/ft}}$$

$\underline{3/109}$ $U_{1-2} = \Delta T$; $2\left(\frac{1}{2}kx^2\right) = \frac{1}{2}mv^2 - 0$

$$k = \frac{1}{2}\frac{mv^2}{x^2} = \frac{1}{2}\frac{3500}{32.2}\left(\frac{5}{30}44\right)^2\frac{1}{(6/12)^2}\frac{1}{12} = \underline{974\ \text{lb/in.}}$$

3/110 $U_{1-2} = \Delta T$: $mgh = \frac{1}{2} m (v^2 - 0)$, $\underline{v = \sqrt{2gh}}$

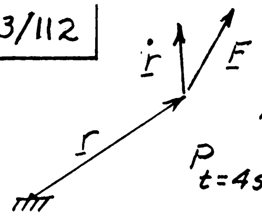
3/111 $U_{1\text{-}2} = \Delta T$: $mgh + Q = \frac{1}{2}m(v_B^2 - 0^2)$

$$0.5(9.81)(1.5) + Q = \frac{1}{2}(0.5)(4.70^2 - 0^2)$$

$$\underline{Q = -1.835 \text{ J}} \quad \left(\text{causes loss of energy}\right)$$

The lost mechanical energy is transformed to heat energy.

$\boxed{3/112}$



Power $P = \underline{F}\cdot\underline{\dot r}$

$$P = (40\underline{i} - 20\underline{j} - 36\underline{k})\cdot(8\underline{i} + 2.4t\underline{j} - 1.5t^2\underline{k})$$

$$P_{t=4s} = (40\underline{i} - 20\underline{j} - 36\underline{k})\cdot(8\underline{i} + 9.6\underline{j} - 24\underline{k})$$

$$= 320 - 192 + 864 = 992 \text{ W}$$

or $\underline{P = 0.992 \text{ kW}}$

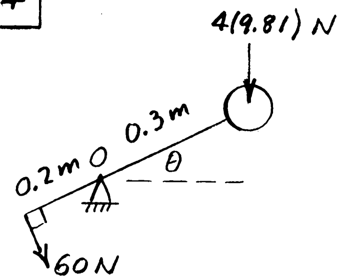
3/113 $P = W\dot{y}$ where $\dot{y} = v \sin\theta$

$\theta = \tan^{-1} 0.05 = 2.86°$, $\sin\theta = 0.0499$

$$P = 200\left(\frac{15}{30}44\right)0.0499 = 219.7 \text{ ft-lb/sec}$$

or $P = \dfrac{219.7}{550} = \underline{0.400 \text{ hp}}$

3/114



For system

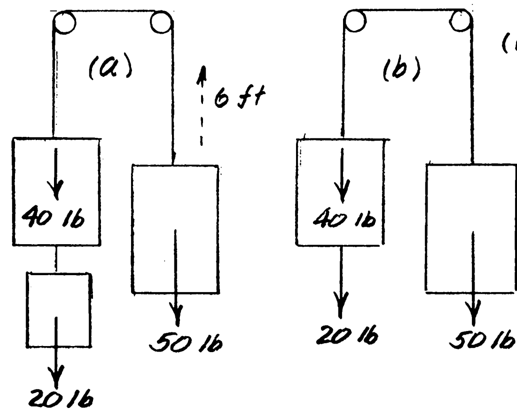
$U_{1-2} = \Delta T$

$60(0.2)(\frac{\pi}{2}) - 4(9.81)(0.3) = \frac{1}{2} 4 v^2$

$v^2 = 3.539 \ (m/s)^2$

$\underline{v = 1.881 \ m/s}$

3/115 Active-force diagrams for entire system



For sytem $U_{1-2} = \Delta T$

(a)

$$(40+20)6 - 50(6) = \frac{1}{2}\,\frac{40+20+50}{32.2}\,v^2$$

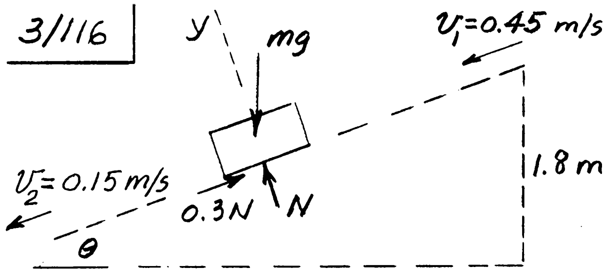
$\underline{v = 5.93 \text{ ft/sec}}$

(b)

$$(40+20)6 - 50(6) = \frac{1}{2}\,\frac{40+50}{32.2}\,v^2$$

$\underline{v = 6.55 \text{ ft/sec}}$

3/116



$\Sigma F_y = 0,\ N - mg\cos\theta = 0$

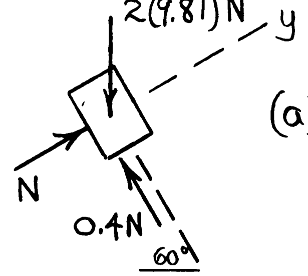
$N = mg\cos\theta$

$U = \Delta T;\ (mg\sin\theta - 0.3mg\cos\theta)\dfrac{1.8}{\sin\theta} = \dfrac{m}{2}(\overline{0.15}^2 - \overline{0.45}^2)$

$1.8(9.81)\left(1 - \dfrac{0.3}{\tan\theta}\right) = -\dfrac{0.180}{2}$

$\tan\theta = 0.2985,\quad \underline{\theta = 16.62^\circ}$

3/117



$\Sigma F_y = 0: \quad N - 2(9.81)\cos 60° = 0$

$N = 9.81 \text{ N}$

(a) $U_{1\text{-}2} = \Delta T: \quad 2(9.81)(0.5 \sin 60°)$

$- 0.4(9.81)(0.5) = \frac{1}{2} 2 v^2$

$\underline{v = 2.56 \text{ m/s}}$

(b) $U_{1\text{-}3} = \Delta T: \quad 2(9.81)(0.5 + x)\sin 60° - 0.4(9.81)(0.5 + x)$

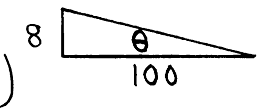
$- \frac{1}{2}(1600)x^2 = 0$

$800x^2 - 13.07x - 6.53 = 0$

$x = 0.0989 \text{ m}$ or $\underline{x = 98.9 \text{ mm}}$

3/118

$$\theta = \tan^{-1}\frac{8}{100} = 4.57^\circ$$



$$U_{1-2} = \Delta T : \; U_f + mgh = \frac{1}{2}m\left(v_2^2 - v_1^2\right)$$

$$U_f = -1200\,(9.81)\,(500 \sin 4.57^\circ) - \frac{1}{2}\,1200\left[\left(\frac{25}{3.6}\right)^2 - \left(\frac{100}{3.6}\right)^2\right]$$

$$U_f = -903\,(10^3)\text{ J} \quad \text{or} \quad -903\text{ kJ}$$

Ans. $\underline{Q = 903\text{ kJ (loss)}}$

$\underline{3/119}$ $P = Wh/\Delta t$

or $P = \dfrac{120(9)}{5} \Big/ 550 = \underline{0.393 \text{ hp}}$

Conversions : $h = 9 \text{ ft} \left(\dfrac{0.3048 \text{ m}}{\text{ft}} \right) = 2.74 \text{ m}$

$W = 120 \text{ lb} \left(\dfrac{4.4482 \text{ N}}{\text{lb}} \right) = 534 \text{ N}$

$P = \dfrac{Wh}{\Delta t} = \dfrac{534(2.74)}{5} = \underline{293 \text{ watts}}$

Check : $0.393 \text{hp} \left(\dfrac{745.7 \text{ watts}}{\text{hp}} \right) = 293 \text{ watts} \checkmark$

3/120 $U_{1-2} = \Delta T$ applied to system:

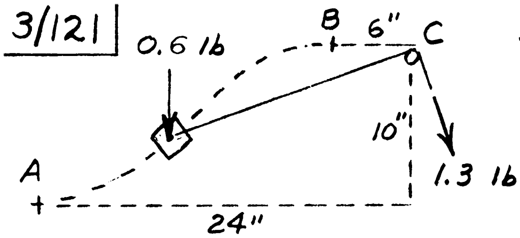
$$U_{1-2} = 40\left[\sqrt{0.4^2 + 0.3^2} - 0.1\right] - 0.8(9.81)(0.4)$$

$$= 12.86 \text{ J}$$

$$\Delta T = T_B - T_A = \frac{1}{2}(0.8)v_B^2 - 0$$

Thus $12.86 = 0.4 v_B^2$, $\underline{v_B = 5.67 \text{ m/s}}$

3/121



System = slider, cord, & pulley at C.

$$U = -0.6\left(\frac{10}{12}\right) + 1.3\,\frac{20}{12}$$

$$= \frac{5}{3} = 1.667 \text{ ft-lb}$$

$\overline{AC} = 26"$; $\overline{AC} - \overline{BC} = 20"$

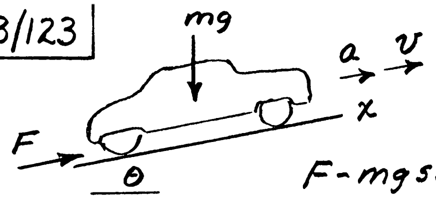
$U = \Delta T$; $1.667 = \frac{1}{2}\,\frac{0.6}{32.2}(v^2 - 0)$, $v^2 = 178.9$, $\underline{v = 13.37 \frac{ft}{sec}}$

$\underline{3/122}$ Net power required $= 30(140)(24)/33{,}000$

$= 3.05$ hp

Mechanical efficiency $= \dfrac{\text{Power required}}{\text{Power supplied}} = \dfrac{3.05}{4.00} = \underline{0.764}$

3/123



$\theta = \tan^{-1} 0.1$, $\sin\theta = 0.0995$

$\Sigma F_x = m a_x$ where $v^2 = 2ax$

$$F - mg\sin\theta = m\frac{v^2}{2x}$$

$$P = Fv = mgv\sin\theta + \frac{mv^3}{2x}$$

$$= 1500(9.81)\frac{50000}{3600}\,0.0995 + \frac{1500\,(50\,000/3600)^3}{2(100)}$$

$$= 20\,336 + 20\,094 = 40\,430 \text{ W}$$

or $\underline{P = 40.4 \text{ kW}}$

3/124 State ① : release ; state ② : final position

$$T_1 + U_{1-2} = T_2$$

$$0 + mg(h+d) - \int_0^d kx^4\,dx = 0$$

$$mg(h+d) - \frac{1}{5}kd^5 = 0$$

$$\Rightarrow \quad k = \frac{5mg(h+d)}{d^5}$$

3/125 Power output = rate of doing work

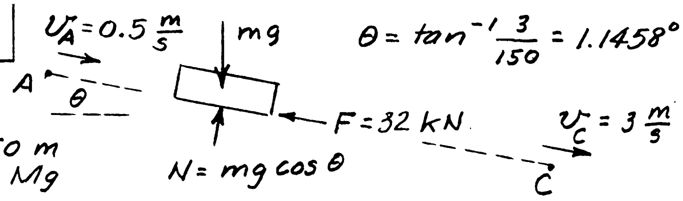
$= 300(9.81)(2) - 100(9.81)(4)$

$= 1962 \text{ J/s (W)}$

$= 1.962 \text{ kW}$

Efficiency $e = \dfrac{\text{Power output}}{\text{Power input}} = \dfrac{1.962}{2.20} = \underline{0.892}$

3/126



$\theta = \tan^{-1}\frac{3}{150} = 1.1458°$

$\overline{AC} = 150$ m

$m = 68$ Mg

$U_{1-2} = \Delta T$; $68(10^3)(9.81)(3) - 32(10^3)x = \frac{1}{2}68(10^3)(3^2 - 0.5^2)$

$2001 - 32x = 297.5$, $\underline{x = 53.2\ m}$

3/127 Work done by weight is $mg(r_{\pi/2} - 0)$

$= 0.5(9.81)(0.3 \times \frac{\pi}{2}) = 2.31$ J

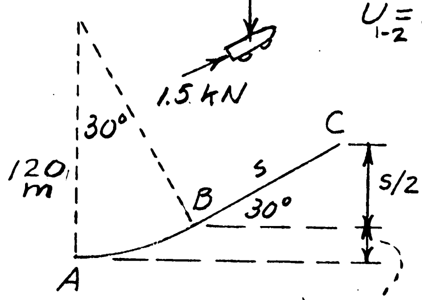
Work done by T is $T(r_{\pi} - r_{\pi/2}) = 10(0.3\pi - 0.3\frac{\pi}{2})$

$= 4.71$ J

$U = \Delta T$; $2.31 + 4.71 = \frac{1}{2}0.5(v^2 - 0)$

$v^2 = 28.10$, $\underline{v = 5.30 \text{ m/s}}$

3/128 $mg = 981\ N$



$120(1-\cos 30°) = 16.08\ m$

$\overline{AB} = r\theta = 120\frac{\pi}{6} = 62.8\ m$

$U_{1-2} = \Delta T = 0$ since $T_C = T_A = 0$

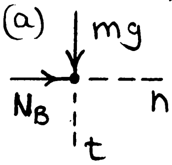
$1500(62.8) - 981\left(16.08 + \frac{s}{2}\right) = 0$

$s = 2(94248 - 15771)/981$

$= \underline{160.0\ m}$

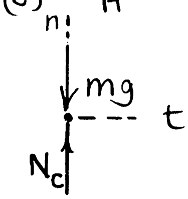
3/129 $T_A + U_{A-B} = T_B: \quad 0 + 2mgR = \frac{1}{2}mv_B^2, \quad v_B^2 = 4gR$

(a)



$$\Sigma F_n = ma_n: \quad N_B = m\,\frac{4gR}{R} = \underline{4mg}$$

(b) $T_A + U_{A-C} = T_C: \quad 0 + 3mgR = \frac{1}{2}mv_C^2, \quad v_C^2 = 6gR$



$$\Sigma F_n = ma_n: \quad N_C - mg = m\,\frac{6gR}{R}$$

$$\underline{N_C = 7mg}$$

(c) Call stopping point E:

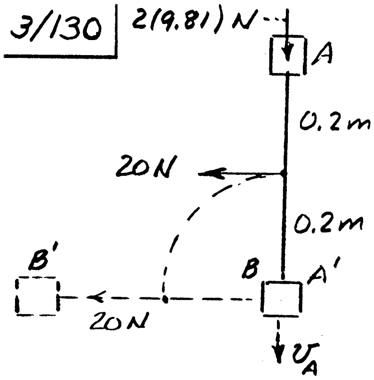
$$T_A + U_{A-E} = T_E$$

$$0 + 2mgR - mg\left(\tfrac{1}{2}s\right) - \mu_k \frac{\sqrt{3}}{2}mgs = 0$$

$$\underline{s = \frac{4R}{1 + \mu_k\sqrt{3}}}$$

(Note: Normal force on incline is $N = mg\cos 30° = \frac{\sqrt{3}}{2}mg$)

3/130



$U_{1-2} = \Delta T$

$T_{initial} = 0$, $T_{final} = \frac{1}{2} m v_A^2 = \frac{2}{2} v_A^2$

$(v_B = 0)$

$U_{1-2} = 20(0.2) + 2(9.81)(0.4)$

$= 11.85 \text{ J}$

$11.85 = v_A^2$, $\underline{v_A = 3.44 \text{ m/s}}$

3/131 $U_{1-2} = \Delta T$; $mg(0.8 - 1.2\cos 60°)$

$= \frac{1}{2}m(v_C^2 - 3^2)$

$9.81(0.20) = \frac{1}{2}(v_C^2 - 9)$, $v_C^2 = 12.92$, $\underline{v_C = 3.59 \text{ m/s}}$

3/132 $U = \Delta T$; $-\int_0^4 (3x^2 + 60x)\,dx = \frac{1}{2}\frac{48}{32.2}(0 - v^2)12$

$x^3 + 30x^2\Big]_0^4 = \frac{288}{32.2}v^2$, v in ft/sec.

$v^2 = \frac{32.2}{288}(64 + 480) = 60.82\ (\text{ft/sec})^2$, $\underline{v = 7.80\ \text{ft/sec}}$

3/133 The power output of the drivetrain is

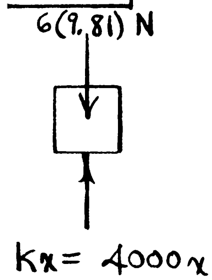
$$P_{out} = Fv = 560\left(\frac{90}{3.6}\right) = 14\,000 \text{ W}$$

The power input to the drivetrain:

$$P_{in} = \frac{P_{out}}{e} = \frac{14\,000}{0.70} = 20\,000 \text{ W}$$

So the motor output $\underline{P = 20 \text{ kW}}$

3/134



$U_{1-2} = \Delta T = 0$:

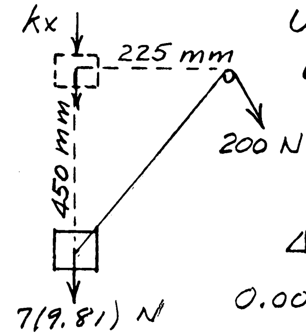
$$6(9.81)(0.1 + \delta) - \int_{0.05}^{0.05+\delta} 4000x\,dx = 0$$

$$2000\,\delta^2 + 141.1\,\delta - 5.89 = 0$$

$\delta = 0.0294\text{m}$ or $\underline{\delta = 29.4 \text{ mm}}$

(Positive result taken from quadratic formula)

3/135 | For system of collar & cable



$U_{1-2} = \Delta T$

$U_{1-2} = 200\left[\sqrt{(0.450)^2 + (0.225)^2} - 0.225\right]$

$- 7(9.81)(0.450) - \frac{1}{2}k(0.075)^2$

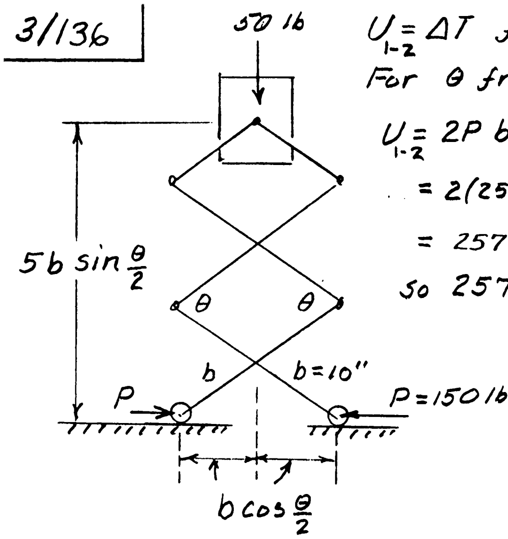
$= 55.6 - 30.9 - 0.00281k$ ✓

$\Delta T = 0$ so

$0.00281k = 55.6 - 30.9$

$k = 8790$ N/m or $\underline{k = 8.79 \text{ kN/m}}$

3/136



$U_{1-2} = \Delta T$ for system

For θ from $60°$ to $180°$,

$$U_{1-2} = 2Pb\cos\frac{\theta}{2} - W(5b)\left(1-\sin\frac{\theta}{2}\right)$$

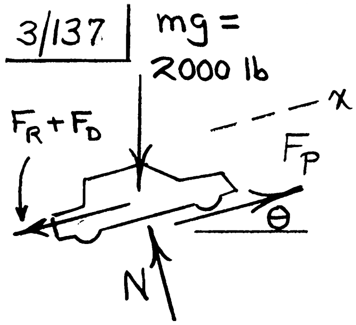
$$= 2(250)\frac{10}{12}\cos 30° - 50\left(\frac{50}{12}\right)(1-\sin 30°)$$

$$= 257 \text{ ft-lb}$$

so $257 = \frac{1}{2}\,\frac{50}{32.2}v^2$, $v^2 = 330.6$

$\underline{v = 18.18 \text{ ft/sec}}$

3/137



$\theta = 0$ or $\theta = \tan^{-1}\frac{6}{100} = 3.43°$

$F_D = kv^2$; $50 = k(60)^2$

$\Rightarrow k = 0.01389 \ \frac{\text{lb-hr}^2}{\text{mi}^2}$

& $F_D = 0.01389 v^2$

$\Sigma F_x = 0$; $F_P - F_R - F_D - mg\sin\theta = 0$

$F_P = F_R + F_D + mg\sin\theta$

(a) $\underline{\theta = 0}$ $v = 30$ mi/hr : $F_D = 0.01389(30^2) = 12.50$ lb

$F_P = F_R + F_D = 50 + 12.50 = 62.5$ lb

$P_{30} = Fv = 62.5\left(30\ \frac{5280}{3600}\right)/550 = \underline{5 \text{ hp}}$

$v = 60$ mi/hr : $F_D = 50$ lb, $F_P = F_R + F_D = 100$ lb

$P_{60} = Fv = 100\left(60\ \frac{5280}{3600}\right)/550 = \underline{16 \text{ hp}}$

(b) $\underline{\theta = 3.43°}$ $F_P = 50 + 50 + 2000 \sin 3.43° = 220$ lb

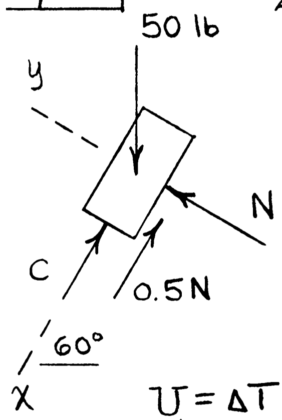
$P_{up} = 220\left(60\ \frac{5280}{3600}\right)/550 = \underline{35.2 \text{ hp}}$

Down: $F_P = 50 + 50 - 2000 \sin 3.43° = -19.78$ lb

$P_{down} = -19.78\left(60\ \frac{5280}{3600}\right)/550 = \underline{-3.17 \text{ hp}}$ (brakes!)

(c) $\Sigma F_x = 0$: $50 + kv^2 - 2000 \sin 3.43° = 0$, $\underline{v = 70.9\ \frac{\text{mi}}{\text{hr}}}$

3/138



$\Sigma F_y = 0: \; N - 50\cos 60° = 0, \; N = 25 \text{ lb}$

Displacement is $3 + \frac{4}{12} = 3.33$ ft

$$U_{1-2} = (50 \sin 60° - 0.5 \cdot 25)\, 3.33 - \frac{1}{12}\int_0^{4''} (100x + 9x^2)\, dx$$

$$= 20.0 \text{ ft-lb}$$

$U_{1-2} = \Delta T: \quad 20.0 = \frac{1}{2}\frac{50}{32.2}(v^2 - 2^2)$

$\underline{v = 5.46 \text{ ft/sec}}$

3/139 $(U_{1-2})_s = -\int_{x_1}^{x_2} (4x - 120x^3)\,dx$

$= \left(-2x^2 + 30x^4\right)\Big|_{x_1}^{x_2} = -2\left(x_2^2 - x_1^2\right) + 30\left(x_2^4 - x_1^4\right)$

With $x_1 = 0.1$ m & $x_2 = 0$, $(U_{1-2})_s = 0.017$ kN·m

or $(U_{1-2})_s = 17$ N·m $= 17$ J

$(U_{1-2})_f = -\mu_k mgd = -0.2(10)(9.81)(0.1) = -1.962$ J

$T_1 + U_{1-2} = T_2: \; 0 + 17 - 1.962 = \frac{1}{2}(10)v^2$

$\underline{v = 1.734 \text{ m/s}}$

For the linear spring, $(U_{1-2})_s = 2\left(x_1^2 - x_2^2\right)$

$= 2(0.1)^2 = 0.02$ kN·m $= 20$ J, $\underline{v' = 1.899 \text{ m/s}}$

3/140 $P = Fv$; $F = ma$, so $P = mav$

& $a = \frac{P}{mv}$

But $v\,dv = a\,ds$, so $mv^2 dv = P\,ds$

$$\int_{v_1}^{v_2} mv^2 dv = \int_0^s P\,ds \; ; \; \frac{m}{3}(v_2^3 - v_1^3) = Ps$$

$$\underline{v_2 = \left(\frac{3Ps}{m} + v_1^3 \right)^{1/3}}$$

3/141 $U = \Delta T$; $\int_x^{x_0} kx\,dx = \frac{1}{2}mv^2$, $v^2 = \frac{k}{m}(x_0^2 - x^2)$

Power $P = Fv$, $P^2 = F^2v^2 = (kx)^2\frac{k}{m}(x_0^2 - x^2)$

$\underline{P = kx\sqrt{\frac{k}{m}(x_0^2 - x^2)}}$, $\frac{dP^2}{dx} = \frac{k^3}{m}(2x_0^2x - 4x^3) = 0$ for max P^2

so $x = 0$ $(P = 0)$, $x = x_0/\sqrt{2}$

so $P_{max} = k\frac{x_0}{\sqrt{2}}\sqrt{\frac{k}{m}}\sqrt{x_0^2 - \frac{x_0^2}{2}} = \underline{\frac{k}{2}\sqrt{\frac{k}{m}}\,x_0^2}$ $\underline{\text{at } x = \frac{x_0}{\sqrt{2}}}$

3/142 State ①: release ; state ②: $x = 0.050$ m

$$T_1 + U_{1-2} = T_2$$

$$\frac{1}{2}m\cancel{v_1}^{2\nearrow 0} + mgx\sin 20° - \frac{1}{2}(3k)x^2 = \frac{1}{2}mv^2$$

$$10(9.81)(0.050)\sin 20° - \frac{3}{2}(120)(0.050)^2 = \frac{1}{2}(10)v^2$$

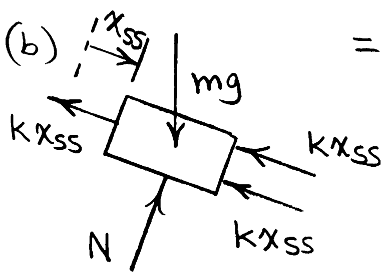
(a) $\underline{v = 0.496 \text{ m/s}}$

Redefine state ②: $x = x_{max}$, $v = 0$

$$T_1 + U_{1-2} = T_2: \quad 0 + mg\,x_{max}\sin 20° - \frac{3}{2}k\,x_{max}^2 = 0$$

$$x_{max} = 0, \quad x_{max} = \frac{2mg\sin 20°}{3k} = \frac{2(10)(9.81)\sin 20°}{3(120)}$$

$$= 0.1864 \text{ m} \quad \text{or} \quad \underline{x_{max} = 186.4 \text{ mm}}$$

(b) 

$$\Sigma F_x = 0:$$

$$-3kx_{ss} + mg\sin 20° = 0$$

$$x_{ss} = \frac{mg\sin 20°}{3k}$$

With numbers, $\underline{x_{ss} = 93.2 \text{ mm}}$

3/143 Note that $\overline{AO} = \sqrt{18^2 + 30^2} = 35.0$ in.

$$U'_{1-2} = \Delta T + \Delta V_e + \Delta V_g$$

$$\Delta T = \frac{1}{2}\frac{2}{32.2}v^2 - 0 = \frac{v^2}{32.2}$$

$$\Delta V_e = \frac{1}{2}(1.60)\left[\left(\frac{20-15}{12}\right)^2 - \left(\frac{35.0-15}{12}\right)^2\right]$$

$$= -2.08 \text{ ft-lb}$$

$$\Delta V_g = 2\left(10/12\right) = 1.667 \text{ ft-lb}$$

$$U'_{1-2} = 0$$

So $0 = \frac{v^2}{32.2} - 2.08 + 1.667$, $\underline{v = 3.65 \text{ ft/sec}}$

$\underline{3/144}$ (a) $\Delta T + \Delta V_g = 0$; $\frac{1}{2}mv^2 - 0 - mgh = 0$

$$v = \sqrt{2gh} = \sqrt{2(9.81)(0.6)} = \underline{3.43 \frac{m}{s}}$$

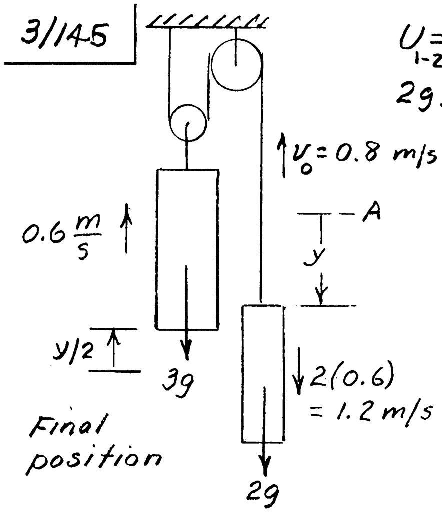
(b) $\Delta V_g + \Delta V_e = 0$ since $\Delta T = 0$

$-mgh + \frac{1}{2}kx^2 = 0$; $x^2 = \frac{2mgh}{k} = \frac{2(4)(9.81)(0.6)}{20(10^3)}$

$$= 0.235(10^{-2})\ m^2$$

$x = 0.0485\ m$ or $\underline{x = 48.5\ mm}$

3/145



$U_{1-2} = \Delta T$

$$2g\,y - 3g\frac{y}{2} = \frac{1}{2}2([1.2]^2 - [0.8]^2) + \frac{1}{2}3([0.6]^2 - [0.4]^2)$$

$$y = \frac{2}{9.81}(0.80 + 0.3)$$

$$\underline{y = 0.224\ \text{m}}$$

3/146 Establish datum @ A.

(a) $T_A + V_A = T_B + V_B$

$0 + 0 = \frac{1}{2}mv_B^2 - mgh_B$

$v_B = \sqrt{2gh_B} = \sqrt{2(9.81)(4.5)} = \underline{9.40 \text{ m/s}}$

(b) State F: spring fully compressed

$T_A + V_A = T_F + V_F$

$0 + 0 = 0 - mgh_f + \frac{1}{2}k\delta^2$

$\delta = \sqrt{\frac{2mgh_f}{k}} = \sqrt{\frac{2(1.2)(9.81)(3)}{24\,000}} = 0.0542 \text{ m}$

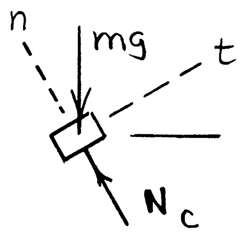
or $\underline{\delta = 54.2 \text{ mm}}$

3/147 Establish datum @ A.

$T_A + V_A = T_C + V_C \; : \; 0 + 0 = \frac{1}{2} m v_C^2 - m g h_C$

$v_C = \sqrt{2 g h_C} = \sqrt{2(9.81)(3 + 1.5\cos 30^\circ)}$

$= 9.18 \text{ m/s}$



(a) $\Sigma F_n = m\frac{v^2}{\rho} \; : \; N_C - 1.2(9.81)\cos 30^\circ = 1.2\frac{9.18^2}{1.5}$

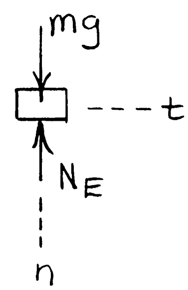
$\underline{N_C = 77.7 \text{ N}}$

(b) $\Sigma F_n = 0 \; : \; N_C - 1.2(9.81)\cos 30^\circ = 0$

$\underline{N_C = 10.19 \text{ N}}$

$T_A + V_A = T_E + V_E \; : \; 0 + 0 = \frac{1}{2} m v_E^2 - m g h_E$

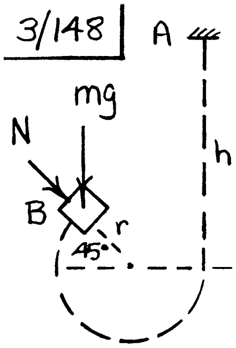
$v_E = \sqrt{2 g h_E} = \sqrt{2(9.81)(3)} = 7.67 \text{ m/s}$



$\Sigma F_n = m\frac{v^2}{\rho} \; : \; -N_E + 1.2(9.81) = 1.2\frac{7.67^2}{1.5}$

$\underline{N_E = -35.3 \text{ N}}$ (down)

3/148



$$U'_{1-2} = \Delta T + \Delta V_g = 0$$

$$\frac{1}{2} m v^2 - mg\left(h - \frac{r}{\sqrt{2}}\right) = 0$$

$$v^2 = 2g\left(h - \frac{r}{\sqrt{2}}\right)$$

$$\Sigma F_n = m a_n: \quad N + \frac{mg}{\sqrt{2}} = m\frac{v^2}{r}$$

$$\Rightarrow N = mg\left[\left(\frac{h}{r} - \frac{1}{\sqrt{2}}\right)2 - \frac{1}{\sqrt{2}}\right]$$

$$= mg\left[2\frac{h}{r} - \frac{3}{\sqrt{2}}\right]$$

With $m = 0.25$ kg, $r = 0.15$ m, & $h = 0.6$ m,

$N = 14.42$ N

3/149

$T_A + V_A = T_B + V_B$, datum @ B

$$0 + mgR + \frac{1}{2}k\left[R\sqrt{2} - R\right]^2 = \frac{1}{2}mv_B^2 + 0$$

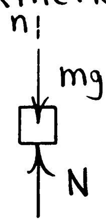
$$\underline{v_B = \sqrt{2gR + \frac{kR^2}{m}\left(3 - 2\sqrt{2}\right)}}$$

$T_A + V_A = T_C + V_C$, datum @ C

$$0 + 2mgR + \frac{1}{2}k\left[R\sqrt{2} - R\right]^2 = \frac{1}{2}mv_C^2 + 0$$

$$\underline{v_C = \sqrt{4gR + \frac{kR^2}{m}\left(3 - 2\sqrt{2}\right)}}$$

Kinetics at C:



$$\Sigma F_n = ma_n: \quad N - mg = m\frac{v_C^2}{R}$$

$$\Rightarrow \underline{N = m\left[5g + \frac{kR}{m}\left(3 - 2\sqrt{2}\right)\right]}$$

3/150 For the system, $U'_{1-2} = 0$ so $\Delta V_g = 0$

$\Delta V_{g_{100}} = -100(7) = -700$ ft-lb

$\Delta V_{g_W} = W\left[2(12\sqrt{2}) - 2\sqrt{5^2+12^2}\right] = 7.94W$

Thus $7.94W - 700 = 0$, $\underline{W = 88.1 \text{ lb}}$

3/151 (a) $\Delta T + \Delta V_g = 0$

$$\frac{1}{2}\frac{5}{32.2}v^2 + \frac{1}{2}\frac{10}{32.2}\left(\frac{12}{18}v\right)^2 + 5\frac{18}{12}\sin 60° - 10\frac{12}{12}\sin 60° = 0$$

$$0.1467 v^2 = 2.165, \quad v^2 = 14.76\ (\text{ft/sec})^2$$

$$\underline{v = 3.84\ \text{ft/sec}}$$

(b) For entire interval $\Delta T = 0$, $\Delta V_g + \Delta V_e = 0$

$$-2.165(12) + \frac{1}{2}(200)x^2 = 0, \quad x^2 = 0.2598\ (\text{in})^2$$

$$\underline{x = 0.510\ \text{in.}}$$

3/152

$\Delta T = \frac{1}{2}(1.5)(3^2 - 2^2) = 3.75$ J

$\Delta V_e = \frac{1}{2}(800)([0.4-0.3]^2 - [0.5-0.3]^2) = -12$ J

$\Delta V_g = 1.5(9.81)(0.4) = 5.89$ J

$U_f = U'_{1-2} = \Delta T + \Delta V_e + \Delta V_g$

$U_f = 3.75 - 12 + 5.89 = \underline{-2.36 \text{ J}}$

$|U_f| = F_{av} s$, $F_{av} = \frac{2.36}{0.7} = \underline{3.38 \text{ N}}$

3/153 Let m be the mass of the car

$$U'_{1-2} = \Delta T + \Delta V_g : \quad 0 = \frac{1}{2} m (v^2 - v_0^2) + mgy$$

$$v^2 = v_0^2 - 2gy$$

$$a_n = \frac{v^2}{\rho} : \quad \frac{v_0^2}{\rho_0} = \frac{v_0^2 - 2gy}{\rho}, \quad \underline{\rho = \rho_0 \left(1 - \frac{2gy}{v_0^2}\right)}$$

For car to remain in contact with the track at the top, $a_n > g$, so for constant a_n, $v_0^2/\rho_0 > g$ so $\underline{v_{0_{min}} = \sqrt{\rho_0 g}}$

3/154 Establish datum at release point.

$$T_A + V_A = T_B + V_B$$

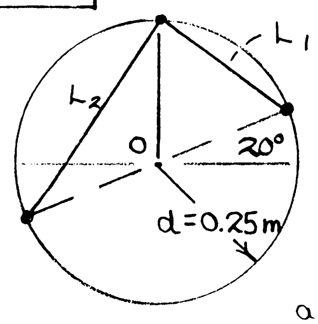
$$0 + \tfrac{1}{2} k_A x_A^2 = 0 + mg(x_A + d + x_B) + \tfrac{1}{2} k_B x_B^2$$

$$\tfrac{1}{2}(48)(12)\left(\tfrac{5}{12}\right)^2 = 14\left(\frac{5 + 14 + x_B}{12}\right) + \tfrac{1}{2}(10)(12)\left(\frac{x_B}{12}\right)^2$$

$$\underline{x_B = 6.89 \text{ in.}}$$

The fact that $x_B > x_A$ is due to the difference in spring stiffnesses (along with the particular value $d = 20 - 6 = 14''$). Note that $d = 14''$ is the distance which the collar moves when out of contact with the springs.

3/155



$$\left\{\begin{array}{l} L_1 = 2d\sin(90^\circ-20^\circ)/2 = 0.287\text{ m} \\ \delta_1 = 0.25\sqrt{2} - L_1 = 0.0668\text{ m} \\ L_2 = 2d\sin\left(\frac{90^\circ+20^\circ}{2}\right) = 0.410\text{ m} \\ \delta_2 = L_2 - 0.25\sqrt{2} = 0.0560\text{ m} \end{array}\right.$$

We may ignore the equal and opposite potential energy changes associated with two of the masses,

$T_1 + V_1 = T_2 + V_2$, datum at O.

$$0 - mgd\cos 20^\circ + \tfrac{1}{2}k\delta_1^2 + \tfrac{1}{2}k\delta_2^2 = 3\left(\tfrac{1}{2}md^2\dot{\theta}^2\right) - mgd$$

$$0 - 3(9.81)(0.25)\cos 20^\circ + \tfrac{1}{2}1200(0.0668)^2$$

$$+\tfrac{1}{2}1200(0.0560)^2 = \tfrac{3}{2}3(0.25)^2\dot{\theta}^2 - 3(9.81)(0.25)$$

Solving, $\underline{\dot{\theta} = 4.22\text{ rad/s}}$

3/156 The system is conservative so

$\Delta T + \Delta V_e + \Delta V_g = 0$; Spring stretch = 13-12=1 in.

$\Delta T = \frac{1}{2}\frac{3}{32.2}(v_B^2 - 8^2) = 0.0466\, v_B^2 - 2.981$ ft-lb

$\Delta V_e = 2\left(\frac{1}{2}10\times 1^2 - 0\right)\frac{1}{12} = 0.833$ ft-lb

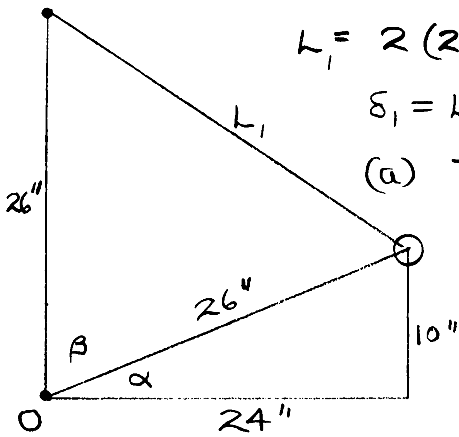
$\Delta V_g = 3(-5/12) = -1.250$ ft-lb

Thus $0.0466\, v_B^2 - 2.981 + 0.833 - 1.250 = 0$

$0.0466\, v_B^2 = 3.398$, $v_B^2 = 72.94$ (ft/sec)^2

$\underline{v_B = 8.54 \text{ ft/sec}}$

3/157



$\alpha = \tan^{-1}\frac{10}{24} = 22.6°$, $\beta = 90 - \alpha = 67.4°$

$L_1 = 2(26)\sin\frac{\beta}{2} = 28.8"$

$\delta_1 = L_1 - 25" = 3.84"$

(a) $T_1 + V_1 = T_2 + V_2$, datum @ O

$0 + 9\left(\frac{10}{12}\right) + \frac{1}{2}1.2(12)\left(\frac{3.84}{12}\right)^2$

$= \frac{1}{2}\frac{9}{32.2}v^2 + \frac{1}{2}(1.2)(12) \times$

$\left(\left[2(26)\sin\frac{90°}{2} - 25\right]/12\right)^2$

$\underline{v = 3.06 \text{ ft/sec}}$

(b) $T_1 + V_1 = T_3 + V_3$, datum @ O

$0 + 9\left(\frac{10}{12}\right) + \frac{1}{2}1.2(12)\left(\frac{3.84}{12}\right)^2 = \frac{1}{2}\frac{9}{32.2}v^2$

$-9\,\frac{26}{12}\sin(35° - \alpha) + \frac{1}{2}(1.2)(12)\left[\frac{2(26)\sin\frac{\beta + 35°}{2} - 25}{12}\right]^2$

$\underline{v = 1.641 \text{ ft/sec}}$

3/158 A force analysis reveals that A will move down & B will move up.

Kinematics : $3v_A = 2v_B$ (speeds)

$T_1 + V_1 = T_2 + V_2$, datum @ initial position

$$0 + 0 = \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B \left(\frac{3}{2} v_A\right)^2 + m_B g h_B - m_A g h_A$$

$$0 = \frac{1}{2}(40) v_A^2 + \frac{1}{2}\, 8\, \frac{9}{4} v_A^2 + 8(9.81)(1) - 40(9.81)\left(\frac{2}{3}(1) \sin 20°\right)$$

$\underline{v_A = 0.616 \text{ m/s}}$, $\underline{v_B = \frac{3}{2} v_A = 0.924 \text{ m/s}}$

3/159 Establish datum at $\theta = 0$ (state A); state B is $\theta = 30°$.

$$T_A + V_A = T_B + V_B$$

$$0 + 0 = 2\left(\frac{1}{2} m v_B^2\right) - 2mgL(1 - \cos\theta) + \frac{1}{2} k \left[2 \frac{L}{2} \sin\theta\right]^2$$

Numbers: $0 = 1.5v^2 - 2(1.5)(9.81)(0.48)(1 - \cos 30°) + \frac{1}{2} 60 \left[0.48 \sin 30°\right]^2$

Solving, $\underline{v = 0.331 \text{ m/s}}$

3/160 $\Delta T + \Delta V_g = 0$

$$\frac{1}{2}m(v^2 - v_0^2) - \frac{mgR^2}{r} + \frac{mgR^2}{R} = 0$$

Let $v = 0$ for $r = \infty$ & get

$$-\frac{v_0^2}{2} + gR, \quad v_0 = \sqrt{2gR}$$

$$= \sqrt{2 \times 9.825 \times 6371 \times 10^3}$$

$$= 11\,190 \text{ m/s} = \underline{11.19 \text{ km/s}}$$

3/161 Constant total energy is $E = T_A + V_{g_A} = T_P + V_{g_A}$

Thus $\frac{1}{2} m v_A^2 - \frac{mgR^2}{r_A} = \frac{1}{2} m v_P^2 - \frac{mgR^2}{r_P}$

$v_A^2 = v_P^2 - 2gR^2\left(\frac{1}{r_P} - \frac{1}{r_A}\right)$, $\underline{v_A = \sqrt{v_P^2 - 2gR^2\left(\frac{1}{r_P} - \frac{1}{r_A}\right)}}$

$\underline{3/162}$ $\Delta T = \frac{1}{2} m (v_B{}^2 - v_A{}^2) = \frac{1}{2}\frac{48}{32.2}[\overline{7600}^2 - \overline{8000}^2]\left(\frac{5280}{3600}\right)^2$

$= -10.00(10^6)$ ft-lb

$\Delta V_g = -mgR^2\left(\frac{1}{r_B} - \frac{1}{r_A}\right) = -48(3959)^2(5280)\left(\frac{1}{4008} - \frac{1}{3983}\right)$

$= 6.221\ (10^6)$ ft-lb

$U = -P(250)(5280) = -1.320\,P\,(10^6)$ ft-lb

$U = \Delta T + \Delta V_g;$ $-1.320\,P = -10.00 + 6.221$, $\underline{P = 2.86\ \text{lb}}$

3/163 $\Delta T + \Delta V_g = 0$, $V_g = -\dfrac{mgR^2}{r}$

Mean radius of earth is $R = 6371$ km

$g = 9.825(3600)^2/1000 = 127.3(10^3)$ km/h^2

Thus

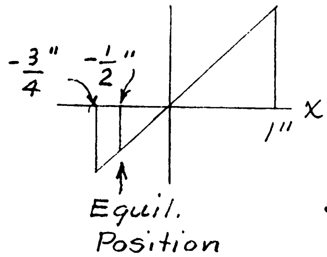
$$\frac{1}{2}m\left(v_B^2 - [24\,000]^2\right) + 127.3(10^3)(6371)^2 m\left(-\frac{1}{6500} + \frac{1}{7000}\right) = 0$$

$$\frac{1}{2}v_B^2 - 288(10^6) + 5167(10^9)(-0.01099)(10^{-3}) = 0$$

$$v_B^2 = 2\left[288 + 56.8\right]10^6 = 690(10^6), \quad \underline{v_B = 26\,300 \text{ km/h}}$$

3/164

$F = 10x$



Let x = stretch of spring from position of zero force

$$\Delta V_g + \Delta V_e + \Delta T = 0$$

$$\Delta V_g = -Wh = -5\frac{1.75}{12} = -0.729 \text{ ft-lb}$$

$$\Delta V_e = \frac{1}{2}k(x_2^2 - x_1^2) = \frac{10}{2}\left(\left[\frac{3}{4}\right]^2 - \left[1\right]^2\right)\frac{1}{12}$$

$$= -0.1823 \text{ ft-lb}$$

$$\Delta T = \frac{1}{2}\frac{W}{g}v^2 = \frac{1}{2}\frac{5}{32.2}v^2 = 0.0776v^2$$

Thus $0.0776v^2 - 0.729 - 0.1823 = 0$, $v^2 = 11.75$, $\underline{v = 3.43 \frac{ft}{sec}}$

3/165 $U'_{1-2} = \Delta T + \Delta V_g + \Delta V_e$ for system

$U'_{1-2} = 0$

$\Delta T = \frac{1}{2} m v^2 = \frac{1}{2} 5 v^2 = 2.5 v^2$ J

$\Delta V_g = mgh = -5 \times 9.81 (0.100 + x)$ J, x in meters

$\Delta V_e = \frac{1}{2} k (x_2^2 - x_1^2) = \frac{1}{2} 1.8 (10^3) x^2 = 900 x^2$ J

For x_{max}, $\Delta T = 0$ so $0 = 0 - 5 \times 9.81 (0.100 + x_{max}) + 900 x_{max}^2$

$x_{max}^2 - 0.0545 x_{max} - 0.00545 = 0$

$x_{max} = 0.1059$ m (or -0.0514)

or $\underline{x_{max} = 105.9 \text{ mm}}$

For v_{max}, $0 = 2.5 v^2 - 5 \times 9.81 (0.100 + x) + 900 x^2$

$v^2 = 1.962 + 19.62 x - 360 x^2$

$\frac{d(v^2)}{dx} = 19.62 - 720 x = 0$ for max v^2 & hence max v

$x = 0.0272$ m or $\underline{x = 27.2 \text{ mm}}$

$v_{max} = \sqrt{1.962 + 19.62(0.0272) - 360(0.0272^2)}$

$= \underline{1.493 \text{ m/s}}$

3/166 For interval from impact to maximum deformation of spring, energy is conserved for system of spring & cylinder

so $\Delta E = \Delta T + \Delta V_g + \Delta V_e = 0$

$\Delta T = 0 - \frac{1}{2} m v^2$, $\Delta V_g = -mg\delta$

Initial stretch of spring is the static deflection under constant-velocity (equil.) condition δ_{ST}

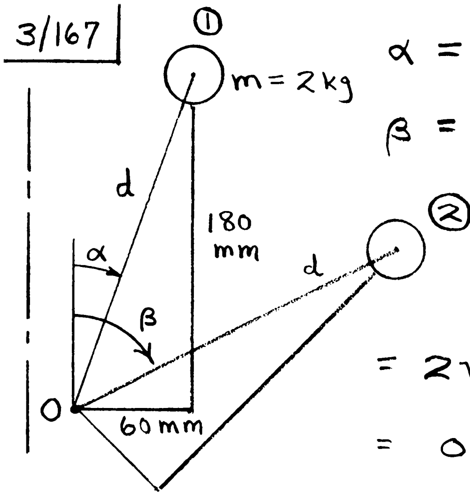
$\delta_{ST} = mg/k$.

$$\Delta V_e = \frac{1}{2}k(\delta_{ST}+\delta)^2 - \frac{1}{2}k\delta_{ST}^2 = k\delta_{ST}\delta + \frac{1}{2}k\delta^2$$
$$= mg\delta + \frac{1}{2}k\delta^2$$

Thus $0 = -\frac{1}{2}mv^2 - mg\delta + mg\delta + \frac{1}{2}k\delta^2$

$mv^2 = k\delta^2$, $\underline{\delta = v\sqrt{m/k}}$

3/167



$\alpha = \tan^{-1}\frac{60}{180} = 18.43°$

$\beta = \alpha + 45° = 63.4°$

Stretch in spring is $\delta = 2d(\sin\beta - \sin\alpha)$

$= 2\sqrt{0.060^2 + 0.180^2}\,(\sin 63.4° - \sin 18.43°)$

$= 0.219$ m

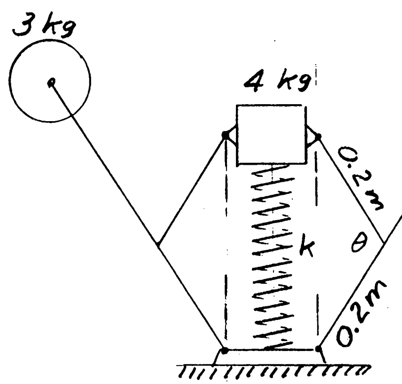
$T_1 + V_1 = T_2 + V_2$, datum at O

$0 + 2mgd\cos\alpha = 0 + 2mgd\cos\beta + \frac{1}{2}k\delta^2$

$2(2)(9.81)(0.1897)\cos 18.43° = 2(2)(9.81)(0.1897)\cos 63.4°$
$+ \frac{1}{2}k(0.219)^2$

Solving, $\underline{k = 155.1 \text{ N/m}}$

3/168



$$\Delta V_e + \Delta V_g + \Delta T = 0$$

$$\Delta T = 0$$

$$\Delta V_e = \frac{1}{2}(900)\left[0.4\left(1-\sin\frac{\theta}{2}\right)\right]^2$$
$$= 72(1-\sin\theta/2)^2 \text{ J}$$

$$\Delta V_g = -2(3)(9.81)(0.5)\left(1-\sin\frac{\theta}{2}\right) - 4(9.81)(0.4)\left(1-\sin\frac{\theta}{2}\right)$$
$$= -45.13\left(1-\sin\frac{\theta}{2}\right)$$

Thus $72\left(1-\sin\frac{\theta}{2}\right)^2 = 45.13\left(1-\sin\frac{\theta}{2}\right)$

$$\sin\frac{\theta}{2} = 1 - \frac{45.13}{72} = 0.3733, \quad \underline{\theta = 43.8°}$$

3/169 Ellipse eccentricity $e = \sqrt{1 - \frac{b^2}{a^2}}$

$= \sqrt{1 - \frac{0.6^2}{0.8^2}} = 0.661$

$r_{min} = a(1-e) = 0.8(1-0.661) = 0.271$ m

$r_{max} = a(1+e) = 0.8(1+0.661) = 1.329$ m

$T_A + V_A = T_C + V_C$

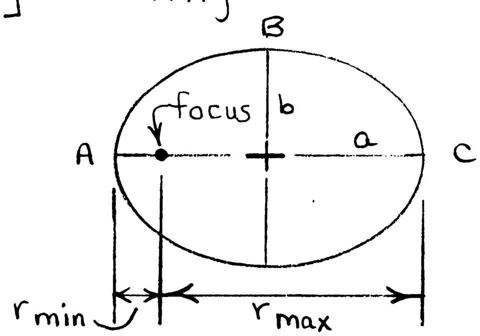
$\frac{1}{2} m v_A^2 + 0 = 0 + \frac{1}{2} k x_C^2$

$\frac{1}{2}(0.4) v_A^2 = \frac{1}{2}(3)\left[1.329 - 0.271\right]^2, \quad v_A = 2.90$ m/s

Then $T_A + V_A = T_B + V_B$

$\frac{1}{2}(0.4)(2.90)^2 + 0 = \frac{1}{2}(0.4) v_B^2 + \frac{3}{2}\left\{\left[(0.8-0.271)^2 + (0.6)^2\right]^{1/2} - 0.271\right\}^2$

$\underline{v_B = 2.51 \text{ m/s}}$



$\boxed{3/170}$ $U'_{1-2} = 0$ so $T_1 + V_{g_1} = T_2 + V_{g_2}$

Take datum $V_g = 0$ at ground level.

$T_1 = \frac{1}{2} \frac{175 + 10}{32.2} v^2 = 2.87 v^2$, $T_2 = 0$

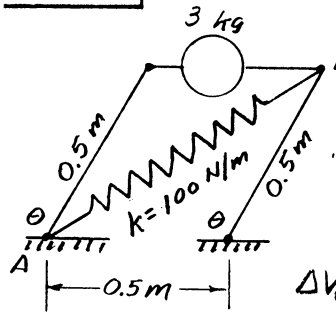
$V_{g_1} = (175 + 10) \frac{42}{12} = 648$ ft-lb

$V_{g_2} = 175(18) + 10(8) = 3230$ ft-lb

So $2.87 v^2 + 648 = 0 + 3230$

$\underline{v = 30.0 \text{ ft/sec}}$ or $\underline{20.4 \text{ mi/hr}}$

3/171 Spring deformation is



$$\delta = \overline{AB} - 0.5\sqrt{2}$$

$$\overline{AB} = \sqrt{2(0.5)^2 - 2(0.5)^2 \cos\theta}$$

$$= 0.5\sqrt{2}\sqrt{1 - \cos 135^\circ}$$

$$\delta = 0.5\sqrt{2}\left[\sqrt{1 + 0.7071} - 1\right]$$

$$= 0.217 \text{ m}$$

$$\Delta V_e = \frac{1}{2} k \delta^2 = \frac{1}{2}(100)(0.217)^2 = 2.35 \text{ J}$$

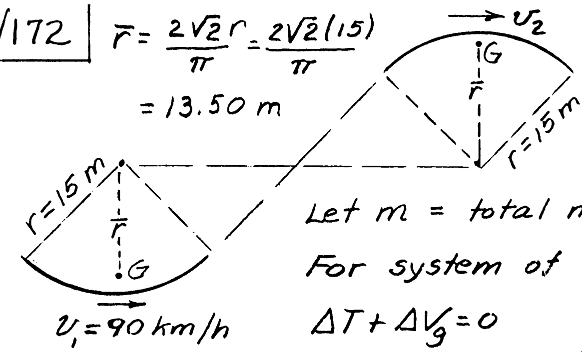
$$\Delta V_g = -mg\Delta h = -3(9.81)(0.5)(1 - \sin 135^\circ) = -4.31 \text{ J}$$

$$\Delta T + \Delta V_g + \Delta V_e = 0; \quad \frac{1}{2} 3 v^2 - 4.31 + 2.35 = 0$$

$$v^2 = 1.307, \quad \underline{v = 1.143 \text{ m/s}}$$

3/172 $\bar{r} = \frac{2\sqrt{2}\,r}{\pi} = \frac{2\sqrt{2}(15)}{\pi}$

$= 13.50\text{ m}$



Let m = total mass of train

For system of cars

$\Delta T + \Delta V_g = 0$

$\frac{1}{2}m(v_2^2 - v_1^2) + mg(2\bar{r}) = 0\ ,\quad v_2^2 = v_1^2 - 4g\bar{r}$

$v_2^2 = \left[\frac{90(1000)}{3600}\right]^2 - 4(9.81)(13.50) = 625 - 529.9 = 95.07\ (\text{m/s})^2$

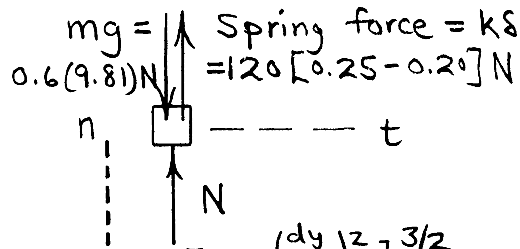
$v_2 = 9.75\text{ m/s}$ or $\underline{v_2 = 35.1\text{ km/h}}$

3/173 $T_A + V_A = T_B + V_B$, datum @ B.

$$0 + 0.6(9.81)(0.5) + \tfrac{1}{2}\,120\left[\sqrt{0.25^2 + 0.5^2} - 0.2\right]^2$$
$$= \tfrac{1}{2}(0.6)\,v_B^2 + \tfrac{1}{2}\,120\,[0.25 - 0.20]^2$$

$\underline{v_B = 5.92 \text{ m/s}}$

Kinetics at B:



Radius of curvature $\rho = \dfrac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{d^2y/dx^2}$

$y = kx^2$; $0.5 = k(0.5)^2 \Rightarrow k = 2$

$y = 2x^2$, $\dfrac{dy}{dx} = 4x$, $\dfrac{d^2y}{dx^2} = 4$

When $x = 0$, $\rho = \dfrac{[1 + 0^2]^{3/2}}{4} = 0.25$ m

$$\Sigma F_n = ma_n : \; N + 120(0.05) - 0.6(9.81) = 0.6\,\frac{5.92^2}{0.25}$$

$\underline{N = 84.1 \text{ N}}$

$\boxed{3/174}$ $x^2 + y^2 = 0.9^2$, $x\dot{x} + y\dot{y} = 0$, $v_A = -\dot{y} = \frac{x}{y}\dot{x} = \frac{x}{y}v_B$

$\Delta T + \Delta V_g = 0$; $\frac{1}{2}m(\dot{x}^2 + \dot{y}^2) + mg(y - \frac{0.9}{\sqrt{2}}) = 0$

$\dot{x}^2(1 + \frac{x^2}{y^2}) = 2(9.81)(\frac{0.9}{\sqrt{2}} - y)$, $\dot{x}^2 \frac{x^2+y^2}{y^2} = 19.62(\frac{0.9}{\sqrt{2}} - y)$

$0.9^2\dot{x}^2 = 19.62(\frac{0.9}{\sqrt{2}}y^2 - y^3)$

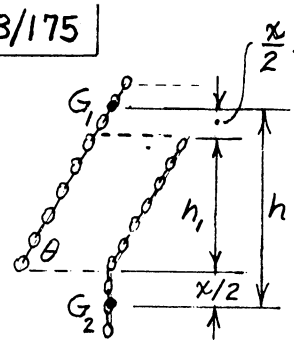
For max. $\dot{x}$, $\frac{d(\dot{x}^2)}{dy} = \frac{19.62}{0.81}(\frac{1.8}{\sqrt{2}}y - 3y^2) = 0$

so $y(\frac{1.8}{\sqrt{2}} - 3y) = 0$, $y = 0.6/\sqrt{2}$ m

& $\dot{x}^2 = \frac{19.62}{0.81}(\frac{0.9}{\sqrt{2}}\frac{0.36}{2} - \frac{0.108}{\sqrt{2}}) = \frac{19.62\sqrt{2}}{30}$

$v_{B_{max}} = \dot{x} = \sqrt{\frac{19.62\sqrt{2}}{30}} = \underline{0.962 \text{ m/s}}$

3/175



$h_1 = (L-x)\sin\theta$

$h = (L-x)\sin\theta + \frac{x}{2}\sin\theta + \frac{x}{2}$

$= L\sin\theta + \frac{x}{2}(1-\sin\theta)$

Let ρ = mass per unit length

$\Delta V_g + \Delta T = 0$

ΔV_g is that of the length x dropping a distance h

$\Delta V_g = -\rho g x h = -\rho g\left[Lx\sin\theta + \frac{x^2}{2}(1-\sin\theta)\right]$

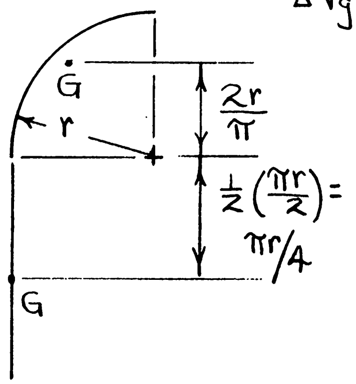
$\Delta T = \frac{1}{2}\rho L v^2$

Thus $-\rho g\left[Lx\sin\theta + \frac{x^2}{2}(1-\sin\theta)\right] + \frac{1}{2}\rho L v^2 = 0$

$v = \sqrt{2gx\left[\sin\theta - \frac{x}{2L}(1-\sin\theta)\right]}$

3/176 For entire chain, $U'_{1-2} = \Delta(T + V_g)$

$U'_{1-2} = 0$, $\Delta T = \frac{1}{2} m v^2 = \frac{1}{2} \rho \left(\frac{\pi r}{2}\right) v^2 = \frac{1}{4} \rho \pi r v^2$



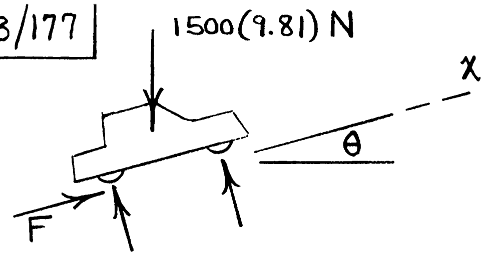
$$\Delta V_g = -\rho\left(\frac{\pi r}{2}\right) g \left[\frac{\pi r}{4} + \frac{2r}{\pi}\right]$$

$$= -\frac{1}{2} \rho g r^2 \pi \left(\frac{\pi}{4} + \frac{2}{\pi}\right)$$

So $\frac{1}{4} \rho \pi r v^2 - \frac{1}{2} \rho g r^2 \pi \left(\frac{\pi}{4} + \frac{2}{\pi}\right) = 0$

$$\underline{v = \sqrt{g r \left(\frac{\pi}{2} + \frac{4}{\pi}\right)}}$$

3/177



$\theta = \tan^{-1}\frac{1}{10} = 5.71°$

$\int \Sigma F_x \, dt = \Delta G_x : \left[F - 1500(9.81)\sin 5.71°\right] 8 = 1500(60-30)\frac{1000}{3600}$

$F = 3030$ N or $\underline{F = 3.03 \text{ kN}}$

3/178

$$\begin{cases} \underline{v} = 1.5t^3\underline{i} + (2.4 - 3t^2)\underline{j} + 5\underline{k} & (\text{m/s}) \\ \dot{\underline{v}} = 4.5t^2\underline{i} - 6t\underline{j} & (\text{m/s}^2) \end{cases}$$

At $t = 2\text{s}$: $\begin{cases} \underline{v} = 12\underline{i} - 9.6\underline{j} + 5\underline{k} \ \text{m/s} \\ \dot{v} = 18\underline{i} - 12\underline{j} \ \text{m/s}^2 \end{cases}$

Then $\underline{G} = m\underline{v} = 1.2(12\underline{i} - 9.6\underline{j} + 5\underline{k})$

$$= \underline{14.40\underline{i} - 11.52\underline{j} + 6\underline{k} \ \text{kg}\cdot\text{m/s}}$$

$$G = \sqrt{14.40^2 + 11.52^2 + 6^2} = \underline{19.39 \ \text{kg}\cdot\text{m/s}}$$

$\Sigma\underline{F} = \dot{\underline{G}}$: $\underline{R} = m\dot{\underline{v}} = 1.2(18\underline{i} - 12\underline{j})$

$$= \underline{21.6\underline{i} - 14.4\underline{j} \ \text{N}}$$

3/179 Conservation of system linear momentum:

$\xrightarrow{+}$ $0.075(600) = 50.075\, v_f$, $v_f = 0.899$ m/s

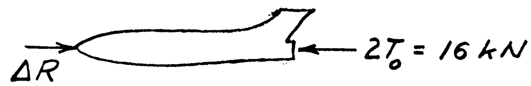
Initial energy $T_1 = \frac{1}{2}(0.075)(600)^2 = 13\ 500$ J

Final energy $T_2 = \frac{1}{2}(50.075)(0.899)^2 = 20.2$ J

Absolute energy loss $|\Delta E| = T_1 - T_2 = \underline{13\ 480 \text{ J}}$

Percent lost: $n = \frac{|\Delta E|}{T_1}(100\%) = \underline{99.9\ \%}$

3/180



$$\int \Sigma F\,dt = m\Delta v; \quad (16\,000 - \Delta R)9 = 10\,000(1050-1000)/3.6$$

$$\underline{\Delta R = 568 \text{ N}}$$

$\underline{3/181}$ $\Delta G = 0$; $150{,}000 \times 2 + 120{,}000 \times 3$

$= (150{,}000 + 120{,}000)\, v$, $v = \underline{2.44 \text{ mi/hr}}$

$$|\Delta E| = \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2 - \frac{1}{2}(m_A + m_B) v^2$$

$$= \frac{1}{2(32.2)} \left(\frac{44}{30}\right)^2 \left[150{,}000 \times 2^2 + 120{,}000 \times 3^2 - 270{,}000 \times \overline{2.44}^2\right]$$

$$\underline{= 2230 \text{ ft-lb loss}}$$

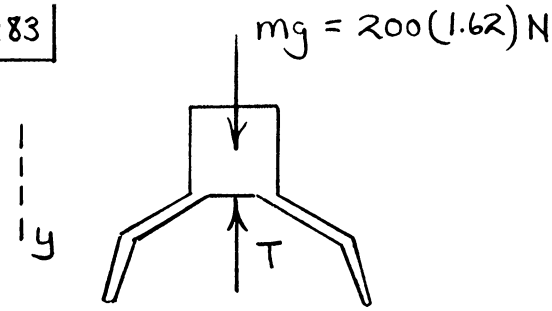
3/182 No difference between cases (a) & (b).

$G_1 = G_2: \quad mv = (3m)v', \quad \underline{v' = \frac{v}{3}}$

$T = \frac{1}{2}mv^2, \quad T' = \frac{1}{2}(3m)\left(\frac{v}{3}\right)^2 = \frac{1}{6}mv^2$

$$n = \frac{T - T'}{T} = \frac{\frac{1}{2}mv^2 - \frac{1}{6}mv^2}{\frac{1}{2}mv^2} = \underline{\frac{2}{3}}$$

3/183



$\int \Sigma F_y \, dt = m \Delta v_y$:

$$200(1.62)(5) - \left[\tfrac{1}{2}2(800) + 2(800)\right] = 200(v-6)$$

$\underline{v = 2.10 \text{ m/s}}$

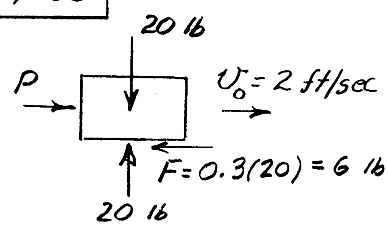
3/184 | $6^{\underline{th}}$ rocket burns out after 7 sec so total impulse on sled during the 10 sec is

$6(8600) - 10R$.

$\int \Sigma F dt = m \Delta v$; $\quad 6(8600) - 10R = \frac{5200}{32.2}\left(200 \frac{44}{30} - 0\right)$

$\underline{R = 423 \text{ lb}}$

3/185

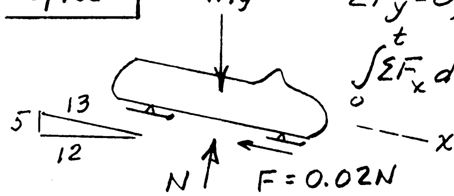


$$\int_0^t \Sigma F\,dt = m\Delta v$$

$$16(0.2) + 8(0.2) - 6(0.4) = \frac{20}{32.2}(v-2)$$

$$\underline{v = 5.86 \text{ ft/sec}}$$

3/186



$\Sigma F_y = 0;\quad N = mg\,\frac{12}{13}$

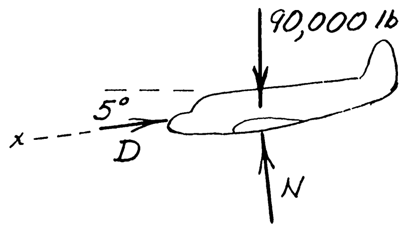
$\int_0^t \Sigma F_x\,dt = m\Delta v_x$

$\left(mg\frac{5}{13} - 0.02\,mg\frac{12}{13}\right)t$

$= m\left(\frac{100\times 1000}{3600} - 0\right)$

$\frac{9.81}{13}(5 - 0.02[12])\,t = 27.78,\quad \underline{t = 7.73\text{ s}}$

3/187



$\int \Sigma F_x \, dt = \Delta G_x$:

$$(90{,}000 \sin 5° - D)\,120 = \frac{90{,}000}{32.2}(360 - 400)\frac{5280}{3600}$$

$\underline{D = 9210 \text{ lb}}$

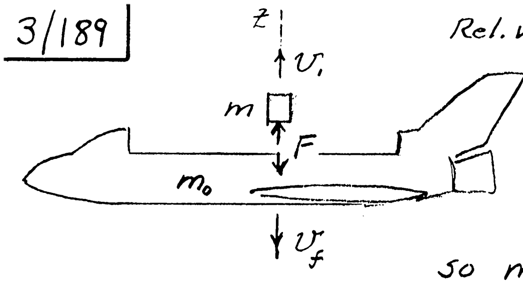
$\underline{3/188}\big|$ $\int F\,dt = m\Delta v$

$$(50{,}000 \cos 20^\circ)\,t = \frac{150{,}000 \times 2240}{32.2}\;\frac{1\times 1.151}{1}\;\frac{44}{30}$$

$$46{,}985\,t = 17.62\times 10^6$$

$t = 375$ sec or $\underline{t = 6.25 \text{ min}}$

3/189



Rel. velocity is

$v_1 + v_f = 0.3$ m/s --(1)

$\int F\,dt = m v_1$

$\int -F\,dt = m_f(-v_f)$

so $m v_1 = m_0 v_f$

$800 v_1 = 90\,000 v_f$ ----- (2)

Solve (1) & (2) & get $v_f = 0.3 - \frac{90000}{800} v_f$

$\underline{v_f = 0.00264 \text{ m/s}}$

so $F_{av}\int_0^4 dt = 90\,000\,(0.00264)$, $F_{av} = \frac{90(2.64)}{4} = \underline{59.5 \text{ N}}$

3/190 $G_1 = G_2$: $mv = (m + pm)v'$, $\underline{v' = \dfrac{v}{1+p}}$

$$\bar{a}_A = \frac{v' - v}{\Delta t} = \frac{\frac{v}{1+p} - v}{\Delta t} = \underline{\frac{-vp}{\Delta t(1+p)}}$$

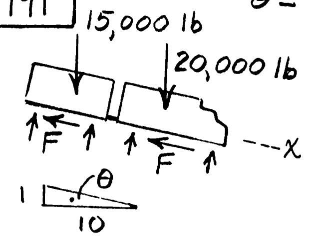
$$\bar{a}_B = \frac{v' - 0}{\Delta t} = \underline{\frac{v}{\Delta t(1+p)}} \qquad \text{— — — —} \oplus$$

For $p = \frac{1}{2}$, $v' = \dfrac{v}{1+1/2} = \underline{\dfrac{2}{3}v}$

$$\bar{a}_A = \frac{-v\left(\frac{1}{2}\right)}{\Delta t\left(1+\frac{1}{2}\right)} = \underline{-\frac{1}{3}\frac{v}{\Delta t}}$$

$$\bar{a}_B = \frac{v}{\Delta t\left(1+\frac{1}{2}\right)} = \underline{\frac{2}{3}\frac{v}{\Delta t}}$$

3/191

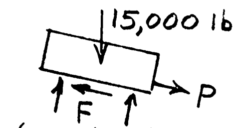


$\theta = \tan^{-1} 0.1 = 5.71°, \quad \sin\theta = 0.0995$

$$\int \Sigma F_x \, dt = m \Delta v_x$$

$$[35{,}000 \times 0.0995 - 2F]\,5 = \frac{35{,}000}{32.2}\left(0 - 20\frac{44}{30}\right)$$

$$F = 4930 \text{ lb}$$

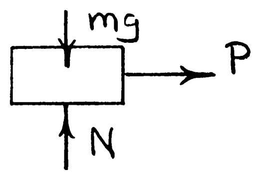


(Trailer)

$$[P - 4930 + 15{,}000 \times 0.0995]\,5 = \frac{15{,}000}{32.2}\left(0 - 20\frac{44}{30}\right)$$

$$\underline{P = 704 \text{ lb (tension)}}$$

3/192



$$\xrightarrow{+} \quad mv_1 + \int \Sigma F\,dt = mv_2 :$$

$$0 + \int_0^t F_0 e^{-bt}\,dt = mv$$

$$v = \frac{F_0}{mb}\left(1 - e^{-bt}\right), \quad v \to \frac{F_0}{mb} \text{ as } t \to \infty$$

$$\frac{ds}{dt} = \frac{F_0}{mb}\left(1 - e^{-bt}\right)$$

$$\int_{s_0=0}^{s} ds = \int \frac{F_0}{mb}\left(1 - e^{-bt}\right) dt$$

$$s = \frac{F_0}{mb}\left[t + \frac{1}{b}\left(e^{-bt} - 1\right)\right]$$

3/193 $F = kt^2: \quad 20 = k(2)^2, \quad k = 5\ \text{N/s}^2$

$F = 5t^2 \qquad (0 \le t \le 2\text{s})$

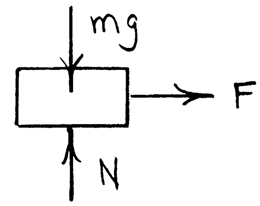
$t = 1\text{ s}: \quad m\cancel{v_0}^{\,0} + \int_0^1 F\,dt = mv_1$

$v_1 = \frac{1}{4}\int_0^1 5t^2\,dt = \frac{1}{4}\,5\,\frac{t^3}{3}\Big|_0^1 = \underline{0.417\ \text{m/s}}$

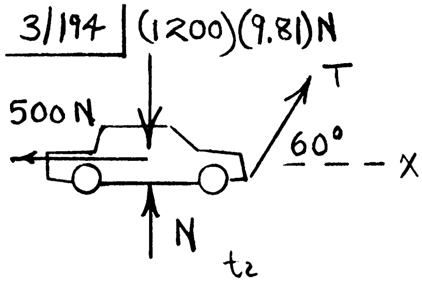
$t = 3\text{ s}: \quad m\cancel{v_0}^{\,0} + \int_0^3 F\,dt = mv_3$

$v_3 = \frac{1}{4}\left[\int_0^2 5t^2\,dt + \frac{3-2}{2}(25-20) + (3-2)(20)\right]$

$= \underline{8.96\ \text{m/s}}$



3/194



$$mv_{x_1} + \int_{t_1}^{t_2} \Sigma F_x \, dt = mv_{x_2}:$$

$$1200\left(\frac{30}{3.6}\right) + \left[-500 + T\cos 60°\right]15 = 1200\left(\frac{70}{3.6}\right)$$

$\underline{T = 2780 \text{ N}}$

3/195 Impact velocity $v_0 = \sqrt{2gh} = \sqrt{2(9.81)(1.4)}$
$= 5.24$ m/s

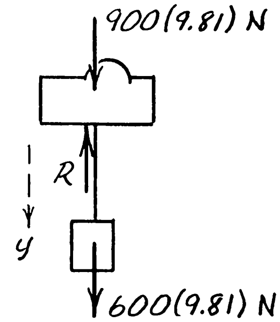
$\Delta G = 0$; $450(5.24) + 0 = (450 + 240)v$
$\underline{v = 3.42 \text{ m/s}}$

Impulse of weights is negligible compared with impulse of impact forces.

3/196 Entire system: $\int \Sigma F_y \, dt = \Delta G_y$

$$\left[(600+900)9.81 - R\right]6 = 600(0.5-3)$$

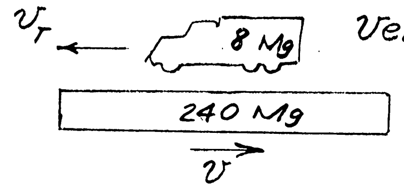
$$\underline{R = 14.96 \text{ kN}}$$



$\underline{3/197}$ $\Delta G = 0; \quad 12(20) + 0 = (350 + 12)v$

$$\underline{v = 0.663 \text{ km/h}}$$

3/198



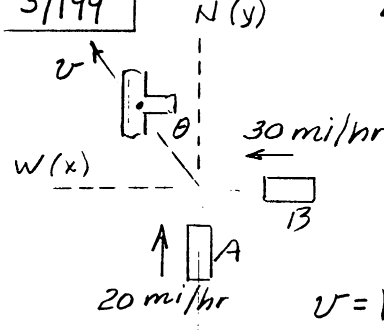
Vel. truck $v_T = v_{rel} - v$

$= 6 - v$ km/h

$\Delta G = 0$, $G_{initial} = G_{final} = 0$

$8(6 - v) = 240v$, $\underline{v = 0.1935 \text{ km/h}}$

3/199



$$\Delta G_x = 0; \quad \frac{3200}{g}(30) = \frac{(3200+3400)}{g} v_x$$

$$v_x = 14.55 \text{ mi/hr}$$

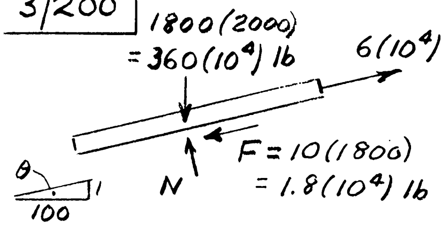
$$\Delta G_y = 0; \quad \frac{3400(20)}{g} = \frac{(3200+3400)}{g} v_y$$

$$v_y = 10.30 \text{ mi/hr.}$$

$$v = \sqrt{(14.55)^2 + (10.30)^2} = \underline{17.82 \text{ mi/hr}}$$

$$\theta = \tan^{-1}\frac{v_x}{v_y} = \tan^{-1}\frac{14.55}{10.30} = \underline{54.7°}$$

3/200



$\sin\theta = 0.0100$

$$\int \Sigma F\,dt = m\Delta v$$

$$[6 - 1.8 - 360(0.0100)]10^4 t = \frac{360(10^4)}{32.2}(30-20)\frac{44}{30}$$

$$0.600\,t = 163.98$$

$t = 273.2$ sec or $\underline{t = 4 \text{ min } 33 \text{ sec}}$

3/201 $\underline{G}_1 = \underline{G}_2$: $m_s \underline{v}_s + m_m \underline{v}_m = (m_s + m_m)\underline{v}$

$$1000\,(2000)\underline{j} + 10\,(5000)\left[\frac{+5\underline{i} - 4\underline{j} - 2\underline{k}}{\sqrt{5^2 + 4^2 + 2^2}}\right] = (1000 + 10)\,\underline{v}$$

$$\underline{v} = 36.9\underline{i} + 1951\underline{j} - 14.76\underline{k} \quad \text{m/s}$$

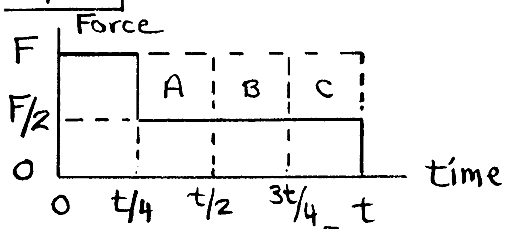
The angle between $\underline{v}_s$ and $\underline{v}$ is

$$\beta = \cos^{-1}\frac{\underline{v}\cdot\underline{v}_s}{v v_s}$$

$$= \cos^{-1}\left[\frac{(36.9\underline{i} + 1951\underline{j} - 14.76\underline{k})\cdot 2000\underline{j}}{\sqrt{36.9^2 + 1951^2 + 14.76^2}\; 2000}\right]$$

$$= 1.167^\circ$$

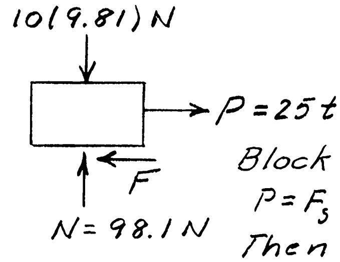
3/202



Solid area is $5/8$ of nominal area, so $\underline{n = 62.5\%}$

In order to compensate, areas A, B, & C must be added after time t, so the extra time $\underline{t' = \frac{3}{4} t}$.

3/203



$F_s = \mu_s N = 0.6(98.1) = 58.9\ N$

$F_k = \mu_k N = 0.4(98.1) = 39.2\ N$

Block does not move until $P = F_s$ or $25t = 58.9$, $t = 2.35\ s$

Then F becomes $39.2\ N$

$$\int \Sigma F dt = m\Delta v; \quad \int_{2.35}^{4} (25t - 39.2)\,dt = 10(v - 0)$$

$$\left.\frac{25t^2}{2} - 39.2t\right]_{2.35}^{4} = 10v, \quad 10v = 66.1, \quad \underline{v = 6.61\ m/s}$$

3/204 $\int_0^t \Sigma F\,dt = m\Delta v$

Area of a semiellipse is $\pi ab/2$ where $a = 30$ N
$b = 0.04$ s

Thus $\dfrac{\pi(30)(0.04)}{2} = 0.20(v - [-6])$

$\underline{v = 3.42 \text{ m/s}}$

3/205 $\int \Sigma F_x \, dt = m \Delta v_x$

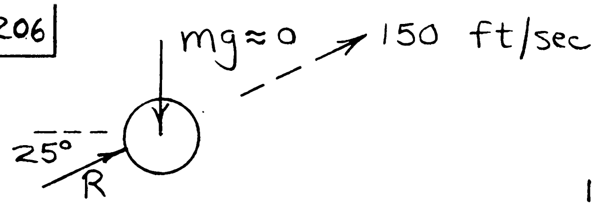
(a) $(0.6 - 0.2)\frac{240}{2} = 4(v_{0.6} - [-10])$

$48 = 4(v_{0.6} + 10)$ $\qquad$ $v_{0.6} = 2 \text{ m/s}$ (+x-dir)

(b) $48 - (0.9 - 0.6)\frac{120}{2} = 4(v_{0.9} - [-10])$

$30 = 4(v_{0.9} + 10)$, $\qquad$ $v_{0.9} = -2.5 \text{ m/s}$ (-x-dir.)

3/206



$$+\nearrow \; R\Delta t = mv : \quad R(0.001) = \frac{1.62/16}{32.2}(150)$$

$$\underline{R = 472 \text{ lb}}$$

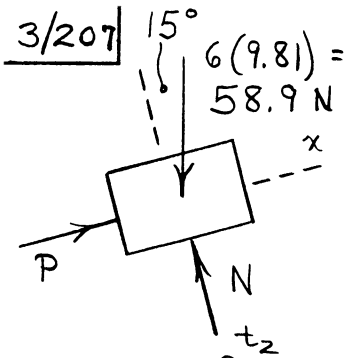
$$+\nearrow \; R = ma : \quad 472 = \frac{1.62/16}{32.2}\, a$$

$$\underline{a = 150{,}000 \text{ ft/sec}^2 \; (4660g)}$$

$$v^2 - v_0^2 = 2ad : \quad 150^2 - 0^2 = 2(150{,}000)\, d$$

$$\underline{d = 0.075 \text{ ft or } 0.900 \text{ in.}}$$

3/207



$0 \le t \le 5\text{ s}: \quad P = 2t^2\text{ N}$

$t \ge 5\text{ s} \quad : \quad P = 50 \quad \text{N}$

$$\int_0^{5\text{ s}} P\,dt = \int_0^{5\text{ s}} 2t^2\,dt$$

$$= \frac{2t^3}{3}\Big|_0^{5\text{ s}} = 83.3\text{ N}\cdot\text{s}$$

$$mv_{x_1} + \int_{t_1=0}^{t_2} \Sigma F_x\,dt = mv_{x_2}:$$

$$-6(20) + 83.3 + 50(8-5) - 58.9\sin 15^\circ(8) = 6v_2$$

$$\underline{v_2 = -1.423\text{ m/s}}$$

For a zero speed:

$$-6(20) + 83.3 + 50(t-5) - 58.9\sin 15^\circ(t) = 0$$

$$\underline{t = 8.25\text{ s}}$$

3/208 $\Delta G = 0, \quad G_1 = G_2$

$$\left(\frac{2/16}{32.2} + 0\right)2000 = \frac{2/16 + 50}{32.2} v_2\,, \quad v_2 = 4.99 \text{ ft/sec}$$

$U = \Delta T$: $v_2 = \sqrt{2gh}$, $4.99^2 = 2(32.2)(6)(1-\cos\theta)$ where $h = 6(1-\cos\theta)$

$$\cos\theta = 0.936\,, \quad \underline{\theta = 20.7^\circ}$$

$$\%\text{ energy loss} = \frac{\frac{1}{2}m_1 v_1^2 - (m_1 + m_2)gh}{\frac{1}{2}m_1 v_1^2} \times 100\% = \left(1 - \frac{m_1 + m_2}{m_1}\frac{2gh}{v_1^2}\right)100\%$$

$$= \left[1 - \frac{2/16 + 50}{2/16}\,\frac{2(32.2)6(1-0.936)}{2000^2}\right]100\% = \underline{99.8\%}$$

3/209 Assume no change in external forces acting on tug-barge system.

v = velocity of tug, v_B = velocity of barge

$v_B = v + 2$

$\Delta G = 0$; $(580 + 1200)6 \times 1.688 = 580v + 1200(v+2)$

$v = 8.78$ ft/sec or $v = \dfrac{8.78}{1.688} = \underline{5.20 \text{ knots}}$

3/210 For plug: $\Delta T + \Delta V_g = 0$; $\frac{1}{2} m_A v^2 - m_A g r = 0$

$v = \sqrt{2gr}$

Plug & block: $\Delta G = 0$; $m_A \sqrt{2gr} = (m_A + m_C) v'$

where v' = velocity of block & plug after impact

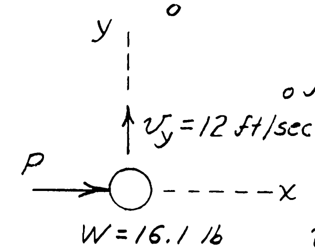
Friction force $F = \mu_k (m_A + m_C) g$

Deceleration $a = F/(m_A + m_C) = \mu_k g$

$v'^2 = 2as$, $\quad s = \left(\frac{m_A}{m_A + m_C}\right)^2 2gr \frac{1}{2\mu_k g} = \underline{\frac{r}{\mu_k}\left(\frac{m_A}{m_A + m_C}\right)^2}$

3/211

$$\int_0^t \Sigma F_x \, dt = m(v_{2_x} - v_{1_x})$$

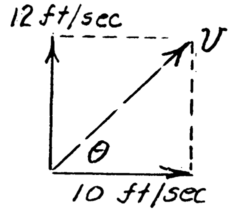


From graph

$$\int_0^2 \Sigma F_x \, dt = \frac{4}{2} + \frac{4}{2} + \frac{2}{2} = 5 \text{ lb-sec}$$

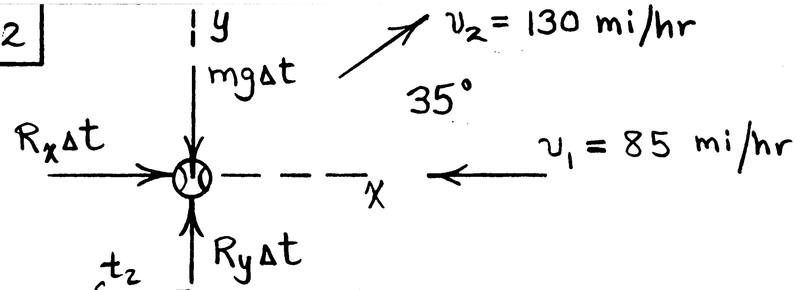
so $5 = \frac{16.1}{32.2}(v_{2_x} - 0)$, $v_{2_x} = 10$ ft/sec

$$v = \sqrt{10^2 + 12^2} = \underline{15.62 \text{ ft/sec}}$$



$$\theta = \tan^{-1}\frac{12}{10} = \underline{50.2°}$$

3/212



$$m v_{x_1} + \int_{t_1}^{t_2} \Sigma F_x \, dt = m v_{x_2} :$$

$$-\frac{5.125/16}{32.2}\left(85 \frac{5280}{3600}\right) + R_x(0.005) = \frac{5.125/16}{32.2}\left(130 \frac{5280}{3600} \cos 35^\circ\right)$$

$\underline{R_x = 559 \text{ lb}}$

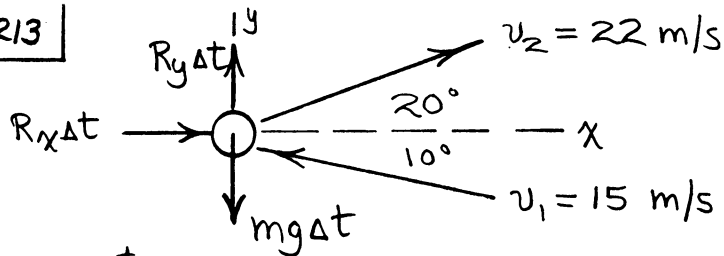
$$m v_{y_1} + \int_{t_1}^{t_2} \Sigma F_y \, dt = m v_{y_2} :$$

$$0 + R_y(0.005) - \frac{5.125}{16}(0.005) = \frac{5.125/16}{32.2}\left(130 \frac{5280}{3600} \sin 35^\circ\right)$$

$\underline{R_y = 218 \text{ lb}}$

If mg is neglected during impact, $R_y = 218$ lb – a good assumption! The quantity mg may <u>not</u> be neglected thereafter – otherwise we obtain a record home-run distance!

3/213



$$mv_{x_1} + \int_{t_1}^{t_2} \Sigma F_x \, dt = mv_{x_2}:$$

$$0.060(-15\cos 10°) + R_x(0.05) = 0.060(22\cos 20°)$$

$R_x = 42.5$ N

$$mv_{y_1} + \int_{t_1}^{t_2} \Sigma F_y \, dt = mv_{y_2}:$$

$$0.060(15\sin 10°) + R_y(0.05) - 0.060(9.81)(0.05) = 0.060(22\sin 20°)$$

$R_y = 6.49$ N

Weight $mg = 0.060(9.81) = 0.589$ N is about 9% of R_y — no need to omit mg from analysis!

$R = \sqrt{R_x^2 + R_y^2} = \underline{43.0 \text{ N}}$

$\beta = \tan^{-1}\dfrac{R_y}{R_x} = \underline{8.68°}$

3/214 $\int \Sigma F_x \, dt = \int_0^2 4e^{-t} \cos 2\pi t \, dt$

$$= 4 \; \frac{e^{-t}(-\cos 2\pi t + 2\pi \sin 2\pi t)}{1 + 4\pi^2} \Bigg]_0^2$$

$$= \frac{4}{1+4\pi^2}\left[e^{-2}(-1+0) - 1(-1+0)\right]$$

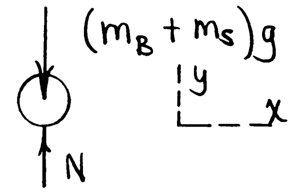
$$= 0.0988\left[1 - e^{-2}\right] = 0.0988(0.8647)$$

$$= 0.0854 \text{ lb-sec}$$

$\int \Sigma F_x \, dt = m \Delta v_x$; $0.0854 = \frac{1}{32.2}(v_x - [-3])$, $v_x = \underline{-0.249 \text{ ft/sec.}}$

(negative x-dir.)

3/215 System :



$$m_B v_{Bx} + m_s \cancelto{0}{v_{Sx}} = (m_B + m_s) v$$

$$v = \frac{m_B v_{Bx}}{(m_B + m_s)} = \frac{80/32.2}{90(32.2)} (16 \cos 30^\circ)$$

$$= \underline{12.32 \text{ ft/sec}}$$

$$m_B v_{By} + m_s \cancelto{0}{v_{Sy}} + \int_0^{\Delta t} [N - (m_B + m_s) g] \, dt = 0$$

$$-\frac{80}{32.2}(16 \sin 30^\circ) + N(0.05) - 90(0.05) = 0$$

$$\underline{N = 488 \text{ lb}}$$

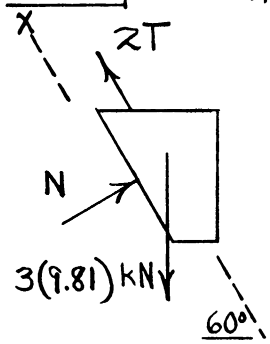
3/216

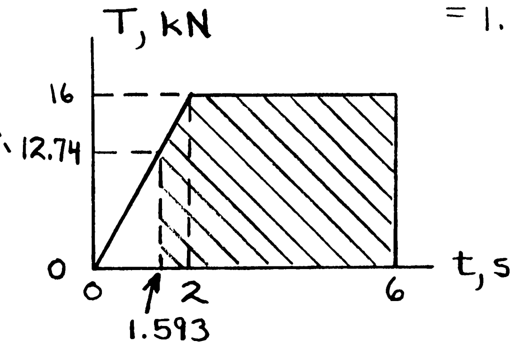
Skip begins to move when

$2T = 3(9.81)\frac{\sqrt{3}}{2}$, $\quad T = 12.74$ kN

which occurs at $t = \frac{12.74}{16}$

$= 1.593$ s





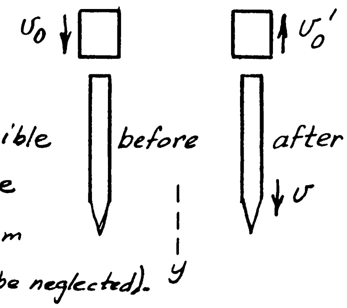
$\int \Sigma F_x \, dt = m \Delta v_x$:

$$2\left[\frac{16 + 12.74}{2}(2 - 1.593) + 16(6-2)\right] - 3(9.81)\frac{\sqrt{3}}{2}(6 - 1.593)$$

$$= 3(v - 0)$$

$\underline{v = 9.13 \text{ m/s}}$

▶3/217 Velocity of ram just before impact is $v_0 = \sqrt{2gh} = \sqrt{2 \times 9.81 \times 1.5}$
$= 5.42$ m/s



$\int \Sigma F_y \, dt = \Delta G_y$ & for ram $W_{ram} \Delta t$ is negligible compared with impulse of extremely large impact force (similarly for pile, vector sum of weight and penetration resistance can be neglected).

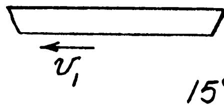
(a) $\Delta G_y = 0$: $(400 + 300)v - 400(5.42) = 0$, $\underline{v = 3.10 \text{ m/s}}$

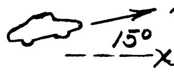
(b) $\Delta G_y = 0$: $400 \times 0 + 300v - 400(5.42) = 0$, $\underline{v = 7.23 \text{ m/s}}$

(c) $\Delta G_y = 0$ where upward velocity of ram immediately after impact is $v_0' = \sqrt{2gh'} = \sqrt{2g(0.100)} = 1.401$ m/s

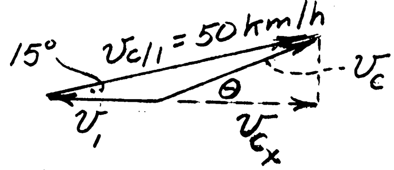
so $(300v - 400 \times 1.401) - 400(5.42) = 0$, $\underline{v = 9.10 \text{ m/s}}$

▸3/218





v_c = absolute velocity of car.



$\underline{v}_c = \underline{v}_1 + \underline{v}_{c/1}$

$v_{c_x} = 50\cos 15° - v_1$

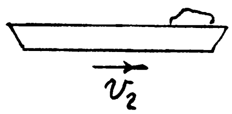
$= 48.296 - v_1$ km/h

$\Delta G_x = 0$; $500 \times 10^3 v_1 = 1500(48.296 - v_1)$, $v_1 = 0.144 \frac{km}{h}$

$v_{c_x} = 48.152$ km/h

or $v_1 = 40.1 \frac{mm}{s}$

$\Delta G_x = 0$



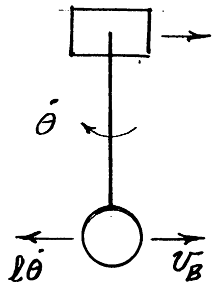
$1500(48.152) = 501500\, v_2$

$v_2 = 0.144$ km/h

or $\underline{v_2 = 40.0 \text{ mm/s}}$

▶3/219 $\Sigma F_x = 0$ on system so $\Delta G_x = 0$

$$m_A(l\dot{\theta} - v_B) - m_B v_B = 0$$



--- x so $l\dot{\theta} = v_B \dfrac{m_A + m_B}{m_A}$

$\Delta T + \Delta V_g = 0;$

$$\frac{1}{2}m_A(l\dot{\theta} - v_B)^2 + \frac{1}{2}m_B v_B^2 - m_A g l = 0$$

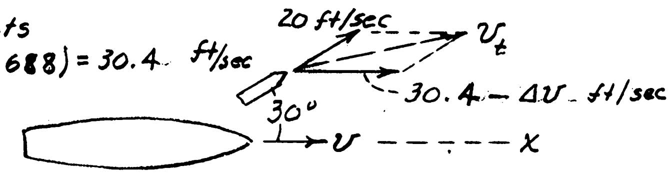
$$\frac{1}{2}m_A\left[v_B^2\left(\frac{m_A+m_B}{m_A}\right)^2 - 2v_B^2\frac{m_A+m_B}{m_A} + v_B^2\right] + \frac{1}{2}m_B v_B^2 = m_A g l$$

simplify & get $\quad v_B^2 = 2\dfrac{m_A^2 g l}{m_B(m_A+m_B)}$

or $\quad v_B = \dfrac{m_A}{m_B}\sqrt{\dfrac{2gl}{1+\dfrac{m_A}{m_B}}}$

▸3/220 18 knots

$= 18(1.688) = 30.4$ ft/sec



$$\Delta G_x = 0;\quad (G_x)_{initial} = (60 \times 2240 + 300)(30.4)/32.2$$
$$= 12.710(10^4) \text{ lb-sec}$$

x-component of torpedo velocity after launch is

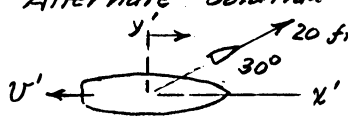
$$(v_t)_x = 30.4 - \Delta v + 20\cos 30° = 47.7 - \Delta v \text{ ft/sec}$$

$$(G_x)_{final} = \frac{60(2240)}{32.2}(30.4 - \Delta v) + \frac{300}{32.2}(47.7 - \Delta v)$$
$$= 12.726(10^4) - 0.41832(10^4)\Delta v$$

Thus $12.710(10^4) - 12.726(10^4) + 0.41832(10^4)\Delta v = 0$

Thus $\underline{\Delta v = 0.0386 \text{ ft/sec}}$

Alternate solution



v' = velocity of boat relative to x'-y' $(= \Delta v)$

$$\Delta G_{x'} = 0;\quad -\frac{60(2240)\Delta v}{g} + \frac{300(20\cos 30° - \Delta v)}{g} = 0$$

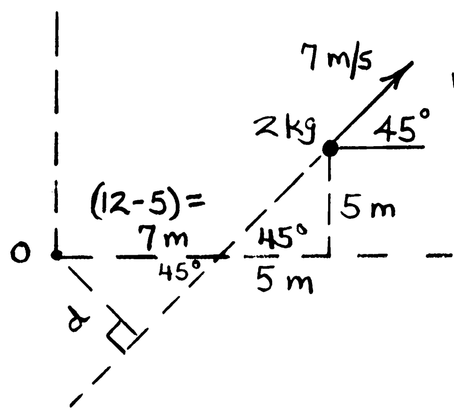
$\underline{v' = 0.0386 \text{ ft/sec}}$

3/221 (a) $\underline{H}_O = \underline{r} \times m\underline{v}$

$= 2(12\underline{i} + 5\underline{j}) \times 7(\cos 45°\,\underline{i} + \sin 45°\,\underline{j})$

$= 69.3\,\underline{k}$ kg·m²/s , so $\underline{H_O = 69.3 \text{ kg·m}^2/\text{s}}$

(b)



$d = 7\sin 45° = 4.95$ m

$\circlearrowleft + \; H_O = mvd$

$= 2(7)(4.95)$

$= \underline{69.3 \text{ kg·m}^2/\text{s}}$

3/222 (a) $\underline{G} = m\underline{v} = 3 \cdot 4 [\cos 45° \underline{i} - \sin 45° \underline{j}]$

$= \underline{8.49\underline{i} - 8.49\,\underline{j} \ \ \text{kg}\cdot\text{m/s}}$

(b) $\underline{H}_o = \underline{r} \times m\underline{v} = \underline{r} \times \underline{G}$

$= 2(\cos 60° \underline{i} + \sin 60° \underline{j}) \times (8.49\underline{i} - 8.49\underline{j})$

$= \underline{-23.2\underline{k} \ \ \text{kg}\cdot\text{m}^2/\text{s}}$

(c) $T = \frac{1}{2} m v^2 = \frac{1}{2}(3)(4)^2 = \underline{24 \text{ J}}$

3/223 $\underline{H}_O = \underline{r} \times \underline{G} = (3\underline{i} + 4\underline{j} - 3\underline{k}) \times (-3\underline{i} - 2\underline{j} + 3\underline{k})$

$= (12 - 6)\underline{i} + (6 - 6)\underline{j} + (-6 + 12)\underline{k}$

$= 6\underline{i} + 6\underline{k}$ ft-lb-sec

$H_O = |\underline{H}_O| = \sqrt{6^2 + 6^2} = \underline{8.49 \text{ ft-lb-sec}}$

3/224 $\underline{r} = 3t^2\underline{i} - 2t\underline{j} - 3t\underline{k}$, $m = 4$ kg

$\underline{v} = \dot{\underline{r}} = 6t\underline{i} - 2\underline{j} - 3\underline{k}$ m/s

$\underline{G} = m\underline{v} = 4(6t\underline{i} - 2\underline{j} - 3\underline{k})$ N·s

$\underline{H} = \underline{r} \times \underline{G} = (3t^2\underline{i} - 2t\underline{j} - 3t\underline{k}) \times 4(6t\underline{i} - 2\underline{j} - 3\underline{k})$

$= 4[(6t - 6t)\underline{i} + (9t^2 - 18t^2)\underline{j} + (12t^2 - 6t^2)\underline{k}]$

$= 12t^2(-3\underline{j} + 2\underline{k})$ N·m·s

$\underline{M} = \dot{\underline{H}} = 24t(-3\underline{j} + 2\underline{k})$ N·m

& for $t = 3$ s, $\underline{H} = 108(-3\underline{j} + 2\underline{k})$ N·m·s

$|\underline{H}| = 108\sqrt{3^2+2^2} = \underline{389 \text{ N·m·s}}$

$|\underline{M}| = 72\sqrt{3^2+2^2} = \underline{260 \text{ N·m}}$

3/225

$$\mathbf{H}_O = \mathbf{r} \times m\mathbf{v}$$

$$= (a\mathbf{i} + b\mathbf{j} + c\mathbf{k}) \times mv\mathbf{j}$$

$$= \underline{mv(-c\mathbf{i} + a\mathbf{k})}$$

$$\dot{\mathbf{H}}_O = \mathbf{M}_O = (a\mathbf{i} + b\mathbf{j} + c\mathbf{k}) \times F\mathbf{k}$$

$$= \underline{F(b\mathbf{i} - a\mathbf{j})}$$

3/226 Angular momentum about O is conserved:

$H_{O_1} = H_{O_2}$: $3mv(L) + 2mv(L) = 3mL^2\omega$

$$\underline{\omega = \frac{5}{3}\frac{v}{L}}$$

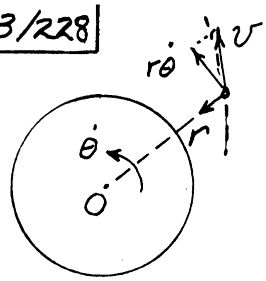
3/227

$$H_1 + \int_{t_1}^{t_2} M\,dt = H_2$$

$$0 + 20(0.1)\,t = 4(3)(0.4)^2\left[150\left(\frac{1}{60}\right)(2\pi)\right]$$

$$\underline{t = 15.08\ \text{s}}$$

3/228



$\Sigma M_O = \dot{H}_O$; $0 = \frac{d}{dt}(mr\dot{\theta} \times r)$

or $\frac{d}{dt}(r^2\dot{\theta}) = 0$

so $r^2\dot{\theta} = const.$

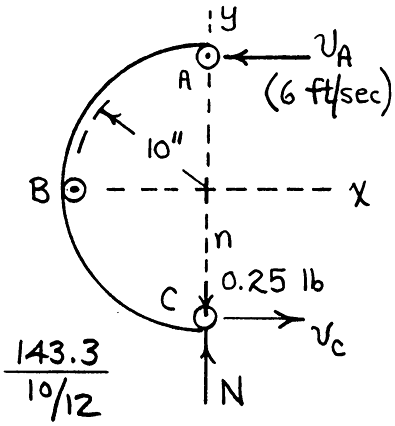
3/229 $T_A + U_{A\text{-}C} = T_C$

$\frac{1}{2} m v_A^2 + mgh_{A\text{-}C} = \frac{1}{2} m v_C^2$

$v_C^2 = v_A^2 + 2g\,h_{A\text{-}C}$

$= 6^2 + 32.2\left(\frac{20}{12}\right)(2)$

$= 143.3 \text{ ft}^2/\text{sec}^2$



$\Sigma F_y = ma_y: \; N - 0.25 = \frac{0.25}{32.2}\,\frac{143.3}{10/12}$

$N = 1.585 \text{ lb}$

$\dot{\underline{H}}_B = \underline{M}_B = (1.585 - 0.25)\,\frac{10}{12}\,\underline{k} = \underline{1.113\,\underline{k} \text{ lb-ft}}$

3/230 Velocity of plug upon impact is

$$v = \sqrt{2gh} = \sqrt{2(9.81)(0.6)} = 3.43 \text{ m/s}$$

For system, $\Delta H_O = 0$. Take C.W. positive

Initial $H_O = -4(0.5)^2(2) - 6(0.3)^2(2) + 2(3.43)(0.5)$

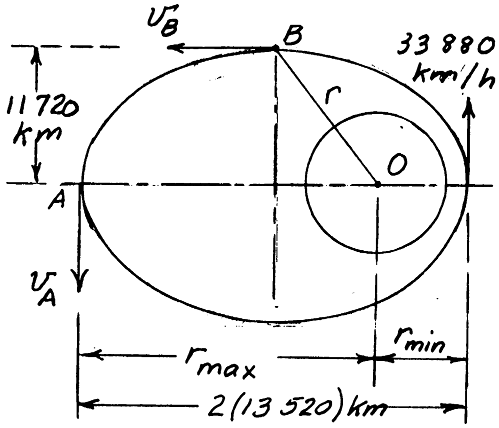
$= -2 - 1.08 + 3.43 = 0.351$ N·m·s

Final $H_O = [(4+2)(0.5)^2 + 6(0.3)^2]\omega$

$= 2.04\omega$

So $0.351 = 2.04\omega$, $\underline{\omega = 0.1721 \text{ rad/s}}$ CW

3/231 $\Sigma M_O = \dot{H}_O = 0$ so H_O = constant



$r_{min} = 6371 + 390 = 6761$ km

$r_{max} = 2(13\,520) - 6761$
$= 20\,279$ km

For H_O constant

$6761(33\,880) = 11720\,v_B$
$= 20\,279\,v_A$

$\underline{v_A = 11\,300 \text{ km/h}}$

$\underline{v_B = 19\,540 \text{ km/h}}$

$\underline{3/232}$ Angular momentum conservation :

$H_A = H_B$: $\quad m v_0 r_0 = m v \left(r_0 - \frac{r_0 z}{h} \right) \cos\theta \quad (1)$

Energy conservation :

$T_A + V_A = T_B + V_B$, datum @ A

$\frac{1}{2} m v_0^2 = \frac{1}{2} m v^2 - mgz \Rightarrow \underline{v = \sqrt{v_0^2 + 2gz}}$ (2)

(2) into (1) yields $\quad \underline{\theta = \cos^{-1} \left[\frac{v_0 h}{(h-z)\sqrt{v_0^2 + 2gz}} \right]}$

$\underline{3/233}$ $\Delta H = 0;\ 2m\, r\omega_0 (r) - 2m(2r)\,\omega(2r) = 0$

$$\underline{\omega = \omega_0/4}$$

$\Delta T = 2\left(\frac{1}{2}m[r\omega_0]^2\right) - 2\left(\frac{1}{2}m\left[2r\frac{\omega_0}{4}\right]^2\right) = mr^2\omega_0^2\,(3/4)$

$n = \Delta T/T = \frac{3}{4}mr^2\omega_0^2 \big/ mr^2\omega_0^2 = \underline{3/4}$

3/234

$$H_1 = H_2$$

$$2mr\dot{\theta}\,(r) = 2mr\cos\beta\,\dot{\theta}'\,r\cos\beta$$

$$\underline{\dot{\theta}' = \dot{\theta}\sec^2\beta}$$

3/235 $\Sigma M_O = \dot{H}_O = 0$, so angular momentum is conserved : $H_{O_1} = H_{O_2}$ (O: any point on axis)

$$0.2\,(0.3\cos 30^\circ)^2\,4 = 0.2(0.2\cos 30^\circ)^2\,\omega$$

$$\underline{\omega = 9 \text{ rad/s}}$$

$$U'_{1-2} = \Delta T + \Delta V_g$$

$$\Delta T = \frac{1}{2}(0.2)\left[(0.2\cos 30^\circ \cdot 9)^2 - (0.3\cos 30^\circ \times 4)^2\right]$$

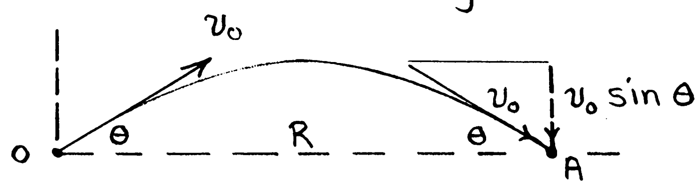
$$= 0.1350 \text{ J}$$

$$\Delta V_g = 0.2\,(9.81)\,(0.1\sin 30^\circ) = 0.0981 \text{ J}$$

So $U'_{1-2} = 0.1350 + 0.0981 = \underline{0.233 \text{ J}}$

3/236 (a) $\underline{H_0 = 0}$ when projectile is at O.

(b) Range $R = \dfrac{2v_0^2 \cos\theta \sin\theta}{g}$

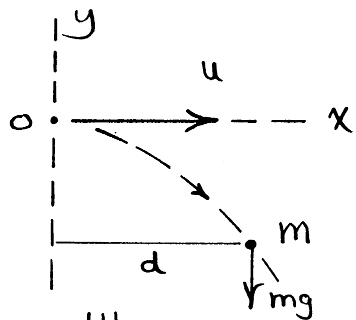


$$H_0 = m v_y R = m v_0 \sin\theta \, \frac{2v_0^2 \cos\theta \sin\theta}{g}$$

$$= \underline{\frac{2 m v_0^3 \sin^2\theta \cos\theta}{g}}$$

The moment of the projectile weight about point O is always increasing the angular momentum about O.

3/237



(Note: $d = ut$)

$$\dot{\underline{H}}_o = \frac{d\underline{H}_o}{dt} = \underline{M}_o = -mgd\,\underline{k}$$

$$\int_0^{\underline{H}_o} d\underline{H}_o = -\int_0^t mgd\,\underline{k}\,dt = -\int_0^t mgut\,\underline{k}\,dt$$

$$\Rightarrow \underline{H}_o = -\tfrac{1}{2}mgut^2\,\underline{k}$$

3/238 Conservation of angular momentum :

$$(\cancel{m} r v_\theta)_A = (\cancel{m} r v_\theta)_B$$

$$50(10^6)(188,500) = 75(10^6)\, v_\theta$$

$$\underline{v_\theta = 125,700 \text{ ft/sec}} \quad @ \ B$$

Energy conservation $T_A + V_A = T_B + V_B$

$$\frac{1}{2}\cancel{m} v_A^2 - \frac{G m_s \cancel{m}}{r_A} = \frac{1}{2}\cancel{m} v_B^2 - \frac{G m_s \cancel{m}}{r_B}$$

$$\frac{1}{2}(188,500)^2 - \frac{1}{2}v_B^2 =$$

$$3.439(10^{-8})(333,000)(4.095)10^{23}\left[\frac{1}{50(10^6)} - \frac{1}{75(10^6)}\right]\frac{1}{5280}$$

$$v_B = 153,900 \text{ ft/sec}$$

$$v_r = \sqrt{v_B^2 - v_\theta^2} = \sqrt{153,900^2 - 125,700^2} = \underline{88,870 \ \frac{\text{ft}}{\text{sec}}}$$

3/239 Forces on particle exert no moment about the central axis, so angular momentum is conserved about this axis. Thus $\Delta H_z = 0$ &

$$m v_0 \cos\beta (r) = m v \cos\theta (r), \quad v_0 \cos\beta = v \cos\theta$$

Also energy is conserved so that

$$\Delta T + \Delta V_g = 0; \quad \frac{1}{2} m v^2 - \frac{1}{2} m v_0^2 - mgh = 0$$

Eliminate v & get $\cos\theta = \dfrac{v_0 \cos\beta}{\sqrt{v_0^2 + 2gh}}$

or $$\theta = \cos^{-1} \frac{\cos\beta}{\sqrt{1 + \dfrac{2gh}{v_0^2}}}$$

3/240 System angular momentum conserved during impact: $\circlearrowleft + \; H_{O_1} = H_{O_2}$:

$$0.050(300)(0.4\cos 20°) - 3.2(0.2)^2 6 - 3.2(0.4)^2 6$$
$$= (0.050 + 3.2)(0.4)^2 \omega' + 3.2(0.2)^2 \omega'$$

$\underline{\omega' = 2.77 \text{ rad/s} \quad (\text{CCW})}$

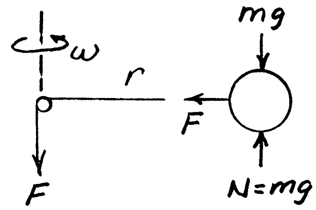
Energy considerations after impact:

$T' + V' = \overset{\nearrow 0}{T} + V$, choose datum @ O:

$$\tfrac{1}{2}(0.05+3.2)[0.4(2.77)]^2 + \tfrac{1}{2}(3.2)[0.2(2.77)]^2$$
$$+ [3.2(0.2) - (3.2+0.05)(0.4)]9.81 = 0 +$$
$$[3.2(0.2) - (3.2+0.05)(0.4)]9.81\cos\theta$$

$\underline{\theta = 52.1°}$

3/241



Angular momentum about central axis is conserved so

$$\dot{H} = \frac{d}{dt}(mr^2\omega) = 0$$

$$m(2r\omega dr + r^2 d\omega) = 0, \quad \underline{\frac{d\omega}{dr} = -\frac{2\omega}{r}}$$

$\Sigma F_n = ma_n$; $F = mr\omega^2$

$dU = dT$; $-F\,dr = d(\frac{1}{2}mr^2\omega^2)$

so $$-mr\omega^2 dr = m(r\omega^2 dr + r^2\omega d\omega)$$
$$= m(r\omega^2 dr + r^2\omega[-\frac{2\omega}{r}dr])$$
$$= -mr\omega^2 dr \quad \text{(check)}$$

3/242 Path form: $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ $\left(\begin{array}{l} a = 5 \text{ ft} \\ b = 4 \text{ ft} \end{array}\right.$

Angular momentum about O is conserved:

$$m r_A v_A = m r_B v_B : \quad v_B = \frac{r_A}{r_B} v_A = \frac{a}{b} v_A$$

$$= \frac{5}{4}(8) = 10 \text{ ft/sec}$$

$$y = b\left[1 - \left(\frac{x}{a}\right)^2\right]^{1/2}$$

$$\frac{dy}{dx} = \frac{1}{2} b \left[1 - \left(\frac{x}{a}\right)^2\right]^{-\frac{1}{2}} \cdot \left(-\frac{2x}{a^2}\right) = -\frac{bx}{a^2}\left[1 - \left(\frac{x}{a}\right)^2\right]^{-\frac{1}{2}}$$

$$\frac{d^2y}{dx^2} = -\frac{b}{a^2}\left[1 - \left(\frac{x}{a}\right)^2\right]^{-\frac{1}{2}} - \frac{bx}{a^2}\left(-\frac{1}{2}\right)\left[1 - \left(\frac{x}{a}\right)^2\right]^{-\frac{3}{2}}\left(-\frac{2x}{a^2}\right)$$

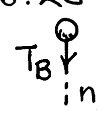
$$= -\frac{b}{a^2}\left[1 - \left(\frac{x}{a}\right)^2\right]^{-\frac{1}{2}} - \frac{bx^2}{a^4}\left[1 - \left(\frac{x}{a}\right)^2\right]^{-\frac{3}{2}}$$

Now, $\left.\dfrac{dy}{dx}\right|_{x=0} = 0$ and $\left.\dfrac{d^2y}{dx^2}\right|_{x=0} = -\dfrac{b}{a^2}$

$$= -\frac{4}{5^2} = -\frac{4}{25}$$

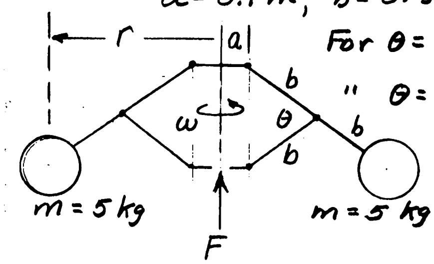
$$\rho_{xy} = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{d^2y/dx^2} = \frac{\left[1+0\right]^{3/2}}{-4/25} = -6.25'$$

So $\rho = 6.25'$; $\Sigma F_n = m\dfrac{v^2}{\rho}$: $T_B = \dfrac{1.5}{32.2}\,\dfrac{10^2}{6.25} = \underline{0.745 \text{ lb}}$



▶ 3/243 $\omega_0 = 40(2\pi)/60 = 4.19$ rad/s

$a = 0.1$ m, $b = 0.3$ m



For $\theta = 90°$, $r_0 = 0.1 + 2(0.3)\cos 45° = 0.524$ m

" $\theta = 60°$, $r = 0.1 + 2(0.3)\cos 30° = 0.620$ m

$\Delta H = 0;\ 2m r_0^2 \omega_0 - 2m r^2 \omega = 0$

$\omega = \frac{r_0^2}{r^2}\omega_0 = \left(\frac{0.524}{0.620}\right)^2 (4.19)$

$= \underline{3.00 \text{ rad/s}}$

$\left(\text{or } \frac{3.00}{2\pi} 60 = 28.6 \text{ rev/min}\right)$

$U = \Delta T + \Delta V_g \ = 2\left(\frac{1}{2}m\right)\left(r^2\omega^2 - r_0^2\omega_0^2\right) + 2mg\,\Delta h$

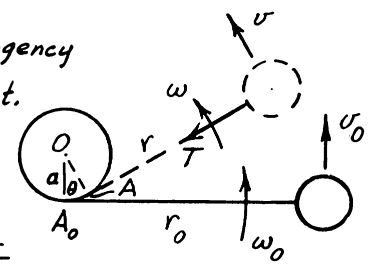
where $\Delta h = 2b\,(\sin 45° - \sin 30°)$

$= 2(0.3)(0.7071 - 0.5) = 0.1243$ m

$U = 5\left([0.620 \times 3.00]^2 - [0.524 \times 4.19]^2\right) + 2(5)(9.81)(0.1243)$

$= -6.850 + 12.190 \ = \underline{5.34 \text{ J}}$

▶3/244 Moment of tension T about tangency point A is zero, but A is not a fixed point. And moment about fixed point O is not zero. Thus angular momentum is not conserved. Also $\Sigma F \neq 0$ so linear momentum is not conserved.



Energy is conserved so $\frac{1}{2}m(r_0\omega_0)^2 = \frac{1}{2}m(r\omega)^2$.

Thus

$$\omega = \frac{r_0}{r}\omega_0 = \frac{r_0}{r_0 - a\theta}\omega_0 \quad \text{or} \quad \underline{\omega = \frac{\omega_0}{1 - a\theta/r_0}}$$

$\circlearrowleft \Sigma M_O = \dot{H}_O$: $-Ta = \frac{d}{dt}(mvr) = mv\dot{r}$ since $v = \text{const.}$

But $\dot{r} = \frac{d}{dt}(r_0 - a\theta) = -a\dot{\theta} = -a\omega$

so $T = -\frac{m}{a}v(-a\omega) = mv\omega$

or $\underline{T = mr_0\omega_0\omega}$ since $v = v_0 = r_0\omega_0$

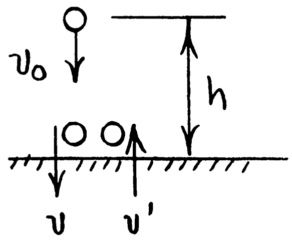
$\boxed{3/245}$ $v = \sqrt{2gh}$, $v' = \sqrt{2gh'}$

$$e = \frac{v'}{v} = \sqrt{\frac{h'}{h}} = \sqrt{\frac{1100}{1600}} = \underline{0.829}$$

$$n = \frac{mgh - mgh'}{mgh}(100\%) = \left(1 - \frac{h'}{h}\right)100\%$$

$$n = \left(1 - \frac{1100}{1600}\right)(100\%) = \underline{31.2\ \%}$$

3/246



$$v' = \sqrt{2gh} \quad , \quad v = \sqrt{v_0^2 + 2gh}$$

$$e = \frac{v'}{v} = \frac{\sqrt{2gh}}{\sqrt{v_0^2 + 2gh}} = 0.8$$

With $h = 1.6$ m, $\underline{v_0 = 4.20 \text{ m/s}}$

3/247 System linear momentum :

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2' \qquad \xrightarrow{} +$$

$$2(7) + 3(-5) = 2v_1' + 3v_2' \qquad (1)$$

Restitution : $e = \dfrac{v_2' - v_1'}{v_1 - v_2}$: $0.6 = \dfrac{v_2' - v_1'}{7-(-5)}$ (2)

Solve (1) & (2) to obtain

$$v_1' = -4.52 \text{ m/s}$$

$$v_2' = 2.68 \text{ m/s}$$

3/248 Conservation of linear momentum:

$$m_1 v_1 + m_2 \cancelto{0}{v_2} = m_1 \cancelto{0}{v_1'} + m_2 v_2'$$

$$v_2' = \frac{m_1}{m_2} v_1$$

Restitution :
$$e = \frac{v_2' - \cancelto{0}{v_1'}}{v_1 - \cancelto{0}{v_2}} = \frac{v_2'}{v_1} = \frac{\frac{m_1}{m_2} v_1}{v_1}$$

$$\underline{\frac{m_1}{m_2} = e}$$

3/249 System linear momentum :

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B' \qquad \xrightarrow{+}$$

$$\cancel{m} v + 0 = \cancel{m} v_A' + p \cancel{m} v_B' \qquad (1)$$

Restitution : $e = \dfrac{v_B' - v_A'}{v_A - v_B}$: $\quad 0.1 = \dfrac{v_B' - v_A'}{v - 0} \quad (2)$

Solve (1) & (2) to obtain

$$\underline{v_A' = \left(\frac{1 - 0.1p}{1+p}\right) v}, \qquad \underline{v_B' = \frac{1.1}{1+p} v}$$

For $p = \frac{1}{2}$: $\underline{v_A' = 0.633 v}$, $\underline{v_B' = 0.733 v}$

3/250 $\Delta G = 0; \quad m_A v_A + 0 = m_A v_A' + m_B v_B'$

$e = 0; \quad v_A' = v_B'$

Thus $\quad m_A v_A = (m_A + m_B) v_A'$

$$|\Delta T| = -\frac{1}{2} m_A v_A'^2 - \frac{1}{2} m_B v_B'^2 + \frac{1}{2} m_A v_A^2$$

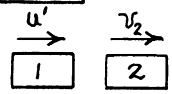
$$= -\frac{1}{2} m_A \left(\frac{m_A}{m_A + m_B} v_A\right)^2 - \frac{1}{2} m_B \left(\frac{m_A}{m_A + m_B} v_A\right)^2 + \frac{1}{2} m_A v_A^2$$

$$= -\frac{1}{2}\left(\frac{m_A}{m_A + m_B} v_A\right)^2 (m_A + m_B) + \frac{1}{2} m_A v_A^2$$

$$= \frac{1}{2} \frac{m_A m_B}{m_A + m_B} v_A^2 \quad (\text{loss})$$

$$\frac{|\Delta T|}{T} = \frac{1}{2} \frac{m_A m_B}{m_A + m_B} v_A^2 \frac{1}{\frac{1}{2} m_A v_A^2} = \underline{\frac{m_B}{m_A + m_B}}$$

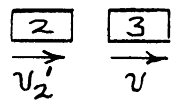
3/251



$$\left.\begin{aligned} \Delta G = 0 &: \quad mu = m(u' + v_2) \\ \text{Also} &: \quad e = \frac{v_2 - u'}{u} \end{aligned}\right\}$$

(After impact)

Solve to obtain $v_2 = \frac{u}{2}(1+e)$



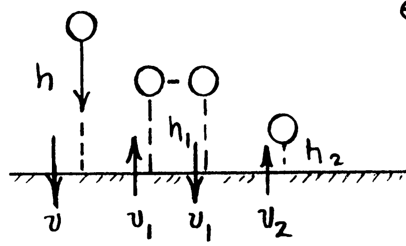
$$\left.\begin{aligned} \Delta G = 0 &: \quad m\frac{u}{2}(1+e) = m(v_2' + v) \\ \text{Also} &: \quad e = \frac{v - v_2'}{\frac{u}{2}(1+e)} \end{aligned}\right\}$$

Solve for $\underline{v = \frac{u}{4}(1+e)^2}$

3/252

$v = \sqrt{2gh}$, $v_1 = \sqrt{2gh_1}$, $v_2 = \sqrt{2gh_2}$

$$e = \frac{v_1}{v} \quad \text{and} \quad e = \frac{v_2}{v_1}$$

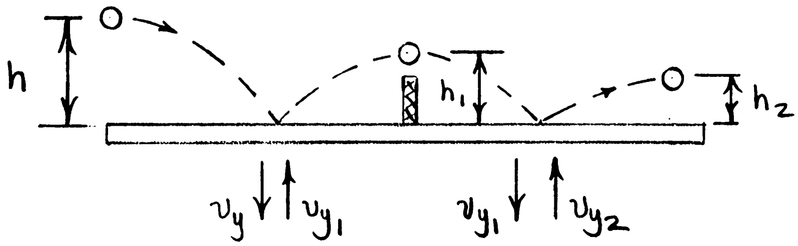


So $v_1{}^2 = v v_2$

and $e = \dfrac{\sqrt{v v_2}}{v} = \sqrt{\dfrac{v_2}{v}}$

$$= \sqrt{\frac{\sqrt{2gh_2}}{\sqrt{2gh}}} = \underline{\left(\frac{h_2}{h}\right)^{1/4}}$$

3/253



(Note: Drop distances reduced by $r = 0.75''$)

$v_y = \sqrt{2g(h-r)}$, $v_{y_1} = e v_y = e\sqrt{2g(h-r)}$

$= \sqrt{2g(h_1 - r)}$

$\therefore \; 0.90^2(2g)(h-0.75) = (2g)(9-0.75)$, $\underline{h = 10.94 \text{ in.}}$

$v_{y_2} = e v_{y_1}$: $\sqrt{2g(h_2 - r)} = 0.9\sqrt{2g(h_1 - r)}$

With $r = 0.75$ in. & $h_1 = 9$ in.: $\underline{h_2 = 7.43 \text{ in.}}$

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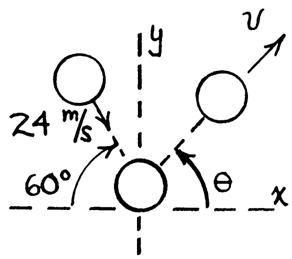


During impact $\Sigma F_x = 0$ so no change in x velocity component.

$v \cos\theta = 24(0.5) = 12$ m/s

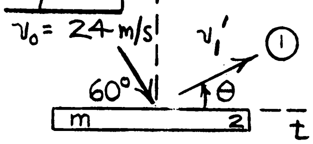
In y-dir., $e = \dfrac{v \sin\theta}{24 \cos 30°} = 0.8$

$\tan\theta = \dfrac{16.63}{12} = 1.386$

$\underline{\theta = 54.2°}$

$v = \dfrac{12}{\cos 54.2°} = \underline{20.5 \text{ m/s}}$

3/2.55



t- momentum conserved:

Ball: $m_1 v_{1t} = m_1 v_{1t}'$

$v_{1t}' = v_{1t} = 24 \cos 60° = 12$ m/s

Plate: $m_2 v_{2t} = m_2 v_{2t}'$, $v_{2t}' = v_{2t} = 0$

n- momentum:

$$\cancel{m}_1 v_{1n} + \cancel{m}_2 v_{2n} = \cancel{m}_1 v_{1n}' + \cancel{m}_2 v_{2n}'$$

$$-24 \sin 60° = v_{1n}' + v_{2n}'$$

Restitution: $e = \dfrac{v_{2n}' - v_{1n}'}{v_{1n} - v_{2n}}$, $0.8 = \dfrac{v_{2n}' - v_{1n}'}{-24 \sin 60° - 0}$

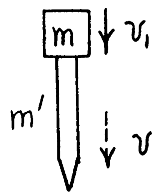
Solve to find $v_{1n}' = -2.08$ m/s, $\underline{v_{2n}' = -18.71 \text{ m/s}}$

$v_1' = \sqrt{v_{1t}'^2 + v_{1n}'^2} = \underline{12.20 \text{ m/s}}$, $\theta = \tan^{-1}\left(\dfrac{v_{1n}'}{v_{1t}'}\right) = \underline{-9.83°}$

3/256 Final velocity of m is to be zero

so for $\Delta G = 0$, $m v_1 + 0 = 300 v + 0$

$e = \dfrac{v - 0}{v_1 - 0}$, so $0.3 = \dfrac{m}{300}$, $\underline{m = 90 \text{ kg}}$



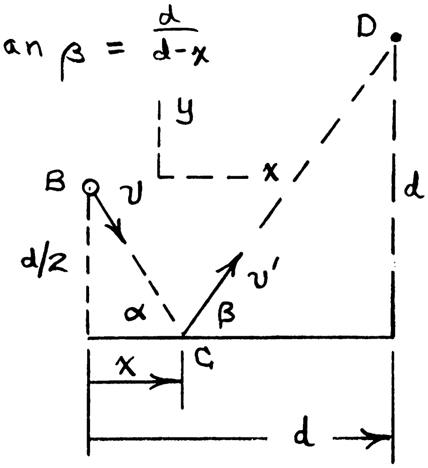
Thus for $v_1 = \sqrt{2(9.81)(4)} = 8.86 \text{ m/s}$

$v = e v_1 = 0.3(8.86) = \underline{2.66 \text{ m/s}}$

$\Delta E = mgh - \frac{1}{2} m' v^2$

$= 90(9.81)(4) - \frac{1}{2}(300)(2.66)^2 = \underline{2470 \text{ J}}$

$\boxed{3/257}$ $\tan\alpha = \dfrac{d/2}{x}$, $\tan\beta = \dfrac{d}{d-x}$



(a) $e = 1$: $\alpha = \beta$

$$\frac{d/2}{x} = \frac{d}{d-x}, \quad \underline{x = \frac{d}{3}}$$

(b) $e = 0.8$: $\alpha \neq \beta$

$$\tan\alpha = \frac{|v_y|}{v_x},$$

$$\tan\beta = \frac{v_y'}{v_x'} = \frac{0.8|v_y|}{v_x}$$

$$\Rightarrow \quad \tan\alpha = \frac{1}{0.8}\tan\beta$$

So $\quad \dfrac{d/2}{x} = \dfrac{1}{0.8}\dfrac{d}{d-x}$, $\quad \underline{x = 0.286d}$

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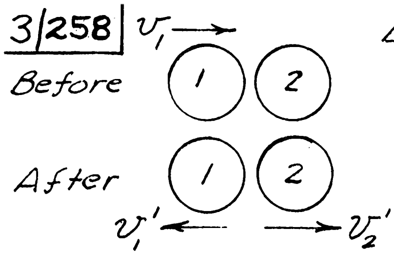


$\Delta G = 0; \; mv_1 = -mv_1' + mv_2'$

$v_2' = v_1 + v_1'$

$e = \dfrac{v_2' + v_1'}{v_1}, \quad v_1' = ev_1 - v_2'$

Combine & get

$v_2' = v_1 + ev_1 - v_2'$

or $v_2' = \dfrac{1+e}{2} v_1$

It follows that $v_3' = \dfrac{1+e}{2} v_2' = \left(\dfrac{1+e}{2}\right)^2 v_1$

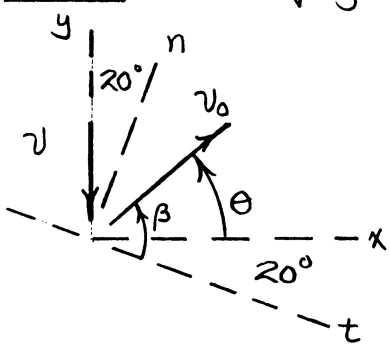
$v_4' = \dfrac{1+e}{2} v_3' = \left(\dfrac{1+e}{2}\right)^3 v_1$

$\vdots \qquad\qquad \vdots$

so $\underline{v_n = \left(\dfrac{1+e}{2}\right)^{n-1} v_1}$

3/259

$v = \sqrt{2gh} = \sqrt{2(9.81)(0.75)} = 3.84$ m/s



$v_{0t} = 3.84 \sin 20° = 1.312$ m/s

$v_{0n} = (3.84 \cos 20°)(0.85)$

$= 3.06$ m/s

$v_0 = \sqrt{v_{0t}^2 + v_{0n}^2} = 3.33$ m/s

$\beta = \tan^{-1} \dfrac{v_{0n}}{v_{0t}} = 66.8°$

$\theta = \beta - 20° = 46.8°$

$x = x_0 + v_{x_0} t$: $R \cos 20° = 0 + 3.33 \cos 46.8° \, t$ (1)

$y = y_0 + v_{y_0} t - \frac{1}{2} g t^2$: $-R \sin 20° = 0 + 3.33 \sin 46.8° t$

$- \dfrac{9.81}{2} t^2$ (2)

Solve (1) for t & substitute into (2) to obtain

$\underline{R = 1.613 \text{ m}}$

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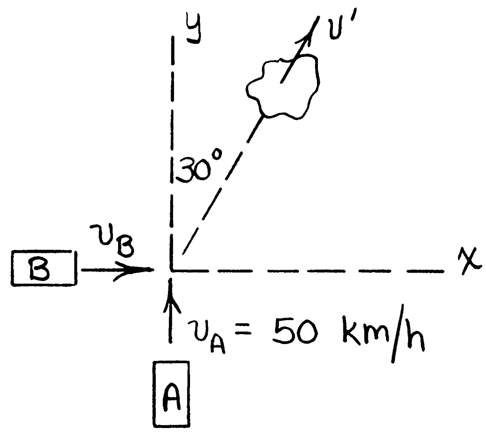

$G_{1x} = G_{2x}$: $m_B v_B + 0 = (m_A + m_B) v' \sin 30°$

$1600\, v_B = 2800 v' (\frac{1}{2})$ (1)

$G_{1y} = G_{2y}$: $m_A v_A + 0 = (m_A + m_B) v' \cos 30°$

$1200 (50) = 2800 v' (0.866)$ (2)

From (2): $v' = 24.7$ km/h

From (1): $\underline{v_B = 21.7 \text{ km/h}}$

3/261 Let v_s and v_b stand for rebound velocities from steel and brass plates.

Impact speed $= \sqrt{2gh} = \sqrt{2(9.81)(0.15)} = 1.716$ m/s

$$\left.\begin{aligned} 0.6 &= \frac{v_s}{1.716}, \quad v_s = 1.029 \text{ m/s} \\ 0.4 &= \frac{v_s}{1.716}, \quad v_b = 0.686 \text{ m/s} \end{aligned}\right\} \quad \begin{aligned} \omega &= \frac{1.029 - 0.686}{0.60} \\ &= 0.572 \text{ rad/s} \\ &\qquad \underline{\text{CCW}} \end{aligned}$$

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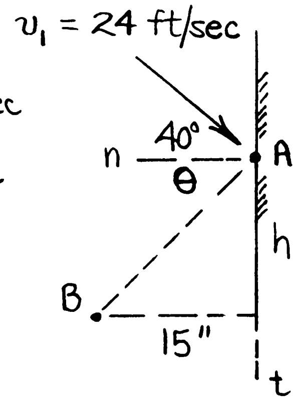


$(v_1)_t = 24 \sin 40° = 15.43$ ft/sec

$(v_1)_n = -24 \cos 40° = -18.39$ ft/sec

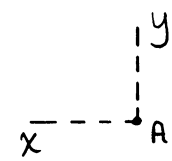
$(v_1')_t = (v_1)_t = 15.43$ ft/sec

$(v_1')_n = -e(v_1)_n = -0.84(-18.39)$

$= +15.44$ ft/sec

(a)

$$\tan\theta = \frac{(v_1')_t}{(v_1')_n} = \frac{h}{15} \Rightarrow \underline{h = 14.98 \text{ in.}}$$



(b) Projectile motion from A to B:

$x_B = x_A + (v_1')_n t: \quad \frac{15}{12} = 0 + 15.44t$

$t = 0.0809$ sec

$y_B = y_A + (-v_1')_t t - \frac{1}{2}gt^2:$

$-h = 0 + (-15.43)(0.0809) - \frac{1}{2}(32.2)(0.0809)^2$

$h = 1.354$ ft or $\underline{h = 16.25 \text{ in.}}$

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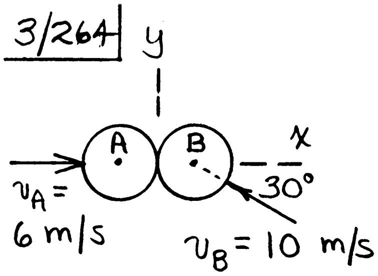


$$v_{A_y}' = v_{A_y} = 0$$

$$v_{B_y}' = v_{B_y} = 10 \sin 30^\circ = 5 \text{ m/s}$$

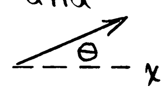
$$m_A v_{A_x} + m_B v_{B_x} = m_A v_{A_x}' + m_B v_{B_x}'$$

$$6 - 10\cos 30^\circ = v_{A_x}' + v_{B_x}' \qquad (1)$$

$$e = \frac{v_{B_x}' - v_{A_x}'}{v_{A_x} - v_{B_x}} : \quad 0.75 = \frac{v_{B_x}' - v_{A_x}'}{6 - (-10\cos 30^\circ)} \qquad (2)$$

Solve Eqs. (1) & (2) : $\begin{cases} v_{A_x}' = -6.83 \text{ m/s} \\ v_{B_x}' = 4.17 \text{ m/s} \end{cases}$

Magnitudes and directions



$$\underline{v_A' = 6.83 \tfrac{m}{s} @ \theta_A = 180^\circ}$$

$$\underline{v_B' = 6.51 \tfrac{m}{s} @ \theta_B = 50.2^\circ}$$

Initial: $T_1 = \frac{1}{2} m (6^2 + 10^2) = 68 m$

Final : $T_2 = \frac{1}{2} m (6.83^2 + 6.51^2) = 44.5 m$

$$n = \frac{68 - 44.5}{68}(100\%) = \underline{34.6\%}$$

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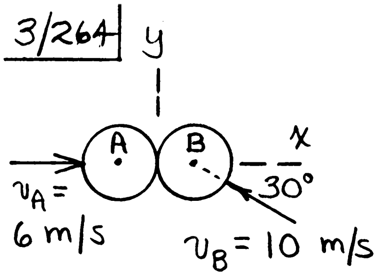


$$v_{A_y}' = v_{A_y} = 0$$

$$v_{B_y}' = v_{B_y} = 10 \sin 30° = 5 \text{ m/s}$$

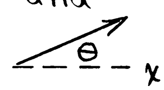
$$m_A v_{A_x} + m_B v_{B_x} = m_A v_{A_x}' + m_B v_{B_x}'$$

$$6 - 10 \cos 30° = v_{A_x}' + v_{B_x}' \qquad (1)$$

$$e = \frac{v_{B_x}' - v_{A_x}'}{v_{A_x} - v_{B_x}} : \quad 0.75 = \frac{v_{B_x}' - v_{A_x}'}{6 - (-10 \cos 30°)} \qquad (2)$$

Solve Eqs. (1) & (2) : $\begin{cases} v_{A_x}' = -6.83 \text{ m/s} \\ v_{B_x}' = 4.17 \text{ m/s} \end{cases}$

Magnitudes and directions



$$\underline{v_A' = 6.83 \tfrac{m}{s} @ \theta_A = 180°}$$

$$\underline{v_B' = 6.51 \tfrac{m}{s} @ \theta_B = 50.2°}$$

Initial: $T_1 = \frac{1}{2} m (6^2 + 10^2) = 68 m$

Final : $T_2 = \frac{1}{2} m (6.83^2 + 6.51^2) = 44.5 m$

$$n = \frac{68 - 44.5}{68} (100\%) = \underline{34.6\%}$$

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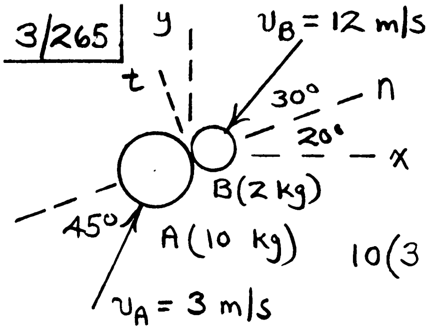


$v_{A_t}' = v_{A_t} = 3 \sin 45° = 2.12 \frac{m}{s}$

$v_{B_t}' = v_{B_t} = -12 \sin 30° = -6 \frac{m}{s}$

$m_A v_{A_n} + m_B v_{B_n} = m_A v_{A_n}' + m_B v_{B_n}'$:

$10(3 \cos 45°) + 2(-12 \cos 30°) = 10 v_{A_n}' + 2 v_{B_n}'$ (1)

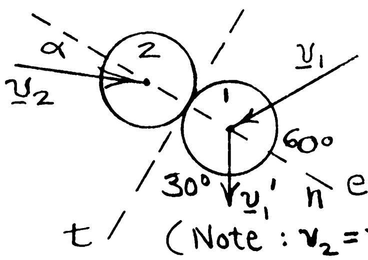
$e = \dfrac{v_{B_n}' - v_{A_n}'}{v_{A_n} - v_{B_n}}$: $0.5 = \dfrac{v_{B_n}' - v_{A_n}'}{3 \cos 45° - (-12 \cos 30°)}$ (2)

Solve (1) & (2) : $v_{A_n}' = -1.007$ m/s , $v_{B_n}' = 5.25$ m/s

Then
$$\begin{cases} v_{A_x}' = -2.12 \sin 20° - 1.007 \cos 20° = -1.672 \text{ m/s} \\ v_{A_y}' = 2.12 \cos 20° - 1.007 \sin 20° = 1.649 \text{ m/s} \\ v_{B_x}' = -(-6 \sin 20°) + 5.25 \cos 20° = 6.49 \text{ m/s} \\ v_{B_y}' = -6 \cos 20° + 5.25 \sin 20° = -3.84 \text{ m/s} \end{cases}$$

3/266 Conservation of n-momentum:

$$m(-v_1 \cos 60°) + m(v_2 \cos\alpha) = m v_{1n}' + m v_{2n}' \qquad (a)$$



Restitution:

$$e = 0.8 = \frac{v_{2n}' - v_{1n}'}{-v_1 \cos 60° - v_2 \cos\alpha} \qquad (b)$$

(Note: $v_2 = v_1$)

Simultaneous solution of Eqs. (a) and (b):

$$v_{1n}' = v_1 [0.9 \cos\alpha - 0.05]$$

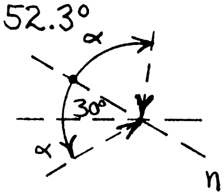
$$v_{1t}' = v_{1t} = v_1 \sin 60° = \frac{3}{2} v_1$$

$$\tan 30° = \frac{v_{1n}'}{v_{1t}'} = \frac{v_1 [0.9 \cos\alpha - 0.05]}{\frac{\sqrt{3}}{2} v_1}$$

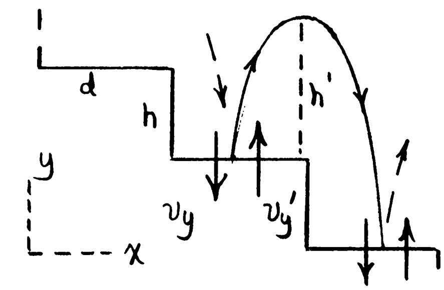
Solving, $\cos\alpha = 0.611$, $\alpha = \pm 52.3°$

So $\theta = 30° + 52.3° = \underline{82.3°}$

or $\theta = 30° - 52.3° = \underline{-22.3°}$



3/267 $\quad v_y = \sqrt{2g(h'+h)}, \quad v_y' = e v_y$



$$= e\sqrt{2g(h'+h)} = \sqrt{2gh'}$$

So $e^2 \cancel{2g}(h'+h) = \cancel{2g}h'$

$$e = \sqrt{\frac{h'}{h'+h}}$$

From y-displacement equation $y = y_0 + v_{y_0}t - \frac{1}{2}gt^2$:

$$-h = \sqrt{2gh'}\,t - \frac{1}{2}gt^2$$

Solve for $t = \sqrt{\frac{2}{g}}\left[\sqrt{h'} \pm \sqrt{h'+h}\right]$

Choose (+) sign, as (−) sign yields $t < 0$.

x-displacement : $x = x_0 + v_{x_0}t$: $d = v_{x_0}t$

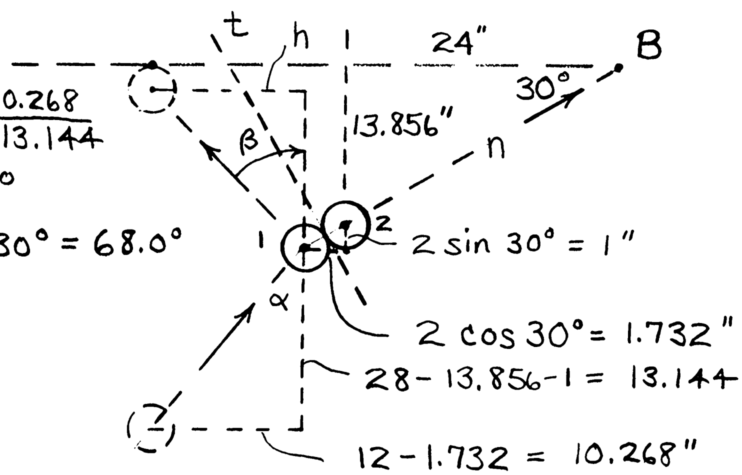
So $v_{x_0} = \frac{d}{t} = \dfrac{\sqrt{\frac{g}{2}}\,d}{\sqrt{h'} + \sqrt{h'+h}}$

3/268

$\alpha = \tan^{-1}\dfrac{10.268}{13.144}$

$= 38.0°$

$\theta_1 = \alpha + 30° = 68.0°$



Mom. : $\cancel{m}_1(v_1)_n + \cancel{m}_2(v_2)_n = \cancel{m}_1(v_1')_n + \cancel{m}_2(v_2')_n$

$v_1 \sin 68.0° = (v_1')_n + (v_2')_n$

Restitution : $e = \dfrac{(v_2')_n - (v_1')_n}{(v_1)_n - (v_2)_n}$

$0.9 = \dfrac{(v_2')_n - (v_1')_n}{v_1 \sin 68.0° - 0}$

Solving, $(v_1')_n = 0.0464 v_1$

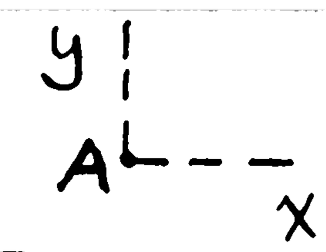
Also, $(v_1')_t = (v_1)_t = v_1 \cos 68.0° = 0.375 v_1$

$\tan\theta_1' = \dfrac{(v_1')_n}{(v_1')_t} = \dfrac{0.0464 v_1}{0.375 v_1}$, $\theta_1' = 7.05°$

$\beta = 30° - \theta_1' = 22.95°$, $\tan\beta = \dfrac{h}{13.856 - 1 + 1} = 0.423$

$h = 5.87"$. Then $x = 24 - 1.732 - 5.87 = \underline{16.40 \text{ in.}}$

▶3/269



$v_{xA} = 50\cos\alpha, \quad v_{yA} = 50\sin\alpha$

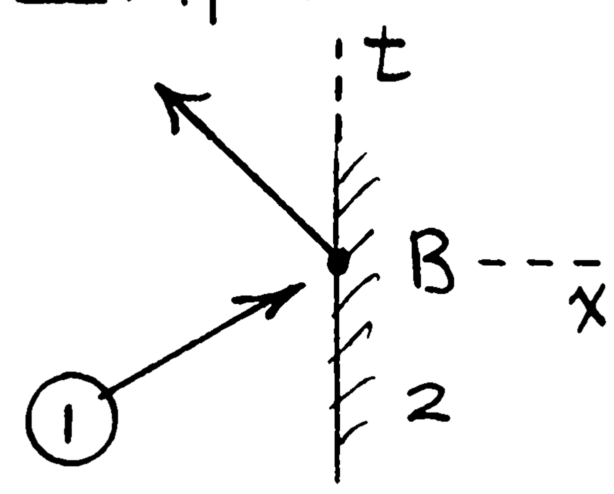
$$t_{AB} = \frac{10}{v_{xA}} = \frac{10}{50\cos\alpha} = \frac{1}{5\cos\alpha}$$

$$v_{xB} = v_{xA} = 50\cos\alpha$$

$$v_{yB} = v_{yA} - gt = 50\sin\alpha - \frac{g}{5\cos\alpha}$$

$$y_B = y_A + v_{yA}t - \tfrac{1}{2}gt^2 = 0 + (50\sin\alpha)\left(\frac{1}{5\cos\alpha}\right) - \frac{g}{2}\left(\frac{1}{25\cos^2\alpha}\right) = 10\tan\alpha - \frac{g}{50\cos^2\alpha}$$

Impact at B:



$$v_{yB}' = v_{yB} = 50\sin\alpha - \frac{g}{5\cos\alpha}$$

$$e = \frac{v_{2x}' - v_{1x}'}{v_{1x} - v_{2x}} = \frac{0 - v_{1x}'}{50\cos\alpha - 0} = 0.5$$

$$v_{1x}' = -25\cos\alpha$$

$$t_{BA} = \frac{10}{25\cos\alpha} = \frac{2}{5\cos\alpha}$$

$$y_A = y_B + v_{yB}'t - \tfrac{1}{2}gt^2$$

$$0 = \left(10\tan\alpha - \frac{g}{50\cos^2\alpha}\right) + \left(50\sin\alpha - \frac{g}{5\cos\alpha}\right)\left(\frac{2}{5\cos\alpha}\right) - \frac{g}{2}\left(\frac{2}{5\cos\alpha}\right)^2$$

Collect terms:

$$30\tan\alpha - \frac{9g}{50}\,\frac{1}{\cos^2\alpha} = 0$$

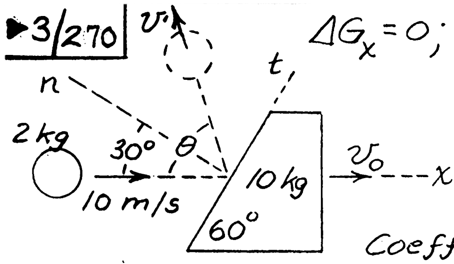
Use $\frac{1}{\cos^2\alpha} = (\tan^2\alpha + 1)$ to obtain

$$5.796\tan^2\alpha - 30\tan\alpha + 5.796 = 0$$

Quadratic solution: $\tan\alpha = 0.201,\ 4.97$

$$\Rightarrow \quad \underline{\alpha = 11.37^\circ \text{ or } 78.6^\circ}$$

▶3/270



$\Delta G_x = 0$; $2(10) + 0 = -2v'\cos\theta + 10v_o$ (For system) (1)

$\Delta G_t = 0$ (for sphere)

$2(10 \sin 30°) = 2v'\sin(\theta - 30°)$ (2)

Coefficient of restitution applies to velocity components in n-dir.

$$0.6 = \frac{v_o \sin 60° + v'\cos(\theta - 30°)}{10\cos 30° + 0} \quad (3)$$

Eq.(3) is $5.196 = 0.866\, v_o + 0.866\, v'\cos\theta + 0.5v'\sin\theta$

Sub. Eq.(1) to eliminate v_o & get

$5.196 = 0.866(2 + 0.2\, v'\cos\theta) + 0.866\, v'\cos\theta + 0.5\, v'\sin\theta$

or $1.039\, v'\cos\theta + 0.5\, v'\sin\theta = 3.464$ (4)

Eq.(2) becomes $0.866\, v'\sin\theta - 0.5\, v'\cos\theta = 5$ (5)

Solve (4) & (5) & get $\underline{v' = 6.04 \text{ m/s}}$, $\underline{\theta = 85.9°}$

From Eq. (1) $v_o = 2.087$ m/s

For carriage $\Delta T + \Delta V_e = 0$; $-\frac{1}{2}10(2.087)^2 + \frac{1}{2}1600\,\delta^2 = 0$

$\delta^2 = 0.02722$, $\delta = 0.1650$ m or $\underline{\delta = 165.0 \text{ mm}}$

3/271

$$v = \sqrt{\frac{Gm_s}{r}} = \sqrt{\frac{(3.439\times10^{-8})(333{,}000)(4.095\times10^{23})}{(93\times10^{6})(5280)}}$$

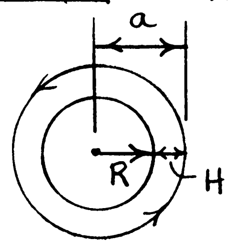
$$= 97{,}725 \text{ ft/sec} = \underline{18.51 \text{ mi/sec}}$$

3/272 For a circular orbit, $r_{min} = r_{max}$ and $a = R + h$, so Eq. 3/44 becomes

$$v = R\sqrt{\frac{g}{R+h}} = 6371(10^3)\sqrt{\frac{9.825}{(6371+590)10^3}}$$

$$= \underline{7569 \text{ m/s}} \text{ or } \underline{27\,250 \text{ km/h}}$$

3/273



Eq. 3/43 with $r = a = R + H$:

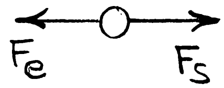
$$v^2 = 2gR^2\left(\frac{1}{a} - \frac{1}{2a}\right) = \frac{gR^2}{R+H}$$

$$= \frac{1.62\,(3476/2)^2}{\frac{3476}{2} + 80}(1000)$$

$\underline{v = 1641 \text{ m/s}}$ or $\underline{5910 \text{ km/h}}$

3/274

Moon m



$$\frac{F_s}{F_e} = \frac{G m_s m / d_{m-s}^2}{G m_e m / d_{m-e}^2}$$

$$= \left(\frac{d_{m-e}}{d_{m-s}}\right)^2 \frac{m_s}{m_e}$$

$$= \left(\frac{384\,398}{149.6(10^6) - 384\,398}\right)^2 333\,000 = 2.21$$

Therefore, the acceleration of the moon is toward the sun, and thus the path is concave toward the sun!

3/275 From Eq. 3/43 for a circular orbit of altitude H & with $a=r=R+H$

$$v^2 = 2gR^2\left(\frac{1}{r}-\frac{1}{2r}\right) = gR^2/(R+H), \quad v = R\sqrt{g/(R+H)}$$

$$v^2_{escape} = 2gR^2\left(\frac{1}{r}-\frac{1}{\infty}\right) = 2gR^2/(R+H), \quad v = R\sqrt{2g/(R+H)}$$

$$\Delta v = v_{escape} - v = R\sqrt{\frac{g}{R+H}}\,(\sqrt{2}-1) = (\sqrt{2}-1)v = 0.414\,v$$

For $R = 3959$ mi, $g = 32.23$ ft/sec^2, $H = 200$ mi,

$$v = 3959\sqrt{\frac{32.23/5280}{3959+200}} = 4.80 \text{ mi/sec}, \quad \Delta v = 0.414(4.80) = \underline{1.987 \frac{mi}{sec}}$$

3/276 $r_{min} = 6371 + 240 = 6611$ km

$r_{max} = 6371 + 400 = 6771$ km

From Eq. 3/39 $\dfrac{r_{min}}{r_{max}} = \dfrac{1-e}{1+e}$

So $(1+e)6611 = (1-e)6771$, $\underline{e = 0.01196}$

From Eq. 3/40 with $a = \frac{1}{2}(r_{max} + r_{min}) = 6691$ km

$$\tau = 2\pi \frac{(6691 \times 10^3)^{3/2}}{(6371 \times 10^3)\sqrt{9.824}} = \underline{5446 \text{ s}} \text{ or}$$

$$\underline{\tau = 1 \text{ h } 30 \text{ min } 46 \text{ s}}$$

3/277 From Appendix D, for the moon

$R = 1080 \text{ mi}$, $g = 5.32 \text{ ft/sec}^2$

so $r_{max} = 1080 + 180 = 1260 \text{ mi}$

$r_{min} = 1080 + 60 = 1140 \text{ mi}$

$a = (r_{max} + r_{min})/2 = (1260 + 1140)/2 = 1200 \text{ mi}$

From Eq. 3/44

Max. vel. is $v_p = R\sqrt{\frac{g}{a}}\sqrt{\frac{r_{max}}{r_{min}}}$

so $v_p = (1080)\sqrt{\frac{5.32}{5280}\frac{1}{1200}}\sqrt{\frac{1260}{1140}} = 1.0404 \text{ mi/sec}$

or $v_p = 1.0404(3600) = \underline{3745 \text{ mi/hr}}$

$\boxed{3/278}$ $r_{min} = 2R, \quad r_{max} = 3R$

$$a = \frac{r_{min} + r_{max}}{2} = 2.5R$$

$$v_p = R\sqrt{\frac{g}{a}}\sqrt{\frac{r_{max}}{r_{min}}} = R\sqrt{\frac{g}{2.5R}}\sqrt{\frac{3R}{2R}} = \sqrt{\frac{3gR}{5}}$$

The velocity in the original circular orbit is

$$v_c = R\sqrt{\frac{g}{a}} = R\sqrt{\frac{g}{2R}} = \sqrt{\tfrac{1}{2}gR}$$

$$\Delta v = v_p - v_c = \sqrt{gR}\left(\sqrt{\tfrac{3}{5}} - \sqrt{\tfrac{1}{2}}\right) = 0.0675\sqrt{gR}$$

Numbers : $\Delta v = 0.0675\sqrt{9.825(6371)(1000)}$

$= \underline{534 \text{ m/s}}$

(Δv to occur opposite B)

$\underline{3/279}$ $r = a = 6371 + 300 = 6671 \text{ km} = 6.671(10^6) \text{ m}$

$$\tau = \frac{2\pi a^{3/2}}{R\sqrt{g}} = \frac{2\pi\,(6.671 \times 10^6)^{3/2}}{6.371 \times 10^6 \sqrt{9.825}}$$

$$= 5421 \text{ s}$$

Speed of ground point on equator

$v_e = R_e \omega_e = (6378)(7.292 \times 10^{-5}) = 0.4651 \text{ km/s}$

Required distance $d = v_e \tau = (0.4651)(5421)$

$$= \underline{2520 \text{ km}}$$

3/280 (a) $v = R\sqrt{\frac{g}{r}} = 6371(1000)\sqrt{\frac{9.825}{(6371+637)(1000)}}$

$= \underline{7544 \text{ m/s}}$

(b) From $r_{min} = a(1-e)$, $a = \frac{r_{min}}{1-e} = \frac{1.1(6371)}{1-0.1}$

$= 7787 \text{ km}$

$v_p = R\sqrt{\frac{g}{a}}\sqrt{\frac{1+e}{1-e}} = 6371(1000)\sqrt{\frac{9.825}{7787(1000)}}\sqrt{\frac{1+0.1}{1-0.1}}$

$= \underline{7912 \text{ m/s} = v}$

(c) $a = \frac{r_{min}}{1-e} = \frac{1.1(6371)}{1-0.9} = 70\,081 \text{ km}$

$v_p = 6371(1000)\sqrt{\frac{9.825}{70\,081(1000)}}\sqrt{\frac{1+0.9}{1-0.9}}$

$= \underline{10\,398 \text{ m/s} = v}$

(d) Eq. 3/43 with $a \to \infty$: $v = R\sqrt{\frac{2g}{r}}$

This is $\sqrt{2}$ times answer for part (a), so

$v = \sqrt{2}\,(7544) = \underline{10\,668 \text{ m/s}}$

3/281 $v_A = R\sqrt{\frac{g}{r}} = (3959)(5280)\sqrt{\frac{32.23}{(4759)(5280)}}$

$= 23{,}676$ ft/sec

$v_B = R\sqrt{\frac{g}{a}}\sqrt{\frac{r_{max}}{r_{min}}}$

$= (3959)(5280)\sqrt{\frac{32.23}{\frac{(2(3959)+1800)(5280)}{2}}}\sqrt{\frac{4959}{4759}}$

$= 23{,}917$ ft/sec

Momentum conservation during impact:

$m_A v_A + m_B v_B = (m_A + m_B) v_C$. But $m_A = m_B$, so

$v_C = \frac{1}{2}(v_A + v_B) = 23{,}796$ ft/sec

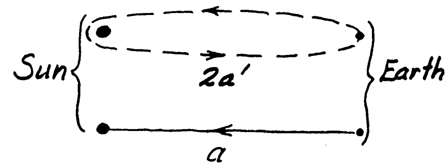
From $v_P = R\sqrt{\frac{g}{a}}\sqrt{\frac{r_{max}}{r_{min}}}$,

$r_{max} = \dfrac{r_{min}}{\left(\dfrac{2gR^2}{v_P^2 r_{min}} - 1\right)} = 2.5652 \times 10^7$ ft
(4858 mi)

$h_{max} = r_{max} - R = 4858 - 3959 = \underline{899 \text{ mi}}$

3/282

Radius of actual orbit around the sun is a, which is the major axis $2a'$ of the degenerate ellipse.



R = radius of sun

g = gravitational accel. on surface of sun

Orbital period Eq. 3/40

For actual orbit $\tau = 2\pi \dfrac{a^{3/2}}{R\sqrt{g}}$

For degenerate ellipse $\tau' = 2\pi \dfrac{(a/2)^{3/2}}{R\sqrt{g}}$

so $\dfrac{\tau'}{\tau} = \dfrac{(\frac{1}{2})^{3/2}}{1}$

But time t to fall is $t = \frac{1}{2}\tau' = \frac{1}{2}\left(\frac{1}{2}\right)^{3/2}\tau = \frac{1}{4\sqrt{2}}\,365.26$

$= \underline{64.6 \text{ days}}$

3/283 The apogee speed at C is

$$v_a = R\sqrt{\frac{g}{a}}\sqrt{\frac{r_{min}}{r_{max}}}$$

$$= 6371(10^3)\sqrt{\frac{9.825}{(2\cdot 6371+240+320)1000/2}}\sqrt{\frac{6371+240}{6371+320}}$$

$$= 7697 \text{ m/s}$$

The circular orbit speed at $h = 320$ km is

$$v_{circ} = R\sqrt{g/r_{max}} = 7720 \text{ m/s}$$

$$\Delta v = v_{circ} - v_a = 7720 - 7697 = 23.25 \text{ m/s}$$

$$F\Delta t = m\Delta v: \quad 2(30\,000)(\Delta t) = 85\,000(23.25)$$

$$\underline{\Delta t = 32.9 \text{ s}}$$

The burn to increase speed is at C.

3/284 The linear impulses from drag and from the thruster must be equal in magnitude, or

$$Dt = \Sigma T t_{burn}$$

$t = 10\tau$, where $\tau = 2\pi \dfrac{a^{3/2}}{R\sqrt{g}}$

or $\tau = 2\pi \dfrac{(6.571 \times 10^6)^{3/2}}{6.371 \times 10^6 \sqrt{9.825}} = 5300 \text{ s}$

$t = 10\tau = 53{,}000 \text{ s}$

$D = \dfrac{\Sigma T t_{burn}}{t} = \dfrac{2(300)}{53{,}000} = \underline{0.01132 \text{ N}}$

3/285 From $r_{min} = a(1-e)$:

$$7959 = [4000 + 3959 + 16{,}000](1-e), \quad e = 0.668$$

$$b = a\sqrt{1-e^2} = 23{,}959\sqrt{1-0.668^2} = \underline{17{,}833 \text{ mi}}$$

At B, $r = \sqrt{16{,}000^2 + 17{,}833^2} = 23{,}959$ mi

$$v_B^{\,2} = 2gR^2\left(\frac{1}{r} - \frac{1}{2a}\right)$$

$$= 2(32.23)3959^2(5280)\left[\frac{1}{23{,}959} - \frac{1}{2(23{,}959)}\right]$$

$$v_B = 10{,}551 \text{ ft/sec}$$

3/286 Speed in circular orbit is

$$v = R\sqrt{\frac{g}{a}} = (3959)(5280)\sqrt{\frac{32.23}{(4459)(5280)}}$$

$$= 24,458 \text{ ft/sec}$$

Time required for B to return to C's burn position : $t = \dfrac{2\pi r - 1000(5280)}{v}$

$$= 5832 \text{ s}$$

$$\tau = \frac{2\pi a^{3/2}}{R\sqrt{g}}, \quad a = \left(\frac{\tau R\sqrt{g}}{2\pi}\right)^{2/3}$$

$$a = \left(\frac{(5832)(3959)(5280)\sqrt{32.23}}{2\pi}\right)^{2/3} = 2.29799\,(10)^7 \text{ ft}$$

At apogee, $v_C = \sqrt{2gR^2\left[\frac{1}{r} - \frac{1}{2a}\right]} = 24,156$ ft/sec

$$\Delta v = v - v_C = 24,458 - 24,156 = \underline{302 \text{ ft/sec}}$$

(Can check to ensure that C does not strike the earth by finding $r_{min} = 2.242 \times 10^7$ ft $> R = 2.090 \times 10^7$ ft !)

3/287 From previous solution, the circular orbit speed is $v = 24{,}458$ ft/sec.

Time required for B to return to C's burn position over almost two circular orbits:

$$t = \frac{4\pi r - (1000)(5280)}{v} = 11{,}881 \text{ s}$$

$$a = \left(\frac{\tau R\sqrt{g}}{2\pi}\right)^{2/3} = \left[\frac{\left(\frac{11{,}881}{2}\right)(3959)(5280)\sqrt{32.23}}{2\pi}\right]^{2/3}$$

$$= 2.32626\,(10^7)\text{ ft}$$

At apogee, $v_c = \sqrt{2gR^2\left(\frac{1}{r} - \frac{1}{2a}\right)} = 24{,}309$ ft/sec

$\Delta v = v - v_c = 24{,}458 - 24{,}309 = \underline{148 \text{ ft/sec}}$

3/288 Circular orbit speed

$$v_0 = R\sqrt{\frac{g}{a}} = R\sqrt{\frac{g}{3R}} = \sqrt{\tfrac{1}{3}gR}$$

Speed at A (apogee) in elliptical orbit:

$$v_A = R\sqrt{\frac{g}{a}}\sqrt{\frac{r_{min}}{r_{max}}} = R\sqrt{\frac{g}{2R}}\sqrt{\frac{R}{3R}} = \sqrt{\tfrac{1}{6}gR}$$

$$v_r = v_0 - v_A = \sqrt{gR}\left[\sqrt{1/3} - \sqrt{1/6}\right] = 0.1691\sqrt{gR}$$

Numbers: $v_r = 0.1691\sqrt{1.62\,\frac{3476}{2}\,(1000)}$

$= \underline{284 \text{ m/s}}$ (directed rearward)

Call the circular orbit period τ_0 and the elliptical orbit period τ_{AB}

$$\tau_0 = \frac{2\pi a^{3/2}}{R\sqrt{g}} = \frac{2\pi(3R)^{3/2}}{R\sqrt{g}}\ ;\ \tau_{AB} = \frac{2\pi(2R)^{3/2}}{R\sqrt{g}}$$

$$\theta = \left(\frac{\tau_{AB}/2}{\tau_0/2}\right)\pi = \left(\frac{2}{3}\right)^{3/2}\pi = 1.710 \text{ rad or}$$

$98.0°$

3/289 Circular orbit : $v = R\sqrt{\frac{g}{r}}$

$$v = (3959)(5280)\sqrt{\frac{32.23}{(4159)(5280)}} = 25{,}324 \text{ ft/sec}$$

During burn : $a_t = \frac{F}{m} = \frac{2(6000)}{(175{,}000)/32.2} = 2.208 \frac{ft}{sec^2}$

$$v_a = v - a_t t = 25{,}324 - 2.208(150) = 24{,}993 \frac{ft}{sec}$$

$$v^2 = 2gR^2\left[\frac{1}{r} - \frac{1}{2a}\right]$$

Substitute conditions at B to find $a = 2.1403(10^7)$ ft

Use $v_A = R\sqrt{\frac{g}{a}}\sqrt{\frac{1-e}{1+e}}$ to obtain $e = 0.02599$

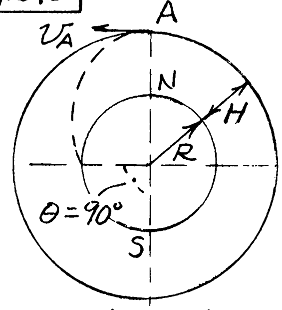
$r = \frac{a(1-e^2)}{1+e\cos\theta}$ utilized at point C:

$$(3959)(5280) = \frac{(2.1403\times10^7)(1-0.02599^2)}{1+0.02599\cos\theta}$$

$\theta = 26.7°$

$\beta = 180 - \theta = \underline{153.3°}$

3/290 From Eq. 3/39 $\dfrac{1}{r} = \dfrac{1+e\cos\theta}{a(1-e^2)}$



when $\theta = 90°$, $r = R$

" $\theta = 180°$, $r = R+H$

Thus $\dfrac{1}{R} = \dfrac{1}{a(1-e^2)}$

& $\dfrac{1}{R+H} = \dfrac{1-e}{a(1-e^2)} = \dfrac{1}{a(1+e)}$

Solve & get $1-e = \dfrac{R}{R+H}$ & $a(1+e) = R+H$

From Eq. 3/44

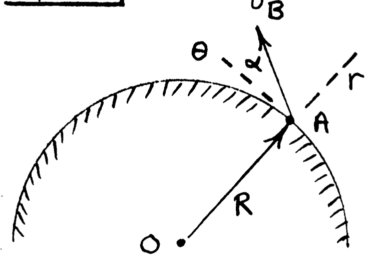
$$v_A = R\sqrt{\frac{g(1-e)}{a(1+e)}} = R\sqrt{g}\sqrt{\frac{R}{R+H}\ \frac{1}{R+H}} = \frac{R\sqrt{gR}}{R+H}$$

For circular orbit

$$v = R\sqrt{\frac{g}{R+H}} \quad \text{so } \Delta v_A = R\sqrt{\frac{g}{R+H}} - \frac{R\sqrt{gR}}{R+H}$$

$$= \underline{R\sqrt{\frac{g}{R+H}}\left(1 - \sqrt{\frac{R}{R+H}}\right)}$$

3/291



$v_\theta = v\cos\alpha = 2000\cos 30°$
$= 1732$ m/s

$v_r = v\sin\alpha = 2000\sin 30°$
$= 1000$ m/s

$$v^2 = 2gR^2\left(\frac{1}{r} - \frac{1}{2a}\right)$$

Using conditions at B:
$a = 3.2906 \times 10^6$ m

$$T_B = \frac{1}{2}mv_B^2 = \frac{1}{2}m(2000)^2 = 2\times10^6\, m$$

$$V_B = \frac{-mgR^2}{r} = \frac{-m(9.825)(6.371\times10^6)^2}{6.371\times10^6}$$

$$= -6.2595\times10^7\, m$$

$E = T_B + V_B = -6.0595\times10^7\, m$

$h = rv_\theta = 6.371(10^6)(1732) = 1.1035\times10^{10}$

Now use $e = \sqrt{1 + \dfrac{2Eh^2}{mg^2R^4}}$ to get $e = 0.9525$

Finally, $r_{max} = a(1+e) = 3.2906(10^6)(1+0.9525)$
$= 6.4249\times10^6$ m

$h_{max} = r_{max} - R = 53\,900$ m or 53.9 km

3/292 Point A is the apogee, so we have $r_{max} = \frac{3R}{2} = a(1+e)$.

$r = \frac{a(1-e^2)}{1+e\cos\theta}$; At B : $R = \frac{a(1-e^2)}{1+e\cos(135°)}$

Solving, $e = 0.6306$, $a = 0.9199R$

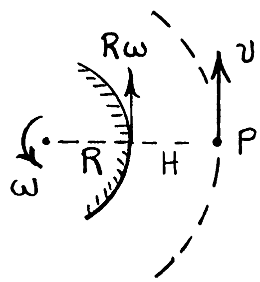
Now, $v_B^2 = 2gR^2\left(\frac{1}{r} - \frac{1}{2a}\right)$

At A : $v_B^2 = 2(9.825)(6.371 \times 10^6)^2 \times$

$$\left(\frac{1}{6.371 \times 10^6} - \frac{1}{2(0.9199)(6.371)(10^6)}\right)$$

$$\underline{v_B = 7560 \text{ m/s}}$$

$\underline{3/293}$ $a = \frac{1}{2}\left[2(6371) + 150 + 1500\right] = 7196$ km



Eq. 3/43 at perigee P:

$$v^2 = 2(9.825)\left[6371(10^3)\right]^2 \times \left[\frac{1}{(6371+150)10^3} - \frac{1}{2(7196)10^3}\right]$$

$v = 8179$ m/s

$R\omega = 6371(10^3)(0.7292 \cdot 10^{-4}) = 465$ m/s

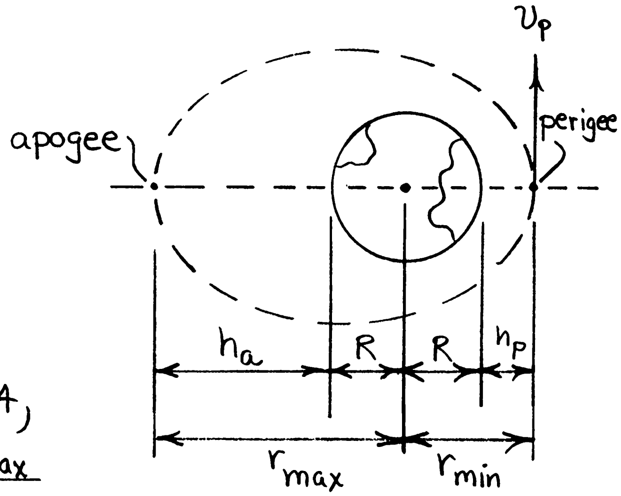
Absolute dish angular velocity $p_a = \dfrac{v - R\omega}{H}$

Relative dish angular velocity $p = p_a - \omega$

$$p = \frac{v - R\omega}{H} - \omega = \frac{8179 - 465}{150(10^3)} - 0.7292(10^{-4})$$

$= \underline{0.0514 \text{ rad/s}}$

3/294



At perigee,

$$a = a_n = \frac{v_p^2}{\rho_p},$$

so $$\rho_p = \frac{v_p^2}{a_n}$$

From Eq. 3/44,

$$v_p^2 = R^2 \frac{g}{a} \frac{r_{max}}{r_{min}}$$

But from Eqs. 3/39: $r_{min} + r_{max} = 2a$, so

$$v_p^2 = gR^2 \frac{r_{max}}{r_{min}} \frac{2}{r_{min} + r_{max}}$$

Also, $a_n = g_{perigee} = g\left(\frac{R}{r_{min}}\right)^2$ (from Chapter 1)

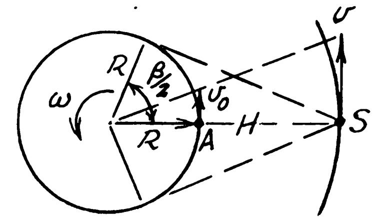
Thus $$\rho_p = 2gR^2 \frac{r_{max}}{r_{min}} \frac{1}{r_{min}+r_{max}} \Big/ g\frac{R^2}{r_{min}^2}$$

$$= 2\frac{r_{max}\, r_{min}}{r_{min} + r_{max}}$$

or $$\rho_p = 2\frac{(R+h_a)(R+h_p)}{2R + h_a + h_p}$$

3/295

Path is limited to an equatorial orbit in order to remain above a point A on the equator.



$\dfrac{v}{R+H} = \omega$ & for circular orbit Eq. 3/43 with $a = r = R+H$ gives $v = R\sqrt{\dfrac{g}{R+H}}$

$\omega = 0.7292 \times 10^{-4}$ rad/s

$R = 6371$ km

Combine & get $R+H = \sqrt[3]{\dfrac{gR^2}{\omega^2}}$, $H = \sqrt[3]{\dfrac{9.825(6371\times10^3)^2}{(0.7292\times10^{-4})^2}} - 6371\times10^3$

$= (42\,170 - 6371)10^3 = 35.8\times10^6$ m

or $\underline{H = 35\,800 \text{ km}}$

$\dfrac{\beta}{2} = \cos^{-1}\dfrac{R}{R+H} = \cos^{-1}\dfrac{6371}{42\,170} = 81.3°$, $\underline{\beta = 162.6° \text{ of longitude}}$

3/296 Eq. (3/40) (m_o fixed): $\tau_f = \dfrac{2\pi a^{3/2}}{\sqrt{gR^2}}$

Eq. (3/45b) (m_o not fixed): $\qquad = \dfrac{2\pi a^{3/2}}{\sqrt{Gm_o}}$

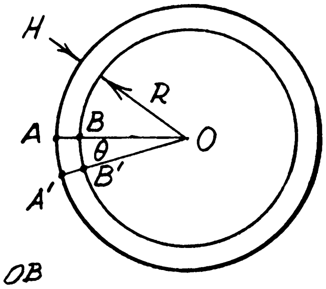
$$\tau_{nf} = \frac{2\pi a^{3/2}}{\sqrt{G(m_o+m)}}$$

Using numbers from Table D/2,

$\tau_f = 658.69$ h

$\tau_{nf} = 654.68$ h

3/297 Let ω_s = angular velocity of OA
$= \frac{v_A}{R+H}$



τ = period of satellite = time for OA to rotate through 2π

τ' = time for observer's radial line to rotate through angle θ

ω_e = angular velocity of earth and line OB

$R = 6371$ km, $H = 300$ km

Thus $\left.\begin{aligned} \theta &= \omega_e \tau' \\ \theta + 2\pi &= \omega_s \tau' \end{aligned}\right\} \tau' = \frac{2\pi}{\omega_s - \omega_e}$

Eq. 3/43 with $a = r = R + H$ gives $v_A = R\sqrt{\frac{g}{R+H}} = 6371\sqrt{\frac{9.825\times 10^{-3}}{6371+300}}$
$= 7.73$ km/s

$\omega_s = \frac{7.73}{6371+300} = 11.59 \times 10^{-4}$ rad/s

From Table D/2 $\quad \omega_e = 0.7292 \times 10^{-4}$ rad/s

So $\tau' = \frac{2\pi \times 10^4}{11.59 - 0.7292} = 5790$ s or $\underline{\tau' = 1\text{ h } 36\text{ min } 25\text{ s}}$

$\tau = \frac{2\pi}{\omega_s} = \frac{2\pi \times 10^4}{11.59} = 5420$ s or $\tau = 1$ h 30 min 21 s

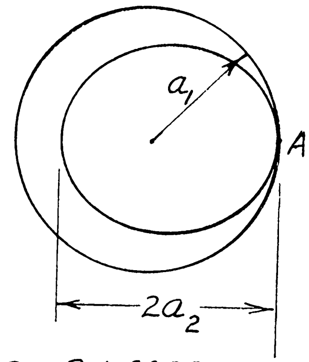
$\underline{\tau' - \tau = 6\text{ min } 4\text{ s}}$

3/298 $Ft = m\Delta v$

For circular orbit, $v_1 = R\sqrt{g/a_1}$

$$= 6371(10^3)\sqrt{\frac{9.825}{12\,371(10^3)}}$$

$$= 5678 \text{ m/s}$$



For elliptical orbit at apogee A

$$v_A = R\sqrt{\frac{g}{a_2}}\sqrt{\frac{r_{min}}{r_{max}}}$$

where $r_{min} = 6371 + 3000 = 9371$ km

$r_{max} = 6371 + 6000 = 12\,371$ km

So $v_A = 6371(10^3)\sqrt{\frac{9.825}{10\,871(10^3)}}\sqrt{\frac{9371}{12\,371}}$

$= 5271$ m/s

Thus $\Delta v = 5678 - 5271 = 406$ m/s

$a_1 = R + 6000$
$= 6371 + 6000$
$= 12371$ km
$2a_2 = 2a_1 - 3000$
$= 21\,742$ km
$a_2 = 10\,871$ km

So $2000\,t = 800(406)$

$\underline{t = 162 \text{ s}}$

3/299 Circular orbit speed v is

$$v = R\sqrt{\frac{g}{r}} = 6371(10^3)\sqrt{\frac{9.825}{(6371+400)(1000)}} = 7674 \ \frac{m}{s}$$

Orbit energy $E = \frac{1}{2}mv^2 - \frac{mgR^2}{r} = m\left[\frac{v^2}{2} - \frac{gR^2}{r}\right]$

$$E = m\left[\frac{7674^2}{2} - \frac{9.825\,(6371\cdot 10^3)^2}{(6371+400)(1000)}\right] = -29.4(10^6)m \text{ J}$$

$$h = rv_\theta = 6771(10^3)\,7674\cos\alpha = 5.20(10^{10})\cos\alpha$$

Substitute $a = -\frac{mgR^2}{2E}$ and $e = \sqrt{1 + \frac{2Eh^2}{mg^2R^4}}$

into $r_{min} = a(1-e)$, where $r_{min} = R$:

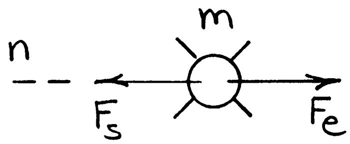
$$R = -\frac{mgR^2}{2E}\left[1 - \sqrt{1 + \frac{2Eh^2}{mg^2R^4}}\right]$$

$$1 = -\frac{mgR}{2E}\left[1 - \sqrt{1 + \frac{2Eh^2}{mg^2R^4}}\right]$$

$$1 = -\frac{m(9.825)(6371)(10^3)}{2(-29.4)(10^6)m}\left[1 - \sqrt{1 + \frac{2(-29.4)10^6 m\,5.20^2\,10^{20}\cos^2\alpha}{m(9.825)^2(6371\cdot 10^3)^4}}\right]$$

Solve for α as $\underline{\alpha = \pm 3.39°}$

*3/300



F_s : force exerted on spacecraft by sun

F_e : force exerted on spacecraft by earth

$$\Sigma F_n = ma_n: \quad F_s - F_e = m\frac{v^2}{\rho} = m\rho\omega^2$$
$$= m(D-h)\left(\frac{2\pi}{T}\right)^2$$

where D is the earth-sun distance and T is the earth orbital period.

$$\frac{Gm_s \cancel{m}}{(D-h)^2} - \frac{Gm_e \cancel{m}}{h^2} = \cancel{m}(D-h)\left(\frac{2\pi}{T}\right)^2$$

With $G = 3.439(10^{-8})\frac{ft^4}{lb\text{-}sec^4}$, $m_s = 333{,}000\, m_e$, $m_e = 4.095(10^{23})$ slugs, $D = 92.96(10^6)(5280)$ ft, and $T = 365.26(24)(3600)$ sec, solve numerically for h as $h = 4.87(10^9)$ ft or $\underline{922{,}000 \text{ mi}}$

▸3/301 For 1, $v_1 = R\sqrt{g/r_1}$; for 2, $v_2 = R\sqrt{g/r_2}$

For transfer ellipse at A, $v_1' = R\sqrt{g/a}\sqrt{r_2/r_1}$

For transfer ellipse at B, $v_2' = R\sqrt{g/a}\sqrt{r_1/r_2}$

$\left.\right\} a = \dfrac{r_1+r_2}{2}$ (Eq. 3/44)

At A, $\Delta v_A = v_1' - v_1 = R\sqrt{g/a}\sqrt{r_2/r_1} - R\sqrt{g/r_1} = \underline{R\sqrt{g/r_1}\left(\sqrt{\dfrac{2r_2}{r_1+r_2}} - 1\right)}$

At B, $\Delta v_B = v_2 - v_2' = R\sqrt{g/r_2} - R\sqrt{g/a}\sqrt{r_1/r_2} = \underline{R\sqrt{g/r_2}\left(1 - \sqrt{\dfrac{2r_1}{r_1+r_2}}\right)}$

$$\Delta v_A = 6371(10^3)\sqrt{\frac{9.825(10^3)}{6871}}\left(\sqrt{\frac{2(42171)}{6871+42171}} - 1\right) = \underline{2370 \text{ m/s}}$$

$$\Delta v_B = 6371(10^3)\sqrt{\frac{9.825(10^3)}{42171}}\left(1 - \sqrt{\frac{2(6871)}{6871+42171}}\right) = \underline{1447 \text{ m/s}}$$

▶ **3/302** Eq. 3/43 : $v^2 = 2gR^2\left(\frac{1}{r} - \frac{1}{2a}\right)$

$$7400^2 = 2(9.825)(6371 \cdot 1000)^2\left[\frac{1}{7371 \cdot 1000} - \frac{1}{2a}\right]$$

$\underline{a = 7462 \text{ km}}$

$$T = \frac{1}{2}mv^2 = \frac{1}{2}m(7400)^2 = 27.38(10^6)m$$

$$V = -\frac{mgR^2}{r} = -\frac{m(9.825)(6371 \cdot 1000)^2}{7371(1000)} = -54.1(10^6)m$$

$$E = T + V = -26.7(10^6)m \quad \text{(in Joules)}$$

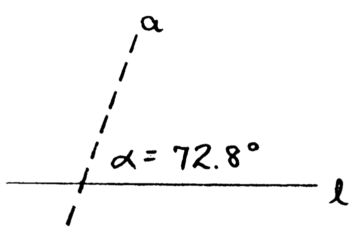
$$h = rv_\theta = 7371(10^3)(7400 \cos 2°) = 5.45(10^{10})\ \frac{m^2}{s}$$

$$e = \sqrt{1 + \frac{2Eh^2}{mg^2R^4}} = \sqrt{1 + \frac{2(-26.7)10^6 m\ 5.45^2\ 10^{20}}{m(9.825)^2(6371 \cdot 1000)^4}}$$

$$= 0.0369$$

From $r = \frac{a(1-e^2)}{1 + e\cos\theta}$: $7371 = \frac{7462(1 - 0.0369^2)}{1 + 0.0369\cos\theta}$

$\theta = \pm 72.8°$ So $\underline{\alpha = 72.8°}$

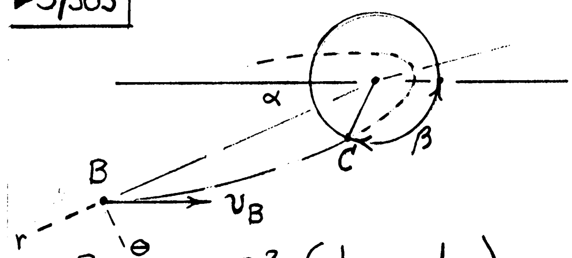


$$r_{min} = a(1-e) = 7462(1 - 0.0369)$$

$$= 7186 > R = 6371 \text{ m}$$

$\therefore$ $\underline{\text{Does not strike earth}}$

▶3/303



At B, $r = \sqrt{29}\,R$

$\alpha = \tan^{-1}\left(\frac{2R}{5R}\right)$

$= 21.8°$

$$v^2 = 2gR^2\left(\frac{1}{r} - \frac{1}{2a}\right)$$

At B : $3200^2 = 2(9.825)(6.371\times10^6)^2\left[\frac{1}{\sqrt{29}\,6.371(10^6)} - \frac{1}{2a}\right]$

$$a = 3.066\times10^7\ \text{m}$$

$$T_B = \tfrac{1}{2}mv_B^2 = \tfrac{1}{2}m\,(3200)^2 = 5.120\times10^6\ m$$

$$V_B = -\frac{mgR^2}{r_B} = -m\,\frac{(9.825)(6.371\times10^6)^2}{\sqrt{29}\,(6.371\times10^6)} = -1.162\times10^7\,m$$

$$E = T_B + V_B = -6.504\times10^6\ m$$

$$v_\theta = 3200\sin\alpha = 1188.5\ \text{m/s}$$

$$h = rv_\theta = \sqrt{29}\,(6.371\times10^6)(1188.5) = 4.077\times10^{10}\ \text{kg-m}^2/\text{s}$$

$$e = \sqrt{1 + \frac{2Eh^2}{mg^2R^4}}$$

$$e = \sqrt{1 + \frac{2(-6.504m)(4.077\times10^{10})^2}{m(9.825)^2(6.371\times10^6)^4}}$$

$$= 0.9295$$

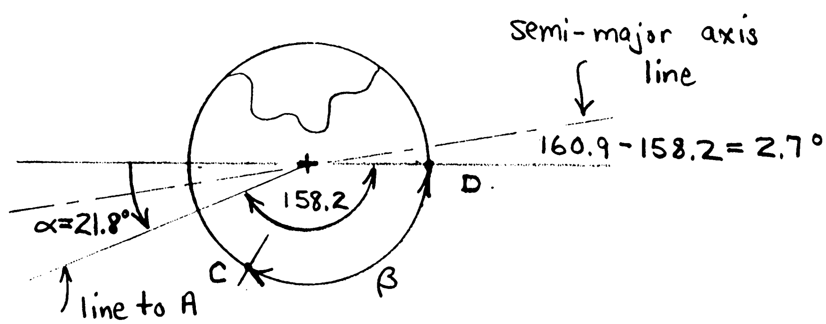
$$r = \frac{a(1-e^2)}{1+e\cos\theta}$$

At B : $$\sqrt{29}(6.371\times10^6) = \frac{(3.066\times10^7)(1-0.9295^2)}{1+0.9295\cos\theta}$$

$$\theta = 160.9^\circ$$

At C : $$6371(10^6) = \frac{(3.066\times10^7)(1-0.9295^2)}{1+e\cos\theta}$$

$$\theta = 111.8^\circ$$



$$\beta = 111.8 - 2.7 = \underline{109.1^\circ}$$

▶3/304 The speed of the orbiter is

$$v_0 = \sqrt{\frac{Gm_e}{r}} = \sqrt{\frac{6.673(10^{-11})(5.976)(10^{24})}{(6371+200)(1000)}} = 7790 \text{ m/s}$$

The speed of the satellite is

$$v = \sqrt{v_0^2 + v_{s/o}^2} = 7791 \text{ m/s}$$

Eq. 3/43: $v^2 = 2gR^2\left(\frac{1}{r} - \frac{1}{2a}\right)$

$$(7791)^2 = 2(9.825)(6371 \cdot 1000)^2\left[\frac{1}{6571(1000)} - \frac{1}{2a}\right]$$

$$\underline{a = 6572 \text{ km}}$$

$$\tau = \frac{2\pi a^{3/2}}{R\sqrt{g}} = \underline{5301 \text{ s}}$$

Energy $E = \frac{1}{2}mv^2 - \frac{Gm_e m}{r} = -30.3m(10^6)$ J

$h = rv_\theta = 6571(1000)7790 = 5.12(10^{10})$ m²/s

Eq. 3/41: $e = \sqrt{1 + \frac{2Eh^2}{mg^2R^4}} = \underline{0.01284}$

From $\frac{1}{r} = \frac{1 + e\cos\theta}{a(1-e^2)}$, $\underline{\theta = 90°}$ exactly
(semimajor axis is parallel to x-axis)

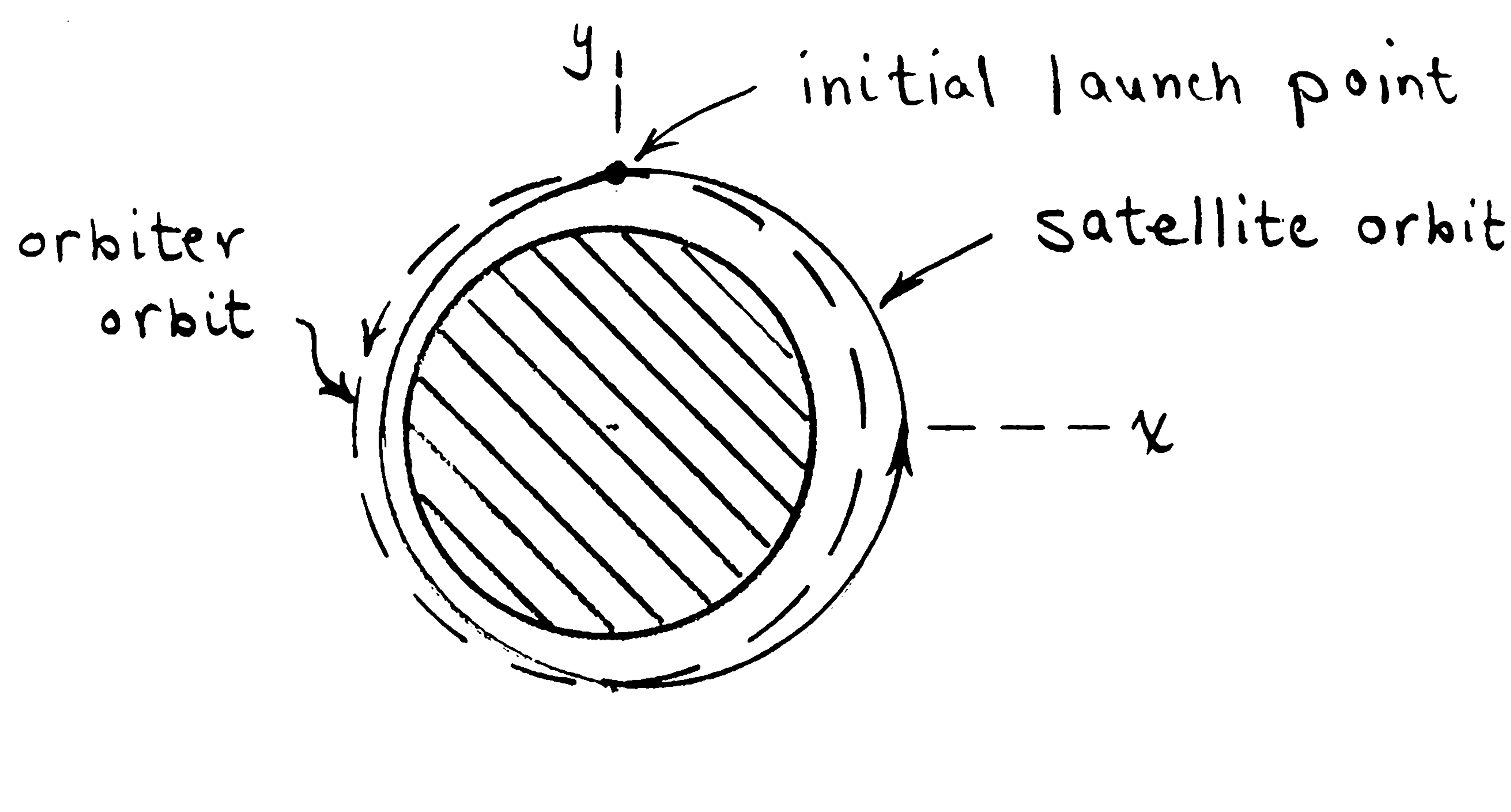
$$r_{min} = a(1-e) = \underline{6.49(10^6) \text{ m}}$$

$$r_{max} = a(1+e) = \underline{6.66(10^6) \text{ m}}$$

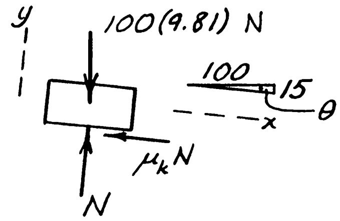
$$v_p = R\sqrt{\frac{g}{a}}\sqrt{\frac{r_{max}}{r_{min}}} = \underline{7890 \text{ m/s}}$$

$$v_a = R\sqrt{\frac{g}{a}}\sqrt{\frac{r_{min}}{r_{max}}} = \underline{7690 \text{ m/s}}$$

Sketch (not to scale):



3/305 Truck bed is a constant-velocity frame of reference so that $U_{rel} = \Delta T_{rel}$ holds.



$\Sigma F_y = 0: \; N - 981 \cos 8.53° = 0$

$N = 970$ N

$U_{rel} = \Delta T_{rel}: \; (981 \sin 8.53° - 970\mu_k)2 = \frac{1}{2} 100(0 - 3^2)$

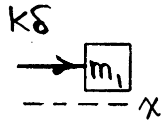
$\theta = \tan^{-1} 0.15 = 8.53°$

$\underline{\mu_k = 0.382}$

3/306

$$\Sigma F_x = m a_x : \quad k\delta = m_1 a_{x_1}$$

$$-k\delta = m_2 a_{x_2}$$



$$a_{1/2} = a_{x_1} - a_{x_2} = \frac{k\delta}{m_1} - \left(-\frac{k\delta}{m_2}\right)$$

or $\underline{a_{1/2} = a_{rel} = k\delta\left(\frac{1}{m_1} + \frac{1}{m_2}\right)}$

$\boxed{3/307}$ $v_{rel} = l\dot{\theta} = 0.5(2) = 1 \text{ m/s} \rightarrow$

$v = v + v_{rel} = 2+1 = 3 \text{ m/s} \rightarrow$

$\underline{G} = m\underline{v} = 3(3\underline{i}) = \underline{9\underline{i} \text{ kg}\cdot\text{m/s}}$

$\underline{G}_{rel} = m\underline{v}_{rel} = 3(1\underline{i}) = \underline{3\underline{i} \text{ kg}\cdot\text{m/s}}$

$T = \frac{1}{2}mv^2 = \frac{1}{2}(3)(3)^2 = \underline{13.5 \text{ J}}$

$T_{rel} = \frac{1}{2}mv_{rel}^2 = \frac{1}{2}(3)(1)^2 = \underline{1.5 \text{ J}}$

$\underline{H}_O = -lmv\underline{k} = -(0.5)(3)(3)\underline{k} = \underline{-4.5\underline{k} \ \frac{\text{kg}\cdot\text{m}^2}{\text{s}}}$

$\underline{H}_{B_{rel}} = -lmv_{rel}\underline{k} = -(0.5)(3)(1)\underline{k} = \underline{-1.5\underline{k} \ \frac{\text{kg}\cdot\text{m}^2}{\text{s}}}$

3/308 Rel. to carrier $U_{rel} = \Delta T_{rel}$

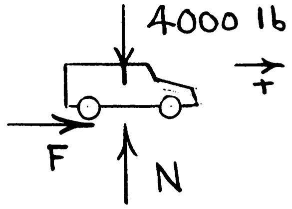
$$(22+P)(10^3)75 = \frac{1}{2}(3)(10^3)\left[(240/3.6)^2 - 0\right]$$

$$22+P = 88.9 \text{ kN}, \quad \underline{P = 66.9 \text{ kN}}$$

3/309 | Barge-fixed frame is Newtonian.

$v^2 = v_0^2 + 2a(s - s_0)$: $\left(15 \frac{5280}{3600}\right)^2 = 2a(80)$

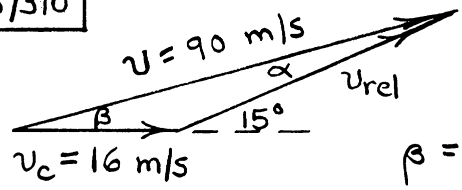
$a = 3.03$ ft/sec$^2 = a_{rel}$



$\overset{+}{\rightarrow} \Sigma F = ma$: $F = \frac{4000}{32.2}(3.03)$

$\underline{F = 376 \text{ lb}}$

3/310



$$\frac{\sin 165°}{90} = \frac{\sin\alpha}{16}$$

$\alpha = 2.64°$

$\beta = 180 - 165 - \alpha = 12.36°$

$$\frac{\sin\beta}{v_{rel}} = \frac{\sin 165°}{90}, \qquad v_{rel} = 74.4 \text{ m/s}$$

$U_{rel} = \Delta T_{rel}$: $Fd = \frac{1}{2} m \left(v_{rel}^2 - 0\right)$

$F(100) = \frac{1}{2}\, 7000\, (74.4^2)$

$\underline{F = 194\,000 \text{ N}}$ or $\underline{194.0 \text{ kN}}$

3/311 $\quad$ --- x

For truck, $a_T = -0.9g$, $t_{stop} = \dfrac{15 \text{ m/s}}{0.9(9.81 \text{ m/s}^2)} = 1.699 \text{ s}$

For crate, $a_c = -0.7g$

$a_{c/T} = a_c - a_T = -0.7g - (-0.9g) = 0.2g$

(As long as truck is moving)

At $t = t_{stop}$,

$$x_{c/T} = (x_{c/T})_0 + (v_{c/T})_0 t_{stop} + \tfrac{1}{2} a_{c/T} t_{stop}^2$$
$$= 0 + 0 + \tfrac{1}{2}(0.2g)(1.699)^2 = 2.83 \text{ m}$$

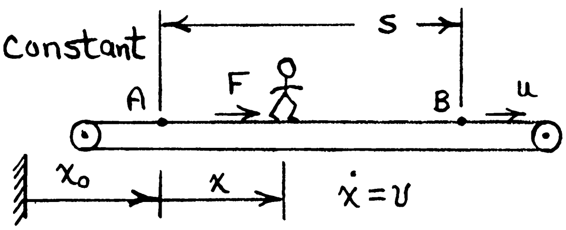
$$v_{c/T} = (v_{c/T})_0 + a_{c/T} t_{stop} = 0 + (0.2g)(1.699) = 3.33 \tfrac{\text{m}}{\text{s}}$$

Then : $\quad v_c^2 = v_{c_0}^2 + 2a_c(x - x_0)$

$$= (3.33)^2 + 2(-0.7g)(3.2 - 2.83)$$

$\underline{v_c = 2.46 \text{ m/s}}$

3/312 $F = \text{constant}$



Absolute: $U = \Delta T$: $F(s + \Delta x_0) = \frac{1}{2}m(u+v)^2 - \frac{1}{2}mu^2$

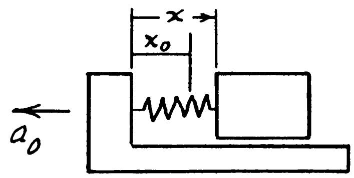
$$Fs + F\Delta x_0 = \frac{1}{2}mv^2 + muv \quad (1)$$

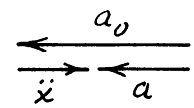
Relative to walkway : $U_{rel} = \Delta T_{rel}$: $Fs = \frac{1}{2}mv^2 - 0$ (2)

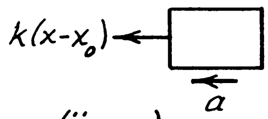
Subtract (2) from (1): $F\Delta x_0 = muv$

The term muv represents the work done by force F due only to the movement of the walkway.

3/313







$\Sigma F_x = ma_x: -k(x-x_o) = m(\ddot{x}-a_o)$

$\dot{x}\,d\dot{x} = \ddot{x}\,dx$ so $\displaystyle\int_0^{\dot{x}} \dot{x}\,d\dot{x} = \int_{x_o}^{x}\left[a_o - \frac{k}{m}(x-x_o)\right]dx$

$$\frac{1}{2}\dot{x}^2 = \left(a_o - \frac{kx_o}{m}\right)(x-x_o) - \frac{k}{2m}(x^2 - x_o^2)$$

$\dfrac{d}{dx}\left(\dfrac{\dot{x}^2}{2}\right) = a_o + \dfrac{kx_o}{m} - \dfrac{kx}{m} = 0$ for max. $\dfrac{\dot{x}^2}{2}$ & hence max $\dot{x}$

so $\dfrac{kx}{m} = a_o + \dfrac{kx_o}{m}$, $x = x_o + \dfrac{ma_o}{k}$

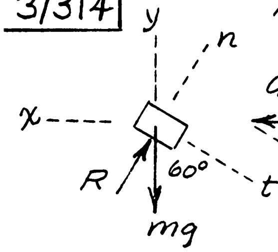
Thus

$$(v_{rel})^2_{max} = \dot{x}^2_{max} = 2\left(a_o + \frac{kx_o}{m}\right)\left(x_o + \frac{ma_o}{k} - x_o\right) - \frac{k}{m}\left(x_o^2 + \frac{2ma_o}{k}x_o + \frac{m^2a_o^2}{k^2} - x_o^2\right)$$

$$= \frac{ma_o^2}{k}$$

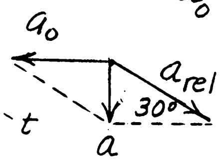
$$\underline{(v_{rel})_{max} = a_o\sqrt{m/k}}$$

3/314



For accel. a vertically down,

$a_o = a_{rel} \cos 30°$



$a_x = 0$ so $\Sigma F_x = 0$

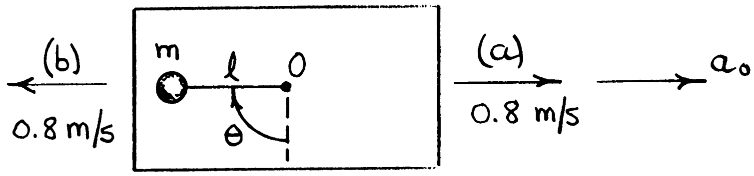
$-R \sin 30° = 0$, $\underline{R = 0}$

$\Sigma F_y = m a_y$, $mg = ma$

$a = g$ & $a_o = a\sqrt{3} = g\sqrt{3}$

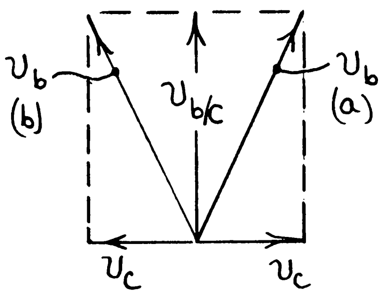
$= 9.81\sqrt{3} = \underline{16.99 \frac{m}{s^2}}$

3/315



$m_b = 10$ kg $\qquad l = 0.8$ m $\qquad \theta = 90°$

$m_c = 250$ kg $\qquad \dot{\theta} = 3$ rad/s



$v_{b/c} = l\dot{\theta} = 0.8(3) = 2.4$ m/s

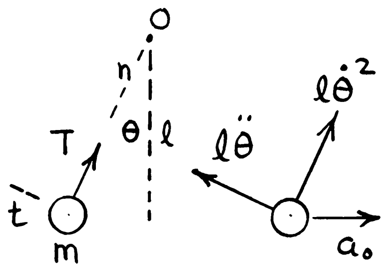
$T_b = \frac{1}{2} m_b v_b^2 = \frac{1}{2}(10)\left[0.8^2 + 2.4^2\right] = 32$ J

(same for cases (a) and (b))

$T_c = \frac{1}{2} m_c v_c^2 = \frac{1}{2}(250)(0.8)^2 = 80$ J

$T = T_b + T_c = 32 + 80 = \underline{112 \text{ J}}$ for both cases

3/316 $\Sigma \underline{F} = m(\underline{a}_0 + \underline{a}_{rel})$. In t-dir., $\Sigma F_t = 0$,



so $a_t = l\ddot{\theta} - a_0 \cos\theta = 0$

$$\ddot{\theta} = \frac{a_0}{l}\cos\theta \qquad (1)$$

In n-dir., $\Sigma F_n = ma_n$

$$T = m\left(l\dot{\theta}^2 + a_0 \sin\theta\right) \quad (2)$$

Integrate (1): $\ddot{\theta} = \dot{\theta}\dfrac{d\dot{\theta}}{d\theta} = \dfrac{a_0}{l}\cos\theta$

$$\int_0^{\dot{\theta}} \dot{\theta}\, d\dot{\theta} = \int_0^{\theta} \frac{a_0}{l}\cos\theta\, d\theta$$

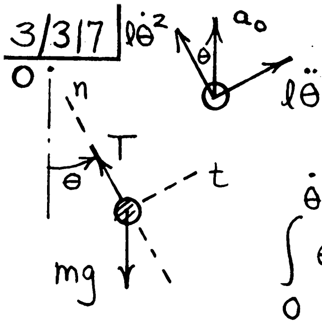
$$\frac{1}{2}\dot{\theta}^2 = \frac{a_0}{l}\sin\theta$$

From (2): $T = m\left[2a_0 \sin\theta + a_0 \sin\theta\right]$

or $\underline{T = 3ma_0 \sin\theta}$

For $\theta = \frac{\pi}{2}$, $T_{\pi/2} = 3ma_0 = 3(10)(3) = \underline{90 \text{ N}}$

3/317



$\Sigma F_t = ma_t$:

$$-mg\sin\theta = m(l\ddot{\theta} + a_0\sin\theta)$$

$$\ddot{\theta} = -\left(\frac{a_0+g}{l}\right)\sin\theta$$

$$\int_0^{\dot{\theta}} \dot{\theta}\,d\dot{\theta} = -\int_{\theta_0}^{\theta}\left(\frac{a_0+g}{l}\right)\sin\theta\,d\theta$$

$$\dot{\theta}^2 = 2\left(\frac{a_0+g}{l}\right)(\cos\theta - \cos\theta_0)$$

$\Sigma F_n = ma_n$: $T - mg\cos\theta = m(l\dot{\theta}^2 + a_0\cos\theta)$

$$T = m\left[g(3\cos\theta - 2\cos\theta_0) + a_0(3\cos\theta - 2\cos\theta_0)\right]$$

$$= m(g+a_0)(3\cos\theta - 2\cos\theta_0)$$

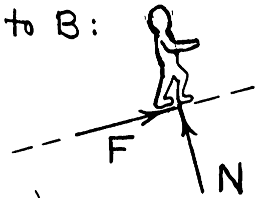
When $\theta = 0$,

$$\underline{T_0 = m(g+a_0)(3 - 2\cos\theta_0)}$$

If $\theta_0 = \pi/2$,

$$\underline{T_0 = 3m(g+a_0)}$$

3/318 For motion from A to B:



Absolute: $U'_{abs} = \Delta T + \Delta V_g$:

$$F(\Delta x_0 + s) = \tfrac{1}{2} m (v_r + u)^2 - \tfrac{1}{2} m u^2 + mg(\Delta x_0 + s)\sin\theta$$
$$= \tfrac{1}{2} m v_r^2 + m v_r u + mg(\Delta x_0 + s)\sin\theta$$

Relative: $U'_{rel} = \Delta T_{rel} + \Delta V_{g_{rel}}$: $Fs = \tfrac{1}{2} m v_r^2 + mgs\sin\theta$

Work done by walkway: $U'_{abs} - U'_{rel} = m v_r u + mg\Delta x_0 \sin\theta$

$m v_r u$ represents the work done by the belt due only to the motion of the walkway.

For $m = \frac{150}{32.2}$ slugs, $v_r = 2.5$ ft/sec, $u = 2$ ft/sec, $\theta = 10°$, $s = 30$ ft:

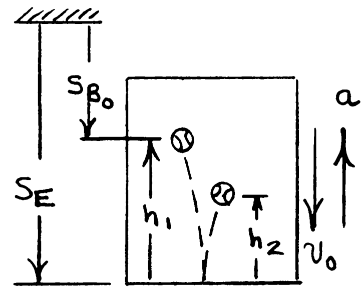
$$\Sigma F_x = m a_{x_{rel}} : \quad a_{x_{rel}} = \frac{v_r^2}{2s} = \frac{2.5^2}{2(30)} = 0.1042 \frac{\text{ft}}{\text{sec}^2}$$

$$F - 150 \sin 10° = \frac{150}{32.2}(0.1042), \quad F = 26.5 \text{ lb}$$

Power by boy: $P_{rel} = F v_r = 26.5(2.5) = 66.3 \ \frac{\text{ft-lb}}{\text{sec}}$

or $P_{rel} = 66.3/550 = \underline{0.1206 \text{ hp}}$

▸3/319 (a) $a = 0$, elevator is Newtonian frame



$$v' = ev = e\sqrt{2gh_1} = \sqrt{2gh_2}$$

$a = \frac{g}{4}$

$$\underline{h_2 = e^2 h_1}$$

(b) $a = \frac{g}{4}$ up

Let B = ball, E = elevator, $\downarrow +$

At impact, $s_B = s_E$: $s_{B_0} + v_{B_0}t + \frac{1}{2}gt^2 = s_{E_0} + v_{E_0}t - \frac{1}{2}\frac{g}{4}t^2$

$$s_{B_0} + v_0 t + \frac{1}{2}gt^2 = (s_{B_0} + h_1) + v_0 t - \frac{1}{8}gt^2, \quad t = 2\sqrt{\frac{2h_1}{5g}}$$

$$v_{B/E} = v_B - v_E = \left(v_0 + g\,2\sqrt{\frac{2h_1}{5g}}\right) - \left(v_0 - \frac{g}{4}2\sqrt{\frac{2h_1}{5g}}\right)$$

$$= \sqrt{\frac{5h_1 g}{2}}$$

After collision, $v'_{B/E_0} = -e\sqrt{\frac{5h_1 g}{2}}$ (up)

$$v'_{B/E} = v'_{B/E_0} + a_{B/E}t = -e\sqrt{\frac{5h_1 g}{2}} + \frac{5}{4}gt$$

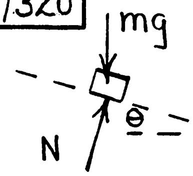
When $v'_{B/E} = 0$, $t = 2e\sqrt{\frac{2h_1}{5g}}$

$$s'_{B/E} = s'_{B/E_0} + v'_{B/E_0}t + \frac{1}{2}\frac{5}{4}gt^2$$

$$= 0 - e\sqrt{\frac{5h_1 g}{2}}\,2e\sqrt{\frac{2h_1}{5g}} + \frac{5}{8}g\,4e^2\frac{2h_1}{5g}$$

$$= -e^2 h_1 \Rightarrow \underline{h_2 = e^2 h_1}$$

▸3/320



$$U_{rel} = \Delta T_{rel}$$
$$mg\,l\sin\theta = \tfrac{1}{2} m v_{rel}^2 - 0$$
$$\underline{v_{rel}^2 = 2gl\sin\theta}$$

$U = \Delta T$: $\quad mg\,l\sin\theta + (N\sin\theta)\,d = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$

where d is the horizontal distance traveled by the block.

Time to slide from B to C : $l = \frac{1}{2}at^2 = \frac{1}{2}g\sin\theta\, t^2$

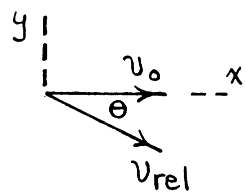
$t = \left(\dfrac{2l}{g\sin\theta}\right)^{1/2}$. So $d = v_0 t = v_0\sqrt{\dfrac{2l}{g\sin\theta}}$

Also, $N = mg\cos\theta$

Solving the work-energy equation for v^2:

$$\underline{v_A = \left(v_0^2 + 2gl\sin\theta + 2v_0\cos\theta\sqrt{2lg\sin\theta}\right)^{1/2}}$$

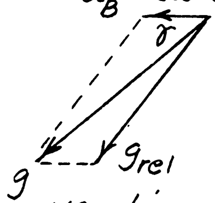
Check :



$$\underline{v}_A = \underline{v}_0 + \underline{v}_{rel}$$
$$= v_0\underline{i} + \sqrt{2gl\sin\theta}\,(\cos\theta\,\underline{i} - \sin\theta\,\underline{j})$$
$$= (v_0 + \sqrt{2gl\sin\theta}\cos\theta)\underline{i} - \sqrt{2gl\sin^3\theta}\,\underline{j}$$
$$v_A^2 = (v_0 + \sqrt{2gl\sin\theta}\cos\theta)^2 + (2gl\sin^3\theta)$$
$$\checkmark\ v_A^2 = v_0^2 + 2gl\sin\theta + 2v_0\cos\theta\sqrt{2gl\sin\theta}$$

3/321 From law of cosines

$a_B = R\omega^2 \cos\gamma$



$$g_{rel}^2 = g^2 + a_B^2 - 2g\,a_B \cos\gamma$$

$$= g^2\left(1 + \left[\frac{a_B}{g}\right]^2 - 2\,\frac{a_B}{g}\cos\gamma\right)$$

$$g_{rel} = g\left[1 + \frac{a_B}{g}\left(\frac{a_B}{g} - 2\cos\gamma\right)\right]^{1/2}$$

use binomial expansion for 1st two terms

$(1+x)^n = 1 + nx + \cdots$ & get

$$g_{rel} = g\left[1 + \frac{a_B}{g}\left(\frac{a_B}{2g} - \cos\gamma\right) + \cdots\right]$$

$$= g + a_B\left(\frac{a_B}{2g} - \cos\gamma\right) + \cdots$$

$$\underline{g_{rel} = g - R\omega^2\cos^2\gamma\left(1 - \frac{R\omega^2}{2g}\right) + \cdots}$$

$R\omega^2 = 6.371(10^6)(0.7292\times10^{-4})^2 = 0.03388\ \text{m/s}^2$

$$g_{rel} = 9.825 - 0.03388\left(1 - \frac{0.03388}{2\times9.825}\right)\cos^2\gamma + \cdots$$

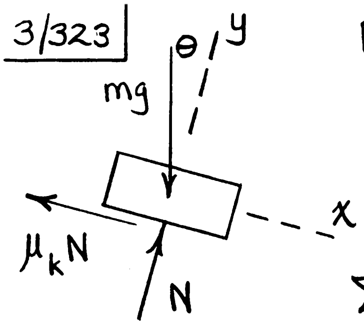
$$= \underline{9.825 - 0.03382\cos^2\gamma}\ \ \text{m/s}^2$$

▸3/322

Case (a): Orbital speed is constant so that $\ddot{x}$ is both the absolute and relative acceleration in the x-direction. Hence $F = m\ddot{x}$ holds.

Case (b): Orbital speed is decreasing in the position shown so that a component of acceleration in the negative x-direction exists so that the true (absolute) acceleration in the x-direction is $\ddot{x}$ minus the tangential orbital deceleration. Consequently $F \neq m\ddot{x}$. Only at the perigee and apogee positions where $\dot{v} = 0$ would $F = m\ddot{x}$ be true.

3/323



Motion would not occur at A without a nudge as

$\mu_s = 0.40 > \tan\theta = \tan 20° = 0.364$

$\Sigma F_y = 0: \quad N = mg\cos\theta$

$\Sigma F_x = ma_x: \quad -\mu_k(mg\cos\theta) + mg\sin\theta = ma_x$

$a_x = g(\sin\theta - \mu_k\cos\theta)$

$A \to B: \quad a_x = 9.81(\sin 20° - 0.30\cos 20°) = 0.590 \text{ m/s}^2$

$B \to C: \quad a_x = 9.81(\sin 10° - 0.22\cos 10°) = -0.422 \text{ m/s}^2$

So $v_B^2 = v_A^2 + 2a_x(x_B - x_A) = 0^2 + 2(0.590)(7)$

$\underline{v_B = 2.87 \text{ m/s}}$

$v_C^2 = v_B^2 + 2a_x(x_C - x_B) = 2.87^2 + 2(-0.422)(7)$

$\underline{v_C = 1.533 \text{ m/s}}$

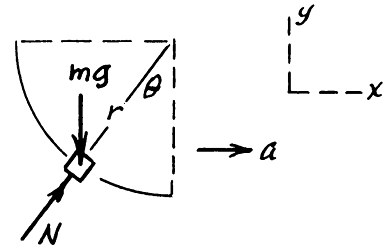
3/324

$\Sigma F_y = 0$: $N\cos\theta - mg = 0$, $N\cos\theta = mg$

$\Sigma F_x = ma_x$: $N\sin\theta = ma$

Divide & get $\tan\theta = a/g$

$\underline{\theta = \tan^{-1}\frac{a}{g}}$



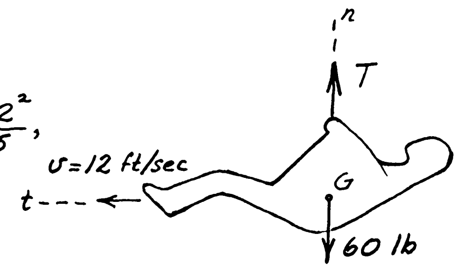
3/325

System:

$$\Sigma F_n = m\frac{v^2}{r}: \quad T - 60 = \frac{60}{32.2}\,\frac{12^2}{15},$$

$$T = 60(1 + 0.298)$$

$$= \underline{77.9\ \text{lb}}$$

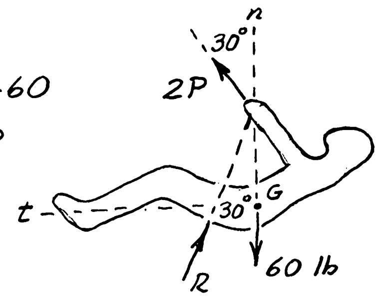


Girl:

$$\Sigma F_n = m\frac{v^2}{r}: \quad 2P\cos 30^\circ + R\cos 30^\circ - 60$$

$$= \frac{60}{32.2}\,\frac{12^2}{15} = 17.89\ \text{lb}$$

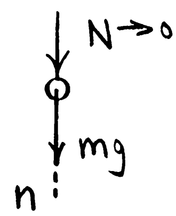
$$\Sigma F_t = 0: \quad 2P\sin 30^\circ - R\sin 30^\circ = 0$$



Solve & get

$$\underline{P = 22.5\ \text{lb}}, \quad \underline{R = 45.0\ \text{lb}}$$

3/326 Dynamics at B (top of loop)



$\Sigma F_n = ma_n: \quad mg = m\frac{v_B^2}{R}$

$v_B^{\,2} = gR$

Work-kinetic energy from A to B:

$T_A + U_{A\text{-}B} = T_B: \quad 0 + \frac{1}{2}k\delta^2 - mg\mu_k R - mg(2R) = \frac{1}{2}m(gR)$

$$\delta = \sqrt{\frac{mgR(5+2\mu_k)}{k}}$$

3/327 $2a = 2(6371) + 600 + 200$

$$a = 6771 \text{ km}$$

$$v_A = R\sqrt{\frac{g}{a}}\sqrt{\frac{r_{min}}{r_{max}}} = 6371(10^3)\sqrt{\frac{9.825}{6771(10^3)}}\sqrt{\frac{6371+200}{6371+600}}$$

$$= \underline{7451 \text{ m/s}}$$

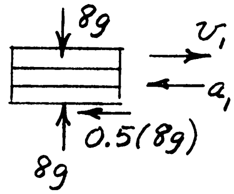
$$r_{max} = a(1+e) : (6371 + 600) = 6771(1+e)$$

$$\underline{e = 0.0295}$$

3/328 $U = \Delta T$; $mgh = \frac{1}{2}mv_B^2$ with no friction so $v_B = v_B' = \sqrt{2gh}$

With friction present, the friction force will be greater in case (a) than in case (b) because the normal reaction between the bead and wire is greater in (a) than in (b). Thus more negative work will be done & kinetic energy will be less at B than B', so $v_B < v_{B'}$.

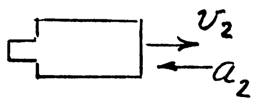
3/329



$F = ma;\quad a_1 = 0.5g$

$v_1 = \sqrt{2a_1 s_1} = \sqrt{2(0.5\times 9.81)\times 0.8}$

$= 2.80$ m/s



$a_2 = 0.5g$

$v_2 = \sqrt{2a_2 s_2} = \sqrt{2(0.5\times 9.81)\times 1.2}$

$= 3.43$ m/s

$\Delta G = 0;\quad 0.060v + 0 = 8(2.80) + (6 + 0.06)3.43$

$\underline{v = 720 \text{ m/s}}$

3/330 $\tau = 2\pi \dfrac{a^{3/2}}{R\sqrt{g}}$, $a = \left(\dfrac{\tau R\sqrt{g}}{2\pi}\right)^{2/3}$

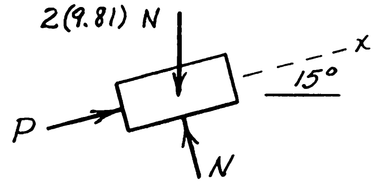
$$a = \left(\frac{(76)(365)(24)(3600)\left(\frac{1,392,000,000}{2}\right)\sqrt{274}}{2\pi}\right)^{2/3}$$

$a = 2.68(10^{12})$ m or $2.68(10^9)$ km

$r_{max} = 2a - r_{min} = 2(2.68[10^9]) - \dfrac{149.6(10^6)}{2}$

$= \underline{5.37(10^9)\text{ km}}$

3/331



$$\int \Sigma F_x \, dt = \Delta G_x :$$

$$\int_0^2 P\,dt = \tfrac{1}{2}(0.5 \times 28) + \tfrac{1}{2}(0.5 \times 20)$$

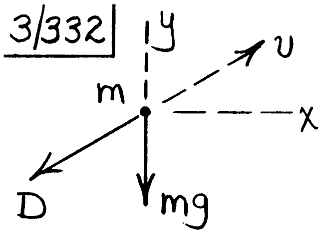
$$= 12 \text{ N·s}$$

$$\int_0^2 \Sigma F_x \, dt = 12 - 2(9.81)\sin 15° \times 2 = 1.844 \text{ N·s}$$

$$\Delta G_x = m \Delta v_x : \quad \Delta G_x = 2(v-2) \text{ kg·m/s}$$

Thus $1.844 = 2(v-2)$, $\underline{v = 2.92 \text{ m/s}}$

3/332



$$\Sigma \underline{F} = m\underline{a} : \; -C_D \tfrac{1}{2}\rho v^2 S \frac{\underline{v}}{v} - mg\,\underline{j} = m\left(a_x \underline{i} + a_y \underline{j}\right)$$

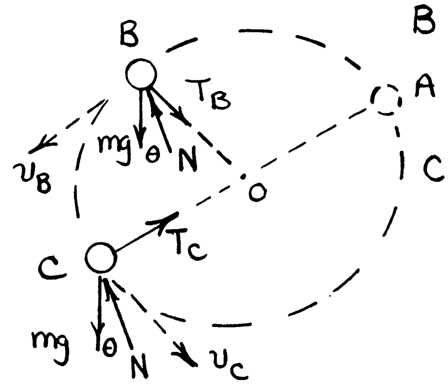
$$-C_D \tfrac{1}{2}\rho v S\left(v_x \underline{i} + v_y \underline{j}\right) - mg\underline{j} = m\left(a_x \underline{i} + a_y \underline{j}\right)$$

So
$$\begin{cases} a_x = -C_D \tfrac{1}{2}\rho S v v_x / m \\ a_y = -C_D \tfrac{1}{2}\rho S v v_y / m - g \end{cases}$$

where $v = \sqrt{v_x^2 + v_y^2}$

The two acceleration expressions are coupled through the speed term. And The expressions are nonlinear.

3/333 $U'_{1-2} = 0$, so $\Delta T + \Delta V_g = 0$



B: $\frac{1}{2}m(v_B^2 - u^2) - mgr\sin\theta = 0$

$a_n = \frac{v_B^2}{r} = \frac{u^2}{r} + 2g\sin\theta$

C: $\frac{1}{2}m(v_C^2 - u^2) - mg(2r\sin\theta) = 0$

$a_n = \frac{v_C^2}{r} = \frac{u^2}{r} + 4g\sin\theta$

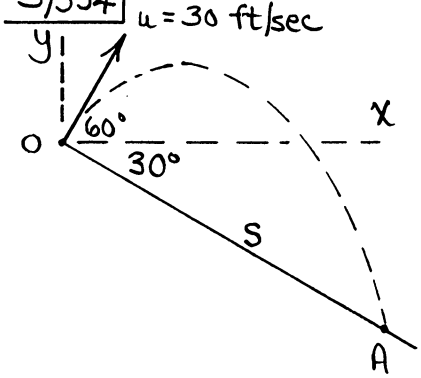
$\Sigma F = ma_n$:

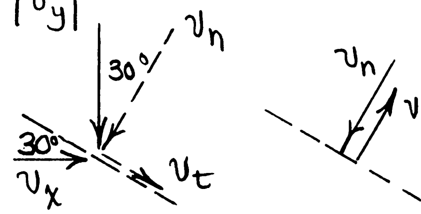
B: $\underline{T_B = m\left(\frac{u^2}{r} + 2g\sin\theta\right)}$

C: $T_C - mg\sin\theta = m\left(\frac{u^2}{r} + 4g\sin\theta\right)$

$T_C = m\left(\frac{u^2}{r} + 5g\sin\theta\right)$

3/334





$x = s\cos 30° = (u\cos 60°)t \; : \; 0.866s = 30(\tfrac{1}{2})t \quad (1)$

$y = -s\sin 30° = (u\sin 60°)t - \tfrac{1}{2}gt^2 :$

$$-\tfrac{1}{2}s = 30(0.866)t - 16.1t^2 \qquad (2)$$

Solve (1) & (2) to obtain $t = 2.15$ s, $s = 37.3$ ft

$v_y = u\sin 60° - gt = 30(0.866) - 32.2(2.15) = -43.3 \frac{\text{ft}}{\text{sec}}$

$v_x = u\cos 60° = 30(\tfrac{1}{2}) = 15$ ft/sec

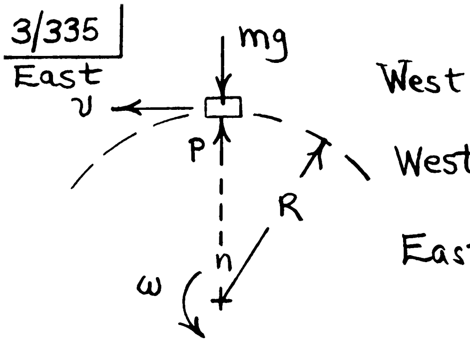
$v_n = 43.3\cos 30° - 15\sin 30° = 30$ ft/sec

$v_t = 43.3\sin 30° + 15\cos 30° = 34.6$ ft/sec

For $e = 0.6$, $\frac{v_n'}{v_n} = 0.6$, $v_n' = 0.6(30) = 18$ ft/sec

Thus $v = \sqrt{18^2 + 34.6^2} = \underline{39.0 \text{ ft/sec}}$

3/335



Westward travel: $v = R\omega - v_r$

Eastward travel: $v = R\omega + v_r$

$$\Sigma F_n = ma_n: \quad mg - P = m\frac{v^2}{R}, \quad P = mg - \frac{mv^2}{R}$$

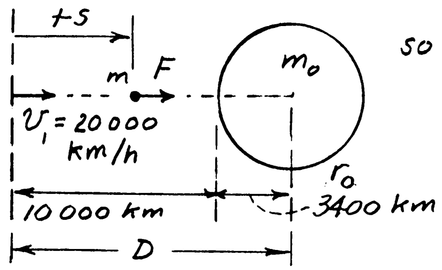
$$\Delta P = P_{east} - P_{west} = \left[mg - \frac{m}{R}(R\omega + v_r)^2\right] - \left[mg - \frac{m}{R}(R\omega - v_r)^2\right]$$

$$\underline{\Delta P = -4m\omega v_r}$$

Numbers: $\Delta P = -4(1500)(0.7292 \cdot 10^{-4})(200)\left(\frac{1000}{3600}\right)$

$= \underline{-24.3 \text{ N}}$

3/336 $\Sigma F = ma;\ G\dfrac{mm_0}{(D-s)^2} = ma$, but when $s = D - r_0$
$a = g_0 = 3.73\ m/s^2$ so



so $G\dfrac{m_0}{r_0^2} = g_0$ & $a = g_0\dfrac{r_0^2}{(D-s)^2}$

$$\int_{v_1}^{v} v\,dv = \int_0^s a\,ds$$

$$\frac{1}{2}(v^2 - v_1^2) = \int_0^{D-r_0} g_0\frac{r_0^2}{(D-s)^2}\,ds = g_0 r_0^2\left(\frac{1}{D-s} - \frac{1}{D}\right)$$

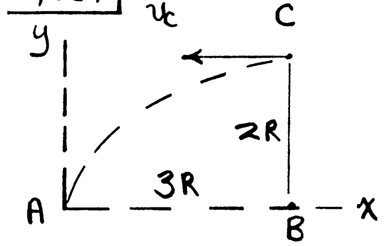
Thus $v^2 = v_1^2 + 2g_0 r_0^2\dfrac{s}{(D-s)D}$

For $s = 10\,000$ km,

$$v^2 = (20\,000)^2 + 2(3.73)\frac{(3600)^2}{1000}(3400)^2\frac{10\,000}{3400(13\,400)}$$
$$= 400(10^6) + 245(10^6) = 645(10^6)\ (km/h)^2$$

$v = 25\,400$ km/h at impact

3/337



$y_A = y_C + v_{y_C} t - \frac{1}{2} g t^2$:

$0 = 2R + 0(t) - \frac{1}{2} g t^2$

$t = 2\sqrt{R/g}$

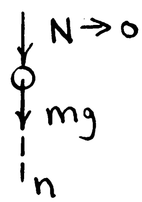
$x_A = x_C + v_{x_C} t$:

$0 = 3R - v_C \, 2\sqrt{R/g}$

$v_C = \frac{3}{2}\sqrt{gR}$

$T_B + U_{B-C} = T_C$: $\frac{1}{2} m u^2 - mg(2R) = \frac{1}{2} m \left[\frac{3}{2}\sqrt{gR}\right]^2$

$$\underline{u = \frac{5}{2}\sqrt{gR}}$$



$\Sigma F_n = m a_n$: $mg = m \frac{v_C^2}{R}$, $v_C = \sqrt{gR}$

$x_A = x_C + v_{x_C} t$:

$0 = x_{min} - \sqrt{gR} \, 2\sqrt{R/g}$

$x_{min} = 2R$

3/338 | Velocity of plug at bottom is

$\sqrt{2gh} = \sqrt{2(32.2)6} = 19.66$ ft/sec

$\Delta G = 0; \quad \frac{2(19.66)}{g} - \frac{(2+4)v}{g} = 0, \quad \underline{v = 6.55 \text{ ft/sec}}$

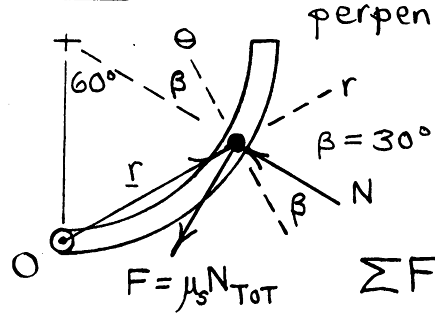
$\Delta T + \Delta V_e = 0; \quad \frac{1}{2}\frac{6}{32.2}(0 - \overline{6.55}^2) + \frac{1}{2}80(x^2 - 0) = 0$

$x^2 = 0.100 \text{ ft}^2, \quad \underline{x = 0.316 \text{ ft}}$

$n = \frac{\Delta T}{T}; \quad n = \left[\frac{1}{2}\frac{2}{g}(19.66)^2 - \frac{1}{2}\frac{6}{g}(6.55)^2\right] \Big/ \frac{1}{2}\frac{2}{g}(19.66)^2$

$= 1 - \frac{6}{2}\left(\frac{6.55}{19.66}\right)^2 = 1 - 3(0.1111) = \underline{0.667}$

3/339 Note that mg and $N_z = mg$ are perpendicular to plane shown.



$$N_{TOT} = \sqrt{N^2 + N_z^2}$$
$$= \sqrt{N^2 + (mg)^2}$$

$$\Sigma F_r = ma_r = m\left(\cancelto{0}{\ddot{r}} - r\dot{\theta}^2\right):$$

$$-N \sin 30° - \mu_s \sqrt{N^2 + [(0.5)(9.81)]^2} \cos 30° = -(0.5)(0.75)(2)^2 t^2$$

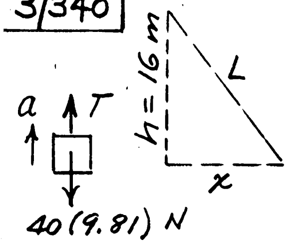
$$\Sigma F_\theta = ma_\theta = m\left(r\ddot{\theta} + 2\dot{r}\cancelto{0}{\dot{\theta}}\right):$$

$$N \cos 30° - \mu_s \sqrt{N^2 + [(0.5)(9.81)]^2} \sin 30° = (0.5)(0.75)(2)$$

Solving second equation: $N = 3.71$ N

First equation: $\underline{t = 2.02 \text{ s}}$

3/340



$a = \ddot{L}$, $\quad L^2 = x^2 + h^2$

$2L\dot{L} = 2x\dot{x}$, $\quad L\dot{L} = x\dot{x}$,

$L\ddot{L} + \dot{L}^2 = x\ddot{x} + \dot{x}^2$ where $\ddot{x} = 0$

$$a = \ddot{L} = (\dot{x}^2 - \dot{L}^2)/L = \left(\dot{x}^2 - \frac{x^2}{L^2}\dot{x}^2\right)/L$$

$$= \frac{L^2 - x^2}{L^3}\dot{x}^2 = \frac{h^2 v^2}{L^3}$$

For $x = 12\text{ m}$, $h = 16\text{ m}$, $L = 20\text{ m}$, $\dot{x} = v = 5\text{ m/s}$

$$a = \frac{16^2(5^2)}{20^3} = 0.80\text{ m/s}^2$$

$\Sigma F = ma$; $\quad T - 40(9.81) = 40(0.80)$, $\quad \underline{T = 424\text{ N}}$

3/341 Final Skidding : $U_{1-2} = \Delta T$

(Prime denotes speed after impact)

$$-\mu_k m g d = 0 - \frac{1}{2} m v'^2$$

$$v' = \sqrt{2\mu_k g d}$$

$$A: \quad v_A' = \sqrt{2(0.9)(32.2)(50)} = 53.8 \text{ ft/sec}$$

$$B: \quad v_B' = \sqrt{2(0.9)(32.2)(100)} = 76.1 \text{ ft/sec}$$

Collision:

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$$

$$\frac{4000}{g} v_A + 0 = \frac{4000}{g}(53.8) + \frac{2000}{g}(76.1)$$

$$v_A = 91.9 \text{ ft/sec}$$

Initial skidding : $U_{1-2} = \Delta T$

$$-\mu_k \cancel{m} g d = \frac{1}{2} \cancel{m} \left(v_A^2 - v_{A_0}^2\right)$$

$$-(0.9)(32.2)(50) = \frac{1}{2}\left(91.9^2 - v_{A_0}^2\right), \quad v_{A_0} = 106.5 \frac{\text{ft}}{\text{sec}}$$

$$= \underline{72.6 \text{ mi/hr}}$$

(Speed limit was exceeded!)

3/342 For the system of man and cord for full fall

(a) $U'_{1-2} = 0 = \Delta V_g + \Delta V_e : \; 0 = 80(9.81)(-44) + \frac{1}{2}k(44-20)^2,$

$k = 119.9$ N/m

(b) $U'_{1-2} = 0 = \Delta T + \Delta V_g + \Delta V_e : \; 0 = \frac{1}{2}80v^2 - 80(9.81)(20+y) + \frac{1}{2}119.9\,y^2$

where y = elongation of bungee cord.

$40\dfrac{d(v^2)}{dy} = 80(9.81) - 119.9y = 0$ for max v^2, $y = 6.55$ m

& $v_{max}^2 = \dfrac{1}{40}\left\{80(9.81)(20+6.55) - \frac{1}{2}119.9(6.55)^2\right\} = 457 \text{ m}^2/\text{s}^2$

$\underline{v_{max} = 21.4 \text{ m/s}}$

(c) Max. acceleration occurs at bottom where tension is greatest

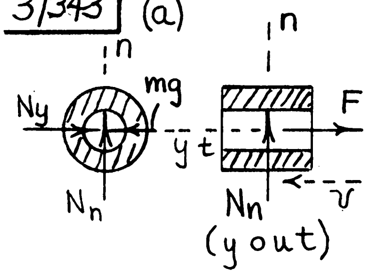
$T_{max} = Ky = 119.9(44-20) = 2880$ N

$\uparrow \Sigma F_y = ma_{max} : \; 2880 - 80(9.81) = 80\,a_{max}$

$\underline{a_{max} = 26.2 \text{ m/s}^2 \text{ or } \frac{8}{3}g}$

80(9.81) N

3/343 (a)



$$\Sigma F_y = 0: \quad N_y = mg$$

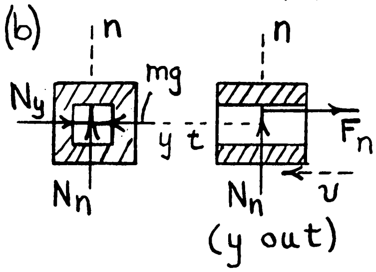
$$\Sigma F_n = m\frac{v^2}{\rho}: \quad N_n = m\frac{v^2}{\rho}$$

$$N_{tot} = \sqrt{N_y^2 + N_n^2} = m\sqrt{g^2 + \frac{v^4}{\rho^2}}$$

$$F = \mu_k N_{tot} = \mu_k m\sqrt{g^2 + \frac{v^4}{\rho^2}}$$

$$\Sigma F_t = ma_t: \; -\mu_k \cancel{m}\sqrt{g^2 + \frac{v^4}{\rho^2}} = \cancel{m}a_t, \quad \underline{a_t = -10.75 \frac{m}{s^2}}$$

(b)



As in part (a), $N_y = mg$ and $N_n = m\frac{v^2}{\rho}$.

But $F_y = \mu_k N_y = \mu_k mg$ and $F_n = \mu_k N_n = \mu_k m\frac{v^2}{\rho}$.

$$a_t = -\frac{F_y + F_n}{m} = -\mu_k g - \mu_k\frac{v^2}{\rho} = \underline{-14.89 \text{ m/s}^2}$$

3/344 D to E : $y = y_0 + v_{y_0} t - \frac{1}{2} g t^2$

$$-\rho = -\frac{1}{2} g t^2 , \quad t = \sqrt{\frac{2\rho}{g}}$$

$x = x_0 + v_{x_0} t$: $\quad d = v_D \sqrt{\frac{2\rho}{g}}$, $\quad v_D = d \sqrt{\frac{g}{2\rho}}$

A to D : $\quad U = \Delta T$

$$\frac{1}{2} k \delta^2 - \mu_k m g \rho - m g \rho = \frac{1}{2} m \left(d^2 \frac{g}{2\rho} \right) - 0$$

$$\underline{\delta = \sqrt{\frac{mg}{k}} \sqrt{\frac{d^2}{2\rho} + 2\rho (1 + \mu_k)}}$$

But speed at top of hill must be ≥ 0 :

$U = \Delta T$: $\quad \frac{1}{2} k \delta^2 - \mu_k m g \rho - 3 m g \rho = \frac{1}{2} m v^2 - 0 \geq 0$

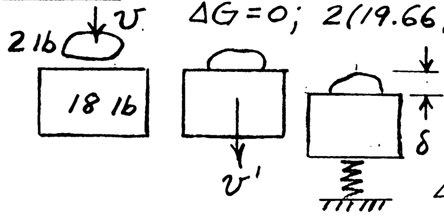
$$\text{or } \delta \geq \sqrt{\frac{2 m g \rho}{k} (3 + \mu_k)}$$

$$\therefore \quad \frac{mg}{k} \left(\frac{d^2}{2\rho} + 2\rho [1 + \mu_k] \right) \geq \frac{2 m g \rho}{k} (3 + \mu_k)$$

or $\quad \underline{d \geq 2\sqrt{2}\, \rho}$

3/345 $v = \sqrt{2gh} = \sqrt{2(32.2)(6)} = 19.66$ ft/sec

$\Delta G = 0;\ 2(19.66) + 0 = (18+2)v',\ v' = 1.966\ \frac{\text{ft}}{\text{sec}}$



Initial spring deflection

$\delta_o = W/(2k) = \frac{18}{2(3)} = 3$ in.

$\Delta T + \Delta V_g + \Delta V_e = 0$

$\Delta T = 0 - \frac{1}{2}\frac{20}{32.2}(1.966)^2 = -1.200$ ft-lb

$\Delta V_g = -20\,\delta/12 = -1.667\delta$ ft-lb where δ is in inches

$\Delta V_e = \frac{1}{2}(2)(3)\left[(3+\delta)^2 - 3^2\right]\frac{1}{12} = \frac{(3+\delta)^2 - 9}{4}$ ft-lb

Thus $-1.200 - 1.667\delta + \frac{(3+\delta)^2 - 9}{4} = 0$

or $\delta^2 - 0.667\delta - 4.800 = 0$

$\delta = \frac{0.667}{2} \pm \frac{1}{2}\sqrt{0.444 + 19.20} = 0.333 \pm 2.216$

$\underline{\delta = 2.55 \text{ in.}}$ (or $\delta = -1.88$ in.)

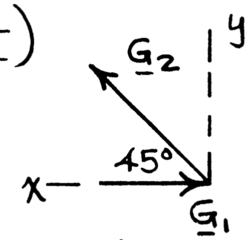
3/346 From S.P. 2/6, $2s = \dfrac{v^2 \sin 2\theta}{g}$

$$350 = \frac{v^2 \sin 90°}{32.2}, \quad v = 106.2 \text{ ft/sec}$$

$$\underline{G}_1 = m\underline{v}_1 = \frac{5/16}{32.2}\left(90\,\frac{5280}{3600}\right)(-\underline{i}) = -1.281\,\underline{i} \text{ lb-sec}$$

$$\underline{G}_2 = m\underline{v} = \frac{5/16}{32.2}\,106.2\left(\frac{\underline{i}}{\sqrt{2}} + \frac{\underline{j}}{\sqrt{2}}\right)$$

$$= 0.729\,(\underline{i} + \underline{j}) \text{ lb-sec}$$



$$\underline{F}_{av}\,\Delta t = \underline{G}_2 - \underline{G}_1 : \underline{F}_{av}(0.005) = 0.729(\underline{i}+\underline{j}) - (-1.281\,\underline{i})$$

$$\underline{F}_{av} = 402\,\underline{i} + 145.7\,\underline{j} \text{ lb}$$

$$F_{av} = \sqrt{402^2 + 145.7^2} = \underline{428 \text{ lb}}$$

(Note: The weight of the baseball is ignored during its impact with the bat. With the weight included, F_{av} still rounds to 428 lb!)

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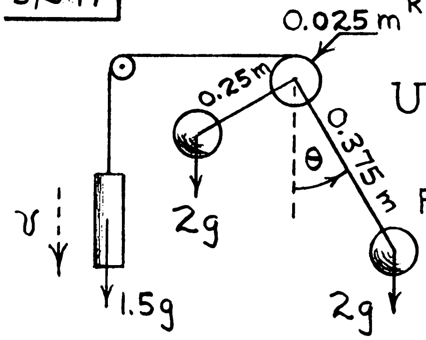


Work done during angular displacement θ is

$$U = 1.5g\,(0.025\,\theta) + 2g\,(0.25\sin\theta) - 2g\,(0.375)(1-\cos\theta)$$

For $\theta = 30^\circ$, $U = 1.659$ J

$$\Delta T = \tfrac{1}{2}\left[1.5\,v^2 + 2\left(\frac{0.25}{0.025}v\right)^2 + 2\left(\frac{0.375}{0.025}v\right)^2\right]$$

$= 325.8\,v^2$. $U_{1\text{-}2} = \Delta T$ yields $\underline{0.0714 \text{ m/s} \left(71.4 \tfrac{\text{mm}}{\text{s}}\right)}$

3/348 Drop of A (state ① → state ②):

$$T_1 + U_{1-2} = T_2: \quad 0 + m_A g\, 1.8\,(1 - \cos 60^\circ) = \frac{1}{2} m_A v_{A2}^2$$

$$v_{A2} = 4.20 \text{ m/s}$$

Collision (② → ③):

$$\begin{cases} m_A v_{A2} + m_B \cancelto{0}{v_{B2}} = m_A v_{A3} + m_B v_{B3} & (1) \\ v_{B3} - v_{A3} = 0.7\,(v_{A2} - \cancelto{0}{v_{B2}}) & (2) \end{cases}$$

Solution: $v_{A3} = 2.42$ m/s, $v_{B3} = 5.36$ m/s

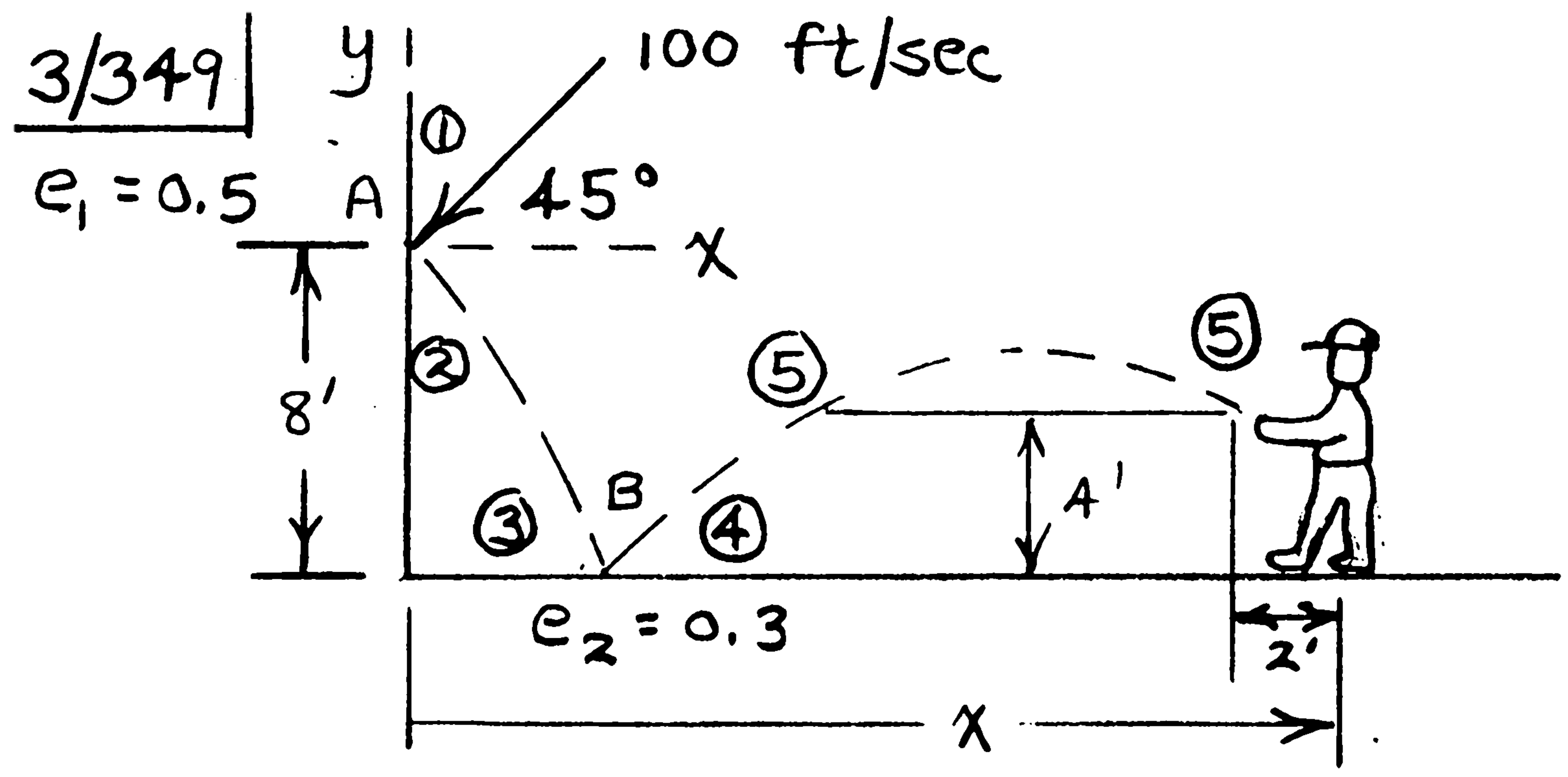
Rise of B (③ → ④):

$$T_3 + U_{3-4} = T_4:$$

$$\frac{1}{2} m_B (5.36)^2 - m_B (9.81)\left[2.4(1 - \cos 30^\circ) + s \sin 30^\circ\right] = 0$$

$$s = 2.28 \text{ m}$$

3/349



Use coordinates & states ① - ⑤ shown.

$v_{1x} = -100 \cos 45° = -70.7$ ft/sec

$v_{1y} = -100 \sin 45° = -70.7$ ft/sec

$v_{2x} = -e_1 v_{1x} = -0.5(-70.7) = 35.4$ ft/sec

$v_{2y} = v_{1y} = -70.7$ ft/sec

$v_{3x} = v_{2x} = 35.4$ ft/sec

$v_{3y} = -\sqrt{v_{2y}^2 + 2g(8)} = -\sqrt{70.7^2 + 2(32.2)(8)} = -74.3 \frac{\text{ft}}{\text{sec}}$

$v_{3y} = v_{2y} - gt_3$: $-74.3 = -70.7 - 32.2\, t_3$, $t_3 = 0.1104$ sec

$v_{4x} = v_{3x} = 35.4$ ft/sec

$v_{4y} = -e_2 v_{3y} = -0.3(-74.3) = 22.3$ ft/sec

$y_5 = y_4 + v_{y_4} t_5 - \frac{1}{2} g t_5^2$:

$-4 = -8 + 22.3\, t_5 - 16.1\, t_5^2$: $t_5 = 0.212,\ 1.172$ sec

Then $x = x_3 + v_{4x} t_5 + 2$, where $x_3 = v_{2x} t_3$

$= 35.4(0.1104) = 3.73$ ft

Thus $x = 3.73 + 35.4(0.212) + 2 = \underline{13.40 \text{ ft}}$

or $x = 3.73 + 35.4(1.172) + 2 = \underline{47.3 \text{ ft}}$

3/350 Results of Prob. 3/301:

$$\Delta v_A = R\sqrt{\frac{g}{r_1}}\left(\sqrt{\frac{2r_2}{r_1+r_2}} - 1\right)$$

Nominally,

$$(\Delta v_A)_n = (3959)(5280)\sqrt{\frac{32.23}{(3959+170)(5280)}} \times$$

$$\left(\sqrt{\frac{2(3959+22{,}300)}{(3959+170)+(3959+22{,}300)}} - 1\right) = 7997 \ \frac{\text{ft}}{\text{sec}}$$

Actually,

$$(\Delta v_A)_a = (3959)(5280)\sqrt{\frac{32.23}{(3959+170)(5280)}} \times$$

$$\left(\sqrt{\frac{2(3959+700)}{(3959+170)+(3959+700)}} - 1\right) = 755 \ \frac{\text{ft}}{\text{sec}}$$

$$\frac{(\Delta v_A)_a}{(\Delta v_A)_n} = \frac{t'}{t}, \quad t' = \frac{(\Delta v_A)_a}{(\Delta v_A)_n}\, t = \frac{755}{7997}(90) = \underline{8.50 \text{ sec}}$$

3/351 Conservation of linear momentum:

$\xrightarrow{x}$ $0 = 6mv_A + mv_B$

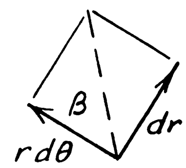
Work-energy $U = \Delta T$: $mgl = \frac{1}{2}mv_B^2 + \frac{1}{2}(6m)v_A^2$

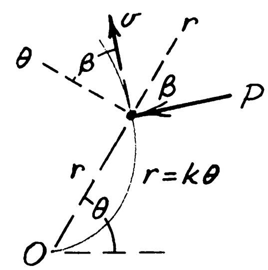
Simultaneous solution: $v_A = \sqrt{\frac{gl}{21}}$ $(\rightarrow)$

$v_B = -6\sqrt{\frac{gl}{21}}$ $(\leftarrow)$

$v_{rel} = v_{B/A} = v_B - v_A = \underline{-\sqrt{\frac{7}{3}gl}}$ $(\leftarrow)$

▸3/352 Eq. 3/27 $\Sigma M_O = \dot{H}_O$





$$H_O = mv\cos\beta \times r$$
$$= mv\frac{r^2}{\sqrt{r^2+k^2}}$$

$$\tan\beta = \frac{dr}{rd\theta} = \frac{k}{r}$$

$$\dot{H}_O = mv\frac{\sqrt{r^2+k^2}\,2r\dot{r} - r^2\frac{r\dot{r}}{\sqrt{r^2+k^2}}}{r^2+k^2} = mv\frac{(r^2+k^2)2r\dot{r} - r^3\dot{r}}{(r^2+k^2)^{3/2}}$$
$$= \frac{r^2+2k^2}{(r^2+k^2)^{3/2}}r\dot{r}mv$$

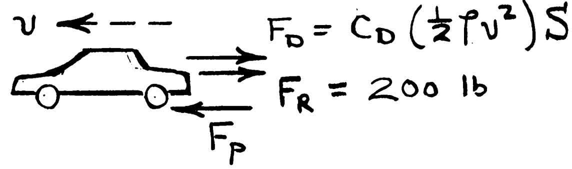
But $v\sin\beta = \dot{r}$, $\dot{r} = v\frac{k}{\sqrt{r^2+k^2}}$

so $\dot{H}_O = \frac{r^2+2k^2}{(r^2+k^2)^2}rv^2km$

$$M_O = Pr\sin\beta = Pr\frac{k}{\sqrt{r^2+k^2}}$$

Thus $Pr\frac{k}{\sqrt{r^2+k^2}} = \frac{r^2+2k^2}{(r^2+k^2)^2}rv^2km$, $\underline{P = \frac{r^2+2k^2}{(r^2+k^2)^{3/2}}mv^2}$

▶3/353



$\overset{+}{\leftarrow}\Sigma F = 0: \quad F_P = F_D + F_R$

Undamaged: $F_P = 0.3\left[\frac{1}{2}\,\frac{0.07530}{32.2}\left(200\cdot\frac{5280}{3600}\right)^2\right]30 + 200 = 1105$ lb

Power required: $P = F_P\cdot v = 1105\cdot\left(200\,\frac{5280}{3600}\right) = 324(10^3)$ ft-lb/sec

Damaged (power available is unchanged)

$P = F_P'\cdot v': \; 324(10^3) = \left[0.4\left(\frac{1}{2}\,\frac{0.07530}{32.2}\,v'^2\right)30 + 200\right]v'$

Solve cubic: $v' = 268$ ft/sec or <u>182.9 mi/hr</u>

▸3/354 $\begin{cases} F_R = -k_1 v, & k_1 = 0.833\ \dfrac{\text{lb-hr}}{\text{mi}} = 0.5682\ \dfrac{\text{lb-sec}}{\text{ft}} \\ F_D = -k_2 v^2, & k_2 = 0.0139\ \dfrac{\text{lb-hr}^2}{\text{mi}^2} = 0.006457\ \dfrac{\text{lb-sec}^2}{\text{ft}^2} \end{cases}$

(a) $P_{30} = Fv = \left[0.833(30) + 0.0139(30)^2\right]\left[30\left(\frac{5280}{3600}\right)\right]$

$= 1650\ \frac{\text{ft-lb}}{\text{sec}} = \underline{3\text{ hp}}$

$P_{60} = Fv = \left[0.833(60) + 0.0139(60)^2\right]\left[60\left(\frac{5280}{3600}\right)\right]$

$= 8800\ \frac{\text{ft-lb}}{\text{sec}} = \underline{16\text{ hp}}$

(b) $-k_1 v - k_2 v^2 = m\frac{dv}{dt}$

$$\int_0^t dt = -m\int_{v_1}^{v_2} \frac{dv}{v(k_1 + k_2 v)}$$

$$t = -\frac{m}{k_1}\ln\left[\frac{v_2(k_1 + k_2 v_1)}{v_1(k_1 + k_2 v_2)}\right]$$

$$t = -\frac{2000/32.2}{0.5682}\ln\left[\frac{7.33(0.5682 + 0.006457(88))}{88\ (0.5682 + 0.006457(7.33))}\right]$$

$= \underline{205\text{ sec}}$

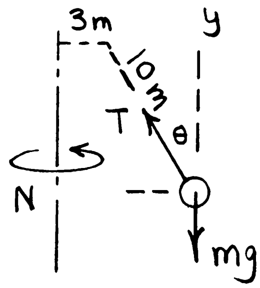
$-k_1 v - k_2 v^2 = mv\frac{dv}{ds}$

$$\int_0^s ds = -m\int_{v_1}^{v_2}\frac{dv}{k_1 + k_2 v}$$

$$s = -\frac{m}{k_2}\ln(k_1 + k_2 v)\Big|_{v_1}^{v_2}$$

$$= -\frac{m}{k_2}\ln\left[\frac{k_1 + k_2 v_2}{k_1 + k_2 v_1}\right] = \underline{5898\text{ ft}}$$

*3/355



$\Sigma F_y = 0: \quad T\cos\theta - mg = 0 \qquad (1)$

$\Sigma F_n = ma_n: \quad T\sin\theta = m(3 + 10\sin\theta)\,\omega^2 \qquad (2)$

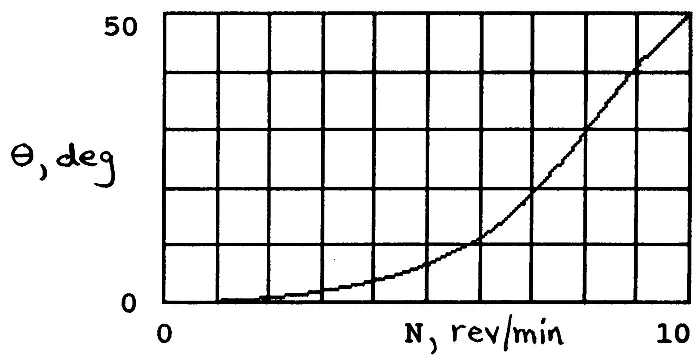
where $\omega = N\frac{\pi}{30}$ (3)

From (1): $T = mg/\cos\theta$

With (3), (2) then becomes

$$g\tan\theta - \left[10\left(\frac{\pi}{30}N\right)^2\right]\sin\theta - 3\left(\frac{\pi}{30}N\right)^2 = 0$$

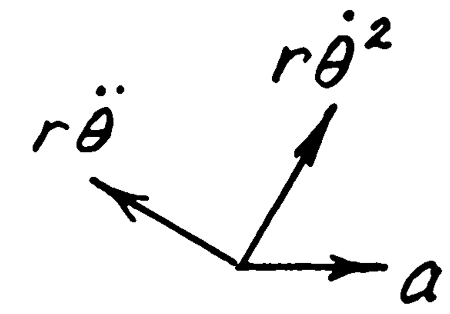
Use Newton's method to solve this equation for θ over $0 \le N \le 10$ rev/min.

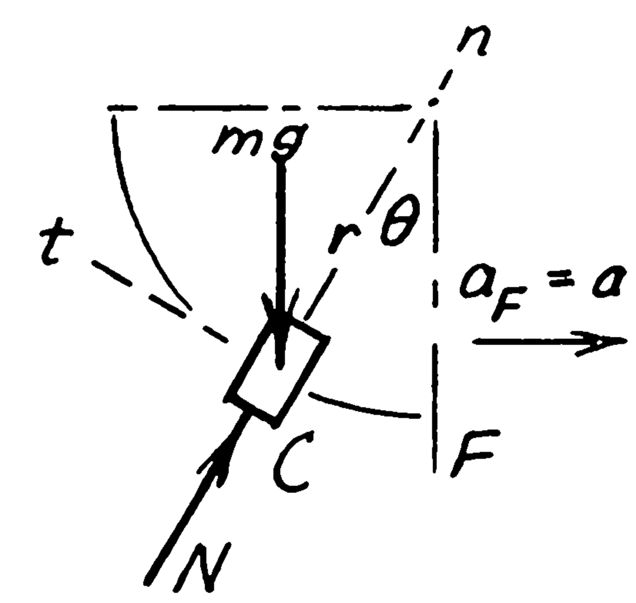


For $N = 8$ rev/min, $\underline{\theta = 29.6°}$.

***3/356** $\underline{a}_C = \underline{a}_F + \underline{a}_{C/F}$

$\underline{a}_{C/F} = r\ddot{\theta}\,\underline{e}_t + r\dot{\theta}^2\,\underline{e}_n$





$\Sigma F_t = ma_t: \; -mg\sin\theta = m(r\ddot{\theta} - a\cos\theta)$

$$r\ddot{\theta} = a\cos\theta - g\sin\theta \qquad (1)$$

$\Sigma F_n = ma_n: \; N - mg\cos\theta = m(r\dot{\theta}^2 + a\sin\theta)$

$$N = m(r\dot{\theta}^2 + a\sin\theta + g\cos\theta) \qquad (2)$$

Multiply (1) by $2\dot{\theta}$ & obtain

$$r\frac{d}{dt}(\dot{\theta}^2) = 2\frac{d\theta}{dt}(a\cos\theta - g\sin\theta)$$

or $r\,d(\dot{\theta}^2) = 2(a\cos\theta - g\sin\theta)\,d\theta$

$r\int_0^{\dot{\theta}^2} d(\dot{\theta}^2) = 2\int_0^{\theta}(a\cos\theta - g\sin\theta)\,d\theta$, $r\dot{\theta}^2 = 2(a\sin\theta + g\cos\theta)\Big|_0^{\theta}$,

$$\dot{\theta} = \sqrt{\frac{2}{r}(a\sin\theta + g\cos\theta - g)}$$

Substitute into (2) & get

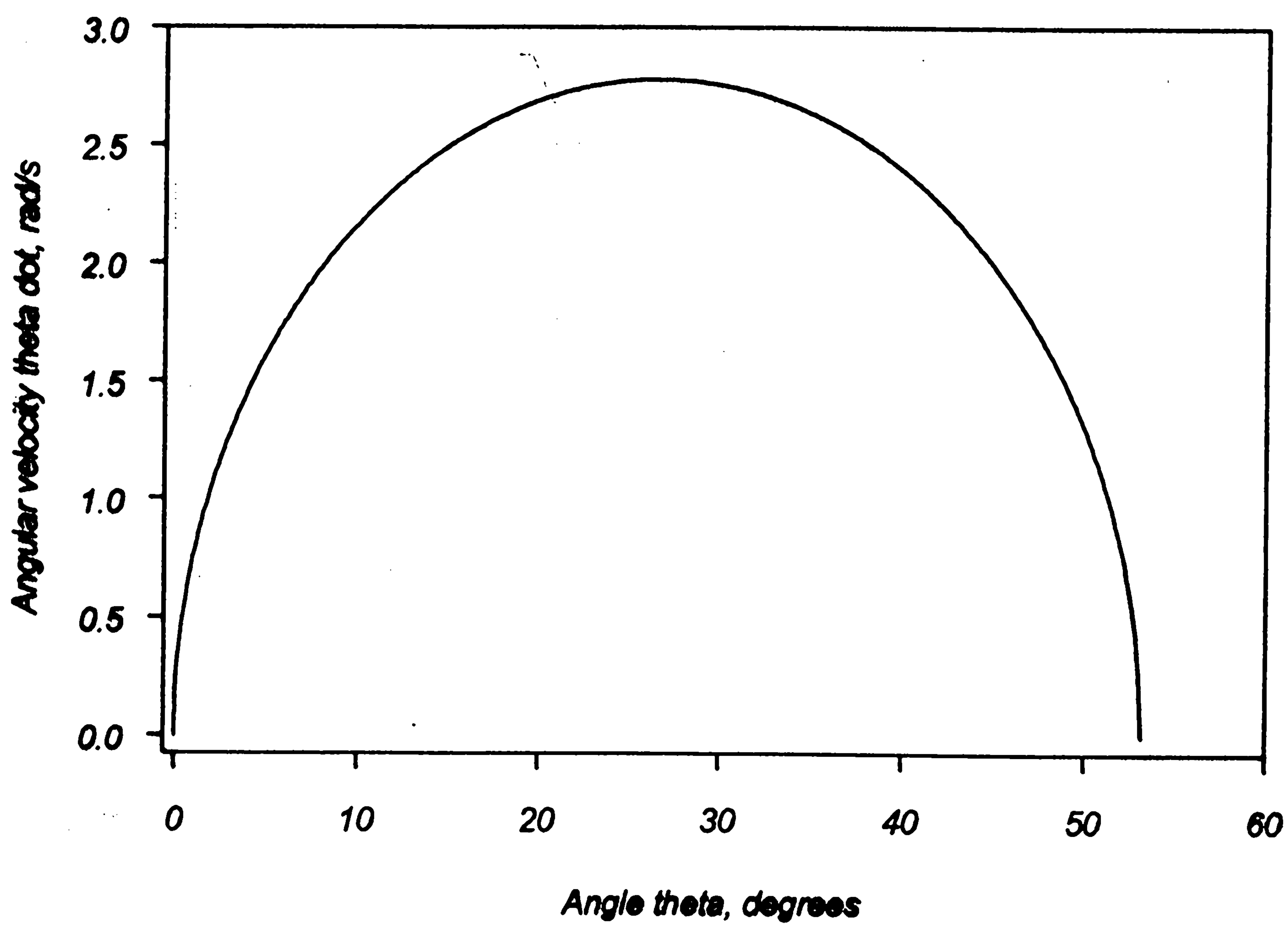
$$N = m(2a\sin\theta + 2g\cos\theta - 2g + a\sin\theta + g\cos\theta)$$
$$= m(3a\sin\theta + 3g\cos\theta - 2g)$$

Max θ occurs when $\dot{\theta} = 0$ so $a\sin\theta + g\cos\theta - g = 0$

so $\underline{\theta_{max} = 53.1°}$ when $a = \frac{g}{2}$.

Thus for $a = \frac{g}{2} = 4.905\ m/s^2$ and $r = 0.3\ m$,

$$\underline{\dot{\theta} = 8.09\sqrt{0.5\sin\theta + \cos\theta - 1}\ \text{rad/s}}$$



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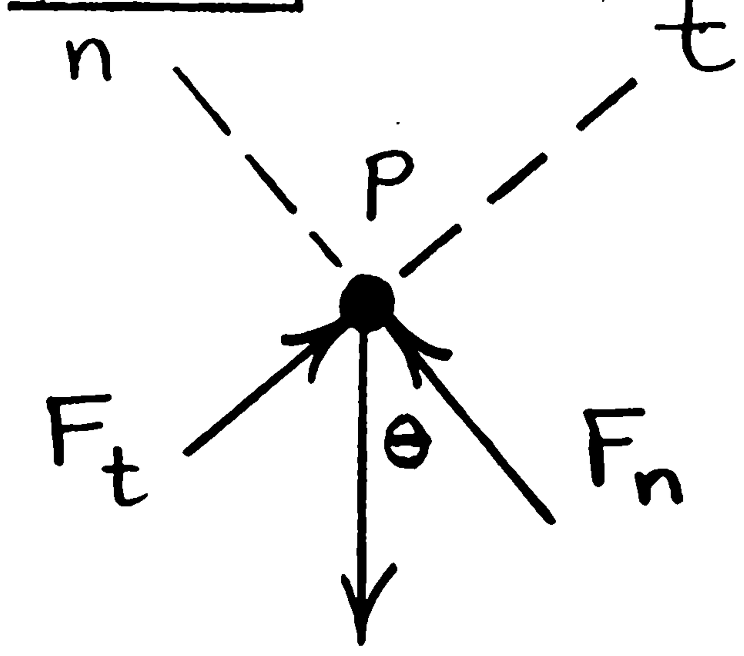


$\theta = t^2, \quad \dot{\theta} = 2t, \quad \ddot{\theta} = 2 \text{ rad/s}^2$

$0.4(9.81) = 3.924$ N

$\Sigma F_n = ma_n: \quad F_n - 3.924\cos t^2 = 0.4(1.5)(2t)^2$

$F_n = 3.924\cos t^2 + 2.4t^2 \quad (\text{N})$

$\Sigma F_t = ma_t: \quad F_t - 3.924\sin t^2 = 0.4(1.5)(2)$

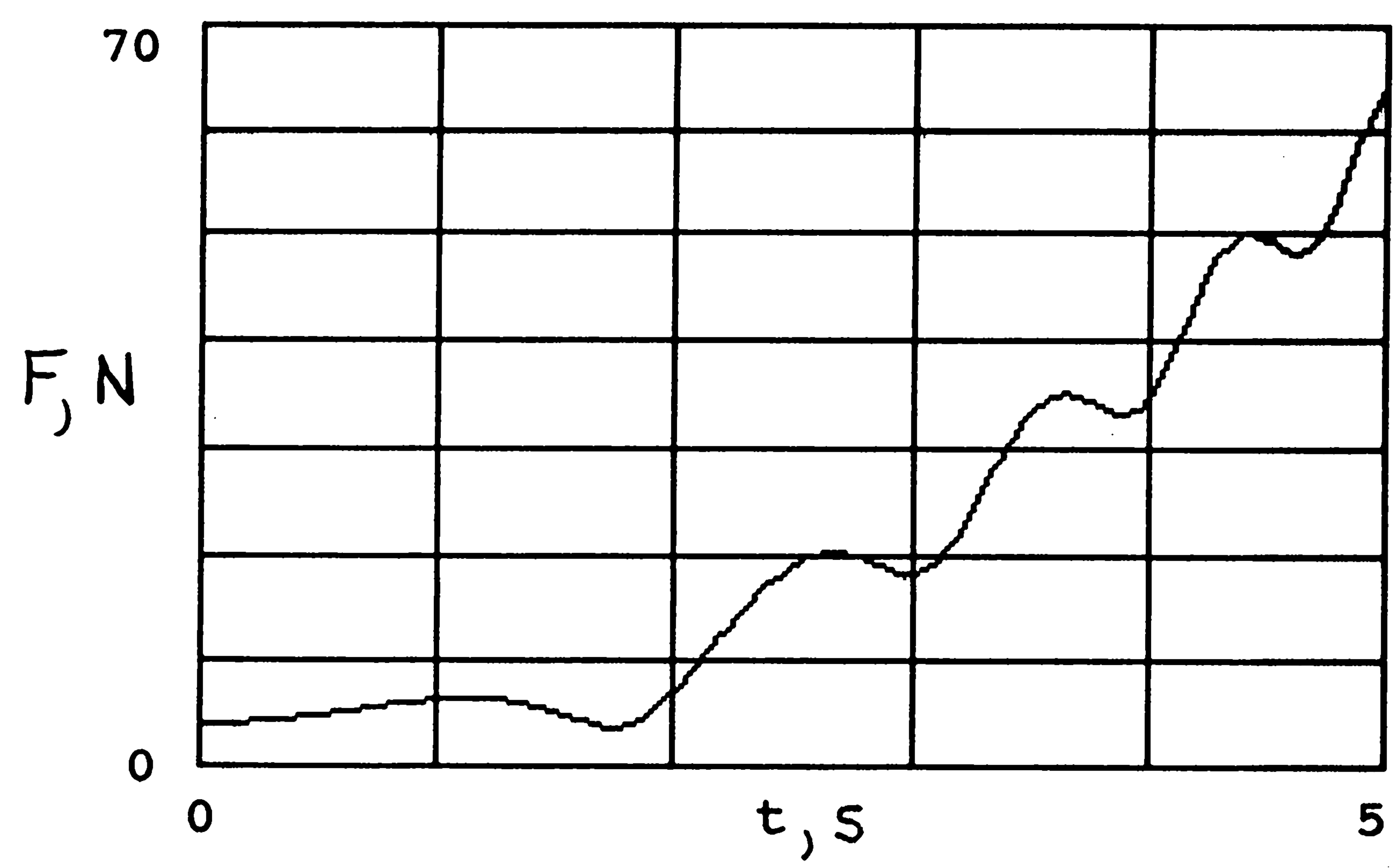
$F_t = 3.924\sin t^2 + 1.2 \quad (\text{N})$

$F = \sqrt{F_n^2 + F_t^2}$; simplify to

$F = \sqrt{16.84 + 18.84t^2\cos t^2 + 9.42\sin t^2 + 5.76t^4}$

When $F = 30$ N, $\underline{t = 3.40 \text{ s}}$ (numerically)

$\theta = 3.40^2 = \underline{11.57 \text{ rad}}$ or $\theta = \underline{663°}$



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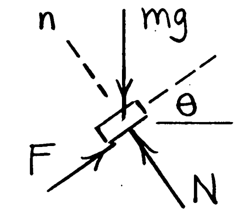


$a_n = r\Omega^2 = \frac{13}{12}(7.5)^2 = 60.9 \frac{ft}{sec^2}$

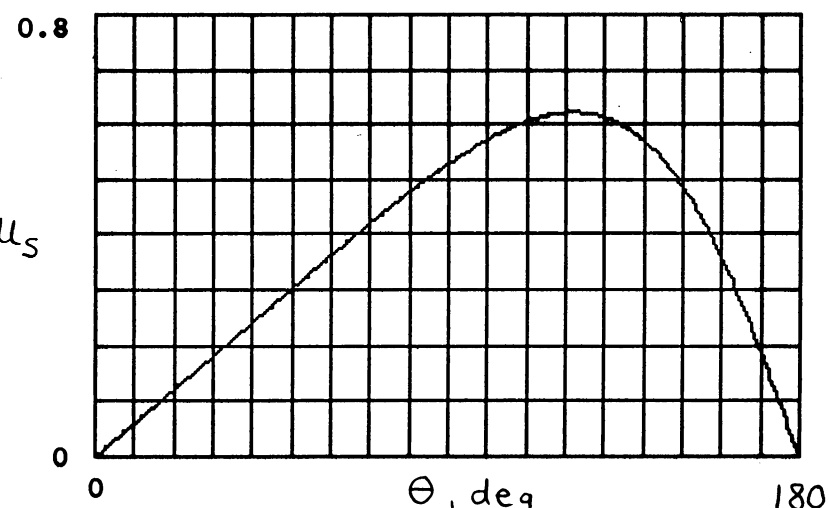
$\begin{cases} \Sigma F_n = ma_n : N - mg\cos\theta = 60.9m \\ \Sigma F_t = ma_t : F - mg\sin\theta = 0 \end{cases}$

Slipping impends when $F = F_{max} = \mu_s N$

Simultaneous solution : $\mu_s = \dfrac{32.2\sin\theta}{60.9 + 32.2\cos\theta}$

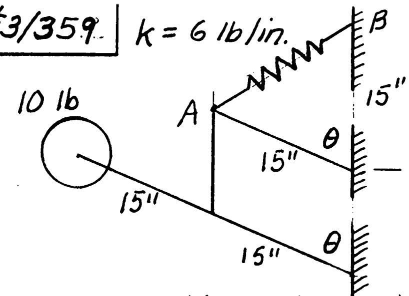
See plot of μ_s vs. θ below. Set $\frac{d\mu_s}{d\theta} = 0$ or numerically determine that

$\underline{\mu_{min} = 0.622}$ @ $\underline{\theta = 121.9°}$



Note that it is impossible to see slippage first occur at any angle greater than $\theta = 121.9°$!

*3/359 $k = 6$ lb/in.



$\overline{AB}^2 = 2(15)^2 - 2(15)^2\cos\theta$

$\overline{AB} = 15\sqrt{2}\sqrt{1-\cos\theta}$

Spring stretch $= \overline{AB} - 15 = \delta$

$\Delta V_e = \frac{1}{2}k\delta^2 - 0$

$= \frac{1}{2}6\left[15(\sqrt{2}\sqrt{1-\cos\theta} - 1)\right]^2$

$= 675(3 - 2\cos\theta - 2\sqrt{2}\sqrt{1-\cos\theta})$ in.-lb

$\Delta V_g = -10(80)(\cos 60° - \cos\theta)$

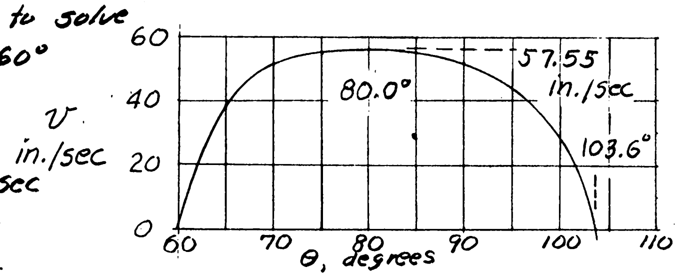
$= -150 + 300\cos\theta$ in.-lb

$\Delta T + \Delta V_g + \Delta V_e = 0$

$\frac{1}{2}\frac{10}{32.2}\frac{1}{12}v^2 - 150 + 300\cos\theta + 675(3 - 2\cos\theta - 2\sqrt{2}\sqrt{1-\cos\theta}) = 0$

$v^2 = 77.28(-1875 + 1050\cos\theta + 1350\sqrt{2}\sqrt{1-\cos\theta})$ $(\text{in./sec})^2$

Set up program to solve for v for $\theta > 60°$ & get

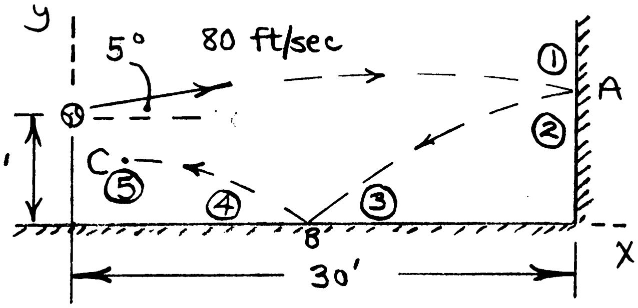


(a) $v_{max} = 57.5$ in./sec

$\theta = 80.0°$

(b) $\theta_{max} = 103.6°$

*3/360



Define states

① - ⑤ as shown.

$$\begin{cases} v_0 = 80 \text{ ft/sec} \\ \theta = 5° \end{cases}$$

$v_{0x} = v_0 \cos\theta = 80 \cos 5° = 79.7$ ft/sec

$v_{0y} = v_0 \sin\theta = 80 \sin 5° = 6.97$ ft/sec

$t_{01} = \dfrac{30}{79.7} = 0.376$ sec

$y_1 = 3 + 6.97(0.376) - 16.1(0.376)^2 = 3.34$ ft

$v_{1x} = v_{0x} = 79.7$ ft/sec

$v_{1y} = v_{0y} - g t_{01} = 6.97 - 32.2(0.376) = -5.15$ ft/sec

Now program the following numbered equations:

$v_{2x} = -e v_{1x}$ (1)

$v_{2y} = v_{1y}$ (2)

$v_{3x} = v_{2x}$ (3)

$v_{3y} = -\sqrt{v_{2y}^2 + 2 g y_1}$ (4)

$t_{23} = (v_{2y} - v_{3y})/g$ (5)

$$x_3 = 30 + v_{2x} t_{23} \tag{6}$$

$$v_{4x} = v_{3x} \tag{7}$$

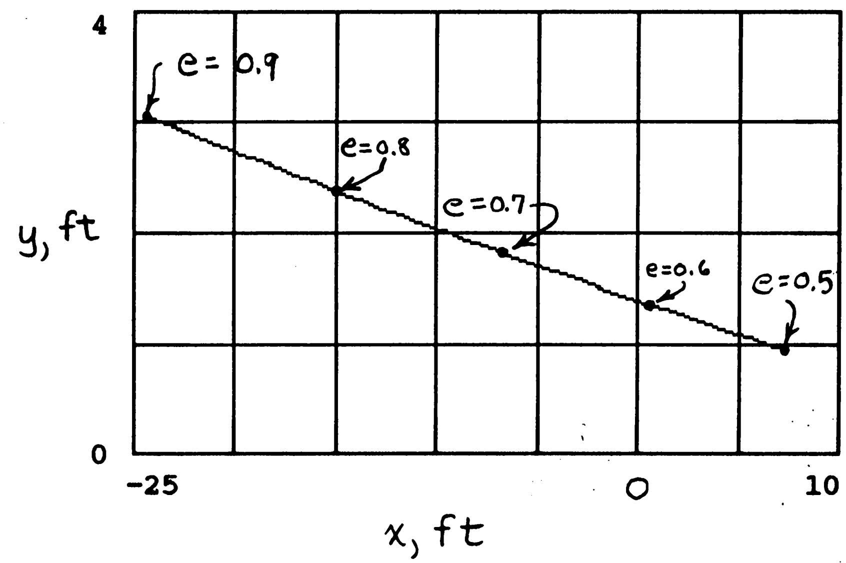
$$v_{4y} = -e v_{3y} \tag{8}$$

$$t_{45} = v_{4y}/g \tag{9}$$

$$x_5 = x_3 + v_{4x} t_{45} = x \tag{10}$$

$$y_5 = v_{4y} t_{45} - \tfrac{1}{2} g t_{45}^2 = y \tag{11}$$

Solve Eqs. (1)–(11) for $0.5 \le e \le 0.9$ to obtain the following plot.



For $x = 0$, $e = 0.610$, $y = 1.396$ ft

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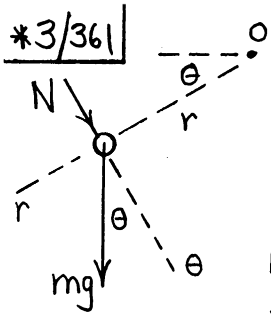


$\Sigma F_r = ma_r = m(\ddot{r} - r\dot{\theta}^2)$:

$mg \sin\theta = m(\ddot{r} - r\omega_0^2)$

$\ddot{r} - \omega_0^2 r = g \sin \omega_0 t$

Assume $r_h = Ce^{st}$ to obtain $s_1 = -\omega_0$, $s_2 = \omega_0$. Assume particular solution of form $r_p = D \sin \omega_0 t$ and find $D = -\frac{g}{2\omega_0^2}$. So

$$r = r_h + r_p = C_1 e^{-\omega_0 t} + C_2 e^{\omega_0 t} - \frac{g}{2\omega_0^2} \sin \omega_0 t$$

Use the initial conditions $r(0) = \dot{r}(0) = 0$ to find C_1 and C_2, allowing us to write the solution as

$$r = \frac{g}{4\omega_0^2}\left(-e^{-\theta} + e^{\theta} - 2\sin\theta\right)$$

Now, set $r = 1$ m and $\omega_0 = 0.5$ rad/s and use Newton's method to solve for θ as $\theta = 0.535$ rad, or $\underline{30.6°}$. From $\theta = \omega_0 t$, $t = \frac{0.535}{0.5} = \underline{1.069\text{ s}}$.

*3/362 I. Drop of 20-lb sphere A

$U = \Delta T$: $20\left(\frac{18}{12}\right)(1-\cos 60^\circ) = \frac{1}{2}\frac{20}{32.2}v_A^2 - 0$

$v_A = 6.95$ ft/sec

II. Collision

$m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$: $\frac{20}{g}(6.95) + 0 = \frac{20}{g}v_A' + \frac{10}{g}v_B'$

$v_B' - v_A' = e(v_A - v_B)$: $v_B' - v_A' = 0.75(6.95 - 0)$

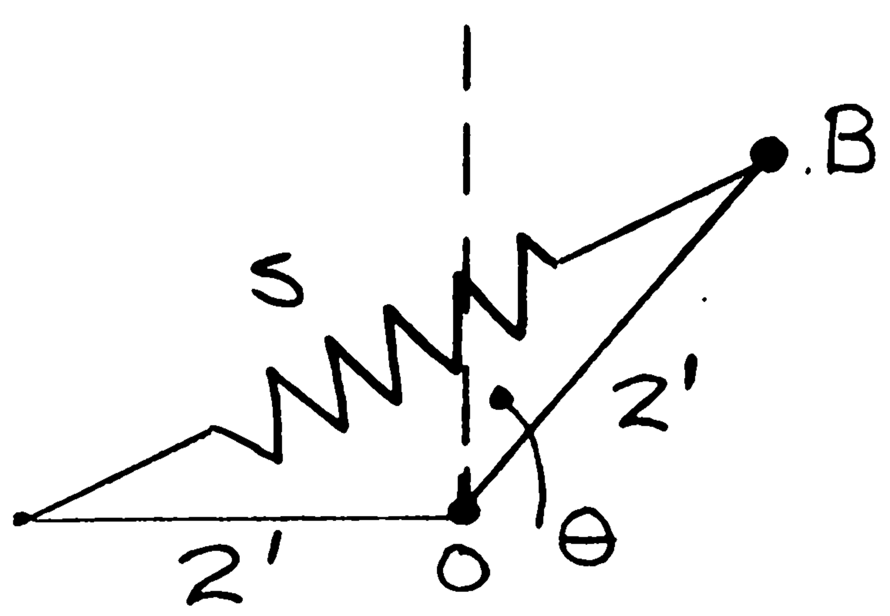
Solving, $v_B' = 8.108$ ft/sec

III. Deflection of B

$U = \Delta T$: $10(2)(1-\cos\theta) + \frac{1}{2}k(\cancelto{0}{x_1^2} - x_2^2)$

$= 0 - \frac{1}{2}\frac{10}{32.2}(8.108)^2$

We need to find x_2 as a function of θ :



$s^2 = 2^2 + 2^2 - 2(2)(2)\cos(\theta + 90^\circ)$

$= 8 + 8\sin\theta$

Stretch $x_2 = s - 2\sqrt{2}$

or $x_2 = (8 + 8\sin\theta)^{1/2} - 2\sqrt{2}$

$x_2^2 = 8\left(2 + \sin\theta - 2\sqrt{1+\sin\theta}\right)$

The previous work-energy equation becomes

$20(1-\cos\theta) - \frac{1}{2}(100)(8)\left(2 + \sin\theta - 2\sqrt{1+\sin\theta}\right)$

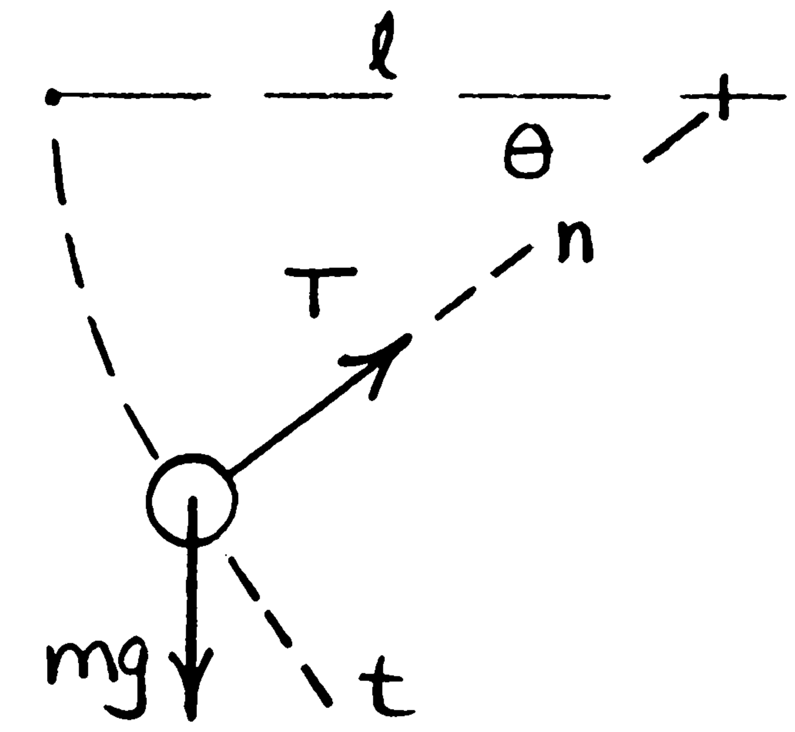
$= -\frac{1}{2}\frac{10}{32.2}(8.108)^2$

Simplification yields

$-769.8 - 20\cos\theta - 400\sin\theta + 800\sqrt{1+\sin\theta} = 0$

Newton's method gives $\underline{\theta = 21.7^\circ}$

*3/363



$v_0 = l\dot{\theta}_0$
$= 0.5\,(0.2)$
$= 0.1$ m/s

$\Sigma F_t = ma_t : \quad mg\cos\theta = ml\ddot{\theta}, \quad \ddot{\theta} = \frac{g}{l}\cos\theta$

$v\,dv = a_t\,ds : \quad v\,dv = l\ddot{\theta}\,(l\,d\theta) = gl\cos\theta\,d\theta$

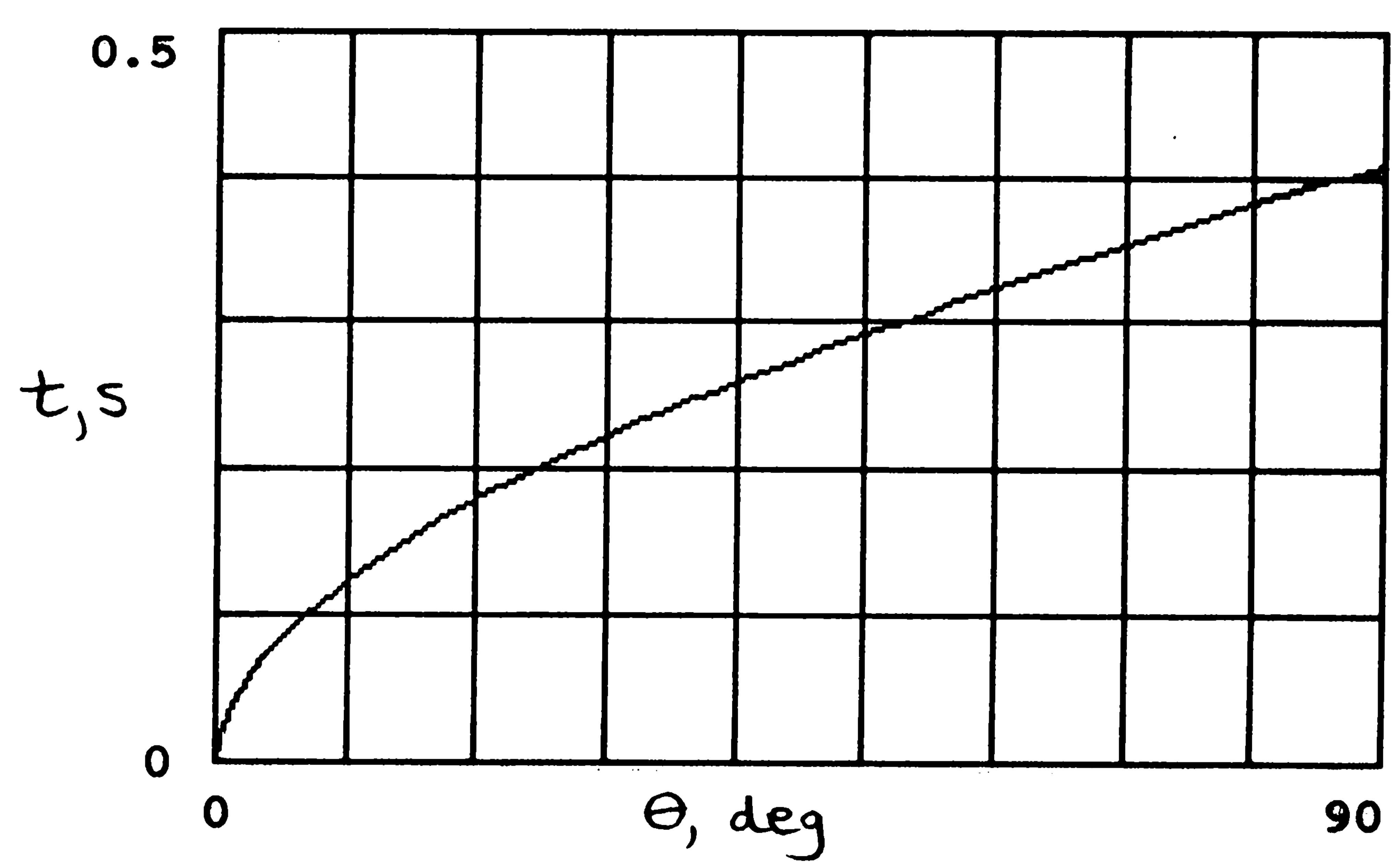
$\int_{v_0=0.1}^{v} v\,dv = \int_{\theta_0=0}^{\theta} gl\cos\theta\,d\theta, \quad v^2 = 2gl\sin\theta + v_0^2$

$v = \frac{ds}{dt} = \frac{l\,d\theta}{dt} : \quad \sqrt{2gl\sin\theta + v_0^2} = l\,\frac{d\theta}{dt}$

Rearrange to $\int_{t_0=0}^{t} dt \quad \int_{\theta_0=0}^{\theta} l\,\frac{d\theta}{\sqrt{2gl\sin\theta + v_0^2}}$

So $t = 0.5 \int_0^{\theta} \frac{d\theta}{\sqrt{9.81\sin\theta + 0.01}}$

Set up a numerical integration scheme (see Appendix C/12) and integrate the above for various upper limits $(0 \le \theta \le \pi/2)$



When $\theta = 90°$, $\underline{t = 0.409 \text{ s}}$.